MECHENG 4K03: A3 - suitorj - 400138679

Setup

```
sympref('AbbreviateOutput', false);
syms theta1 theta2 theta3 d1 d2 d3 a1 a2 a3
prismatic = [0; 0; 0];
revolute = [0; 0; 1];
```

Question 1

```
A1 = [
    cos(theta1) 0  sin(theta1) 0;
    sin(theta1) 0 -cos(theta1) 0;
        0       1     0        0;
        0       0     0        1;

];

A2 = [
    0 0 -1  0;
    1 0  0  0;
    0 -1 0 d2;
    0 0  0  1;
];

A3 = [
    1 0 0 0;
    0 1 0 0;
    0 0 1 d3;
    0 0 0 1
];

T = simplify(A1 * A2 * A3);

JA = jacobian(T(1:3, end), [theta1, d2, d3]);

z0 = revolute;
z1 = extract_r(A1) * prismatic;
z2 = extract_r(A1) * extract_r(A3) * prismatic;
JB = [z0 z1 z2];

J = [JA; JB]
```

J =

$$\begin{pmatrix} d_2\cos(\theta_1)+d_3\sin(\theta_1) & \sin(\theta_1) & -\cos(\theta_1) \\ d_2\sin(\theta_1)-d_3\cos(\theta_1) & -\cos(\theta_1) & -\sin(\theta_1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

```
J_simple = remove_zero_rows(J)
```

J_simple =
$$\begin{pmatrix} d_2\cos(\theta_1) + d_3\sin(\theta_1) & \sin(\theta_1) & -\cos(\theta_1) \\ d_2\sin(\theta_1) - d_3\cos(\theta_1) & -\cos(\theta_1) & -\sin(\theta_1) \\ 1 & 0 & 0 \end{pmatrix}$$

```
J_det = det(J_simple)
```

J_det = $-\cos(\theta_1)^2 - \sin(\theta_1)^2$

```
J_det = simplify(J_det)
```

J_det = -1

The determinant is -1 therefore the robot is never singular.

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \begin{pmatrix} d_2\cos(\theta_1) + d_3\sin(\theta_1) & \sin(\theta_1) & -\cos(\theta_1) \\ d_2\sin(\theta_1) - d_3\cos(\theta_1) & -\cos(\theta_1) & -\sin(\theta_1) \\ 1 & 0 & 0 \end{pmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Question 2

```
A1 = [
    -1 0  0  0;
     0  0 1  0;
     0  1 0 d1;
     0  0 0  1;
];

A2 = [
    0  0 -1   0;
    -1 0  0   0;
    0  1  0  d2;
    0  0  0   1
];

A3 = [
    cos(theta3) -sin(theta3) 0 a3*cos(theta3);
    sin(theta3)  cos(theta3) 0 a3*sin(theta3);
        0            0       1        0      ;
        0            0       0        1      ;
];

T = simplify(A1 * A2 * A3);

JA = jacobian(T(1:3, end), [d1, d2, theta3]);

z0 = prismatic;
z1 = extract_r(A1) * prismatic;
z2 = extract_r(A1) * extract_r(A2) * revolute;
JB = [z0 z1 z2];

J = [JA; JB]
```

J =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & a_3 \cos(\theta_3) \\ 1 & 0 & a_3 \sin(\theta_3) \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
J_simple = remove_zero_rows(J)
```

J_simple =

$$\begin{pmatrix} 0 & 1 & a_3 \cos(\theta_3) \\ 1 & 0 & a_3 \sin(\theta_3) \\ 0 & 0 & 1 \end{pmatrix}$$

```
J_det = det(J_simple)
```

J_det = -1

The determinant is -1 therefore the robot will never be singular.

$$\begin{bmatrix} v_y \\ v_z \\ \omega_x \end{bmatrix} = \begin{pmatrix} 0 & 1 & a_3 \cos(\theta_3) \\ 1 & 0 & a_3 \sin(\theta_3) \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{\theta}_3 \end{bmatrix}$$