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Uncertainty representation, quantification and evaluation for data and information fusion

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***Abstract*—Mathematical and uncertainty modelling is an important component of data fusion (the fusion of unprocessed sensor data) and information fusion (the fusion of processed or interpreted data). If uncertainties in the modelling process are not or are incorrectly accounted for, fusion processes may provide under- or overconfident results, or in some cases incorrect results. These are often owing to incorrect or invalid simplifying assumptions during the modelling process. The authors investigate the sources of uncertainty in the modelling process. In particular, four processes of abstraction are identified where uncertainty may enter the modelling process. These are isolation abstraction (where uncertainty is introduced by isolating a portion of the real world to be modelled), datum uncertainty (where uncertainty is introduced by representing real world information by a mathematical quantity), data generation abstraction (where uncertainty is introduced through a mathematical representation of the mapping between a real-world process and an observable datum), and process abstraction (where uncertainty is introduced through a mathematical representation of real world entities and processes). The uncertainties associated with these abstraction processes are characterised according to the uncertainty representation and reasoning evaluation framework (URREF) ontology. A Bayesian network information fusion use case that models the rhino poaching problem is utilised to demonstrate the taxonomies introduced in this paper.**

I. Introduction

Fusion is the combination of information from multiple sources to draw more comprehensive, specific and accurate inferences about the world than are achievable from the individual sources in isolation. There are many sources of uncertainty that can affect the performance of a fusion system. Data from sensors are inherently noisy. Information from human sources may be imprecise, ambiguous, irrelevant and lack credibility. Algorithms for information fusion are based on models: of the real-world entities and processes of interest and of how those entities and processes generate the information being fused. Models are always abstractions of the phenomena being modelled, excluding some aspects of reality and simplifying or approximating others. These abstractions introduce inaccuracies and uncertainties in fusion results. Poorly understood entities, processes and information-generation mechanisms introduce additional uncertainty due to lack of model fidelity to the real world. Models typically contain tunable parameters such as physical constants, assumptions about the operating environment, or characteristics of the sensors. These parameters may be only imprecisely known, introducing additional uncertainty. Other sources of error include amplification of small errors in inputs through nonlinear dynamical models, and numerical errors from finite precision operations on digital computers.

As fusion systems address problems of greater complexity, difficulties of uncertainty assume greater importance. The Joint Directors of Laboratories (JDL) [1], [2], [3] and Data Fusion Interest Group (DFIG) [4], [3] fusion models allow for several levels of abstraction. As such, fusion systems may model not only physical entities and processes based on clearly defined and physically interpretable models, but also higher level entities and processes such as situation and impact assessment, for which behaviors are less well-understood and models are much more uncertain. In situation and impact assessment, there exists uncertainty in how meaning of symbols (tokens) are acquired and how these symbols should be chosen to represent objects and relations between objects in the real world. These uncertainties become prominent in machine assisted situation assessment. The problem of symbolic representation and reasoning in fusion is known as the semantic challenge [5], [6], [7]. It is necessary to account for such semantic uncertainties, but this further adds to the complexities of reasoning and model generation, which creates yet more uncertainty.

The International Society of Information Fusion (ISIF) Evaluation Techniques for Uncertainty Representation Working Group (ETURWG) studies the quantification and evaluation of all types of uncertainty in the information fusion process. The group introduced the Uncertainty Representation and Reasoning Evaluation Framework (URREF) ontology, which represents concepts and criteria needed to evaluate uncertainty management aspects of a fusion system [8]. The URREF ontology continues to evolve, and has been applied to characterize uncertainty in a variety of fusion problems (e.g., [9], [10], [11], [12]).

The purpose of this paper is to consider, within the context of the URREF ontology, the different places where uncertainty enters into the information fusion process, to analyse the extent to which these uncertainty sources are addressed within URREF, and to identify any modifications to URREF that are needed to fully address these uncertainties. The next section gives a brief overview of the discipline of Uncertainty Quantification, which has a similar motivation of characterizing and addressing uncertainties in complex models and simulations. Then we discuss the process of modelling a real-world process that is the subject of a fusion system, as well as modelling the generation of information input to the fusion system. The uncertainties in modelling and information generation are discussed and related to the URREF ontology. Finally, a case study is provided of a fusion system to make inferences about rhino poaching.

II. Uncertainty Quantification

The discipline of Uncertainty Quantification (UQ) is devoted to the problem of characterizing and properly addressing uncertainties in the use of mathematical and computational models of complex processes and data [13], [14]. This literature had its genesis in the recognition of the need to address uncertainty in computer models and simulations of highly complex physical processes, such as petroleum reservoir prediction, groundwater flows, or nuclear reactor safety, and to study how uncertainty propagates through complex models to affect conclusions drawn from the models.

The UQ literature classifies uncertainties as *epistemic* or *aleatoric* [15], [13]. Epistemic uncertainty is derived from the Greek word "episteme" and relates to uncertainty owing to a lack of knowledge or ignorance about the modelled process or entity. As such this uncertainty lies *outside* of the entity or process being modelled. Aleatoric uncertainty is derived from the Latin word "alea" which refers to the casting of dice. Aleatoric uncertainty refers to random events *within* the entity or process being modelled. A number of sources of uncertainty through which uncertainty enters the modelling process are considered in [16]. These include parameter uncertainty, parametric variability, structural uncertainty, algorithmic uncertainty, experimental uncertainty and interpolation uncertainty. In [16], uncertainties as a result of the mathematical modelling and variable abstraction process are not discussed separately from the computer model and its respective abstractions.

UQ is concerned with how uncertainty is propagated through a model to affect conclusions drawn from the model. The literature distinguishes two varieties: forward uncertainty propagation and inverse uncertainty quantification [17], [14]. Forward uncertainty propagation quantifies the effect of uncertainty in model parameters and input variables (parametric variability) on the output variables of the model. This typically involves how measurement errors propagate through the mathematical model and how they influence the output variables. Typical methods to perform such forward uncertainty propagation analyses are random and deterministic sampling methods. Sensitivity analysis and response surface methods are other ways in which the effect of perturbations of input variables on the output can be quantified [14]. In the information fusion domain, forward uncertainty propagation may involve how uncertainty in certain inputs affect the fusion results and ultimately the decisions and outcomes. On the other hand, inverse uncertainty quantification can be seen as a generalisation of parameter estimation error analysis [17] and falls within the category of inverse problems [18]. The objective of inverse uncertainty quantification is to study discrepancies between the model and its parameters on the one hand, and observed outcomes on the other. Inverse uncertainty quantification can be used for bias correction (which quantifies discrepancies between model and outcomes), parameter calibration (which quantifies parameter uncertainty) or both [19]. Methods for performing inverse uncertainty quantification include frequentist, modular Bayesian or fully Bayesian approaches.

Apart from difficulties with dimensionality scaling, which are by no means unique to uncertainty quantification methods, the identifiability problem is particularly interesting [20]. It may happen that multiple combinations of unknown parameters and discrepancy functions can yield the same experimental predictions, or in other words fit well to the data. In this case, different values of parameters or different model functions cannot be distinguished or identified. In such cases, explicitly characterising the uncertainty of the parameters and/or models by probability distributions, belief functions, fuzzy sets and so on may assist in quantifying such ambiguities.

III. Modelling Real World Processes

The modelling process entails several different processes of abstraction. Within the context of this paper, abstraction is meant to represent both the domain description and the uncertainties associated with this description. A typical modelling problem considers a portion of interest of the real world to be modelled (Figure 1), and here is termed a real world entities and processes (RWEPs). Ideally, all interactions between all processes would need to be considered (indicated by dotted arrows), but as this clearly infeasible, processes are split and such interactions are replaced by boundary conditions as well as input and output interactions. This is the first abstraction that takes place and is here labelled as real world entity and process *isolation abstraction*. Although not explicitly shown in Figure 1, these processes also have a temporal component, and causal dependencies (dotted arrows) also exist between different and the same RWEPs at different time instances.

Consider without a loss of generality such an isolated nth set of entities and processes within the real world denoted by RWEPn that generates a datum $D_{n,k}$ at time instant k. The nth real world process has physical properties that are represented by the symbol Ω_n. The way in which observable effects are generated by the RWEP, is represented by the transformation $\{D_n|\Omega_n\}$, and can be read as D_n given Ω_n.

The datum is a real world effect that is observed. It is mathematically meaningless, since a process of abstraction is needed to convert it into a mathematical quantity such as a integer, real number, complex vector, a first order logic statement etc. This process is labelled *datum abstraction*. In some cases, a datum may also be the output of another fusion process (such as the output of a filter), and as such dependencies exist between data points. A datum should also not be confused with a *measurement* which has already been transduced by a sensor into an instantiation of a mathematical quantity. The RWEP is understood as including the whole process which provides data or information, and as such includes the source with perception means, internal reasoning process and possible uncertainty assignment as suggested in [21]. The datum can consequently take many different forms, for example ranging from a raw signal, image, or power levels, possibly transformed or filtered to remove noise. It can be natural language sentences such as provided by intelligence reports, the internet, or any

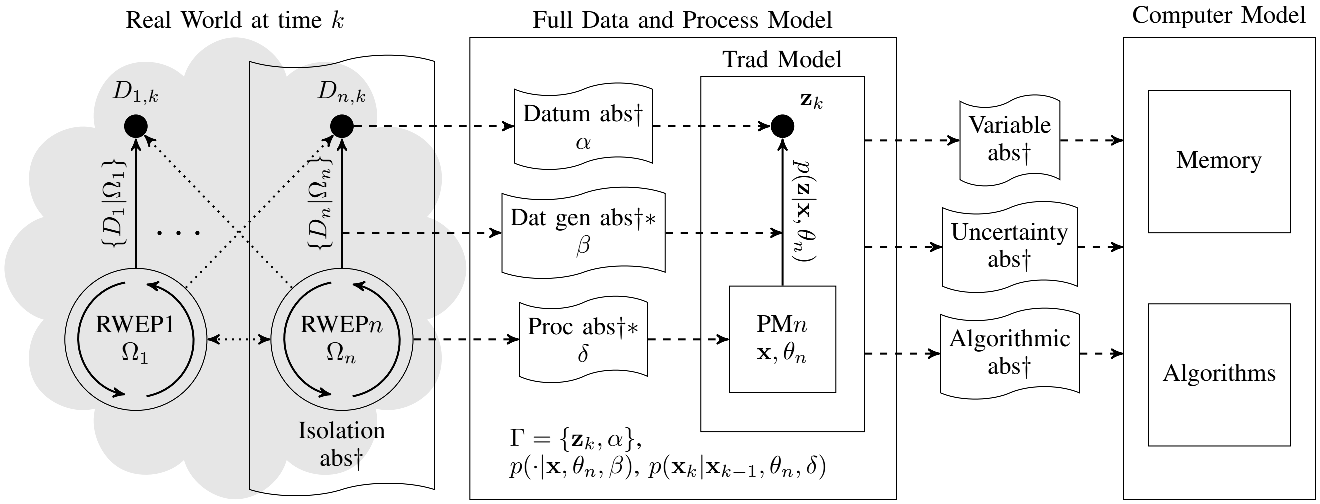

Fig. 1. The modelling process at time k. Solid arrows indicate how data is generated. Dotted arrows indicate how real world processes influence each other. Dashed arrows indicate the flow of abstraction during the modelling process. Ribbons indicate processes of abstraction. The symbol † indicates epistemic uncertainty, whereas the symbol $*$ indicates aleatoric uncertainty.

TABLE I. A TABLE DEPICTING DIFFERENT TYPES OF ABSTRACTION IN THE MODELLING PROCESS, THEIR DESCRIPTIONS AND EXAMPLES

Abtsraction Type	Description	Example
Isolation	Isolating the RWEP or multiple RWEPs by choosing the domain, processes and entities of interest in the real world	The features, dynamics and sensing of multiple targets that are observable or can be inferred indirectly from measurements within the coverage area of multiple radars. This isolation could explicitly be represented by an ontology.
Datum	Choosing a mathematical or numeric representation of a measurement $\mathbf{z}_k$ to represent a real world datum $D_{n,k}$ or data	Integer, natural number, real number, vector, matrix, complex number, tensor, norm, first order logic expression, etc.
Data Generation	Choosing a mapping between RWEPs, and data and an uncertainty representation for representing uncertainty in the data generation process *as well as* characterising the real world data generation process	Choosing a probabilistic uncertainty representation and specifying a Gaussian model of data generation with mean and covariance parameters to model the generation of range and Doppler measurements by a radar
Process	Choosing states, parameters, a mapping between states and parameters and an uncertainty representation for states, parameters and mappings	Choosing a hidden Markov model to represent the time evolution of a target state, where the plant noise captures both uncertainties in knowledge of the motion model and real world randomness such as air pockets, and imprecise control inputs by the pilot of an aircraft.

human operator. It also covers the cases of simple numerical values encoding the signal, pre-modelled information like score vectors, not yet a probability vector but encoding some uncertainty provided by some internal algorithm of the source. It may finally be an uncertainty representation with dedicated mathematical framework such as a probability distribution, belief function, possibility distribution issued from another system with elaborated uncertainty representation processing and communication. As such, the sensing or perception process which can produce data is subsumed within RWEPs. The modelling of such sensing processes which generate data (sensor models) is discussed next, and this type of abstraction is labelled *data generation abstraction*.

In addition to isolation and datum abstraction, another two processes of abstraction are needed to result in a mathematical model. The third process of abstraction is the modelling of the real world data generation procedure. In the case of traditional probabilistic modelling, this relation is characterised by the likelihood function $L_z(\mathbf{x}) = p(\mathbf{z}|\mathbf{x}, \theta_n)$. There is an important distinction that should be made. If $\mathbf{z}_k$ is kept constant and θ_n is the variable, the function $p(\mathbf{z}|\mathbf{x}\theta_n)$ represents the likelihood and is a function, not a probability distribution. However if θ_n is kept constant and $\mathbf{z}_k$ is the variable, then $p(\mathbf{z}|\mathbf{x}, \theta_n)$ represents the probabilistic model of data generation, and it is a proper distribution. Note that $p(\mathbf{z}|\mathbf{x}, \theta_n)$ should include the sensor model or the model of perception, as the sensor forms part of the RWEPs and also generates data. The fourth process of abstraction encapsulates the relevant aspects of real world processes that generate the data, and result in process models (PMs). In general, such models are time dependent, and describe the stochastic evolution of future states based on past states $\mathbf{x}$ and model parameters θ. These states and parameters are typically abstractions of the real world physical attributes contained in Ω_n. The uncertainty in the PM evolution is represented by $p(\mathbf{x}_k|\mathbf{x}_{k-1}, \theta_n)$.

In summary, this view depicts four abstraction procedures that take place in order to arrive at a traditional mathematical model: real world process isolation, datum abstraction, data generation process abstraction and real world process abstraction.

A. The modelling process

Along the lines of the discussion in Chapter 3 of [22], traditional modelling approaches do not explicitly consider many of the uncertainties that enter into the modelling process. Each process of abstraction is a potential entry point for uncertainty about the underlying real world process. When considering RWEP isolation, if non-negligible links between real world processes are severed, there is uncertainty about how these will influence the fidelity and performance of the model. As such, the nature of the uncertainty is epistemic (i.e. owing to a lack of knowledge about how the RWEP in question is affected by other RWEPs), and is indicated by a † in Figure 1.

In the case of datum abstraction, if a datum is not sufficiently enumerated, for example if an inherently complex natured datum is represented by real number instead of a complex number, some underlying structure in the RWEP may in turn not be encapsulated by the model. Hence, there is uncertainty about how well the mathematical quantity captures the important or relevant properties of the datum. This is also a form of epistemic uncertainty owing to the lack of knowledge about the true nature of the datum.

The procedure of data generation abstraction causes epistemic uncertainty, since there may be lack of knowledge about the nature of the transformation from a RWEP to a datum. In addition to epistemic uncertainty, aleatoric uncertainty (denoted by a $*$ in Figure 1) is expressed through the random nature by which data are generated. Hence the measurement process is depicted in Figure 1 to contain both epistemic and aleatoric uncertainties.

A process model relates parameters and states to each other over time. Epistemic uncertainty enters into the PM through incomplete knowledge about the corresponding RWEP. Aleatoric uncertainty enters into the model through random perturbations in the time evolution of the model. Consider for example a discrete time varying equation $\mathbf{x}_k = f(\mathbf{x}_{k-1}) + \epsilon$, where $\mathbf{x}$ is the system state. In many cases both epistemic and aleatoric uncertainties are incorrectly lumped together in ϵ.

The final layers of abstraction, when proceeding from the mathematical model to a computer model, are beyond the scope of this discussion. In the case of digital computers, the use of established scientific libraries and vector-matrix mathematical programming environments make *variable abstraction* fairly well characterised. Uncertainties may enter through *algorithmic abstraction* in the form of possible incorrect implementation, numerical instabilities or strange behaviour in untested states. However, most cases of numerical instabilities in digital computer code are well characterised [23], and examples include the inversion of an ill-conditioned matrix, or numerical instabilities owing to Euler numerical integration. In this case incorrect implementation would be owing to oversight by the programmer. *Uncertainty abstraction* is characterised by pseudo number generators and Taylor series expansions to represent continuous probability distributions. Uncertainties for this type of abstraction are also well characterised in the literature. If on the other hand, analogue computers were used, this abstraction would have needed particular care in characterizing uncertainties, as the results would be noisy.

B. The complete data and process models

In traditional statistical modelling, $\mathbf{z_k}$ is seen as the "datum" and $p(\mathbf{z}|\mathbf{x}, \theta_n)$ is seen as the complete encapsulation of the uncertainty model of $\mathbf{z}$. This procedure ignores the fact that $\mathbf{z_k}$ is itself an abstraction of $D_{n,k}$, and similarly $p(\mathbf{z}|\mathbf{x}, \theta_n)$ is an abstraction of $\{D_n|\Omega_n\}$ and as such, any uncertainties associated with these abstraction processes are ignored. Higher order uncertainty (uncertainty about uncertainty) is modelled by imprecise probability models, belief functions or credal sets. For instance: Rather than a single probability distribution, a set of probability distributions is considered, and the probability of an event is defined by upper and lower bounds. As stated in [22], a complete model of data generation must have the form $p(\Gamma|\mathbf{x}, \theta_n, \alpha)$, where $\Gamma = \{\mathbf{z}_k, \alpha\}$ is a mathematical model for $\mathbf{z}_k$ *as well as* the uncertainties associated with constructing Γ, denoted by α. Furthermore, the uncertainty representation denoted by $p(\cdot|\mathbf{x}, \theta_n, \beta)$ must be a mathematical model of both the data generation process, as well as the uncertainties β associated with the construction of $p(\cdot|\mathbf{x}, \theta_n, \beta)$. This uncertainty representation is referred to as the *generalised likelihood* in [22]. Finally the complete process model $p(\mathbf{x}_k|\mathbf{x}_{k-1}, \theta_n, \delta)$ should encapsulate the uncertainty in the evolution of states as well as the uncertainties δ associated with the construction of that model.

IV. Uncertainty during data generation

It may be worth considering the different kinds of data generated by RWEPs, since there may be uncertainties about the relationships between the RWEP and the data (the data generation process is ambiguous) as well as to what is observed (the RWEP generating the datum is ambiguous). Figure 2 shows data generation and data source ambiguity in relation to the generation of data by RWEPs in Figure 1. Mahler [22] defines four possible combinations of data generation and data source ambiguities:

1) Unambiguously generated unambiguous (UGU) measurements - These are conventional measurements where the data are unambiguously generated (the relationship $p(\mathbf{z}|\mathbf{x}, \theta_n)$ between states/parameters and measurements is clearly defined) and the data are unambiguous (it is known exactly what is observed). A single target range and Doppler measurement are examples of a UGU measurement.
2) Ambiguously generated unambiguous (AGU) measurements - These are measurements where the data are ambiguously generated (the relationship $p(\mathbf{z}|\mathbf{x}, \theta_n)$ between states/parameters and measurements is poorly understood, but the data are unambiguous - it is known what is observed). A high range resolution radar (HRR) of synthetic aperture radar (SAR) is an AGU data generation process, since their likelihood functions are ambiguously defined owing to real world variations that are difficult to characterise.
3) Unambiguously generated ambiguous (UGA) measurements - These are measurements where the data are unambiguously generated (the relationship $p(\mathbf{z}|\mathbf{x}, \theta_n)$ between states/parameters and measurements is exactly understood, but it is not exactly known what is observed). Examples include features extracted by humans or digital signal processors from RWEP data, some natural language statements, and rules.
4) Ambiguously generated ambiguous (AGA) measurements - These are measurements where the data are ambiguously generated (the relationship

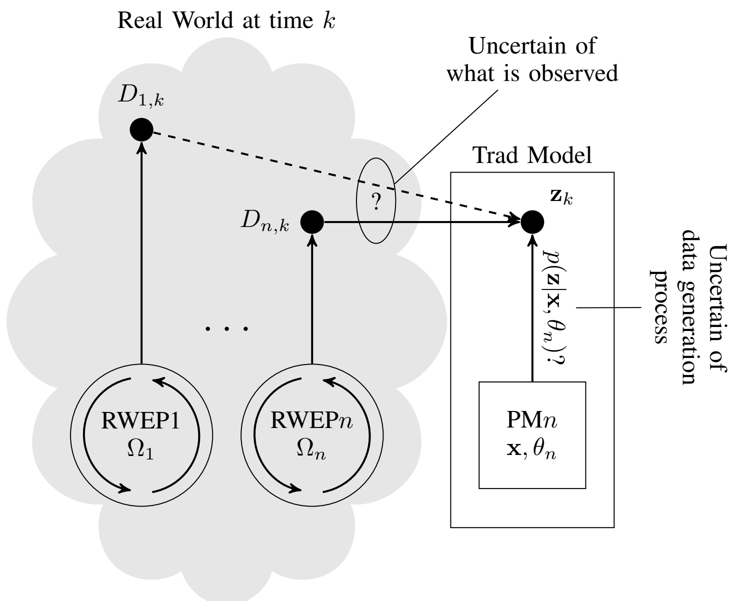

Fig. 2. A diagram depicting uncertainty in the measurement owing to ambiguity in the relationship between states/parameters and the measurement through $p(\mathbf{z}|\mathbf{x}, \theta_n)$, and ambiguity in what is actually being observed (datum source).

$p(\mathbf{z}|\mathbf{x}, \theta_n)$ between states/parameters and measurements is poorly understood and it is not exactly known what is observed. Examples are the same as for the UGA case, but here sensor models are constructed using domain expert knowledge.

V. Uncertainty classes according to the URREF ontology

According to Version 2d of the URREF ontology, the uncertainties that enter into the model through the four abstraction procedures can be classified according to nature (epistemic or aleatoric), type (inconsistency, vagueness, randomness, ambiguity and incompleteness) and uncertainty representation (probability, belief functions, rough sets, fuzzy sets and random sets).

In the evaluation of uncertainty in the modelling of data and RWEPs there seem to be three classes of criteria that are relevant. The first is *DataHandlingCriterion* which consists of *Interpretation* and *Traceability* as evaluation criteria. The second is *DataCriterion*, which consists of *RelevanceToProblem*, *WeightOfEvidence*, *Credibility* and *Quality*. The criterion *Credibility* is further expanded to include *ObservationalSensitivity*, *SelfConfidence* and *Objectivity*. The criterion *Quality* is further expanded to include *Accuracy*, *Veracity* and *Precision*. The third class of criteria which seems to be relevant is *RepresentationCriterion* which consists of *Compatibility*, *Simplicity*, *Adaptability*, *Knowledge handling* and *Expressiveness*. The criterion of *Expressiveness* is further expanded to include *Relational*, *Assessment*, *Dependency*, *HigherOrderUncertainty*, *Outcomes* and *Configurality*.

VI. Uncertainty evaluation in mathematical modelling

The different processes of abstraction require evaluation criteria that capture the uncertainty that enters into the model through these processes of abstraction. These different processes depicted in Figure 1 will be treated in turn below.

A. Isolation Abstraction

Isolation abstraction identifies the variables of interest (such as the nodes in a Bayesian networks) as having an observable impact on the decision variables. The isolation process also separates context and situation and is thus the first level of abstraction and modelling. This abstraction is guided by the *Relevance DataCriterion*, which can be assessed *a priori* (set of relevant variables) and *a posteriori* based on other *DataCriteria* of *DataOutput* (e.g., accuracy, precision) or *ReasoningCriteria* (e.g., timeliness), to keep only the most relevant features to accelerate the computation process with barely decreased performances. The main source of uncertainty here is an inaccurate representation of the underlying RWEP and its interaction with the rest of the real world. As such, the *RepresentationCriterion* is of interest here. There is an inherent trade-off between the three criteria of *Simplicity*, *Adaptability* and *Expressiveness* when deciding how much of the real world should be modelled for the task at hand. In particular *Simplicity* is typically sacrificed for *Adaptability* and *Expressiveness*. Maximum simplicity should be strived for while adequately representing (in terms of adaptability and expressiveness) the part of the real world of interest. This is reminiscent of Occam's razor [24]. *KnowledgeHandling* may be of interest if the RWEP is a human generating a human language statement datum.

B. Datum Abstraction

Datum abstraction selects the scales and domains of the previously identified observable variables. The *Outcomes* criterion under *Expressiveness* could be relevant or useful here. The key question in this type of epistemic abstraction, is whether the mathematical representation of the measurement $\mathbf{z}_k$ (not the measurement itself), captures the essence of the observable datum in the real world, i.e. what type and level of uncertainty is introduced by representing $\mathbf{z}_k$ as a particular type of number (integer, natural number, real number, vector, matrix, complex number, tensor, norm, first order logic expression, etc.). This is again a representation uncertainty, and as such *RepresentationCriterion* is of interest, along with *Compatibility*, *Simplicity*, *Adaptability*, *Knowledge handling* and *Expressiveness*. If vector and matrix are considered here, it should be mentioned that they have not been assigned specific semantics nor satisfy specific axioms. At the next level of abstraction (data generation abstraction), they are framed in the mathematical model (probabilistic, evidential, fuzzy sets, etc) and assigned the corresponding semantics which meet the epistemic uncertainty meaning (*UncertaintyDerivation*, *UncertaintyType*).

C. Data Generation Abstraction

Data generation abstraction defines the uncertainty measure over the domain of the observed variables of interest, over the links between them especially between the observation and inference spaces. As mentioned, data generation inherently has two types of uncertainty associated with it. Starting with representation again, epistemic uncertainty enters through uncertainty about the model of data generation $p(\mathbf{z}|\mathbf{x}, \theta_n)$. When considering epistemic uncertainty, the question to ask is whether the most suitable uncertainty representation (probabilistic, evidential, rough sets, fuzzy sets and random sets) is used for the model of data generation. Again, here the trade-off between *Simplicity*, *Adaptability* and *Expressiveness* may give a clue as to the suitability of the uncertainty representation of the model of data generation. Secondly, consideration of the aleatoric uncertainty is necessary to characterise the model of data generation $p(\mathbf{z}|\mathbf{x}, \theta_n)$, given a particular uncertainty representation. This characterisation involves the decision about the parameterisation of $p(\mathbf{z}|\mathbf{x}, \theta_n)$ which may be a belief function, probability density function, and so on. The criteria here should capture uncertainties that are inherent in the sensing or measurement process (provide a sensing or measurement uncertainty model). Thus the *DataCriterion* is of interest here, together with the sub-criteria of *RelevanceToProblem*, *WeightOfEvidence*, *Credibility* and *Quality*. These are criteria which evaluate uncertainties that enter as a result of the sensing or perception process. As such, these uncertainties should be modelled as part of the measurement generation process. The three levels of abstraction above are sequentially embedded in each other and the data generation abstraction includes the two previous ones.

D. Process Abstraction

As with data generation abstraction there are both epistemic and aleatoric uncertainties that should be represented and evaluated in process abstraction. Here, epistemic uncertainty relates to the model of the underlying entities and processes (the PM) which is usually dynamic. Such entities and processes may be hidden, or be indirectly or directly observed through the data generation process which is described in VI-C. Process models are characterised by temporal transition functions, parameters and states and relationships between parameters and states. Each of these may have some form of epistemic uncertainty associated with it. These uncertainties are again evaluated according to *RepresentationCriterion*, with the same trade-offs that characterise epistemic uncertainties that were mentioned before. The uncertainty representation must be chosen which characterises the temporal transitions functions, the parameters and the states and relationships between parameters and states of the model. Typically this uncertainty representation is the same as that which characterises the data generation abstraction in section VI-C. The implications of conversions between uncertainty representations require some investigation.

Aleatoric uncertainty is generated by the PM and may be evaluated according to *RepresentationCriterion*. The fidelity of the representation of the random events in the RWEP must be evaluated in this case.

In most fusion systems, the hidden parameters of the PM need to be inferred and are as such the outputs of a fusion system. This means that the uncertainties in the outputs should be evaluated according to *DataCriterion*, since the outputs of a fusion process may form the input of another fusion process and should have a "sensor model" in the second fusion process. In a sense the outputs become data for a higher level of fusion.

VII. Modelling within the context of fusion

A fusion system draws inferences about a set of real-world entities and processes from reports provided by a set of information sources. This is accomplished through fusion algorithms that are based, whether implicitly or implicitly, on models of the subject RWEPs and their relationship to the reports provided to the fusion system. As discussed above, a probabilistic fusion system bases its algorithms on a statistical model of the process $\{D_n|\Omega_n\}$ by which RWEPn generates observable data D_n. In such a model, the properties Ω_n of RWEPn are abstracted as a state $\mathbf{x}$; the observable effects D_n are abstracted as observations $\mathbf{z}$ from different sources, and the data-generating process $\{D_n|\Omega_n\}$ is abstracted as a statistical model $p(\mathbf{z}|\mathbf{x}, \theta_n)$ that represents the manner in which RWEPn produces these observable effects, including the uncertainties associated with each of the different information sources.

This statistical model $p(\mathbf{z}|\mathbf{x}, \theta_n)$ forms the basis for fusion algorithms that draw conclusions about $\mathbf{x}$ from the multi-source observations $\mathbf{z}$. In a Bayesian system, for example, this is accomplished by defining a prior distribution $p(\mathbf{x})$, representing what is known about $\mathbf{x}$ prior to observing $\mathbf{z}$, and using Bayes rule to find a posterior distribution $p(\mathbf{x}|\mathbf{z}, \theta_n)$. Bayesian inference can also be used to refine knowledge of the parameters θ_n of the information generation process, yielding a posterior distribution $p(\mathbf{x}, \theta_n|\mathbf{z})$. Conclusions about the abstractions $\mathbf{x}$ and θ_n are then translated into conclusions about RWEPn and $\{D_n|\Omega_n\}$.

VIII. Case Study - Bayesian Network Fusion

The paper [9], introduces a URREF interpretation of Bayesian network (BN) information fusion. It considers a model which captures the causal factors that lead to rhino poachings on an operational level. It lists three types of uncertainty that enter the BN network modelling process. These are model structure uncertainty (uncertainty in the causal links between random variables), parameter uncertainty (uncertainty in the conditional probability distributions) and observational uncertainty (uncertainty in the evidence provided to the network).

Structure uncertainty is epistemic and has to do with the isolation abstraction. Firstly, we might not have a perfect knowledge of all the relevant variables significantly influencing the variables representing the hypothesis (e.g. poaching event). Consequently, the BN will not explicitly model all the relevant variables. The inference process will thus be influenced by confounders. Secondly, we might not have a perfect knowledge of all the relevant direct influences between the variables in a BN. Thirdly, some of the dependencies might be left out on purpose, for the sake of simplicity. In this way the expressiveness will be influenced.

The parameter uncertainty is related to process abstraction, since these parameters characterise PMs. The choice of the BN paradigm and the representation based on conditional probabilities might not be the most suitable for the problem at hand. This is due to the epistemic uncertainty. Moreover, the parameters defining the conditional probability tables represent inherently random relations between the phenomena that directly influence each other. It is often difficult to know the true conditional probabilities due to aleatoric uncertainty.

Five additional types of modelling uncertainty that are not mentioned in [9] are variable choice uncertainty, variable type uncertainty, variable range uncertainty, variable semantic uncertainty and ambiguity conversion uncertainty.

Variable choice uncertainty refers to uncertainty introduced by the choice of variables to represent a domain or problem in the real world. It is therefore a form of isolation abstraction, since it involves choosing which factors will represent real-world processes and entities.

The following variable uncertainties, apart form observational uncertainty, appear within the PMs, and are as such results of process abstraction. Only *observational uncertainty* relates to data generation abstraction, and can take the form of either hard, soft or likelihood evidence. *Variable type uncertainty* is the uncertainty introduced by choosing the type of states, i.e. whether a variable is discrete, continuous, or if the variable is continuous in the real world but can be discretised for the BN. Furthermore, in the case of discrete variables, the variable type can be Boolean, labelled, numbered or interval. *Variable range* uncertainty refers to uncertainty introduced by making a choice of the range of the variable. If the variable is characterised by a continuous distribution with tails stretching into +-∞, then the range is $(-\infty, \infty)$, unless truncated or defined over a finite interval. In the discrete case there is uncertainty about the number of states and intervals associated with each of the states, if the underlying quantity is continuous. *Variable semantic uncertainty* relates to uncertainty introduced by the definition of the variable as it relates to a entity in a RWEP. This is of particular importance if the underlying random variable is abstract (for example a sentiment, anomaly etc.). In the rhino poaching case the “Vegetation” variable indicates whether the vegetation is palatable to a rhino or not. It could be confused with other meanings, for example it may indicate vegetation availability, density or proximity. The meaning of the random variable must be scoped to fit the context of its parent variables and/or child variables. If these variables are observable, this could correspond to AGA or UGA cases in that it is not exactly known what is observed. *Ambiguity conversion uncertainty* captures the case where the underlying entity in the RWEP has different type of uncertainty (for example inconsistency, vagueness, ambiguity, imprecision or incompleteness), and has to be converted to a conditional probability distribution.

IX. Summary and Conclusion

In this paper, the approaches and abstractions that are implicitly performed as part of the modelling process have been made explicit. This explicit representation of these approaches and abstractions was used to demonstrate all the possible entry points for uncertainty during the modelling process, and to demonstrate the relevant URREF criteria for evaluating such uncertainties. Broad categorisation of these uncertainties fall into two classes. Epistemic uncertainty represents a lack of knowledge of the underlying real-world entities and processes being modelled, and lies outside of such entities and processes. Aleatoric uncertainty represents randomness which is generated inside of the underlying real-world entities and processes.

Four processes of abstraction are presented. Isolation abstraction is the process of isolating part of the real world that is of interest to be modelled, together with specifying boundary conditions, inputs and outputs. Datum abstraction converts a real-world piece of information to a mathematical interpretable quantity that can be entered into mathematical functions, transformations and other operations. Data generation abstraction creates a mathematical transformation that describes how data are generated by real world entities and processes, including sensors. Lastly, process abstraction creates mathematical transformations that describe how states and parameters are related over time.

Uncertainties in isolation abstraction can be evaluated according to the sub-criteria in the class *RepresentationCriterion*, as these are epistemic in nature. The same goes for datum abstraction. The uncertainties that enter owing to the data generation and process abstraction actions are both epistemic and aleatoric, and as such can be evaluated according to both *RepresentationCriterion* and *DataCriterion*. In all cases where *RepresentationCriterion* is used, there is a trade-off between *Simplicity* on the one hand, and *Adaptability* and *Expressiveness* on the other.

Bayesian network fusion for a rhino poaching modelling problem was the focus of previous papers [9], [25] and is used as a case study for demonstrating some of the concepts and ideas in this paper. Modelling uncertainty enters a Bayesian Network through eight choices about the structure, variables and parameters of the network. Model structure uncertainty represents uncertainty in the causal links between random variables in the Bayesian network. Parameter uncertainty represents uncertainty in the conditional probability distributions. Observational uncertainty represents uncertainty in the evidence provided to the network. Variable type uncertainty represents uncertainty introduced by making a choice whether the variable is discrete, continuous, or if the variable is continuous in the real world but can be discretised for the BN. Variable range uncertainty represents uncertainty introduced by making a choice regarding the range of the variable. Variable semantic uncertainty represents uncertainty of exactly what a variable represents in the real world. Finally, ambiguity conversion uncertainty enters if an underlying entity in the RWEP has different types of uncertainty (for example inconsistency, vagueness, ambiguity, imprecision or incompleteness) and has to be converted to a conditional probability distribution.

This focus of this paper is on the uncertainty that enters in the modelling and abstraction processes, and not explicitly on uncertainty that enters during the fusion process. Fusion and reasoning processes could be represented explicitly in this taxonomy as RWEPs, but then the entire URREF would apply. A unification of this taxonomy and the process of fusion would make all sources of uncertainty in the reasoning process explicit. Thus future work would involve unifying the Atomic Decision Process of [9] with the taxonomy presented in this paper. Biases of different agents in multi-agent fusion systems could be handled through characterisation of data generation abstraction uncertainties. If such agents have different conceptual frameworks (ontologies) representing RWEPs, then differential isolation abstraction and process abstraction uncertainties come into play. This could be investigated in future work. The effect of uncertainties entering through the abstraction processes on the output of a fusion system can then also be considered in more detail. As indicated by the BN fusion example, uncertainties in choices of the parameter variable (continuous/discrete, ranges, semantics, and uncertainty type conversion) in PMs still need to be more explicitly addressed. Another task for future work would be to establish better defined links to the field of UQ. A more explicit consideration of process abstraction, and how uncertainties are associated with the abstraction of parameters and states of models are needed.

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