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RESEARCH ARTICLE

A new probability transformation method based on a correlation coefficient of belief functions

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Abstract

The Dempster-Shafer evidence theory is widely used in many fields of information fusion because of its advantage in handling uncertain information. One of the key issues in this theory is how to make decision based on a basic probability assignment (BPA). Currently, a feasible scheme is transforming a BPA to a distribution of probabilities. However, little attention was paid to the correlation between BPA and probability distribution. In this paper, a novel method about the probability transformation based on a correlation coefficient of belief functions is proposed. The correlation coefficient is a new measurement, which can effectively measure the correlation between BPAs. The proposed method aims at maximizing the correlation coefficient between the given BPA and the transformed probability distribution. On the basis of this idea, the corresponding probability distribution can be obtained and could reflect the original information of the given BPA to the maximum extent. It is valid to consider that the proposed probability transformation method is reasonable and effective. Numerical examples are given to show the effectiveness of the proposed method.

KEYWORDS

Dempster-Shafer evidence theory, evidential correlation coefficient, probability transformation

1 | INTRODUCTION

Dempster-Shafer evidence theory, as well known as D-S theory,[1,2] has its special advantage in dealing with uncertain information.[3,4] This theory is widely used in classification and clustering,[5,6] knowledge reasoning,[7,8] fault diagnosis,[9,10] decision making,[11,13] and other fields.[14] One of the important issues in the D-S theory is how to make decision based on basic probability assignment (BPA), which has attracted considerable attention. A useful scheme is transforming a BPA to a probability distribution. At present, there are some probability transformation methods. For example, pignistic probability transformation (PPT) from the transferable belief model[15] is widely used. In PPT, the belief assigned to the subset containing multiple focal elements is equally redistributed to single elements in that set. Researchers also proposed some other probability transformation methods. A well-known method is the plausibility transformation method proposed by Cobb.[16] The main idea of plausibility transformation is to distribute beliefs according to the plausibility function with normalization. Cuzzolin[17] discussed some semantics and properties of the relative belief transform. Another often mentioned method is the proportional probability transformation.[18] In this method, the belief defined on subset X, which containing multiple focal elements, is reassigned to $X's$ elements. This method is influenced by the proportion of BPAs assigned to singletons.

However, there are little research focusing on the relevance between BPA and probability distribution in probability transformation methods. Since the basis of decision making is BPA, when using the probability transformed by BPA to make probabilistic decision, the relevance of the given BPA and probability distribution should be taken into consideration. In this paper, the basic idea of the proposed probability transformation method is to maximize the relevance based on a correlation coefficient between BPAs. Jiang [19] proposed a correlation coefficient of belief functions, which is a new method measuring relevance between two BPAs. When transforming a given BPA to a probability distribution, we think it is reasonable that the obtained probability distribution has the maximum relevance with the given BPA. Because it keeps a high fidelity of the probability distribution to the original BPA in a sense. And the probability distribution can be considered as a special BPA, which is a Bayesian mass function, so we can use the correlation coefficient measure the relevance between the probability distribution and the given BPA. So given a BPA, according to the maximum correlation coefficient, the objective function can be established, and then the corresponding probability distribution can be obtained by the mathematical derivation.

The rest of this paper is organized as follows. Section 2 introduces some background knowledge. In Section 3, the proposed method about probability transformation and the mathematical derivation of the probability distribution is shown in detail. Section 4 uses some examples to illustrate the effectiveness of the proposed method. Conclusion is given in Section 5.

2 | PRELIMINARIES

2.1 | Dempster-Shafer evidence theory

The Dempster-Shafer theory is an effective tool to express uncertainty information and make information fusion.[20-23] The research related to the Dempster-Shafer theory[1,2] has attracted much interest, including uncertainty measure,[24,25] evidential network,[26,27] evidence combination,[28-30] conflict management,[31,32] likelihood function,[33,34] and so on.[35-37] On the basis of the Dempster-Shafer theory, many new models handling uncertainty information have also been proposed, like D numbers.[38] In the Dempster-Shafer theory, a problem domain denoted by a finite nonempty set Θ of mutually exclusive and exhaustive hypotheses is named as the frame of discernment. Let 2^Θ denotes the power set of Θ; a BPA is a mapping $m : 2^\Theta \rightarrow [0, 1]$, satisfying

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Theta} m(A) = 1. \tag{1}$$

The belief function and plausibility function of a BPA are, respectively, defined in Equations (2) and (3):

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B) \quad \forall A \subseteq \Theta \tag{2}$$

$$\mathrm{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B) \quad \forall A \subseteq \Theta. \tag{3}$$

2.2 | Pignistic probability transformation

Let m be a BPA on the frame of discernment Θ. The PPT for the singletons is defined as follows:

$$\mathrm{Bet}P_m(\theta_i) = \sum_{\theta_i \in B, B \subseteq \Theta} \frac{m(B)}{|B|}, \tag{4}$$

where $|\cdot|$ is the cardinality of subset B.

2.3 | Plausibility probability transformation

The plausibility transformation method[16] is defined as follows:

$$\mathrm{PnPl}(\theta_i) = \frac{\mathrm{Pl}_m(\theta_i)}{\sum_j \mathrm{Pl}_m(\theta_j)}. \tag{5}$$

2.4 | Proportional probability transformation

Proportional probability transformation[18] is defined on the frame of discernment Θ as follows:

$$\mathrm{Pro}P(\theta_i) = \sum_{\theta_i \in B, B \subseteq \Theta} \frac{m(\theta_i)}{\sum_{\theta_j \in B} m(\theta_j)} \cdot m(B), \tag{6}$$

if $\sum_{\theta_j \in B} m(\theta_j) = 0$, then $|B|$ is used instead, and thus $m(B)$ is equally distributed to the elements of B in such a case.

3 | CORRELATION COEFFICIENT PROBABILITY TRANSFORMATION

In this section, a new probability transformation method based on the evidential correlation is proposed. The correlation coefficient,[19] is introduced first, and then the corresponding probability distribution based on the correlation coefficient of belief functions is presented. The final result of probability distribution is analyzed in the end. The correlation coefficient is defined as follows. Let m_1 and m_2 be two BPAs on the frame of discernment Θ with N elements. A correlation coefficient is defined as

$$r_{\mathrm{BPA}}(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1) \cdot c(m_2, m_2)}}, \tag{7}$$

where $c(m_1, m_2)$ has the following form:

$$c(m_1, m_2) = \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|}, \tag{8}$$

where $i, j = 1, 2, \ldots, 2^N$; A_i, A_j are the focal elements of BPA, and $|\cdot|$ is the cardinality of a subset. $r_{\mathrm{BPA}}(m_1, m_2)$ is used to measure the relevance between two BPAs.

In the proposed method, the correlation coefficient is used to measure the relevance between the probability distribution and the given BPA. To keep the maximum correlation coefficient, the objective function is established based on the given BPA, and then the probability distribution can be obtained. The proposed probability transformation method based on the evidential correlation is given as follows.

Let the frame of discernment be $\Theta = \{\theta_1, \theta_2 \ldots, \theta_n\}$; given a BPA m_1, assume the transformed probability distribution is a Bayesian mass function m_2:

$$m_2 : m_2(\theta_1) = \mathrm{Cor}P(\theta_1), \ \ m_2(\theta_2) = \mathrm{Cor}P(\theta_2), \ldots, m_2(\theta_n) = \mathrm{Cor}P(\theta_n), \tag{9}$$

where m_2 satisfies Equation (10)

$$\text{s.t.} \begin{cases} \mathrm{Bel}(B) \le \sum\limits_{\theta_i \in B} \mathrm{Cor}P(\theta_i) \le \mathrm{Pl}(B) \\ 0 \le \mathrm{Cor}P(\theta_i) \le 1 \quad \forall\, \theta_i \in \Theta \\ \sum\limits_{\theta_i \in \Theta} \mathrm{Cor}P(\theta_i) = 1. \end{cases} \tag{10}$$

By analyzing, Equation (10) can be simplified as follows.

First, $r_{\mathrm{BPA}}(m_1, m_2)$ can be simplified. In Equation (7), A_j, which is the focal element of $m_2(A_j)$, is the singleton, that is $A_j \in \{\theta_1, \theta_2, \ldots, \theta_n\}$, so when $\theta_j \in A_i$, $\frac{|A_i \cap \theta_j|}{|A_i \cup \theta_j|} = \frac{|\theta_j|}{|A_i|} = \frac{1}{|A_i|}$, while $\theta_j \notin A_i$, $\frac{|A_i \cap \theta_j|}{|A_i \cup \theta_j|} = 0$. $c(m_1, m_2)$ is simplified as follows:

$$
\begin{aligned}
c(m_1, m_2) &= \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \\
&= \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(\theta_j) \frac{|A_i \cap \theta_j|}{|A_i \cup \theta_j|} \\
&= \mathrm{Cor}P(\theta_1) \sum_{i=1,\theta_2 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} + \cdots \mathrm{Cor}P(\theta_1) \sum_{i=1,\theta_n \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} \\
&= \left(\sum_{i=1,\theta_1 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|}, \ldots, \sum_{i=1,\theta_2 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} \right) \cdot (\mathrm{Cor}P(\theta_1), \ldots, \mathrm{Cor}P(\theta_n)),
\end{aligned}
$$

so $c(m_1, m_2)$ can be denoted as follows:

$$
c(m_1, m_2) = \vec{r} \cdot \vec{s}, \tag{11}
$$

where $\vec{r}$ and $\vec{s}$ are two sets as follows:

$$
\begin{aligned}
\vec{r} &= \left(\sum_{i=1,\theta_1 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|}, \ldots, \sum_{i=1,\theta_n \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} \right) \\
\vec{s} &= (\mathrm{Cor}P(\theta_1), \ldots, \mathrm{Cor}P(\theta_n)),
\end{aligned}
$$

$c(m_2, m_2)$ is simplified as follows:

$$
\begin{aligned}
c(m_2, m_2) &= \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_2(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{Cor}P(\theta_i) \mathrm{Cor}P(\theta_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \\
&= \mathrm{Cor}P(\theta_1)^2 + \cdots \mathrm{Cor}P(\theta_i)^2 + \cdots \mathrm{Cor}P(\theta_n)^2 \\
&= |\vec{s}|^2.
\end{aligned} \tag{12}
$$

Then substituting Equation (11) and Equation (12) into Equation (8), Equation (8) is finally denoted as follows:

$$
r_{\mathrm{BPA}}(m_1, m_2) = \frac{\vec{r} \cdot \vec{s}}{|\vec{s}|} \cdot \frac{1}{\sqrt{c(m_1, m_1)}}. \tag{13}
$$

Next we use the model of the angular cosine of two n-dimensional vectors to solve this optimization problem. Assume two n-dimensional vectors be $\vec{r_1} = (x_1, x_2, \ldots, x_n)$ and $\vec{s_1} = (y_1, y_2, \ldots, y_n)$; the angular cosine of two vectors can be denoted as

$$
\cos\alpha = \frac{\vec{r_1} \cdot \vec{s_1}}{|\vec{r_1}| \cdot |\vec{s_1}|}, \tag{14}
$$

when the value of $\cos\alpha$ is 1, which is the maximum value, the value of α is 0, so $\vec{r}$ and $\vec{s}$ are parallel:

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} = \cdots \frac{x_n}{y_n} = k_1, \tag{15}$$

where k_1 is a nonzero constant, specially if $x_i = 0$, then $y_i = 0$. And according to Equation (13), we can get Equation (16):

$$r_{\text{BPA}}(m_1, m_2) \cdot \sqrt{c(m_1, m_1)} = \frac{\vec{r} \cdot \vec{s}}{|\vec{s}|}. \tag{16}$$

When given a BPA m_1, vector $\vec{r}$, $|\vec{r}|$ and $c(m_1, m_1)$ can be calculated specifically, so $\vec{r}$, $|\vec{r}|$ and $c(m_1, m_1)$ are known values. In Equation (14), if the value of $\cos\beta$ takes the maximum value, $(\vec{r} \cdot \vec{s})/(|\vec{r}| \cdot |\vec{s}|)$ takes the maximum value because $|\vec{r}|$ is a certain value. Then in Equation (16), $r_{\text{BPA}}(m_1, m_2) \cdot \sqrt{c(m_1, m_1)}$ is the maximum value, at the same time, $c(m_1, m_1)$ is also a known value, so $r_{\text{BPA}}(m_1, m_2)$ takes the maximum value. In a word, when $\cos\beta$ takes the maximum value, $r_{\text{BPA}}(m_1, m_2)$ takes the maximum value. For another part, as mentioned above, if $\cos\beta$ takes the maximum value, then vector $\vec{r}$ and vector $\vec{s}$ are parallel:

$$\frac{\sum_{i=1,\theta_1 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|}}{\text{Cor}P(\theta_1)} = \cdots = \frac{\sum_{i=1,\theta_n \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|}}{\text{Cor}P(\theta_n)} = k. \tag{17}$$

According to Equation (17), Equation (18) can be obtained

$$\sum_{j=1}^{n} \sum_{i=1,\theta_j \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} = k(\text{Cor}P(\theta_1) + \text{Cor}P(\theta_2) + \cdots + \text{Cor}P(\theta_n)) \tag{18}$$

And we know that

$$\begin{cases} \sum_{j=1}^{n} \sum_{i=1,\theta_j \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} = 1 \\ \text{Cor}P(\theta_1) + \text{Cor}P(\theta_2) + \cdots + \text{Cor}P(\theta_n) = 1 \end{cases}. \tag{19}$$

So according to Equation (17) and Equation (19), we can see $k = 1$, so the final probability distribution based on the proposed method is obtained as follows:

$$\begin{cases} \text{Cor}P(\theta_1) = \sum_{i=1,\theta_1 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} \\ \text{Cor}P(\theta_2) = \sum_{i=1,\theta_2 \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|} \\ \vdots \\ \text{Cor}P(\theta_n) = \sum_{i=1,\theta_n \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|}. \end{cases} \tag{20}$$

This result can be written as follows:

$$\mathrm{Cor}P(\theta_j) = \sum_{i=1,\theta_j \in A_i}^{2^N} \frac{m_1(A_i)}{|A_i|}. \tag{21}$$

According to Equation (9), Equation (21) is finally obtained. Given a BPA, when using Equation (21) to transform BPA, the value of correlation coefficient between this BPA and transformed probability distribution is maximum. That is to say the proposed probability transformation method keeps the greatest relevance between probability and BPA. By analyzing Equation (21), it is easy to find that this transformation result is the same as PPT. Therefore, the proposed method shows the rationality and efficiency of PPT from a point of view of relevance and giving a new explanation to PPT.

4 | NUMERICAL EXAMPLES

In this section, examples are given to compare the values of correlation coefficient when using the proposed method and other three probability transformation methods, including PPT, plausibility transformation, and proportional probability transformation method. Assume a frame of discernment is $\Theta = \{1, 2, \ldots, 10\}$, and a BPA is given as follows:

$$m(\{3, 4, 5\}) = 0.15,\ m(\{6\}) = 0.05,\ m(\{\Theta\}) = 0.1,\ m(\{A\}) = 0.7.$$

This example shows 10 cases where subset A adds one additional element at a time, starting from case 1, with $A = 1$, and ending with case 10, when $A = \Theta$. The probability distribution resulting of the proposed method Cor$P$, PPT Bet$P$, plausibility transformation method PnPl, proportional probability transformation Pro$P$ and the values of corresponding correlation coefficient r_{BPA} in this example are shown in Tables 1 to 4, respectively. Figure 1 compares the corresponding correlation coefficient when using four probability transformation methods.

As shown in Tables 1 to 4 and Figure 1, the proposed method and PPT have the same results. And the proposed method has the maximal value of correlation coefficient. In case 1, the

TABLE 1 The results of the proposed method Cor$P(\{\cdot\})$

Case	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	$\mathbf{r_{BPA}}$
$A = 1$	0.71	0.01	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01	0.9723
$A = 1,2$	0.36	0.36	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01	0.6976
$A = 1,2,3$	0.2433	0.243	0.294	0.06	0.06	0.06	0.01	0.01	0.01	0.01	0.5903
$A = 1,..,4$	0.185	0.185	0.235	0.235	0.06	0.06	0.01	0.01	0.01	0.01	0.5256
$A = 1,..,5$	0.15	0.15	0.2	0.2	0.2	0.06	0.06	0.01	0.01	0.01	0.4773
$A = 1,..,6$	0.126	0.126	0.177	0.177	0.177	0.177	0.01	0.01	0.01	0.01	0.4625
$A = 1,..,7$	0.11	0.11	0.16	0.16	0.16	0.16	0.11	0.01	0.01	0.01	0.4355
$A = 1,..,8$	0.0975	0.0975	0.1475	0.1475	0.1475	0.1475	0.0975	0.0975	0.01	0.01	0.4129
$A = 1,..,9$	0.0877	0.0877	0.1378	0.1378	0.1378	0.1378	0.0878	0.0878	0.0878	0.01	0.3938
$A = 1,..,10$	0.08	0.08	0.13	0.13	0.13	0.13	0.08	0.08	0.08	0.08	0.3772

TABLE 2 The results of $\mathrm{Bet}P(\{\cdot\})$

Case	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	r_{BPA}
$A = 1$	0.71	0.01	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01	0.9723
$A = 1,2$	0.36	0.36	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01	0.6976
$A = 1,2,3$	0.2433	0.243	0.294	0.06	0.06	0.06	0.01	0.01	0.01	0.01	0.5903
$A = 1,..,4$	0.185	0.185	0.235	0.235	0.06	0.06	0.01	0.01	0.01	0.01	0.5256
$A = 1,..,5$	0.15	0.15	0.2	0.2	0.2	0.06	0.06	0.01	0.01	0.01	0.4773
$A = 1,..,6$	0.126	0.126	0.177	0.177	0.177	0.177	0.01	0.01	0.01	0.01	0.4625
$A = 1,..,7$	0.11	0.11	0.16	0.16	0.16	0.16	0.11	0.01	0.01	0.01	0.4355
$A = 1,..,8$	0.0975	0.0975	0.1475	0.1475	0.1475	0.1475	0.0975	0.0975	0.01	0.01	0.4129
$A = 1,..,9$	0.0877	0.0877	0.1378	0.1378	0.1378	0.1378	0.0878	0.0878	0.0878	0.01	0.3938
$A = 1,..,10$	0.08	0.08	0.13	0.13	0.13	0.13	0.08	0.08	0.08	0.08	0.3772

TABLE 3 The results of $\mathrm{PnPl}(\{\cdot\})$

Case	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	r_{BPA}
$A = 1$	0.3635	0.0455	0.1136	0.1136	0.1136	0.0682	0.0455	0.0455	0.0455	0.0455	0.8919
$A = 1,2$	0.2759	0.2759	0.0862	0.0862	0.0862	0.0516	0.0345	0.0345	0.0345	0.0345	0.6831
$A = 1,2,3$	0.2286	0.2286	0.2429	0.0713	0.0713	0.0429	0.0286	0.0286	0.0286	0.0286	0.5855
$A = 1,..,4$	0.1860	0.1860	0.2209	0.2209	0.0581	0.0366	0.0243	0.0243	0.0243	0.0243	0.5234
$A = 1,..,5$	0.16	0.16	0.19	0.19	0.19	0.03	0.02	0.02	0.02	0.02	0.4783
$A = 1,..,6$	0.1404	0.1404	0.1667	0.1667	0.1667	0.1491	0.0175	0.0175	0.0175	0.0175	0.4602
$A = 1,..,7$	0.126	0.126	0.1484	0.1484	0.1484	0.1328	0.125	0.0156	0.0156	0.0156	0.4324
$A = 1,..,8$	0.1127	0.1127	0.1338	0.1338	0.1338	0.1196	0.1127	0.1127	0.0141	0.0141	0.4092
$A = 1,..,9$	0.1026	0.1026	0.1218	0.1218	0.1218	0.1089	0.1026	0.1026	0.1026	0.0127	0.3891
$A = 1,..,10$	0.0941	0.0941	0.1118	0.1118	0.1118	0.1	0.0941	0.0941	0.0941	0.0941	0.3717

TABLE 4 The results of $\mathrm{Pro}P(\{\cdot\})$

Case	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	r_{BPA}
$A = 1$	0.7933	0	0.05	0.05	0.05	0.0567	0	0	0	0	0.9706
$A = 1,2$	0.35	0.35	0.05	0.05	0.05	0.15	0	0	0	0	0.6862
$A = 1,2,3$	0.2333	0.2333	0.2834	0.05	0.05	0.15	0	0	0	0	0.5780
$A = 1,..,4$	0.175	0.175	0.225	0.225	0.05	0.15	0	0	0	0	0.5130
$A = 1,..,5$	0.14	0.14	0.19	0.19	0.19	0.15	0	0	0	0	0.4768
$A = 1,..,6$	0.1666	0.1166	0.1667	0.1667	0.1667	0.2667	0	0	0	0	0.4189
$A = 1,..,7$	0	0	0.05	0.05	0.05	0.85	0	0	0	0	0.2411
$A = 1,..,8$	0	0	0.05	0.05	0.05	0.85	0	0	0	0	0.2014
$A = 1,..,9$	0	0	0.05	0.05	0.05	0.85	0	0	0	0	0.1876
$A = 1,..,10$	0	0	0.05	0.05	0.05	0.85	0	0	0	0	0.1763

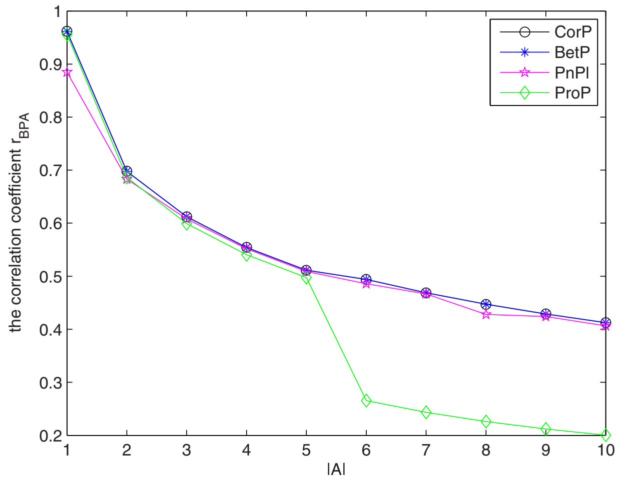

FIGURE 1 The correlation coefficient in different cases [Color figure can be viewed at wileyonlinelibrary.com]

proposed method, PPT, and proportional probability transformation have the correlation coefficient of belief functions with values close to 1, while plausibility transformation method has the correlation coefficient with value less than 0.9. In the rest of nine cases, the calculated values are close, but proportional probability transformation has smaller value of correlation coefficient than others. Especially, in case 6 to case 10, the calculated value of correlation coefficient by using the proportional probability transformation method is around 0.2, which indicates very small relevance between BPA and the probability distribution.

5 | CONCLUSION

In this paper, we focused on the probability transformation based on the correlation coefficient. In the proposed method, given a BPA, obtained probability distribution has the greatest correlation with original BPA. By mathematical derivation in the proposed method, an equation used to transform BPA to probability distribution is obtained. From the analytical equation, the obtained result is the same as PPT. That is to say, PPT keeps the greatest correlation between the obtained probability distribution and given BPA. Therefore, a new theoretical meaning for the classical probability transformation method PPT is found in this paper. This conclusion may further promote the application of PPT and expand the combination of PPT and other theories in the future, like Physarum-inspired network.[39]

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CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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