Distributed Bayesian Filters for Multi-Vehicle Network by Using Latest-In-and-Full-Out Exchange Protocol of Measurements

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Abstract— **This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a network of unmanned ground vehicles (UGVs). The DBF utilizes the Latest-In-and-Full-Out (LIFO) local exchange protocol of sensor measurements for data communication within the network. Different from existing statistics dissemination-based approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only exchanges with neighboring UGVs a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs to scale linearly with the size of the network. Under the condition of fixed and undirected topology, LIFO can guarantee non-intermittent dissemination of all measurements over the network within a finite time. Two types of LIFO-based DBF algorithms are presented to estimate individual probability density function (PDF) for a static target and for a moving target, respectively. For the static target, each UGV locally fuses the newly received measurements while for the moving target, a set of measurement history is stored and sequentially fused. The consistency of LIFO-based DBF is proved that the estimated target position converges in probability to the true position. The effectiveness of this method is demonstrated by comparing with a consensus-based distributed filter and a centralized filter in the simulation of target localization.**

I. INTRODUCTION

Distributed filtering that focuses on using a group of networked UGVs to collectively infer environment status has been used for various applications, such as intruder detection [1], pedestrian tracking [2] and micro-environmental monitoring [3]. Several techniques have been developed for distributed filtering, including the distributed Kalman filter (DKF) [4], distributed extended Kalman filter [5], and distributed particle filter [6]. As a generic filtering scheme for nonlinear systems with arbitrary noise distributions, the distributed Bayesian filter (DBF) has received increasing interest during past years [7], [8]. This work focuses on a communication-efficient DBF for networked UGVs.

The design of distributed filtering algorithms depends on the communication topology of multi-UGV network, which can be classified into two types: fusion center (FC)-based and neighborhood (NB)-based. In FC-based approaches, each UGV uses a filter to estimate local statistics of environment status based on its own measurement. The local statistics is then transmitted to a single FC, where a global posterior distribution (or statistical moments) is calculated at each filtering cycle after receiving all local information [9]. In NB-based approaches, a set of UGVs execute distributed filters to estimate individual posterior distribution. Consensus of individual estimates is achieved by solely communicating statistics and/or observations within local neighbors of these UGVs. The NB-based methods have become popular in recent years since such approaches do not require complex routing protocols or global knowledge of the network and therefore are robust to changes in network topology and to link failures.

So far, most studies on NB-based distributed filtering have mainly focused on the so-called *statistics dissemination* strategy that each UGV actually exchanges statistics, including posterior distributions and likelihood functions, with neighboring UGVs [10]. For example, Sheng et al. (2005) proposed a multiple leader-based distributed particle filter for target tracking [11]. Sensor group leaders run particle filters and exchange particle information with each other. Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-linear-average consensus algorithm [12]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [7].

Despite the popularity of statistics dissemination strategy, exchanging statistics can consume high communication resources. One promising remedy is to disseminate measurement instead of statistics among neighbors, which, however, has not been fully exploited. One pioneering work was done by Coates et al. (2004), who used adaptive encoding of measurements to minimize communication overhead [13]. Ribeiro et al. (2006) exchanged quantized measurements along with error-variance limits considering more pragmatic signal models [14]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each UGV has a complete set of measurements of other UGVs for executing particle filtering [15]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a complete graph that every pair of distinct UGVs is directly connected by a unique edge, which is not always feasible in reality.

This paper extends existing works by introducing a Latest-In-and-Full-Out (LIFO) protocol into distributed Bayesian filters (DBF) for networked UGVs. Each UGV is only allowed to broadcast measurements to its neighbors by using

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single-hopping and then implements individual Bayesian filter locally after receiving transmitted measurements. The main benefit of using LIFO is on the reduction of communication burden, with the transmission data volume scaling linearly with the UGV number, while a statistics dissemination-based strategy can suffer from the order of environment size. The proposed LIFO-based DBF has following properties: (1) For a fixed and undirected network, LIFO guarantees the global dissemination of measurements over the network in a non-intermittent manner. (2) The corresponding DBF ensures the consistency of the estimated target position, i.e., the estimated position converges in probability to the true target position as measurements are continually fused.

The rest of this paper is organized as follows: The problem of distributed Bayesian filtering is formulated in Section II. The LIFO-based DBF algorithm is described in Section III, followed by the proof of consistency in Section IV. Simulation results are presented in Section V.

II. PROBLEM FORMULATION

Consider a network of N UGVs in a bounded two-dimensional space S. The aim of UGVs is to efficiently localize a target in S. For the purpose of simplicity, each UGV is assumed to be equipped with a binary sensor for environmental perception. Due to the limit of communication range, each UGV can only exchange observations with its neighbors. The Bayesian filter is run locally on each UGV based on its own and received observations via single-hopping to estimate true position of target.

A. Target and Sensor Model

The target motion takes a discrete-time model that can be described by

$$x^g_{k+1} = f(x^g_k, w^g_k), \tag{1}$$

where $x^g_k \in \mathbb{R}^2$ denotes the target position at time k and w^g_k is the process noise.

Each UGV's sensor constantly measures the target position and the measurement by i^{th} UGV is modeled as

$$z^i_k = \begin{cases} 1 & h^i(x^g_k; x^i_k) \geq \gamma \\ 0 & h^i(x^g_k; x^i_k) < \gamma \end{cases},$$

where z^i_k denotes the measurement by the i^{th} sensor at time k; h^i is the sensor property that characterizes the target position. For example, h^i can represent the power received by an ultrasonic sensor. When the received signal y^i_k is greater than a threshold γ, indicating that the target is detected, the sensor returns 1; otherwise, 0 is returned by the sensor.

We use a likelihood function to represent the probability of the target being detected by the binary sensor:

$$p^i_{1,k} = P(z^i_k = 1 | x^g_k; x^i_k) \in [0, 1]\,,\ x^g_k \in S, \tag{2}$$

where x^i_k is the i^{th} sensor's position. Correspondingly, the likelihood function for no target being detected is:

$$p^i_{0,k} = P(z^i_k = 0 | x^g_k; x^i_k) = 1 - p^i_{1,k}. \tag{3}$$

The combination of Eq. (2) and Eq. (3) forms the probabilistic model for a binary sensor, and the measurement follows a Bernoulli distribution $B(1, p^i_{1,k})$, i.e.

$$P(z^i_k | x^g; x^i_k)^1 = (p^i_{1,k})^{z^i_k} (p^i_{0,k})^{1 - z^j_l}.$$

The commonly used likelihood functions for binary sensors include the Gaussian function [16], [17] and step function [18]. For example, a Guassian function sensor model [19] is defined as

$$p^i_{1,k} = e^{-\frac{1}{2}(x^g_k - x^i_k)^T \Sigma^{-1} (x^g_k - x^i_k)}, \tag{4}$$

where Σ is a positive definite covariance matrix characterizing the sensing range and uncertainty.

Remark 1: Given the knowledge of current target and UGV positions, current observation by each UGV can be considered conditionally independent from its own past observations and those by other UGVs [20].

B. Graphical Model of Communication Topology

The UGV network is assumed to be connected, i.e., there exists a path, either direct or indirect, between every pair of UGVs. Under this assumption, consider an undirected and fixed graph $G = (V, E)$, where $V = \{1, \ldots, N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. The adjacency matrix $A = \left[A_{(ij)}\right]$ describes the communication topology of G:

$$A_{(ij)} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where $A_{(ij)}$ denotes the entity of adjacency matrix. The notation $A_{(ij)} = 1$ indicates that a communication link exists between i^{th} and j^{th} UGV and $A_{(ij)} = 0$ indicates no communication between them.

The *direct neighborhood* of i^{th} UGV is defined as $\mathcal{N}_i = \left\{j | A_{(ij)} = 1, \forall j \in \{1, \ldots, N\}\right\}$. All the UGVs in $\mathcal{N}_i$ can directly exchange information with i^{th} UGV in one step. We also define another set $\mathcal{Q}_i$, called *available neighborhood*, that contains indices of UGVs whose observations can be received by the i^{th} UGV within single or multiple steps. Note that $\mathcal{N}_i \subseteq \mathcal{Q}_i$.

C. Distributed Bayesian Filter for Multiple UGVs

The generic distributed Bayesian filter (DBF) is introduced in this section. Each UGV has its individual estimation of probability density function (PDF) of target position, called *individual PDF*. The individual PDF of i^{th} UGV at time k is defined as $P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k})$, where $\mathbf{z}^i_{1:k}$ denotes the set of measurements by i^{th} UGV and by UGVs in $\mathcal{Q}_i$ that are transmitted to i^{th} UGV by time k. The individual PDF is initialized by using all available prior information including past experience and environmental knowledge. Under the framework of DBF, the individual PDF is recursively estimated by two steps, i.e., prediction step and updating step, based on observations of i^{th} UGV and UGVs in $\mathcal{Q}_i$.

[1]For the purpose of simplicity, we will not explicitly write x^i_k in the sensor model for the rest of the paper.

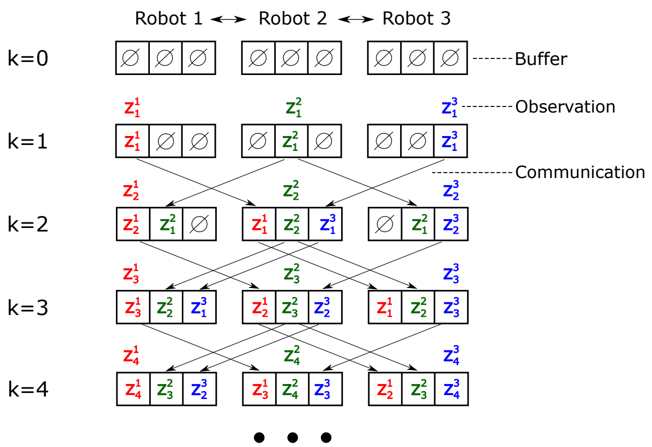


Fig. 1: Example of LIFO with three UGVs using a line communication topology. UGVs send their own CBs and receive CBs from neighboring UGVs. CBs are updated using Algorithm 1.

1) Prediction: At time k, the prior individual PDF $P^i_{pdf}(x^g_{k-1}|\mathbf{z}^i_{1:k-1})$ is first predicted forward by using the Chapman-Kolmogorov equation:

$$P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k-1}) = \int P(x^g_k|x^g_{k-1})P^i_{pdf}(x^g_{k-1}|\mathbf{z}^i_{1:k-1})dx^g_{k-1} \quad (5)$$

where $P(x^g_k|x^g_{k-1})$ is a Markov motion model of the target defined by Eq. (1), which maps the state transition probability of the target. For a static target, its Markov motion model is simplified to be

$$P(x^g_k|x^g_{k-1}) = \begin{cases} 1 & \text{if } x^g_k = x^g_{k-1} \\ 0 & \text{if } x^g_k \neq x^g_{k-1} \end{cases}.$$

2) Updating: The i^{th} individual PDF is then updated by Bayes' theorem using the set of newly received observations at time k, $\mathbf{z}^i_k$:

$$P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k}) = K_i P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k-1})P(\mathbf{z}^i_k|x^g_k) \quad (6)$$

where K_i is a normalization factor, given by:

$$K_i = 1/\int P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k-1})P(\mathbf{z}^i_k|x^g_k)dx^g_k$$

and $P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k})$ is called posterior individual PDF; $P(\mathbf{z}^i_k|x^g_k)$ comes from the sensor model Eq. (2) and Eq. (3).

III. Distributed Bayesian Filter via Latest-In-and-Full-Out Protocol

This study proposes a Latest-In-and-Full-Out (LIFO) protocol for observation exchange and derives two corresponding distributed Bayesian filtering (DBF) algorithms, shorted as LIFO-DBF. The data communication in LIFO is synchronized with the execution of DBF. In each step, LIFO only allows single-hopping communication within the direct neighborhood, but is able to broadcast observations of each UGV to any other agent after a finite number of steps. The individual PDF is forward predicted and updated in DBF after each LIFO cycle. The theoretical analysis show that LIFO-DBF can ensure the consistency and consensus of distributed estimation while requiring much less communication burden than statistics dissemination-based methods.

Algorithm 1 LIFO Protocol

(1) Initialization: The CB of i^{th} UGV is initialized when $k=0$:

$$z^j_{k^i_j} = \varnothing,\ k^i_j = 0,\ j = 1,\ldots,N$$

(2) At k^{th} step for i^{th} UGV :

(2.1) Receiving Step:

The i^{th} UGV receives all CBs of its direct neighborhood $\mathcal{N}_i$, each of which corresponds to the (k-1)-step CB of a UGV in $\mathcal{N}_i$. The received CB from l^{th} ($l \in \mathcal{N}_i$) UGV is denoted as

$$\mathbf{z}^{CB,l}_{k-1} = \left[z^1_{(k-1)^l_1},\ldots,z^N_{(k-1)^l_N}\right],\ l \in \mathcal{N}_i$$

(2.2) Observation Step:

The i^{th} UGV updates $z^j_{k^i_j}$ $(j=i)$ by its own observation at current step:

$$z^j_{k^i_j} = z^i_k,\ k^i_j = k,\ \text{if } j = i.$$

(2.3) Comparison Step:

The i^{th} UGV updates other elements of its own CB, i.e., $z^j_{k^i_j}$ $(j \neq i)$, by selecting the latest information among all received CBs from $\mathcal{N}_i$. For all $j \neq i$,

$$l_{\text{latest}} = \underset{l\in\mathcal{N}_i,\ i}{\operatorname{argmax}} \left\{(k-1)^i_j, (k-1)^l_j\right\}$$

$$z^j_{k^i_j} = z^j_{(k-1)^{l_{\text{latest}}}_j},\ k^i_j = (k-1)^{l_{\text{latest}}}_j$$

(2.4) Sending Step:

The i^{th} UGV broadcasts its updated CB to all of its neighbors defined in $\mathcal{N}_i$.

(3) $k \leftarrow k+1$ until stop

A. Latest-In-and-Full-Out (LIFO) Protocol

Under LIFO, each UGV contains a communication buffer (CB) to store its latest knowledge of observations of all UGVs:

$$\mathbf{z}^{CB,i}_k = \left[z^1_{k^i_1},\ldots,z^N_{k^i_N},x^1_{k^i_1},\ldots,x^N_{k^i_N}\right]$$

where $z^j_{k^i_j}$ represents the observation made by j^{th} UGV at time k^i_j and $x^j_{k^i_j}$ denotes the sensor position when the associated measurement $z^j_{k^i_j}$ is made. We will not explicitly write $\left[x^1_{k^i_1},\ldots,x^N_{k^i_N}\right]$ in $\mathbf{z}^{CB,i}_k$ for the rest of the paper for the purpose of simplicity. Note that under LIFO, $\mathcal{Q}_i = \{1,\ldots,N\}\setminus\{i\}$, which will be proved in Corollary 1. At time k, $z^j_{k^i_j}$ is received and stored in i^{th} UGV CB, in which k^i_j is the latest observation time of j^{th} UGV available to i^{th} UGV. Due to the communication delay, $k^i_j < k, \forall j \neq i$ and $k^i_i = k$ always holds. The **LIFO protocol** is stated in Algorithm 1.

Fig. 1 illustrates the LIFO cycles with 3 UGVs using a line topology. For general graphs, we have the following proposition:

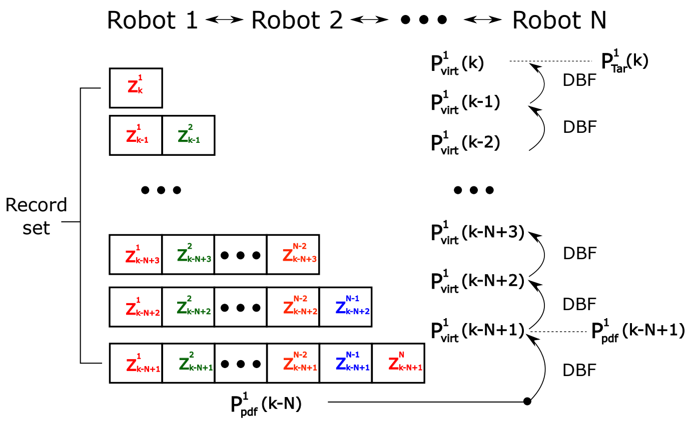


Fig. 2: Example of LIFO-DBF for 1st UGV at time k. Networked UGVs take a line topology. The stored individual PDF is $P^1_{pdf}(k-N)$. The UGV first calculates $P^1_{virt}(k-N+1)$ using DBF and stores it as $P^1_{pdf}(k-N+1)$. Repeating DBF until obtaining $P^1_{pdf}(k)$, which is then used as the target PDF estimation of 1st UGV at time k. In this example, $\Omega^1_\xi = \{1, 2, \ldots, N+1-\xi\}$, $\xi = 1, \ldots, N$.

Proposition 1: For a fixed and undirected network of N UGVs, the latest observations of i^{th} and j^{th} UGV are exchanged via the shortest path(s) under LIFO. The delay of exchange $\tau_{i,j}$ is equivalent to the length of the shortest path(s) between them.

Proof: Since the network is connected, there exists a minimum integer $\tau_{i,j}$ such that $A^{\tau_{i,j}}_{(i,j)} > 0$ and $\tau_{i,j}$ is the length of a shortest path between i^{th} and j^{th} UGV. Under the LIFO, the latest observation of i^{th} UGV will always be received and then propagated in the communication buffer of the UGVs on a shortest path. Therefore, the latest observation that j^{th} UGV receives from i^{th} UGV is delayed by $\tau_{i,j}$ iterations of communication. ■

Corollary 1: For the same topology assumption in Proposition 1, all elements in $\mathbf{z}^{CB,i}_k$ under LIFO become filled when $k \geq N$, i.e., $\mathcal{Q}_i = \{1, \ldots, N\} \setminus \{i\}$.

Corollary 2: For the same topology assumption in Proposition 1, once all elements in $\mathbf{z}^{CB,i}_k$ are filled, the updating of each element is non-intermittent.

Compared to statistics dissemination, LIFO is generally more communication-efficient for distributed filtering. To be specific, we consider an $M \times M$ grid environment with a network of N UGVs. The transmitted data of LIFO between each pair of UGVs are only the CB of each UGV and the corresponding UGV positions where observations were made, the size of which is $O(N)$, scaling linearly with UGV number. On the contrary, an unparameterized posterior distribution or likelihood function is usually represented by an $M \times M$ matrix [21]. Therefore, the size of transmitted data for a statistics dissemination approach is $O(M^2)$, on the order of environmental size. Since M is generally much larger than N in applications such as target localization, LIFO requires much less communication resources.

Algorithm 2 LIFO-DBF Algorithm

For i^{th} UGV at k^{th} step:
After updating CB by Algorithm 1,
(1) The stored individual PDF for time $(k-N)$ is:

$$P^i_{pdf}(x^g_{k-N} | z^1_{1:k-N}, \ldots, z^N_{1:k-N})$$

(2) Initialize a virtual PDF by assigning the individual PDF to it:

$$P^i_{virt}(x^g_{k-N}) = P^i_{pdf}(x^g_{k-N} | z^1_{1:k-N}, \ldots, z^N_{1:k-N})$$

(3) From $\xi = 1$ to N, iteratively repeat two steps of Bayesian filtering:
(3.1) Prediction

$$\begin{aligned} &P^{pre}_{virt}(x^g_{k-N+\xi}) \\ &= \int P(x^g_{k-N+\xi} | x^g_{k-N+\xi-1}) P^i_{virt}(x^g_{k-N+\xi-1}) dx^g_{k-N+\xi-1} \end{aligned}$$

(3.2) Updating

$$P^i_{virt}(x^g_{k-N+\xi}) = K_\xi P^{pre}_{virt}(x^g_{k-N+\xi}) \prod_{j \in \Omega^i_\xi} P(z^j_{k-N+\xi} | x^g_{k-N+\xi})$$

$$K_\xi = 1 / \int P^{pre}_{virt}(x^g_{k-N+\xi}) \prod_{j \in \Omega^i_\xi} P(z^j_{k-N+\xi} | x^g_{k-N+\xi}) dx^g_{k-N+\xi}$$

(3.3) When $\xi = 1$, store the virtual PDF as the individual PDF for time $(k-N+1)$

$$P^i_{pdf}(x^g_{k-N+1} | z^1_{1:k-N+1}, \ldots, z^N_{1:k-N+1}) = P^i_{virt}(x^g_{k-N+1}).$$

(4) Individual PDF of i^{th} UGV at time k is $P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k}) = P^i_{virt}(x^g_k)$.

B. Algorithm of LIFO-DBF for Static Target

This section derives the LIFO-DBF algorithm for localizing a static target. It is assumed that, all UGV know the sensor models of other UGVs. Each UGV stores last-step individual PDF, i.e., $P^i_{pdf}(x^g | \mathbf{z}^i_{1:k-1})$. According to Corollary 2, $\mathbf{z}^i_k = \mathbf{z}^{CB,i}_k$ and $\mathbf{z}^i_{1:k} = \mathbf{z}^{CB,i}_{1:k} = \left[z^1_{1:k^i_1}, \ldots, z^N_{1:k^i_N} \right]$. The assumption of static target can simplify the Bayesian filter as the prediction step becomes unnecessary. Therefore, the i^{th} individual PDF is only updated by

$$\begin{aligned} P^i_{pdf}(x^g | \mathbf{z}^i_{1:k}) &= K_i P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k-1}) P(\mathbf{z}^i_k | x^g_k) \\ &= K_i P^i_{pdf}(x^g | \mathbf{z}^i_{1:k-1}) \prod_{j=1}^N P(z^j_{k^i_j} | x^g) \end{aligned} \quad (7)$$

where

$$K_i = 1 / \int P^i_{pdf}(x^g | \mathbf{z}^i_{1:k-1}) \prod_{j=1}^N P(z^j_{k^i_j} | x^g) dx^g$$

C. Algorithm of LIFO-DBF for Moving Target

This section derives the LIFO-DBF for localizing a moving target. Instead of storing last-step PDF, at time k each UGV maintains an individual PDF of time $(k-N)$ and a collection of historical observations, called the *record set*, from time $(k-N+1)$ to k. The i^{th} individual PDF is then alternatively predicted and updated by using the aforementioned Bayesian filter (Eq. (5) and Eq. (6)) from

$(k-N)$ to k. Fig. 2 illustrates the LIFO-DBF procedure for the 1[st] UGV as an example. Let Ω_ξ^i $(\xi = 1, \ldots, N)$ denote the index set of UGVs whose observation at time $(k-N+\xi)$ is stored in i^{th} UGV's record set. The **LIFO-DBF algorithm** for moving target is then stated in Algorithm 2.

IV. Proof of Consistency

This section proves the consistency of LIFO-DBF for localizing a static target using static UGVs with heterogeneous sensors. The proof of LIFO-DBF using moving UGVs is similar but requires extra algebraic manipulation, and is omitted due to space limit. Assume S is finite and x^{g^*} is the true position of target, the consistency of LIFO-DBF for static UGVs is stated as:

Theorem 1: When UGVs are static, the estimated target position converges to the true position of target in probability using LIFO-DBF, i.e.,

$$\lim_{k\to\infty} P(x^g = x^{g^*}|\mathbf{z}_{1:k}^i) = 1, \; i = 1, \ldots, N.$$

Proof: Considering the conditional independence of observations given $x^g \in S$, the batch form of DBF is:

$$P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i) = P_{pdf}^i(x^g|z_{1:k_1^i}^1, \ldots, z_{1:k_N^i}^N)$$
$$= \frac{P_{pdf}^i(x^g) \prod_{j=1}^{N} \prod_{l=1}^{k_j^i} P(z_l^j|x^g)}{\sum_{x^g\in S} P_{pdf}^i(x^g) \prod_{j=1}^{N} \prod_{l=1}^{k_j^i} P(z_l^j|x^g)},$$

where $P_{pdf}^i(x^g)$ is i^{th} UGV's initial individual PDF.

Comparing $P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)$ with $P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)$ yields

$$\frac{P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)}{P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)} = \frac{P_{pdf}^i(x^g) \prod_{j=1}^{N} \prod_{l=1}^{k_j^i} P(z_l^j|x^g)}{P_{pdf}^i(x^{g^*}) \prod_{j=1}^{N} \prod_{l=1}^{k_j^i} P(z_l^j|x^{g^*})}. \tag{8}$$

Take the logarithm of Eq. (8) and average it over k steps:

$$\frac{1}{k}\ln\frac{P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)}{P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)} = \frac{1}{k}\ln\frac{P_{pdf}^i(x^g)}{P_{pdf}^i(x^{g^*})} + \sum_{j=1}^{N}\frac{1}{k}\sum_{l=1}^{k_j^i}\ln\frac{P(z_l^j|x^g)}{P(z_l^j|x^{g^*})}. \tag{9}$$

Since $P_{pdf}^i(x^g)$ and $P_{pdf}^i(x^{g^*})$ are bounded, $\lim_{k\to\infty}\frac{1}{k}\ln\frac{P_{pdf}^i(x^g)}{P_{pdf}^i(x^{g^*})} = 0$. Define $p_1^{j^*} = P(z_l^j = 1|x^{g^*})$and $p_1^j = P(z_l^j = 1|x^g)$. The law of large numbers yields $\frac{1}{k}\sum_{l=1}^{k_j^i} z_l^j \xrightarrow{P} p_1^{j^*}$, and $\frac{1}{k}(k_j^i - \sum_{l=1}^{k_j^i} z_l^j) \xrightarrow{P} 1 - p_1^{j^*}$, where "$\xrightarrow{P}$" denotes "convergence in probability". Then,

$$\frac{1}{k}\sum_{l=1}^{k_j}\ln\frac{P(z_l^j|x^g)}{P(z_l^j|x^{g^*})} \xrightarrow{P} p_1^{j^*}\ln\frac{p_1^j}{p_1^{j^*}} + (1-p_1^{j^*})\ln\frac{1-p_1^j}{1-p_1^{j^*}}. \tag{10}$$

Note that the right-hand side of Eq. (10) achieves maximum value 0 if and only if $p_1^j = p_1^{j^*}$.

Define $c(x^g) = \sum_{j=1}^{N} p_1^{j^*}\ln\frac{p_1^j}{p_1^{j^*}} + (1-p_1^{j^*})\ln\frac{1-p_1^j}{1-p_1^{j^*}}$. Considering Eq. (10), the limit of Eq. (9) is

$$\frac{1}{k}\ln\frac{P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)}{P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)} \xrightarrow{P} c(x^g) \tag{11}$$

It follows from Eq. (11) that

$$\frac{P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)}{P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)e^{c(x^g)k}} \xrightarrow{P} 1. \tag{12}$$

Define the set $\bar{X}^T = S \setminus \{x^{g^*}\}$ and $c_M = \max_{x^g\in\bar{X}^T} c(x^g)$. Then $c_M < 0$. Summing Eq. (12) over $\bar{X}^T$ yields

$$\frac{\sum_{x^g\in\bar{X}^T} P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)e^{[c_M - c(x^g)]k}}{P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)e^{c_M k}} \xrightarrow{P} |\bar{X}^T|, \tag{13}$$

where $|\bar{X}^T|$ denotes the cardinality of $\bar{X}^T$.

Since $c_M < 0$, $P_{pdf}^i(x^{g^*}|\mathbf{z}_{1:k}^i)e^{c_M k} \longrightarrow 0$ and Eq. (13) implies $\sum_{x^g\in\bar{X}^T} P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i)e^{[c_M - c(x^g)]k} \xrightarrow{P} 0$.

Therefore $\sum_{x^g\in\bar{X}^T} P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i) \xrightarrow{P} 0$, and it follows that

$$\lim_{k\to\infty} P(x^g = x^{g^*}|\mathbf{z}_{1:k}^i) = 1 - \lim_{k\to\infty}\sum_{x^g\in\bar{X}^T} P_{pdf}^i(x^g|\mathbf{z}_{1:k}^i) = 1.$$

■

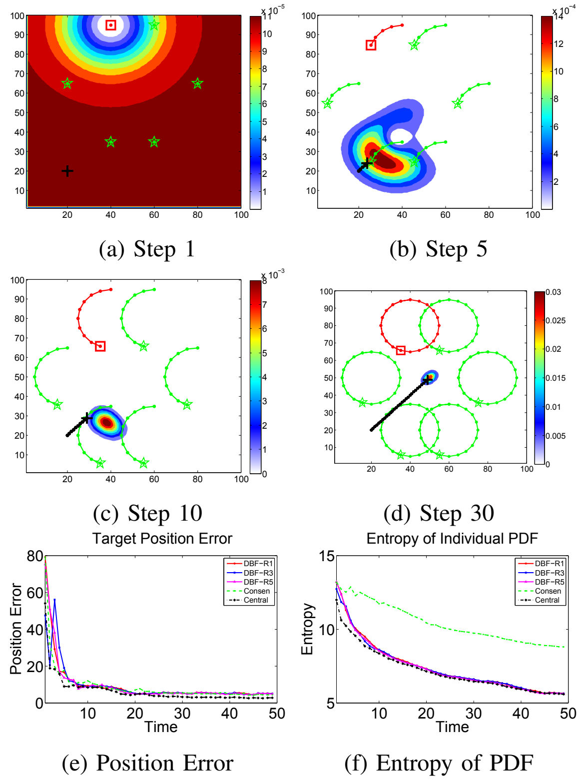
V. Simulation

This section simulates a scenario of target localization to evaluate the effectiveness of LIFO-DBF. The networked UGVs take a ring communication topology that each UGV can communicate with two fixed neighbors. The probabilistic sensor model takes the form defined in Eq. (4).

The LIFO-DBF is compared with two commonly adopted approaches in multi-agent filtering: the consensus-based distributed filtering (CbDF) method and the centralized filtering (CF) method. The CbDF requires UGVs to continually exchange their individual PDFs with direct neighbors, computing the average of all received and its own target PDFs. Multiple rounds of communication and averaging are conducted at each time step to ensure the convergence of each UGV's individual PDFs. The CF assumes a central unit that can constantly receive and fuse all UGVs' latest observations into a single PDF. 10 test trials with randomly generated initial target positions are run and each trial is terminated after 50 time steps. The average error between the estimated and true target position and the average entropy of individual PDFs of all 10 trials are compared.

Figures 3a to 3d shows the evolution of a UGV's individual PDF. Each UGV moves along a pre-defined circular trajectory. The target motion is modeled as a single-integrator without process noise. The LIFO-DBF described in Section III-C is utilized for target localization. It can be noticed that the individual PDF asymptotically concentrates to the true target location. Figs. 3e and 3f compares LIFO-DBF with CbDF and CF[2]. Unsurprisingly, the CF achieves the best performance in terms of both small position estimation error and fast reduction of entropy. This happens because the central unit has access to the latest observations of all UGVs, thus making most use of all available information. It is worth noting that, LIFO-DBF achieves similar asymptotic

[2]Since the average estimation errors of all six UGVs' LIFO-DBF are very similar, we only include three UGVs to make figures easier to read.



(a) Step 1 (b) Step 5 (c) Step 10 (d) Step 30 (e) Position Error (f) Entropy of PDF

Fig. 3: (a)-(d) The individual PDF of a UGV (red square) at different times. The stars represent other UGVs. The black cross shows the target position. (e) Average position estimation errors of the 1st, 3rd and 5th UGV's LIFO-DBF, CbDF and CF. (f) Average entropy of individual PDFs.

performance as the CF, both in position estimation error and entropy reduction; this is achieved even though each UGV only communicates with its two neighboring UGVs. CbDF has the slowest entropy reduction among all three filtering approaches. We note that CbDF requires multiple rounds of exchanging individual PDFs, which incurs much higher communication burden than LIFO-DBF at each time step. Considering the small difference in position estimation error and significantly faster entropy reduction, LIFO-DBF is preferable over CbDF for moving target scenario.

VI. Conclusion

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a multi-UGV network, utilizing the Latest-In-and-Full-Out (LIFO) protocol for measurement exchange. By exchanging full communication buffers among neighboring UGVs, LIFO significantly reduces the transmission burden between each pair of UGVs. It should be noted that LIFO is a general measurement exchange protocol and thus applicable to various sorts of sensors. Two types of LIFO-based DBF algorithms are proposed to estimate individual PDFs for a static target and a moving target, respectively and its consistency property is proved. Future work includes considering the imperfect communication between UGVs and applying the proposed method for distributed localization and mapping.

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