Asymptotic Property of Bayesian Filters

Chang Liu

February 1, 2017

Assume there exist a moving target in the field S. We want to use a sensor network to localize the target. We can model the target dynamics and the sensor measurement model as follows:

$$x_{k+1} = f(x_k, w_k) \tag{1}$$
$$y_k = h(x_k, v_k), \tag{2}$$

where x_k is the target state at time k and y_k is the sensor measurement. w_k and v_k are random noise.

Assume the placement of sensors enables the unique localization of the target, e.g., three triangularly placed range sensors. A centralized Bayesian filter can utilize the sensor measurements to estimate the target position, which involves two steps:

Prediction Step

$$P^i_{pdf}(X_k|\mathbf{z}_{1:k-1}) = \int\limits_{X_{k-1}\in S} P(X_k|X_{k-1})P^i_{pdf}(X_{k-1}|\mathbf{z}_{1:k-1})dX_{k-1}.$$

Updating Step

$$P^i_{pdf}(X_k|\mathbf{z}_{1:k}) = K_i P^i_{pdf}(X_k|\mathbf{z}_{1:k-1})P(\mathbf{z}_k|X_k).$$

$\mathbf{z}_k$ is the set of measurements from the sensors of time k.

At each time, we use a point estimate, e.g., MAP, to estimate the target position:

$$\hat{x}_k = \arg\max_{X\in S} P^i_{pdf}(X_k|\mathbf{z}_{1:k}).$$

Can we show that $\|\hat{x}_k - x_k\|_2 \xrightarrow{P} 0$? If the MAP does not have this property, can some other point estimate, e.g., the mean value or the median of $P^i_{pdf}(X_k|\mathbf{z}_{1:k})$, have this property?

This seems to be a parameter identification problem, where the target state is the (time-varying) parameter.