Linear Algebra

Lecture 1-5

Lecture 2

$$AB = R$$

- 单个元素计算

$$R_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$$

- 列线性组合
  结果矩阵的每一列都可以看成是矩阵A的列的线性组合。

$$Column_j\ of\ R = A * Column_j\ of\ B$$

- 行线性组合
  结果矩阵的每一行都可以看成是矩阵B的行的线性组合。

$$Row_j\ of\ R = Row_j\ of\ A * B$$

Lecture 3

矩阵乘法$AB = C$

1. columns of C are combinations of columns of A
2. rows of C are combinations of rows of B
3. $C_{ij} = \sum_{k=1}^{n} a_{ik} * b_{kj}$
4. $AB = \sum_{k=1}^{n} (column\ k\ of\ A) * (row\ k\ of\ B)$
5. Block multiply

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}\begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} A_1B_1 + A_2B_2 & \cdots \\ \cdots & \cdots \end{bmatrix}$$

Inverse

If A is square and invertible, $A^{-1}A = I = AA^{-1}$

> **Note:** Invertible = nonsingular

存在$A\vec{x} = 0(\vec{x} \neq 0)$，则$A$不可逆。

Find Inverse

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, find A^{-1}$$

$$\begin{bmatrix} a & c & 1 & 0 \\ b & b & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & e & f \\ 0 & 1 & g & h \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

Prove:
suppose $EA = I$, E is the emlination matrix.
so, we get $E = A^{-1}$
then, $E[A\ I] = [I\ E] = [I\ E^{-1}]$

> **Note:** The inverse of AB is $B^{-1}A^{-1}$ since $ABB^{-1}A^{-1} = I$

Lecture 4

Transpose

formula: $(A^T)_{ij} = A_{ji}$

formula: $AB = C \Rightarrow B^TA^T = C^T$

> **Note:** $(A^{-1})^T = (A^T)^{-1}$
> prove: $AA^{-1} = I \Rightarrow (A^{-1})A^T = I^T = I \Rightarrow (A^{-1})^T = (A^T)^{-1}$

Permutations

置换矩阵，用于行交换，通过交换Identity的行得到。

Example:
$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$交换了A的第一行和第二行。

对于某一维度的矩阵，所有的Permutation形成一个族群。在族群内，$P^{-1} = P^T$，P^{-1}和P^T都在族群内。

Symmetric Matrix

formaul: $A^T = A$

> **Note:** $R^T R$ is always symetric.
> prove: $(R^T R)^T = R^T (R^T)^T = R^T R$

Product of elimination matrices $PA = LU$

$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$

Do elimination
$E_{21} A = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = U$

Figure out L
$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = LU$

> **Note:** $L = E^{-1}$

also

$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} = LDU$

> **Note:** D is diagonal matrix，非对角线上的元素都为0

Lecture 5

Vector Space

combinations still in space. — ***CLOSE**

Requirements

$V + W$ and cV are in the space
all combinations $cV + dW$ are in the space

Subspace

That's a space: some vectors inside the given vector space, but still make up a **vector space** of their own.

The **union** of two subspace P and L is **not a subspace**. (不一定)
The **intersection** of two subspace P and L is **a subspace**.

> **Note:** All subspace of R^2:
>
> - R^2
> - any line through $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
> - $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ alone
