Lecture 15

Bayesian Reasoning

Linear Least Sqaures

State Space Control Principles

- Controllable
- Observable
- Luneberg observer

Regression

Expected Value, Variance

- Matrix version
- Scalar version

Fusion (optimal linear mix) + Scalar Kalman

How to Practice

Bayesian Reasoning Example

- P(A | B) -> B has happened
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- Partitioning is when you split into sections
  - $\cup B_k = u$
  - $B_k \cap b_l =$ Disjoint
  - $P(k) = \sum P(A|B_k)P(K_k)$

Linear Least Squares Example

- (x^k, y^k)
- Linear Model F(x) = $\sum \alpha_k J_k(x)$
- $\frac{Minimize}{\alpha_1 \alpha_n} \sum_{k=1}^{M} ||F(x^k) - y^k||$
- Given (x^k, y^k), k=1 $\dots$ N
- Model is f(x) = aX
- $\begin{bmatrix} x^1 \\ x^2 \\ x^N \end{bmatrix} * \begin{bmatrix} a \end{bmatrix} = \begin{bmatrix} y^1 \\ y^2 \\ y^N \end{bmatrix}$
- $A^T Ax = A^T y$
- $\begin{bmatrix} x^1 & ... & x^n \end{bmatrix} * \begin{bmatrix} x^1 \\ ... \\ x^N \end{bmatrix} a = \begin{bmatrix} x^1 & ... & x^n \end{bmatrix} y$
- $\sum x_k^2 a = \sum x_k y_k$
- $a = \frac{\sum x_k^2}{\sum x_k y_k}$

Probability Example

- Random Var X -> x = scalar
- Random Vector (X_1, X_2) = matrix
- E(x) -> Definition u(x) = $\frac{1}{N} \sum x^k$
- $VAR(x) = E((E(x) - x)^2)$
- $CO - VAR(x, y) = E((E(x) - x)(E(y) - y))$

Scalar Example

- x is a random variable, C is a constant

- VAR(x + C) = VAR(x)
- f(x) is a linear mapping
- $\Re - > \Re$
- I know the variance of x.
- What is the VAR(f(x))
- f(x) = ax
- VAR(ax) = a^2 VAR(x)

Vector Example

- $VAR(X) = E((E(x) - x)(E(x) - x)^T)$
- $(AB)^T = B^T A^T$
- $X^T y = y^T X$

State Space Control Example

- Controllable, Observable
- $x_{m+1} = Ax_m + Bu_m$
- $y_m = Cx_m$