

# A FIELD EXPERIMENT ON CONTRAST REDUCTION LAW

M. GAZZI, V. VICENTINI and U. BONAFÉ FISBAT-C.N.R., Bologna, Italy

Abstract—Atmospheric opacity monitoring employs visibility meters based on various physical principles. All the meters currently in use suffer from systematic errors typical of photometric measurements as well as systematic effects due to airborne particle size distribution. Rühle's luminance contrast meter, which avoids all these sources of error, is not considered a wholly reliable comparison instrument for the other meters because the restrictive conditions on atmosphere and illumination required by the visibility theory on which it relies would not be verifiable during measurements.

A re-examination of contrast reduction by the atmosphere in both ideal and non-ideal visibility conditions, and a re-interpretation of Rühle's hypotheses on errors in apparent luminance measurements, show that Rühle's instrumental apparatus itself makes it possible to verify the presence of ideal conditions a posteriori of a prolonged measurement period in as broad an opacity range as possible. Field experiment results confirming this fact are reported and discussed.

Key word index: Visibility, contrast reduction, telephotometer.

### INTRODUCTION

Atmospheric opacity monitoring employs visibility meters that rely on various physical principles. The differences between these latter arise from the part played in their working by the atmosphere itself, i.e. as a means of light attenuation (transmissometer), as a source of scattered light (integrating nephelometer) or both at the same time (backscatter meter).

The commonly employed visibility meters consist of a light source illuminating a sample of atmosphere and a photometer looking at the same air volume. All these meters suffer limitations and undetectable systematic errors which are mainly due to artificial lighting of the atmosphere [e.g. forward scattering in transmissometer measurements (Gumprecht and Sliepcevic, 1953), integrating nephelometer truncation error (Ensor and Waggoner, 1970)], absolute measurements (all meters) and setting method itself [e.g. transmissometer calibration by comparison to sight observations of landscape objects causes an overestimate of the measured atmospheric transparency with respect to the true one (Gazzi et al., 1985)]. A reference meter would be therefore necessary to test the reliability of visibility meters in use, as evidenced by the intercomparison of visibility measurements recently organised by the World Meteorological Organization (Griggs et al., 1989).

Thus such a reference instrument must not suffer from systematic errors typical of absolute photometric measurements (calibration and zero losses, stray light) nor systematic effects due to airborne particle size distribution. The oldest experimental apparatus employed to measure atmospheric opacity meets all these requisites. It consists of a telephotometer and a black target placed at some distance from it. Its

governing equation is Koschmieder's air-light equation (Koschmieder, 1925)

$$B(x) = B_h(1 - e^{-\sigma x}) \tag{1}$$

which gives the apparent luminance acquired by a black object due to the daylight scattered into the path of sight when it is observed at distance x through an atmosphere with extinction coefficient  $\sigma$  against the horizon sky of luminance  $B_h$ . As the solution of equation (1) gives the extinction coefficient as a function of the ratio of two luminances observed by the same photometer, measurements are relative and no calibration is needed. The only drawback of this meter resides in the fact that the measured luminances can be affected by the stray light in the photometer and by any weak reflectivity of the target.

To account for these causes of systematic error Rühle replaced equation (1) by the following modified air-light equation (Rühle, 1930)

$$B'(x) = B'_{h}[1 - (1 - \delta)e^{-\sigma x}]$$
 (2)

where B'(x) and  $B'_h$  are the measured luminances,  $B'_h = B_h + B_t$ ,  $\delta = (B_0 + B)/B'_h$ ,  $B_0$  is the target's inherent luminance due to its imperfect blackness and  $B_t$  the increase in luminance due to stray light. Moreover, to exclude the effects of stray light and target reflectivity in atmospheric opacity evaluation, he employed two identical targets (i.e. with the same weak inherent luminance) set at two distances from the photometer and deduced the extinction coefficient value from the ratio between the two measured horizon-target luminance differences

$$\frac{B'_{h} - B'(x_{2})}{B'_{h} - B'(x_{1})} = e^{-\sigma(x_{2} - x_{1})}$$
(3)

thereby eliminating parameter  $\delta$ .

902 M. GAZZI et al.

Equation (3), which avoids all causes of systematic error in the measurement of atmospheric opacity, is the governing equation of the most reliable visibility meter. Nevertheless, Rühle's apparatus, i.e. a photometer and two black targets, is not considered a wholly reliable comparison meter for the other instruments because of the very restrictive conditions, i.e. both light and atmosphere must be uniform along the entire path of sight, in which the air-light equation holds true.

The present study shows that Rühle's apparatus itself makes it possible to verify the presence of these conditions *a posteriori* of a prolonged measurement period in as broad an opacity range as possible.

### CONTRAST REDUCTION

In accordance with Middleton (1952), the apparent luminance of an object observed through the atmosphere can be derived from the equation

$$\frac{\mathrm{d}B(x)}{\mathrm{d}x} = -\sigma(x)B(x) + \frac{\mathrm{d}B_{\bullet}(x)}{\mathrm{d}x} \tag{4}$$

which states that the object's luminance on the one hand diminishes because of atmospheric extinction and on the other increases due to daylight coming from all directions and scattered towards the observer. In the most simple case atmosphere is lighted from one direction only (single scattering) and the increase of air luminance  $B_n(x)$  per unit of sight path is  $\beta'E$ , where  $\beta'$  is the value of the air volume scattering function for the angle between light propagation and sight direction and E the received illumination. By contrast, the general situation in which illumination is incident from all directions (the increase in luminance is due to multiply scattered light) requires integration over the sphere of product  $\beta' dE = \beta' B_s d\omega$ , where  $B_s$  is the luminance of the surroundings as seen in any direction from a point in the path of sight and  $d\omega$  an element of solid angle. The luminance enhancement due to the scattered daylight is thus

$$\frac{\mathrm{d}B_{a}}{\mathrm{d}x} = \int_{0}^{4\pi} \beta' B_{s} \mathrm{d}\omega = b \int_{0}^{4\pi} \frac{\beta'}{b} B_{s} \mathrm{d}\omega \tag{5}$$

where b is the air scattering coefficient,  $\beta'/b$  the relative scattering function and the integral in the last right-hand expression is the so-called source function.

By using as boundary condition the object inherent luminance  $B_0$  at x = 0, the formal solution of equation (4) is

$$B(x) = B_0 e^{-\int_0^x \sigma(\xi) d\xi} + \int_0^x \frac{dB_a(\xi)}{d\xi} e^{-\int_\xi^x \sigma(\zeta) d\zeta} d\xi \qquad (6)$$

where the second term in the right-hand-side expression represents the apparent luminance of a black object  $(B_0=0)$  located in 0 as seen at distance x through an atmosphere in which prevailing light and attenuation conditions noticeably differ from the ideal

ones required by Koschmieder's visibility theory. Such a complex analytical expression of black target apparent luminance prevents any interpretation of telephotometric measurements in terms of atmospheric opacity. Moreover, this expression cannot be usefully simplified solely by assuming uniform atmosphere, prevailing scattering with respect to absorption, or isotropic illumination at any point along the path of sight. In fact, Koschmieder's "polished" airlight equation does not hold true mainly because of non-uniform daylight illumination along the sight direction. A very thorough physical discussion of the consequences of non-uniform illumination on distant contrast observations is provided by Horvath (1981). Here our discussion will be limited to a few mathematical remarks on the subject.

The most important consequence of the unevenness in lighting conditions is that the horizon-sky luminance value changes along with observation point, as does sky luminance observed along a slant path. Indeed, horizon luminance, which is the integral of black target apparent luminance from  $-\infty$  (horizon) to observation point x (see Fig. 1), has the following constant value

$$B_{\rm h} = \frac{1}{\sigma} \frac{\mathrm{d}B_{\rm a}}{\mathrm{d}x} \tag{7}$$

as seen from any point in the line-of-sight only if all ideal conditions are met. By contrast, if non-uniform illumination prevails, two horizon luminance values observed in the same direction are linked by equation

$$B_{h}(x_{2}) = B_{h}(x_{1}) e^{-\sigma(x_{2} - x_{1})} + e^{-\sigma(x_{2} - x_{1})} \int_{x_{1}}^{x_{2}} \frac{dB_{a}(\xi)}{d\xi} e^{\sigma(\xi - x_{1})} d\xi$$
(8)

which states that the horizon observed from  $x_2$  appears as a target in  $x_1$  endowed with inherent luminance  $B_h(x_1)$ .

Equations (6) and (8) can be combined to yield

$$B(x) = B_h(x) - [B_h(0) - B_0] e^{-\sigma x}$$
 (9)

where  $B_h(0)$  is the horizon luminance observed from object location x=0. By using equation (9) the object-horizon luminance contrast as seen from x can be written as follows

$$C(x) = \frac{B_{h}(x) - B(x)}{B_{h}(x)} = \frac{B_{h}(0) - B_{0}}{B_{h}(0)} \frac{B_{h}(0)}{B_{h}(x)} e^{-\sigma x}$$
(10)

i.e. as a function of both inherent contrast  $C(0) = [B_h(0) - B_0]/B_h(0)$  and the ratio of horizon inherent  $B_h(0)$  to apparent  $B_h(x)$  luminance.

Equation (10), which states that the apparent contrast value depends on object location because of both distance x and horizon inherent luminance  $B_h(0)$ , clearly evidences the difference between ideal and non-ideal lighting conditions. Indeed, if uniform illumination prevails, equations (4) and (7) can be com-

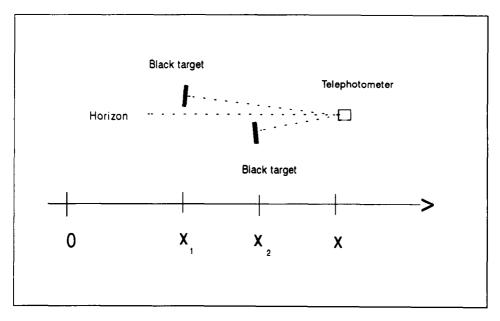


Fig. 1. Schematic of the experiment.

bined as

$$\frac{\mathrm{d}B(x)}{\mathrm{d}x} = -\sigma B(x) + \sigma B_{\mathrm{h}} \tag{11}$$

whose solution is

$$B(x) = B_0 e^{-\sigma x} + B_h (1 - e^{-\sigma x})$$
 (12)

which in turn reduces to air-light equation (1) if the object is black. Consequently, the object-horizon luminance contrast becomes

$$C(x) = \frac{B_{h} - B_{0}}{B_{h}} e^{-\sigma x}$$
 (13)

its value here depending on object location solely because of the exponential term since  $B_h$  is both apparent and inherent luminance at the same time. Equation (13) represents Koschmieder's contrast reduction law, whose main feature is that the luminance contrast between a black object and the horizon–sky does not depend on field luminance value, as it does in equation (10). Visibility theory completely relies on this law and on the air-light equation, neither of which holds in non-uniform lighting conditions because of horizon luminance's dependence on the observation point.

In order to verify the existence of ideal conditions, equation (13) has been tested experimentally using more than two identical targets, i.e. with the same inherent luminance, set at various distances from the photometer and nearly in the same straight line (Duntley, 1948; Coleman et al., 1949). This same end can also be achieved, however, exploiting the difference between equations (10) and (13) as follows. Let us consider two targets with the same inherent luminance  $B_0$  at different distances from observation

point x and in the same line-of-sight. Let us furthermore double one target location distance  $vis-\dot{a}-vis$  from the other to facilitate calculation. If uniform illumination prevails, both targets will have the same inherent contrast value  $C_0 = (B_{\rm h} - B_0)/B_{\rm h}$  with the horizon and their apparent contrasts will be linked by the following simple relation

$$C_1(x) = \frac{C_2^2(x)}{C_0} \tag{14}$$

where subscripts 1 and 2 refer to far and near target locations, respectively.

By contrast, if the illumination is not uniform, the targets will have different inherent contrasts with the horizon; the same will also hold for the inherent-to-apparent horizon luminance ratio values. As a result, instead of the simple equation (14) we get the very bulky, over-complicated relation

$$C_1(x) = \frac{C_0(x_1)}{C_0^2(x_2)} \frac{B_h(x_1)B_h(x)}{B_h^2(x_2)} C_2^2(x)$$
 (15)

which cannot be simplified even if black targets are assumed because three different horizon luminance values are involved.

Controlling the existence of ideal conditions can thus be effected by verifying a posteriori of luminance contrast measurements if the readings agree with equation (14). Note that the experimental test of equation (13), which involves only one opacity value at a time, relies on the fact that the target's inherent contrast with the horizon does not vary during a measurement set. By contrast, verification of equation (14) has to be done in the broadest possible opacity range, the inherent contrast thereby coming to depend on field luminance, which can noticeably

904 M. GAZZI et al.

change during measurements. Targets thus have to be truly black, or at any rate inherent contrast has to be independent of lighting conditions (see below).

## RÜHLE'S APPARATUS

In order experimentally to verify via equation (14) the existence of ideal conditions during measurements, an interpretation of a nearly black target's measured (by a stray-light-affected photometer) apparent luminance that differs from Rühle's is necessary. His modified air-light equation (2) is not in effect accounted for, unlike equation (14), by visibility theory. We will therefore seek a formal analogy with non-black target luminance equation (12), i.e. we will attempt to make the target's inherent luminance dependent on stray light, as is the horizon's.

The first step is to express the measured luminances and contrasts after Coleman (1947) in his paper on stray light in telephotometers, i.e. by making use of the photometer's transmission factor T, scattering coefficient K and contrast rendition R. These parameters, which characterize an optical system, are to a great extent independent of the viewing conditions encountered in distant contrast measurements, as shown by Coleman's experimental findings. They are defined as the image-to-object luminance, stray light-to-background luminance and measured-to-true contrast ratio, respectively. Accordingly, a nearly black target's measured luminance is

$$B'(x) = T[B_{h}(1 - e^{-\sigma x}) + B_{0}e^{-\sigma x}] + KB_{h}$$
 (16)

and that of the horizon-sky becomes

$$B_{\rm h}' = TB_{\rm h} + KB_{\rm h} \tag{17}$$

where  $KB_h$  is the increase in luminance due to stray light. By rearranging these equations, the measured target luminance can be written as

$$B'(x) = B'_{h} - (TB_{h} - TB_{0})e^{-\sigma x}$$
 (18)

where  $TB_0$  and  $TB_h$  would be the measured target and horizon inherent luminances, respectively, if the photometer is not affected by stray light.

Adding and subtracting the stray light contribution in brackets yields either of two equations:

(a) Rühle's modified air-light equation (2), in which parameter  $\delta$  now becomes

$$\delta = \frac{TB_0 + KB_h}{(T + K)B_h} \tag{19}$$

which takes into account both photometer's transmission factor and scattering coefficient.

(b) Non-black target apparent luminance equation

$$B'(x) = B'_0 e^{-\sigma x} + B'_h (1 - e^{-\sigma x})$$
 (20)

in which the "false" light due to the target's reflectance and photometer's stray light

$$B_0' = TB_0 + KB_h \tag{21}$$

plays the role of "false" (accounting for stray light too) inherent luminance of a target observed against a horizon-sky whose "false" luminance is given by equation (17).

The advantage of (b) with respect to (a) is evident if we consider that, unlike equation (2), equation (20) makes it possible to handle a nearly black target's luminance observed by an instrument which suffers from stray light as the luminance of a target which is not at all black measured by a perfect photometer. As a consequence, the luminance contrast with the horizon-sky becomes

$$C'(x) = C_0' e^{-\sigma x} \tag{22}$$

where the "false" inherent contrast is

$$C_0' = \frac{B_h' - B_0'}{B_h'} = C_0 R \tag{23}$$

 $C_0$  being the true intrinsic contrast  $(B_h - B_0)/B_h$  due to target reflectance and

$$R = \frac{T}{T + K} \tag{24}$$

being the photometer's contrast rendition, which is due solely to stray light.

Likewise, the underestimate of measured contrast as compared to the case in which stray light and reflectance are absent as sources of error is given by

$$\frac{\Delta C}{C} = -(1 - C_0 R) \tag{25}$$

whose value does not depend on target location. To this corresponds an overestimate of the extinction coefficient

$$\Delta \sigma = \frac{1}{x} \ln \left( \frac{1}{C_0 R} \right) \tag{26}$$

which is all the greater the shorter the photometertarget distance.

Note that equations (20) and (22) yield neither results that differ from those of equation (2) or that can be obtained by calculating the contrast by means of it (e.g.  $1-\delta$  coincides with the photometer's contrast rendition if the target is black). What we do have are the conditions to which the results of measurements must be subject if there exist the ideal conditions required by visibility theory. In fact, given that two targets with the same weak reflectance acquire the same "false" inherent luminance as observed by the same photometer, "false" inherent contrasts with the horizon also are the same. As a consequence, if one target location is twice the other, the measured contrasts must obey equation (14) when ideal conditions prevail. Moreover, another important feature of equation (20) it that it holds true even if the target has no reflectance at all, or if  $TB_0$  can be disregarded at least with respect to  $KB_h$  (e.g. hole in a black box). In this case, as "false" inherent luminance  $B'_0$  is the stray light itself, the equation accounts for the effect of a photometer's source of error as if it were a target characteristic. The result is that the "false" inherent contrast reduces to the photometer's contrast rendition and equation (14) becomes

$$C'_1(x) = \frac{1}{R}C'_2(x)$$
 (27)

which, because R is independent of field luminance, represents a parabola through the origin of the axes in the plane  $C'_2$ ,  $C'_1$ .

Ideal conditions can thus be verified by Rühle's apparatus (a telephotometer and two targets) itself, which can be envisaged as two identical (a photometer and a truly black target) yet distinct instruments, i.e. two luminance meters which differ from each other in the baseline length alone, a posteriori of a prolonged measurement period in a range  $C_2$  as wide as possible.

Finally, if the results of the intercomparison between the two contrast meters agree with equation (27), the true extinction coefficient values can be found by using Rühle's method which, by employing contrasts instead of luminance differences, can be stated as follows: given that two objects of equal "false" inherent luminance have the same "false" intrinsic contrast with the horizon when ideal conditions exist, the ratio between the measured contrasts is equal to that between the contrasts which two true black targets situated at the same distances would have, i.e.

$$\frac{C_1'(x)}{C_2'(x)} = e^{-\sigma(x_2 - x_1)}$$
 (28)

which thus substitutes Rühle's apparatus-governing equation (3).

## EXPERIMENTAL RESULTS AND CONCLUSIONS

These assumptions were tested in measurements taken in a rural area (San Pietro Capofiume) of the Po Valley during winter. The instrumental apparatus comprised an old model 1970-PR of Pritchard's photometer (Stimson, 1974) and two black targets set 50 and 100 m away from it in a northerly direction (Fig. 1). A baseline as short as 50 m was chosen in order to collect data in the range in which most visibility meters operate (meteorological range less than 2 km). The photometer had a 7" F 3.5 PD objective, and the field of view was 6'. The original phototube was replaced by a PIN silicon photodiode (UDT photop) (Stimson, 1974) for protracted use of the instrument, and the measurements were taken with a photopic filter. The targets were black boxes with center holes. In order to have a substantial number of data, the measurements were run automatically by setting the photometer on a sort of turn-table; a small motor would turn it towards the horizon and then towards each of the two targets. A microprocessor system controlled the measurement runs and recorded the data. Each run lasted 20 s with a 3-min interval in-between; a zero check was made before each.

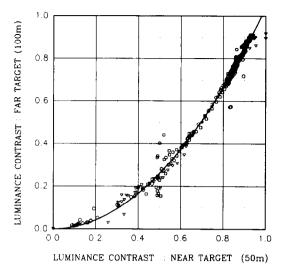


Fig. 2. Comparison between the two luminance contrast meters during three haze and thin fog episodes: (o) fog, 171 observations; (V) haze, 200 observations; (I) haze, 208 observations. The solid curve is the parabola represented by equation (27) in which the photometer's contrast rendition is assumed to be 0.96.

The results of the comparison between the two luminance contrast meters (a photometer and a target) are shown in Fig. 2. They refer to data collected during three thick haze and thin fog episodes. Meteorological visibility ranged from roughly 100 to 3000 m. No information on the presence of any clouds along the path-of-sight is available.

The experimental points fit quite well the parabola represented by equation (27), in which the photometer's contrast rendition is assumed to be 0.96 (solid curve), especially at  $C_2 > 0.60$  values (meteorological range greater than 400 m). The scattering of the data around the curve which occurs at lower visibilities is probably due to the non-simultaneity of measurements, which can strongly affect the calculated values for contrasts because of rapid variations in the extinction coefficient and, hence, of the three luminances during fog evolution.

These results are purely indicative, given the inadequacy to detect rapid luminance variations of the instrumentation employed and the few episodes investigated. They nevertheless show that (i) during our measurements the prevailing light and atmosphere conditions were those required by visibility theory; (ii) "false" inherent contrast with the horizon of our targets is a constant, regardless of the field luminance value, i.e. targets can be considered black; and (iii) the contrast rendition of our photometer is about 96%. As a consequence, Rühle's method can be applied a posteriori to the data collected in order to obtain the true extinction coefficient values.

As an example of the entire process (test of ideal conditions and Rühle's method application), the results of measurements taken during a fog dissipation episode are presented. Figure 3 shows luminances as

906 M. GAZZI et al.

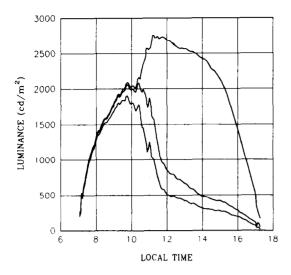


Fig. 3. Apparent luminances measured from sunrise to sunset during a fog dissipation episode. From the top: horizon, far target and near target luminance.

measured from sunrise to sunset [although the absolute luminances are not required, their values are the true ones as the photometer was calibrated for comparison with a commercially available telephotometer (UBD Spectra Spotmeter) (Stimson, 1974)]. The values are running means of five data. Fog is shown by the unsuitableness of the photometer to distinguish one reading from another, given that the apparent luminances of the targets are virtually equal to their liminal value  $B'_{h}$ . Data collected through such an extremely opaque atmosphere are thus to be disregarded, as are those gathered at sunrise and sunset which strongly reduces the precision of the contrast calculation because the  $B'_h$  values are too low (Gazzi et al., 1985). Figure 4 shows the experimental comparison between the two contrast meters. A contrast range as large as  $0.10 \le C_2' < 0.90$ , which corresponds to meteorological visibilities from roughly 100 to 1500 m, evidences a parabola through the origin of the axes. The presence of ideal conditions being thus verified, Rühle's method equation (28) can be applied to the results of the measurements and the extinction coefficient values deduced from the data collected by each of the two contrast meters via equation (22) can be compared with the true ones (Fig. 5). The figure shows that, according to equation (26), the absolute errors are constant and the error of the meter which makes use of the closer target (0.82 km<sup>-1</sup>) is twice that of the one aimed at the farther target (0.41 km<sup>-1</sup>). Figure 5 is thus an example of the use of Rühle's apparatus as a comparison meter to test the reliability of a visibility sensor which, in this case, is a contrast meter employing only one target.

If these experimental results are confirmed by a larger data set collected during a thorough measurement test, in which instrumentation suitable to detect the rapid variations that occur in apparent luminance

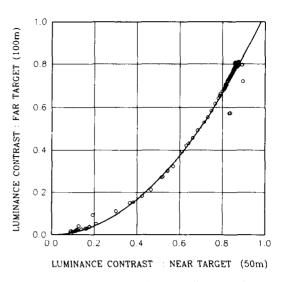


Fig. 4. Comparison between the data collected by the two contrast meters during a fog dissipation. Data are part of the data in Fig. 2 [(0), 171 observations], the solid curve is the same as in Fig. 2.

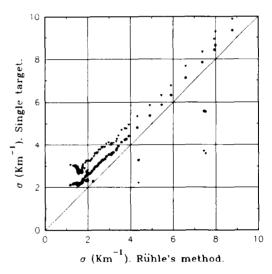


Fig. 5. Application of Rühle's method. Extinction coefficient values deduced from near target (V) and far target (•) measured apparent contrast with horizon as compared with the true ones (171 observations).

values during foggy conditions is employed, it can be concluded that Rühle's apparatus is the required reference meter for the visibility sensors currently in use in that (i) it eliminates all sources of systematic error of a photometric nature (calibration, zero losses, stray light), (ii) it does not suffer from systematic effects due to artificial lighting of the atmosphere (e.g. forward scattering) and (iii) it enables its underlying physical principle (contrast reduction law) to be verified a posteriori of measurements.

## REFERENCES

- Coleman H. S. (1947) Stray light in optical systems. J. Opt. Soc. Am. 37, 434-451.
- Coleman H. S., Morris F. J., Rosemberg H. E. and Walker M. J. (1949) A photoelectric method of measuring the atmospheric attenuation of brightness contrasts along a horizontal path for the visible region of the spectrum. J. Opt. Soc. Am. 39, 515-521.
- Duntley S. Q. (1948) The reduction of apparent contrast by the atmosphere. J. Opt. Soc. Am. 38, 179-191.
- Ensor D. S. and Waggoner A. P. (1970) Angular truncation error in the integrating nephelometer. Atmospheric Environment 4, 481-487.
- Gazzi M., Vicentini V., Pesci C. and Bonafé U. (1985) Diagnosis of the causes of systematic errors in atmospheric transparency measurements and some experimental verifications in Po Valley fogs. J. atmos. ocean. Tech. 2, 201-211.
- Griggs D. J., Jones D. W., Ouldridge M. and Sparks W. R.

- (1989) The first WMO international intercomparison of visibility measurements. Part I: the instruments, siting, data logging and data processing techniques. WMO Tech. Conf. Instr. and Methods of Obs. (TECIMO-IV), Brussels, September.
- Gumprecht R. O. and Sliepcevic C. M. (1953) Scattering of light by large spherical particles. J. chem. Phys. 57, 90-95. Horvath H. (1981) Atmospheric visibility. Atmospheric Environment 15, 1785-1796.
- Koschmieder H. (1925) Theorie der horizontalen Sichtweite. Beitr. Phys. Atmos. 12, 33-55 and 171-181.
- Middleton W. E. K. (1952) Vision Through the Atmosphere. University of Toronto Press, Toronto.
- Rühle H. (1930) Danziger Sichtmessungen II: Die scheinbare Flachenhelligkeit einer schwarzen Flache in Abhangigkeit vom Sonnenazimut bei grossen Sichtweiten. Forschungsarbeiten staatl. Obs. Danzig 3, 7-11.
- Stimson A. (1974) Photometry and Radiometry for Engineers. John Wiley, New York.