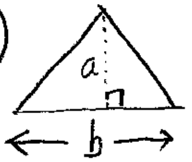
Review for Chapter 4 Exam

4.1 ⑲



$\frac{da}{dt} = 1\,\frac{cm}{min}$ $\frac{dA}{dt} = 2\,\frac{cm^2}{min}$ Find $\frac{db}{dt}$ when $a = 10cm$, $A = 100cm^2$

$b = 20$ cm

$$A = \frac{1}{2}ba \xrightarrow{\frac{d}{dt}} \frac{dA}{dt} = \frac{1}{2}a\frac{db}{dt} + \frac{1}{2}b\frac{da}{dt}$$

$$2 = \frac{1}{2}(10)\frac{db}{dt} + \frac{1}{2}(20)(1) \Rightarrow \frac{db}{dt} = \boxed{-\frac{8}{5}\ \frac{cm}{min} \quad -1.6}$$

4.3 ⑭ $f(x) = x^2 \ln x$ Concave Up?

$$f'(x) = 2x\ln x + \left(\frac{1}{x}\right)x^2 = \underline{2x\ln x + x}$$

$$f''(x) = 2\ln x + \left(\frac{1}{\cancel{x}}\right)\underset{2}{(\cancel{2x})} + 1 = \underline{2\ln x + 3} = 0$$

$\ln x = -\frac{3}{2}$

$-\ \ |\ \ +$ f''

$e^{-3/2}$

$f(x)$ is Concave Up on the interval $(e^{-3/2}, \infty)$

4.5 ㊶ $\lim_{x \to 0} (1-2x)^{1/x} = 1^{\infty}$

$$y = (1-2x)^{1/x}$$

$$\ln y = \frac{1}{x}\ln(1-2x) = \frac{\ln(1-2x)}{x}$$

$$\lim_{x \to 0} \frac{\ln(1-2x)}{x} = \frac{0}{0} \xrightarrow{L} \lim_{x \to 0}\left[\frac{\frac{-2}{1-2x}}{1}\right] = \boxed{-2}$$

If $\lim_{x \to 0}[\ln y] = -2$

Then $\lim_{x \to 0}[y] = \boxed{e^{-2}}$

4.6 ㊼

# TVs x	Demand p(x)
1000	\$ 450
1100	440
1200	430

$\boxed{p(x) = -\frac{1}{10}x + 550}$

GOAL: Maximize PROFIT

$$P = R - C$$

$$R = x \cdot p$$

$\boxed{C(x) = 68{,}000 + 150x}$ $\boxed{R = -\frac{1}{10}x^2 + 550x}$

$$P = -\frac{1}{10}x^2 + 400x - 68{,}000$$

$$P' = -\frac{1}{5}x + 400$$

$+\ \ |\ \ -$ P'

2000

PRICE? $p(2000) = \$350$

\$100 Rebate Maximizes Profit

4.8 ⑫ $f(x) = 3e^x + 7\sec^2 x$

$F(x) = 3e^x + 7\tan x + C$

㉗ $f'(t) = 2\cos t + \sec^2 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2} \qquad f\left(\frac{\pi}{3}\right) = 4$

I.C.

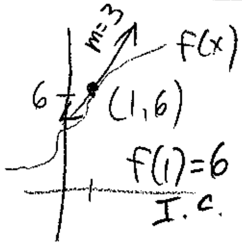
$f(t) = 2\sin t + \tan t + C$

I.C.: $f\left(\frac{\pi}{3}\right) = 4 = 2\sin\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right) + C$

$4 = \sqrt{3} + \sqrt{3} + C \Rightarrow \boxed{C = 4 - 2\sqrt{3}}$

$f(t) = 2\sin t + \tan t + 4 - 2\sqrt{3}$

㊲



$m_{TAN} = 2x + 1 = f'(x)$

$x^2 + x + C = f(x)$

$(1)^2 + (1) + C = 6$

$C = 4$

$f(x) = x^2 + x + 4$

$f(2) = 2^2 + 2 + 4 = \boxed{10}$

4.R ② $f(x) = x(1-x)^{1/2} \quad [-1, 1]$

Absolute Extrema?

$f'(x) = 1 \cdot (1-x)^{1/2} + \left[-\frac{1}{2}(1-x)^{-1/2}\right]x = \frac{1}{2}(1-x)^{-\frac{1}{2}}\left[\cancel{2(1-x)' - x}\right]$

$2 - 3x$

$\overset{+}{}\,|\,\overset{-}{} \quad f'$

$1/2$

x	$f(x)$
-1	$-\sqrt{2}$
$\frac{2}{3}$	$\frac{2}{3\sqrt{3}}$
1	0

$f\left(\frac{2}{3}\right) = \frac{2}{3}\left(\frac{1}{3}\right)^{1/2}$

Abs MIN $f(-1) = -\sqrt{2}$

Abs MAX $f\left(\frac{2}{3}\right) = \frac{2}{3\sqrt{3}}$ OR $\frac{2\sqrt{3}}{9}$

(54) $f'(u) = \frac{u^2 + \sqrt{u}}{u}$

$f(1) = 3$

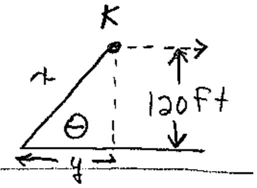
$= u + u^{-1/2}$

$\Rightarrow f(u) = \frac{1}{2}u^2 + 2u^{1/2} + C$

$\longrightarrow f(u) = \frac{1}{2} + 2 + C = 3 \Rightarrow C = \frac{1}{2}$

$f(u) = \frac{1}{2}u^2 + 2\sqrt{u} + \frac{1}{2}$ OR $\frac{u^2 + 4\sqrt{u} + 1}{2}$

8 ADDITIONAL PROBLEMS (Not from Textbook)



① A kite is moving horizontally 120 ft above the ground at a speed of $12 \frac{ft}{s}$. At what rate is θ decreasing when $80\sqrt{3}$ ft of string has been let out?

$\tan\theta = \frac{120}{y} \Rightarrow y \tan\theta = 120 \Rightarrow \tan\theta \frac{dy}{dt} + y \sec^2\theta \frac{d\theta}{dt} = 0$

When $x = 80\sqrt{3}$, $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°$ $\quad \tan 60°(12) + (40\sqrt{3})\sec^2 60° \frac{d\theta}{dt} = 0$

$6\sqrt{3} + 160\sqrt{3} \frac{d\theta}{dt} = 0 \Rightarrow \boxed{\frac{d\theta}{dt} = -\frac{3}{80} \frac{Rad}{s}}$

② $f'(x) = (x+4)^3(x-1)^4(x-5)^5$ when is $f(x)$ Increasing?

$+ \quad | \quad - \quad | \quad - \quad | \quad +$

$-4 \qquad 1 \qquad 5$

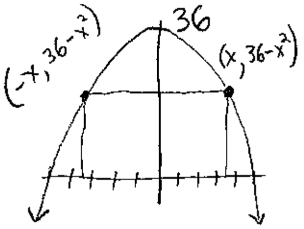
f is increasing on the intervals $(-\infty, -4) \cup (5, \infty)$

③ $\boxed{\text{Find } \lim_{x \to 4} \left[\frac{x^a - 4^a}{x^b - 4^b}\right]} = \frac{0}{0} \xrightarrow{L} \lim_{x \to 4} \frac{a x^{a-1}}{b x^{b-1}} = \frac{a 4^{a-1}}{b 4^{b-1}} = \frac{a}{b} 4^{a-b}$

④ $\boxed{f'(x) \text{ is continuous. } f(3) = 2. \ f'(3) = 5. \text{ Evaluate } \lim_{x \to 0} \frac{f(3+7x) - f(3+4x)}{x}} = \frac{f(3) - f(3)}{0} = \frac{0}{0}$

$L \rightarrow \lim_{x \to 0} \frac{7 f'(3+7x) - 4 f'(3+4x)}{1} = \frac{7(5) - 4(5)}{1} = 15$

⑤ Find the dimensions of the largest rectangle that has its base on the x-axis and its other two vertices above the x-axis and lying on the graph of $y = 36 - x^2$



$A = 2xy = 2x(36 - x^2) = 72x - 2x^3$

$A'(x) = 72 - 6x^2$

$A'(x) = 0 \Rightarrow 6x^2 = 72$

$x^2 = 12$

$x = 2\sqrt{3}$

$+ \quad | \quad - \quad A'(x)$

$2\sqrt{3}$

Base $= 2x = 4\sqrt{3}$

Height $= y = 36 - x^2 = 24$

Area $= 96\sqrt{3}$

⑥ The AVERAGE COST of producing x units of a commodity is $c(x) = 45 - .009x$. Find the MARGINAL COST at a production level of 1200 units.

$c(x) = \frac{C(x)}{x} = 45 - .009x \Rightarrow C(x) = 45x - .009x^2 \Rightarrow C'(x) = 45 - .018x \Rightarrow C'(1200) = 23.4 \frac{\$}{\text{unit}}$

⑦ The line $y = 5x - 8$ is tangent to the curve $y = f(x)$ when $x = 6$. Newton's method is used to locate a root of the equation $y = f(x) = 0$ with an initial approximation $x_1 = 4$. Find the second approximation.

$x_2 = 4 - \frac{12}{5} = \frac{8}{5}$ or 1.6

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

⑧ Use Newton's Method to find x_3 if $f(x) = x^3 + 40$ with $x_1 = -4$

$f'(x) = 3x^2$

$x_2 = -4 - \frac{-24}{48} = -3.5$ or $-\frac{7}{2}$

$x_3 = -\frac{7}{2} - \frac{-23/8}{\frac{147}{4}} = -\frac{7}{2} + \frac{23}{294} = \frac{-1029 + 23}{294} = \frac{-1006}{294} = -\frac{503}{147} \approx -3.4217687$