Times Series Multivariate Analysis of the Federal Funds Rate

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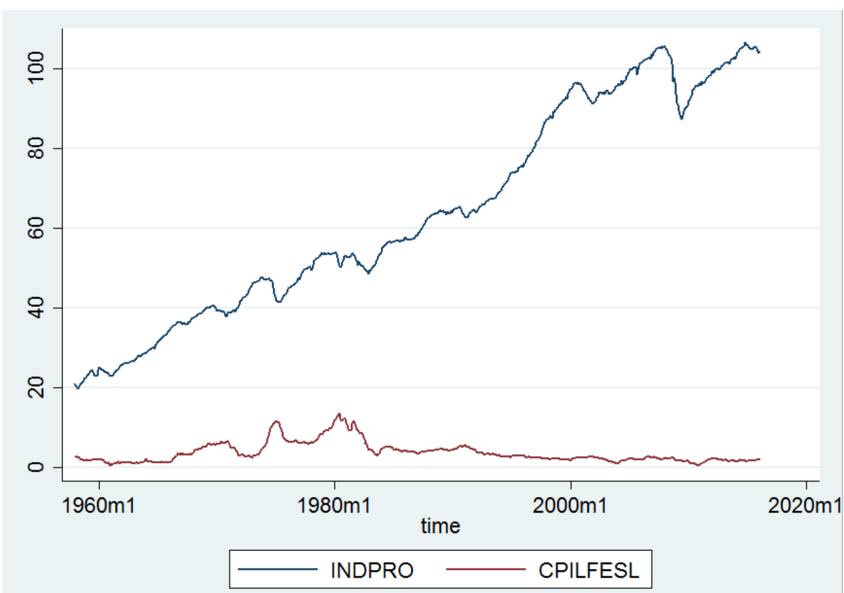
University of the Pacific, School of Engineering and Computer Science
In fulfilment of the requirements for the degree of Master of Science in Analytics

Data Sources:
(https://fred.stlouisfed.org/series/FEDFUNDS), Federal Funds rate
(http://research.stlouisfed.org/fred2/series/CPILFESL), Consumer Price Index
(http://research.stlouisfed.org/fred2/series/UNRATE), US unemployment rate
(https://fred.stlouisfed.org/series/INDPRO), US Industrial Production rate

Section 1: Multivariate analysis

1. The federal funds rate is the overnight interest on loans between banks. It is the key interest rate that our Federal Reserve uses to control the economy.

. twoway (line ip time) (line infl time), name(IP_INFL, replace)



FFNDS VARIABLE
. dfuller ffnds
MacKinnon approximate p-value for Z(t) = 0.3444
. gen ffdif=d.ffnds
. dfuller ffdif
MacKinnon approximate p-value for Z(t) = 0.0000

IP VARIABALE

. dfuller ip
MacKinnon approximate p-value for Z(t) = 0.8774
. gen gy=d.ip
(1 missing value generated)
. dfuller gy
MacKinnon approximate p-value for Z(t) = 0.0000

INFL VARIABLE
. dfuller infl
MacKinnon approximate p-value for Z(t) = 0.6328
. gen pidif=d.infl
. dfuller pidif
MacKinnon approximate p-value for Z(t) = 0.0000

UNRATE VARIABLE
. dfuller unrate
MacKinnon approximate p-value for Z(t) = 0.5438
. gen udif=d.unrate
. dfuller udif
MacKinnon approximate p-value for Z(t) = 0.0000

INDEPENDANT VARIABLES ARE NOW STATIONARY AT GROWTH RATES

REGRESSING FFDIF WITH LAGS OF FFDIF

```
. reg ffdif l(1,2,4).ffdif

      Source |       SS           df       MS      Number of obs   =       692
-------------+----------------------------------   F(3, 688)       =     50.24
       Model |  34.1186427         3  11.3728809   Prob > F        =    0.0000
    Residual |  155.739262       688  .226365206   R-squared       =    0.1797
-------------+----------------------------------   Adj R-squared   =    0.1761
       Total |  189.857904       691  .274758183   Root MSE        =    .47578

------------------------------------------------------------------------------
       ffdif |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ffdif |
         L1. |   .4414178   .0375491    11.76   0.000     .3676932    .5151424
         L2. |  -.1635806   .0374285    -4.37   0.000    -.2370685   -.0900928
         L4. |  -.0852319   .0345793    -2.46   0.014    -.1531254   -.0173383
             |
       _cons |   -.002179   .0180866    -0.12   0.904    -.0376905    .0333326
------------------------------------------------------------------------------
. estat durbinalt

Durbin's alternative test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          0.101               1                   0.7508
---------------------------------------------------------------------------
                        H0: no serial correlation
```

Growth rate of federal funds is significantly explained by its lags 1,2, and 4, and there is no auto correlation.
Adjusted r squared - how close the data are to the fitted regression line - is not very good (.179)

. estat ic - as a point of reference
939.7602 957.9186

INTRODCING GY, LOG GROWTH OF INDUSTRIAL PRODUCTION

```
reg ffdif l(1,2,4).ffdif l(3).gy

      Source |       SS           df       MS      Number of obs   =       692
-------------+----------------------------------   F(4, 687)       =     39.26
       Model |  35.3235312         4  8.83088279   Prob > F        =    0.0000
    Residual |  154.534373       687  .224940863   R-squared       =    0.1861
-------------+----------------------------------   Adj R-squared   =    0.1813
       Total |  189.857904       691  .274758183   Root MSE        =    .47428

------------------------------------------------------------------------------
       ffdif |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ffdif |
         L1. |   .4384589   .0374526    11.71   0.000     .3649236    .5119943
         L2. |  -.1681988   .0373639    -4.50   0.000    -.2415599   -.0948376
         L4. |  -.0869334   .0344781    -2.52   0.012    -.1546286   -.0192383
             |
          gy |
         L3. |   .0874005   .0377637     2.31   0.021     .0132544    .1615466
             |
       _cons |  -.0128867   .0186137    -0.69   0.489    -.0494333      .02366
------------------------------------------------------------------------------
936.3857    959.0836
```

INTRODCING PIDIF, GROWTH IN THE INFLATION RATE

```
 reg ffdif l(1,2).ffdif l(3).gy l(1).pidif

      Source |       SS           df       MS      Number of obs   =       693
-------------+----------------------------------   F(4, 688)       =     41.02
       Model |  36.6007433         4  9.15018582   Prob > F        =    0.0000
    Residual |    153.4803       688  .223081831   R-squared       =    0.1926
-------------+----------------------------------   Adj R-squared   =    0.1879
       Total |  190.081043       692  .274683588   Root MSE        =    .47232

------------------------------------------------------------------------------
       ffdif |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ffdif |
         L1. |   .4384738   .0372213    11.78   0.000     .3653928    .5115548
         L2. |  -.1600935   .0373038    -4.29   0.000    -.2333364   -.0868506
             |
          gy |
         L3. |   .0715706    .037707     1.90   0.058    -.0024641    .1456052
             |
       pidif |
         L1. |    .243372   .0689759     3.53   0.000     .1079435    .3788004
             |
       _cons |  -.0098821   .0185208    -0.53   0.594    -.0462461    .0264819
------------------------------------------------------------------------------
931.9806    954.6857
```

After including pidif, the pvalue of gy was > .05 and thus insignificant and removed

```
 reg ffdif l(1,2).ffdif l(1).pidif

      Source |       SS           df       MS      Number of obs   =       694
-------------+----------------------------------   F(3, 690)       =     53.36
       Model |  35.7932519         3   11.931084   Prob > F        =    0.0000
    Residual |  154.289556       690  .223608051   R-squared       =    0.1883
-------------+----------------------------------   Adj R-squared   =    0.1848
       Total |  190.082807       693  .274289765   Root MSE        =    .47287

------------------------------------------------------------------------------
       ffdif |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ffdif |
         L1. |   .4403072   .0372509    11.82   0.000     .3671684    .5134459
         L2. |   -.155458   .0372127    -4.18   0.000    -.2285216   -.0823944
             |
       pidif |
         L1. |   .2550161   .0687847     3.71   0.000     .1199637    .3900684
             |
       _cons |  -.0010553   .0179502    -0.06   0.953    -.0362989    .0341882
------------------------------------------------------------------------------
933.9599    952.1298
```

INTRODCING UDIF, GROWTH IN UNEMPLOYMENT

```
. reg ffdif l(1,2).ffdif l(1).pidif l(3).udif

      Source |       SS           df       MS      Number of obs   =       693
-------------+----------------------------------   F(4, 688)       =     42.39
       Model |  37.5855389         4  9.39638473   Prob > F        =    0.0000
    Residual |  152.495504       688  .221650442   R-squared       =    0.1977
-------------+----------------------------------   Adj R-squared   =    0.1931
       Total |  190.081043       692  .274683588   Root MSE        =     .4708

------------------------------------------------------------------------------
       ffdif |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ffdif |
         L1. |   .4380181   .0370977    11.81   0.000     .3651798    .5108564
         L2. |  -.1567205   .0371172    -4.22   0.000     -.229597   -.0838439
             |
       pidif |
         L1. |   .2352944   .0688331     3.42   0.001     .1001461    .3704426
             |
        udif |
         L3. |  -.2742921   .0965617    -2.84   0.005    -.4638831   -.0847011
             |
       _cons |  -.0015057   .0178848    -0.08   0.933    -.0366211    .0336096
------------------------------------------------------------------------------
927.5197    950.2248
```

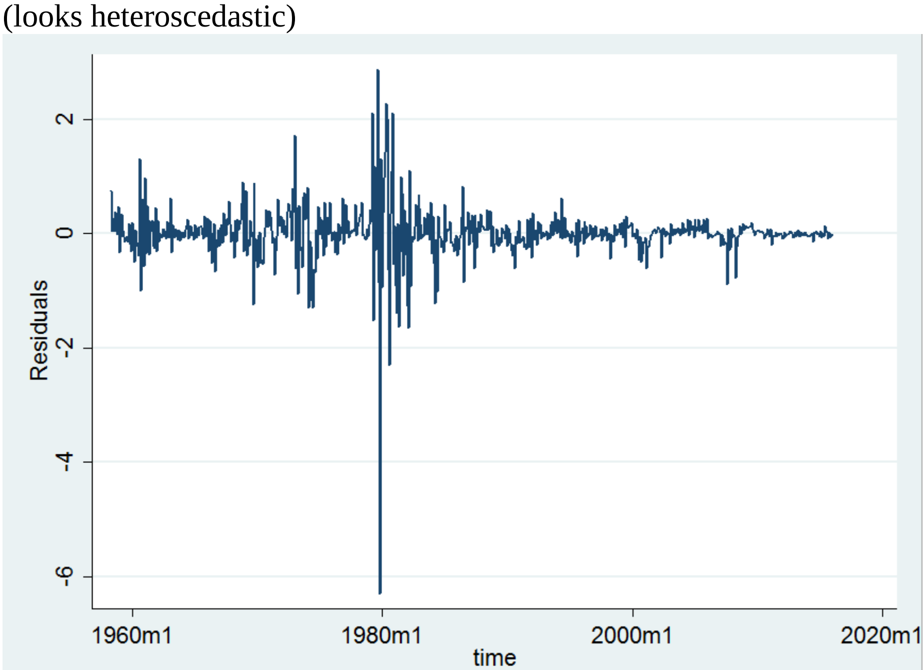
Lowest estat ic and increased adjusted r squared (but still not great)

```
. estat durbinalt

Durbin's alternative test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          0.491               1                   0.4835
---------------------------------------------------------------------------
                        H0: no serial correlation
```

This model gives the lowest BIC and exhibits no autocorrelation.

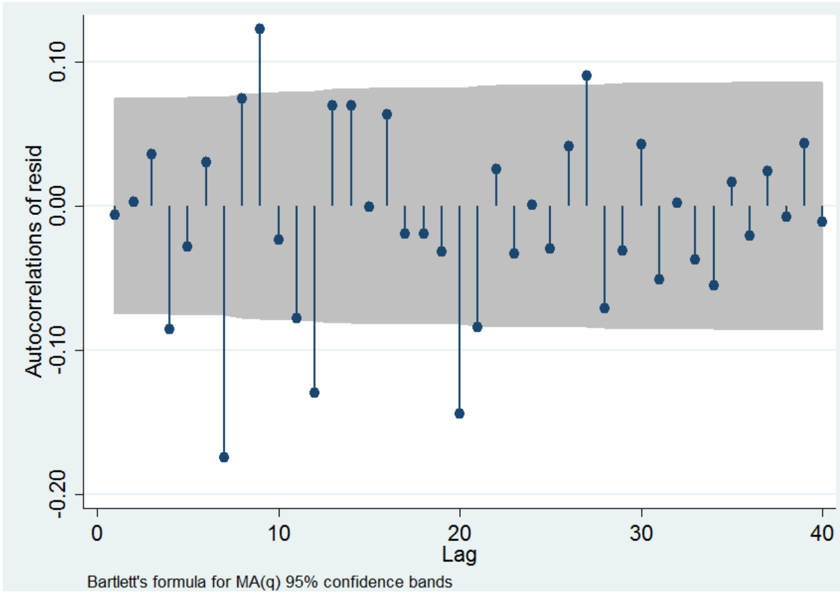
PREDICT RESIDUALS

. predict resid, resid
. tsline resid
(looks heteroscedastic)

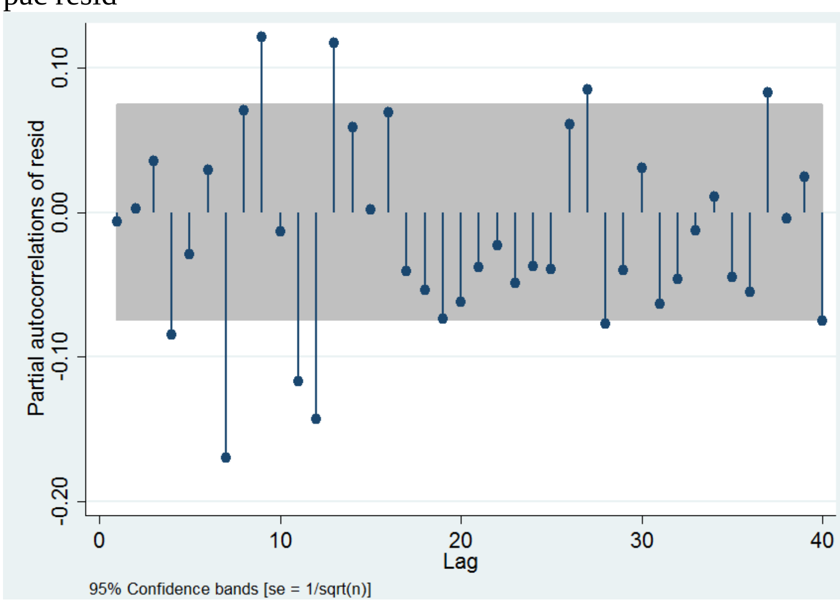


RESIDUAL ANALYSIS

ac resid



pac resid



few outside of bands. not white noise yet.

ARIMA MODELS

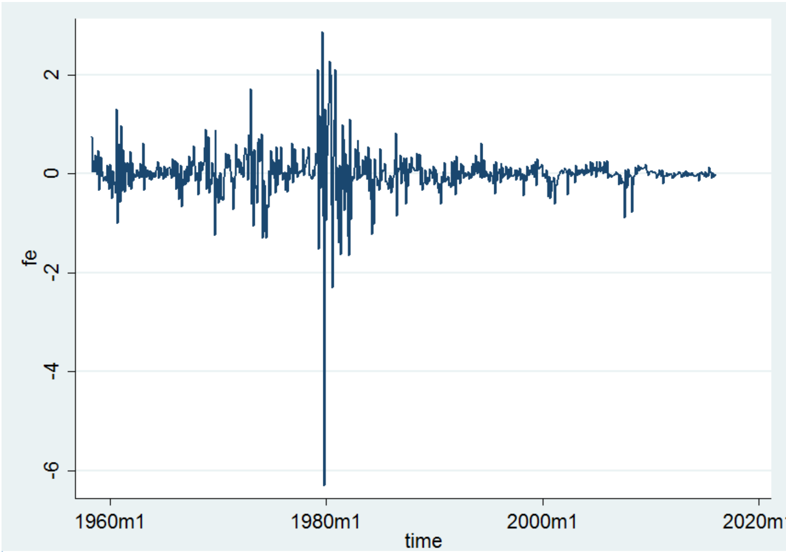
```
 arima ffdif l(1,2).ffdif l(1).pidif l(3).udif, nolog

ARIMA regression

Sample:  1958m5 - 2016m1                        Number of obs     =        693
                                                Wald chi2(4)      =     754.76
Log likelihood = -458.7598                      Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
       ffdif |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ffdif        |
       ffdif |
         L1. |   .4380181   .0168522    25.99   0.000     .4049884    .4710478
         L2. |  -.1567205    .016717    -9.37   0.000    -.1894851   -.1239558
             |
       pidif |
         L1. |   .2352944   .0452887     5.20   0.000     .1465302    .3240585
             |
        udif |
         L3. |  -.2742921   .1072661    -2.56   0.011    -.4845298   -.0640544
             |
       _cons |  -.0015057   .0215168    -0.07   0.944    -.0436779    .0406664
-------------+----------------------------------------------------------------
      /sigma |   .4690962    .004121   113.83   0.000     .4610192    .4771732
------------------------------------------------------------------------------
. estat ic
929.5197    956.7658
```

First version with gy=d.log(ip)
. reg ffdif l(1,2,4).ffdif l(1,3).pidif l(3).udif
ADJRSQ 0.2045
918.4742   950.2513

Second version with gy=d.ip (Plots and models shown above)
. reg ffdif l(1,2).ffdif l(1).pidif l(3).udif
ADJRSQ 0.1931
927.5197   950.2248
Using gy=d.ip gives slightly better BIC but slightly worst adjusted r squared.

2. Forecast with 95% confidence bands.

. reg ffdif l(1,2,4).ffdif l(1,3).pidif l(3).udif
. predict pred_ffdif, xb
. gen fe=ffdif- pred_ffdif
. tsline fe




```
. sum fe

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
          fe |        693   -8.16e-11     .469435  -6.303317   2.869722
```

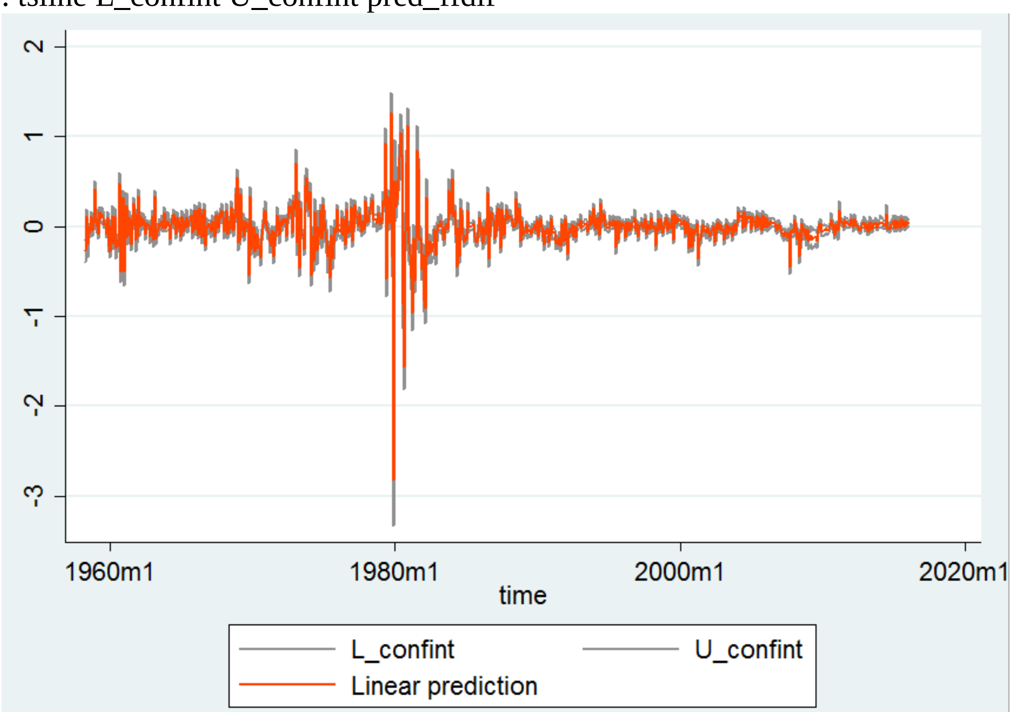
CONFIDENCE INTERVAL CALCULATION

. predict se_cycle, stdp

. gen L_confint=pred_ffdif-1.96*se_cycle

. gen U_confint=pred_ffdif+1.96*se_cycle

. tsline L_confint U_confint pred_ffdif



Section 2: Vector auto regression (VAR)

1) **VAR to obtain IRF's as a stationary series.**

Variable Stationarity: (testing and correction)

HOUSEP
. dfuller housep
MacKinnon approximate p-value for Z(t) = 0.1475
. gen log_housep=log(housep)
. dfuller log_housep
MacKinnon approximate p-value for Z(t) = 0.0038

FFR (Federal Funds Rate)
. dfuller ffr
MacKinnon approximate p-value for Z(t) = 0.4455
. gen ffrdif=d.ffr
. dfuller ffrdif
MacKinnon approximate p-value for Z(t) = 0.0000

IP (Industrial Production)
. dfuller ip
MacKinnon approximate p-value for Z(t) = 0.8496
. gen ipdif=d.ip
. dfuller ipdif
MacKinnon approximate p-value for Z(t) = 0.0000

VARSOC to pick number of lags.
varsoc ffrdif ipdif log_housep

```
. varsoc ffrdif ipdif log_housep

   Selection-order criteria
   Sample:  2000m6 - 2016m5                     Number of obs      =       192
```

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-48.283				.000342	.534198	.554812	.585096
1	641.071	1378.7	9	0.000	2.9e-07	-6.55282	-6.47036	-6.34923
2	857.103	432.06	9	0.000	3.3e-08*	-8.7094*	-8.5651*	-8.35311*
3	862.496	10.787	9	0.291	3.4e-08	-8.67184	-8.46569	-8.16285
4	872.92	20.847*	9	0.013	3.4e-08	-8.68666	-8.41868	-8.02498

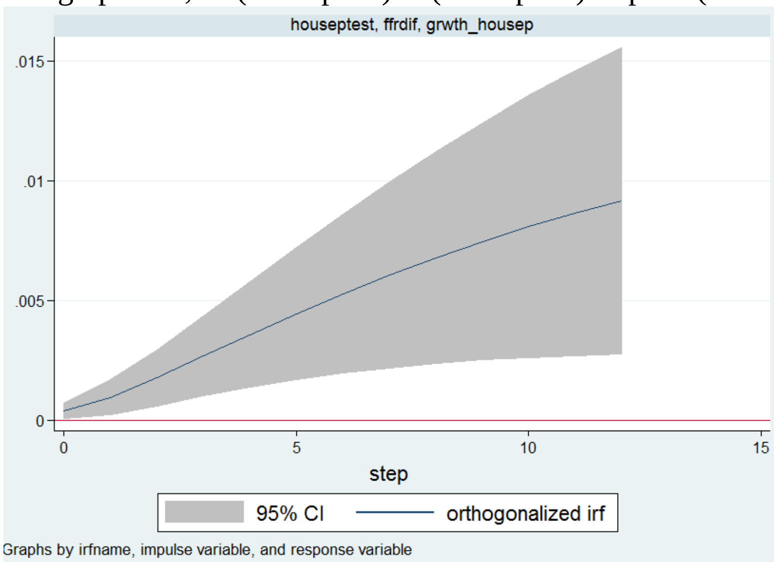
```
   Endogenous:  ffrdif ipdif log_housep
    Exogenous:  _cons
```

BIC indicates 2 lags (as do FPE, ACI, and HQIC)

Impulse response function (IRF):

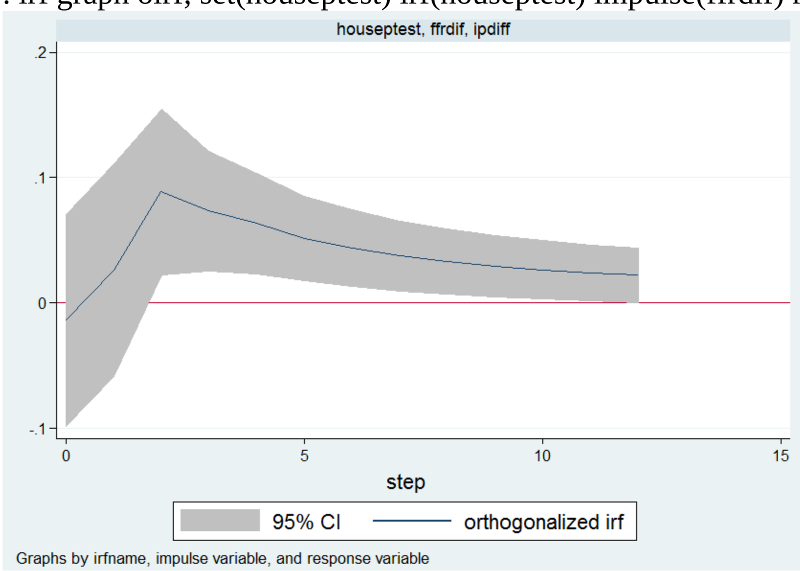
. quietly var ffrdif ipdiff grwth_housep, lags(1 2)
. irf create houseptest, set(houseptest, replace) step(12) order(ffrdif ipdiff grwth_housep)

. irf graph oirf, set(houseptest) irf(houseptest) impulse(ffrdif) response(grwth_housep) yline(0)



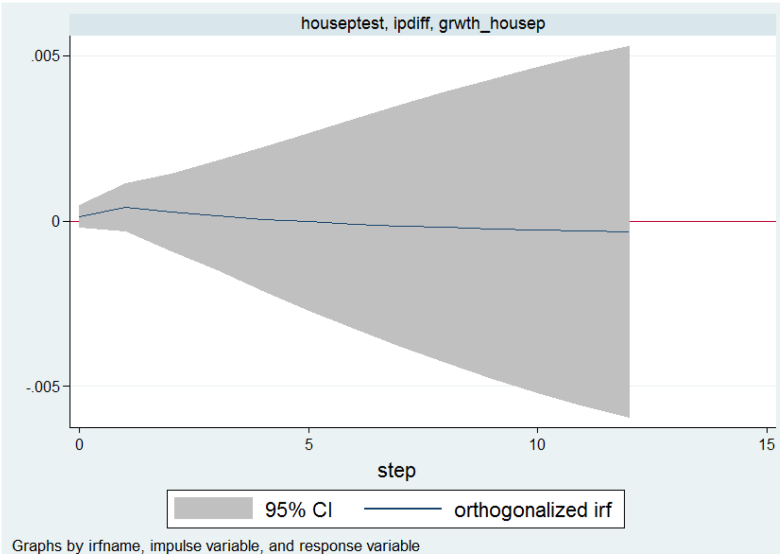
A shocked difference in Federal funds rate (d.ffr) has a statistical significance correlation with log of housing prices.

. irf graph oirf, set(houseptest) irf(houseptest) impulse(ffrdif) response(ipdiff) yline(0)



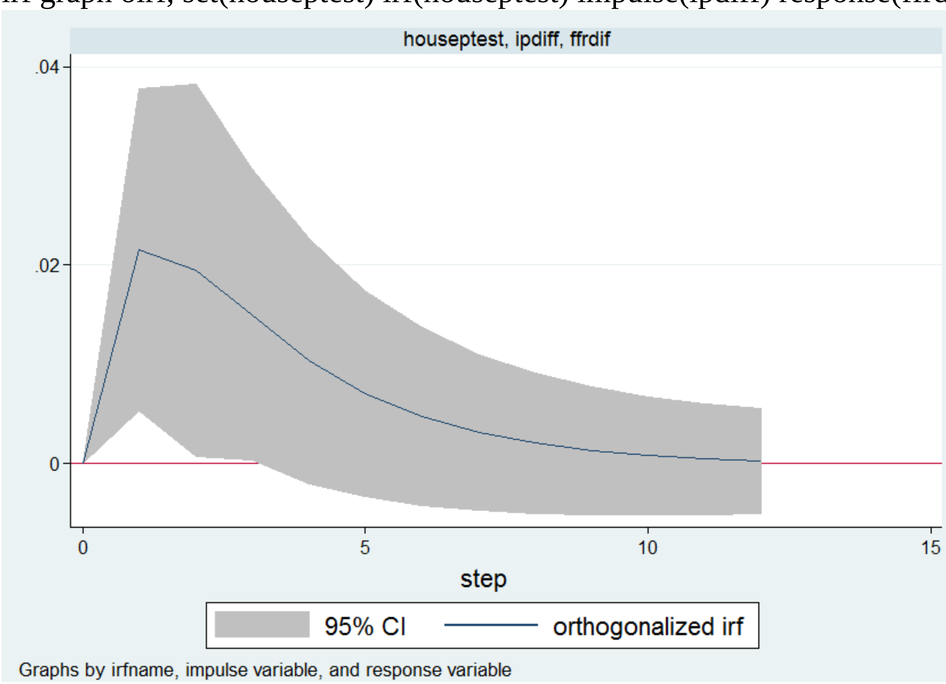
A shocked difference in Federal funds rate (d.ffr) has a statistical significance correlation with change in industrial production from months 2 to 12.

irf graph oirf, set(houseptest) irf(houseptest) impulse(ipdiff) response(grwth_housep) yline(0)



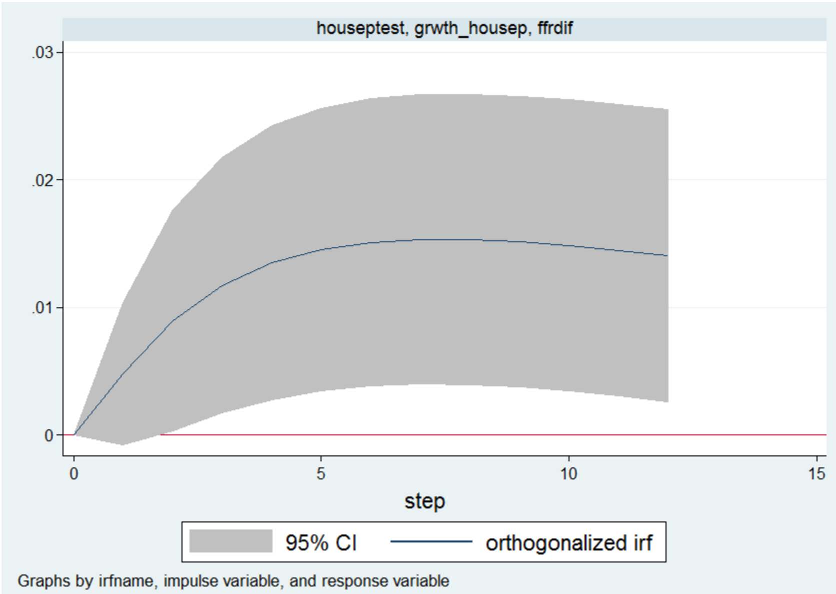
A shock to change in industrial production (ipdiff) has no statistical significance correlation with the log of housing prices.

irf graph oirf, set(houseptest) irf(houseptest) impulse(ipdiff) response(ffrdif) yline(0)



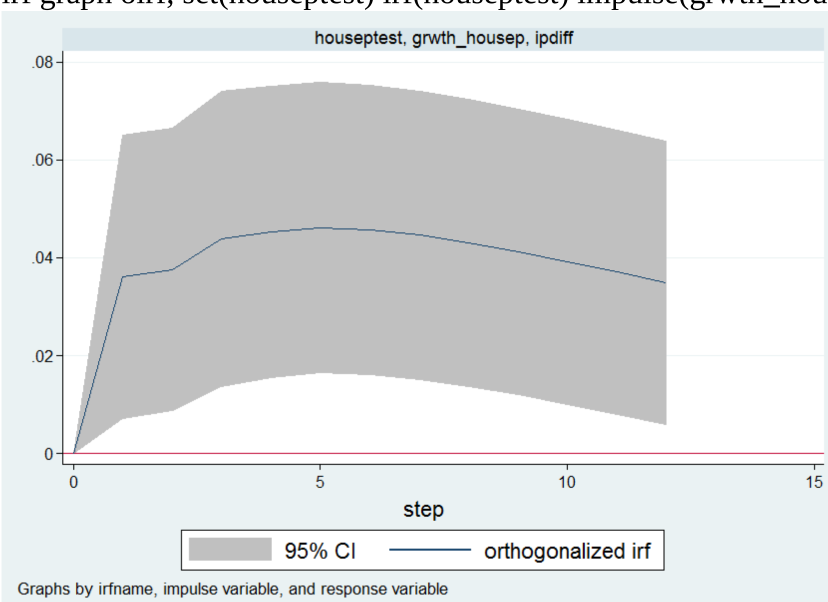
A shock to change in industrial production has a statistical significance correlation with the change in federal funds rate for the first three months.

irf graph oirf, set(houseptest) irf(houseptest) impulse(grwth_housep) response(ffrdif) yline(0)



Impulse of the log of house price has a statistical significance correlation with the change in federal funds rate

irf graph oirf, set(houseptest) irf(houseptest) impulse(grwth_housep) response(ipdiff) yline(0)



Impulse of the log of house price has a statistical significance correlation with the change in industrial production.

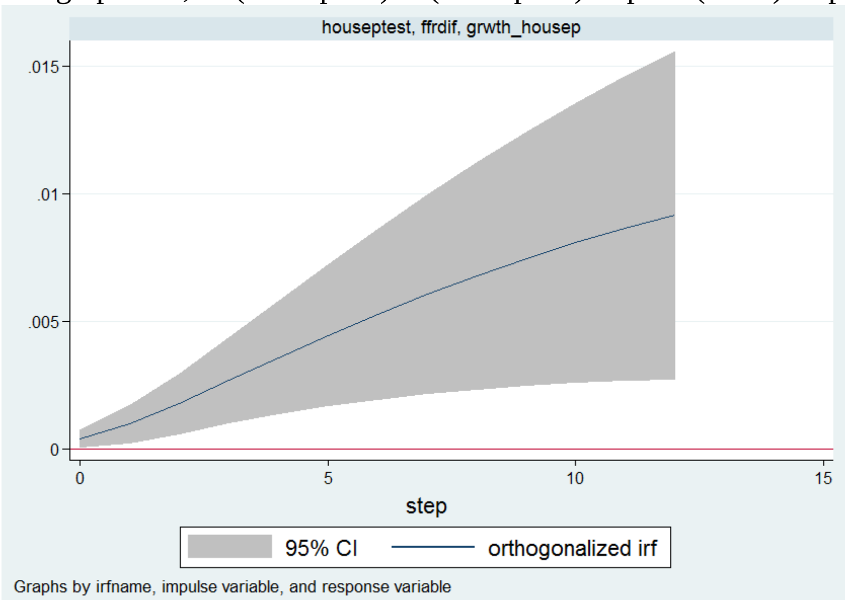
2) Re ordering variables and reinterpretation of IRF.

From the above impulse response exploration with the order (ffrdif ipdiff grwth_housep) it was seen that a shock to the difference in federal funds rate (d.ffr) has a statistical significance correlation with the log of housing prices.

In addition, a shocked difference in industrial production (d.ip) has a statistical significance correlation with the difference in federal funds rate in months 1 to 3, while a shocked difference in federal funds rate (d.ffr) has a statistical significance correlation with the change in industrial production from months 2 to 12 after the shock.

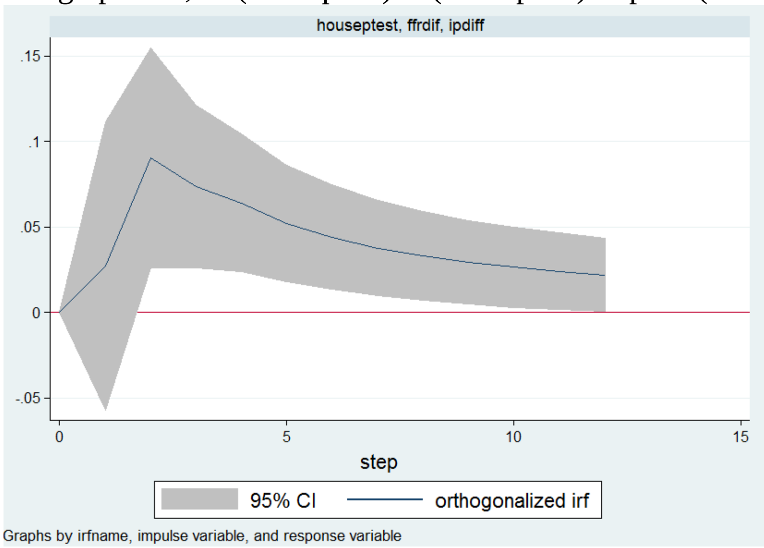
Based on these finding, a Choleski ordering of (ipdiff ffrdif grwth_housep) was chosen for further exploration.

. irf graph oirf, set(houseptest) irf(houseptest) impulse(ffrdif) response(grwth_housep) yline(0)



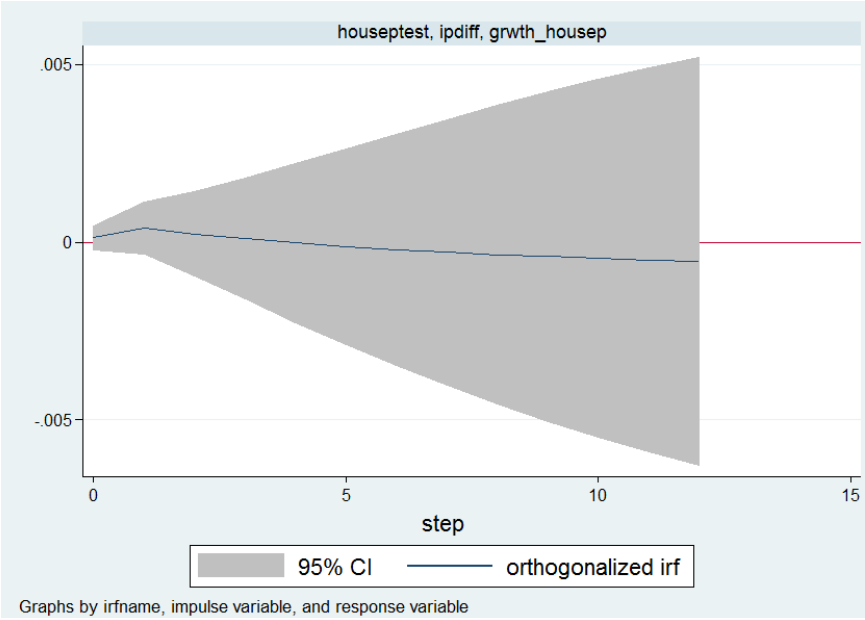
Appears identical to the same plot with order (ffrdif ipdiff grwth_housep).

. irf graph oirf, set(houseptest) irf(houseptest) impulse(ffrdif) response(ipdiff) yline(0)



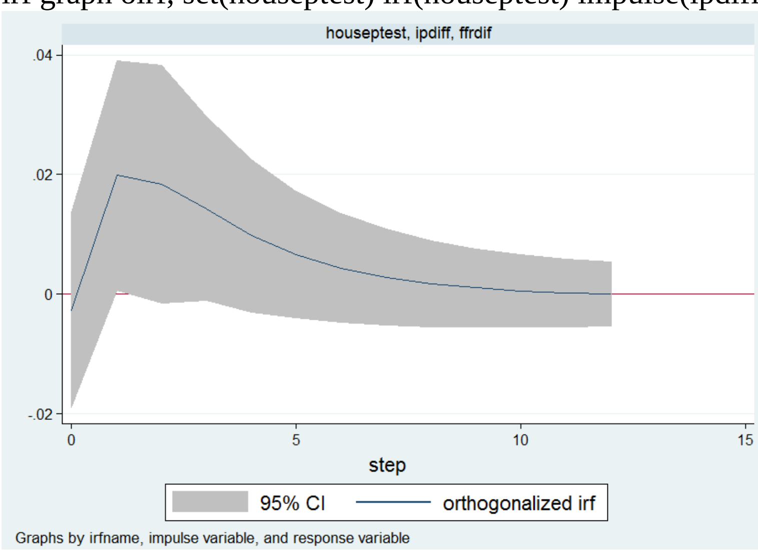
Conveys the same statistically significance correlation as the plot when ordered (ffrdif ipdiff grwth_housep), however there are differences in the confidence boundaries at time zero.

irf graph oirf, set(houseptest) irf(houseptest) impulse(ipdiff) response(grwth_housep) yline(0)



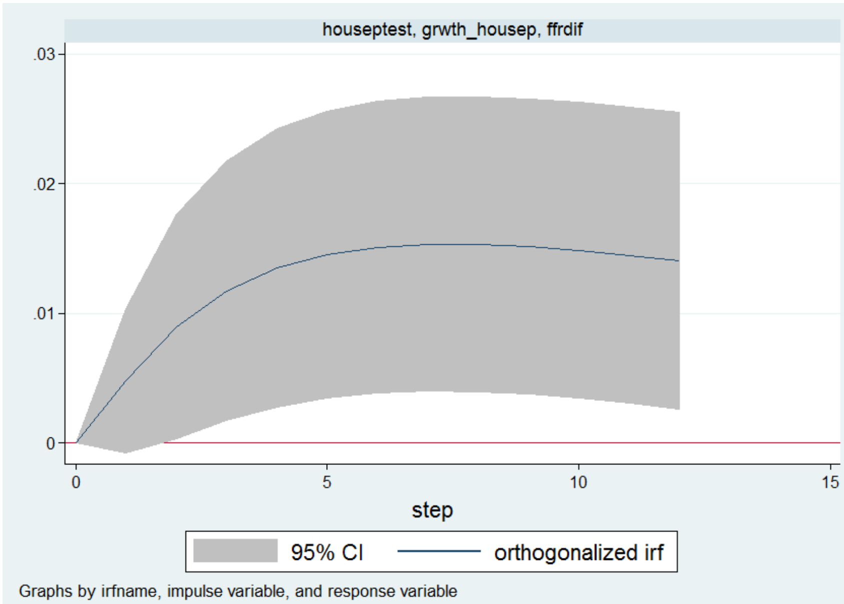
As in the prior ordering, a shock to change in industrial production has no statistical significance correlation with the log of housing prices.

irf graph oirf, set(houseptest) irf(houseptest) impulse(ipdiff) response(ffrdif) yline(0)



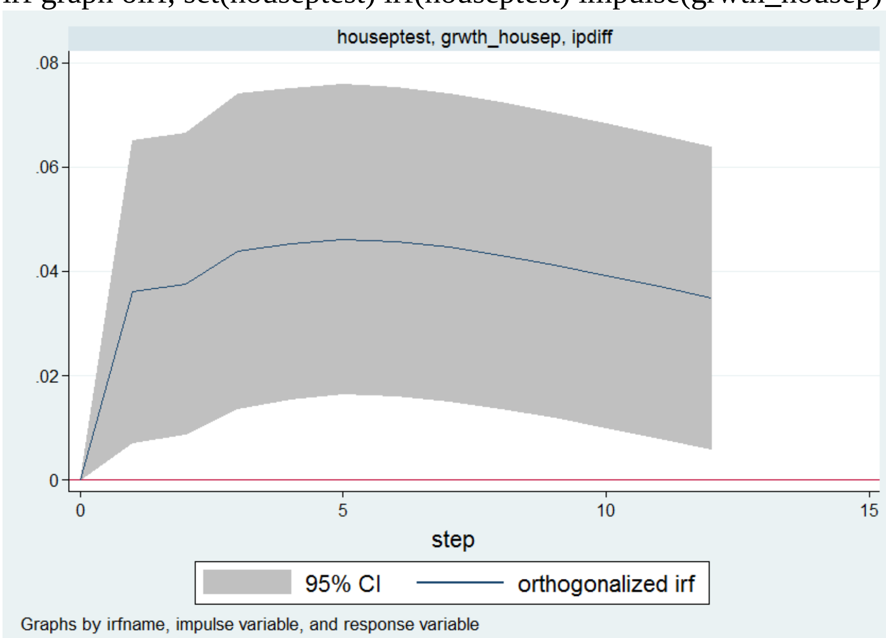
A shock to change in industrial production has a statistical significance correlation with the change in federal funds rate except for a fraction of a period between period 1 and 2.

. irf graph oirf, set(houseptest) irf(houseptest) impulse(grwth_housep) response(ffrdif) yline(0)



Impulse of the log of house price has a statistical significance correlation with the change in federal funds rate and appears unaffected by the different ordering.

irf graph oirf, set(houseptest) irf(houseptest) impulse(grwth_housep) response(ipdiff) yline(0)



Impulse of the log of house price has a statistical significance correlation with the change in industrial production and appears unaffected by the different ordering.

Re ordering with industrial production last is appropriate since the IRF plots indicate that it has the least correlation with housing and federal funds in either of the two prior order explorations.

The causal relationship between change in federal funds rate and log of house prices should determine which is ordered first. The Granger Causality Test can be used to get an indication of the direction of this relationship.

3) Granger causality test for ordering of impulse response functions.

Granger causality test for grwth_housep ffrdif :

. quietly var grwth_housep ffrdif, lags(2)

```
. vargranger

  Granger causality Wald tests
  +------------------------------------------------------------------+
  |          Equation           Excluded |   chi2     df Prob > chi2 |
  |--------------------------------------+---------------------------|
  |      grwth_housep             ffrdif |  58.525     1    0.000    |
  |      grwth_housep                ALL |  58.525     1    0.000    |
  |--------------------------------------+---------------------------|
  |            ffrdif       grwth_housep |  3.0169     1    0.082    |
  |            ffrdif                ALL |  3.0169     1    0.082    |
  +------------------------------------------------------------------+
```

H0: grwth_housep granger causes ffrdif can be rejected (Prob of H0 rejection > .05) and
Ha: ffrdif granger causes grwth_housep can be accepted.

Thus the we can conclude that the change in the federal funds rate granger causes growth rate in housing prices and the best ordering is (ffrdif grwth_housep ipdiff).