Lecture 9

Stochastic Process

- $E(x) = \int xP_X(x)dx$
- $E(x) linear$
- $E(\lambda_1 + lambda_2 y)$
- $= \lambda_1 E(x) + \lambda_2 E(y)$
- E(c) = c

Variance

- $VAR(x) = E((E(x) - x)^2)$
- $= E(x^2) - E(x)^2$
- $VAR(x) >= 0, VAR(c) = 0$ bi-linear quadractic
- $VAR(ax) = a^2 VAR(x)$
- $CO - VAR(x, y) = E((E(x) - x) * (E(y) - y))$
- $VAR(x + y) = VAR(X) + VAR(Y) + 2CO - VAR(x, y)$

2 Dimension Example

- $(\begin{matrix} X \\ Y \end{matrix}) * (X \quad Y)^T$
- $= (\begin{matrix} X^2 & YX \\ YX & Y^2 \end{matrix})$
- $= (\begin{matrix} VAR(X) & CO - VAR(X, Y) \\ CO - VAR(X, Y) & VAR(Y) \end{matrix})$ = covariance matrix
- $$\begin{matrix} x \\ y * x \quad y \quad z \\ z \end{matrix} = (\begin{matrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 \end{matrix})$$
- The above matrix is symmetric

Noise

- $Whitenoise \frac{1}{\epsilon(t)}$
- All frequencies have same probability
- Gaussian noise shaped like gauss
- Pink noise $\frac{12DB}{octave}$

Stochastic (Model Fitting)

- Model Y = aX + b

Equations

- $Y_k = ax_k + b + \epsilon_k$
- $\epsilon_k = N(0, \sigma_x^2)$
- $VAR = \sigma_x^2$
- $$CO - VAR(\epsilon_k, \epsilon_l) = \begin{matrix} 0 & k! = l \\ \sigma_x^2 & k = l \end{matrix}$$

Monte Carlo

- $\frac{1}{N} \sum_{k=1}^{N} y_k$
- $= \frac{1}{N} \sum_{k=1}^{N} aX_k + \epsilon_k$
- $= \frac{1}{N} \sum_{k=1}^{N} aX_k + \frac{1}{N} \sum_{k=1}^{N} \epsilon_k$

Ensemble Averaging

- By adding together all the noise, due to the noise being guassian the noise is equal to 0. So that gives you a meaningful measurement.

Maximum Likelihood (Estimator)

- $Y_k = aX_k + b$ -> Guass

- $P(Y_k - aX_kb)$ -> Want to maximize the probability
- $L = \prod_{k=1}^{N} P(y_k - ax_k - b)$
- $\frac{MaxL}{a,b} = \prod P(y_k - ax_k - b)$
- $Y_k - aX_k - b = \epsilon_k$
- $\epsilon_k = N(0, \sigma_x^2)$
- $\prod e^{\frac{\frac{-1}{2}(y_k - ax_k - b)^2}{\sigma_x^2}}$
- Maximize L
- First pull log
- Max(f) = Max(log(f))
- $e^{\sum_{k=1}^{N} \frac{\frac{-1}{2}(y_k - ax_k - b)^2}{\sigma_x^2}}$
- Pull Log
- $\frac{-1}{2sigma_x^2} \sum (y_k - ax_k - b)^2$ -> same as before

Stochastic System

- $x_{m+1} = f(x_m) = \epsilon_k$
- $y_m = h(x_m) + u_k$