Lecture 5

Euclidean Space

- $\Re^m$
- m equations
- m < n, x + y = 1 = Space / undeterminant
- m = n there is one solution
- m > n there are more parameters than solutions

Model Fitting

- Model -> Functional
- $F_\theta(x), (x^k, y^k)$
- $\theta, x = parameters$
- $\frac{ArgMin}{\theta} \sum_{k=1}^{N} ||F_\theta(x^k) - y^k||^2$
- $F_\theta(x) = \sum_{m=0}^{N} \theta_m \phi_m(x)$
- $\phi_m(x) = x^m$
- $\phi_m(x) = e^{-iwm}$
- Basis
  - $\lambda_1 \phi_m(x) = \lambda_2 \phi_k(x)$
  - $\lambda_1 = lambda_2 = 0$
  - $m = k$
  - $\phi_0 = \begin{matrix} 1 \\ 0 \end{matrix}$
  - $\phi_1 = \begin{matrix} 0 \\ 1 \end{matrix}$
  - $\begin{matrix} 2 \\ 3 \end{matrix} = 2\phi_0 + 3\phi_1$
- $F_\theta(x^k) = y^k$

- $\theta_0\phi_0(x^1) + \theta_1\phi_1(x^1) + \theta_2\phi_2(x^1) = y^1$

- $$\begin{matrix} \phi_0(x^1) & phi_1(x^1) & ... & \phi_m(x^1) & \theta_0 & y^1 \\ \phi_0(x^2) & phi_1(x^2) & ... & \phi_m(x^2) * & \theta_1 & * y^2 \\ \phi_0(x^N) & phi_1(x^N) & ... & \phi_m(x^N) & theta_m & y^k \end{matrix}$$

- $||A\theta 0y||2$

- k = horizontal, n = vertical

- N » K, significantly larger

Linear Algebra

- $||x||^2 = X^TX$

- Norm Scalar Product
- $||x||^2 = \sum x_i^2$

- $\sum x_i^2 = variance$

- $\frac{ArgMin}{\theta}||A\theta - y||^2$

- $(A\theta - y)^t(A\theta - y)$

- $\theta^TA^TA\theta - \theta^TA^Ty - y^TA\theta + y^Ty$

  - $(AB)^T = B^TA^T$

  - $X^TY = Y^TX$

  - $Y^tA\theta = A^T\theta^Ty$

  - $\theta 6TA^TA\theta - 2\theta^TA^Ty + y^Ty$

  - $\frac{d}{d\theta}2A^TA\theta - 2A^Ty = 0$

  - $A^TA\theta = A^Ty =$ normal equations

  - $A\theta = y$

Example

- $F(x) = ax + b$
- (x^k, y^k)
  - $ax^1 + b = y^1$
  - $ax^2 + b = y^2$

$$\begin{matrix} 1 & x^1 \\ 1 & x^2 \\ 1 & x^N \end{matrix} * \begin{matrix} b \\ a \end{matrix} * \begin{matrix} y^1 \\ y^2 \\ y^N \end{matrix}$$

- $A^T A = \begin{matrix} N & \sum x^k \\ \sum x^k & \sum {x^k}^2 \end{matrix}$
- $(\begin{matrix} a & b \\ c & d \end{matrix})^{-1} = \frac{1}{det(A)} * (\begin{matrix} d & -b \\ -c & a \end{matrix})$
- $N \sum (x^k)^2 - (\sum x_k)^2$
- $a = \frac{N(\sum x^k y^k) - (\sum x^k)(\sum y^k)}{N \sum (x^k)^2 - \sum (x_k)^2}$

Example 2

- $f(x) = ax + b$
- (1, 2) (1, 3) (2, 5)
- $\begin{matrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{matrix} * \begin{matrix} b \\ a \end{matrix} = \begin{matrix} 1 \\ 3 \\ 5 \end{matrix}$
- $\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{matrix} * \begin{matrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{matrix} = A^T A$
- $\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{matrix} * \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} = A^T y$