MATH 1210 – Chapter 2 Review

2.2 (32) $\lim\limits_{x\to 1} \dfrac{x^3-1}{\sqrt{x}-1}\left[\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right] = \dfrac{\cancel{(x-1)}(x^2+x+1)(\sqrt{x}+1)}{\cancel{(x-1)}} = (3)(2) = \boxed{6}$

2.3 (19) $\lim\limits_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} = \dfrac{\frac{x+4}{4x}}{\frac{4+x}{1}} = \dfrac{\cancel{(x+4)}}{4x}\left(\dfrac{1}{\cancel{4+x}}\right) = \boxed{-\dfrac{1}{16}}$

2.5 (23) $\lim\limits_{x\to\infty} \dfrac{x^3+5x}{2x^3-x^2+4} = \boxed{\dfrac{1}{2}}$ (24) $\lim\limits_{t\to-\infty} \dfrac{t^2+2}{t^3+t^2-1} = \boxed{0}$

(26) $\lim\limits_{x\to\infty} \dfrac{x+2}{\sqrt{9x^2+1}} = \lim\limits_{x\to\infty} \dfrac{x+2}{3x} = \boxed{\dfrac{1}{3}}$ (37) $\lim\limits_{x\to\infty} \dfrac{x+x^3+x^5}{1-x^2+x^4} = \boxed{+\infty}$

(43) $\lim\limits_{x\to-\infty} \dfrac{\left(\sqrt{x^2+x+1}+x\right)}{1}\left[\dfrac{\sqrt{x^2+x+1}-x}{\sqrt{x^2+x+1}-x}\right] = \dfrac{x^2+x+1-x^2}{\sqrt{x^2+x+1}-x}\ \lim\limits_{x\to-\infty} \dfrac{x+1}{\underbrace{\sqrt{x^2}}_{|x| = -x}-x} = \dfrac{x+1}{-2x} = \boxed{-\dfrac{1}{2}}$

2.6 (5) $\boxed{y = 4x-3x^2 \quad (2,-4)}$

$m_{TAN} = \lim\limits_{x\to 2} \dfrac{[4x-3x^2]-(-4)}{x-2} = \lim\limits_{x\to 2} \dfrac{-(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = -8 \Rightarrow y+4 = -8(x-2)$ OR $\boxed{y=-8x+12}$

(20) $f(x)$ & Tangent Line pass through $(4,3) \Rightarrow \boxed{f(4)=3}$

$m_{TAN} = \dfrac{\Delta y}{\Delta x} = \dfrac{3-2}{4-0} = \dfrac{1}{4} \Rightarrow \boxed{f'(4) = \dfrac{1}{4}}$

(34) $\lim\limits_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h}$

Compare to $f'(a) = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

$\Rightarrow f(x) = \sqrt[4]{x}$ and $a = 16$

2.R (1) a) i) 3 ii) 0 iii) DNE iv) 2 v) $+\infty$ vi) $-\infty$ vii) 4 viii) -1

b) $y=-1$ & $y=4$ c) $x=0$ & $x=2$ d) Jump: $x=-3$, Infinite: $x=0$ & $x=2$, Removable: $x=4$

(5) $f(x) = \dfrac{x^2-9}{x^2+2x-3} = \dfrac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)} \Rightarrow \lim\limits_{x\to-3} f(x) = \dfrac{-6}{-4} = \boxed{\dfrac{3}{2}}$

(6) $\lim\limits_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3} = \dfrac{-8}{1^+ + 2 - 3} = \boxed{-\infty}$

(10) $\lim\limits_{v\to 4^+} \dfrac{4-v}{|4-v|} = \lim\limits_{v\to 4^+} \dfrac{\cancel{(4-v)}}{-\cancel{(4-v)}} = \boxed{-1}$

(12) $f(x) = \dfrac{\sqrt{x+6}-x}{x^3-3x^2}\left[\dfrac{\sqrt{x+6}+x}{\sqrt{x+6}+x}\right] = \dfrac{x+6-x^2}{x^2(x-3)[\sqrt{x+6}+x]} = \dfrac{-\cancel{(x-3)}(x+2)}{x^2\cancel{(x-3)}[\sqrt{x+6}+x]}$

$\Rightarrow \lim\limits_{x\to 3} f(x) = \dfrac{-5}{9(3+3)} = \boxed{-\dfrac{5}{54}}$

(13) $\lim\limits_{x\to\pi^-} \ln[\sin x] = \ln[0^+] = \boxed{-\infty}$

(21) $\underbrace{2x-1}_{g(x)} \le f(x) \le \underbrace{x^2}_{h(x)}$ $\lim\limits_{x\to 1} g(x) = 1$ & $\lim\limits_{x\to 1} h(x) = 1 \Rightarrow 1 \le \lim\limits_{x\to 1} f(x) \le 1 \Rightarrow \boxed{\lim\limits_{x\to 1} f(x) = 1}$ SQUEEZE THEOREM

(26) Show $e^{-x^2} = x$ has a root on $(0,1)$

let $f(x) = e^{-x^2}-x$

$f(0) = 1$ & $f(1) = \frac{1}{e}-1 \approx -.632$

Since $-.632 < 0 < 1$ and $f(x)$ is continuous on $\mathbb{R}$, The Int Value Thrm guarantees $0<c<1$ such that $f(c)=0$.

2.R 27 a)i) $s = 1+2t+\frac{1}{4}t^2$ $[1,3]$ | $V_{ave} = \dfrac{s(3)-s(1)}{3-1} = \dfrac{9\frac{1}{4} - 3\frac{1}{4}}{2} = \boxed{3}$

37) $\boxed{f(x)=\sqrt{3-5x}}$ a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{3-5(x+h)}-\sqrt{3-5x}}{h}\left[\dfrac{\sqrt{3-5x-5h}+\sqrt{3-5x}}{\sqrt{3-5x-5h}+\sqrt{3-5x}}\right] = \dfrac{3-5x-5h-(3-5x)}{h\left[\sqrt{3-5x-5h}+\sqrt{3-5x}\right]}$

$= \lim\limits_{h\to 0} \dfrac{-5\cancel{h}}{\cancel{h}\left[\sqrt{3-5x-5h}+\sqrt{3-5x}\right]} = \boxed{\dfrac{-5}{2\sqrt{3-5x}}}$

Domain of $f(x)$: $x \le 3/5$

Domain of $f'(x)$: $x < 3/5$

39)

Point where NOT DIFFERENTIABLE:	$x=-4$	$x=-1$	$x=2$	$x=5$
Reason why:	Jump Discontinuity	Cusp	Infinite Discontinuity	Vertical Graph

44)

Interval:	$(-3,-2)$	$(-2,-1)$	$(-1,0)$	$(0,1)$	$(1,2)$	$(2,3)$	$(3,4)$	$(4,5)$	$(5,8)$
Sign of f':	$-$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
Sign of f'':	$+$	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$-$
Description of graph of f:	Decreasing Concave UP	Incr Conc UP	Incr Conc DOWN	Decr Conc DOWN	Decr Conc UP	Decr Conc DOWN	Decr Conc UP	Incr Conc UP	Incr Conc DOWN