

# Linear Algebra

Lecture 1-5

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## Lecture 2

$$AB = R$$

- 单个元素计算

$$R_{ij} = \sum_{k=1}^n A_{ik} B_{kj}$$

- 列线性组合  
结果矩阵的每一列都可以看成是矩阵A的列的线性组合。

$$\text{Column}_j \text{ of } R = A * \text{Column}_j \text{ of } B$$

- 行线性组合  
结果矩阵的每一行都可以看成是矩阵B的行的线性组合。

$$\text{Row}_j \text{ of } R = \text{Row}_j \text{ of } A * B$$

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## Lecture 3

矩阵乘法  $AB = C$

1. columns of C are combinations of columns of A
2. rows of C are combinations of rows of B
3.  $C_{ij} = \sum_{k=1}^n a_{ik} * b_{kj}$
4.  $AB = \sum_{k=1}^n (\text{column } k \text{ of } A) * (\text{row } k \text{ of } B)$
5. Block multiply

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} A_1 B_1 + A_2 B_2 & \dots \\ \dots & \dots \end{bmatrix}$$

## Inverse

If A is square and invertible,  $A^{-1}A = I = AA^{-1}$

**Note:** Invertible = nonsingular

存在  $A\vec{x} = 0 (\vec{x} \neq 0)$ , 则A不可逆。

## Find Inverse

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, \text{ find } A^{-1}$$

$$\begin{bmatrix} a & c & 1 & 0 \\ b & d & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & e & f \\ 0 & 1 & g & h \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

Prove:

suppose  $EA = I$ ,  $E$  is the elimination matrix.

so, we get  $E = A^{-1}$

then,  $E[A \ I] = [I \ E] = [I \ E^{-1}]$

**Note:** The inverse of  $AB$  is  $B^{-1}A^{-1}$  since  $ABB^{-1}A^{-1} = I$

# Lecture 4

## Transpose

formula:  $(A^T)_{ij} = A_{ji}$

formula:  $AB = C \Rightarrow B^T A^T = C^T$

**Note:**  $(A^{-1})^T = (A^T)^{-1}$

prove:  $AA^{-1} = I \Rightarrow (A^{-1})A^T = I^T = I \Rightarrow (A^{-1})^T = (A^T)^{-1}$

## Permutations

置换矩阵，用于行交换，通过交换Identity的行得到。

Example:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \text{ 交换了 } A \text{ 的第一行和第二行。}$$

对于某一维度的矩阵，所有的Permutation形成一个族群。在族群内， $P^{-1} = P^T$ ， $P^{-1}$ 和 $P^T$ 都在族群内。

## Symmetric Matrix

formaul:  $A^T = A$

**Note:**  $R^T R$  is always symetric.

prove:  $(R^T R)^T = R^T (R^T)^T = R^T R$

Product of elimination matrices  $PA = LU$

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$$

Do elimination

$$E_{21}A = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = U$$

Figure out  $L$

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = LU$$

**Note:**  $L = E^{-1}$

also

$$A = \begin{bmatrix} 2 & 1 & 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} = LDU$$

**Note:** D is diagonal matrix，非对角线上的元素都为0

# Lecture 5

## Vector Space

combinations still in space. — **CLOSE**

### Requirements

$V + W$  and  $cV$  are in the space

all combinations  $cV + dW$  are in the space

## Subspace

That's a space: some vectors inside the given vector space, but still make up a **vector space** of their own.

The **union** of two subspace P and L is **not a subspace**. (不一定)

The **intersection** of two subspace P and L is **a subspace**.

**Note:** All subspace of  $R^2$ :

- $R^2$
  - any line through  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
  - $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$  alone
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