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Multi-sensor data fusion based on the belief divergence measure of evidences and the belief entropy

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ABSTRACT

Multi-sensor data fusion technology plays an important role in real applications. Because of the flexibility and effectiveness in modeling and processing the uncertain information regardless of prior probabilities, Dempster–Shafer evidence theory is widely applied in a variety of fields of information fusion. However, counter-intuitive results may come out when fusing the highly conflicting evidences. In order to deal with this problem, a novel method for multi-sensor data fusion based on a new belief divergence measure of evidences and the belief entropy was proposed. First, a new Belief Jensen–Shannon divergence is devised to measure the discrepancy and conflict degree between the evidences; then, the credibility degree can be obtained to represent the reliability of the evidences. Next, considering the uncertainties of the evidences, the information volume of the evidences are measured by making use of the belief entropy to indicate the relative importance of the evidences. Afterwards, the credibility degree of each evidence is modified by taking advantage of the quantitative information volume which will be utilized to obtain an appropriate weight in terms of each evidence. Ultimately, the final weights of the evidences are applied to adjust the bodies of the evidences before using the Dempster's combination rule. A numerical example is illustrated that the proposed method is feasible and effective in handling the conflicting evidences, where the belief value of target increases to 99.05%. Furthermore, an application in fault diagnosis is given to demonstrate the validity of the proposed method. The results show that the proposed method outperforms other related methods where the basic belief assignment (BBA) of the true target is 89.73%.

1. Introduction

Multi-sensor data fusion technology plays an important role in real applications, such as the risk analysis [1,2], fault diagnosis [3–6], wireless sensor networks [7–11], health prognosis [12], image processing [13], target tracking [14], and so on [15–18]. Because of the complexity of the targets, the data collected from a single sensor is not enough in decision making process. Additionally, by reason of the environment's impacts, like, sensor failure, bad weather conditions, shortage of energy supply, data communication problems, etc., the data gathered from multi-sensors could be unreliable or even incorrect so that it may make the wrong decision. Hence, multi-sensor data fusion technologies are widely applied in many fields of real applications [19,20]. Whereas, the imprecision and uncertainty are inevitable for the practical applications in the real world. It is still an open issue about how to model and handle these kinds of imprecise and uncertain information. To address this issue, a number of theories have been presented on multi-sensor data fusion, including the rough sets theory [21,22], fuzzy sets theory [23–30], evidence theory [31–35], Z numbers [36,37], D numbers theory [38–42], evidential reasoning [43–45], and so on [46–49].

Dempster–Shafer evidence theory, as an uncertainty reasoning method, was firstly proposed by Dempster [31] and had been developed by Shafer [32]. Dempster–Shafer evidence theory has many advantages; on the one hand, it has the possibility of expressing ignorance explicitly by allocating masses not only to the propositions consisting of single objects, but also to the unions of such objects; on the other hand, it can begin with complete ignorance and has the acceptance of an incomplete model without prior probabilities. Because of the flexibility and effectiveness in modelling both of the uncertainty and imprecision regardless of prior information, Dempster–Shafer evidence theory is widely applied in various fields of information fusion, such as decision making [50–55], pattern recognition [56–58], risk analysis [59,60], human reliability analysis [61], supplier selection [62], aphasia diagnosis [63], fault diagnosis [64–67], and so on [68,69]. Although Dempster–Shafer evidence theory has many advantages, the counter-intuitive results may be generated when fusing the highly conflicting evidences [70]. To solve this problem, a lot of methods have been

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developed which are mainly divided into two types [71–75]. The first type is to modify the Dempster's combination rule. The second type is to pre-process the bodies of evidences. The main research works focusing on the first type consist of Smets's unnormalized combination rule [76], Dubois and Prade's disjunctive combination rule [77], and Yager's combination rule [78]. Nevertheless, some good properties, like the commutativity and associativity are often destructed through modifying the combination rule. What's more, if the counter-intuitive results are caused by the sensor failure, such a modification would have no effect. Therefore, many research works are inclined to pre-process the bodies of evidences to resolve the problem of fusing the highly conflicting evidences. The main research works focusing on the second type include Murphy's simple average approach of the bodies of evidences [79], Deng et al.'s weighted average of the masses based on the evidence distance [80], Zhang et al.'s cosine theorem-based method [81], and Yuan et al.'s entropy-based method [82]. Deng et al.'s weighted average approach [80] overcame the weakness of Murphy's method [79] to some extent. Later on, Zhang et al. [81] made an improvement based on [80] and introduced the concept of vector space to handle the conflicting evidences. However, the effect of evidence itself on the weight was ignored. By taking this into account, Yuan et al. [82] introduced the belief entropy to express the effect of evidence itself. But, there is still some room for improvement to achieve more accurate fusing results.

In this paper, a new Belief Jensen–Shannon divergence is first proposed for measuring the distance between the bodies of the evidences. Based on that, a novel multi-sensor data fusion method is proposed by integrating the Belief Jensen–Shannon divergence with the belief entropy. The proposed method considers both of the credibility degree between the evidences and the uncertainty measure of the evidences on the weight, so that it can obtain a more appropriately weighted average evidence before using the Dempster's combination rule. Consequently, the proposed method consists of the following procedures. Firstly, in order to measure the credibility degree between the evidences, a new Belief Jensen–Shannon divergence is proposed which represents the reliability of the evidence. After that, the relative importance of the evidences are indicated by making advantage of the belief entropy to obtain the uncertainty measure of each evidence. Whereafter, the credibility degree of each evidence is modified which is regard as the final weight for each evidence. Based on that, the weighted average evidence can be obtained; then, it will be fused by using the Dempster's combination rule. A numerical example and an application in fault diagnosis are illustrated to demonstrate the rationality and effectiveness of the proposed method.

The rest of this paper is organized as follows. Section 2 briefly introduces the preliminaries of this paper. A new Belief Jensen–Shannon divergence is proposed for measuring the distance between the bodies of the evidences in Section 3. A novel multi-sensor data fusion method which is based on the belief divergence measure of evidences and the belief entropy is proposed in Section 4. Section 5 illustrates a numerical example to show the effectiveness of the proposed method. In Section 6, the proposed method is applied to an application in fault diagnosis. Finally, Section 7 gives a conclusion.

2. Preliminaries

2.1. Dempster–Shafer evidence theory

Dempster–Shafer evidence theory [31,32] is applied to deal with uncertain information, belonging to the category of artificial intelligence. Because of the flexibility and effectiveness in modelling both of the uncertainty and imprecision without prior information, Dempster–Shafer evidence theory requires more weaker conditions than the Bayesian theory of probability. When the probability is confirmed, Dempster–Shafer evidence theory could convert into Bayesian theory, so it is considered as an extension of the Bayesian theory. Dempster–Shafer evidence theory has the advantage that it can directly express the "uncertainty" by allocating the probability into the subsets of the set which consists multiple objects, rather than to an individual object. Furthermore, it is capable of combining the bodies of evidences to derive a new evidence. The basic concepts are introduced as below.

Definition 2.1. (Frame of discernment).

Let U be a set of mutually exclusive and collectively exhaustive events, indicted by

$$U = \{E_1, E_2, \ldots, E_i, \ldots, E_N\}. \tag{1}$$

The set U is called a frame of discernment. The power set of U is indicated by 2^U, where

$$2^U = \{\emptyset, \{E_1\}, \ldots, \{E_N\}, \{E_1, E_2\}, \ldots, \{E_1, E_2, \ldots, E_i\}, \ldots, U\}, \tag{2}$$

and $\emptyset$ is an empty set. If $A \in 2^U$, A is called a proposition.

Definition 2.2. (Mass function).

For a frame of discernment U, a mass function is a mapping m from 2^U to [0, 1], formally defined by

$$m: 2^U \to [0, 1], \tag{3}$$

which satisfies the following condition:

$$m(\emptyset) = 0 \textit{ and } \sum_{A \in 2^U} m(A) = 1. \tag{4}$$

In the Dempster–Shafer evidence theory, a mass function can be also called as a basic belief assignment (BBA). If $m(A)$ is greater than 0, A will be called as a focal element, and the union of all of the focal elements is called as the core of the mass function.

Definition 2.3. (Belief function).

For a proposition $A \subseteq U$, the belief function Bel: $2^U \to [0, 1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B). \tag{5}$$

The plausibility function Pl: $2^U \to [0, 1]$ is defined as

$$Pl(A) = 1 - Bel(\overline{A}) = \sum_{B \cap A \neq \emptyset} m(B), \tag{6}$$

where $\overline{A} = U - A$.

Apparently, $Pl(A)$ is equal or greater than $Bel(A)$, where the function Bel is the lower limit function of proposition A and the function Pl is the upper limit function of proposition A.

Definition 2.4. (Dempster's rule of combination).

Let two BBAs m_1 and m_2 on the frame of discernment U and assuming that these BBAs are independent, Dempster's rule of combination, denoted by $m = m_1 \oplus m_2$, which is called as the orthogonal sum, is defined as below:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B) m_2(C), & A \neq \emptyset, \\ 0, & A = \emptyset, \end{cases} \tag{7}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C), \tag{8}$$

where B and C are also the elements of 2^U, and K is a constant that presents the conflict between two BBAs.

Notice that, the Dempster's combination rule is only practicable for the two BBAs with the condition $K < 1$.

2.2. Jensen–Shannon divergence measure

Lin [83] introduced an information-theoretical based divergence measure between two or more probability distributions, called as Jensen–Shannon (JS) divergence. Unlike others divergence measures, the main property of JS divergence is that, it does not require the condition of absolute continuity for the probability distributions involved. JS divergence defines a true metric in the space of probability distributions - actually it is the square of a metric [84]. The main concepts are defined as below.

Definition 2.5. (The JS divergence between two probability distributions) [83,85].

Let us consider a discrete random variable X, and let $P_1 = \{p_{11}, p_{12}, \ldots, p_{1M}\}$ and $P_2 = \{p_{21}, p_{22}, \ldots, p_{2M}\}$ be two probability distributions for X. The JS divergence between the probability distributions P_1 and P_2 is denoted as:

$$JS(P_1, P_2) = \frac{1}{2}\left[S\left(P_1, \frac{P_1 + P_2}{2}\right) + S\left(P_2, \frac{P_1 + P_2}{2}\right)\right], \tag{9}$$

where $S(P_1, P_2) = \sum_i p_{1i} \log \frac{p_{1i}}{p_{2i}}$ $(i = 1, 2, \ldots, M)$ is the Kullback–Leibler divergence and $\sum_i p_{ji} = 1$ $(i = 1, 2, \ldots, M; j = 1, 2)$.

$JS(P_1, P_2)$ can be also expressed in the following form

$$\begin{aligned} JS(P_1, P_2) &= H\left(\frac{P_1 + P_2}{2}\right) - \frac{1}{2}H(P_1) - \frac{1}{2}H(P_2), \\ &= \frac{1}{2}\left[\sum_i p_{1i} \log\left(\frac{2p_{1i}}{p_{1i} + p_{2i}}\right) + \sum_i p_{2i} \log\left(\frac{2p_{2i}}{p_{1i} + p_{2i}}\right)\right], \end{aligned} \tag{10}$$

where $H(P_j) = -\sum_i p_{ji} \log p_{ji}$ $(i = 1, 2, \ldots, M; j = 1, 2)$ is the Shannon entropy.

There are some properties for the JS divergence:

(1) $JS(P_1, P_2)$ is symmetric and always well defined;
(2) $JS(P_1, P_2)$ is bounded, $0 \leq JS(P_1, P_2) \leq 1$;
(3) its square root, $\sqrt{JS(P_1, P_2)}$ verifies the triangle inequality.

2.3. Belief entropy

A novel belief entropy which is called as the Deng entropy is first proposed by Deng [51]. As the generalization of the Shannon entropy [86,87], the Deng entropy is an efficient method to measure the uncertain information. It can be used in evidence theory, in which the uncertain information is expressed by the BBA. In such a situation that the uncertainty is expressed by probability distribution, the uncertain degree that is measured by the Deng entropy will be the same as the uncertain degree that is measured by the Shannon entropy. The basic concepts are introduced below.

Let A_i be a hypothesis of the belief function m, $|A_i|$ is the cardinality of set A_i. Deng entropy E_d of set A_i is defined as follows:

$$E_d = -\sum_i m(A_i) \log \frac{m(A_i)}{2^{|A_i|} - 1}. \tag{11}$$

When the belief value is only allocated to the single element, Deng entropy degenerates to Shannon entropy, i.e.,

$$E_d = -\sum_i m(A_i) \log \frac{m(A_i)}{2^{|A_i|} - 1} = -\sum_i m(A_i) \log m(A_i). \tag{12}$$

The greater the cardinality of hypotheses is, the greater the Deng entropy of evidence is, so that the evidence contains more information. When an evidence has a big Deng entropy, it is supposed to be better supported by other evidences, which indicates that this evidence plays an important role in the final combination.

3. Belief divergence measure

In Dempster–Shafer evidence theory, how to measure the discrepancy and conflict among evidences is still an open issue that is critical for the fusion of evidences. Obviously, Dempster–Shafer evidence theory is a generalization of probability theory. By integrating the Dempster–Shafer evidence theory with above mentioned Jensen–Shannon divergence, a novel divergence measure named Belief Jensen–Shannon (BJS) divergence which is designed for the belief function is defined as below.

Definition 3.1. (The BJS divergence between two BBAs).

Let A_i be a hypothesis of the belief function m, and let m_1 and m_2 be two BBAs on the same frame of discernment Ω, containing N mutually exclusive and exhaustive hypotheses. The BJS divergence between the two BBAs m_1 and m_2 is denoted as:

$$BJS(m_1, m_2) = \frac{1}{2}\left[S\left(m_1, \frac{m_1 + m_2}{2}\right) + S\left(m_2, \frac{m_1 + m_2}{2}\right)\right], \tag{13}$$

where $S(m_1, m_2) = \sum_i m_1(A_i) \log \frac{m_1(A_i)}{m_2(A_i)}$ and $\sum_i m_j(A_i) = 1$ $(i = 1, 2, \ldots, M; j = 1, 2)$. $\sum_i m_j(A_i) = 1$ $(i = 1, 2, \ldots, M; j = 1, 2)$

$BJS(m_1, m_2)$ can be also expressed in the following form

$$\begin{aligned} BJS(m_1, m_2) &= H\left(\frac{m_1 + m_2}{2}\right) - \frac{1}{2}H(m_1) - \frac{1}{2}H(m_2), \\ &= \frac{1}{2}\left[\sum_i m_1(A_i) \log\left(\frac{2m_1(A_i)}{m_1(A_i) + m_2(A_i)}\right)\right. \\ &\quad \left. + \sum_i m_2(A_i) \log\left(\frac{2m_2(A_i)}{m_1(A_i) + m_2(A_i)}\right)\right], \end{aligned} \tag{14}$$

where $H(m_j) = -\sum_i m_j(A_i) \log m_j(A_i)$ $(i = 1, 2, \ldots, M; j = 1, 2)$ is the Shannon entropy.

It is obvious that the fraction value tends to infinity when the BBA assignment is zero and the value of its logarithm also tends to infinity. The proposed method will fail in this case, so a very small number 1×10^{-12} is used to replace zero value when the above case occurs. It has been proven that this will not affect the calculation results [88].

The Belief Jensen–Shannon divergence is similar with Jensen–Shannon divergence in form, however, the Belief Jensen–Shannon divergence utilizes the mass function by taking the place of probability distribution function. In such a situation that all of the belief function's hypothesis are assigned to the single elements, the BBA will turn into probability; the Belief Jensen–Shannon divergence degenerates to Jensen–Shannon divergence in this case.

The property can be inferred as below:

(1) $BJS(m_1, m_2)$ is symmetric and always well defined;
(2) $BJS(m_1, m_2)$ is bounded, $0 \leq BJS(m_1, m_2) \leq 1$;
(3) its square root, $\sqrt{BJS(m_1, m_2)}$ verifies the triangle inequality.

Example 1. Supposing that there are two BBAs m_1 and m_2 in the frame of discernment $\Omega = \{A, B, C\}$ which is complete, and the two BBAs are given as follows:

m_1: $m_1(A) = 0.6$, $m_1(B) = 0.2$, $m_1(C) = 0.2$;
m_2: $m_2(A) = 0.6$, $m_2(B) = 0.2$, $m_2(C) = 0.2$.

As shown in Example 1, it can be see that m_1 has the same BBAs as m_2, where $m_1(A) = m_2(A) = 0.6$, $m_1(B) = m_2(B) = 0.2$ and $m_1(C) = m_2(C) = 0.2$. Then, the specific calculation processes of Belief Jensen–Shannon divergence $BJS(m_1, m_2)$ are listed as follows:

$$
\begin{aligned}
BJS(m_1, m_2) = {} & \frac{1}{2} \times 0.6 \times log\left(\frac{2 \times 0.6}{0.6 + 0.6}\right) + \frac{1}{2} \times 0.6 \times log\left(\frac{2 \times 0.6}{0.6 + 0.6}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) = 0.
\end{aligned}
$$

This example verifies that when m_1 has the same BBAs as m_2, the Belief Jensen–Shannon divergence between m_1 and m_2 is 0 which accords with an intuitionistic result.

Example 2. Supposing that there are two BBAs m_1 and m_2 in the frame of discernment $\Omega = \{A, B, C\}$ which is complete, and the two BBAs are given as follows:

m_1: $m_1(A) = 0.6$, $m_1(B) = 0.2$, $m_1(C) = 0.2$;
m_2: $m_2(A) = 0.7$, $m_2(B) = 0.2$, $m_2(C) = 0.1$.

As shown in Example 2, we can notice that m_1 and m_2 have relatively large belief values to support the object A, where $m_1(A) = 0.6$ and $m_2(A) = 0.7$. The Belief Jensen–Shannon divergence between m_1 and m_2 $BJS(m_1, m_2)$ is calculated as follows:

$$
\begin{aligned}
BJS(m_1, m_2) = {} & \frac{1}{2} \times 0.6 \times log\left(\frac{2 \times 0.6}{0.6 + 0.7}\right) + \frac{1}{2} \times 0.7 \times log\left(\frac{2 \times 0.7}{0.6 + 0.7}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.1}\right) + \frac{1}{2} \times 0.1 \times log\left(\frac{2 \times 0.1}{0.2 + 0.1}\right) \\
& = 0.0150.
\end{aligned}
$$

On the other hand, the Belief Jensen–Shannon divergence between m_2 and m_1 $BJS(m_2, m_1)$ is produced below:

$$
\begin{aligned}
BJS(m_2, m_1) = {} & \frac{1}{2} \times 0.7 \times log\left(\frac{2 \times 0.7}{0.6 + 0.7}\right) + \frac{1}{2} \times 0.6 \times log\left(\frac{2 \times 0.6}{0.6 + 0.7}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.2}\right) \\
& + \frac{1}{2} \times 0.1 \times log\left(\frac{2 \times 0.1}{0.2 + 0.1}\right) + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.1}\right) = 0.0150.
\end{aligned}
$$

From the above results, it can be see that the Belief Jensen–Shannon divergence between m_1 and m_2 $BJS(m_1, m_2)$ is equal to the divergence measure between m_2 and m_1 $BJS(m_2, m_1)$.

Consequently, the symmetric property of Belief Jensen–Shannon divergence measure method is verified in this example.

Example 3. Supposing that there are three BBAs m_1, m_2 and m_3 in the frame of discernment $\Omega = \{A, B\}$ which is complete, and the three BBAs are given as follows:

m_1: $m_1(A) = 0.99$, $m_1(B) = 0.01$;
m_2: $m_2(A) = 0.90$, $m_2(B) = 0.10$;
m_3: $m_3(A) = 0.01$, $m_3(B) = 0.99$.

As shown in Example 3, we can see that m_1 and m_2 have great belief values to support the object A, where $m_1(A) = 0.99$ and $m_2(A) = 0.90$. On the contrary, m_3 has a great belief value to support the object B, where $m_3(B) = 0.99$. The Belief Jensen–Shannon divergence between m_1 and m_2 $BJS(m_1, m_2)$ is calculated below:

$$
\begin{aligned}
BJS(m_1, m_2) = {} & \frac{1}{2} \times 0.99 \times log\left(\frac{2 \times 0.99}{0.99 + 0.90}\right) + \frac{1}{2} \times 0.90 \times log\left(\frac{2 \times 0.90}{0.99 + 0.90}\right) \\
& + \frac{1}{2} \times 0.01 \times log\left(\frac{2 \times 0.01}{0.01 + 0.11}\right) + \frac{1}{2} \times 0.11 \times log\left(\frac{2 \times 0.11}{0.01 + 0.11}\right) = 0.0324.
\end{aligned}
$$

On the other hand, the Belief Jensen–Shannon divergence between m_2 and m_3 $BJS(m_2, m_3)$ is computed as follows:

$$
\begin{aligned}
BJS(m_2, m_3) = {} & \frac{1}{2} \times 0.90 \times log\left(\frac{2 \times 0.90}{0.90 + 0.01}\right) + \frac{1}{2} \times 0.01 \times log\left(\frac{2 \times 0.01}{0.90 + 0.01}\right) \\
& + \frac{1}{2} \times 0.10 \times log\left(\frac{2 \times 0.10}{0.10 + 0.99}\right) + \frac{1}{2} \times 0.99 \times log\left(\frac{2 \times 0.99}{0.10 + 0.99}\right) = 0.7193.
\end{aligned}
$$

Moreover, the Belief Jensen–Shannon divergence between m_1 and m_3 $BJS(m_1, m_3)$ is computed as follows:

$$
\begin{aligned}
BJS(m_1, m_3) = {} & \frac{1}{2} \times 0.99 \times log\left(\frac{2 \times 0.99}{0.99 + 0.01}\right) + \frac{1}{2} \times 0.01 \times log\left(\frac{2 \times 0.01}{0.99 + 0.01}\right) \\
& + \frac{1}{2} \times 0.01 \times log\left(\frac{2 \times 0.01}{0.01 + 0.99}\right) + \frac{1}{2} \times 0.99 \times log\left(\frac{2 \times 0.99}{0.01 + 0.99}\right) = 0.9192.
\end{aligned}
$$

After that, their corresponding square root values can be calculated as follows:

$\sqrt{BJS(m_1, m_2)} = \sqrt{0.0324} = 0.1799$;
$\sqrt{BJS(m_2, m_3)} = \sqrt{0.7193} = 0.8481$;
$\sqrt{BJS(m_1, m_3)} = \sqrt{0.9192} = 0.9588$.

It can be noticed that $\sqrt{BJS(m_1, m_2)} + \sqrt{BJS(m_2, m_3)} = 1.0280$, so that $\sqrt{BJS(m_1, m_3)} < \sqrt{BJS(m_1, m_2)} + \sqrt{BJS(m_2, m_3)}$ which satisfies the triangle inequality property of Belief Jensen–Shannon divergence measure method.

Example 4. Supposing that there are two BBAs m_1 and m_2 in the frame of discernment $\Omega = \{A, B, C\}$ which is complete, and the two BBAs are given as follows:

m_1: $m_1(A) = 0.5$, $m_1(B) = 0.1$, $m_1(C) = 0.2$, $m_1(A, B, C) = 0.2$;
m_2: $m_2(A) = 0.6$, $m_2(B) = 0.2$, $m_2(C) = 0.1$, $m_2(A, B, C) = 0.1$.

As shown in Example 4, it can be see that m_1 and m_2 have belief values $m_1(A) = 0.5$ and $m_2(A) = 0.6$ supporting the object A, while they also have a BBA with multiple objects, where $m_1(A, B, C) = 0.2$ and $m_2(A, B, C) = 0.1$. The specific calculation processes of Belief Jensen–-Shannon divergence $BJS(m_1, m_2)$ are given as follows:

$$
\begin{aligned}
BJS(m_1, m_2) = {} & \frac{1}{2} \times 0.5 \times log\left(\frac{2 \times 0.5}{0.5 + 0.6}\right) + \frac{1}{2} \times 0.6 \times log\left(\frac{2 \times 0.6}{0.5 + 0.6}\right) \\
& + \frac{1}{2} \times 0.1 \times log\left(\frac{2 \times 0.1}{0.1 + 0.2}\right) + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.1 + 0.2}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.1}\right) + \frac{1}{2} \times 0.1 \times log\left(\frac{2 \times 0.1}{0.2 + 0.1}\right) \\
& + \frac{1}{2} \times 0.2 \times log\left(\frac{2 \times 0.2}{0.2 + 0.1}\right) + \frac{1}{2} \times 0.1 \times log\left(\frac{2 \times 0.1}{0.2 + 0.1}\right) \\
= {} & 0.0693.
\end{aligned}
$$

Example 5. Supposing that there are two BBAs m_1 and m_2 in the frame of discernment $\Omega = \{A, B\}$ which is complete, and the two BBAs are given as follows:

m_1: $m_1(A) = \alpha$, $m_1(B) = 1 - \alpha$;
m_2: $m_2(A) = 0.9999$, $m_2(B) = 0.0001$.

As shown in Example 5, m_2 has a great belief value to support the object A, where $m_2(A) = 0.9999$. As the parameter α changes from [0, 1], the variation of Belief Jensen–Shannon divergence measure between m_1 and m_2 is depicted in Fig. 1.

It is obvious that as α tends to 1, the Belief Jensen–Shannon divergence between m_1 and m_2 is going to 0. It explains the phenomenon intuitively where m_1 and m_2 are almost the same at this situation with a great belief value that supports the object A as the target.

In the case that when α is close to 0, the Belief Jensen–Shannon divergence measure between m_1 and m_2 is going to 1. This elucidates the phenomenon intuitively where m_1 and m_2 are completely different. To be specific, m_1 has a great belief value that supports the object B as the target, while m_2 has a great belief value that supports the object A as the target.

In a word, the bounded property of Belief Jensen–Shannon divergence measure method [0, 1] is verified in this example.

4. The proposed method

In this paper, a new multi-sensor data fusion approach is presented. The proposed method is based on the Belief Jensen–Shannon

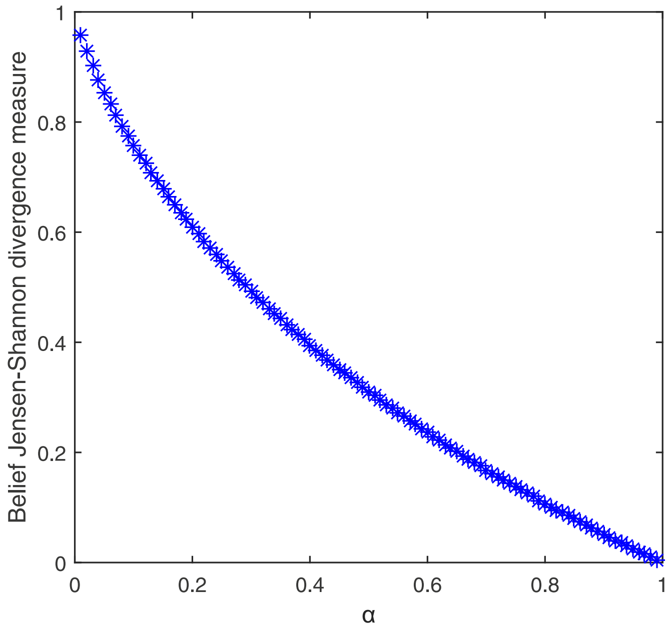


Fig. 1. An example of Belief Jensen-Shannon divergence measure with changing parameter α.

divergence measure of evidences and the belief entropy which consists of the following parts. The Belief Jensen–Shannon (BJS) divergence is first devised to measure the discrepancy and conflict degree among the evidences; then the credibility degree deriving from the BJS divergence measure is obtained to denote the reliability of the evidences. When an evidence is well supported by other evidences, it is supposed to less conflict with other evidences so that a big weight should be allocated to this evidence. Instead, when an evidence is poorly supported by other evidences, it is supposed to highly conflict with other evidences so that a small weight should be allocated to this evidence. Next, the information volume of the evidence is calculated by making use of the belief entropy to express the uncertainties of the evidences. Whereafter, the credibility degree of the evidence is modified by taking advantage of the information volume of the evidences which is considered as the final weight. At last, the final weights of the evidences are applied to adjust the body of the evidences before using the Dempster's combination rule. The flowchart of the proposed method is shown in Fig. 2.

4.1. Calculate the credibility degree of the evidences

Step 1-1: By making use of the Belief Jensen–Shannon divergence measure Eq. (13), the distance measure between the bodies of evidences m_i $(i = 1, 2, \ldots, k)$ and m_j $(j = 1, 2, \ldots, k)$, denoted as BJS_{ij} $(i \neq j)$ can be obtained; a Belief Jensen–Shannon divergence measure matrix, namely, a distance measure matrix $DMM = (BJS_{ij})_{k\times k}$ can be constructed as follows:

$$DMM = \begin{bmatrix} 0 & \cdots & BJS_{1i} & \cdots & BJS_{1k} \\ \vdots & \cdots & \vdots & \vdots & \vdots \\ BJS_{i1} & \cdots & 0 & \cdots & BJS_{ik} \\ \vdots & \cdots & \vdots & \vdots & \vdots \\ BJS_{k1} & \cdots & BJS_{ki} & \cdots & 0 \end{bmatrix}. \tag{15}$$

Step 1-2: The average evidence distance $\widetilde{BJS}_i$ of the body of evidence m_i can be calculated by

$$\widetilde{BJS}_i = \frac{\sum_{j=1, j\neq i}^{k} BJS_{ij}}{k-1}, \quad 1 \leq i \leq k; 1 \leq j \leq k. \tag{16}$$

Step 1-3: The support degree Sup_i of the body of evidence m_i is defined as follows:

$$Sup_i = \frac{1}{\widetilde{BJS}_i}, \quad 1 \leq i \leq k. \tag{17}$$

Step 1-4: The credibility degree Crd_i of the body of the evidence m_i is defined as follows:

$$Crd_i = \frac{Sup(m_i)}{\sum_{s=1}^{k} Sup(m_s)}, \quad 1 \leq i \leq k. \tag{18}$$

4.2. Measure the information volume of the evidences

Step 2-1: The belief entropy of the evidence m_i $(i = 1, 2, \ldots, k)$ is calculated by leveraging Eq. (11).

Step 2-2: In order to avoid allocating zero weight to the evidences in some cases, we use the information volume IV_i to measure the uncertainty of the evidence m_i as below:

$$IV_i = e^{E_d} = e^{-\sum_i m(A_i) \log \frac{m(A_i)}{2^{|A_i|}-1}}, \quad 1 \leq i \leq k. \tag{19}$$

Step 2-3: The information volume of the evidence m_i is normalized as below, which is denoted as $\widetilde{IV}_i$:

$$\widetilde{IV}_i = \frac{IV_i}{\sum_{s=1}^{k} IV_s}, \quad 1 \leq i \leq k. \tag{20}$$

4.3. Generate and fuse the weighted average evidence

Step 3-1: Based on the information volume $\widetilde{IV}_i$, the credibility degree Crd_i of the evidence m_i will be adjusted, denoted as $ACrd_i$:

$$ACrd_i = Crd_i \times \widetilde{IV}_i, \quad 1 \leq i \leq k. \tag{21}$$

Step 3-2: The adjusted credibility degree which is denoted as $\widetilde{ACrd}_i$ is normalized that is considered as the final weight in terms of each evidence m_i:

$$\widetilde{ACrd}_i = \frac{ACrd_i}{\sum_{s=1}^{k} ACrd_s}, \quad 1 \leq i \leq k. \tag{22}$$

Step 3-3: On account of the final weight $\widetilde{ACrd}_i$ of each evidence m_i, the weighted average evidence $WAE(m)$ will be obtained as follows:

$$WAE(m) = \sum_{i=1}^{k} (\widetilde{ACrd}_i \times m_i), \quad 1 \leq i \leq k. \tag{23}$$

Step 3-4: The weighted average evidence $WAE(m)$ is fused via the Dempster's combination rule Eq. (7) by $k-1$ times, if there are k number of evidences. Then, the final combination result of multi-evidences can be obtained.

5. Experiment

In this section, in order to demonstrate the effectiveness of the proposed method, a numerical example is illustrated.

5.1. Problem statement

Example 6. Consider a multi-sensor-based target recognition problem associated with the sensor reports that are collected from five different

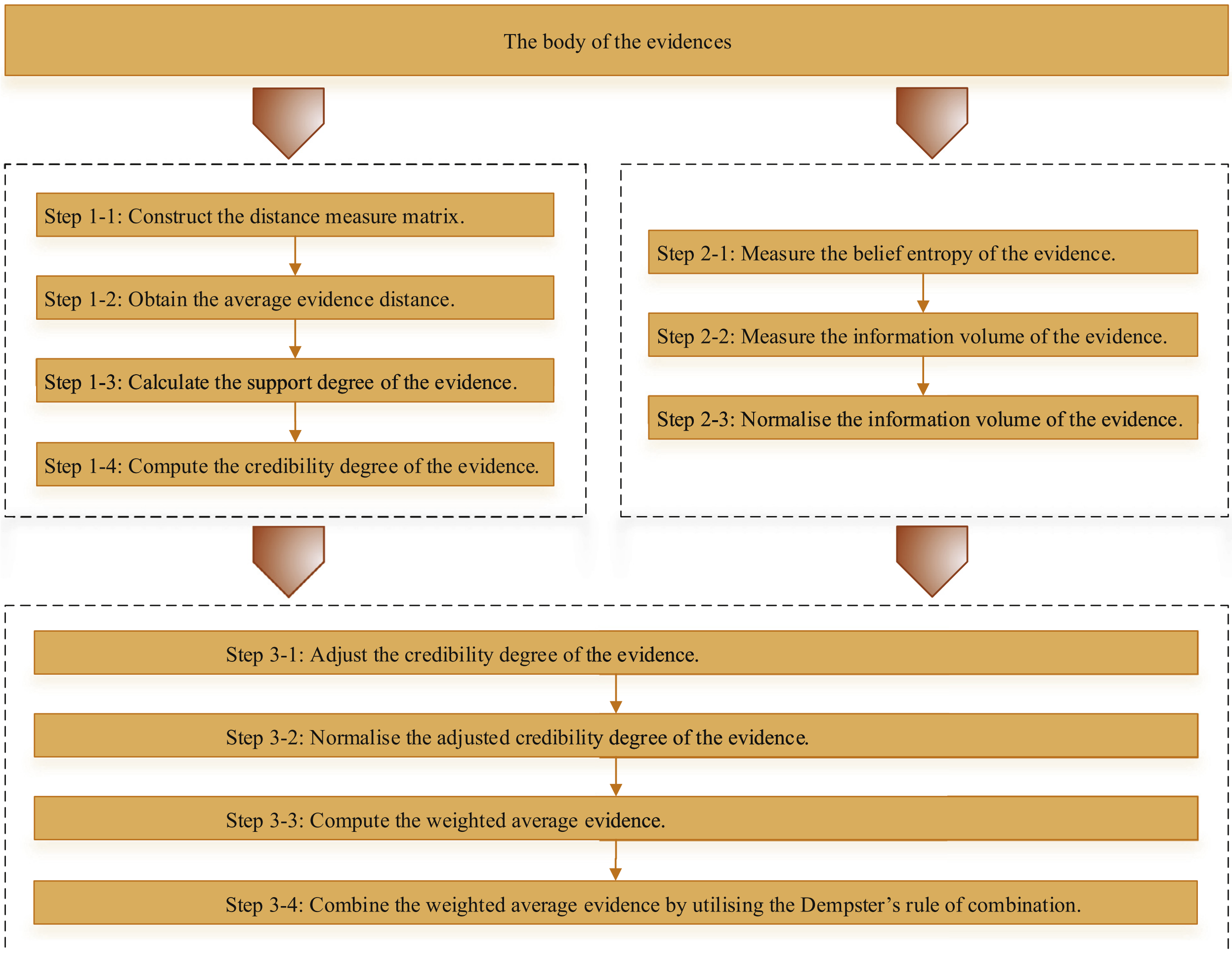


Fig. 2. The flowchart of the proposed method.

types of sensors. These sensor reports which are modeled as the BBAs are given in Table 1 from [80], where the frame of discernment Θ that consists of three potential objects is given by $\Theta = \{A, B, C\}$.

5.2. Implementation based on the proposed method

Step 1: Construct the distance measure matrix $DMM = (BJS_{ij})_{k\times k}$ as follows:

$$DMM = \begin{pmatrix} 0 & 0.3611 & 0.3877 & 0.3672 & 0.3478 \\ 0.3611 & 0 & 0.8186 & 0.7655 & 0.7655 \\ 0.3877 & 0.8186 & 0 & 0.0022 & 0.0034 \\ 0.3672 & 0.7655 & 0.0022 & 0 & 0.0022 \\ 0.3478 & 0.7655 & 0.0034 & 0.0022 & 0 \end{pmatrix}.$$

Step 2: Obtain the average evidence distance $\widetilde{BJS}_i$ of the evidence m_i as follows:

$\widetilde{BJS}_1 = 0.3659,$
$\widetilde{BJS}_2 = 0.6777,$
$\widetilde{BJS}_3 = 0.3030,$
$\widetilde{BJS}_4 = 0.2843,$
$\widetilde{BJS}_5 = 0.2797.$

Step 3: Calculate the support degree of the evidence m_i as below:

$Sup_1 = 2.7326,$
$Sup_2 = 1.4756,$
$Sup_3 = 3.3005,$
$Sup_4 = 3.5177,$
$Sup_5 = 3.5749.$

Step 4: Compute the credibility degree of the evidence m_i as follows:

$Crd_1 = 0.1872,$
$Crd_2 = 0.1011,$
$Crd_3 = 0.2260,$
$Crd_4 = 0.2409,$
$Crd_5 = 0.2448.$

Step 5: Measure the belief entropy of the evidence m_i as below:

$E_{d1} = 1.5664,$
$E_{d2} = 0.4690,$
$E_{d3} = 1.8092,$
$E_{d4} = 1.8914,$
$E_{d5} = 1.7710.$

Step 6: Measure the information volume of the evidence m_i as below:

$IV_1 = 4.7894,$
$IV_2 = 1.5984,$
$IV_3 = 6.1056,$
$IV_4 = 6.6286,$
$IV_5 = 5.8767.$

Step 7: Normalise the information volume of the evidence m_i as follows:

Table 1
The BBAs for a multi-sensor-based target recognition.

BBA	$\{A\}$	$\{B\}$	$\{C\}$	$\{A, C\}$
S_1: $m_1(\cdot)$	0.41	0.29	0.30	0.00
S_2: $m_2(\cdot)$	0.00	0.90	0.10	0.00
S_3: $m_3(\cdot)$	0.58	0.07	0.00	0.35
S_4: $m_4(\cdot)$	0.55	0.10	0.00	0.35
S_5: $m_5(\cdot)$	0.60	0.10	0.00	0.30

$\widetilde{IV}_1 = 0.1916,$
$\widetilde{IV}_2 = 0.0639,$
$\widetilde{IV}_3 = 0.2442,$
$\widetilde{IV}_4 = 0.2652,$
$\widetilde{IV}_5 = 0.2351.$

Step 8: Adjust the credibility degree of the evidence m_i based on the information volume of the evidence as below:
$ACrd_1 = 0.0359,$
$ACrd_2 = 0.0065,$
$ACrd_3 = 0.0552,$
$ACrd_4 = 0.0639,$
$ACrd_5 = 0.0576.$

Step 9: Normalise the adjusted credibility degree of the evidence m_i as below:
$\widetilde{A}Crd_1 = 0.1638,$
$\widetilde{A}Crd_2 = 0.0295,$
$\widetilde{A}Crd_3 = 0.2521,$
$\widetilde{A}Crd_4 = 0.2917,$
$\widetilde{A}Crd_5 = 0.2629.$

Step 10: Compute the weighted average evidence as follows:
$m(\{A\}) = 0.5316,$
$m(\{B\}) = 0.1472,$
$m(\{C\}) = 0.0521,$
$m(\{A, C\}) = 0.2692.$

Step 11: Combine the weighted average evidence via the Dempster's rule of combination with 4 times, and the fusing results are shown in Table 2 and Fig. 3.

5.3. Discussion

From Example 6, we can notice that the evidence m_2 highly conflicts with other evidences. The fusing results that are obtained by different combination approaches are presented in Table 2. The comparisons of the BBA of the target A based on different combination rules are shown in Fig. 3.

As shown in Table 2, Dempster's combination rule generates counter-intuitive result and recognizes the object C as the target, even though the other four evidences support the target A. Whereas, Murphy's method [79], Deng et al.'s method [80], Zhang et al.'s method [81], Yuan et al. [82] and the proposed method present reasonable results and recognize the target A. Additionally, the proposed method is more efficient in dealing with the conflicting evidences with the highest belief (99.05%) as shown in Fig. 3. The reason is that the proposed method not only makes use of the function of Belief Jensen–Shannon divergence to obtain the credibility degree of the evidences, but also considers the uncertainty of the evidences by adopting the belief entropy to measure the information volume among the evidences. After considering the above aspects, the reliable evidence's weight is increased while unreliable evidence's weight is decreased, so that its negative effect was relieved on the final fusing results than other methods.

6. Application

In this section, the proposed method is applied to a case study on fault diagnosis of machines, where the data in [65] is used for the comparison with the related method.

6.1. Problem statement

Supposing that the frame of discernment Θ which consists of three types of faults for the machines is given by $\Theta = \{F_1, F_2, F_3\}$. The set of sensors given by $S = \{S_1, S_2, S_3\}$ are positioned on different places for gathering the reports. The collected sensor reports which are modeled as BBAs are provided in Table 3, where $m_1(\cdot)$, $m_2(\cdot)$ and $m_3(\cdot)$ represent the BBAs reported from the three sensors S_1, S_2 and S_3, respectively.

Table 2
Combination results of the evidences in terms of different combination rules.

Method	$\{A\}$	$\{B\}$	$\{C\}$	$\{AC\}$	Target
Dempster [31]	0	0.1422	0.8578	0	C
Murphy [79]	0.9620	0.0210	0.0138	0.0032	A
Deng et al. [80]	0.9820	0.0039	0.0107	0.0034	A
Zhang et al. [81]	0.9820	0.0034	0.0115	0.0032	A
Yuan et al. [82]	0.9886	0.0002	0.0072	0.0039	A
Proposed method	0.9905	0.0002	0.0061	0.0043	A

6.2. Fault diagnosis based on the proposed method

Step 1: Construct the distance measure matrix $DMM = (BJS_{ij})_{k\times k}$ as follows:

$$DMM = \begin{pmatrix} 0 & 0.4398 & 0.0150 \\ 0.4398 & 0 & 0.4722 \\ 0.0150 & 0.4722 & 0 \end{pmatrix}.$$

Step 2: Obtain the average evidence distance $\widetilde{BJS}_i$ of the evidence m_i as follows:
$\widetilde{BJS}_1 = 0.2274,$
$\widetilde{BJS}_2 = 0.4560,$
$\widetilde{BJS}_3 = 0.2436.$

Step 3: Calculate the support degree of the evidence m_i as below:
$Sup_1 = 4.3976,$
$Sup_2 = 2.1932,$
$Sup_3 = 4.1052.$

Step 4: Compute the credibility degree of the evidence m_i as follows:
$Crd_1 = 0.4111,$
$Crd_2 = 0.2050,$
$Crd_3 = 0.3838.$

Step 5: Measure the belief entropy of the evidence m_i as below:
$E_{d1} = 2.2909,$
$E_{d2} = 1.3819,$
$E_{d3} = 1.7960.$

Step 6: Measure the information volume of the evidence m_i as below:
$IV_1 = 9.8838,$
$IV_2 = 3.9825,$
$IV_3 = 6.0255.$

Step 7: Normalise the information volume of the evidence m_i as follows:
$\widetilde{IV}_1 = 0.4969,$
$\widetilde{IV}_2 = 0.2002,$
$\widetilde{IV}_3 = 0.3029.$

Step 8: Adjust the credibility degree of the evidence m_i based on the information volume of the evidence as below which denotes the dynamic reliability of the sensor report:
$w(DR)_1 = ACrd_1 = 0.2043,$
$w(DR)_2 = ACrd_2 = 0.0411,$
$w(DR)_3 = ACrd_3 = 0.1163.$

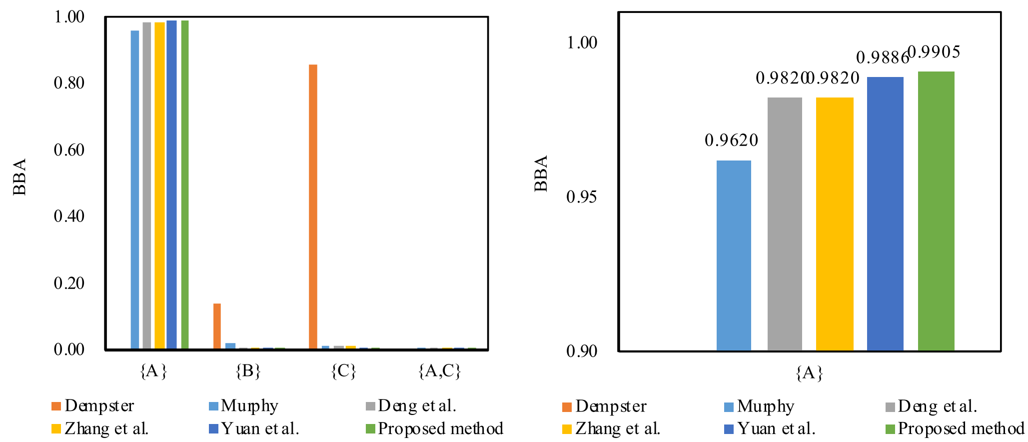
Step 9: Acquire the parameters in the fault diagnosis application given in Table 4 from [65] in terms of the sufficiency index $\mu(m)$ and importance index $\nu(m)$ of the evidences; the static reliability of the evidences can be calculated by the following formula as:

$$w(SR)_i = \mu_i \times \nu_i, \quad 1 \le i \le k. \tag{24}$$

$w(SR)_1 = 1.0000, w(SR)_2 = 0.2040, w(SR)_3 = 1.0000.$

Step 10: Compute the final weight of the evidence m_i on basis of the static reliability and the dynamic reliability of the evidences as follows:
$w_1 = w(DR)_1 \times w(SR)_1 = 0.2043,$
$w_2 = w(DR)_2 \times w(SR)_2 = 0.0084,$
$w_3 = w(DR)_3 \times w(SR)_3 = 0.1163.$

Step 11: Normalise the final weight of the evidence m_i as below:
$\widetilde{w}_1 = 0.6211,$
$\widetilde{w}_2 = 0.0255,$



(a) BBAs for different objectives (b) BBAs for target A

Fig. 3. The comparison of BBAs generated by different methods in Example 6.

Table 3
The collected sensor reports modeled as BBAs in the fault diagnosis problem.

BBA	$\{F_1\}$	$\{F_2\}$	$\{F_2, F_3\}$	$\{F_1, F_2, F_3\}$
S_1: $m_1(\cdot)$	0.60	0.10	0.10	0.20
S_2: $m_2(\cdot)$	0.05	0.80	0.05	0.10
S_3: $m_3(\cdot)$	0.70	0.10	0.10	0.10

$\widetilde{w}_3 = 0.3535$.

Step 12: Compute the weighted average evidence as follows:
$m(\{F_1\}) = 0.6213$,
$m(\{F_2\}) = 0.1178$,
$m(\{F_2, F_3\}) = 0.0987$,
$m(\{F_1, F_2, F_3\}) = 0.1621$.

Step 13: Combine the weighted average evidence via the Dempster's rule of combination with 2 times, and the fusing results are shown in Table 5 and Fig. 4.

6.3. Discussion

As shown in Table 5, the proposed method can diagnose the fault type F_1, which is consistent with Fan and Zuo's method [65] and Yuan et al.'s method [66]. Even facing the conflicting sensor reports m_2, Fan and Zuo's method, Yuan et al.'s method and the proposed method can well manage the conflicting evidences. Whereas, the Dempster's rule of combination method [31] cannot handle the conflicting evidences very well and comes to the wrong result that the fault type is $m\{F_2\}$. Additionally, the proposed method has the highest belief degree on fault type F_1 (89.73%) which is higher than Fan and Zuo's method and Yuan et al.'s method as shown in Fig. 4. This is because that the distance measure of the proposed method is based on the proposed Belief Jensen–Shannon divergence measure, while the method in the work by Yuan et al. is based on the Jousselme's distance function. Furthermore, the proposed method takes the uncertainty of the sensor reports into account by making use of the belief entropy, so that it outperforms Fan and Zuo's method. As a results, these reasons contribute to the effectiveness and superiority of the proposed method.

Table 4
Parameters in the fault diagnosis application.

Evidence	m_1	m_2	m_3
Sufficiency index $\mu(m)$	1.00	0.60	1.00
Importance index $v(m)$	1.00	0.34	1.00

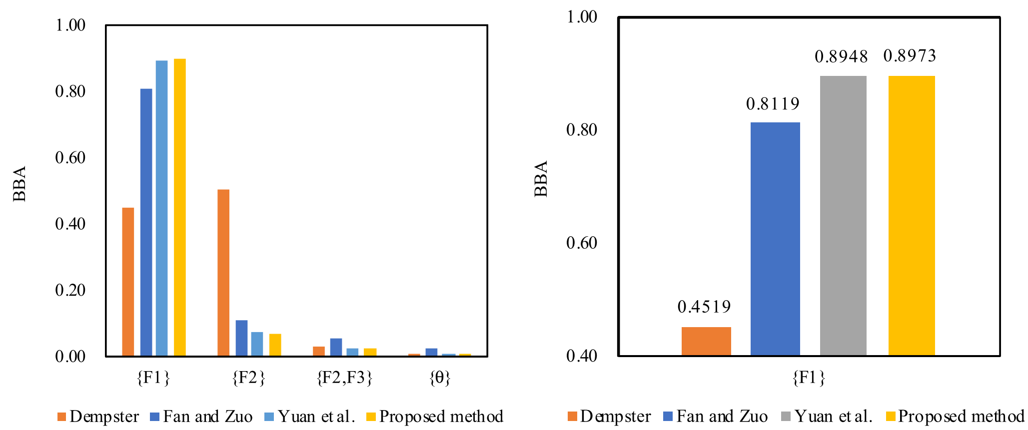
Table 5
Fusion results in terms of different combination rules for fault diagnosis.

Method	$\{F_1\}$	$\{F_2\}$	$\{F_2, F_3\}$	$\{\Theta\}$	Target
Dempster [31]	0.4519	0.5048	0.0336	0.0096	F_2
Fan and Zuo's method [65]	0.8119	0.1096	0.0526	0.0259	F_1
Yuan et al. [66]	0.8948	0.0739	0.0241	0.0072	F_1
Proposed method	0.8973	0.0688	0.0254	0.0080	F_1

7. Conclusion

In this paper, by considering both of the credibility degree between the evidences and the effect of the uncertainty of evidences on the weight, a novel method for multi-sensor data fusion based on the presented Belief Jensen–Shannon divergence and the belief entropy was proposed. The proposed method consisted of three main procedures. Firstly, a new Belief Jensen–Shannon divergence was proposed for measuring the distance between the bodies of the evidences; then, the credibility degree of the evidences were calculated to represent the reliability of the evidences. Secondly, the information volume of the evidences were generated for indicating the relative importance of the evidences. Thirdly, based on the first two processes, the final weight of the evidences was computed which was used to produce the weighted average evidence; it could be fused by applying the Dempster's combination rule. Finally, a numerical example was illustrated that the proposed method was more effective and feasible than other related methods to handle the conflicting evidence combination problem under multi-sensor environment. In addition, an application in fault diagnosis were presented to demonstrate that the proposed method could diagnose the faults more accurate.

In the near future work, we intend to develop a generalized BJS divergence measure method to make it more applicable and efficient to fit the practical applications. Especially, considering those applications where different weights are assigned to decision makers, how can we develop an improved generalized BJS divergence measure method and apply it in reality will be investigated in the near future.



(a) BBAs for different fault types (b) BBAs for fault type F_1

Fig. 4. The comparison of BBAs generated by different methods for fault diagnosis.

Conflict of Interest

The author states that there are no conflicts of interest.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.inffus.2018.04.003

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