

Random Permutation Set

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Abstract

For exploring the meaning of the power set in evidence theory, a possible explanation of power set is proposed from the view of Pascal's triangle and combinatorial number. Here comes the question: what would happen if the combinatorial number is replaced by permutation number? To address this issue, a new kind of set, named as random permutation set (RPS), is proposed in this paper, which consists of permutation event space (PES) and permutation mass function (PMF). The PES of a certain set considers all the permutation of that set. The elements of PES are called the permutation events. PMF describes the chance of a certain permutation event that would happen. Based on PES and PMF, RPS can be viewed as a permutation-based generalization of random finite set. Besides, the right intersection (RI) and left intersection (LI) of permutation events are presented. Based on RI and LI, the right orthogonal sum (ROS) and left orthogonal sum (LOS) of PMFs are proposed. In addition, numerical examples are shown to illustrate the proposed conceptions. The comparisons of probability theory, evidence theory, and RPS are discussed and summarized. Moreover, an RPS-based data fusion algorithm is proposed and applied in threat assessment. The experimental results show that the proposed RPS-based algorithm can reasonably and efficiently deal with uncertainty in threat assessment with respect to threat ranking and reliability ranking.

Keywords: Dempster-Shafer evidence theory, Permutation number, Random permutation set, Permutation mass function, Orthogonal sum, Random finite set, Threat assessment

1 Introduction

Since uncertainty exists in everywhere of our life, many theories have been developed to deal with uncertainty, such as probability theory [9], fuzzy set theory [22], Dempster-Shafer evidence theory (evidence theory) [2, 11], rough sets [10], and Z-numbers [18, 23].

Most existing theories are based on set theory. Set theory provides a tool for describing the collection of objects or elements, which is important and fundamental in math science [8]. For instance, the sample space of an experiment in probability theory is the set of all possible outcomes of that experiment [9]. In evidence theory, the power set is the set considering all possible subsets of frame of discernment, which is a basis for basic probability assignment [2, 11].

Recently, a possible physical explanation of power set is proposed [12]. Given a set S with N hypotheses, its corresponding power set, denoted as 2^S , can be seen as all possible k -combination, where k ranges from 0 to N [12]. In addition, the relationship between combinatorial number, Pascal's triangle, and power set is quantitatively revealed by Deng entropy [5] from the view of information measure [12].

Here comes the straightforward question: **if we consider all possible k -permutation of a set S with N hypotheses instead of the k -combination, namely, if the combinatorial number in the Pascal's triangle is replaced by permutation number, what would happen?**

To address this issue, a new kind of set, named as random permutation set (RPS), is proposed in this paper, which consists of permutation event space (PES) and permutation mass function (PMF). The PES of a certain set Θ considers all the permutation of Θ . The elements of PES are named as the permutation events. PMF describes the chance of a certain permutation event that would happen. Based on PES and PMF, RPS can be viewed as a permutation-based generalization of random finite set. In addition, the right intersection (RI) and left intersection (LI) of permutation events are presented. Based on RI and LI, the right orthogonal sum (ROS) and left orthogonal sum (LOS) of PMFs are proposed. Next, numerical examples are shown to illustrate the proposed conceptions about RPS. The comparisons of probability theory, evidence theory, and RPS are discussed and summarized. Moreover, an RPS-based data fusion algorithm is proposed for threat assessment. The experimental results show that, with respect to threat ranking and reliability ranking, the proposed RPS-based algorithm can handle uncertainty of threat assessment reasonably and efficiently.

The rest of this article is as follows. Section 2 introduces the preliminaries. Section 3 proposes the conceptions of RPS. Section 4 compares probability theory, evidence theory, and RPS. Section 5 proposes an RPS-based data fusion algorithm for threat assessment. Section 6 makes a brief conclusion.

2 Preliminaries

In this section, we briefly review some preliminaries of this paper, including Dempster-Shafer evidence theory, Pascal's triangle, and entropic explanation of power set.

2.1 Dempster-Shafer evidence theory

Dempster-Shafer evidence theory [2, 11] is an efficient tool for dealing with uncertainty [7] and has broad applications, such as decision making [15], engineering budget [1], and fault diagnosis [17]. Researchers have developed evidence theory theoretically and practically. In 1919, Yager proposed the generalized Dempster-Shafer (D-S) structures, providing a view of the D-S structure based on a two-step process [21]. This generalized D-S structure is further applied in fuzzy rule bases for fuzzy system modeling [19]. Xiao presented complex evidence theory in 2020, which is a complex-valued generalization of evidence theory [16]. In order to handle the uncertainty in open world, Deng present generalized evidence theory in 2015 [4]. For measuring the uncertainty of evidence theory, Yager presented an interval valued entropy [20]. In 2016, Deng proposed a novel entropy for BPA, called Deng entropy [5]. Deng entropy is further developed into information volume [6], which has been applied in many areas [3, 14]. Some basic conceptions about evidence theory are summarized as follows.

Definition 2.1 (Frame of discernment). Let Θ , called the frame of discernment (FOD), denotes a fixed set of N mutually exclusive and exhaustive elements, indicated by $\Theta = \{\theta_1, \theta_2, \dots, \theta_N\}$. The power set of Θ , denoted as 2^Θ , contains all possible subsets of Θ , which is indicated by $2^\Theta = \{A_1, A_2, \dots, A_{2^N}\} = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \dots, \{\theta_N\}, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \dots, \{\theta_1, \theta_N\}, \dots, \Theta\}$.

Definition 2.2 (Mass function). Given a FOD of Θ , a mass function, also called a basic probability assignment (BPA), is a mapping from 2^Θ to $[0, 1]$, formally defined by:

$$m : 2^\Theta \rightarrow [0, 1] \quad (1)$$

satisfying $m(\emptyset) = 0$ and $\sum_{A \in 2^\Theta} m(A) = 1$, where $m(A)$ is the belief value with the proposition A . If $m(A) > 0$, then A is called a focal element.

Definition 2.3 (Dempster's combination rule). Given two mass functions indicated by m_1 and m_2 , their combination $m_1 \oplus m_2$ is mathematically defined as:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B)m_2(C), & A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \quad (2)$$

where $A, B, C \in 2^\Theta$ and K is calculated by $K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C)$.

2.2 Pascal's triangle

In mathematics, Pascal's triangle is a triangular array of the binomial coefficients that arises in probability theory [13]. As is illustrated in Figure 1 (a), Pascal's triangle is a geometric arrangement of binomial coefficients in a triangle, which satisfies:

- The x -th number in the n -th row can be expressed as $C(n-1, x-1)$, meaning the combinatorial number of drawing $x-1$ elements from $n-1$ elements.
- In the triangle, the sum of the x -th and the $x+1$ -th number in the n -th row equals to the $x+1$ -th number in the $n+1$ -th row, namely, $C(n, x) = C(n-1, x-1) + C(n-1, x)$.
- The sum of all the numbers in the n -th row is 2^n .

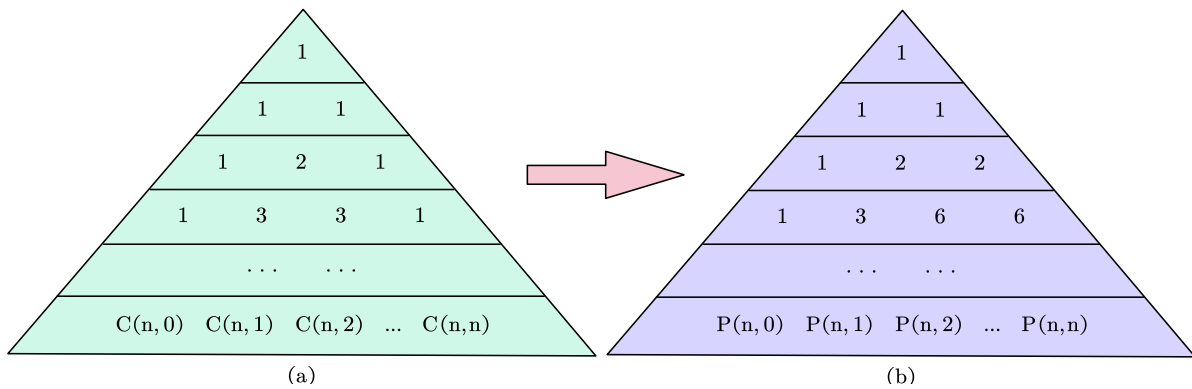


Figure 1: (a) The illustration of Pascal's triangle; (b) The illustration of Pascal's triangle where combinatorial number is replaced by permutation number

2.3 Entropic explanation of power set

A power set of a FOD Θ is defined as the set of all subsets of Θ , including Θ itself and empty set, denoted as 2^Θ . Dempster-Shafer evidence theory extends the FOD to power set. However, the meaning of power set in evidence theory needs exploration.

Hence, an entropic explanation of power set in evidence theory is proposed [12]. In evidence theory, a power set of FOD whose cardinality is N can be seen as all the possible k -combination of FOD, where k ranges from 0 to N . With respect to different N , all the k -combination can form Pascal's triangle as is shown in Figure 1 (a). In addition, the relationship between combinatorial

number, Pascal's triangle, and power set is quantitatively revealed by Deng entropy [5] from the view of information measure [12].

Here comes the question: if we consider all possible k -permutation of a set S with N hypotheses instead of the k -combination, namely, if the combinatorial number in the Pascal's triangle is replaced by permutation number as is shown in Figure 1 (b), what would happen? This will be detailed in Section 3.

3 Random permutation set

In this section, some definitions about random permutation set (RPS) are proposed.

Definition 3.1 (Permutation event space). Given a fixed set of N mutually exclusive and exhaustive elements $\Theta = \{\theta_1, \theta_2, \dots, \theta_N\}$, its **permutation event space (PES)** is a set of all possible permutations of Θ defined as follows:

$$PES(\Theta) = \{A_{ij} \mid i = 0, \dots, N; j = 1, \dots, P(N, i)\} \quad (3)$$

$$= \{ \emptyset, (\theta_1), (\theta_2), \dots, (\theta_N), (\theta_1, \theta_2), (\theta_2, \theta_1), \dots, (\theta_{N-1}, \theta_N), (\theta_N, \theta_{N-1}), \dots, (\theta_1, \theta_2, \dots, \theta_N), \dots, (\theta_N, \theta_{N-1}, \dots, \theta_1) \} \quad (4)$$

in which $P(N, i)$ is the i -permutation of N defined as $P(N, i) = \frac{N!}{(N-i)!}$. The element A_{ij} in PES is called the **permutation event**, which is a tuple representing a possible permutation of Θ , where i indicates the index for the cardinality of A_{ij} and j denotes the index for the possible permutation.

For better understanding of PES, let us consider the following example.

Example 3.1. Assume there are three balls in a box, whose colors are red, blue, and green, respectively. With respect to different colors, these balls in the box can be denoted by $\Theta = \{R, B, G\}$. Then, we **randomly** take out a number of balls from the box **in sequence without replacement**.

For example, if we **firstly** take out the red ball and **secondly** take out the blue ball, this result will be represented by (R, B) . By contrast, if we **firstly** take out the blue ball and **secondly** take out the red ball, this result will be represented by (B, R) . Hence, (R, B) and (B, R) are two different events.

All the possible results are illustrated in Figure 2. In fact, the result are all the permutations of Θ , which can be represented by the PES of Θ , detailed as follows:

$$PES(\{R, B, G\}) = \{A_{01}, A_{11}, A_{12}, A_{13}, \quad (5)$$

$$A_{21}, A_{22}, A_{23},$$

$$A_{24}, A_{25}, A_{26},$$

$$A_{31}, A_{32}, A_{33},$$

$$A_{34}, A_{35}, A_{36} \}$$

$$= \{ \emptyset, (R), (B), (G), \quad (6)$$

$$(R, B), (R, G), (B, G),$$

$$(B, R), (G, R), (G, B),$$

$$(R, B, G), (R, G, B), (B, R, G),$$

$$(B, G, R), (G, B, R), (G, R, B) \}$$

Example 3.1 shows that the PES of Θ is a set containing all the possible permutations of Θ . Besides, permutation event (R, B) is different from (B, R) , because they have different orders of elements in Θ .

Definition 3.2 (Random permutation set). Given a fixed set of N mutually exclusive and exhaustive elements $\Theta = \{\theta_1, \theta_2, \dots, \theta_N\}$, its **random permutation set (RPS)** is a set of pairs defined as follows:

$$RPS(\Theta) = \{ \langle A, \mathcal{M}(A) \rangle \mid A \in PES(\Theta) \} \quad (7)$$

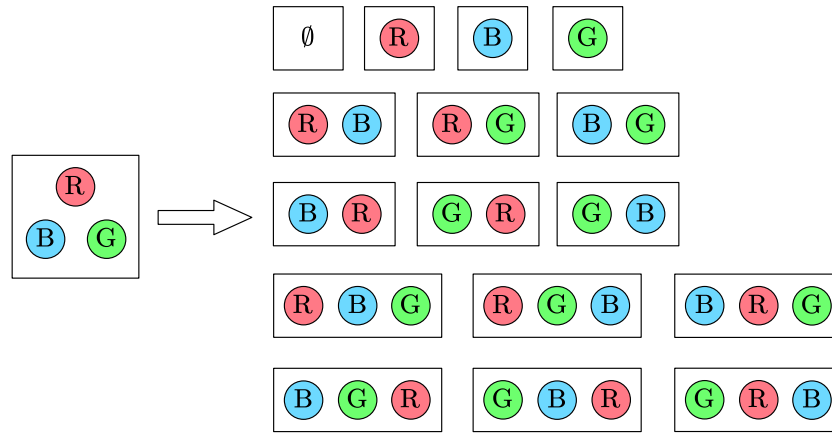


Figure 2: The illustration for **Example 3.1**: all the possible results of randomly taking out a number of balls from the box in sequence without replacement

where \mathcal{M} is called the **permutation mass function (PMF)**, which is defined as:

$$\mathcal{M} : PES(\Theta) \rightarrow [0, 1] \quad (8)$$

constrained by $\mathcal{M}(\emptyset) = 0$ and $\sum_{A \in PES(\Theta)} \mathcal{M}(A) = 1$.

For better understanding of RPS, let us consider the following example.

Example 3.2. Following Example 3.1, given $\Theta = \{R, B, G\}$, its corresponding PES is shown in Eq. (5). Then, one possible RPS can be given as:

$$RPS(\Theta) = \{\langle (R), 0.4 \rangle, \langle (R, B), 0.3 \rangle, \langle (B, R), 0.3 \rangle\} \quad (9)$$

where $\langle (R, B), 0.3 \rangle$ means that the PMF of the permutation event (R, B) , i.e., one takes out **the red ball for the first time**, then takes out **the blue ball for the second time**, is 0.3. By contrast, $\langle (B, R), 0.3 \rangle$ means that the PMF of the permutation event (B, R) , i.e., one takes out **the blue ball for the first time**, then takes out **the red ball for the second time**, is also 0.3.

Besides, comparing $\langle (R), 0.4 \rangle$ and $\langle (R, B), 0.3 \rangle$, we can get the information that the permutation event (R) , i.e., one **only take out the red ball**, has a higher chance to happen compared with the permutation event (R, B) , i.e., one takes out **the red ball for the first time**, then takes out **the blue ball for the second time**.

Example 3.2 shows that, $\langle (R, B), 0.3 \rangle$ and $\langle (B, R), 0.3 \rangle$ are two different PMFs, since permutation events (R, B) and (B, R) are distinguished from the different orders of elements in Θ . In addition, PMF in RPS describes the chance of a certain permutation event that would happen, which is similar to the probability distribution in probability theory and the BPA in evidence theory.

Definition 3.3 (Intersection of permutation events). Given two permutation events $A, B \in PES(\Theta)$, the **right intersection (RI)** and the **left intersection (LI)** of A and B are defined as follows:

$$A \overrightarrow{\cap} B = B \setminus \bigcap_{\theta \in B, \theta \notin A} \{\theta\} \quad (RI) \quad (10)$$

$$A \overleftarrow{\cap} B = A \setminus \bigcap_{\theta \in A, \theta \notin B} \{\theta\} \quad (LI) \quad (11)$$

where $X \setminus Y$ denotes removing Y from X .

For better understanding of RI and LI, let us consider the following example.

Example 3.3. Let two RPSs defined on $\Theta = \{R, B, G\}$ be as follows:

$$RPS_1(\Theta) = \{\langle (R), 0.4 \rangle, \langle (R, B), 0.3 \rangle, \langle (B, R), 0.3 \rangle\} \quad (12)$$

$$RPS_2(\Theta) = \{\langle (G), 0.4 \rangle, \langle (R, B), 0.1 \rangle, \langle (R, G, B), 0.15 \rangle, \langle (G, B, R), 0.35 \rangle\}.$$

Take two permutation events as an instance, i.e., (R, B) of $RPS_1(\Theta)$ and (G, B, R) of $RPS_2(\Theta)$. The RI and LI of these two permutation events can be respectively obtained as:

$$(R, B) \overrightarrow{\cap} (G, B, R) = (G, B, R) \setminus \{G\} = (B, R) \quad (13)$$

$$(R, B) \overleftarrow{\cap} (G, B, R) = (R, B) \setminus \emptyset = (R, B) \quad (14)$$

Next, all intersection results (including RI and LI) of permutation events of $RPS_1(\Theta)$ and $RPS_2(\Theta)$ are shown in Table 1.

Table 1: RI and LI of permutation events of $RPS_1(\Theta)$ and $RPS_2(\Theta)$

$RPS_1(\Theta)$	$RPS_2(\Theta)$	LI	RI	$RPS_1(\Theta)$	$RPS_2(\Theta)$	LI	RI	$RPS_1(\Theta)$	$RPS_2(\Theta)$	LI	RI
	(G)	\emptyset	\emptyset		(G)	\emptyset	\emptyset		(G)	\emptyset	\emptyset
(R)	(R,B)	(R)	(R)	(R,B)	(R,B)	(R,B)	(R,B)	(B,R)	(R,B)	(B,R)	(R,B)
	(R,G,B)	(R)	(R)		(R,G,B)	(R,B)	(R,B)		(R,G,B)	(B,R)	(R,B)
	(G,B,R)	(R)	(R)		(G,B,R)	(R,B)	(B,R)		(G,B,R)	(B,R)	(B,R)

Example 3.3 shows that, because permutation event considers the order of the elements in Θ , the result of RI for two permutation events is different from that of LI.

Definition 3.4 (Orthogonal sum of permutation mass functions). Given two PMFs \mathcal{M}_1 and \mathcal{M}_2 , the **right orthogonal sum (ROS)** of these two PMFs is denoted as $\mathcal{M}_1 \overrightarrow{\oplus} \mathcal{M}_2$ and defined by:

$$\mathcal{M}^R(A) = \begin{cases} \frac{1}{1-\overrightarrow{K}} \sum_{B \overrightarrow{\cap} C=A} \mathcal{M}_1(B) \cdot \mathcal{M}_2(C), & A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \quad (15)$$

where $A, B, C \in PES(\Theta)$, $\overrightarrow{\cap}$ denotes the right intersection, and \overrightarrow{K} is defined as

$$\overrightarrow{K} = \sum_{B \overrightarrow{\cap} C=\emptyset} \mathcal{M}_1(B) \cdot \mathcal{M}_2(C). \quad (16)$$

The **left orthogonal sum (LOS)** of these two PMFs is denoted as $\mathcal{M}_1 \overleftarrow{\oplus} \mathcal{M}_2$ and defined by:

$$\mathcal{M}^L(A) = \begin{cases} \frac{1}{1-\overleftarrow{K}} \sum_{B \overleftarrow{\cap} C=A} \mathcal{M}_1(B) \cdot \mathcal{M}_2(C), & A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \quad (17)$$

where $A, B, C \in PES(\Theta)$, $\overleftarrow{\cap}$ denotes the left intersection, and \overleftarrow{K} is defined as

$$\overleftarrow{K} = \sum_{B \overleftarrow{\cap} C=\emptyset} \mathcal{M}_1(B) \cdot \mathcal{M}_2(C). \quad (18)$$

For better understanding of ROS and LOS, let us consider the following example.

Example 3.4. Following Example 3.3, given two RPSs defined on $\Theta = \{R, B, G\}$ be as follows:

$$RPS_1(\Theta) = \{ \langle (R), 0.4 \rangle, \langle (R, B), 0.3 \rangle, \langle (B, R), 0.3 \rangle \} \quad (19)$$

$$RPS_2(\Theta) = \{ \langle (G), 0.4 \rangle, \langle (R, B), 0.1 \rangle, \langle (R, G, B), 0.15 \rangle, \langle (G, B, R), 0.35 \rangle \}$$

whose PMFs are respectively denoted by \mathcal{M}_1 and \mathcal{M}_2 , then we will show the calculating procedure of ROS $\mathcal{M}_1 \overrightarrow{\oplus} \mathcal{M}_2$ and LOS $\mathcal{M}_1 \overleftarrow{\oplus} \mathcal{M}_2$.

Take $\mathcal{M}^R(R, B)$ of $\mathcal{M}_1 \overrightarrow{\oplus} \mathcal{M}_2$ and $\mathcal{M}^L(R, B)$ of $\mathcal{M}_1 \overleftarrow{\oplus} \mathcal{M}_2$ as examples. They are respectively calculated as:

$$\begin{aligned} \overrightarrow{K} &= \mathcal{M}_1(R) \cdot \mathcal{M}_2(G) + \mathcal{M}_1(R, B) \cdot \mathcal{M}_2(G) + \mathcal{M}_1(B, R) \cdot \mathcal{M}_2(G) \\ &= 0.4 * 0.4 + 0.3 * 0.4 + 0.3 * 0.4 = 0.4 \end{aligned} \quad (20)$$

$$\begin{aligned}
\mathcal{M}^R(R, B) &= \frac{1}{1 - \overrightarrow{K}} [\mathcal{M}_1(R, B) \mathcal{M}_2(R, B) + \mathcal{M}_1(R, B) \mathcal{M}_2(R, G, B) \\
&\quad + \mathcal{M}_1(B, R) \mathcal{M}_2(R, B) + \mathcal{M}_1(B, R) \mathcal{M}_2(R, G, B)] \\
&= \frac{1}{1 - 0.4} (0.3 * 0.1 + 0.3 * 0.15 + 0.3 * 0.1 + 0.3 * 0.15) \\
&= 0.25
\end{aligned} \tag{21}$$

$$\begin{aligned}
\overleftarrow{K} &= \mathcal{M}_1(R) \mathcal{M}_2(G) + \mathcal{M}_1(R, B) \mathcal{M}_2(G) + \mathcal{M}_1(B, R) \mathcal{M}_2(G) \\
&= 0.4 * 0.4 + 0.3 * 0.4 + 0.3 * 0.4 = 0.4
\end{aligned} \tag{22}$$

$$\begin{aligned}
\mathcal{M}^L(R, B) &= \frac{1}{1 - \overleftarrow{K}} [\mathcal{M}_1(R, B) \mathcal{M}_2(R, B) + \mathcal{M}_1(R, B) \mathcal{M}_2(R, G, B) \\
&\quad + \mathcal{M}_1(R, B) \mathcal{M}_2(G, B, R)] \\
&= \frac{1}{1 - 0.4} (0.3 * 0.1 + 0.3 * 0.15 + 0.3 * 0.35) \\
&= 0.3
\end{aligned} \tag{23}$$

Next, the results for the orthogonal sum (including ROS and LOS) of \mathcal{M}_1 and \mathcal{M}_2 are shown in Table 2.

Table 2: The results for ROS and LOS of \mathcal{M}_1 and \mathcal{M}_2

Permutation event	ROS $\mathcal{M}_1 \oplus \mathcal{M}_2$	LOS $\mathcal{M}_1 \oplus \mathcal{M}_2$
(R)	$\mathcal{M}^R(R) = 0.4$	$\mathcal{M}^L(R) = 0.4$
(R, B)	$\mathcal{M}^R(R, B) = 0.25$	$\mathcal{M}^L(R, B) = 0.3$
(B, R)	$\mathcal{M}^R(B, R) = 0.35$	$\mathcal{M}^L(B, R) = 0.3$

Example 3.4 shows that ROS is different from LOS, since they take different intersection of permutation events into account, namely, ROS considers RI while LOS uses LI.

4 Numerical examples and discussions

In this section, some numerical examples are shown to compare RPS, evidence theory, and probability theory.

Example 4.1. Given a fixed set of N mutually exclusive and exhaustive elements $\Theta = \{X, Y\}$, its corresponding PES is that

$$PES(\Theta) = \{\emptyset, (X), (Y), (X, Y), (Y, X)\} \tag{24}$$

Consider the following two scenarios:

- If the order of the element in permutation event is ignored, then permutation events (X, Y) and (Y, X) will be the same. To be specific, both of (X, Y) and (Y, X) will degenerate into the set $\{X, Y\}$, since the order of X and Y is regardless. As a result, if the order of the element in $PES(\Theta)$ is ignored, $PES(\Theta)$ will degenerate into the power set in evidence theory [2, 11]: $2^\Theta = \{\emptyset, \{X\}, \{Y\}, \{X, Y\}\}$.
- If each permutation event is limited to containing just one element, then permutation events (X, Y) and (Y, X) will disappear, so that $PES(\Theta)$ will degenerate into the sample space in probability theory [9]: $\Omega = \{\{X\}, \{Y\}\}$.

Example 4.1 shows that PES is compatible with the power set in evidence theory and the sample space in probability theory.

Example 4.2. Let two RPSs defined on $\Theta = \{X, Y, Z\}$ be as follows:

$$RPS_1(\Theta) = \left\{ \left\langle (X), \frac{1}{7} \right\rangle, \left\langle (Y), \frac{1}{7} \right\rangle, \left\langle (Z), \frac{1}{7} \right\rangle, \left\langle (X, Y), \frac{1}{7} \right\rangle, \right. \\ \left. \left\langle (Y, Z), \frac{1}{7} \right\rangle, \left\langle (X, Z), \frac{1}{7} \right\rangle, \left\langle (X, Y, Z), \frac{1}{7} \right\rangle \right\} \quad (25)$$

$$RPS_2(\Theta) = \left\{ \left\langle (X), \frac{1}{3} \right\rangle, \left\langle (Y), \frac{1}{3} \right\rangle, \left\langle (Z), \frac{1}{3} \right\rangle \right\} \quad (26)$$

whose PMFs are respectively denoted by \mathcal{M}_1 and \mathcal{M}_2 , which are discussed as follows:

- Since the PES of $RPS_1(\Theta)$ degenerates into the power set in evidence theory: $2^\Theta = \{\emptyset, \{X\}, \{Y\}, \{Z\}, \{X, Y\}, \{Y, Z\}, \{X, Z\}, \{X, Y, Z\}\}$, the PMF \mathcal{M}_1 of $RPS_1(\Theta)$ is actually a BPA in evidence theory [2, 11]:

$$m(\{X\}) = m(\{Y\}) = m(\{Z\}) = m(\{X, Y\}) = \\ m(\{Y, Z\}) = m(\{X, Z\}) = m(\{X, Y, Z\}) = \frac{1}{7}. \quad (27)$$

- Because the PES of $RPS_2(\Theta)$ degenerates into the sample space in evidence theory: $\Omega = \{\{X\}, \{Y\}, \{Z\}\}$, \mathcal{M}_2 of $RPS_2(\Theta)$ is actually a probability distribution in probability theory [9]:

$$P(\{X\}) = P(\{Y\}) = P(\{Z\}) = \frac{1}{3}. \quad (28)$$

Example 4.2 shows that PMF of RPS is compatible with the BPA in evidence theory and the probability distribution in probability theory.

Example 4.3. Following Example 3.4, given two RPSs defined on $\Theta = \{R, B, G\}$ be as follows:

$$RPS_1(\Theta) = \{\langle (R), 0.4 \rangle, \langle (R, B), 0.3 \rangle, \langle (B, R), 0.3 \rangle\} \quad (29)$$

$$RPS_2(\Theta) = \{\langle (G), 0.4 \rangle, \langle (R, B), 0.1 \rangle, \langle (R, G, B), 0.15 \rangle, \langle (G, B, R), 0.35 \rangle\}$$

whose PMFs are respectively denoted by \mathcal{M}_1 and \mathcal{M}_2 . Consider the following two scenarios:

- If the order of the element in permutation event is ignored, then \mathcal{M}_1 and \mathcal{M}_2 will degenerate into BPAs:

$$m_1 : m_1(\{R\}) = 0.4, m_1(\{R, B\}) = 0.6 \quad (30)$$

$$m_2 : m_2(\{G\}) = 0.4, m_2(\{R, B\}) = 0.1, m_2(\{R, G, B\}) = 0.5 \quad (31)$$

Use Dempster's rule [2, 11] to combine m_1 and m_2 . The result is that:

$$m : m(\{R\}) = 0.4, m(\{R, B\}) = 0.6 \quad (32)$$

- The associated ROS and LOS of \mathcal{M}_1 and \mathcal{M}_2 are calculated in Example 3.4, which are shown as follows:

$$\mathcal{M}^R : \mathcal{M}^R(R) = 0.4, \mathcal{M}^R(R, B) = 0.25, \mathcal{M}^R(B, R) = 0.35 \quad (33)$$

$$\mathcal{M}^L : \mathcal{M}^L(R) = 0.4, \mathcal{M}^L(B, R) = 0.3, \mathcal{M}^L(R, B) = 0.3 \quad (34)$$

If the order of the element in permutation event is ignored, then \mathcal{M}^R and \mathcal{M}^L will degenerate into BPAs:

$$m^R : m^R(\{R\}) = 0.4, m^R(\{R, B\}) = 0.6 \quad (35)$$

$$m^L : m^L(\{R\}) = 0.4, m^L(\{R, B\}) = 0.6 \quad (36)$$

Comparing Eq. (32), Eq. (35), and Eq. (36), we can find that all of them are the same, which shows that, if PMF becomes BPA, both ROS and LOS will degenerate into Dempster's combination rule.

Example 4.3 shows that ROS and LOS are compatible with Dempster's combination rule in evidence theory. For clearance, the comparisons of probability theory, evidence theory, and RPS discussed from Example 4.1 to 4.3 are summarized in Table 3.

Table 3: Comparisons of probability theory, evidence theory, and RPS

Theory	Probability theory	Evidence theory	Random permutation set (RPS)
Space	Sample space	Power set	Permutation event space (PES)
Mapping function	Probability distribution	BPA	Permutation mass function (PMF)
Fusion rule	Bayes fusion rule	Dempster's rule	Right orthogonal sum (ROS) Left orthogonal sum (LOS)

5 Application in sensor data fusion and threat assessment

In this section, to illustrate the performance of RPS, an RPS-based data fusion algorithm is proposed and applied in threat assessment.

5.1 Problem statement

Threat assessment is a real practice of determining the seriousness of a potential threat, and evaluating the probability that a threat will become a reality. Threat assessment has several major procedures, including identification of potential threatening target, initial assessment for the seriousness of the targets, and follow-up assessment, i.e., data fusion and decision making. Threat assessment can be applied in many fields, such as business, education, risk management, and so on.

In the scenario of enemy aircraft detection, assume there are three types of aircraft (potential threatening target) represented by $\{A, B, C\}$. In order to detect the aircraft, assume there are three radars remarked from 1 to 3. For threat assessment of the aircraft, the report provided by each radar should includes the following three aspects:

(I) Representation of the threat event

There three types of aircraft, and each type of aircraft may pose a threat, which is regarded as a *one-type threat event*. However, in a real world scenario, the enemy may not just send one type of aircraft, but allocate an aircraft fleet with several different types of aircraft, which is referred as a *multi-types threat event*.

For example, if the enemy sent an aircraft fleet with two types of aircraft (A and C), this threat is called the *two-types threat event*, which is denoted as $\{A, C\}$.

(II) Threat ranking of the threat event

In aircraft fleet, different aircraft have different levels of threat. Hence, there exists *threat ranking* in the threat event. For example, the threat event $\{A, C\}$ mentioned in aspect (I) has two kinds of threat ranking, which can be represented by $A \succ C$ and $C \succ A$. Based on permutation event of RPS, these two rankings can be denoted as (A, C) and (C, A) . In this sense, according to the seriousness of the aircraft, all the possible threat ranking of threat event can be represented as PES:

$$\begin{aligned}
 PES(\{A, B, C\}) = \{ & \emptyset, (A), (B), (C), \\
 & (A, B), (A, C), (B, C), \\
 & (B, A), (C, A), (C, B), \\
 & (A, B, C), (A, C, B), (B, A, C), \\
 & (B, C, A), (C, B, A), (C, A, B) \}
 \end{aligned} \tag{37}$$

To better understand, let's consider the following example. (A, C, B) indicate that the threat ranking of threat event $\{A, B, C\}$ is $A \succ C \succ B$, which means that aircraft A is very threatening, while the threat of B is the lowest (but can not be ignored) compared with A and C . Besides, (A, B) means that the threat of C is trivial which can be ignored, while the threat of A and B should be taken into consideration: their ranking of threat is $A \succ B$.

For clearance and concise, in the rest of the part, we use tuple (\cdot) to represent threat event with threat ranking, inside of using $\{\cdot\}$.

(III) Threat degree of the threat event

Threat degree of a certain threat event is regarded as the chance that the threat event will become a reality, which can be represented by PMF $\mathcal{M} : PES(\{A, B, C\}) \rightarrow [0, 1]$. For instance, this is a PMF

obtained by radar:

$$\mathcal{M} : \mathcal{M}(C) = 0.1 \quad \mathcal{M}(C, A) = 0.3 \quad \mathcal{M}(C, A, B) = 0.6 \quad (38)$$

where $\mathcal{M}(C) = 0.1$ indicates that aircraft C may cause a potential threat, but the threat degree is relatively low. Besides, $\mathcal{M}(C, A) = 0.3$ means that aircraft C and A are likely to threaten the security, and the threat of C is more serious than that of A , while the threat of B can be ignored. $\mathcal{M}(C, A, B) = 0.6$ denotes that an aircraft fleet with three types of aircraft may constitute a big threat, the threat ranking of them is $C \succ A \succ B$.

In order to representing the three aspects mentioned above, the threat assessment reports by radars should be based on RPS, i.e., PES of RPS for indicating aspect (I) and (II), and PMF of RPS for representing aspect (III).

Besides, in real practice, due to difference of resolution and mechanical structure, the reliability of the radars would be different. Hence, the reliability ranking of the radar should be taken into account. It should be noted that, the order in threat ranking of threat event is for aircraft (targets), while the order in reliability ranking is for radars (information sources).

To sum up, to make full use of the threat assessment reports and the reliability ranking, as well as to recognize the most threatening aircraft of the three, the data should be suitably treated and fused. In the next subsection, an RPS-based data fusion algorithm is proposed for threat assessment.

5.2 RPS-based data fusion algorithm for threat assessment

In this subsection, an RPS-based data fusion algorithm is proposed, where RPS and its orthogonal sum are used to process the data. Given a sample space $\Theta = \{\theta_1, \dots, \theta_N\}$ and M RPSs based on $PES(\Theta)$, the proposed algorithm is as follows.

Step 1: Input threat assessment reports, and obtain N RPSs by different radars which are denoted as

$$RPS_i, \quad i = 1, 2, \dots, M. \quad (39)$$

Step 2: Based on the reliability ranking $RPS_x \succ RPS_y \succ \dots \succ RPS_z$, combine the PMF associated with the M RPS by using the left orthogonal sum (LOS) for $M - 1$ times, and then get the left-combined PMF:

$$\mathcal{M}^L = \mathcal{M}_x \overset{\leftarrow}{\oplus} \mathcal{M}_y \overset{\leftarrow}{\oplus} \dots \overset{\leftarrow}{\oplus} \mathcal{M}_z \quad (40)$$

Step 3: Based on \mathcal{M}^L , calculate the relative threat degree (RTD) for the j -th threat event A_{ij} with i types of aircraft:

$$RTD(A_{ij}) = \frac{\mathcal{M}^L(A_{ij})}{\sum_{j=1}^{P(N,i)} \mathcal{M}^L(A_{ij})} \quad A_{ij} \in PES(\Theta) \quad (41)$$

where $i = 1, \dots, N$ is the number of types for aircraft, and $j = 1, \dots, P(N, i)$ is the index for the i -types threat event (the threat event with i types of aircraft). $P(N, i) = \frac{N!}{(N-i)!}$ is the permutation number.

Step 4: Based on RTD, output the final threat assessment (FTA) for different i -types threat events:

$$FTA_i = \arg \max_{1 \leq j \leq P(N,i)} [RTD(A_{ij})] \quad 1 \leq i \leq N, \quad A_{ij} \in PES(\Theta) \quad (42)$$

Table 4: The RPS-based threat assessment reports by three radars

$\mathcal{M}(\cdot)$	(A)	(B)	(A, B)	(B, A)	(A, C)	(A, B, C)	(A, C, B)	(B, A, C)	(C, A, B)
RPS_1	0.12	0.10	0.15	0.15	0.03	0.10	0.25	0.08	0.02
RPS_2	0.10	0.15	0.07	0.23	0.10	0.10	0.05	0.12	0.08
RPS_3	0.18	0.05	0.15	0.05	0.00	0.12	0.30	0.10	0.05

Table 5: The left-combined PMF based on LOS

	(A)	(B)	(A, B)	(B, A)	(A, C)	(C, A)	(A, B, C)	(A, C, B)	(B, A, C)	(C, A, B)
$\mathcal{M}^{L'}(\cdot)$	0.267	0.123	0.260	0.069	0.016	0.002	0.055	0.139	0.046	0.023
$\mathcal{M}^L(\cdot)$	0.350	0.201	0.251	0.063	0.033	0.003	0.021	0.052	0.017	0.009

5.3 Experiment and result

The threat assessment reports for the aircraft by three radars are collected in Table 4, and The reliability ranking of the three radars can be obtained by experts and is shown as follows: $RPS_3 \succ RPS_1 \succ RPS_2$.

To fuse the information in threat assessment reports and the reliability ranking, as well as to obtain the final threat assessment, the data is processed based on the proposed RPS-based data fusion algorithm. The procedure is as follows.

Step 1: Input threat assessment reports, and obtain three RPSs: RPS_1, RPS_2, RPS_3 .

Step 2: Based on the reliability ranking $RPS_3 \succ RPS_1 \succ RPS_2$, combine their associated PMF by using LOS for two times, and then get the left-combined PMF:

$$\mathcal{M}^L = \mathcal{M}_3 \stackrel{\leftarrow}{\oplus} \mathcal{M}_1 \stackrel{\leftarrow}{\oplus} \mathcal{M}_2 \quad (43)$$

which can be calculated by:

$$\mathcal{M}^{L'} = \mathcal{M}_3 \stackrel{\leftarrow}{\oplus} \mathcal{M}_1 \quad (44)$$

$$\mathcal{M}^L = \mathcal{M}^{L'} \stackrel{\leftarrow}{\oplus} \mathcal{M}_2 \quad (45)$$

whose results are summarized in Table 5.

Step 3: Based on \mathcal{M}^L , calculate the relative threat degree (RTD). For one-type threat events (A) and (B), their corresponding RTDs are that:

$$RTD(A) = \frac{\mathcal{M}^L(A)}{\mathcal{M}^L(A) + \mathcal{M}^L(B)} = \frac{0.350}{0.350 + 0.201} = 0.635 \quad (46)$$

$$RTD(B) = \frac{\mathcal{M}^L(B)}{\mathcal{M}^L(A) + \mathcal{M}^L(B)} = \frac{0.201}{0.350 + 0.201} = 0.365 \quad (47)$$

For two-types threat events (A, B), (B, A), (A, C), and (C, A), their RTDs are that:

$$\begin{aligned} RTD(A, B) &= 0.717 & RTD(B, A) &= 0.180 \\ RTD(A, C) &= 0.094 & RTD(C, A) &= 0.009 \end{aligned} \quad (48)$$

For three-types threat events (A, B, C), (A, C, B), (B, A, C), and (C, A, B), their RTDs are that:

$$\begin{aligned} RTD(A, B, C) &= 0.212 & RTD(A, C, B) &= 0.525 \\ RTD(B, A, C) &= 0.172 & RTD(C, A, B) &= 0.091 \end{aligned} \quad (49)$$

Step 4: Based on RTD, output the final threat assessment (FTA) for one-type threat events, two-type threat events, and three-type threat events, which are respectively shown as follows:

$$FTA_1 = \arg \max [RTD(A), RTD(B)] = (A) \quad (50)$$

$$FTA_2 = \arg \max [RTD(A, B), \dots, RTD(C, A)] = (A, B) \quad (51)$$

$$FTA_3 = \arg \max [RTD(A, B, C), \dots, RTD(C, A, B)] = (A, C, B) \quad (52)$$

From these FTAs, it can be concluded that:

- If the enemy just sends **one type of aircraft**, namely one-type threat event, then aircraft A is the most threatening aircraft to us.
- If the enemy sends **an aircraft fleet with two types of aircraft**, namely two-types threat event, then aircraft fleet with A and B is the most threatening fleet to us. The threat ranking is $A \succ B$, which means that, in this fleet, aircraft A is more threatening than B .
- If the enemy sends **an aircraft fleet with three types of aircraft**, namely three-types threat event, then aircraft fleet with A , C , and B is the most threatening fleet to us. The threat ranking is $A \succ C \succ B$, which indicates that, in this fleet, aircraft A is the most threatening aircraft, C also poses a potential threat to us, while the threat of B is the lowest.

5.4 Analysis and discussion

In this subsection, to show the efficiency of the proposed RPS-based algorithm in threat assessment, we compare it with the method based on BPA and Dempster's rule of combination [2].

As is illustrated in **Example 4.1**, if the order of the element in $PES(\Theta)$ is ignored, $PES(\Theta)$ will degenerate into the power set in evidence theory, so that RPS in Table 4 will boil down to body of evidence: E_1 , E_2 , and E_3 , which are listed in Table 6.

Table 6: The BPA-based threat assessment reports by three radars

$m(\cdot)$	$\{A\}$	$\{B\}$	$\{A, B\}$	$\{A, C\}$	$\{A, B, C\}$
E_1	0.12	0.10	0.30	0.03	0.45
E_2	0.10	0.15	0.30	0.10	0.35
E_3	0.18	0.05	0.20	0.00	0.57

Table 7: Data fusion results based on two methods

BPA+D's rule ^a	$\{A\}$	$\{B\}$	$\{A, B\}$	$\{A, C\}$	$\{A, B, C\}$
$m^D(\cdot)$	0.350	0.201	0.314	0.036	0.099
$RTD^{BPA}(\cdot)$	0.635	0.365	0.897	0.103	1.000
FTA_i^{BPA}	$\{A\}$	$\{A, B\}$			$\{A, B, C\}$

RPS+LOS ^b	(A)	(B)	(A, B)	(B, A)	(A, C)	(C, A)	(A, B, C)	(A, C, B)	(B, A, C)	(C, A, B)
$\mathcal{M}^L(\cdot)$	0.350	0.201	0.251	0.063	0.033	0.003	0.021	0.052	0.017	0.009
$RTD^{RPS}(\cdot)$	0.635	0.365	0.717	0.180	0.094	0.009	0.212	0.525	0.172	0.091
FTA_i^{RPS}	(A)	(A, B)					(A, C, B)			

^a: "BPA+D's rule" denotes BPA and Dempster's rule of combination.

^b: "RPS+LOS" indicates RPS and left orthogonal sum (LOS).

Then, E_1 , E_2 , and E_3 are combined based on Dempster's rule of combination, and the data fusion result m^D is summarized in Table 7. For comparison, the data fusion result \mathcal{M}^L in Table 5 is listed again in Table 7. Besides, the results of RTD and FTA based on the two methods are also summarized in Table 7. It can be seen from Table 7 that:

- LOS (as well as ROS) considers the order of combination, which is a generalization of Dempster's rule. If the order of the element in $PES(\Theta)$ is ignored, RPS will degenerate into body of evidence, so that \mathcal{M}^L based on LOS will further degenerate into m^D based on Dempster's rule, such as $m^D(\{A, B\}) = \mathcal{M}^L(A, B) + \mathcal{M}^L(B, A)$. Besides, LOS can combine RPS based on the reliability ranking. For example, if the reliability ranking is $RPS_3 \succ RPS_1 \succ RPS_2$, then the combination of RPSs based on LOS is that $\mathcal{M}_3 \oplus \mathcal{M}_1 \oplus \mathcal{M}_2$. However, Dempster's rule does not have this sense of reliability ranking.
- RPS takes the order of the element into consideration, so that it can distinguish from (A, B, C) and (A, C, B) , and assign different PMF to them. However, BPA of evidence theory does not have the sense of order. Hence, power set just has the element $\{A, B, C\}$, which is a set not a tuple, so that element $\{A, B, C\}$ just has a single BPA.
- As for RTD and FTA, since RPS assigns different PMF for (A, C, B) and (A, B, C) , which can be further used to represent relative threat degree (RTD), such as $RTD^{RPS}(A, C, B) = 0.525$, so that $FTA_3^{RPS} = (A, C, B)$, from which we can obtain the threat ranking $A \succ C \succ B$. Nevertheless, the element $\{A, B, C\}$ in power set just has a single BPA, which is not suitable for representing RTD. We can see from Table 7, $RTD^{BPA}(\{A, B, C\}) = 1.000$, so that FTA_3^{BPA} is always $\{A, B, C\}$. Hence, BPA is not capable to represent RTD and is not suitable to be used for generating FTA.

To summarize, since RPS takes the ranking and order of element into consideration, the proposed RPS-based data fusion algorithm is able to reasonably and efficiently handle uncertainty in threat assessment with respect to threat ranking and reliability ranking. The comparisons of evidence theory and RPS with respect to threat assessment are summarized in Table 8.

Table 8: Comparisons of evidence theory and RPS with respect to threat assessment

	Evidence theory	Random permutation set (RPS)
Reliability ranking	×	✓ (based on LOS or ROS)
Threat ranking	×	✓ (based on PES)
Threat degree	✓ (based on BPA)	✓ (based on PMF)
RTD	×	✓ (based on RPS and LOS)
FTA	×	✓ (based on RPS and LOS)

6 Conclusion

For exploring the meaning of the power set in evidence theory, a possible explanation of power set is proposed from the view of Pascal's triangle and combinatorial number. Here comes the question: what would happen if the combinatorial number is replaced by permutation number? To address this issue, a new kind of set, named as random permutation set, is proposed in this paper. The main contributions and some remarks of this paper are summarized as follows:

- Random permutation set (RPS) is proposed, which consists of permutation event space (PES) and permutation mass function (PMF). The PES of a certain set considers all the permutation of that set. The elements of PES are called the permutation events. PMF describes the chance of a certain permutation event that would happen.
- The right intersection (RI) and left intersection (LI) of permutation events are presented. Based on RI and LI, the right orthogonal sum (ROS) and left orthogonal sum (LOS) of PMFs are proposed.
- The comparisons of probability theory, evidence theory, and RPS are discussed and summarized. The PES of RPS is compatible with the power set in evidence theory and sample space

in probability theory. The PMF of RPS are compatible with the BPA and the probability distribution. The ROS and LOS of RPS is compatible with Dempster's rule. Also, since RPS extends power set (a collection for combination of elements) into PES (a collection for permutation of elements), RPS is actually a permutation-based generalization of random finite set.

- A RPS-based data fusion algorithm is proposed for threat assessment. The results show that, because RPS considers the ranking and order of element, the proposed RPS-based algorithm can reasonably and efficiently process uncertain information in threat assessment with threat ranking and reliability ranking.

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