a) $m(x) = x$ is Murderer

$f(x,y) = x$ is friends with y

$w(x,y) = x$ was with y at the time of murder

$a = $ Arthur

$b = $ Bertram

$c = $ Carleton

$v = $ Victor

1) $m(a) \lor m(b) \lor m(c)$

2) $\left(\neg(m(a) \land m(b))\right) \land \neg \cdot \left(m(a) \land m(c)\right) \land \neg\left(m(b) \land m(c)\right)$

3) $\neg m(a) \rightarrow f(b,v)$

4) $\neg m(a) \rightarrow \neg f(c,v)$

5) $\neg m(b) \rightarrow \neg w(b,v)$

6) $\neg m(b) \rightarrow \neg f(b,v)$

7) $\neg m(c) \rightarrow w(a,v)$

8) $\neg m(c) \rightarrow w(b,v)$

b)
1) $(m(a), m(b), m(c))$
2) $(\neg m(a), \neg m(b))$ ⟵
3) $(\neg m(a), \neg m(c))$ ⟵
4) $(\neg m(c), \neg m(d))$
5) $(m(a), f(b,v))$ ⟵
6) $(m(a), \neg f(c,v))$
7) $(m(b), \neg f(b,v))$
8) $(m(b), \neg w(b,v))$ ⟵
9) $(m(c), w(a,v))$
10) $(m(c), w(b,v))$

$(\neg m(a), m(a))$

$(\neg m(a), m(b))$

$(m(a), m(b))$

$(m(b), m(c))$

c) $\exists x\, m(x)$ There is atleast one murder
   $(m(x))$ clausal form

11) $(\neg m(x), x)$
12) $R[5b, 7b]\ (m(a), m(b))$
13) $R[8b, 10b]\ (m(b), m(c))$
14) $R[13b, 3b]\ (\neg m(a), m(b))$
15) $R[14b, 12a]\ (m(b))$
16) $R[15, 11a]\ \{x=b\}\ (b)$

d) remove:

$\neg(m(a) \land m(b)) \land \neg(m(a) \land m(c)) \land \neg(m(b) \land m(c))$

So remove $(\neg m(a), \neg m(b))$
$\quad\quad\quad\quad (\neg m(a), \neg m(c))$
$\quad\quad\quad\quad (\neg m(b), \neg m(c))$

$I = \langle D, \Phi, \Psi, \nu \rangle_{new}$

$\neg D = \{a, b, c, \nu\}$

$\Phi = $ for every $x \in D$.

phi → $\Psi(m) = \{a, c\}$

$\Psi(f) = \{\}$

$(\Psi(\omega) = \{\}$

↑
psy

1) $(m(a), m(b), m(c))$
2) $(m(a), f(b,\nu))$ ←
3) $(m(b), \neg f(c,\nu))$
4) $(m(b), \neg f(b,\nu))$
5) $(m(b), \neg \omega(b,\nu))$
6) $(m(c), \omega(a,\nu))$
7) $(m(c), \omega(b,\nu))$

∵ $I \models m(a)$ ∴ I satisfy (1), (2), (3)

∵ $I \models m(c)$ ∴ I satisfy (6), (7)

∵ $I \nvDash \neg f(b,\nu)$ ∴ I satisfy (4)

∵ $I \nvDash \neg \omega(b,\nu)$ ∴ I satisfy (5)