2.6 $\boxed{1}$ $m = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ $\boxed{2}$ $m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ $\boxed{3}$ $v(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

$\boxed{5}$ $f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ $\boxed{4}$ $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

(29) $\boxed{f(t) = \frac{2t+1}{t+3}}$ $\boxed{4}$ $f'(a) = \lim_{h \to 0} \left[\frac{\frac{2(a+h)+1}{a+h+3} - \frac{2a+1}{a+3}}{h} \right]$, $DQ = \frac{\frac{2a^2+6a+2ah+6h+a+3 - (2a^2+a+2ah+h+6a+3)}{(a+h+3)(a+3)}}{h}$

$DQ = \frac{5\cancel{h}}{(a+h+3)(a+3)\cancel{h}}$ $\lim_{h \to 0} [DQ] = \boxed{\frac{5}{(a+3)^2}} = f'(a)$

(31) $f(x) = \sqrt{1-2x}$ $\boxed{5}$ $\lim_{x \to a} \left(\frac{\sqrt{1-2x} - \sqrt{1-2a}}{x-a} \right) \frac{\sqrt{\ } + \sqrt{\ }}{\sqrt{\ } + \sqrt{\ }} = \frac{\cancel{1-2x-(1-2a)}}{\cancel{(x-a)}\left[\sqrt{1-2x}+\sqrt{1-2a}\right]}$ DQ $-2\cancel{(x-a)}$

$\lim_{x \to a} [DQ] = \frac{-2}{\sqrt{1-2a}+\sqrt{1-2a}} = \frac{-2}{2\sqrt{1-2a}} = \boxed{-\frac{1}{\sqrt{1-2a}}}$

$f'(a)$

(5) $\boxed{y = 4x - 3x^2 \quad (2, -4)}$ $a=2$ $f(a)=-4$

$\boxed{1}$ $m = \lim_{x \to 2} \left[\frac{(4x-3x^2)-(-4)}{x-2} \right]$ $DQ = \frac{-(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$ $\lim_{x \to 2} \left[-(3x+2) \right] = \boxed{-8}$

$y - y_1 = m(x-x_1) \Rightarrow y+4 = -8(x-2) \Rightarrow \boxed{y = -8x+12}$

(13) $\boxed{y = 40t - 16t^2 \quad \text{Find } v(2)} \to a=2$

$\boxed{3}$ $v(2) = \lim_{h \to 0} \left\{ \frac{[40(2+h)-16(2+h)^2] - [80-64]}{h} \right\}$ $DQ = \frac{80+40h-64-64h-16h^2-16}{h}$

$= \frac{-16h^2-24h}{h} = -16h-24$

$v(2) = \lim_{h \to 0} [-16h-24] = \boxed{-24 \ \frac{ft}{s}}$

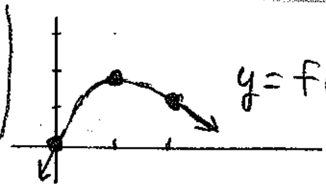
(15) $\boxed{s = \frac{1}{t^2}}$ $\boxed{3}$ $v(a) = \lim_{h \to 0} \left[\frac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h} \right]$ $DQ = \frac{\frac{a^2-(a^2+2ah+h^2)}{(a+h)^2 a^2}}{h} = \frac{\cancel{-2ah-h^2}\ -h(2a+h)}{h(a+h)^2 a^2}$ $\frac{-(2a+h)}{(a+h)^2 a^2}$

$\lim_{h \to 0} \left[\frac{-(2a+h)}{(a+h)^2 a^2} \right] = \frac{-2a}{a^4} = \boxed{\frac{-2}{a^3}}$

Therefore

$v(1) = -2 \ \frac{m}{s}$ $v(2) = -\frac{1}{4} \ \frac{m}{s}$ $v(3) = -\frac{2}{27} \ \frac{m}{s}$

(21) $f(0)=0 \quad f'(0)=3 \quad f'(1)=0 \quad f'(2)=-1$



$y=f(x)$

(39) $f(t) = 100 + 50t - 4.9t^2$

Find velocity and speed when $t=5$

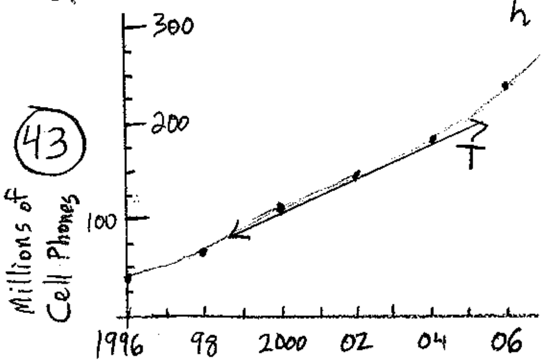
NOTE: SPEED $= |v(t)|$ (See pg 140 in book)

$\boxed{3}$ $$v(5) = \lim_{h\to 0}\left\{\frac{[100+50(5+h)-4.9(5+h)^2]-[100+50(5)-4.9(5)^2]}{h}\right\}$$

$$DQ = \frac{100+250+50h-122.5-49h-4.9h^2 - 227.5}{h} = 1-4.9h \Rightarrow \lim_{h\to 0}[1-4.9h] = 1 \ \text{m/s}$$

$$\text{Speed} = |1| = 1 \ \text{m/s}$$

(43)



$$m_{SEC} = \frac{182-141}{2004-2002} = 20.5$$

$$m_{SEC} = \frac{141-109}{2002-2000} = 16.0$$

$$\text{Ave} = 18.25 \ \frac{\text{millions of cell phones}}{\text{Year}}$$

$\approx$ Instantaneous rate of change for year 2002 (Slope of Tangent Line).

(45) $\boxed{C(x) = 5000 + 10x + .05x^2}$

x	100	105	101
$C(x)$	6500	6601.25	6520.05

Average Rate of Change

$$\frac{6601.25-6500}{105-100} = 20.25 \ \frac{\$}{\text{Unit}} \qquad \frac{6520.05-6500}{101-100} = 20.05 \ \frac{\$}{\text{unit}}$$

$\boxed{4}$ $$f'(100) = \lim_{h\to 0}\left\{\frac{[5000+10(100+h)+.05(100+h)^2]-[5000+10(100)+.05(100)^2]}{h}\right\}$$

$$DQ = \frac{10h+10h+.05h^2}{h} = 20+.05h \qquad \lim_{h\to 0}[20+.05h] = 20 \ \frac{\$}{\text{Unit}}$$

MARGINAL COST

2.7 "The Derivative as a function"

$$\boxed{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

OTHER NOTATIONS of DERIVATIVES

$$f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}[f(x)]$$ (Pg 150 in Book)

THEOREM $\boxed{4}$

If f is differentiable at a, Then f is continuous at a.

The CONTRAPOSITIVE is even more useful:

If f is NOT continuous at a, Then f is NOT differentiable at a.

Functions from PHYSICS

DIFFERENTIATING

$s(t) =$ POSITION

$v(t) = s'(t)$ VELOCITY

$a(t) = v'(t) = s''(t)$ ACCELERATION

$j(t) = a'(t) = v''(t) = s'''(t)$ JERK