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ITAI-1370 / MOD 3 Puzzle

Monty Hall Problem and Bayes Theorem

The Monty Hall Problem is a probability puzzle named after the host of the American television game show 'Let's Make a Deal.' In this puzzle, a contestant is faced with three doors. Behind one of the doors is a car (the prize), and behind the other two are goats. The contestant picks one door. Then, the host, who knows what's behind each door, opens one of the two remaining doors, revealing a goat. The contestant is then given a choice: stick with their original pick or switch to the other remaining closed door.

Problem Discussion

The key to this problem lies in understanding conditional probabilities. At the beginning, the probability that the car is behind the chosen door is 1/3, while the probability that the car is behind one of the other two doors is 2/3.

When the host opens a door to reveal a goat, the probability distribution changes based on the additional information provided. This is where Bayes' theorem helps us understand the updated probabilities. The probability that the car is behind the initially chosen door remains 1/3, while the probability that it's behind the remaining door increases to 2/3.

The formula is P(A I B) = (P(B I A) * P(A)) / P(B)

Should You Switch?

Yes, switching doors increases the probability of winning the car from 1/3 to 2/3. The intuitive thought might be that after one goat is revealed, the odds should be 50/50 between the two remaining doors. However, that is not the case. By sticking to your original choice, you maintain the initial 1/3 chance, whereas switching takes advantage of the 2/3 probability that the car is behind one of the unchosen doors.

Conclusion

In conclusion, by switching doors, a contestant significantly increases their chance of winning. This counterintuitive result highlights the importance of understanding conditional probability and using updated information to guide decision-making. Bayes' theorem is crucial in this reasoning as it adjusts probabilities based on the new information (the host opening a door). Therefore, switching doors is the optimal strategy.