Supplementary Material

I. Simulation Verification for Target Localization

Simulation has been conducted to evaluate the effectiveness of LIFO-DBF for target localization using a team of six UGVs. The networked UGVs take a ring communication topology that each UGV can communicate with two fixed neighbors. DBF is implemented using the histogram filter method, by which the continuous space is finely discretized into finitely many regions and the individual PDF is approximated by the cumulative probability of each region [1]. Two types of scenarios are used: in the first scenario, both UGVs and the target are static; the second scenario subsequently deals with moving UGVs for localizing a moving target. In each scenario, we also test two different kinds of UGV team: the first UGV team, called the homogeneous team, is equipped with bearing-only sensors; in the second team, called the heterogeneous team, three UGVs are equipped with bearing-only sensor and the other three equipped with range-only sensors. The bearing-only sensors are assumed to have large enough sensing range to cover the simulation field and the measurement noise is a zero-mean Gaussian white noise with standard deviation being 0.5. The range-only sensors are assumed to have 360° field of view and the measurement noise is a zero-mean Gaussian white noise with standard deviation being 5.

In both scenarios, LIFO-DBF is compared with two commonly adopted approaches in multi-agent filtering: the consensus-based distributed filtering (CbDF) method [2] and the centralized filtering (CF) method [3]. The CbDF requires UGVs to continually exchange their individual PDFs with direct neighbors, computing the average of its own and the received PDFs. Multiple rounds of communication and averaging are conducted during the "Sending Step" and "Receiving Step" in Algorithm 1 at each step to ensure the convergence of UGV's individual PDFs. The CF assumes a central unit that can constantly receive and fuse all UGVs' latest measurements into a single PDF. Ten test trials with randomly generated initial UGV and target positions are run and each trial is terminated after 50 time steps. The average error between the estimated (using the MAP estimator) and true target position and the average entropy of individual PDFs of all 10 trials using these three approaches are compared.

A. Static UGVs, Static Target

The individual PDF of each UGV is initialized as a uniform distribution over a bounded space. At each time step, each UGV executes the LIFO-DBF for static target (Section III-B) to update their estimation of target position. The evolution of the 1st robot's individual PDF is shown in Figures 1a to 1d. Since the robot is equipped with a noisy bearing-only sensor, the measurement at step 1 results in a wide distribution with the peak centered along the noise-corrupted measured direction from the sensor to the target, as Figure 1a illustrates. As more measurements are fused, the individual PDF asymptotically concentrates to the true location of the target, which accords with the consistency of LIFO-DBF.

The Figures 1e and 1f shows the comparison of LIFO-DBF with CbDF and CF. Unsurprisingly, the CF achieves the best performance in terms of both small position estimation error and fast reduction of entropy. This happens because the central unit has access to the latest measurements of all UGVs, thus being able to make the most use of all available information. It is worth noting that, LIFO-DBF achieves similar asymptotic performance as the CF, both in position estimation error and entropy reduction; this is achieved even though each UGV only communicates with its two neighboring UGVs, which requires less communication burden than the CF. The CbDF has the worst performance among these three filtering approaches. It results in slow reduction of entropy and the position error remains large. This happens because Bayes filtering (Eq. (3) and (5)) is a nonlinear process. Using the linear average consensus law to fuse individual PDFs thus deviates from the actual Bayesian filtering and therefore cannot fully exploit the information of new measurements to reduce the uncertainty of estimation.

The Figure 2 shows the simulation results of the heterogeneous team. The individual PDF of the UGV with a noisy range-only sensor is presented in Figures 2a to 2d. At step 1, the target is detected and the individual PDF is updated such that the peak of the distribution centers at the positions whose distance to the sensor equals the noise-corrupted measured value. Similar to the case of using the homogeneous team, the individual PDF asymptotically concentrates to the true target position. Due to the use of various sensors, the estimation accuracy increases faster than the homogeneous team case, as shown in Figures 2e and 2f. It can also be noticed that, LIFO-DBF achieves similar performance as CF, while the performance of CbDF is the worst.

B. Moving UGVs, Moving Target

In this scenario, each UGV follows a pre-defined circular trajectory. The target motion is modeled as a single-integrator with unit velocity on both directions. Each UGV executes the LIFO-DBF for moving target (Section III-C) for estimation. Figures 3a to 3d and Figures 4a to 4d illustrate the evolution of individual PDF for homogeneous and heterogeneous teams, respectively. They all present similar asymptotic behavior of individual PDFs as in aforementioned simulations.

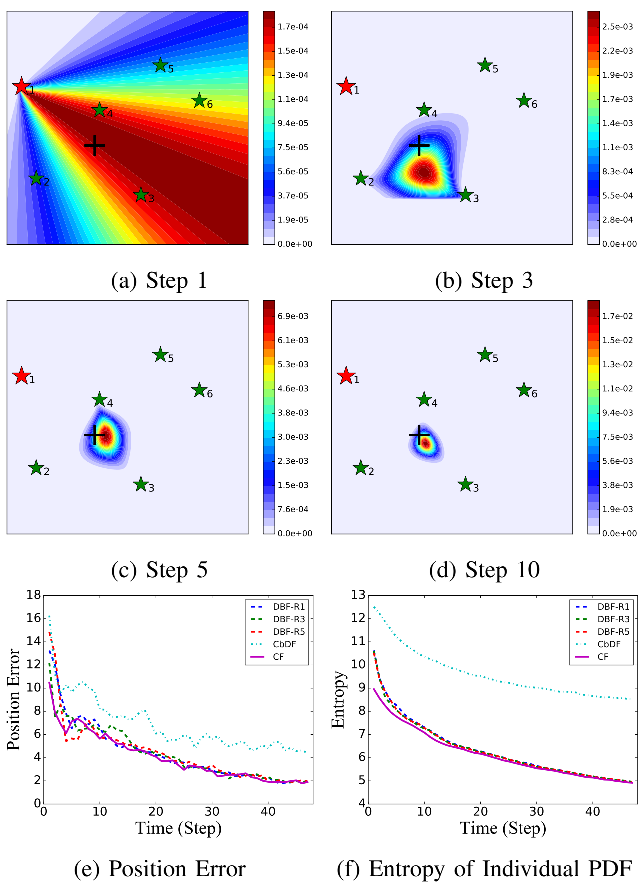
Figs. 3e and 3f and Figs. 4e and 4f compare LIFO-DBF with CbDF and CF. It is worth noting that, for the heterogeneous team, CbDF achieves comparable position estimation error performance as the CF and LIFO-DBF. However, CbDF requires multiple rounds of exchanging individual PDFs, which incurs much higher communication burden than LIFO-DBF at each time step. Considering the small difference in position estimation error and significantly faster entropy reduction, LIFO-DBF is still preferable over CbDF for moving target scenario.

References

[1] S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics*. MIT press, 2005.

[2] R. Olfati-Saber, E. Franco, E. Frazzoli, and J. S. Shamma, "Belief consensus and distributed hypothesis testing in sensor networks," in *Networked Embedded Sensing and Control*. Springer, 2006, pp. 169–182.

[3] V. V. Veeravalli and P. K. Varshney, "Distributed inference in wireless sensor networks," *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 370, no. 1958, pp. 100–117, 2012.

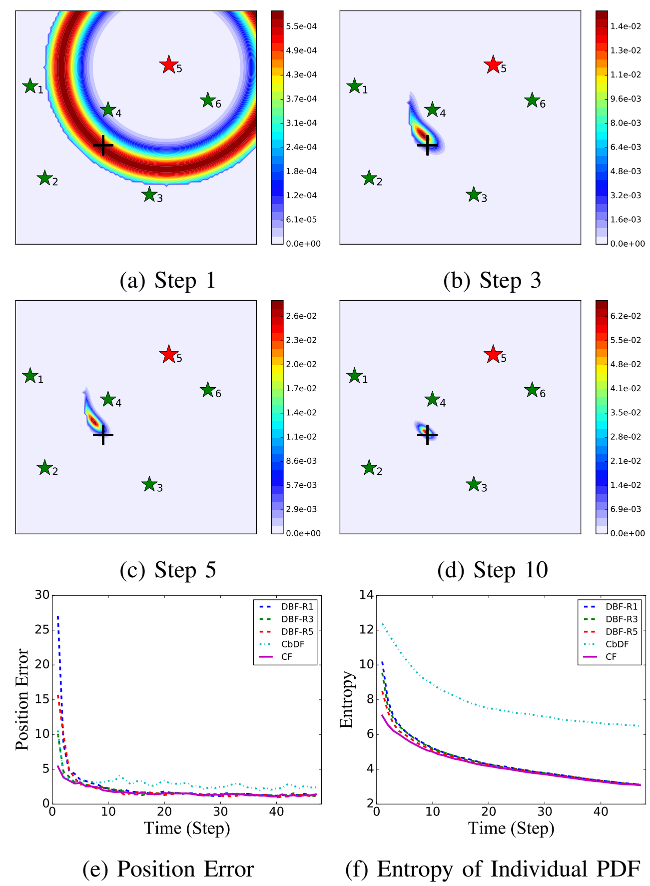


(a) Step 1 (b) Step 3

(c) Step 5 (d) Step 10

(e) Position Error (f) Entropy of Individual PDF

Fig. 1: (a)-(d) The 1st UGV's individual PDF at different times. The cross stands for the target. Note that the value of the color bar varies among different figures. (e) Average position estimation error of the 1st, 3rd and 5th UGV's LIFO-DBF, the CbDF and the CF. (f) Average entropy of individual PDFs.

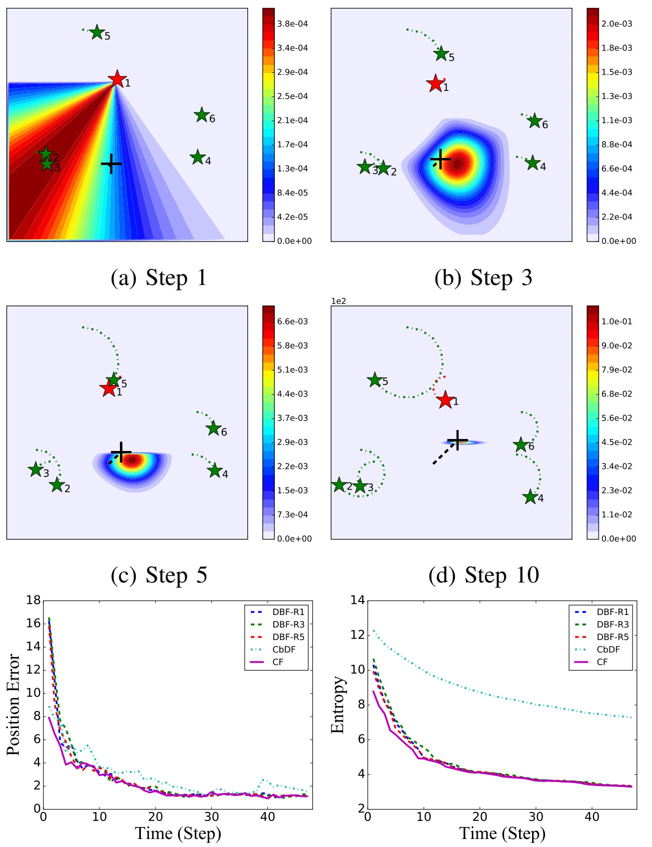


(a) Step 1 (b) Step 3

(c) Step 5 (d) Step 10

(e) Position Error (f) Entropy of Individual PDF

Fig. 2: (a)-(d) The 5th UGV's individual PDF at different times. (e) Average position estimation error. (f) Average entropy of individual PDFs.

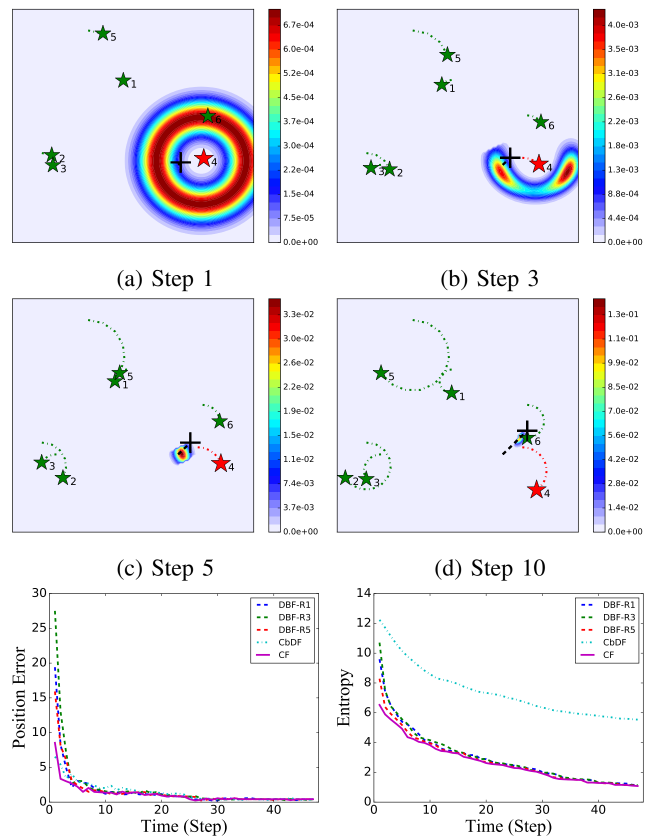


(a) Step 1 (b) Step 3

(c) Step 5 (d) Step 10

(e) Position Error (f) Entropy of Individual PDF

Fig. 3: (a)-(d) The 1st UGV's individual PDFs at different times. The green dashed lines represent robots' trajectories. (e) Average position estimation error. (f) Average entropy of individual PDFs.



(a) Step 1 (b) Step 3

(c) Step 5 (d) Step 10

(e) Position Error (f) Entropy of Individual PDF

Fig. 4: (a)-(d) The 4th UGV's individual PDF at different times. (e) Average position estimation error. (f) Average entropy of individual PDFs.