Lecture 13

Predictor & Observer

- Use the prediction and observation to determine state
- $P(x_m|x_{m-1})$ = prediction
- $P(x_m|y_m)$ = measurement of state
- Guassian assumption
- $E(x_m) = \widetilde{x}_m$
- $VAR(x_m) = P_m$
- $Given \widetilde{x}_{m-1} P_{m-1}$
- $Predict \widetilde{x}_m^+ P_m^+$
- $Observe y_m$
- Update to get $\widetilde{x}_m P_m$

Kalman Filter

- Scalar
- $x_{m+1} = ax_m + v_m$
- $y_m = cx_m + q_m$
- $N(0, \sigma_v^2)$
- $N(0, \sigma_q^2)$
- $\widetilde{x}_m = E(x_m)$
- $P_m = VAR(x_m - \widetilde{x}_m)$

1. Predict -> given $\widetilde{x}_{m-1} P_{m-1}$
    - $\widetilde{x}_m^+ = A\widetilde{x}_{m-1}$
    - $\widetilde{P}_m^+ = VAR(x_m - \widetilde{x}_m^+)$

- $= VAR(Ax_{m-1} + v_m - A\widetilde{x}_{m-1})$
- $= a^2 VAR(x_{m-1} - \widetilde{x}_{m-1}) + VAR(v_m) + 2CO - VAR(x_{m-1}, -\widetilde{x}_{m-1}, v_m)$
- $= a^2 P_{,-1} + \sigma_v^2$
- $\widetilde{x}_m^+ = a\widetilde{x}_{m-1}$
- $P_m^+ = a^2 P_{m-1} + \sigma_v^2$

2. Observe y_m
   - $\widetilde{x}_m = \widetilde{x}_m^+ + k(y_m - \widetilde{y}_m)$
   - $\widetilde{x}_m = \widetilde{x}_m^+ + kcx_m - kc\widetilde{x}_m^+ + kq_m$
   - $P_m = VAR(x_m - \widetilde{x_m}) = E((x_m - \widetilde{x}_m)^2)$
   - $x_m - \widetilde{x}_m = x_m - \widetilde{x}_m^+ - k(cx_m + q - \widetilde{y}_m)$
   - $x_m - \widetilde{x}_m^+ - kcx_m - kc\widetilde{x}_m^+ - kq_m$
   - $(1 - kc)(x_m - \widetilde{x}_m^+) - kq_m$
   - $(ax + b)2$
     - $a = (1 - kc)$
     - $a = (x_m - \widetilde{x}_m^+)$
     - $k = -kq_m$

- $E((1 - kc)^2 (x_m \ widetildex_m^+) + k^2 q_m^2 + 2(a - kc)(x - \widetilde{x}_m^+))$
- $(1 - kc)^2 E((x_m - \widetilde{x}_m^+)^2) = k^2 E(q_m^2)$
- $(1 - 2kc + k^2c^2)P_m^+ + k^2\sigma_q^2$
- $\frac{d}{dk} = -2cP_m^+ + 2kc^2 P_m^+ + 2k\sigma_q^2 = 0$
- $k = \frac{cP_m^+}{c^2 P_m^+ + \sigma_q^2}$
- $\widetilde{x}_m = \widetilde{x}_m^+ + k(y_m - c\widetilde{x}_m^+)$
- $P_m = (1 - kc)P_m^+$

Scalar Kalman

- $Given a, c, \sigma_v^2 = model uncertainty, \sigma_q^2 = observation uncertainty$
- $Start at \widetilde{x}_0 with P_0$
- Predict
  - $\widetilde{x}_m^+ = a x_{m-1}$
  - $P_m^+ = a^2 P_{m-1} + \sigma_v^2$
- Update y_m
  - $\widetilde{x}_m = \widetilde{x}_m^+ + k(y_m - c\widetilde{x}_m^+)$
  - $P_m = (1 - kc)P_m^+$
  - $k = \frac{cP_m^+}{c^2 P_m^+ + \sigma_q^2}$