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Probability transformation of mass function: A weighted network method based on the ordered visibility graph

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ABSTRACT

Transform of basic probability assignment to probability distribution is an important aspect of decision making process. To address this issue, a weighted network method based on the ordered visibility graph is proposed in this paper, named OVGWP. In this proposed method, the information volume of focal elements is calculated by belief entropy. The entropy value is used to determine the rank of each proposition. After generating the rank, a weighted network corresponding to the given basic probability assignment can be constructed. The global ratio for proportional belief transformation is determined by the degree of nodes and its weighted edges in the network. Compared with existing ordered visibility graph probability, we have considered not only the belief value itself, but also the cardinality of basic probability assignment. Hence the proposed OVGWP considers a much more comprehensive information for transformation. Experimental results reveal that OVGWP produces an effective and reasonable transformation performance compared with existing methods. If the basic probability assignment is given as $m(\Theta) = 1$, the proposed OVGWP has the same result with pignistic probability transformation. The proposed OVGWP satisfies the consistency of the upper and lower boundaries.

1. Introduction

In engineering applications, uncertainty often exists and plays an important role in decision making, such as reliability analysis (Wang et al., 2021; Chen and Deng, 2018; Pan et al., 2020), target tracking (Altan and Hacıoğlu, 2020; Cheong et al., 2019), and pattern recognition (Wen et al., 2020; Altan and Karasu, 2020). How to express and handle uncertainty has attracted the attention of many researchers (Wang and Song, 2018). Many theories have been developed, for instances, fuzzy sets (Wang et al., 2020a; Deng and Deng, 2021), rough sets (Wang et al., 2020b; Luo et al., 2020), Dempster–Shafer evidence theory (DSET) (Xiao, 2021), and Z numbers (Tian et al., 2020). Among these theories, DSET is based on basic probability assignment (BPA). Compared with probability theory, BPA can assign belief values both in singleton propositions and multi-subset propositions, which provides a flexible way to deal with more uncertain information represented by probability distribution.

Though BPA in DSET can address the uncertainty, one shortcoming is in the lack of efficient decision making model directly based on BPA. An obvious way to address this issue is to transform BPA into probability. Generally, the existing probability transformation (PT) methods can be categorized into two kinds. The first kind is the proportional belief transformation method. The main idea is to allocate the belief of multi-element propositions to singletons proportionally. For example, the most well-known pignistic probability transformation (PPT) (Smets and Kennes, 1994) presented the idea of equal distribution on multi-subset propositions. Sudano and Martin (Martin and Sudano, 2006) proposed several proportional transformation methods using belief functions and plausible functions. Huang et al. (2021) applied the Shapley value approach to make a transformation. Other methods include Cuzzolin's method (Cuzzolin, 2012) and Daniel's method (Daniel, 2006). The second kind is based on the optimization principle. The main idea is to construct certain objective functions. For example, Han et al. (2010) designed a PT objective model based on the uncertainty minimization (or minimum entropy). In general, proportional belief transformation methods are more commonly used than optimization methods, due to the low complexity and easy realization.

Recently, Li et al. (2016) propose a new method, named the ordered visibility graph probability (OVGP), which is an efficient proportional belief transformation method. The main idea is to convert BPA into a complex network based on visibility graph (VG), which can transform time series into a network. The order of belief values is considered

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to construct a network. Then, the degree of each node is used to determine the ratio for proportional belief transformation. The experimental results proves the effectiveness of OVGP. However, OVGP still has two shortcomings. One is that OVGP only considers a part of BPA information, i.e., the values of BPA, to construct a network. While the cardinality in BPA in OVGP is neglected. The other is the use of unweighted network, which does not consider the edge weight. Compared with the weighted network, the unweighted network loses some information. To address these issues, an ordered visibility graph weighted network, referred to as OVGWP, is proposed in this paper to transform BPA into probability. In our proposed method, the network is generated by the order of information volume of each focal element. A weighted network is then constructed to measure the importance of focal elements. Finally, a global ratio for proportional belief transformation is presented to obtain the probability distribution. The experimental results are shown that the proposed OVGWP can get more reasonable results than OVGP.

The contributions of the proposed OVGWP are listed as follows:

(1) In the network construction of focal elements, OVGP (Li et al., 2016) ranks focal elements based on its belief values, while OVGWP ranks focal elements using the information volume, calculated by belief entropy. The information volume considers not only the belief value itself, but also the cardinality of focal elements. Compared with OVGP, the proposed OVGWP takes into account a more comprehensive BPA information to construct networks.

(2) Different from the unweighted network generated by OVGP (Li et al., 2016), the proposed OVGWP presents the weighted network to measure the importance of nodes (focal elements). Two different methods are proposed to calculate the weight of nodes in this paper, using the node distance and belief entropy, respectively. The weighted network can better measure the importance of nodes to determine the global ratio, which provides an efficient proportional belief transformation to obtain the final probability distribution for decision making.

The rest of the paper is organized as follows. Section 2 introduces some definitions about DSET and VG. In Section 3, the proposed OVGWP is presented, along with a detailed example. Section 4 lists some examples to show the effectiveness of the proposed OVGWP, with comparison and discussion with other existing methods. Finally, Section 5 concludes the paper.

2. Preliminaries

In this section, some preliminaries including Dempster–Shafer evidence theory, belief entropy and visibility graph are briefly introduced.

2.1. Dempster-Shafer evidence theory

Dempster–Shafer evidence theory (DSET), also known as belief function theory, provides an efficient way to manage with vague, incomplete and uncertain information (Xue and Deng, 2021a; Yager and Alajlan, 2015). DSET is considered as the generalization of Bayesian probability since it can express one's belief on multi-subset propositions (Deng and Jiang, 2020). Owing to the powerful ability, DSET achieves a wide application in the decision domain (Liu et al., 2019; Wang et al., 2021; Gao and Xu, 2019).

Assume a random variable X taking values from $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$, Θ is called a frame of discernment (FOD) in DSET. The power set of Θ, formed by $2^\Theta = \{\emptyset, \{\theta_1\}, \dots, \{\theta_n\}, \{\theta_1, \theta_2\}, \dots, \{\theta_1, \theta_2, \dots, \theta_n\}, \dots, \Theta\}$, includes all possible subsets of Θ. A basic probability assignment (BPA), also called a mass function, is a mapping from 2^Θ to the interval [0, 1], which satisfies (Dempster, 2008; Shafer, 1976)

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Theta} = 1 \tag{1}$$

m(A) represents the belief value that supports the proposition A. If m(A) > 0, then A is called a focal element.

Based on a BPA m, its associated belief function $Bel_m(A)$ and plausibility function $Pl_m(A)$ are defined as (Dempster, 2008; Shafer, 1976):

$$Bel_m(A) = \sum_{B \in \Theta, B \subseteq A} m(B) \tag{2}$$

$$Pl_m(A) = \sum_{B \in \Theta, B \cap A \neq \emptyset} m(B) \tag{3}$$

$Bel_m(A)$ represents the total amount of justified support to A, $Pl_m(A)$ is the maximum amount of potential support to A.

2.2. Belief entropy

How to measure the uncertainty of BPA is still an open issue, and many methods have been proposed to quantify uncertainty (Maeda et al., 1993; Harmanec and Klir, 1994; Jousselme et al., 2006). Entropy function is one of the important methods for uncertainty modelling (Wang et al., 2017; Xue and Deng, 2021b; Babajanyan et al., 2020; Zhang and Deng, 2021). Recently, Deng (Deng, 2020) put forward a kind of belief entropy, named Deng entropy, to address such issue. The related definition is given as (Deng, 2020)

$$E_d(m) = -\sum_{A \subseteq \Theta} m(A) log_2 \frac{m(A)}{2^{|A|} - 1} \tag{4}$$

2.3. Visibility graph

Time series forecasting models are very important in many fields, for instances, crude oil price estimation (He et al., 2012; Karasu et al., 2020), wind speed forecasting (Cadenas et al., 2016; Altan et al., 2021), tourism (Jiao and Chen, 2019; Chen et al., 2019) and digital currency (Indera et al., 2017; Altan et al., 2019). Visibility graph (VG) is a simple and fast computational method to transform time series into a graph (Lacasa et al., 2008, 2009). VG can deal with time series from a new point of complex network theory (Wen and Cheong, 2021; Liu et al., 2020). VG has been widely studied both in theory and application (Iacovacci and Lacasa, 2019; Li et al., 2020). For example, Lacasa and Just (2018) explored the relation between VG and symbolic dynamics. Zhu and Wei (2021) studied the characteristics of stock network based on VG and entropy.

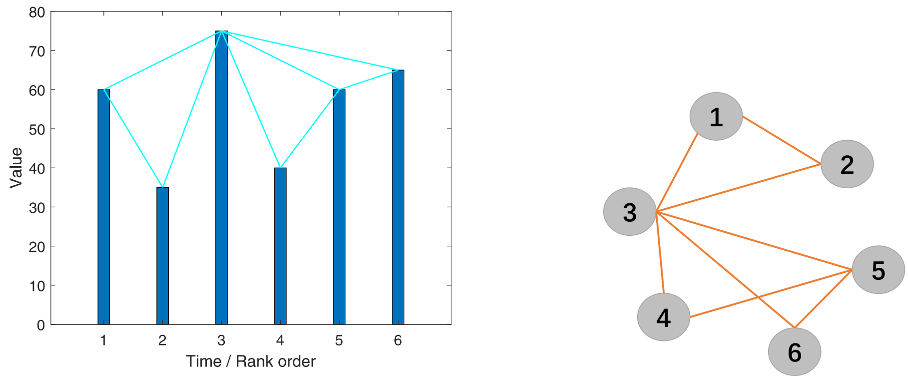
Given a set of time series data $X = \{(t_1, x_1), (t_2, x_2), \dots, (t_m, x_m)\}$, VG plots them as many pillars arranged on the time axis, the height of pillars represents the value of x_i. Any two pillars are linked if they can see each other without being blocked by other pillars, which means they are visible. Mathematically, x_i and x_j are visible if they satisfy the visibility criteria (Lacasa et al., 2008):

$$\frac{x_i - x_j}{t_i - t_j} > \frac{x_i - x_k}{t_i - t_k} \tag{5}$$

where t_k can be any value between t_i and t_j. Finally, the visibility graph is generated, with the nodes corresponding to the pillars and the edges corresponding to the links.

However, in the case when the obtained data is not time series, how can we get a complex network? In Li et al. (2016), the idea of ordered visibility graph (OVG) is inspired. The value of mass function is ranked to obtain an order. Using the order, a complex network is generated. Specifically, for an ordered data sequence $X = \{(r_1, x_1), (r_2, x_2), \dots, (r_m, x_m)\}$, r_m means the rank value, then the network is constructed by Eq. (6). Fig. 1 shows an example of VG/OVG with six data.

$$\frac{x_i - x_j}{r_i - r_j} > \frac{x_i - x_k}{r_i - r_k} \tag{6}$$



(a) The data (b) The related network

Fig. 1. An example of time series data (ordered data) and the related visibility graph.

3. The proposed method

3.1. The proposed OVGWP

In this section, a weighted network method based on the visibility graph (OVGWP) is presented to manage with probabilistic transformation for BPA in DSET. The main idea of OVGWP is to transform an ordered BPA to a network, each focal element is a node. Then the weight of focal elements can be measured according to degrees and weighted edges. Finally, based on the weight distribution, a global ratio for proportional belief transformation is calculated to get the probability distribution. Fig. 2 shows the framework of OVGWP, the detailed steps are illustrated below.

Step 4-1: Calculate the information volume of focal elements using belief entropy. Given a body of BPA m with s focal elements, for each focal element $A_i \subseteq \Theta, i \in [1, s]$, the information volume of A_i, denoted as $IV(A_i)$, is computed using belief entropy (see Eq. (4)). The specific formula is given as

$$IV(A_i) = -m(A_i)log_2 \frac{m(A_i)}{2^{|A_i|} - 1} \tag{7}$$

Step 4-2: Rank focal elements based on information volume.

Based on IV values of all focal elements, an ordered BPA sequence can be obtained using the descending principle, denoted as $\{(1, IV(A'_1)), (2, IV(A'_2)), \dots, (s, IV(A'_s))\}$, note that A'_1 is the focal element with the largest information volume.

Step 4-3: Construct the network of focal elements using the OVG algorithm.

Given the ordered sets of BPA in Step 4–2, the related network of focal elements can be constructed using Eq. (6), denoted as G(V, E), $|V|$ = s. The adjacency matrix is $B = [b_{ij}]_{s\times s}$, if A'_i is connected to A'_j, then $b_{ij} = 1$, otherwise $b_{ij} = 0$.

Step 4-4: Calculate the weighted adjacency matrix (WA) based on node distance or belief entropy.

For the network G, a crucial step is to calculate the weight of edges between nodes. The weighted adjacency matrix is denoted as $WA = [w_{ij}]_{s\times s}$, i and j are the indexes of A'_i and A'_j, respectively. Two different methods are introduced to measure edge weights using node distance and belief entropy, respectively.

$$w_{ij} = w(A'_i, A'_j) = \frac{b_{ij}}{|i-j|} \tag{8}$$

$$w_{ij} = w(A'_i, A'_j) = \frac{1}{2}\left(\frac{IV(A'_i)}{\sum_{a_{kj}=1} IV(A'_k)} + \frac{IV(A'_j)}{\sum_{a_{ki}=1} IV(A'_k)}\right) \tag{9}$$

Eq. (8) shows that the larger the rank order of two nodes, the smaller weight of the related edge. Eq. (9) shows that the larger the normalized information volume of the connected edges, the larger weight of the related edge.

Step 4-5: Calculate the degree of focal elements.

The degree of one focal element is defined by the sum of weighted edges of connected edges, as shown in Eq. (10).

$$D(A'_i) = \sum_{j=1}^{s} w(A'_i, A'_j) \tag{10}$$

Step 4-6: Compute the global weight of each singleton element in FOD.

In this step, by distributing the degrees of focal elements to the contained single elements, the global weight of singleton elements is calculated, as shown in Eq. (11).

$$w'(\theta) = \sum_{\theta \in A' \subseteq \Theta} \frac{D(A')}{|A'|} \tag{11}$$

Step 4-7: Calculate the transformed probability values.

For each focal element, a global ratio to its contained singleton elements is obtained based on global weights. Therefore, the proportional belief transformation is calculated as:

$$OVGWP(\theta_i) = \sum_{\theta_i \in A \subseteq 2^\Theta} \frac{w'(\theta_i)}{\sum_{\theta_j \in A} w'(\theta_j)} * m(A) \tag{12}$$

$\frac{w'(\theta_i)}{\sum_{\theta_j \in A} w'(\theta_j)}$ is the global ratio of θ_i in the focal element A.

Proposition 1. *The proposed OVGWP method satisfies the consistency of the upper and lower boundaries (Daniel, 2006), namely* $Bel_m(\theta_i) \leq OVGWP(\theta_i) \leq Pl_m(\theta_i)$.

Proof. Eq. (12) can be rewritten as:

$$OVGWP(\theta_i) = m(\theta_i) + \sum_{\theta_i \in A \subseteq 2^\Theta, A \neq \theta_i} \frac{w'(\theta_i)}{\sum_{\theta_j \in A} w'(\theta_j)} * m(A)$$

$$\text{Since} \quad 0 \leq \Lambda(\theta_i) = \frac{w'(\theta_i)}{\sum_{\theta_j \in A} w'(\theta_j)} \leq 1$$

$$\Lambda(\theta_i) \geq 0, \quad OVGWP(\theta_i) \geq m(\theta_i) = Bel_m(\theta_i)$$

$$\Lambda(\theta_i) \leq 1, \quad OVGWP(\theta_i) \leq m(\theta_i) + \sum_{\theta_i \in A \subseteq 2^\Theta, A \neq \theta_i} m(A) = Pl_m(\theta_i)$$

Hence $Bel_m(\theta_i) \leq OVGWP(\theta_i) \leq Pl_m(\theta_i)$.

3.2. A simple example

Following is a simple example to show the calculation of OVGWP.

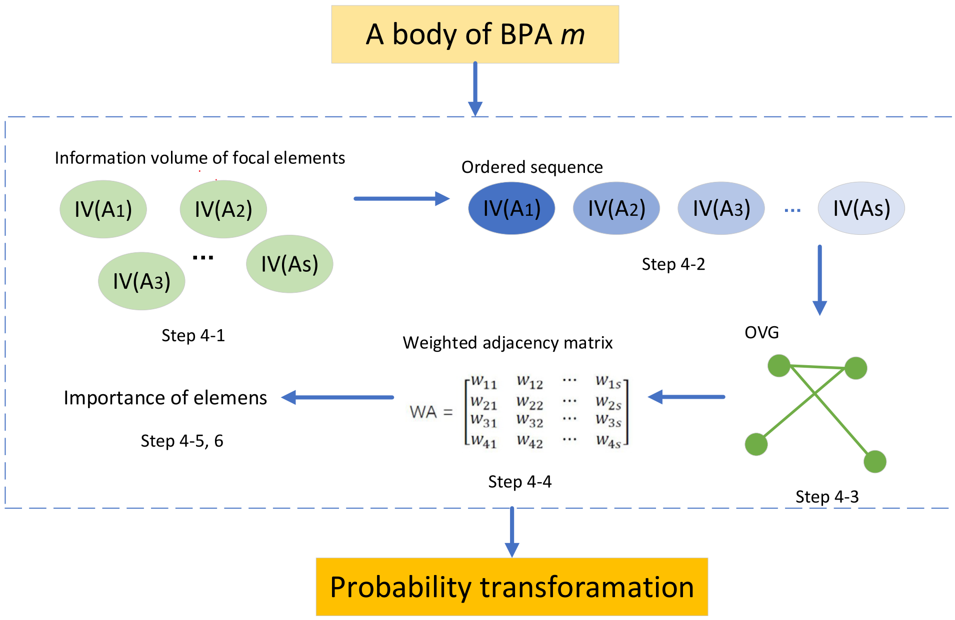


Fig. 2. The flowchart of the proposed OVGWP.

Example 3.1. Let m be a BPA defined on the FOD $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and the distribution is given as:

$$m(\{\theta_1\}) = 0.1, m(\{\theta_1, \theta_2\}) = 0.2$$
$$m(\{\theta_2, \theta_3\}) = 0.3, m(\{\Theta\}) = 0.4$$

Based on Eq. (7), the information volume of focal elements are calculated as follows:

$$IV(\{\theta_1\}) = 0.3322, IV(\{\theta_1, \theta_2\}) = 0.7814$$
$$IV(\{\theta_2, \theta_3\}) = 0.9966, IV(\Theta) = 1.6517$$

Hence the ordered BPA sequence is obtained as: {(1, 1.6517), (2, 0.9966), (3, 0.7814), (4, 0.3322)}. According to Eq. (6), the related OVG can be generated, as shown in Fig. 3.

The adjacency matrix B of the network is shown as:

$$B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

According to Eqs. (8)–(9), the weighted adjacency matrix WA based on node distance is calculated as:

$$\mathrm{WA} = \begin{bmatrix} 0 & 1 & 0.5 & 0 \\ 1 & 0 & 1 & 0 \\ 0.5 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Therefore, based on Eq. (10), the degrees of focal elements are computed as:

$$D(\{\theta_1\}) = 1, D(\{\theta_1, \theta_2\}) = 2.5, D(\{\theta_2, \theta_3\}) = 2, D(\Theta) = 1.5$$

Using Eq. (11) to measure the global weights of all single elements in Θ:

$$w'(\{\theta_1\}) = 2.75, w'(\{\theta_2\}) = 2.75, w'(\{\theta_3\}) = 1.5$$

Finally the transformation probabilistic values are obtained using Eq. (12).

$$OVGWP_1(\{\theta_1\}) = 0.1 + \frac{2.75}{2.75 + 2.75} * 0.2 + \frac{2.75}{2.75 + 2.75 + 1.5} * 0.4 = 0.3571$$

Table 1
Transformation results of different methods for Example 4.1.

	θ_1	θ_2	θ_3
OVGP (Li et al., 2016)	0.3333	0.3333	0.3333
ITP (Deng and Wang, 2020)	0.3333	0.3333	0.3333
$OVGWP_1$	0.3333	0.3333	0.3333
$OVGWP_2$	0.3333	0.3333	0.3333

$$OVGWP_1(\{\theta_2\}) = \frac{2.75}{2.75 + 2.75} * 0.2 + \frac{2.75}{2.75 + 1.5} * 0.3 + \frac{2.75}{2.75 + 2.75 + 1.5} * 0.4 = 0.4513$$

$$OVGWP_1(\{\theta_3\}) = \frac{1.5}{2.75 + 1.5} * 0.3 + \frac{1.5}{2.75 + 2.75 + 1.5} * 0.4 = 0.1916$$

4. Examples and discussions

In this section, through some examples from Li et al. (2016), the proposed OVGWP is compared with two recent methods, i.e., OVGP (Li et al., 2016) and ITP (Deng and Wang, 2020). Note that $OVGWP_1$ and $OVGWP_2$ denote the results using node distance and belief entropy, respectively.

Example 4.1. Let m be a BPA defined on $\Theta = \{\theta_1, \theta_2, \theta_3\}$, m($\Theta$) = 1.

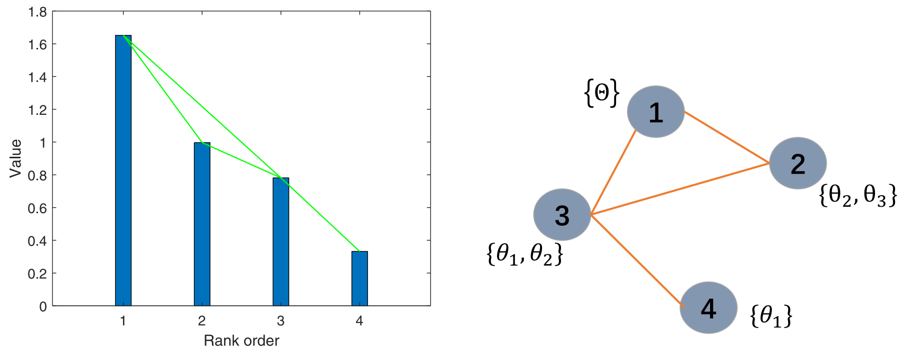
Table 1 shows the transformation results of three different methods, all of which give equal distribution to $\{\theta_1\}$, $\{\theta_2\}$ and $\{\theta_3\}$. Since no prior information about three singleton elements is provided, it is reasonable to assign equal belief value to $\{\theta_1\}$, $\{\theta_2\}$ and $\{\theta_3\}$. In addition, the transformation result by PPT is also $m(\theta_1) = m(\theta_2) = m(\theta_3) = 0.3333$.

Remark 1. Given a FOD with n elements, $n \geq 1$. If the BPA is given as $m(\Theta) = 1$, the proposed OVGWP has the same result with PPT.

Example 4.2. Let m be a BPA defined on $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and the BPA is described below:

$$m(\{\theta_1\}) = 0.2, m(\{\theta_2, \theta_3\}) = 0.8$$

Table 2 shows that three methods have the same transformation results. The element $\{\theta_1\}$ keeps the original value of 0.2, the other



(a) The data (b) The related network

Fig. 3. The OVG of Example 3.1.

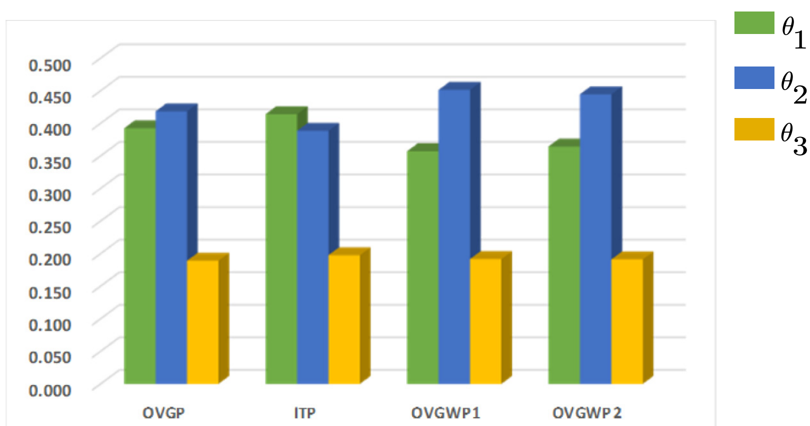


Fig. 4. The transformation results of Example 4.3.

two elements $\{\theta_2\}$ and $\{\theta_3\}$ have the same probability of 0.4. $\{\theta_2,\theta_3\}$ does not contain the information of the single focal element $\{\theta_1\}$, hence the belief value of $\{\theta_2,\theta_3\}$ makes no contribution on $\{\theta_1\}$ in assigning process, $\{\theta_1\}$ keeps the same belief value. Besides, no additional information is provided for the elements $\{\theta_2\}$ and $\{\theta_3\}$, the two elements are indistinguishable. Therefore, the results by three methods are reasonable.

Remark 2. Given a BPA m, for a focal element with multiple elements, if there is no additional BPA information about the contained singleton elements, an equal distribution is for its singleton elements.

Example 4.3. Let m be a BPA defined on $\Theta = \{\theta_1,\theta_2,\theta_3\}$, and the BPA is described below:

$$m(\{\theta_1\}) = 0.1, m(\{\theta_1,\theta_2\}) = 0.2, m(\{\theta_2,\theta_3\}) = 0.3, m(\Theta) = 0.4$$

Compared with Examples 4.1 and 4.2, this example shows a general BPA distribution. Table 3 and Fig. 4 depicts the transformation results by three different methods. It is observed that only ITP gives the highest probability to the element $\{\theta_1\}$, while both OVGP and the proposed OVGWP suggest that $\{\theta_2\}$ has the highest probability. The reason explained by ITP is that the belief value of $\{\theta_2\}$ is vacuous while $\{\theta_1\}$ has the value with 0.1. However, this evidence is sided. The other three focal elements all contain the information of $\{\theta_2\}$, and the total belief value is 0.9. Therefore, $\{\theta_2\}$ should has a higher probability than $\{\theta_1\}$ intuitively. In the proposed OVGWP, $\{\theta_2\}$ has the highest probability, which is a more reasonable result than ITP. In conclusion, this example shows that the proposed OVGWP has a better transform performance than ITP.

Table 2
Transformation results of different methods for Example 4.2.

	θ_1	θ_2	θ_3
OVGP (Li et al., 2016)	0.2	0.4	0.4
ITP (Deng and Wang, 2020)	0.2	0.4	0.4
$OVGWP_1$	0.2	0.4	0.4
$OVGWP_2$	0.2	0.4	0.4

Table 3
Transformation results of different methods for Example 4.3.

	θ_1	θ_2	θ_3
OVGP (Li et al., 2016)	0.3925	0.4184	0.1891
ITP (Deng and Wang, 2020)	0.4140	0.3885	0.1975
$OVGWP_1$	0.3571	0.4513	0.1916
$OVGWP_2$	0.3644	0.4445	0.1911

Table 4
Transformation results of different methods for Example 4.4.

	θ_1	θ_2
OVGP (Li et al., 2016)	0.75	0.25
ITP (Deng and Wang, 2020)	1	0
$OVGWP_1$	0.875	0.125
$OVGWP_2$	0.875	0.125

Example 4.4. Let m be a BPA defined on $\Theta = \{\theta_1,\theta_2\}$, and the BPA is described below:

$$m(\{\theta_1\}) = 0.5, m(\{\theta_1,\theta_2\}) = 0.5$$

The transformation results by three different methods are presented in Table 4. This example has the feature with equal belief values on all

Table 5
Probabilistic transformation results of $OVGWP_1$ in ten cases for Example 4.5.

	Element	1	2	3	4	5	6	7	8	9	10
case 1	X = {1}	**0.7310**	0.0617	0.0617	0.0617	0.0021	0.0021	0.0733	0.0021	0.0021	0.0021
case 2	X = {1,2}	0.2488	**0.5518**	0.0564	0.0564	0.0033	0.0033	0.0700	0.0033	0.0033	0.0033
case 3	X = {1,2,3}	0.1491	**0.3593**	**0.3593**	0.0493	0.0026	0.0026	0.0700	0.0026	0.0026	0.0026
case 4	X = {1,2,3,4}	0.0984	**0.2729**	**0.2729**	**0.2729**	0.0026	0.0026	0.0700	0.0026	0.0026	0.0026
case 5	X = {1,2,3,4,5}	0.0818	**0.2520**	**0.2520**	**0.2520**	0.0818	0.0026	0.0700	0.0026	0.0026	0.0026
case 6	X = {1,2,3,4,5,6}	0.0706	**0.2368**	**0.2368**	**0.2368**	0.0706	0.0706	0.0700	0.0026	0.0026	0.0026
case 7	X = {1,2,3,4,5,6,7}	0.0518	0.1952	0.1952	0.1952	0.0518	0.0518	**0.2512**	0.0026	0.0026	0.0026
case 8	X = {1,2,3,4,5,6,7,8}	0.0469	0.1881	0.1881	0.1881	0.0469	0.0469	**0.2428**	0.0469	0.0026	0.0026
case 9	X = {1,2,3,4,5,6,7,8,9}	0.0431	0.1821	0.1821	0.1821	0.0431	0.0431	**0.2355**	0.0431	0.0431	0.0026
case 10	X = {1,2,3,4,5,6,7,8,9,10}	0.0226	0.1756	0.1756	0.1756	0.0226	0.0226	**0.2374**	0.0226	0.0226	0.0226

Table 6
Probabilistic transformation results of $OVGWP_2$ in ten cases for Example 4.5.

	Element	1	2	3	4	5	6	7	8	9	10
case 1	X = {1}	**0.7233**	0.0629	0.0629	0.0629	0.0035	0.0035	0.0705	0.0035	0.0035	0.0035
case 2	X = {1,2}	0.2719	**0.5358**	0.0533	0.0533	0.0036	0.0036	0.0678	0.0036	0.0036	0.0036
case 3	X = {1,2,3}	0.1889	**0.3475**	**0.3475**	0.0374	0.0024	0.0024	0.0667	0.0024	0.0024	0.0024
case 4	X = {1,2,3,4}	0.1335	**0.2628**	**0.2628**	**0.2628**	0.0023	0.0023	0.0665	0.0023	0.0023	0.0023
case 5	X = {1,2,3,4,5}	0.1106	**0.2345**	**0.2345**	**0.2345**	0.1106	0.0023	0.0664	0.0023	0.0023	0.0023
case 6	X = {1,2,3,4,5,6}	0.0946	**0.2144**	**0.2144**	**0.2144**	0.0946	0.0946	0.0663	0.0022	0.0022	0.0022
case 7	X = {1,2,3,4,5,6,7}	0.0717	0.1791	0.1791	0.1791	0.0717	0.0717	**0.2409**	0.0022	0.0022	0.0022
case 8	X = {1,2,3,4,5,6,7,8}	0.0642	0.1694	0.1694	0.1694	0.0642	0.0642	**0.2306**	0.0642	0.0022	0.0022
case 9	X = {1,2,3,4,5,6,7,8,9}	0.0582	0.1616	0.1616	0.1616	0.0582	0.0582	**0.2221**	0.0582	0.0582	0.0022
case 10	X = {1,2,3,4,5,6,7,8,9,10}	0.0403	0.1479	0.1479	0.1479	0.0403	0.0403	**0.2144**	0.0403	0.0403	0.0403

focal elements. OVGP has an equal distribution for $\{\theta_1, \theta_2\}$, ignoring the prior information of $m(\{\theta_1\}) = 0.5$. As for ITP (Deng and Wang, 2020), it assigns all belief of $\{\theta_1, \theta_2\}$ to $\{\theta_1\}$, which seems a too optimistic result. However, the proposed OVGWP considers the prior information of $m(\{\theta_1\}) = 0.5$, it assigns a larger weight to $\{\theta_2\}$ than $\{\theta_1\}$ in distributing $\{\theta_1, \theta_2\}$. Therefore, OVGWP has a more reasonable result than ITP and OVGP.

Example 4.5. Given a FOD with 10 elements, $\Theta = \{1,2,3,4,5,6,7,8,9,10\}$. The BPA distribution is described below:

$m(\{2,3,4\}) = 0.15, m(\{7\}) = 0.05, m(\Theta) = 0.1, m(\{X\}) = 0.7.$

$\{X\}$ has 10 cases which is varied from $\{1\}$ to $\{1,2,3,\ldots,10\}$. This example comes from the paper of OVGP (Li et al., 2016) and we use it again to make a detailed comparison with the proposed OVGWP. The probabilistic transformation results by OVGWP and OVGP are shown in Tables 5–7, the visualized results are shown in Figs. 5 and 6.

From Figs. 5 and 6, it can be easily found that the change trend of probabilities by the proposed OVGWP and OVGP is different. Specifically, Tables 5–7 highlight the largest probability in bold for each case. As the size increase of X, OVGP always assigns the highest probability to $\{1\}$. While the proposed OVGWP has different decision elements with the change of X. Which one is more reasonable? Intuitively, with the rise of the size of X, the assigned probability to $\{1\}$ from X decrease generally. Therefore, it is unreasonable that $\{1\}$ always has the highest probability in OVGP. While in the proposed OVGWP, it can fully consider the prior information, dynamically achieve the most reasonable results with the change of X. For example, in Cases 4–6, three elements $\{2\}$, $\{3\}$, $\{4\}$ have the same highest transform probability. The reason is that these three elements have the most informative prior information of $m(\{2,3,4\}) = 0.15$, so a larger global weight is obtained from X. In addition, the three elements $\{2\}$, $\{3\}$, $\{4\}$ have the same probabilities by the proposed OVGWP. While in OVGP, $\{2\}$ has a larger probability than $\{3\}$ and $\{4\}$, which is also unreasonable since no prior information can be used to weight them. It can be concluded that the proposed OVGWP has a more reasonable transformation performance than OVGP.

5. Conclusion

In this paper, an ordered visibility graph weighted network, named OVGWP, is developed to transform basic probability assignment to

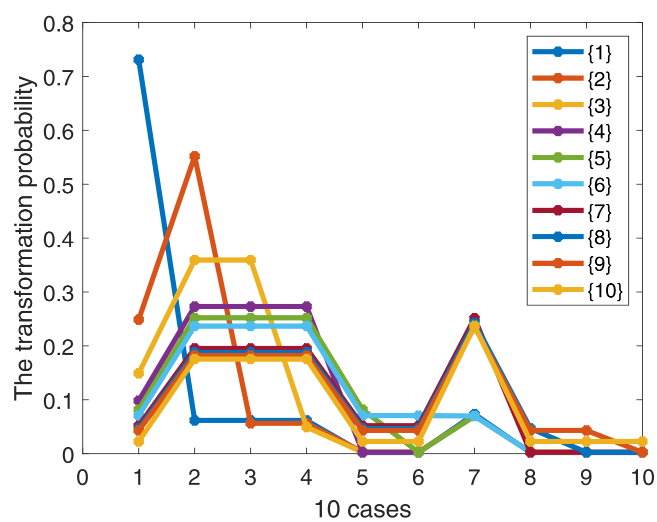


Fig. 5. The transformation probability trend of OVGWP in 10 cases for Example 4.5.

probability. In our proposed method, a complex network is generated by the information volume of each focal element. The weight of focal elements is then measured by the degrees and the edges. Finally, a global ratio for proportional belief transformation is presented to obtain the probability distribution. The proposed method meets the consistency of lower and upper boundaries, several examples are given to illustrate the reasonability and efficiency of the proposed OVGWP compared with other existing methods. The proposed method has potential contributions to decision making under uncertain environments.

Compared with other existing methods, the proposed OVGWP has two advantages. One is that OVGWP calculates information volume to rank focal elements. The information volume considers not only the belief value itself, but also the cardinality of focal elements. OVGWP considers more information of BPA to construct the network. The other is that the proposed OVGWP takes into account the weighted network. The weighted network also considers more information of degrees and edges. It can better calculate the importance of focal elements to determine the global ratio, which provides an efficient proportional

Table 7
Probabilistic transformation results of OVGP (Li et al., 2016) in ten cases for Example 4.5.

	Element	1	2	3	4	5	6	7	8	9	10
case 1	X = {1}	**0.7316**	0.1023	0.0387	0.0387	0.0031	0.0031	0.0735	0.0031	0.0031	0.0031
case 2	X = {1,2}	**0.5652**	0.2687	0.0387	0.0387	0.0031	0.0031	0.0735	0.0031	0.0031	0.0031
case 3	X = {1,2,3}	**0.4628**	0.2368	0.1732	0.0387	0.0031	0.0031	0.0735	0.0031	0.0031	0.0031
case 4	X = {1,2,3,4}	**0.3933**	0.2151	0.1515	0.1515	0.0031	0.0031	0.0735	0.0031	0.0031	0.0031
case 5	X = {1,2,3,4,5}	**0.3761**	0.2097	0.1461	0.1461	0.0364	0.0031	0.0735	0.0031	0.0031	0.0031
case 6	X = {1,2,3,4,5,6}	**0.3604**	0.2049	0.1413	0.1413	0.0348	0.0348	0.0735	0.0031	0.0031	0.0031
case 7	X = {1,2,3,4,5,6,7}	**0.2754**	0.1783	0.1147	0.1147	0.0267	0.0267	0.2544	0.0031	0.0031	0.0031
case 8	X = {1,2,3,4,5,6,7,8}	**0.2675**	0.1759	0.1123	0.1123	0.0259	0.0259	0.2485	0.0259	0.0031	0.0031
case 9	X = {1,2,3,4,5,6,7,8,9}	**0.2600**	0.1736	0.1100	0.1100	0.0252	0.0252	0.2429	0.0252	0.0252	0.0031
case 10	X = {1,2,3,4,5,6,7,8,9,10}	**0.2530**	0.1713	0.1077	0.1077	0.0245	0.0245	0.2378	0.0245	0.0245	0.0245

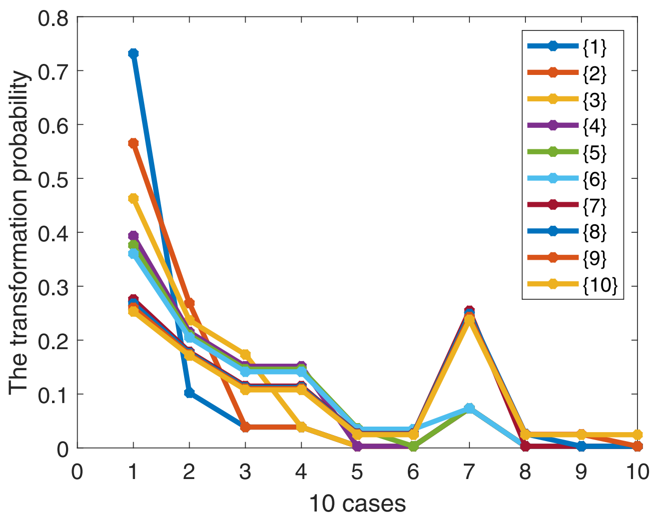


Fig. 6. The transformation probability trend of OVGP (Li et al., 2016) in 10 cases for Example 4.5.

belief transformation to obtain the final probability distribution for decision making.

CRediT authorship contribution statement

Luyuan Chen: Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **Yong Deng:** Validation, Resources, Writing – review & editing, Supervision, Funding acquisition. **Kang Hao Cheong:** Formal analysis, Validation, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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