The Pseudo-Pascal Triangle of Maximum Deng Entropy

Xiaozhuan Gao, Yong Deng

Xiaozhuan Gao [1]
1. Institute of Fundamental and Frontier Science
University of Electronic Science and Technology of China, Chengdu, 611731
Yong Deng[12]*
1. Institute of Fundamental and Frontier Science
University of Electronic Science and Technology of China, Chengdu, 611731
2. Medical Center, Vanderbilt University, Nashville, TN. USA, 37205
*Corresponding author: dengentropy@uestc.edu.cn, prof.deng@hotmail.com

Abstract

Pascal triangle (known as Yang Hui Triangle in Chinese) is an important model in mathematics while the entropy has been heavily studied in physics or as uncertainty measure in information science. How to construct the the connection between Pascal triangle and uncertainty measure is an interesting topic. One of the most used entropy, Tasllis entropy, has been modelled with Pascal triangle. But the relationship of the other entropy functions with Pascal triangle is still an open issue. Dempster-Shafer evidence theory takes the advantage to deal with uncertainty than probability theory since the probability distribution is generalized as basic probability assignment, which is more efficient to model and handle uncertain information. Given a basic probability assignment, its corresponding uncertainty measure can be determined by Deng entropy, which is the generalization of Shannon entropy. In this paper, a Pseudo-Pascal triangle based the maximum Deng entropy is constructed. Similar to the Pascal triangle modelling of Tasllis entropy, this work provides the a possible way of Deng entropy in physics and information theory.

Keywords: Deng entropy, Maximum Deng Entropy, Pascal Triangle, Dempster-Shafer evidence theory, Basic Probability Assignment

1 Introduction

Pascal triangle (known as the Yang Hui Triangle in China) is a triangular arrangement of the binomial coefficients and that it contains many remarkable numerical relations [6, 64]. Pascal-Triangle has also become a suitable model system for the exploration of statistical mechanical structures [53, 64]. Pascal Triangle has attracted many people attention and been used in many fields [20, 46], such as network [29], cellular automata[79] and so on[5, 7, 44].

However, how to set connection between Pascal triangle and uncertainty is an interesting issue. In uncertainty, entropy plays an important role [9, 82] and has been used in many fields [10, 23, 58, 75]. There are various entropies, such as Shannon entropy, Renyi entropy [51], Kolmogorov entropy and so on. Hence, setting the connection between Pascal triangle and entropy is an issue with considering.

Tsallis proposed the Tsallis entropy based on the probability theory as generalization of Boltzmann-Gibbs statistics [61] and analyzed the connection between Tsallis entropy and Pascal-triangle [62]. As the extension of probability theory, Dempster-Shafer evidence theory (D-S evidence theory) has attracted many people attention. About uncertainty in D-S evidence theory, there are also various measurement. Deng proposed Deng entropy to measure the uncertainty of D-S evidence theory [11, 57]) [15], which is compatible with Shannon entropy. Deng entropy has attracted many people attention since proposed [3, 16, 26, 27, 28, 54, 66], which has been used in many fields, such as information fusion [36, 47], quantum information [21], uncertainty measurement [42] and so on [63, 73]. Recently, Kang and Deng proposed the maximum Deng entropy, which BPA satisfies some conditions with the changes of frame of discernment [25]. Based on the maximum Deng entropy, the paper proposed the pseudo-Pascal triangle to further explore Deng entropy.

The paper is organized as follows. The preliminaries Dempster-Shafer evidence theory and Deng entropy are briefly introduced in Section 2. Section 3 introduced the maximum Deng entropy and discussed the some properties of BPA, besides, analysed the Semi-Pascal-Triangle of BPA. Finally, this paper is concluded in Section 4.

2 Preliminaries

In this section, the preliminaries of D-S theory [11, 57] and Deng entropy [15] will be briefly introduced.

2.1 Dempster-Shafer evidence theory

D-S evidence theory offers a useful fusion tool for uncertain information [19, 77]. D-S evidence theory needs weaker conditions than the Bayesian theory of probability, D-S evidence theory assigns the probability into the power set of events[17]. D-S evidence theory has been used to many applications, such as data fusion [1, 49, 60], conflict management [34, 55], pattern recognition [45, 52, 56], evidential reasoning [18, 80, 81] and so on [2, 22, 35, 74]. Some preliminaries in D-S theory are introduced as follows. For additional details about D-S theory, refer to [11, 57].

Definition 1. (Frame of discernment)
Let Θ be the set of mutually exclusive and collectively exhaustive events A_i, namely

$$\Theta = \{A_1, A_2, \cdots, A_n\} \tag{1}$$

The power set of Θ composed of 2^N elements of is indicated by 2^Θ, namely:

$$2^\Theta = \{\phi, \{A_1\}, \{A_2\}, \cdots, \{A_1, A_2\}, \cdots, \Theta\} \tag{2}$$

Definition 2. (Mass Function)
For a frame of discernment $\Theta = \{A_1, A_2, \cdots, A_n\}$, the mass function m is defined as a mapping of m from 0 to 1, namely:

$$m : 2^\Theta \to [0, 1] \tag{3}$$

which satifies

$$m(\phi) = 0 \tag{4}$$

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{5}$$

In D-S theory, a mass function is also called a basic probability assignment (BPA) or a piece of evidence or belief structure. The $m(A)$ measures the belief exactly assigned to A and represents how strongly the piece of evidence supports A. If $m(A) > 0$, A is called a focal element, and the union of all focal elements is called the core of a mass function. There are some studies about BPA, such as association [48], negation [41, 76], divergence [75] and information quality [8, 30, 31].

2.2 Deng entropy

Uncertainty modelling is still an open issue [33, 39, 40, 50, 59, 65]. Many models are presented to model the complexity such as network models [32, 68, 69, 70, 71], Bayesian [24, 43, 78] and D numbers [13, 14, 37, 38]. Among many math tools and models, entropy function plays an important role in real applications [12, 72]. Deng proposed an *Deng Entropy*, which is an generalization of Shannon entropy [15].

Definition 3. *(Deng entropy)*
Given a BPA, Deng entropy can be defined as:

$$H_D = - \sum_{A \subseteq \Theta} m(A) log_2 \frac{m(A))}{2^{|A|} - 1} \tag{6}$$

Through a simple transformation, Deng Entropy can be rewrite as follows:

$$H_D = \sum_{A \subseteq \Theta} m(A) log_2(2^{|A|} - 1) - \sum_{A \subseteq \Theta} m(A) log_2 m(A) \tag{7}$$

where m is a BPA defined on the frame of discernment Θ, and A is the focal element of m, $|A|$ is the cardinality of A. Besides, the term $\sum m(A) \times log_2(2^{|A|} - 1)$ could be interpreted as a measure of total nonspecificity in the mass function m, and the term $-m(A) \times log_2 m(A)$ is the measure of discord of the mass function among various focal elements.

Definition 4. *(The Maximum Deng entropy)*
Given a BPA, the maximum Deng entropy is as follows:

$$H_{M-D} = - \sum m(A) \times log_2 \frac{m(A)}{2^{|A|} - 1} \tag{8}$$

if and only if

$$m(A) = \frac{2^{|A|} - 1}{\sum 2^{|A|} - 1} \tag{9}$$

The more details about maximum Deng entropy refer to [25].

3 The Pseudo-Pascal Triangle of Maximum Deng Entropy

3.1 The Example of Maximum Deng Entropy

The specific BPA of having maximum Deng entropy was showed as follows.

Example 1. *Given the frame of discernment $\Theta = \{A\}$, there is only one phenomenon, as follows:*

$$m(A) = 1$$

$$H_D = 0$$

From the above, it can be easily found that only having one element means no uncertainty, that is to say, the event is certain.

Example 2. *Given the frame of discernment $\Theta = \{A, B\}$, the BPA and maximum Deng entropy are shown below:*

$$m(A) = \frac{1}{5}, m(B) = \frac{1}{5}, m(A, B) = \frac{3}{5}$$
$$H_{M-D} = 2.3219$$

From the above, the BPA of having maximum Deng entropy has the phenomenon that $m(A)$ and $m(B)$ has the same distribution.

Example 3. *Given the frame of discernment* $\Theta = \{A, B, C\}$, *the BPA and maximum Deng entropy are as follows:*

$$m(A) = \frac{1}{19}, m(B) = \frac{1}{19}, m(C) = \frac{1}{19}$$

$$m(A, B) = \frac{3}{19}, m(B, C) = \frac{3}{19}, m(A, C) = \frac{3}{19}, m(A, B, C) = \frac{7}{19}$$

$$H_{M-D} = 4.2474$$

It can known that the mass function having the same cardinality has same distribution. Besides, from the above *Example* 1, *Example* 2 and *Example* 3, it can be seen that the BPA follow a certain rules with changes of frame of discernment. More details, it can be discussed in the next.

3.2 Distribution of maximum Deng entropy

Frame of Discrement	$|X|= 1$	$|X|= 2$	$|X|= 3$	$|X|= 4$
$\Theta = \{A\}$	1			
$\Theta = \{A, B\}$	(1/5,1/5)	3/5		
$\Theta = \{A, B, C\}$	(1/19,1/19,1/19)	(3/19,3/19,3/19)	7/19	
$\Theta = \{A, B, C, D\}$	(1/65,1/65,(3/65,3/65,3/65, 1/65,1/65) 3/65,3/65,3/65)	(7/65,7/65, 7/65,7/65)	15/65	

Table 1: The BPA of having Maximum Deng entropy

Table 1 shows BPA of having maximum Deng entropy in various frame of discernment. In *Table* 1, $|X|$ represents the cardinality of focal element, Θ represents the frame of discernment, the other information represents the BPA. Such as, for $\Theta = \{A, B\}$, in the second column, $(\frac{1}{5}, \frac{1}{5})$ means the $m(A) = \frac{1}{5}$ and $m(B) = \frac{1}{5}$, $(\frac{3}{5})$ means the $m(A, B) = \frac{3}{5}$. From the *Table* 1, it can be known focal element having the same cardinality has same BPA in the case of maximum Deng entropy. Besides, the BPA can be rewrite the *Fig* .1. In *Fig* .1, the first column, N corresponds to the number of frame of discernments. Besides, the second column corresponds to the BPA with having maximum Deng entropy. For example, in $(2, 3/5)$, 2 represents that the cardinality of focal element, $3/5$ represents the BPA.

N=1   (1,1)

N=2   (1,1/5)   (2,3/5)

N=3   (1,1/19)   (2,3/19)   (3,7/19)

N=4   (1,1/65)   (2,3/65)   (3,7/65)   (4,15/65)

Figure 1: The BPA of having Maximum Deng entropy

3.3 Pseudo-Pascal Triangle of maximum Deng entropy

Based the above discussion, it can be seen that the BPA has some regularity. *Fig*, 2 shows the connection between Pseudo-Pascal triangle and the maximum Deng entropy. In *Fig*. 2, N represents the number of frame of discernment. $|m|$ represents the cardinality of BPA, the others represents the number of having the $2^{|m|} - 1$. For better understanding, it can be considered that a system can be divided with some subsystems. That is to say, a system is evenly distributed, every subsystem occupies a certain proportion. When the frame of Discernment is N, the system can be divided into $\sum_{m=1}^{m=N}(2^{|m|} - 1) \times C_N^m$ parts, the focal elements whose cardinality is $|m|$ has $2^{|m|} - 1$ parts.

From the *Fig*. 2, when the $N = 2$, the system can be divided 5 parts. In this system, the number having $2^{|1|} - 1 = 1$ parts is 2, the number having $2^{|2|} - 1 = 3$ parts is 1. Through the above analyse, it can be known that the distribution of the maximum Deng entropy is similarly with Pascal triangle,

$2^{|m|-1}$	$2^{|1|-1}$	$2^{|2|-1}$	$2^{|3|-1}$	$2^{|4|-1}$	$2^{|5|-1}$	$2^{|6|-1}$	$2^{|7|-1}$	$2^{|8|-1}$	$2^{|9|-1}$	$2^{|10|-1}$	$2^{|11|-1}$	$2^{|12|-1}$
N=1	1											
N=2	2	1										
N=3	3	3	1									
N=4	4	6	4	1								
N=5	5	10	10	5	1							
N=6	6	15	20	15	6	1						
N=7	7	21	35	35	21	7	1					
N=8	8	28	56	70	56	28	8	1				
N=9	9	36	84	126	126	84	36	9	1			
N=10	10	45	120	210	252	210	120	45	10	1		
N=11	11	55	165	330	462	462	330	165	55	11	1	
N=12	12	66	220	495	792	924	792	495	220	66	12	1

Figure 2: The Pseudo-Pascal Triangle

called Pseudo-Pascal triangle. Hence, the paper proposed the connection between Pseudo-Pascal triangle and the maximum Deng entropy.

4 Conclusions

Pascal triangle is an important tool in mathematics to deal with some issue. Uncertainty plays an essential role in modern lief. Hence, how to generate the connection between Pascal triangle and uncertainty is an interesting issue. Deng entropy can provide a method to measure the uncertainty in D-S evidence theory, which is compatible with Shannon entropy. The paper discussed the BPA of the maximum Deng entropy, which changes with N according to evolutionary rules. By discussion, it can be found that the BPA of the maximum Deng entropy can construct Pseudo-Pascal triangle. The paper analysis the BPA of maximum Deng entropy, which is useful for uncertainty measurement.

The paper have set the relationship between Pseudo-Pascal triangle and maximum Deng entropy. However, the relationship between maximum Deng entropy and Pascal triangle still needs further exploration.

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