Distributed Bayesian Filters for Multi-Vehicle Network by Using Latest-In-and-Full-Out Exchange Protocol of Observations

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***Abstract*—This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a network of unmanned ground vehicles (UGVs). The DBF utilizes the Latest-In-and-Full-Out (LIFO) local exchange protocol of sensor measurements for data communication within the network. Different from existing statistics dissemination-based approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only exchanges with neighboring UGVs a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs to scale linearly with the size of the network. Under the condition of fixed and undirected topology, LIFO can guarantee non-intermittent dissemination of all measurements over the network within finite time. Two types of LIFO-based DBF algorithms are then derived to estimate individual probability density function (PDF) for a static target and for a moving target, respectively. For the static target, each UGV locally fuses the newly received measurements while for the moving target, a set of historical measurements is stored and sequentially fused. The consistency of LIFO-based DBF is proved that the estimated target position converges in probability to the true target position. The effectiveness of this method is demonstrated in both simulation and experiment.**

I. INTRODUCTION

Distributed filtering that focuses on using a group of networked UGVs to collectively infer environment status has been used for various applications, such as intruder detection [1], moving target tracking [2] and object localization [3]. Several techniques have been developed for distributed filtering. For example, Olfati-Saber (2005) proposed a distributed Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [4]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Madhavan et al. (2004) presented a distributed extended Kalman filter for nonlinear systems [5]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with environmental features. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [6]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. As a generic filtering scheme for general system dynamics and arbitrary noise distributions, distributed Bayesian filters (DBF) have received increasing interest during past years [7], [8] and is the focus of this study. It is worth noting that Bayesian filters can be reduced to Kalman filters and particle filters under appropriate conditions [9].

The design of distributed filtering algorithms depends on the communication topology of multi-UGV network, which can be classified into two types: fusion center (FC)-based and neighborhood (NB)-based. In FC-based approaches, each UGV uses a filter to estimate local statistics of environment status based on its own measurement. The local statistics is then transmitted (possibly via multi-hopping) to a single FC, where a global posterior distribution (or statistical moments in DKF [10]) is calculated at each filtering cycle after receiving all local information [11], [12]. In NB-based approaches, a set of UGVs execute distributed filters to estimate individual posterior distribution. Consensus of individual estimates is achieved by solely communicating statistics and/or measurements within local neighbors of these UGVs. The NB-based methods have become popular in recent years since such approaches do not require complex routing protocols or global knowledge of the network and therefore are robust to changes in network topology and to link failures.

So far, most studies on NB-based distributed filtering have mainly focused on the so-called *statistics dissemination* strategy that each UGV actually exchanges statistics, including posterior distributions and likelihood functions, with neighboring UGVs [13]. This strategy can be further categorized into two types: leader-based and consensus-based. In the former, statistics is sequentially passed and updated along a path formed by active UGVs, called leaders. Only leaders perform filtering based on its own measurement and received measurements from local neighbors. For example, Sheng et al. (2005) proposed a multiple leader-based distributed particle filter with Gaussian Mixer for target tracking [14]. Sensors are grouped into multiple uncorrelated cliques, in each of which a leader is assigned to perform particle filtering and the particle information is then exchanged among leaders. In consensus-based distributed filters, every UGV diffuses statistics among neighbors, via which global agreement of the statistics is achieved by using consensus protocols [10], [15], [16]. For example, Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-linear-average con-

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sensus algorithm when local likelihood functions belong to the exponential family of distributions [17]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [7]. Other examples can be found in [8], [18].

Despite the popularity of statistics dissemination strategy, exchanging statistics can consume high communication resources. One promising remedy is to disseminate measurement instead of statistics among neighbors, which, however, has not been fully exploited. One pioneering work was done by Coates et al. (2004), who used adaptive encoding of measurements to minimize communication overhead [19]. Ribeiro et al. (2006) exchanged quantized measurements along with error-variance limits considering more pragmatic signal models [20]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each UGV has a complete set of measurements of other UGVs for executing particle filtering [21]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a complete graph that every pair of distinct UGVs is directly connected by a unique edge, which is not always feasible in reality.

This paper extends existing works by introducing a Latest-In-and-Full-Out (LIFO) protocol into distributed Bayesian filters (DBF) for networked UGVs. Each UGV is only allowed to broadcast measurements to its neighbors by using single-hopping and then implements individual Bayesian filter locally after receiving transmitted measurements. The main benefit of using LIFO is on the reduction of communication burden, with the transmission data volume scaling linearly with the UGV number, while a statistics dissemination-based strategy can suffer from the order of environmental size. The proposed LIFO-based DBF has following properties: (1) For a fixed and undirected network, LIFO guarantees the global dissemination of measurements over the network in a non-intermittent manner. (2) The corresponding DBF ensures the consistency of estimated target position, i.e., the estimated position converges in probability to the true target position when the number of measurements tends to infinity.

The rest of this paper is organized as follows: The problem of distributed Bayesian filtering is formulated in Section II. The LIFO-based DBF algorithm is described in Section III, followed by the proof of consistency in Section IV. Simulation results are presented in Section V and Section VI concludes the paper.

II. PROBLEM FORMULATION

Consider a network of N UGVs in a bounded two-dimensional space S. The aim of UGVs is to efficiently localize a target in S. Each UGV is equipped with a noisy sensor for environmental perception. Due to the limit of communication range, each UGV can only share information locally by exchanging measurements with its neighbors. The Bayesian filter is run locally on each UGV based on its own and received measurements via single-hopping to estimate true position of the target.

A. Target and Sensor Model

The target motion takes a deterministic discrete-time model that can be described by

$$x^g_{k+1} = f(x^g_k, u^g_k), \tag{1}$$

where $x^g_k \in \mathbb{R}^2$ denotes the target position at time k; u^g_k represents the control input of the target.

Each UGV constantly measures the target position and the sensor measurement of i^{th} UGV can be described by a stochastic model:

$$z^i_k = h_i(x^g_k, w^i_k; y^i_k), \tag{2}$$

where w^i_k is the white measurement noise and $y^i_k = [x^i_k; \theta^i_k]$ represents the sensor state, consisting of the sensor position x^i_k and direction θ^i_k. The measurement function h_i depends on the type of the sensor. Let $\mathcal{F}(y^i_k)$ denote the sensor's sensing domain, h_i for several typical sensor types are defined as follows [22]:

Range-only sensors: when the target is within the sensor's sensing domain, the measurement only depends on the relative distance between the sensor and the target.

$$h_i(x^g_k, w^i_k; y^i_k) = \begin{cases} \|x^g_k - x^i_k\|_2 + w^i_k & \text{if } x^g_k \in \mathcal{F}(y^i_k) \\ \emptyset & \text{if } x^g_k \notin \mathcal{F}(y^i_k) \end{cases}. \tag{3}$$

Bearing-only sensors: when the target is within the sensor's sensing domain, the measurement only depends on the relative bearing between the sensor and the target.

$$h_i(x^g_k, w^i_k; y^i_k) = \begin{cases} \measuredangle(x^g_k - x^i_k) + w^i_k & \text{if } x^g_k \in \mathcal{F}(y^i_k) \\ \emptyset & \text{if } x^g_k \notin \mathcal{F}(y^i_k) \end{cases}, \tag{4}$$

where $\measuredangle$ denotes the relative angle from the sensor to the target.

A probabilistic sensor model that describes the conditional probability of a certain measurement given sensor and target state is a key component for Bayesian filtering [23]. We define a likelihood function to represent the probability that the target is detected by the sensor:

$$p^i_{1,k} = P(Z^i_k \neq \emptyset | x^g_k; y^i_k) \in [0,1]\,,\ x^g_k \in S, \tag{5}$$

where Z^i_k is a random variable denoting the possible measurement result; x^i_k denotes the i^{th} sensor's position. Correspondingly, the likelihood function for no target being detected is:

$$p^i_{0,k} = P(Z^i_k = \emptyset | x^g_k; y^i_k) = 1 - p^i_{1,k}. \tag{6}$$

The combination of Eq. (5) and (6) forms the probabilistic model for a sensor. If w^i_k is a zero-mean Gaussian white noise, then the probabilistic sensor model can be explicitly described as

$$p^i_{1,k} = \begin{cases} \mathcal{N}(\bar{z}^i_k, \Sigma^i_k) & \text{if } x^g_k \in \mathcal{F}(y^i_k) \\ 0 & \text{if } x^g_k \in \mathcal{F}(y^i_k) \end{cases}, \tag{7}$$

where $\bar{z}^i_k$ is the nominal value of the measurement and equals $\|x^g_k - x^i_k\|$ and $\measuredangle(x^g_k - x^i_k)$ for range-only and bearing-only sensors, correspondingly.

Consequently,

$$p^i_{0,k} = \begin{cases} 0 & \text{if } x^g_k \in \mathcal{F}(y^i_k) \\ 1 & \text{if } x^g_k \in \mathcal{F}(y^i_k) \end{cases}. \tag{8}$$

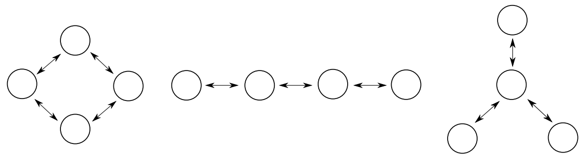


Fig. 1: Three types of topologies: (a) ring topology; (b) line topology; (c) star topology

For the purpose of simplicity, we will not explicitly write y_k^i in the probabilistic sensor model for the rest of the paper.

Remark 1. *Given the knowledge of current target position and UGV state, current measurement by each UGV can be considered conditionally independent from its own past measurements and those by other UGVs [24].*

Remark 2. *The proposed LIFO protocol to be described in Section III and the consistency property proved in Section IV are applicable for general sensors, not limited to the ones described in this section.*

B. Graphical Model of Communication Topology

The UGV network is assumed to be connected, i.e., there exists a path, either direct or indirect, between every pair of UGVs. Under this assumption, consider an undirected and fixed graph $G = (V, E)$, where $V = \{1, \ldots, N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. The adjacency matrix $M = \left[M_{(ij)}\right]$ describes the communication topology of G:

$$M_{(ij)} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where $M_{(ij)}$ denotes the entity of adjacency matrix. The notation $M_{(ij)} = 1$ indicates that a communication link exists between i^{th} and j^{th} UGV and $M_{(ij)} = 0$ indicates no communication between them.

The *direct neighborhood* of i^{th} UGV is defined as $\mathcal{N}_i = \left\{j | M_{(ij)} = 1, \forall j \in \{1, \ldots, N\}\right\}$. All the UGVs in $\mathcal{N}_i$ can directly exchange information with i^{th} UGV via single hopping. We also define another set $\mathcal{Q}_i$, called *available neighborhood*, that contains indices of UGVs whose measurements can be received by the i^{th} UGV via single or multiple hoppings. Therefore $\mathcal{N}_i \subseteq \mathcal{Q}_i$. Fig. 1 illustrates three types of typical topologies: ring [25], line [26], and star [27]. All of them are undirected and connected topologies.

C. Distributed Bayesian Filter for Multiple UGVs

The generic distributed Bayesian filter (DBF) is introduced in this section. Let $X_k^g \in S$ be a random variable that each UGV uses to represent the position of the target at time k. The probability density function (PDF) of X_k^g, called *individual PDF*, of i^{th} UGV is then defined as $P_{pdf}^i(X_k^g|\mathbf{z}_{1:k}^i)$, where $\mathbf{z}_{1:k}^i$ denotes the set of measurements by i^{th} UGV and by UGVs in $\mathcal{Q}_i$ that have been transmitted to i^{th} UGV by time k. The initial individual PDF $P_{pdf}^i(X_0^g|\mathbf{z}_0^i)$ is constructed given all available prior information including past experience and environmental knowledge. It is important to initialize the individual PDF such that the probability density of true target position is nonzero, i.e., $P_{pdf}^i(X_0^g|\mathbf{z}_0^i) \neq 0$.

Under the framework of DBF, the individual PDF is recursively estimated by two steps: prediction step and updating step, based on measurements of i^{th} UGV and UGVs in $\mathcal{Q}_i$.

1) Prediction: At time k, the prior individual PDF $P_{pdf}^i(X_{k-1}^g|\mathbf{z}_{1:k-1}^i)$ is first predicted forward by using the Chapman-Kolmogorov equation:

$$P_{pdf}^i(X_k^g|\mathbf{z}_{1:k-1}^i) = \int\limits_{X_{k-1}^g \in S} P(X_k^g|X_{k-1}^g) P_{pdf}^i(X_{k-1}^g|\mathbf{z}_{1:k-1}^i) dX_{k-1}^g, \tag{9}$$

where $P(X_k^g|X_{k-1}^g)$ represents the state transition probability of the target, based on the Markovian motion model (Eq. (1)). For the deterministic motion model, the state transition probability is simplified to be

$$P(X_k^g = c_1|X_{k-1}^g = c_2) = \begin{cases} 1 & \text{if} \quad c_1 = f(c_2, u_1) \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

2) Updating: The i^{th} individual PDF is then updated by Bayes' theorem using the set of newly received measurements at time k, i.e., $\mathbf{z}_k^i$:

$$P_{pdf}^i(X_k^g|\mathbf{z}_{1:k}^i) = K_i P_{pdf}^i(X_k^g|\mathbf{z}_{1:k-1}^i) P(\mathbf{z}_k^i|X_k^g) \tag{11}$$

where K_i is a normalization factor, given by:

$$K_i = \left[\int\limits_{X_k^g \in S} P_{pdf}^i(X_k^g|\mathbf{z}_{1:k-1}^i) P(\mathbf{z}_k^i|X_k^g) dX_k^g\right]^{-1};$$

$P_{pdf}^i(X_k^g|\mathbf{z}_{1:k}^i)$ is the posterior individual PDF; $P(z_k^i|X_k^g)$ is the likelihood function of measurement z_k^i, as described in Eq. (7) and (8).

III. Distributed Bayesian Filter via Latest-In-and-Full-Out Protocol

This study proposes a Latest-In-and-Full-Out (LIFO) protocol for measurement exchange and derives two corresponding distributed Bayesian filtering (DBF) algorithms, shorted as LIFO-DBF. The data communication in LIFO is synchronized with the execution of DBF. In each step, LIFO only allows single-hopping communication within the direct neighborhood, but is able to broadcast measurements of each UGV to any other agent after a finite number of steps. The individual PDF is forward predicted and updated in DBF after each LIFO cycle. The theoretical analysis show that LIFO-DBF can ensure the consistency of distributed estimation while requiring much less communication burden than statistics dissemination-based methods.

A. Latest-In-and-Full-Out (LIFO) Protocol

Under LIFO, each UGV contains a communication buffer (CB) to store its latest knowledge of measurements of all UGVs:

$$\mathbf{z}_k^{CB,i} = \left[z_{k_1^i}^1, \ldots, z_{k_N^i}^N, y_{k_1^i}^1, \ldots, y_{k_N^i}^N\right],$$

where $z_{k_j^i}^j$ represents the measurement made by j^{th} UGV at time k_j^i and $y_{k_j^i}^j$ denotes the sensor state when the associated

Algorithm 1 LIFO Protocol

(1) Initialization: The CB of i^{th} UGV is initialized when $k = 0$:

$$z^j_{k^i_j} = \varnothing,\ k^i_j = 0,\ j = 1, \dots, N$$

(2) At k^{th} step for i^{th} UGV :

(2.1) Receiving Step:

The i^{th} UGV receives all CBs of its direct neighborhood $\mathcal{N}_i$, each of which corresponds to the (k-1)-step CB of a UGV in $\mathcal{N}_i$. The received CB from l^{th} ($l \in \mathcal{N}_i$) UGV is denoted as

$$\mathbf{z}^{CB,l}_{k-1} = \left[z^1_{(k-1)^l_1}, \dots, z^N_{(k-1)^l_N} \right],\ l \in \mathcal{N}_i$$

(2.2) Observation Step:

The i^{th} UGV updates $z^j_{k^i_j}$ ($j = i$) by its own measurement at current step:

$$z^j_{k^i_j} = z^i_k,\ k^i_j = k,\ \text{if } j = i.$$

(2.3) Comparison Step:

The i^{th} UGV updates other elements of its own CB, i.e., $z^j_{k^i_j}$ ($j \neq i$), by selecting the latest information among all received CBs from $\mathcal{N}_i$. For all $j \neq i$,

$$l_{\text{latest}} = \underset{l \in \mathcal{N}_i,\ i}{\operatorname{argmax}} \left\{ (k-1)^i_j, (k-1)^l_j \right\}$$

$$z^j_{k^i_j} = z^j_{(k-1)^{l_{\text{latest}}}_j},\ k^i_j = (k-1)^{l_{\text{latest}}}_j$$

(2.4) Sending Step:

The i^{th} UGV broadcasts its updated CB to all of its neighbors defined in $\mathcal{N}_i$.

(3) $k \leftarrow k + 1$ until stop

measurement $z^j_{k^i_j}$ is made. For the purpose of simplicity, we will not explicitly write $\left[y^1_{k^i_1}, \dots, y^N_{k^i_N} \right]$ in $\mathbf{z}^{CB,i}_k$ for the rest of the paper. Note that under LIFO, $\mathcal{Q}_i = \{1, \dots, N\} \setminus \{i\}$, which will be proved in Corollary 1. At time k, $z^j_{k^i_j}$ is received and stored in i^{th} UGV's CB, in which k^i_j is the latest time of j^{th} UGV's measurement that is available to i^{th} UGV. Due to the communication delay, $k^i_j < k, \forall j \neq i$ and $k^i_i = k$ always hold. The **LIFO protocol** is stated in Algorithm 1.

Fig. 2 illustrates the LIFO cycles with 3 UGVs using a line topology. For general graphs, we have the following proposition:

Proposition 1. *For a fixed and undirected network of N UGVs, the latest measurements of i^{th} and j^{th} UGV are exchanged via the shortest path(s) under LIFO. The delay of exchange $\tau_{i,j}$ is equivalent to the length of the shortest path(s) between them.*

Proof. Since the network is connected, there exists a minimum integer $\tau_{i,j}$ such that $M^{\tau_{i,j}}_{(i,j)} > 0$ and $\tau_{i,j}$ is the length of a shortest path between i^{th} and j^{th} UGV. Under the LIFO, the latest measurement of i^{th} UGV will always be propagated via the CBs of UGVs along a shortest path between i^{th} and j^{th} UGV. Therefore, the latest measurement that j^{th} UGV receives from i^{th} UGV is delayed by $\tau_{i,j}$ iterations of communication. □

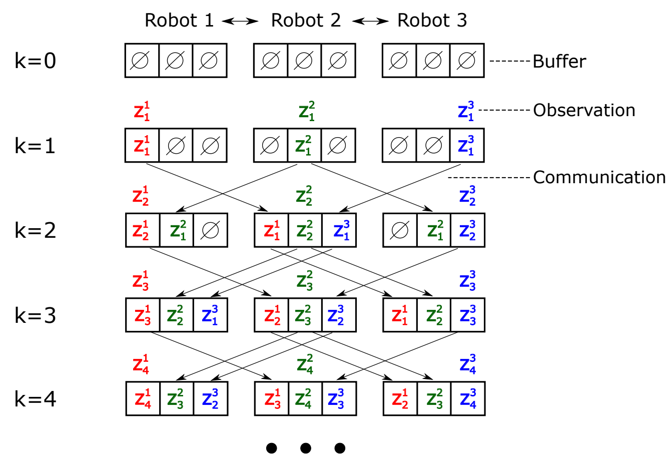


Fig. 2: Example of LIFO with three UGVs using line communication topology

Corollary 1. *For the same topology assumption in Proposition 1, all elements in $\mathbf{z}^{CB,i}_k$ under LIFO become filled when $k \geq N$. This also implies $\mathcal{Q}_i = \{1, \dots, N\} \setminus \{i\}$.*

Corollary 2. *For the same topology assumption in Proposition 1, once all elements in $\mathbf{z}^{CB,i}_k$ are filled, the updating of each element is non-intermittent.*

Compared to statistics dissemination, LIFO is generally more communication-efficient for distributed filtering. To be specific, assume the environment S is represented by an $M \times M$ grid. For a network of N UGVs, the transmitted data of LIFO between each pair of UGVs are only the CB of each UGV and the corresponding sensor states, the size of which is $O(N)$, scaling linearly with UGV number. On the contrary, the length of transmitted data for a statistics dissemination approach that transmits unparameterized posterior distributions or likelihood functions is $O(M^2)$, which is in the order of environmental size. Since M is generally much larger than N in applications such as target localization and environment exploration, LIFO requires much less communication resources.

B. Algorithm of LIFO-DBF for Static Target

This section derives the LIFO-DBF algorithm for localizing a static target. It is assumed that, all UGVs know the probabilistic sensor model of other UGVs. According to Corollary 2, $\mathbf{z}^i_k = \mathbf{z}^{CB,i}_k$ and $\mathbf{z}^i_{1:k} = \mathbf{z}^{CB,i}_{1:k} = \left[z^1_{1:k^i_1}, \dots, z^N_{1:k^i_N} \right]$. The assumption of static target can simplify the Bayesian filter as the prediction step becomes trivial. At time k, the i^{th} UGV starts from the last-step individual PDF that is locally stored, i.e., $P^i_{pdf}(x|\mathbf{z}^i_{1:k-1})$. The i^{th} individual PDF is then updated by fusing all measurements in $\mathbf{z}^i_k$:

$$\begin{aligned} P^i_{pdf}(X^g|\mathbf{z}^i_{1:k}) &= K_i P^i_{pdf}(X^g|\mathbf{z}^i_{1:k-1}) P(\mathbf{z}^i_k|X^g) \\ &= K_i P^i_{pdf}(X^g|\mathbf{z}^i_{1:k-1}) \prod_{j=1}^{N} P(z^j_{k^i_j}|X^g) \end{aligned} \tag{12}$$

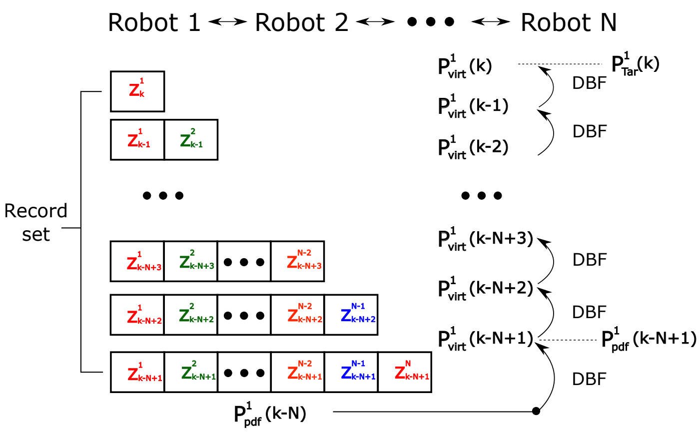


Fig. 3: Example of LIFO-DBF for 1st UGV at time k. Networked UGVs take a line topology. The stored individual PDF is represented by $P^1_{pdf}(k-N)$. The UGV first calculates $P^1_{virt}(k-N+1)$ using DBF and stores it as $P^1_{pdf}(k-N+1)$. Repeating DBF until obtaining $P^1_{pdf}(k)$, which is the individual PDF at time k. In this example, $\Omega^1_\xi = \{1, 2, \ldots, N+1-\xi\}$, $\xi = 1, \ldots, N$.

where

$$K_i = \left[\int\limits_{X^g \in S} P^i_{pdf}(X^g|\mathbf{z}^i_{1:k-1}) \prod_{j=1}^{N} P(z^j_{k^i_j}|X^g)dX^g \right]^{-1}.$$

C. Algorithm of LIFO-DBF for Moving Target

This section derives the LIFO-DBF for localizing a moving target. Instead of storing last-step PDF, at time k each UGV maintains an individual PDF of time $(k-N)$ and a collection of historical measurements, called the *record set*, from time $(k-N+1)$ to k. The i^{th} individual PDF is then alternatively predicted and updated by using the aforementioned Bayesian filter (Eq. (9) and (11)) from $(k-N)$ to k. Fig. 3 illustrates the LIFO-DBF procedure for the 1st UGV as an example.

Let Ω^i_ξ $(\xi = 1, \ldots, N)$ denote the index set of UGVs whose measurement at time $(k-N+\xi)$ is stored in i^{th} UGV's record set. The **LIFO-DBF algorithm** for moving target is then stated in Algorithm 2.

It is worth noting that, for the static target, each UGV only needs current step CB to update individual PDFs. Therefore, besides storing its own individual PDF, only current-step CB is stored in an UGV's memory and all previous CBs can be discarded, which means that the size of occupied memory is $O(N+M^2)$. On the contrary, for the moving target, each UGV needs to store a set of historical measurements with size of $O(N^2)$ and an individual PDF with size $O(M^2)$, therefore the size of the occupied memory for each UGV is $O(M^2+N^2)$. This is generally larger that that of statistics dissemination-based methods, the storage size of which is $O(M^2)$. In addition, additional computation power is needed for LIFO-DBF compared to statistics dissemination-based methods. Therefore, LIFO-DBF sacrifices storage size and computation efficiency for reducing communication burden.

Algorithm 2 LIFO-DBF Algorithm

For i^{th} UGV at k^{th} step:
After updating CB by Algorithm 1,
(1) The stored individual PDF for time $(k-N)$ is:

$$P^i_{pdf}(x_{k-N}|z^1_{1:k-N}, \ldots, z^N_{1:k-N})$$

(2) Initialize a virtual PDF by assigning the individual PDF to it:

$$P^i_{virt}(x_{k-N}) = P^i_{pdf}(x_{k-N}|z^1_{1:k-N}, \ldots, z^N_{1:k-N})$$

(3) From $\xi = 1$ to N, iteratively repeat two steps of Bayesian filtering:
(3.1) Prediction

$$P^{pre}_{virt}(x_{k-N+\xi}) = \int P(x_{k-N+\xi}|x_{k-N+\xi-1}) P^i_{virt}(x_{k-N+\xi-1}) dx_{k-N+\xi-1}$$

(3.2) Updating

$$P^i_{virt}(x_{k-N+\xi}) = K_\xi P^{pre}_{virt}(x_{k-N+\xi}) \prod_{j \in \Omega^i_\xi} P(z^j_{k-N+\xi}|x_{k-N+\xi})$$

$$K_\xi = 1 / \int_S P^{pre}_{virt}(x_{k-N+\xi}) \prod_{j \in \Omega^i_\xi} P(z^j_{k-N+\xi}|x_{k-N+\xi}) dx_{k-N+\xi}$$

(3.3) When $\xi = 1$, if $z^j_{k_N+1} \neq \emptyset$ for $\forall j \in \{1, \ldots, N\}$, store the virtual PDF as the individual PDF for time $(k-N+1)$:

$$P^i_{pdf}(x_{k-N+1}|z^1_{1:k-N+1}, \ldots, z^N_{1:k-N+1}) = P^i_{virt}(x_{k-N+1}).$$

(4) Individual PDF of i^{th} UGV at time k is $P^i_{pdf}(x_k|\mathbf{z}^i_{1:k}) = P^i_{virt}(x_k)$.

This is desirable for real applications as local memory of vehicles are usually abundant compared to the limited bandwidth for communication.

IV. Proof of Consistency

This section proves the consistency of the maximum a posteriori (MAP) estimator of LIFO-DBF under unbiased sensors (sensors without offset). An estimator of a state is said to be consistent if it converges in probability to the true value of the state [28]. Consistency is an important metric for stochastic filtering approaches [9] and it differs from the concept of consensus; consensus implies that the estimation results of all sensors converge to a same value, while consistency not only implies achieving consensus asymptotically, but also requires the estimated value converge to the true value (true target position).

We first prove the consistency for static UGVs and static target. The consistency for moving UGVs and moving target are subsequently proved. For simplicity and clarity, we assume S is a finite set (e.g. a finely discretized field) and x^g is the true position of target.

A. Static UGVs and Static Target

The consistency of LIFO-DBF for static UGVs and target is stated as follows:

Theorem 1. *Assume both UGVs and the target are static, and the sensors are unbiased, the MAP estimator of target position converges in probability to the true position of the target using LIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X_{MAP} = x^g|\mathbf{z}^i_{1:k}) = 1,\ i = 1, \ldots, N,$$

where

$$X^{MAP} = \arg\max_{X^g} P^i_{pdf}(X^g|\mathbf{z}^i_{1:k}).$$

Proof. First look at the asymptotic behavior of $P^i_{pdf}(X^g|\mathbf{z}^i_{1:k})$. The batch form of DBF at k^{th} step is:

$$P^i_{pdf}(X^g|\mathbf{z}^i_{1:k}) = P^i_{pdf}(X^g|z^1_{1:k^i_1}, \ldots, z^N_{1:k^i_N}) \tag{13a}$$

$$= \frac{P^i_{pdf}(X^g) \prod_{j=1}^{N} \prod_{l=1}^{k^i_j} P(z^j_l|X^g)}{\sum_{X^g\in S} P^i_{pdf}(X^g) \prod_{j=1}^{N} \prod_{l=1}^{k^i_j} P(z^j_l|X^g)}. \tag{13b}$$

The decomposition into multiplication in Eq. (13b) results from the conditional independence of measurements given X^g.

Comparing the conditional probability of an arbitrary position $x \in S$ with that of target position x^g:

$$\frac{P^i_{pdf} X = x|\mathbf{z}^i_{1:k})}{P^i_{pdf}(X = x^g|\mathbf{z}^i_{1:k})} = \frac{P^i_{pdf}(x) \prod_{j=1}^{N} \prod_{l=1}^{k^i_j} P(z^j_l|x)}{P^i_{pdf}(x^g) \prod_{j=1}^{N} \prod_{l=1}^{k^i_j} P(z^j_l|x^g)}. \tag{14}$$

Take the logarithm of Eq. (14) and average it over k steps:

$$\frac{1}{k} \ln \frac{P^i_{pdf}(x|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})} = \frac{1}{k} \ln \frac{P^i_{pdf}(x)}{P^i_{pdf}(x^g)} + \sum_{j=1}^{N} \frac{1}{k} \sum_{l=1}^{k^i_j} \ln \frac{P(z^j_l|x)}{P(z^j_l|x^g)}. \tag{15}$$

Since $P^i_{pdf}(x)$ and $P^i_{pdf}(x^g)$ are bounded and we can initialize $P^i_{pdf}(x^g)$ to be nonzero, then

$$\lim_{k\to\infty} \frac{1}{k} \ln \frac{P^i_{pdf}(x)}{P^i_{pdf}(x^g)} = 0. \tag{16}$$

Utilizing the facts: (1) z^j_l are conditionally independent samples from $P(z^j_l|x)$ and (2) $k - N < k^i_j \leq k$, the law of large numbers yields

$$\frac{1}{k} \sum_{l=1}^{k^i_j} \ln \frac{P(z^j_l|x)}{P(z^j_l|x^g)} \xrightarrow{P} \mathbb{E}_{z^j_l} \left[\ln \frac{P(z^j_l|x)}{P(z^j_l|x^g)} \right] \tag{17a}$$

$$= \int_{z^j_l} P(z^j_l|x^g) \ln \frac{P(z^j_l|x)}{P(z^j_l|x^g)} dz^j_l \tag{17b}$$

$$= -D_{KL} \left(P(z^j_l|x^g) \| P(z^j_l|x) \right), \tag{17c}$$

where "$\xrightarrow{P}$" represents "convergence in probability" and $D_{KL}(P_1\|P_2)$ denotes the Kullback-Leibler (KL) divergence between two probability distribution P_1 and P_2. The KL divergence has the property [29] that

$$\forall P_1,\, P_2,\ D_{KL}(P_1\|P_2) \leq 0,$$
$$\text{equality holds iff } \ P_1 = P_2.$$

This leads to the following conclusion:

$$\lim_{k\to\infty} \frac{1}{k} \sum_{l=1}^{k^i_j} \ln \frac{P(z^j_l|x)}{P(z^j_l|x^g)} \begin{cases} < 0, & x \neq x^g \\ = 0, & x = x^g \end{cases}. \tag{19}$$

Then by considering the limiting case of Eq. (15), we get:

$$\lim_{k\to\infty} \frac{1}{k} \ln \frac{P^i_{pdf}(x|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})} \begin{cases} < 0, & x \neq x^g \\ = 0, & x = x^g \end{cases}. \tag{20}$$

Eq. (20) implies that

$$\frac{P^i_{pdf}(x|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})} \xrightarrow{P} \begin{cases} 0, & x \neq x^g \\ 1, & x = x^g \end{cases}. \tag{21}$$

Therefore,

$$\lim_{k\to\infty} P^i_{pdf}(X = x^g|\mathbf{z}^i_{1:k}) = 1.$$

It is then straightforward to show

$$\lim_{k\to\infty} P(X^{MAP} = x^g|\mathbf{z}^i_{1:k}) = 1.$$

□

B. Static UGVs and Moving Target

The consistency of LIFO-DBF for static UGVs and moving target is stated as follows:

Theorem 2. *Assume UGVs are static, sensors are unbiased and the target is moving deterministically, the MAP estimator of the target position converges in probability to the true position of the target using LIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X^{MAP}_k = x^g_k|\mathbf{z}^i_{1:k}) = 1,\ i = 1, \ldots, N.$$

Proof. Since the target motion model is deterministic, as defined in Eq. (10), the integration in the prediction step (Eq. (9)) can be removed, as no uncertainty exists in $P(X_k|X_{k-1})$.

Eq. (13) now becomes

$$P^i_{pdf}(X_k|\mathbf{z}^i_{1:k}) = P^i_{pdf}(X_k|z^1_{1:k^i_1}, \ldots, z^N_{1:k^i_N})$$
$$= \frac{P^i_{pdf}(X_0) \prod_{j=1}^{N} \prod_{l=1}^{k^i_j} P(z^j_l|X_l) P(X_l|X_{l-1})}{\sum_{X_k\in S} P^i_{pdf}(X_0) \prod_{j=1}^{N} \prod_{l=1}^{k^i_j} P(z^j_l|X_l) P(X_l|X_{l-1})}.$$

The updating terms $P(X_l|X_{l-1})$ cancel when comparing $P^i_{pdf}(X_k = x_k|\mathbf{z}^i_{1:k})$ with $P^i_{pdf}(X = x^g_k|\mathbf{z}^i_{1:k})$, yielding the same result as Eq. (14). Therefore, the rest of the proof is similar to that of Theorem 1. □

C. Moving UGVs

This subsection considers the case of using moving UGVs to localize a target, either static or moving. The difficulty of consistency proof for this case lies in the fact that each UGV makes measurements at multiple positions. Here, the main idea of proof is to classify UGV measurement positions into two disjoint sets: *infinite-measurement spots* that contain positions where a UGV makes infinitely many measurements as time tends to infinity, and *finite-measurement spots* that contain positions where the UGV makes finitely many measurements (i.e., the positions where the UGV does not visit again after a finite time period). Before stating the main theorem, the following lemma is introduced.

Lemma 1. *For a set of UGVs moving within a collection of finite positions, each UGV has at least one position where infinite measurements are made as k tends to infinity.*

Proof. Let $n_j^{i,k}$ denote the times that i^{th} UGV visits j^{th} position up to time k. Then, $\sum_j n_j^{i,k} = k$. It is straightforward to see that $\exists n_j^{i,k}$, such that $n_j^{i,k} \to \infty$, as $k \to \infty$. □

Theorem 3. *Assume UGVs move within a collection of finite positions and sensors are unbiased, the MAP estimator of target position converges in probability to the true position of the target using LIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X_k^{MAP} = x_k^g | \mathbf{z}_{1:k}^i) = 1, \; i = 1, \dots, N.$$

Proof. Without loss of generality, we assume the target is static. The proof when target is moving deterministically is similar. According to Theorem 1, the batch form of DBF at k^{th} step is

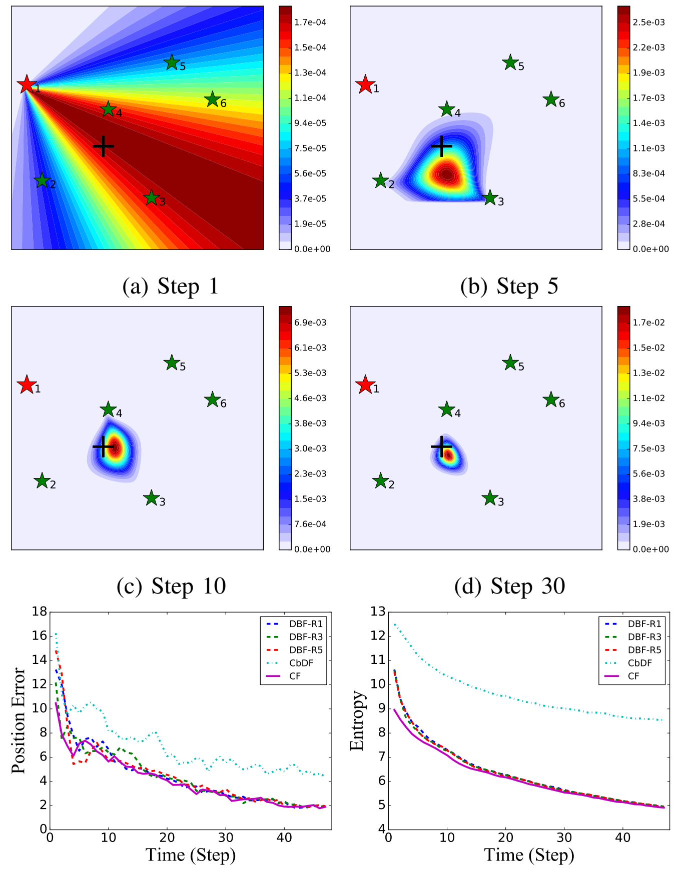
$$\frac{P_{pdf}^i(X = x|\mathbf{z}_{1:k}^i)}{P_{pdf}^i(X = x^g|\mathbf{z}_{1:k}^i)} = \frac{P_{pdf}^i(x) \prod_{j=1}^{N} \prod_{l=1}^{k_j^i} P(z_l^j|x; x_l^R)}{P_{pdf}^i(x^g) \prod_{j=1}^{N} \prod_{l=1}^{k_j^i} P(z_l^j|x^g; x_l^R)}. \tag{23}$$

The only difference from Eq. (14) is that $P(z_l^j|x; x_l^R)$ in Eq. (23) varies as the UGV moves. For each UGV, there exists at least one position where infinite measurements are made as $k \to \infty$, according to Lemma 1. All positions can be classified into finite-measurement spots and infinite-measurement spots. For the former, by referring to Eq. (17), it is easy to know that their contribution to Eq. (23) diminishes when $k \to \infty$. Therefore, Eq. (23) can be reduced to only considering infinite-measurement spots and the rest of proof is similar to that of Theorem 1. □

Remark 3. *The assumption of unbiased sensors are important for the consistency. In fact, with unknown non-zero bias, the distribution of z_l^j differs from $P(z_l^j|x^g)$, which invalidates the derivation in Eq. (17) and the consistency proof. This assumption also makes intuitive sense. In the extreme case, if each sensor has a very large unknown measurement offset, then the estimated target position of each sensor (without communicating with other sensors) will be very different from each other's. Therefore, no common target position can be obtained when they fuse measurements.*

V. Simulation and Experiment

Simulation has been conducted to evaluate the effectiveness of LIFO-DBF for target localization using a team of six UGVs. The networked UGVs take a ring communication topology that each UGV can communicate with two fixed neighbors. DBF is implemented using the histogram filter method, by which the continuous space is finely discretized into finitely many regions and the individual PDF is approximated by

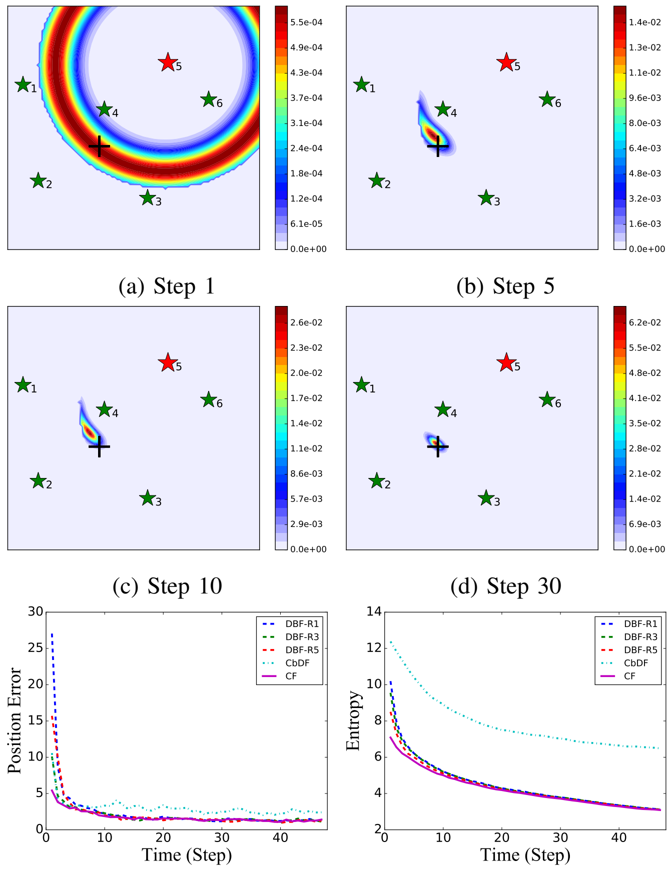


(a) Step 1 (b) Step 5 (c) Step 10 (d) Step 30 (e) Position Error (f) Entropy of Individual PDF

Fig. 4: (a)-(d) The 1st UGV's individual PDF at different times. The square denotes the current UGV and stars represent other UGVs. The cross stands for the target. (e) Average position estimation errors of 1st, 3rd and 5th UGV's LIFO-DBF, CbDF and CF. (f) Average entropy of individual PDFs.

the cumulative probability of each region [23]. Two types of scenarios are used: in first scenario, both UGVs and the target are static; the second scenario subsequently deals with six moving UGVs for localizing a moving target. In each scenario, we also test two different kinds of UGV team: the first UGV team, called the homogeneous team, is equipped with bearing-only sensors; in the second team, called the heterogeneous team, three UGVs are equipped with bearing-only sensor and the other three equipped with range-only sensors. The bearing-only sensors are assumed to have large enough sensing range to cover the simulation field and the measurement noise is a zero-mean Gaussian white noise with standard deviation being 0.5. The range-only sensors are assumed to have $360°$ field of view but limited sensing range ($50m$). The measurement noise is a zero-mean Gaussian white noise with standard deviation being 5.

In both scenarios, LIFO-DBF is compared with two commonly adopted approaches in multi-agent filtering: the consensus-based distributed filtering (CbDF) method [30] and the centralized filtering (CF) method [31]. The CbDF requires UGVs to continually exchange their individual PDFs with direct neighbors, computing the average of its own and the received target PDFs. Multiple rounds of communication



(a) Step 1 (b) Step 5

(c) Step 10 (d) Step 30

(e) Position Error (f) Entropy of Individual PDF

Fig. 5: (a)-(d) The 1$^{\text{st}}$ UGV's individual PDF at different times. The square denotes the current UGV and stars represent other UGVs. The cross stands for the target. (e) Average position estimation errors of 1$^{\text{st}}$, 3$^{\text{rd}}$ and 5$^{\text{th}}$ UGV's LIFO-DBF, CbDF and CF. (f) Average entropy of individual PDFs.

and averaging are conducted during the "Sending Step" and "Receiving Step" in Algorithm 1 at each step to ensure the convergence of UGV's individual PDFs. The CF assumes a central unit that can constantly receive and fuse all UGVs' latest measurements into a single PDF. Ten test trials with randomly generated initial UGV and target positions are run and each trial is terminated after 50 time steps. The average error between the estimated and true target position and the average entropy of individual PDFs of all 10 trials using these three approaches are compared.

A. Static UGVs, Static Target

The individual PDF of each UGV is initialized as a uniform distribution over a bounded search space. At each time step, each UGV executes the LIFO-DBF for static target (Section III-B) to update their estimation of target position. The evolution of the 1$^{\text{st}}$ robot's individual PDF is shown in Figures 4a to 4d. Since the robot is equipped with a noisy bearing-only sensor, the measurement at step 1 results in a wide distribution of the target position, with the peak centered along the noise-corrupted measured direction from the sensor to the target, as Figure 4a illustrates. As more measurements are fused, the individual PDF asymptotically concentrates to the true location of the target, which accords with the consistency of LIFO-DBF.

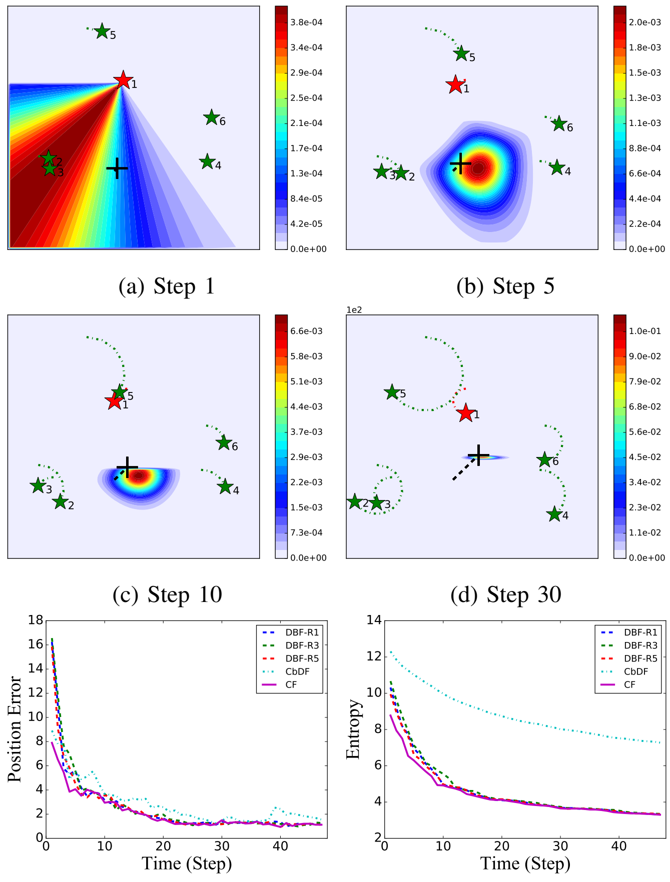
The Figures 4e and 4f shows the comparison of LIFO-DBF with CbDF and CF. Unsurprisingly, the CF achieves the best performance in terms of both small position estimation error and fast reduction of entropy. This happens because the central unit has access to the latest measurements of all UGVs, thus making most use of all available information. It is worth noting that, LIFO-DBF achieves similar asymptotic performance as the CF, both in position estimation error and entropy reduction; this is achieved even though each UGV only communicates with its two neighboring UGVs, which requires less communication burden than the CF. The CbDF has the worst performance among these three filtering approaches. It results in slow reduction of entropy and the position error remains large. This happens because for nonlinear systems, Bayes filtering (Eq. (9) and (11)) is a nonlinear process. Using the linear average consensus law to fuse individual PDFs thus deviates from the actual Bayesian filtering and therefore cannot fully exploit the information of new measurements to reduce the uncertainty of estimation.

The Figure 5 shows the simulation results of the heterogeneous team. The individual PDF of the UGV with a noisy range-only sensor is presented in Figures 5a to 5d. At step 1, the target is detected and the individual PDF is updates such that the peak of the distribution centers at the positions the distance from which to the sensor equals the noise-corrupted measured value. Similar to the case of using the homogeneous team, the individual PDF asymptotically concentrates to the true target position. Due to the use of various sensors, the estimation accuracy increases faster than the homogeneous team case, as shown in Figures 5e and 5f. It can also be noticed that, LIFO-DBF achieves similar performance as CF, while the performance of CbDF is worst.

B. Moving UGVs, Moving Target

In this scenario, each UGV follows a pre-defined circular trajectory. The target motion is modeled as a single-integrator with unit velocity on both directions. Each UGV executes the LIFO-DBF for moving target (Section III-C) to estimation. Figures 6a to 6d and Figures 7a to 7d illustrate the evolution of individual PDF for homogeneous and heterogeneous teams, respectively. They all present similar asymptotic behavior of individual PDFs as in aforementioned simulations. Note that Figure 7a shows the individual PDF at step 1 when the target is out of the sensing domain of the 5$^{\text{th}}$ UGV's range-only sensor.

Figs. 6e and 6f and Figs. 7e and 7f compare LIFO-DBF with CbDF and CF. It is worth noting that, for the heterogeneous team, CbDF achieves comparable position estimation error performance as the CF and LIFO-DBF. However, CbDF requires multiple rounds of exchanging individual PDFs, which incurs much higher communication burden than LIFO-DBF at each time step. Considering the small difference in position estimation error and significantly faster entropy reduction, LIFO-DBF is still preferable over CbDF for moving target scenario.



(a) Step 1 (b) Step 5

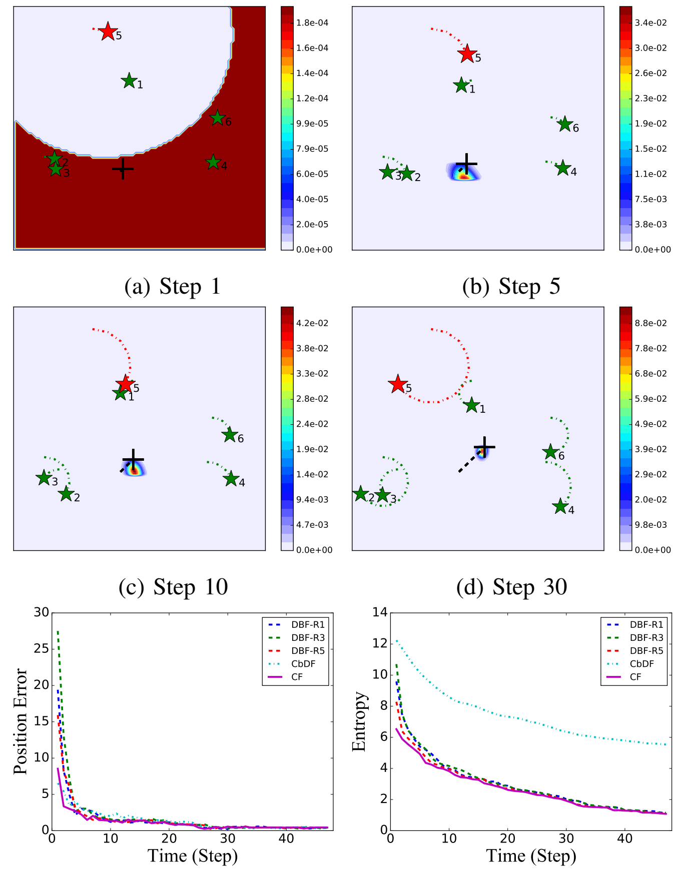
(c) Step 10 (d) Step 30

(e) Position Error (f) Entropy of Individual PDF

Fig. 6: (a)-(d) The 6th UGV's individual PDFs at different times. (e) Average position estimation errors. (f) Average entropy of individual PDFs.

C. Experiment

We have conducted an experiment to validate the LIFO-DBF approach using sonar sensors mounted on a P3DX robot platform for localizing a static target(**??**). The sonar is tested to have a sensing field of view of 25° and a range of $5m$ and its measurement noise can be modeled as a zero-mean Gaussian random variable with standard deviation $2mm$. The experiment is conducted in a square room and the probability map is constructed on a grid that is evenly separated with $0.1m$ interval on each axis. To avoid the inaccuracy caused by sensor state estimation, we manually measure the sensing positions. Due to the limited robot platform, we collect the sensor measurement from different different positions and postprocess these data, treating them as the data from three different robots. The effect of the target size has been considered during the postprocessing. Experiment results show fast convergence between the MAP estimate of target position to the true position value. Figure 9b illustrates the entropy reduction over time. In the end, entropy reduces to zero and this corresponds to the case that all probability mass concentrates to the position of the target. The sudden drop of the entropy at step 48 is because the sensor comes closer to the target and obtains a more accurate measurement of the target. Experiment results have shown the effectiveness of the



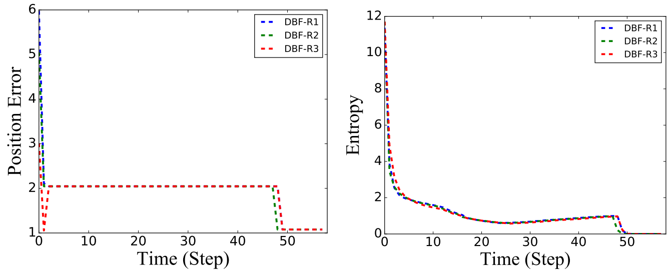
(a) Step 1 (b) Step 5

(c) Step 10 (d) Step 30

(e) Position Error (f) Entropy of Individual PDF

Fig. 7: (a)-(d) The 1st UGV's individual PDF at different times. The square denotes the current UGV and stars represent other UGVs. The cross stands for the target. (e) Average position estimation errors of 1st, 3rd and 5th UGV's LIFO-DBF, CbDF and CF. (f) Average entropy of individual PDFs.

Fig. 8: Experiment Setup



(a) Position Error (b) Entropy of Individual PDF

LIFO-DBF for target localization.

VI. CONCLUSION

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a multi-UGV network, utilizing the Latest-In-and-Full-Out (LIFO) protocol for measurement exchange. By exchanging full communication buffers among neighboring UGVs, LIFO significantly reduces the transmission burden between each pair of UGVs to scale linearly with the network size. It should be noted that LIFO is a general measurement exchange protocol and thus applicable to various sorts of sensors. Two types of LIFO-based DBF algorithms are proposed to estimate individual PDFs for a static target and a moving target, respectively. The consistency of LIFO-based DBF is proved by utilizing the law of large numbers, which ensures that the estimated target position converges in probability to the true target position.

Future work includes considering other types of sensors and imperfect communication between UGVs. Other types of sensors may have biased measurement , which complicates the design and analysis of LIFO-DBF. Imperfect communication, including package loss and transmission delay, requires extensions of current LIFO-DBF approach. (**TODO:** can list some benefits of concensus method as mentioned by CDC reviewer. also mention that sensor placement may help improve the result.)

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