400138679 - A3 - 4AX3

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October 20 2021

Q1

$VAR(S_1) = \sigma_1^2$ $VAR(S_2) = \sigma_2^2$
Chose some k such that 0 <= k <= 1
To get the best estimate minimize the variance

$kS_1 + (1-k)S_2$
$VAR(kS_1 + (1-k)S_2)$
$a = k, b = (1-k)$

$$
\begin{aligned}
VAR(aS_1 + bS_2) &= VAR(aS_1 + bS_2) \\
&= E((E(aS_1 + bS_2) - aS_1 - bS_2)^2) \\
&= E((aE(S_1) + bE(S_2) - aS_1 - bS_2)^2) \\
&= (a(E(S_1) - S_1) + b(E(S_2) - S_2))^2 \\
&= a^2VAR(S_1) + b^2VAR(S_2) + 2abCOVAR(S_1, S_2) \\
&= a^2VAR(S_1) + b^2VAR(S_2) && \text{(Independent so covariance cancels)} \\
&= k^2\sigma_1^2 + (1-k)^2\sigma_2^2 && \text{(Substitute for } \sigma\text{)} \\
&= k^2\sigma_1^2 + \sigma_2^2 - k\sigma_2^2 + k^2\sigma_2^2
\end{aligned}
$$

Need to minimize so take the derivative

$\frac{d}{dk} = 2k\sigma_1^2 - \sigma_2^2 + 2k\sigma_2^2 = 0$
$k = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$

Q2

Plant
$x_{n+1} = Ax_n + Bu_n$
$y_n = Cx_n$

Model $\widetilde{x}_{n+1} = A\widetilde{x}_n + Bu_n + l(y_n - \widetilde{y}_m)$
$\widetilde{y}_n = C\widetilde{x}_n$

Error

$$\begin{aligned} e_{n+1} &= x_{n+1} - \widetilde{x}_{n+1} \\ &= Ax_n + Bu_n - (A\widetilde{x}_n + Bu_n + L(y_n - \widetilde{y}_n)) \\ &= Ax_n + Bu_n - A\widetilde{x}_n - Bu_n - L(y_n - \widetilde{y}_n) \\ &= Ax_n + -A\widetilde{x}_n - L(Cx_n - C\widetilde{x}_n) \\ &= (A - LC)(x_n - \widetilde{x}_n) \\ &= (A - LC)e \\ &= (1 - LC)e \end{aligned}$$

By selecting L so that the poles are within the unit circle L will go to 0.

$e_{n+1} = (1 - 0 * C)e$
$e_{n+1} = e$

If the error at time 0 is 0 then the observation controlled error is 0.

Q4

$x_{n+1} = Ax_n + Bu_n$

$y_n = Cx_n$

$x_0 = 0$

$x_1 = Bu(0)$

$x_2 = ABu(0) + Bu(1)$

$[B, AB, A^2B] =$ full rank

Yes, the solution exists

Q5

$f(x) = ax + b$

$(x_1, y_1), (x_2, y_2), (x_3, y_3)$

$$a = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} b \\ a \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{aligned}
a^T ab &= a^T y \\
b &= (a^T a)^{-1} a^T y \\
\begin{bmatrix} b \\ a \end{bmatrix} &= (\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{bmatrix})^{-1} \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \\
&= (\begin{bmatrix} k & \sum x^k \\ \sum x^k & \sum (x^k)^2 \end{bmatrix})^{-1} \begin{bmatrix} \sum y^k \\ \sum x^k y^k \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{k-1} & \frac{1}{(1-k)\sum x^k} \\ \frac{1}{(1-k)\sum x^k} & \frac{k}{(k-1)\sum (x^k)^2} \end{bmatrix} \begin{bmatrix} \sum y^k \\ \sum x^k y^k \end{bmatrix} \\
b &= \frac{\sum y^k}{k-1} + \frac{\sum x^k y^k}{(1-k)\sum x^k} \\
a &= \frac{\sum y^k}{(1-k)\sum x^k} + \frac{k \sum x^k y^k}{(k-1)\sum (x^k)^2}
\end{aligned}$$