ECEN 4532: Digital Signal Processing Lab

Lecture Notes: Discrete Cosine Transform for Compression

Instructor: Prof. Farhad Pourkamali-Anaraki

University of Colorado at Boulder

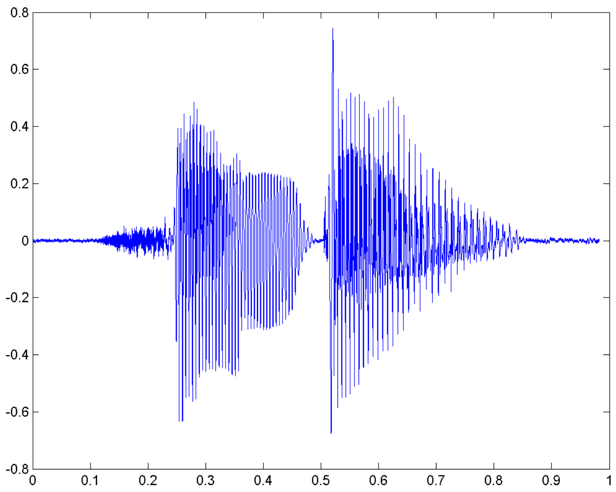
Spring 2017

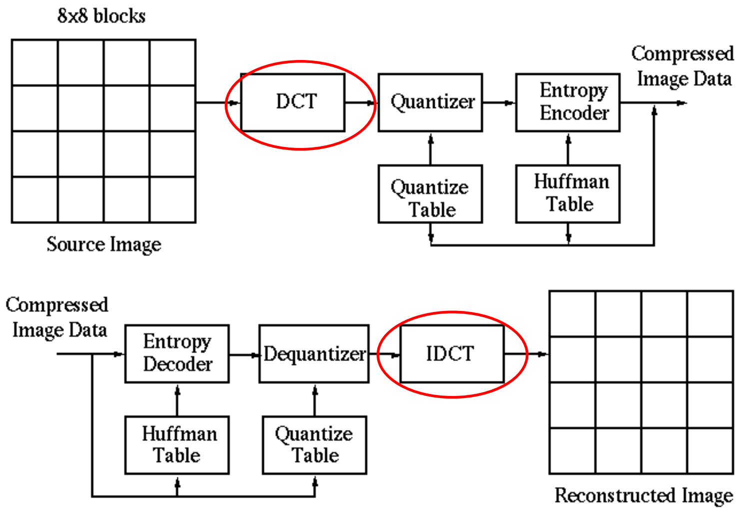
University of Colorado
Boulder

Introduction

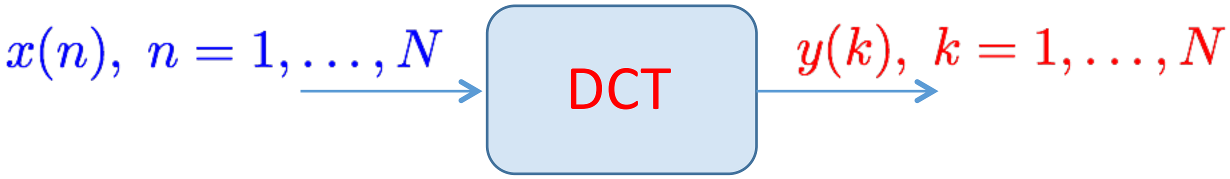
- Discrete Cosine Transform (DCT) expresses a finite sequence of data points in terms of a sum of cosine functions oscillating at different frequencies.

- DCT has many applications in signal and image processing:
  - Audio compression
  - JPEG compression





DCT



$$y(k) = w(k) \sum_{n=1}^{N} x(n) \cos \frac{\pi(2n-1)(k-1)}{N}, k = 1, \ldots, N$$

$$w(k) = \begin{cases} 1/\sqrt{N} & k = 1 \\ \sqrt{2}/\sqrt{N} & k = 2, \ldots, N \end{cases}$$

Inverse DCT

$$y(k),\ k = 1, \ldots, N \longrightarrow \boxed{\text{IDCT}} \longrightarrow x(n),\ n = 1, \ldots, N$$

$$x(n) = \boldsymbol{w(n)} \sum_{k=1}^{N} y(k) \cos \frac{\pi(2n-1)(k-1)}{N}, n = 1, \ldots, N$$

$$w(n) = \begin{cases} 1/\sqrt{N} & n = 1 \\ \sqrt{2}/\sqrt{N} & n = 2, \ldots, N \end{cases}$$

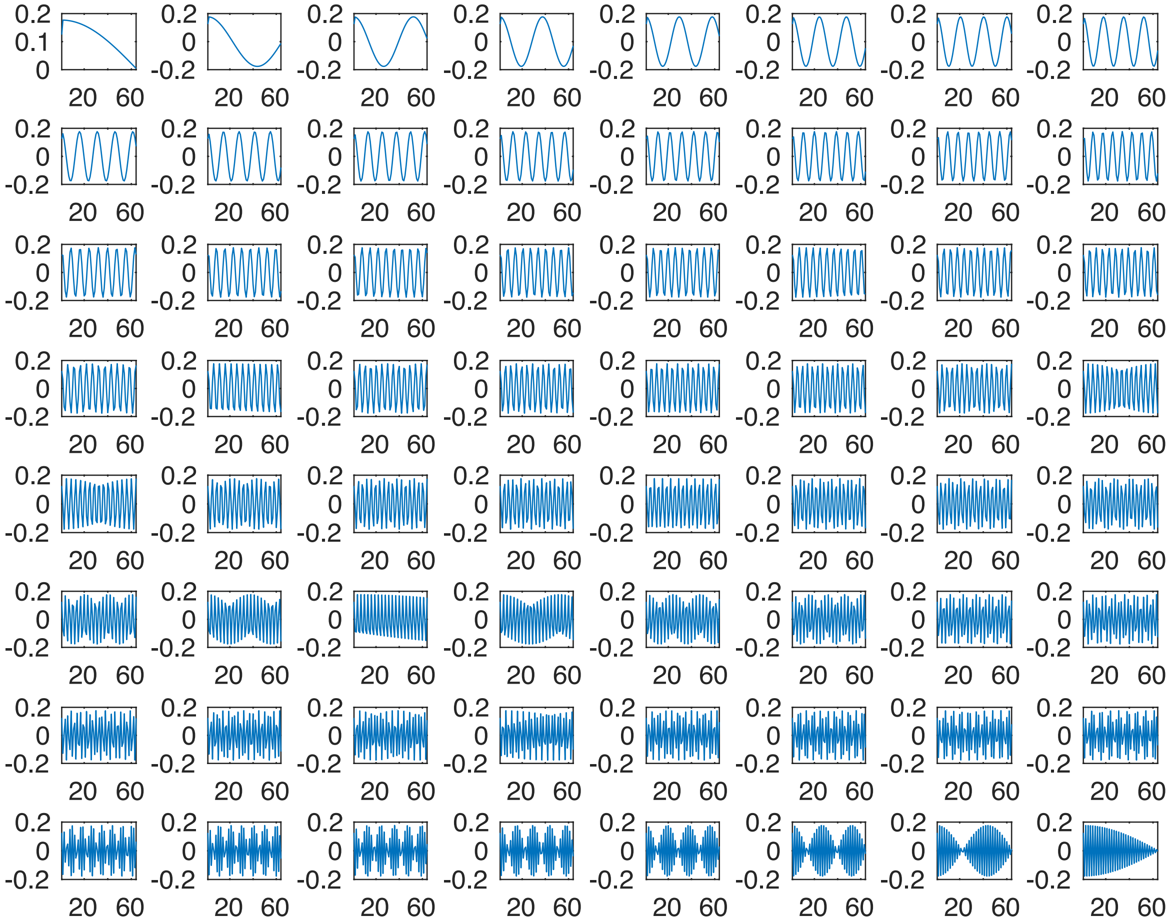
DCT Basic Elements (1)

$$y(k) = w(k) \sum_{n=1}^{N} x(n) \boxed{\cos \frac{\pi(2n-1)(k-1)}{N}}, \quad k = 1, \ldots, N$$

```
N = 64;

for i = 1 : N
    x = zeros(1,N);
    x(i) = 1;
    y = dct(x);
    subplot(8,8,i)
    plot(1:N,y)
end
```

DCT Basic Elements (2)

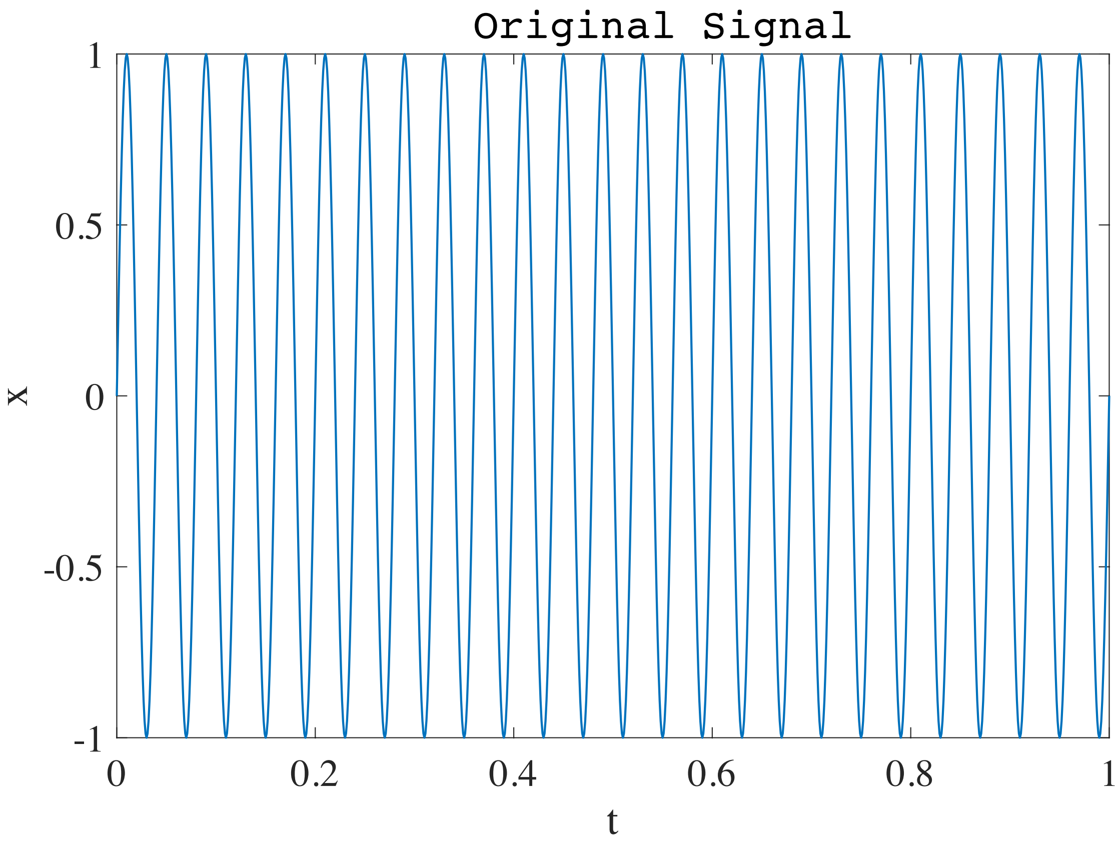


Simple Example (1)

```
t = 0:1/1000:1;
x = sin(2*pi*25*t);
```

N=1,001

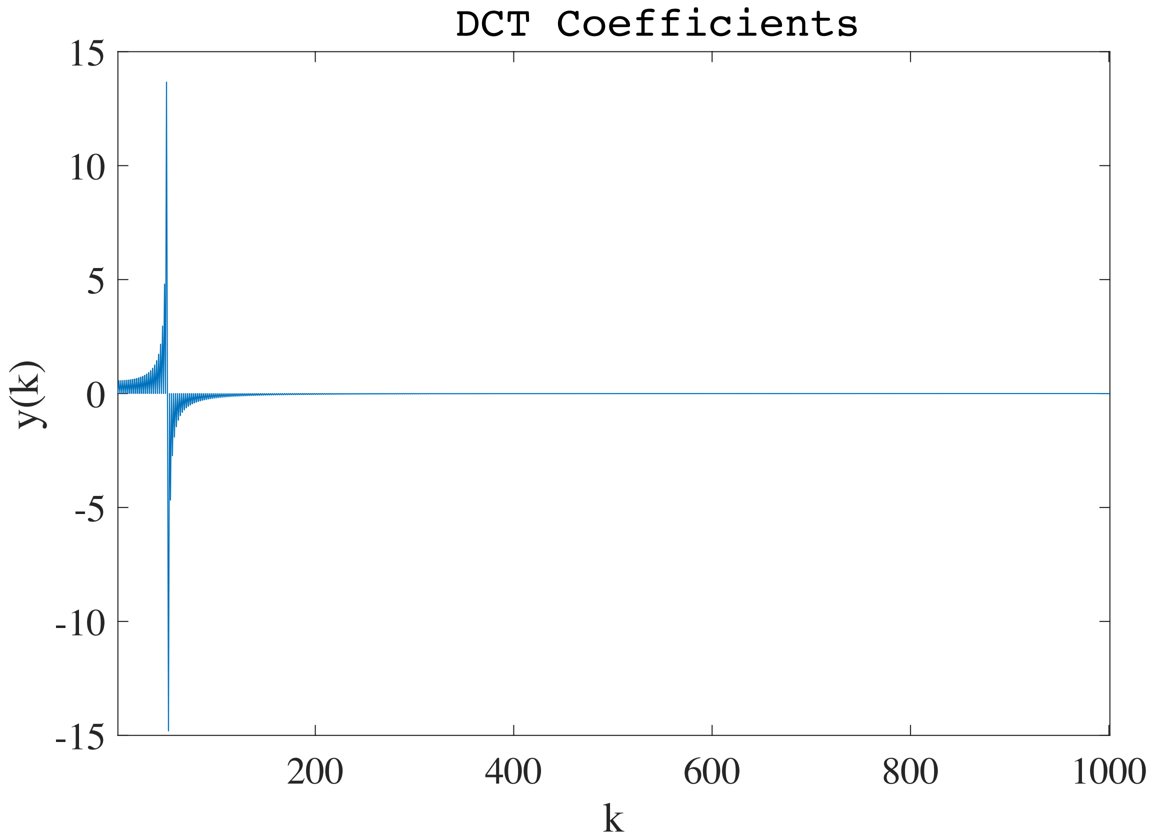
frequency 25Hz



Simple Example (2)

```
y = dct(x);
```

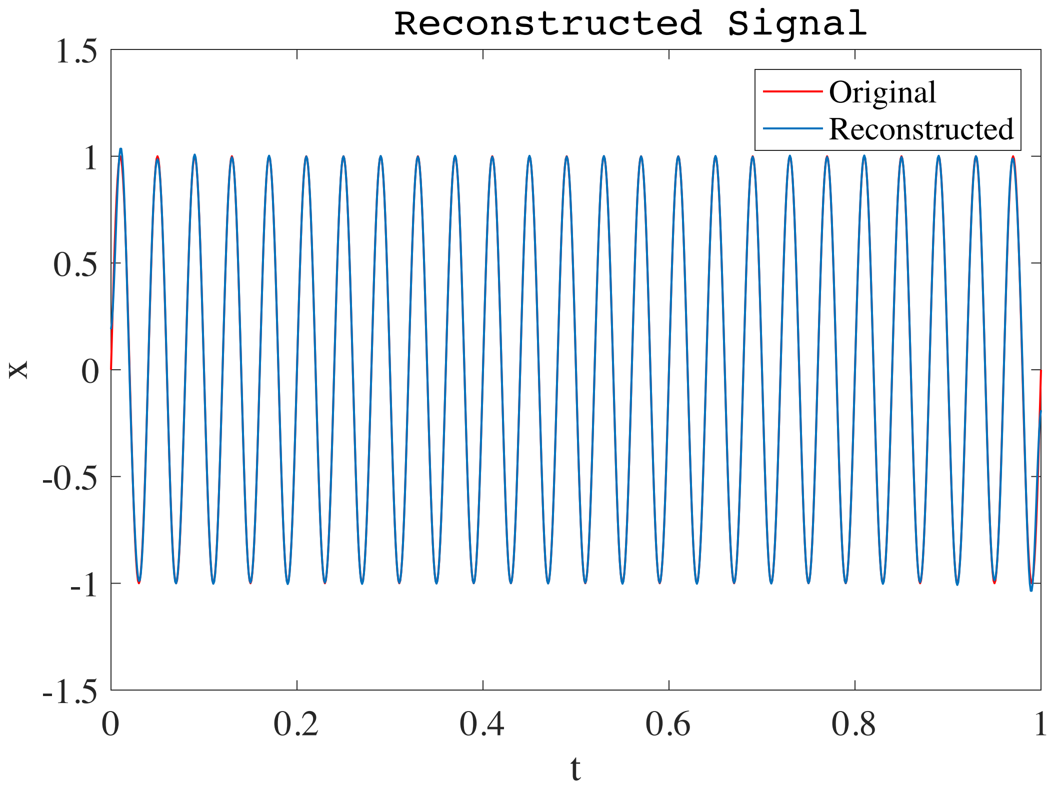
y(k) are real-valued coefficients



Simple Example (3)

- Reconstruct the signal using only those components with value greater than 0.1

```
y2 = find(abs(y) < 0.1);
y(y2) = zeros(size(y2));
z = idct(y);
```



Simple Example (4)

- How many coefficients are nonzero for the reconstruction step?

```
y2 = find(abs(y) < 0.1);
y(y2) = zeros(size(y2));
z = idct(y);
```

64 nonzero coefficients!

- Compression ratio?

64/1001 = 0.0639

Simple Example (5)

- How to measure accuracy?

- Normalized Error:

$$\frac{\|x_{original} - x_{reconstructed}\|}{\|x_{original}\|}$$

- In our example:

0.0194

The reconstructed signal keeps approximately 98% of the energy in the original signal.

Homework

- This HW is optional! (10 bonus points for lab 3)

- It investigates how to compress a speech signal using the discrete cosine transform (DCT).

- Download the mat file "lab3speech":
  - x: vector of length 4899
  - fs: sampling frequency

Homework

1. Decompose the signal into DCT basis vectors.

2. Sort the coefficients from largest to smallest.

3. Find how many DCT coefficients represent 99.9% of the energy in the signal.

4. Set to zero the coefficients that contain the remaining 0.1% of the energy.

5. Plot the original signal, its reconstruction, and the difference between the two.

2-D Discrete Cosine Transform