Lecture 14

Kalman

- $x_{m+1} = Ax_m = w_m$
- $y_m = cx_m + v_m$
- $E(w) = 0, E(v) = 0$
- $VAR(w) = Q$
- $XX^T =$ CO-VAR matrix
- $E(ww^T) = Q, E(vv^T) = R$

Observer

- $\widetilde{x}_m = E(x_m)$
- $P_m = VAR(x_m - E(x_m)$
- $= E((x_m - \widetilde{x}_m)(x_m - \widetilde{x}_m)^T)$

Predict

- $\widetilde{x}_m = A\widetilde{x}_{m-1}$
- $P_m^+ = E((x_m - \widetilde{x}_m^+)(x_m - \widetilde{x}_m^+)^T)$
- $E(A()()^T A^T + 2A()v_m^T + v_m v_m^T)$
- $E(A()()^T A^T = v_m v_m^T)$
- $P_m^+ = AP_{m-1}A^T + Q$
- $\widetilde{x}_m^+ = A\widetilde{x}_{m-1}$

Update

- Observe y_m
- $\widetilde{x}_m = \widetilde{x}_m^+ = k(y - m - c\widetilde{x}_m^+)$

- $\widetilde{x}_m = \widetilde{x}_m^+ + kcx_m + kv_m - kc\widetilde{x}_m^+$
- $= (I + kc)(x_m - \widetilde{x}_m^+) + kv_m$
- Minimize P choosing K
- $E((x_m - \widetilde{x}_m)(x_m - widetildex_m)^T)$
- $\widetilde{x}_m = \widetilde{x}_m^+ + kcx_m + kv_m - kc\widetilde{x}_m^+$
- $x_m - \widetilde{x}_m = x_m - \widetilde{x}_m^+ - kcx_m - kv_m + kc\widetilde{x}_m^+$
- $(I - kc)(x_m - \widetilde{x}_m^+) - kv_m$
- $aa^T + ba^T = ab^T + bb^T$
- $(I - kc)(x_m - \widetilde{x}_m^+)(x_m - \widetilde{x}_m^+)^T(I - kc)^T + kv_m v_m^T k^T$

Kalman Steps

- Given $\widetilde{x}_0, P_0, Q, R$
- $\widetilde{x}^+(m) = A\widetilde{x}(m-1)$
- $P^+(m) = AP(m-1)A^T + Q$
- Update with y(m)
  - $k = \frac{P^+(m)C^T}{cP^+(m)C^T + R}$
  - $\widetilde{x}_m = \widetilde{x}_m^+ + k(y(m) - c\widetilde{x}^+(m))$
  - $P(m) = (1 - kc)P^+(m)$