Name:	ID:

McMaster University
MECHTRON 4AX3
Dr. M. v. Mohrenschildt

1 Linear Least Square

To estimate the DC b of a signal with a known slope a we use a linear least square with M points to estimate b in model $y(n) = ax + b$. Set up your LLS and give your estimate for b.

2 Bayesian position estimation

having either a R or G marker as given in F.
Initially we know we are in the first row, so x_0 is given below. We observe an R, the probability distribution of that is given in S_R. What is your estimate position x_1 ?

$$F = \begin{array}{|c|c|}\hline G & R \\ \hline R & G \\ \hline\end{array},\ x_0 = \begin{array}{|c|c|}\hline 0 & 0 \\ \hline .5 & .5 \\ \hline\end{array},\ p(R|x) = \begin{array}{|c|c|}\hline .1 & .4 \\ \hline .4 & .1 \\ \hline\end{array}$$

3 Kalman Filter

During lectures we try to estimate the class size. Each lecture some students might not show up, or we might have guests listening, causing a variance of $\sigma_l^2 = 10$. Additionally students can drop or add with a variance of $\sigma_c^2 = 5$. Initially, we know for sure that 100 students registered. In the first lecture we observe 91 students.
Here are the Kalman filter equations:

- **Predict**

$$\begin{aligned}\tilde{x}^+(n+1) &= A\tilde{x}(n) + Bu(n) \\ P^+(n+1) &= AP(n)A^T + Q\end{aligned}$$

- **Update** after measurement $y(n)$

$$\begin{aligned}K &= P^+(n)C^T(CP^+(n)C^T + R)^{-1} \\ \tilde{x}(n) &= \tilde{x}^+(n) + K\big(y(n) - C\tilde{x}^+(n)\big) \\ P(n) &= (I - KC)P^+(n)\end{aligned}$$

Give the value of all qualities $A, B, C, Q, R, x(0), P(0)$ as you set up and compute the class estimate $\tilde{x}(1)$ and $P(1)$.

4 Bellman Equation

Given a Markov system with states $s_i \in S$ and actions $a_i \in A$ and the probability $\mathcal{P}_{ss'}^a$ and the function $\mathcal{R}_{ss'}^a$.

- Give a value function $V(s)$, how can we compute the corresponding policy $\pi(s,a)$?
- For a policy π_1 we know that for some specific state s we have $V^{\pi_1}(s) = v_1$, and for some other policy π_2 we know that for the same state s $V^{\pi_2}(s) = v_2$. What do we know of the value of $V^{\pi^*}(s)$ of the optimal policy π^* ?
- Given the Q-function $Q(s,a)$, how can we obtain the value function $V(s)$?
- We are in state s and $V(s) = V(s')$ for all states s' with $\mathcal{P}_{ss'}^a > 0$. What is the value of the optimal policy $\pi^*(s,a)$ in s ?
- Given a greedy policy $\pi(s)$ (its a function $S \to A$ not a probability) How can we compute the Q-function $Q^\pi(s,a)$, meaning how can one evaluate the policy π.

Give formulas and short precise statements, not essays !