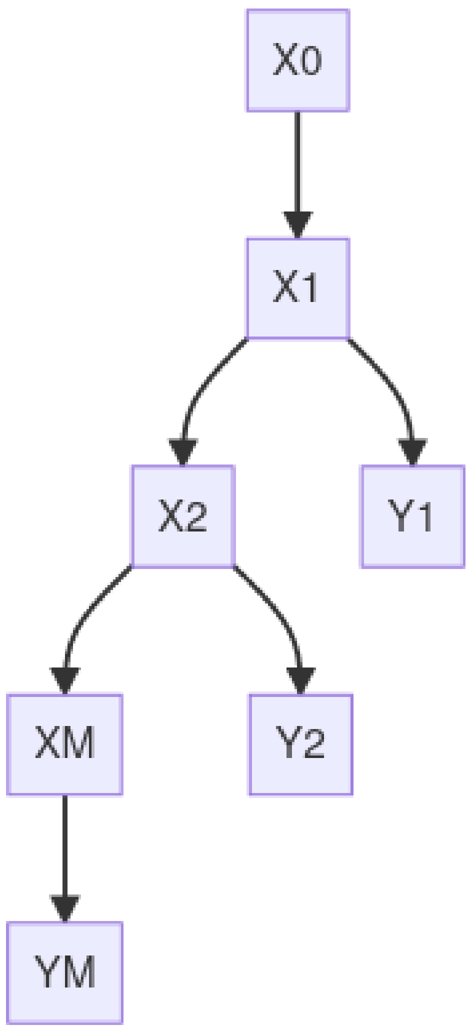
Lecture 10

Stochastic Systems

- Markov Chains



- $x_{m+1} = f(x_m)$ -> deterministic
- $x_{m+1} = f(x_m) + \epsilon_m$
- $P(x_{m+1}|x_m)$

- $E(x_{m+1} = \int xP(x_{m+1})x_m dx)$
- $P(y_m|x_m)$

State Space Control

- $x' = f(x, u)$
- $y = h(x)$
- These equations are constraints on the system

Performance Measurement

- $J = \int_0^{\inf} g(x, x', u)dt$ = cost function

Particle Example

- f(x)
- g(x) = 0
- $L = g(x, u) + \lambda f(x - u) - x'$
- $\frac{dL}{dx}$
- $\frac{dL}{d\lambda}$
- $x_{m+1} = Ax_m + Bu_m + \epsilon_m$
- $y_m = Cx_m$
- $J = \sum_{m=0}^{T-1}(x_m^T Q x_m + u_m^T R u_m) + x_T^T Q_T x_T$
- Infinite horizon -> for stability, T = inf
- Finite -> positioning
- $x^T y = scalarproduct = x^T Q y$
- $x^T x = ||x||$

Optimal Control

Controlability

- $x_{m+1} = (\begin{matrix}1 & 0 \\ 0 & 1\end{matrix})x_m + (\begin{matrix}1 \\ 1\end{matrix})u_m$
- $y(u) = (1,1)x_m$
- Pick some x such that, go from 0 to any x
- (2, 2) works but (2, 3) doesn't because the system only works on the diagonal
- $x_{m+1} = Ax_m + B_m$
  - $x_0 = (\begin{matrix}0 \\ 0\end{matrix})$
  - $x_1 = Bu(0)$
  - $x_2 = ABu(0) + Bu(1)$
  - $B, AB, A^2B, ...$
  - If the matrix is full rank it is controllable

Observable

- Reconstruct x(0) from observing $x_1, x_2, x_3, ...x_m$
- $x_{m+1} = (\begin{matrix}1 & 0 \\ 1 & 0\end{matrix})x + (\begin{matrix}1 \\ 1\end{matrix})u$
- y(u) = (0, 1) x
- This system is not observable because you can't view all states