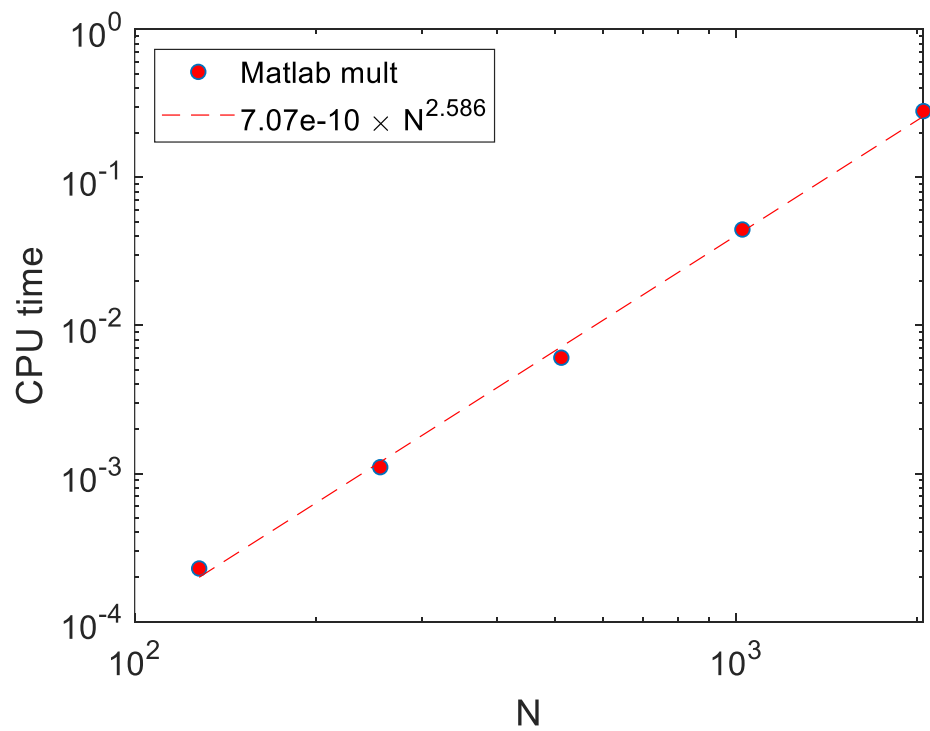
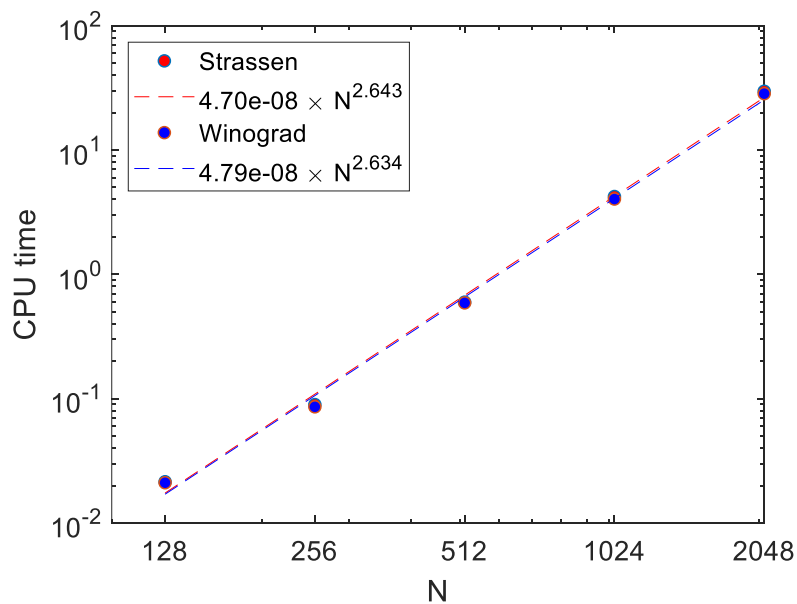


4X03: A3

1.



```

function [a,b,n,time] = bigOconstants(alg)
disp(alg)
n = 7:11;
n = 2.^n;
l = length(n);
time = zeros(l, 1);
for i=1:length(n)
    A = rand(n(i));
    B = rand(n(i));
    if alg == 1
        tic
        A*B;
        time(i) = toc;

    elseif alg == 2
        tic
        strassen(A,B);
        time(i) = toc;

    elseif alg == 3
        tic
        strassenw(A,B);
        time(i) = toc;
    end
end

end

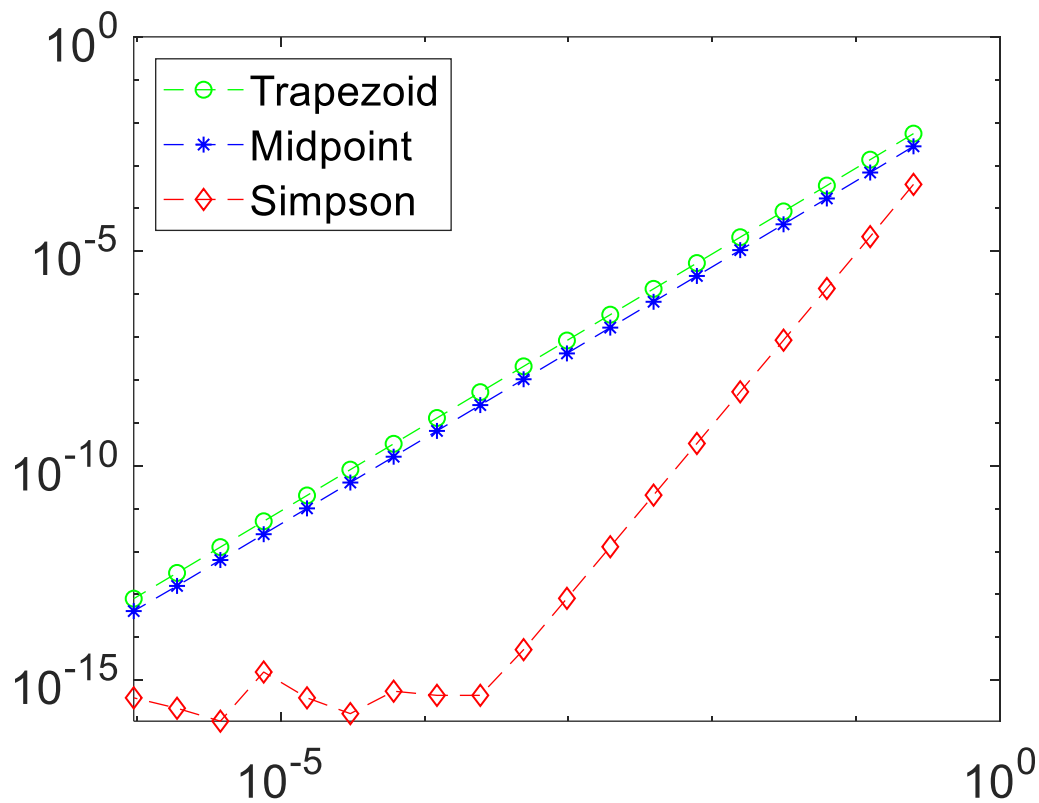
B = [ones(l,1), log(n)'];
coeff = B\log(time)';
a = exp(coeff(1));
b = coeff(2);
return
end

```

The a constants are much higher for the Strassen algorithms because while the overall b coefficient is lower, there is reduced numeric stability caused by the lowering of the b coefficient which is accounted for by utilizing a higher a coefficient to compensate.

2.

- a. The two lines are parallel because both trapezoid and midpoint are of the same function order. They just have different constants, but the slopes of the lines are the same.
- b. The slopes of trapezoid and midpoint are h^2 while Simpsons is irregular in this case h^2 .
- c. The wiggles the error of simpson having a higher order derivative (4^{th}) which can cause the function to oscillate where as both trapezoid and midpoint only utilize a second order derivative in their error term result in less pronounced oscillation.



```
function [eT, eM, eS] = error_int(f, a, b, n, ref)
eT = zeros(1,length(n));
eM = zeros(1,length(n));
eS = zeros(1,length(n));
for i=1:length(n)
    eT(i) = abs(ref - trapezoid(f,a,b,n(i)));
    eM(i) = abs(ref - midpoint(f,a,b,n(i)));
    eS(i) = abs(ref - simpson(f,a,b,n(i)));
end
end
```

```
function val = trapezoid(f, a, b, n)
h = (b-a)/n;
x = linspace(a,b,n+1);
range = x(2:n);
sum_val = sum(f(range));
val = h/2*(f(a) + 2*sum_val + f(b));
end
```

```
function val = midpoint(f, a, b, n)
h = (b-a)/n;
x = linspace(a,b,2*n+1);
range = x(2:2:2*n);
val = h * sum(f(range));
end
```

```

function val = simpson(f, a, b, n)
h = (b-a)/n;
x = linspace(a,b,n+1);
range2 = x(3:2:n-1);
range4 = x(2:2:n);
sum2 = 2 * sum(f(range2));
sum4 = 4 * sum(f(range4));
val = h/3 * (f(a) + f(b) + sum2 + sum4);
end

```

3.

$$f(x) = \sin(\pi x^2/3)$$

$$f''(x) = \left[-4x^2 \pi^2 \sin\left(\frac{\pi x^2}{3}\right) - 6\pi \cos\left(\frac{\pi x^2}{3}\right) \right] / 9$$

max @ 1

$$|f''(1)| = 2.757614$$

$$\text{Error} = \frac{-f''(x)}{12} (b-a)h^2 \leq 10^{-10}$$

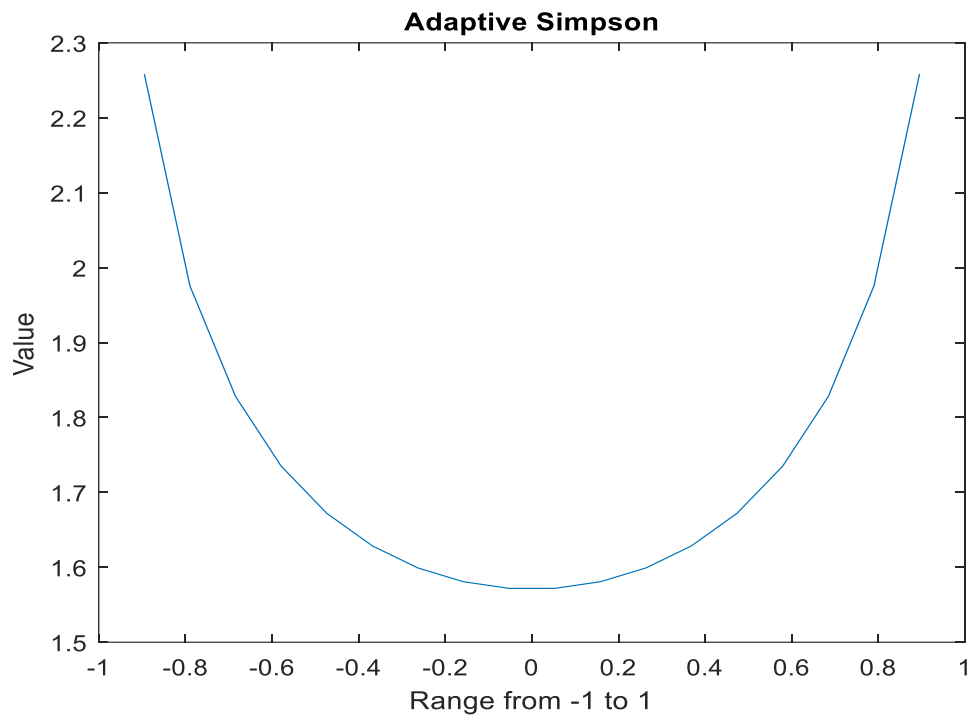
$$h = \frac{b-a}{n}$$

$$h \leq 2.088 \times 10^{-5}$$

$$\frac{1-0}{n} \leq 2.088 \times 10^{-5}$$

$$47893 \leq n$$

4.



5.

```

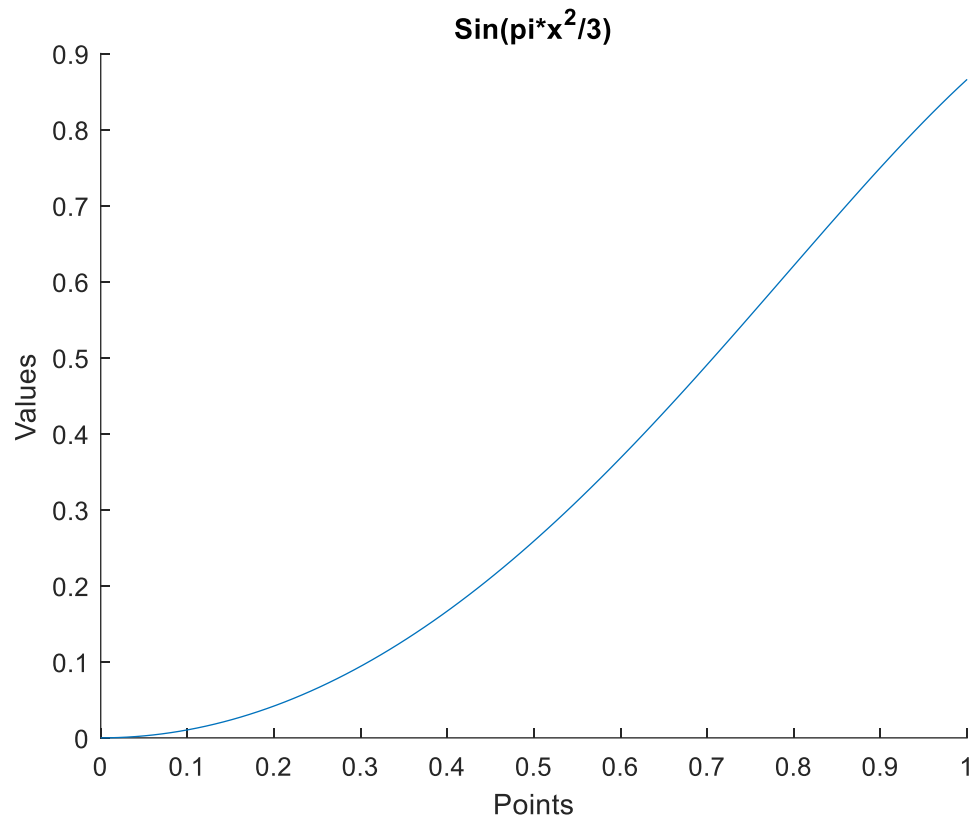
function [S, count] = inner_integral(f, a, b, tol, count, count_max)
global xpoints
count = count + 1;
h = b - a;
c = (a + b) / 2;
d = (a + c) / 2;
e = (c + b) / 2;
syms x
fa = f(x,a);
fb = f(x,b);
fc = f(x,c);
fd = f(x,d);
fe = f(x,e);
S1 = h/6*(fa + 4*fc + fb);
S2 = h/12 * (fa + 4*fd + 2*fc + 4*fe + fb);
E2 = (S2 - S1)/15;
xpoints = unique([xpoints, a, b, c, d, e]);
if count >= count_max
    S = S2 + E2;
else
    [Q1, count] = inner_integral(f, a, c, tol/2, count, count_max);
    [Q2, count] = inner_integral(f, c, b, tol/2, count, count_max);
    S = Q1 + Q2;
end
end

```

```
function Q = myquad2d(fun, a, b, c, d, tol)
f = matlabFunction(inner_integral(fun,c,d,tol,0,20));
Q = adaptive_simpson(f,a,b,tol, 0, 20);
end
```

Error from script = 1.1277e-07

6.



C1 = 16651

C2 = 165

C1 / C2 = 100.9152

Adaptive simpson error = 3.1489×10^{-11}

Composite simpson error = $5.5511e^{-17}$

7.

