Estimation of Moving Targets Using Distributed Bayesian Filter Under Dynamically Changing Networks

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This paper presents a distributed Bayesian filtering (DBF) method for a network of multiple unmanned ground vehicles (UGVs) under dynamically changing interaction topologies. The information exchange among UGVs relies on a measurement dissemination scheme, called Latest-In-and-Full-Out (FIFO) protocol. Different from statistics dissemination approaches that transmit posterior distributions or likelihood functions, each UGV under FIFOonly sends a buffer that contains latest available measurements to neighboring nodes, which significantly reduces the transmission burden between each pair of UGVs to scale linearly with the size of the network. Under the condition that the union of undirected switching topologies is connected frequently enough, FIFOcan disseminate observations over the network within finite time. The FIFO-based DBF algorithm is then derived to estimate individual probability density function (PDF) for target localization in a static environment. The consistency of this algorithm is proved that each individual estimate of target position converges in probability to the true target position. The effectiveness of this method is demonstrated by comparing with consensus-based distributed filters and the centralized filter in simulations.

1 INTRODUCTION

(**TODO:** mention the relation between BF and KF.) (**TODO:** mention the OOSM issue.)

Unmanned ground vehicles (UGV) that operate without on-board operators have been used for many applications that are inconvenient, dangerous, or impossible to human. Distributed estimation using a group of networked UGVs has been applied to collectively infer status of complex environment, such as intruder detection [1] and object tracking [2]. Several techniques have been developed for distributed estimation, including distributed linear Kalman filters (DKF) [3], distributed extended Kalman filters [4] and distributed particle filters [5], etc. The most generic filtering scheme is distributed Bayesian filters (DBF), which can be applied for nonlinear systems with arbitrary noise distributions [6,7]. This paper focuses on a communication-efficient DBF for networked UGVs.

The interaction topology plays a central role on the design of DBF, of which two types are widely investigated in literature: fusion center (FC) and neighborhood (NB). In the former, local statistics estimated by each agent is transmitted to a single FC, where a global posterior distribution is calculated at each filtering cycle [8,9]. In the latter, each agent individually executes distributed estimation and the agreement of local estimates is achieved by certain consensus strategies [10–12]. In general, the NB-based distributed filters are more suitable in practice since they do not require a fusion center with powerful computation capability and are more robust to changes in network topology and link failures. So far, the NB-based approaches have two mainstream schemes according to the transmitted data among agents, i.e., *statistics dissemination* (SD) and *measurement dissemination* (MD). In the SD scheme, each agent exchanges statistics such as posterior distributions and likelihood functions within neighboring nodes [13]. In the MD scheme, instead of exchanging statistics, each agent sends its observations to neighboring nodes.

Statistics dissemination scheme has gained increasing interest and been widely investigated during last decade. Madhavan et al. (2004) presented a distributed extended Kalman filter for nonlinear systems [4]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with en-

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vironmental features. Olfati-Saber (2005) proposed a distributed linear Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [3]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [5]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-linear-average consensus algorithm when local likelihood functions belong to the exponential family of distributions [14]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [6]. Other examples can be found in [7] and [15].

Despite the popularity of statistics dissemination, exchanging statistics can consume high communication resources. One remedy is to approximate statistics with parametric models, e.g., Gaussian Mixture Model [16], which can reduce communication burden to a certain extent. However, such manipulation increases the computation burden of each agent and sacrifices filtering accuracy due to approximation. The measurement dissemination scheme is an alternative solution to address the issue of exchanging statistics. An early work on measurement dissemination was done by Coates et al. (2004), who used adaptive encoding of observations to minimize communication overhead [17]. Ribeiro et al. (2006) exchanged quantized observations along with error-variance limits considering more pragmatic signal models [18]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each agent has a complete set of observations of other agents for executing particle filtering [19]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a fixed and complete graph that every pair of distinct agents is constantly connected by a unique edge. In many real applications,the interaction topology may change dynamically due to unreliable links, external disturbances and/or range limits [20]. In such cases, dynamically changing topologies can cause random packet loss and variable transmission delay, thus decreasing the performance of distributed estimation, and even leading to inconsistency and non-consensus. Leung et al. (2010) explored a decentralized filter for dynamic robot networks [21]. The algorithm was shown to achieve centralized-equivalent filtering performance in simulations. However, it required the communication of both measurements and statistics, which could incur large communication overhead.

The main contribution of the paper is that we present a measurement dissemination-based distributed Bayesian filtering (DBF) method for a group of networked UGVs with dynamically changing interaction topologies. In our previous work, we have proposed a Latest-In-and-Full-Out (LIFO) protocol for data exchange and developed a LIFO-based DBF. However, it only applies to static target. In this work, we introduce the concept of the track list and extend our methods to time-varying topologies. The measurement dissemination scheme uses the so-called Full-In-and-Full-Out (FIFO) protocol, under which each UGV is only allowed to broadcast observations to its neighbors by using single-hopping. Individual Bayesian filter is implemented locally by each UGV after exchanging observations using FIFO. Under the condition that the union of undirected switching topologies is connected frequently enough, two properties are achieved: (1) FIFO can disseminate observations over the network within finite time; (2) FIFO-based DBF guarantees the consistency of estimation that each individual estimate of target position converges in probability to the true target position as the number of observations tends to infinity. The main benefit of using FIFOis on the reduction of communication burden, with the transmission data volume scaling linearly with the size of the UGV network.

The rest of this paper is organized as follows: the FIFO protocol for dynamically changing interaction topologies is formulated in **??**; the FIFO-based DBF algorithm is described in Section 4, where the consistency of estimation is proved; simulation results are presented in Section 6 and Section 7 concludes the paper.

2 Problem Formulation

Consider a network of N UGVs in a bounded two-dimensional space S. The interaction topology can be dynamically changing due to limited communication range, varying team formation or link failure. Each UGV is equipped with a sensor for environmental perception. Due to the limit of communication range, each UGV can only exchange sensor measurements with its local neighbors. Every UGV locally runs a Bayesian filter to estimate the target position in S utilizing its own measurements and the received measurements from other UGVs. Since this work is focused on the distributed Bayesian filter for target localization, we assume that UGVs' states are accurately known.

2.1 Target and Sensor Model

The target motion takes a deterministic discrete-time model:

$$x^g_{k+1} = f(x^g_k), \tag{1}$$

where the superscript g represents the target and $x^g_k \in S$ is the target position at time k.

The sensor measurement is described by a stochastic model:

$$z^i_k = h_i(x^g_k, x^i_k) + w^i_k, \tag{2}$$

where the superscript $i \in \{1,\dots,N\}$ represents the index of the UGV; $x^i_k \in S$ is the sensor position and w^i_k is the white measurement noise. The measurement function h_i depends on the type of the sensor.

The conditional probability, $P(z_k^i|x_k^g;x_k^i)$, of obtaining a certain measurement z_k^i conditioning on the target and sensor states is critical to designing the Bayesian filter [22]. It also depends on the distribution of the measurement noise. For example, if w_k^i is a zero-mean Gaussian white noise with covariance Γ_k^i, then, according to Eq. (2), $P(z_k^i|x_k^g;x_k^i)$ can be described as

$$P(z_k^i|x_k^g;x_k^i) = \mathcal{N}(h_i(x_k^g,x_k^i),\Gamma_k^i).$$

For non-Gaussian noise distributions, such as the Poisson noise or Cauchy noise [23], $P(z_k^i|x_k^g;x_k^i)$[1] can also be similarly defined. It should be noted that, the approach presented in this work does not rely on the specific distribution of the noise.

The h_i for several typical sensor are defined as follows [24]:

Range-only sensors: h_i only depends on the relative Euclidean distance between the sensor and the target:

$$h_i(x_k^g,x_k^i) = \|x_k^g - x_k^i\|_2,$$

where $\|\cdot\|_2$ is the Euclidean distance in S.

Bearing-only sensors: h_i only depends on the relative bearing between the sensor and the target:

$$h_i(x_k^g,x_k^i) = \measuredangle(x_k^g - x_k^i),$$

where $\measuredangle$ denotes the angle from the sensor to the target.

Range-bearing sensors: h_i includes both the relative distance and bearing:

$$h_i(x_k^g,x_k^i) = x_k^g - x_k^i.$$

Remark 1. *Given the knowledge of current target and UGV positions, the current measurement by each UGV can be considered conditionally independent from its own past measurements and those by other UGVs [25].*

2.2 Graphical Model of Interaction Topology

We consider a simple[2] graph $G = (V,E)$ to represent the interaction topology of N networked UGVs, where the vertex set $V = \{1,\dots,N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. For the purpose of clarity and generalizability, we use directed graphs to describe our approach in this work. However, the approach can conveniently apply to undirected graphs, which can actually be treated as bidirectional directed graphs.

The *adjacency matrix* $A = [a_{ij}]$ of the graph G describes the interaction topology:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where a_{ij} is the entry on the i^{th} row and j^{th} column of the adjacency matrix. The notation $a_{ij} = 1$ indicates that the i^{th} UGV can directly communicate to the j^{th} UGV and $a_{ij} = 0$ indicates no direct communication from i to j. A directed graph is *strongly connected* if there is a directed path connecting any two arbitrary vertices in V[3].

Let $\bar{G}$ denote the set of all possible simple directed graphs[4] defined over the network of UGVs. It is easy to know that $\bar{G}$ has finite elements. The adjacency matrix associated with a graph $G_l \in \bar{G}$ is denoted as $A_l = [a_{ij}^l]$. Define the *union* of a collection of graphs $\{G_{i_1}, G_{i_2}, \dots, G_{i_l}\} \subset \bar{G}$ as the graph with the vertices in V and the edge set given by the union of edge sets of G_{i_j}, $j = 1\dots,l$. Such collection is *jointly strongly connected* if the union of its members forms a strongly connected graph [5].

We define two concepts of neighbors in a UGV network. The *direct neighbors* of i^{th} UGV under topology G_l is defined as $\mathcal{N}_i(G_l) = \left\{ j | a_{ij}^l = 1,\, j \in V \right\}$. All UGVs in $\mathcal{N}_i(G_l)$ can directly receive information from the i^{th} UGV via the single hopping.

3 Full-In-and-Full-Out (FIFO) Protocol

This study proposes a Full-In-and-Full-Out (FIFO) protocol for measurement exchange in time-varying topologies. Let $Y_{\mathcal{K}}^i = \left\{ \left[x_k^i, z_k^i\right] | k \in \mathcal{K} \right\}$ be the set of state-measurement pairs of robot i, where $\mathcal{K}$ is an index set of time steps. Each UGV contains a communication buffer (CB) that stores state-measurement pairs consisting of measurements and the corresponding states of all UGVs:

$$\mathcal{B}_k^i = \left[Y^1_{\mathcal{K}_k^{i,1}}, \dots, Y^N_{\mathcal{K}_k^{i,N}} \right],$$

where $\mathcal{B}_k^i$ is the CB of i^{th} UGV at time k and $\mathcal{K}_k^{i,j} (j \in V)$ is the time index set. $Y^j_{\mathcal{K}_k^{i,j}}$ represents the set of j^{th} UGV's measurements of time steps in $\mathcal{K}_k^{i,j}$ that are stored in i^{th} UGV's CB. The **FIFO protocol** is stated in Algorithm 1. Note that each section in the algorithm contains the CB and TL parts. For the purpose of clarity, we ignore the TL parts at this stage and will describe them in Section 4.2.

(**TODO:** modify the whole paragraph when new plot is made.) Fig. 1 illustrates the FIFO cycles of a network of 3 UGVs with switching line topologies. There are two types of topologies: under the first one only UGV 1 and UGV 2 can directly communicate and under second one only UGV 2 and UGV 3 can directly communicate. Several facts can be noticed in Fig. 1: (1) the two topologies are jointly connected within each time intervals $[0,3)$, $[3,5)$, $[5,7)$; (2) (**TODO:** may need to change) CBs of all UGVs are filled within 5 steps; (3) after being filled, each CB keeps updated every

[1] For the purpose of simplicity, we will not explicitly write the parameter x_k^i in $P(z_k^i|x_k^g;x_k^i)$ for the rest of the paper.

[2] A (directed/undirected) graph $G = (V,E)$ is *simple* if it has no self-loops (i.e., $(i,j) \in E$ only if $i \neq j$) or multiple edges with the same source and target nodes (i.e., E only contains distinct elements).

[3] An undirected graph with this property is called a *connected* graph.

[4] For undirected graphs, we consider the set of simple undirected graphs.

[5] For undirected graphs, such collection is jointly connected [10]

Algorithm 1 FIFO Protocol

(1) Initialization.

CB: The CB of i^{th} UGV is initialized at $k=0$:

$$B_0^i = \left[Y^1_{\mathcal{K}_0^{i,1}}, \ldots, Y^N_{\mathcal{K}_0^{i,N}}\right], \text{ where } Y^j_{\mathcal{K}_0^{i,j}} = \{[\varnothing, \varnothing]\}.$$

TL: The TL of i^{th} UGV is initialized at $k=0$:

$$P_0^i = \mathbf{0}, \text{ i.e. } p_0^{j,l} = 0, \forall j, l \in \{1 \ldots, N\}.$$

(2) At time $k \, (k \geq 1)$ for i^{th} UGV:

(2.1) Receiving Step.

CB: The i^{th} UGV receives all CBs of its direct neighbors $\mathcal{N}_i(G_{k-1})$. The received CBs are totally $|\mathcal{N}_i(G_{k-1})|$ groups, each of which corresponds to the $(k-1)^{\text{th}}$ step CB of a UGV in $\mathcal{N}_i(G_{k-1})$. The received CB from l^{th} UGV is

$$\mathcal{B}^l_{k-1} = \left[Y^1_{\mathcal{K}_{k-1}^{l,1}}, \ldots, Y^N_{\mathcal{K}_{k-1}^{l,N}}\right], \, l \in \mathcal{N}_i(G_{k-1})$$

TL: The i^{th} UGV receives all TLs of its direct neighborhood $\mathcal{N}_i(G_{k-1})$. The received TL from l^{th} $(l \in \mathcal{N}_i(G[k-1]))$ UGV is $P^l_{t_l}$.

(2.2) Observation Step.

CB: The i^{th} UGV updates $Y^i_{\mathcal{K}_k^{i,i}}$ by its own state-measurement pair at current step:

$$Y^i_{\mathcal{K}_k^{i,i}} = Y^i_{\mathcal{K}_{k-1}^{i,i}} \cup \left\{ \left[x_k^i, z_k^i\right] \right\}.$$

(2.3) Updating Step.

CB: The i^{th} UGV updates other elements of its own CB, i.e., $Y^j_{\mathcal{K}_k^{i,j}} \, (j \neq i)$, by merging with all received CBs:

$$Y^j_{\mathcal{K}_k^{i,j}} = Y^j_{\mathcal{K}_{k-1}^{i,i}} \cup Y^j_{\mathcal{K}_{k-1}^{l,j}}, \, \forall j \neq i, \, \forall l \in \mathcal{N}_i(G_{k-1}).$$

TL: The i^{th} UGV updates its own TL using all the received TLs:

$$\begin{aligned} &\text{if } k^{i,j} > k^{i,l}, \text{ keep current } \mathbf{p}^j_{k^{i,j}}; \\ &\text{if } k^{i,j} = k^{i,l}, \mathbf{p}^j_{k^{i,j}} = \mathbf{p}^j_{k^{i,j}} \vee \mathbf{p}^j_{k^{l,j}}; \qquad \forall j \in \{1 \ldots, N\} \\ &\text{if } k^{i,j} < k^{i,l}, \mathbf{p}^j_{k^{i,j}} = \mathbf{p}^j_{k^{l,j}} \text{ and } k^{i,j} = k^{i,l}. \end{aligned}$$

Trim the CB based on the updated track lists, see Algorithm 3.

(2.4) Sending Step:

CB: The i^{th} UGV broadcasts its updated CB, $\mathcal{B}^i_k = \left[Y^1_{\mathcal{K}_k^{l,1}}, \ldots, Y^N_{\mathcal{K}_k^{l,N}}\right]$, to all of its neighbors defined in $\mathcal{N}_i(G_k)$.

TL: The i^{th} UGV broadcasts its updated track list to all of its neighbors defined in $\mathcal{N}_i(G_k)$.

(3) $k \leftarrow k+1$ until stop

finite time steps, which means each UGV receives new observations of other UGVs with finite delay. Extending these facts to a network of N UGVs, we have the following proposition:

Theorem 1. *Consider a network of N UGVs with switching interaction topologies. If the following two conditions are satisfied:*

1. *there exists an infinite sequence of time intervals $[k_m, k_{m+1})$, $m = 1, 2, \ldots$, starting at $k_1 = 0$ and are contiguous, nonempty and uniformly bounded;*
2. *the union of graphs across each such interval is jointly strongly connected,*

then any pair of UGVs can exchange measurements under FIFO. And the communication delay between each pair of UGVs is no greater than $(N-1)T_u$, where $T_u = \sup_{m=1,2,\ldots} (k_{m+1} - k_m)T$ is the upper bound of interval lengths.

Proof. Without loss of generality, we consider the transmission of B_1^i from the i^{th} UGV to an arbitrary j^{th} UGV $(j \in V \setminus \{i\})$. Since each UGV will receive direct neighbors' CBs and send the merged one to its neighbors at the next time step, the i^{th} UGV can transmit B_1^i to j if and only if there is a path from vertex i to j. As the union of graphs across the time interval $[k_1, k_2)$ is jointly connected, i^{th} UGV can directly send B_1^i to at least one another UGV at a time instance, i.e., $\exists l_1 \in V \setminus \{i\}, \exists t_1 \in [k_1, k_2)$ s.t. $l_1 \in \mathcal{N}_i(G_{t_1})$. If $l_1 = j$, then B_1^i has been sent to j. If $l_1 \neq j$, B_1^i has been merged into $B^{l_1}_{t_1}$ and will be sent out in the next time step.

By using the similar derivation for time intervals $[k_m, k_{m+1})$, $, m = 2, 3, \ldots,$ it can be shown that all UGVs can receive the state-measurement pairs in B_1^i no later by k_N. Therefore, the transmission delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$.

Similar to the definition in [10], we define an interaction topology that satisfies the two conditions in Theorem 1 as a *frequently jointly strongly connected* network.

Corollary 1. *For a frequently jointly strongly connected network, each UGV receive the CBs of all other UGVs under FIFO within finite time.*

Proof. According to Theorem 1, the transmission delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$. Therefore, each UGV is guaranteed to receive B_t^j, $\forall t \geq 0$, $j \in V$ when $k \geq t + (N-1)T_u$.

Remark 2. *The frequency that each UGV receive other UGVs' CBs depends on the property of the network. Corollary 1 gives an upper bound on the transmission time for all frequently jointly strongly connected networks under FIFO.*

4 Distributed Bayesian Filter via FIFO Protocol

We first introduce the generic distributed Bayesian filter (DBF). Let $X_k \in S$ be the random variable representing the position of the target at time k. Define $\mathbf{z}_k^i$ to be the set of all measurements of time k in the i^{th} UGV's CB, i.e., $\mathbf{z}_k^i = \left\{ z_k^j \middle| \left[x_k^i, z_k^j\right] \in \mathcal{B}_k^i, \, \forall j \in V \right\}$ and let $\mathbf{z}^i_{1:k} = \bigcup_{t=1}^{k} \mathbf{z}^i_t$. The probability density function (PDF) of X_k, called *individual PDF*, of the i^{th} UGV is represented by $P^i_{pdf}(X_k | \mathbf{z}^i_{1:k})$ and is the estimation

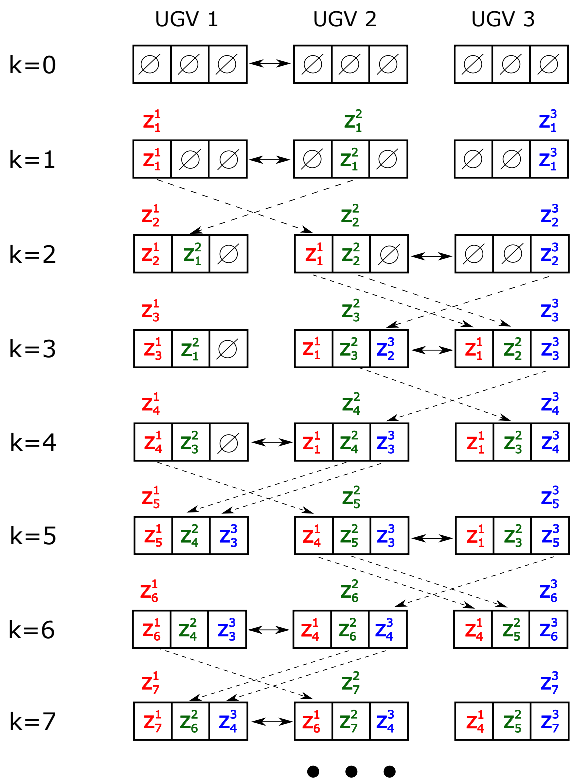


Fig. 1: Example of FIFOwith three UGVs using switching line interaction topologies. The double-headed arrow represents a communication link between two UGVs.

of the target position given all the measurements that the i^{th} UGV has received. The initial individual PDF, $P^i_{pdf}(X_0)$, is constructed given all available prior information including past experience and environment knowledge. It is necessary to initialize $P^i_{pdf}(X_0)$ such that the probability density of the true target position is nonzero, i.e., $P^i_{pdf}(X_0 = x^g_0) > 0$.

Under the framework of DBF, the individual PDF is recursively estimated by two steps: the prediction step and the updating step.

Prediction. At time k, the prior individual PDF $P^i_{pdf}(X_{k-1}|\mathbf{z}^i_{1:k-1})$ is first predicted forward by using the Chapman-Kolmogorov equation:

$$P^i_{pdf}(X_k|\mathbf{z}^i_{1:k-1}) = \int\limits_{X_{k-1}\in S} P(X_k|X_{k-1})P^i_{pdf}(X_{k-1}|\mathbf{z}^i_{1:k-1})dX_{k-1}, \tag{3}$$

where $P(X_k|X_{k-1})$ represents the state transition probability of the target, based on the Markovian motion model (Eq. (1)). For the deterministic motion model, the state transition probability is simplified to be

$$P(X_k = c_k|X_{k-1} = c_{k-1}) = \begin{cases} 1 & \text{if} \quad c_k = f(c_{k-1}, u^g_{k-1}) \\ 0 & \text{otherwise} \end{cases}. \tag{4}$$

Updating. The i^{th} individual PDF is then updated by the Bayes' theorem using the set of newly received measurements at time k, i.e., $\mathbf{z}^i_k$:

$$\begin{aligned} P^i_{pdf}(X_k|\mathbf{z}^i_{1:k}) &= K_i P^i_{pdf}(X_k|\mathbf{z}^i_{1:k-1})P(\mathbf{z}^i_k|X_k) \\ &= K_i P^i_{pdf}(X_k|\mathbf{z}^i_{1:k-1}) \prod_{j\in\Omega^i_k} P(z^j_k|X_k) \end{aligned}$$

where $P(\mathbf{z}^i_k|X_k)$ is the sensor model and K_i is a normalization factor, given by:

$$K_i = \left[\int\limits_{X_k\in S} P^i_{pdf}(X_k|\mathbf{z}^i_{1:k-1})P(\mathbf{z}^i_k|X_k)dX_k\right]^{-1}.$$

Ω^i_k denotes the index set of UGVs whose state-measurement pair of time k is stored in the i^{th} UGV's CB, i.e. $\Omega^i_k = \left\{j \in V | \left[x^j_k, z^j_k\right] \in \mathcal{B}^i_k\right\}$. The factorization of $P(\mathbf{z}^i_k|X_k)$ comes from the conditional independence of measurements between each UGV given the target position and the corresponding UGV's position.

Algorithm 2 FIFO-DBF Algorithm

For i^{th} UGV at k^{th} step ($\forall i \in V$):
After the updating step in Algorithm 1,
(1) Initialize a *temporary PDF* by assigning the stored individual PDF to it:

$$P^i_{tmp}(X_t) = P^i_{stored,t},$$

where the stored individual PDF is for time t:

$$P^i_{stored,t} = P^i_{pdf}(X_t|z^1_{1:t}, \dots, z^N_{1:t}).$$

(2) For $\xi = t+1$ to k, iteratively repeat two steps of Bayesian filtering:
(2.1) Prediction

$$P^{pre}_{tmp}(X_\xi) = \int_S P(X_\xi|X_{\xi-1})P^i_{tmp}(X_{\xi-1})dX_{\xi-1}.$$

(2.2) Updating

$$P^i_{tmp}(X_\xi) = K_\xi P^{pre}_{tmp}(X_\xi)P(\mathbf{z}^j_\xi|X_\xi),$$
$$K_\xi = \left[\int_S P^{pre}_{tmp}(X_\xi)P(\mathbf{z}^j_\xi|X_\xi)dX_\xi\right]^{-1}.$$

(2.3) When $\xi = t+1$, if $z^j_{t+1} \neq \emptyset$ for $\forall j \in V$, then the stored PDF will be updated to be the temporary PDF of time $t+1$:

$$P^i_{stored,t+1} = P^i_{tmp}(X_{t+1}),$$

where

$$P^i_{tmp}(X_{t+1}) = P^i_{pdf}(X_{t+1}|z^1_{1:t+1}, \dots, z^N_{1:t+1}).$$

(3) The individual PDF of i^{th} UGV at time k is $P^i_{pdf}(X_k|\mathbf{z}^i_{1:k}) = P^i_{tmp}(X_k)$.

4.1 FIFO-DBF

The generic DBF is not directly applicable to time-varying interaction topologies. This is because changing topologies can cause intermittent and out-of-sequence reception of measurements from different UGVs, which can give rise to OOSM problem. One solution is to ignore all OOSM, which is undesirable since this will cause information loss. Another solution is to start from the initial individual PDF and incorporate all measurements at every time step, which can cause excessive computational burden. To avoid the OOSM problem while reducing unnecessary computational complexity, we add a new PDF, call the *stored PDF*. The stored PDF, $P^i_{sto}(X_t)$, is updated from the i^{th} UGV's initial PDF by fusing the state-measurement pairs of *all* UGVs up to a certain time $t \leq k$. The choice of t is described in Section 4.2. The individual PDF, $P^i_{pdf}(X_k|\mathbf{z}^i_{1:k})$, is then computed from $P^i_{sto}(X_t)$ by fusing the measurements of times from $t+1$ to k in the CB, running the Bayesian filter (Eq. (3) and **??**). Note that initially, $P^i_{sto}(X_0) = P^i_{pdf}(X_0)$.

The **FIFO-DBF algorithm** is stated in Algorithm 2. At the beginning, we assign the stored PDF to a temporary PDF, which will then be updated by sequentially fusing measurements in the CB to obtain the individual PDF. The temporary PDF is then assigned as the individual PDF of time k. It should be noted that, when the UGV's CB contains all UGVs' state-measurement pairs of time $t+1$, the stored PDF will be updated to be the temporary PDF of $t+1$. Fig. 2 illustrates the FIFO-DBF procedure for the 1st UGV as an example. It can be noticed that, the purpose of using the stored PDF is to avoid running the Bayesian filtering from the initial PDF at every time step. Since the stored PDF has incorporated all UGVs' measurements up to some time step t, the information loss problem no longer exists when computing the individual PDF. We point out that the time t of each UGV's stored PDF can be different from others. The stored PDF is saved locally by each UGV and not transmitted to others[6].

4.2 Track Lists for Trimming Communication Buffers

The size of CBs can keep increasing as measurements cumulate over time. The use of the stored PDF has made it feasible to trim excessive measurements from the CBs. A state-measurement pair can be trimmed from a UGV's CB only when *all* UGVs have received it. To track each UGV's reception of other UGVs' measurements, every UGV maintains a *track list* (TL), $Q^i_k = \left[\mathbf{q}^{i,1}_{k^1}, \dots, \mathbf{q}^{i,N}_{k^N}\right]^T$ $(\forall i \in V)$, where $\mathbf{q}^{i,j}_{k^j} = \left[q^{j,l}_{k^j}, l \in V\right]$ is a binary vector with size N. For the i^{th} UGV, the TL Q^i_k represents its knowledge of the oldest measurements, in terms of the measurement time, by all UGVs. Each element $q^{j,l}_{k^j}$ equals 1 if the i^{th} robot knows that $\left[x^l_{k^j}, z^l_{k^j}\right]$ has been received by the j^{th} robot, and equals 0 if the i^{th} robot cannot determine whether $\left[x^l_{k^j}, z^l_{k^j}\right]$ has been

[6](**TODO:** change notations here) Due to the space limit, in this figure we use $P^i_{pdf}(k)$, $P^i_{pdf}(k-N)$ and $P^i_{pdf}(k-N+1)$ to represent $P^i_{pdf}(X|\mathbf{z}^i_{1:k})$ $P^i_{pdf}(X|\mathbf{z}^i_{1:k-N})$ and $P^i_{pdf}(X|\mathbf{z}^i_{1:k-N+1})$, respectively.

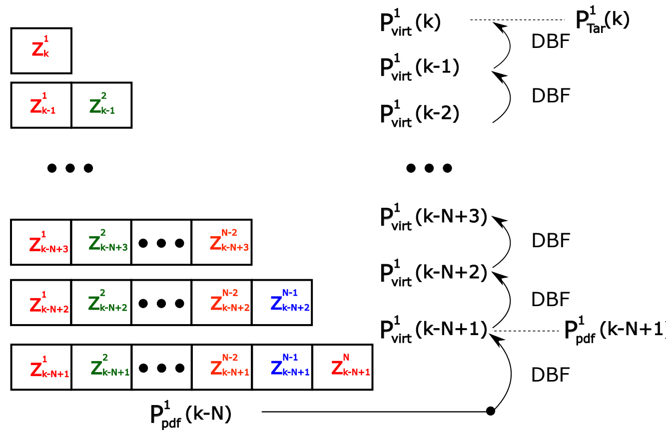


Fig. 2: Example of FIFO-DBF for 1st UGV at time k. Networked UGVs take a line topology. The stored individual PDF is represented by $P^1_{pdf}(k-N)$. The UGV first calculates $P^1_{tmp}(k-N+1)$, defined in Algorithm 2, and then stores it as $P^1_{pdf}(k-N+1)$. Repeating DBF until obtaining $P^1_{pdf}(k)$. In this example, $\Omega^1_\xi = \{1, 2, \dots, N+1-\xi\}$, $\xi = 1, \dots, N$.

received by the j^{th} robot, i.e.,

$$q^{j,l}_{k^j} = \begin{cases} 1 & \text{if} \quad \exists t \in \left[k^{i,j}, k\right] \text{ s.t. } \left[x^l_{k^{i,j}}, z^l_{k^{i,j}}\right] \in B^j_t, \\ 0 & \text{if} \quad \nexists t \in \left[k^{i,j}, k\right] \text{ s.t. } \left[x^l_{k^{i,j}}, z^l_{k^{i,j}}\right] \in B^j_t. \end{cases}$$

It can happen that $\left[x^l_{k^{i,j}}, z^l_{k^{i,j}}\right]$ has been received by the j^{th} UGV but the i^{th} UGV does not know this and thus $q^{j,l}_{k^j} = 0$. When all elements of Q^i_k are 1's, the i^{th} UGV can be sure that the j^{th} UGV $(\forall j \in V)$ has received the state-measurement pairs of time $k^{i,j}$ from all UGVs.

The exchange and updating of TLs are described in the TL part in Algorithm 1. The Algorithm 3 describes the approach to trim CBs using TLs. Since a TL only keeps track of the earliest measurements in CBs, the CBs are only trimmed by one time step each time the trim happens.

The use of TLs can avoid the excessive size of CBs and guarantee that fusing the measurements and trimming the CBs will not lose any information; the trimmed measurements have been encoded into the stored PDF. The following theorem formalizes this property.

Theorem 2. *For a frequently jointly strongly connected network using FIFO-DBF, each UGV's individual PDF is updated with measurements from all UGVs. Each UGV's estimation result using the trimmed CB by the TL is the same as that using the non-trimmed CB.*

Proof. According to Corollary 1, each UGV receives the state-measurement pairs from all other UGVs within finite time steps that are used for updating the individual PDF.

Let $k_m = \min_j k^{i,j}$. Trimming P^i_k happens when all entries are 1. This indicates that each UGV has received the state-measurement pairs of time k_m from all UGVs, i.e.,

$\left[x^l_{k_m}, z^l_{k_m}\right]$, $l \in V$. A UGV has either saved the pairs in its CB or already fused them to update its individual PDF. In both cases, such pairs are no longer needed to be transmitted since it will not add any unused information to the team. Therefore, it causes no loss to trim theses measurements.

The following theorem describes when CBs get trimmed. Consider trimming all the state-measurement pairs of time t in the i^{th} UGV's CB. Let $k^{lj}_t (> t)$ be the first time that the l^{th} UGV communicates to the j^{th} UGV in the time interval (t, ∞). Similarly, define $k^{lji}_t (> k^{lj}_t)$ as the first time that the j^{th} UGV communicates to the i^{th} UGV in the time interval (k^{lj}_t, ∞). The following theorem gives the lower bound on when the i^{th} UGV ($\forall i \in V$) trims all state-measurement pairs of time t in its own CB.

Theorem 3. *The i^{th} UGV trims $\left\{\left[x^l_t, z^l_t\right] (\forall l \in V)\right\}$ from its CB at time k^i_t, where $\max_{l,j \in V} k^{lji}_t \leq k^i_t \leq t + 2(N-1)T_u$ and $T_u = \sup_{m=1,2,\ldots} (k_{m+1} - k_m)T$.*

Proof. The first time for the j^{th} UGV to receive B^l_t from l^{th} UGV is time t_{lj} and the j^{th} UGV's TL is updated so that $q^{j,l}_{t'} = 1$ for some $t' \leq t$. If $t' = t$, then all measurements before t from the l^{th} UGV has already been fused to generate $P^j_{sto}(X_{t-1})$. The first time the i^{th} UGV knows $g^{j,l}_{t'} = 1$ is at time k^{lji}_t, when the j^{th} UGV's TL is transmitted to i^{th} UGV. Therefore, $\max_{l,j \in V} k^{lji}_t$ is the earliest time when the i^{th} UGVs know about each UGV's reception of all measurements of time t.

If i^{th} UGV's stored PDF corresponds to the time step $t-1$, then the trim happens at $k^i_t = \max_{l,j \in V} k^{lji}_t$. If the stored PDF corresponds to a time step less than $t-1$, then the trim happens at $k^i_t > \max_{l,j \in V} k^{lji}_t$. Therefore $k^i_t \geq \max_{l,j \in V} k^{lji}_t$. The upper bound can be obtained using Theorem 1.

Corollary 2. *The first time to trim the i^{th} UGV's CB occurs at $k^i_1 = \max_{l,j \in V} k^{lji}_1$.*

Proof. Notice that the initial stored PDF of each UGV corresponds to the time 0. Therefore, the first time a TL is all 1's the trim occurs.

Corollary 3. *The length of the i^{th} UGV's CB, L^i, is $[1, t + 2(N-1)T_u]$ (**TODO:** double check whether the conclusion is correct.)*

4.3 Complexity of FIFO-DBF

Compared to statistics dissemination, FIFO is generally more communication-efficient for distributed filtering. To be specific, consider a grid representation of the environment of the size $D \times D$. The transmitted data between each pair of UGVs are the CB and TL of each UGV. The size of the CB is $O(NL)$ and the size of the TL is $O(N^2)$. According to Theorem 3, the worst-case size of the CB is $O(N^2 T_u)$. Therefore, the overall communication complexity is $O(N^2 T_u)$. On the contrary, the communicated data of a statistics dissemination approach that transmits unparameterized posterior distributions or likelihood functions is $O(D^2)$. Since D is generally much larger than N in applications such as target localization, the FIFO protocol requires much less communication burden.

It is worth noting that each UGV needs to store an individual PDF and a sotred PDF, each of which has size $O(D^2)$. In addition, each UGV needs to keep the CB and TL. This is generally larger than that of statistics dissemination-based methods, which only stores the individual PDF. Therefore, the FIFO-DBF sacrifices the local memory for reducing the communication burden. This is actually desirable for real applications as local memory of vehicles is usually abundant compared to the limited bandwidth for communication.

Algorithm 3 Trimming CBs using TLs

For the i^{th} UGV: find the smallest time in the track list: $k^i_m = \min\left\{k^1, \ldots, k^N\right\}$. If all elements associated with time k^i_m in Q^i_k are 1's, then

1. set all these items in the track list to be 0, increase k_m by 1.
2. update the track list with the measurements in the current CB;
3. remove all corresponding measurements in the i^{th} UGV's CB.

Remark 3. *Under certain interaction topologies, CBs can grow to undesirable sizes and causes excessive communication burden if the trim cannot happen frequently. In this case, we can use a time window to constrain the measurements that are saved in CBs. This will cause information loss to the measurements. However, with a decently long time window, FIFO-DBF can still effectively estimate the target position.*

5 Proof of Consistency

This section proves the consistency of the maximum a posteriori (MAP) estimator of LIFO-DBF under unbiased sensors (sensors without offset). A state estimator is *consistent* if it converges in probability to the true value of the state [26]. Consistency is an important metric for stochastic filtering approaches [27] and it differs from the concept of consensus; the consensus implies that all UGVs' estimation results converge to a same value, while the consistency not only implies achieving consensus asymptotically, but also requires the estimated value converge to the true value. We first prove the consistency for static UGVs and then for moving UGVs. For simplicity and clarity, we assume S is a finite set (e.g. a finely discretized field).

5.1 Static UGVs

The consistency of FIFO-DBF for static UGVs is stated as follows:

Theorem 4. *Assume the UGVs are static and the sensors are unbiased. If the network of N UGVs is frequently jointly strongly connected, then the MAP estimator of target position converges in probability to the true position of the target using FIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X_k^{MAP} = x_k^g) = 1,\ i \in V,$$

where

$$X_k^{MAP} = \arg\max_X P^i_{pdf}(X_k|\mathbf{z}^i_{1:k}).$$

Proof. Define the time set of i^{th} UGV, $\mathcal{K}_k^{i,j}\,(j\in V)$, that contains the time steps of measurements by the j^{th} UGV that are contained in B_k^i. The batch form of DBF at k^{th} step is

$$P^i_{pdf}(X_k|\mathbf{z}^i_{1:k}) = \frac{P^i_{pdf}(X_0)\prod_{j=1}^{N}\prod_{t\in\mathcal{K}_k^{i,j}} P(z_t^j|X_t)P(X_t|X_{t-1})}{\sum_{X_0,\dots,X_k\in S} P^i_{pdf}(X_0)\prod_{j=1}^{N}\prod_{t\in\mathcal{K}_k^{i,j}} P(z_t^j|X_t)P(X_t|X_{t-1})}.$$

Comparing $P^i_{pdf}(X_k = x_k|\mathbf{z}^i_{1:k})$ with $P^i_{pdf}(X_k = x_k^g|\mathbf{z}^i_{1:k})$[7] yields

$$\frac{P^i_{pdf}(x_k|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x_k^g|\mathbf{z}^i_{1:k})} = \frac{P^i_{pdf}(x_0)\prod_{j=1}^{N}\prod_{t\in\mathcal{K}_k^{i,j}} P(z_t^j|x_t)}{P^i_{pdf}(x_0^g)\prod_{j=1}^{N}\prod_{t\in\mathcal{K}_k^{i,j}} P(z_t^j|x_t^g)}. \tag{5}$$

Take the logarithm of Eq. (5) and average it over k steps:

$$\frac{1}{k}\ln\frac{P^i_{pdf}(x_k|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x_k^g|\mathbf{z}^i_{1:k})} = \frac{1}{k}\ln\frac{P^i_{pdf}(x_0)}{P^i_{pdf}(x_0^g)} + \sum_{j=1}^{N}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{i,j}}\ln\frac{P(z_t^j|x_t)}{P(z_t^j|x_t^g)}. \tag{6}$$

Since $P^i_{pdf}(x_0)$ and $P^i_{pdf}(x_0^g)$ are bounded, then $\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x_0)}{P^i_{pdf}(x_0^g)} = 0$.

The law of large numbers yields

(**TODO:** big mistake: I need to differentiate the moving target and static target. For moving target, the LLN does not apply directly. Gosh, this part seems to be a huge problem. Not sure if it's worth to prove anything in this case...)

$$\frac{1}{k}\sum_{t\in\mathcal{K}_k^{i,j}}\ln\frac{P(z_t^j|x_t)}{P(z_t^j|x_t^g)} \xrightarrow{P} \mathbb{E}_{z_t^j}\left[\frac{P(z_t^j|x_t)}{P(z_t^j|x_t^g)}\right] \tag{7a}$$

$$= \int_{z_t^j} P(z_t^j|x_t^g)\frac{P(z_t^j|x_t)}{P(z_t^j|x_t^g)}dz_t^j \tag{7b}$$

$$= -D_{KL}\left(P(z_t^j|x_t)\|P(z_t^j|x_t^g)\right), \tag{7c}$$

where "$\xrightarrow{P}$" represents "convergence in probability" and $D_{KL}(P_1\|P_2)$ denotes the Kullback-Leibler (KL) divergence between two probability distribution P_1 and P_2. KL divergence has the property that $\forall P_1, P_2,\ D_{KL}(P_1\|P_2) \le 0$ and equality holds if and only if $P_1 = P_2$. This leads to the following conclusion:

$$\lim_{k\to\infty}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{i,j}}\ln\frac{P(z_t^j|x_t)}{P(z_t^j|x_t^g)} < 0,\quad x_t \ne x_t^g$$

$$\lim_{k\to\infty}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{i,j}}\ln\frac{P(z_t^j|x_t)}{P(z_t^j|x_t^g)} = 0,\quad x_t = x_t^g.$$

Then by considering the limiting case of Eq. (6), we can get:

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x_l|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x_l^g|\mathbf{z}^i_{1:k})} < 0,\quad x_l \ne x_l^g \tag{8}$$

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x_l|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x_l^g|\mathbf{z}^i_{1:k})} = 0,\quad x_l = x_l^g. \tag{9}$$

Eq. (8) and (9) imply that

$$\frac{P^i_{pdf}(x_l|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x_l^g|\mathbf{z}^i_{1:k})} \xrightarrow{P} \begin{cases} 0 & x_l \ne x_l^g, \\ 1 & x_l = x_l^g. \end{cases}$$

Therefore,

$$\lim_{k\to\infty} P(X_k^{MAP} = x_k^g) = 1.$$

5.2 Moving UGVs

The consistency proof for the moving UGVs case is different from the static UGVs case in that each moving UGV makes measurements at multiple different positions. We classify UGV measurement positions into two disjoint sets: *infinite-measurement spots* that contain positions where a UGV keeps revisiting as time tends to infinity, and *finite-measurement spots* that contain positions where the UGV visits finitely many times (i.e., the UGV does not visit again after a finite time period). It is easy to know that each UGV has at least one position where it revisits infinitely many times as k tends to infinity.

Theorem 5. *Assume UGVs move within a collection of finite positions and sensors are unbiased, then the MAP estimator of target position converges in probability to the true position of the target using FIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X_k^{MAP} = x_k^g) = 1,\ i \in V.$$

Proof. Similar to Eq. (5), comparing $P^i_{pdf}(x_k^g|\mathbf{z}^i_k)$ and $P^i_{pdf}(x_k^g|\mathbf{z}^i_k)$ yields

$$\frac{P^i_{pdf}(x_k|\mathbf{z}^i_k)}{P^i_{pdf}(x_k^g|\mathbf{z}^i_k)} = \frac{P^i_{pdf}(x_0)\prod_{j=1}^{N}\prod_{l\in\mathcal{K}^i_{j,k}} P(z_l^j|x;x_l^j)}{P^i_{pdf}(x_0^g)\prod_{j=1}^{N}\prod_{l\in\mathcal{K}^i_{j,k}} P(z_l^j|x^g;x_l^j)}. \tag{10}$$

The only difference from Eq. (5) is that $P(z_l^j|x;x_l^j)$ in Eq. (10) varies as the UGV moves. For the finite-measurement spots, by referring to Eq. (7), it is easy to know that their contribution to Eq. (6) diminishes when $k\to\infty$. Therefore, proof using Eq. (10) can be reduced to only considering the infinite-measurement spots and the rest of the proof is similar to that of Theorem 4.

[7]For the purpose of simplicity, we use $P^i_{pdf}(x_k|\mathbf{z}^i_{1:k})$ to represent $P^i_{pdf}(X_k = x_k|\mathbf{z}^i_{1:k})$ in this proof.

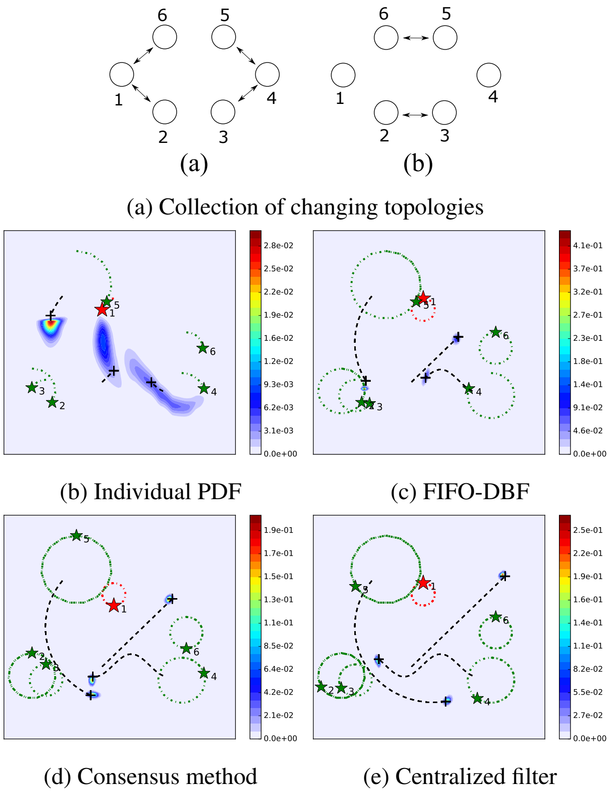


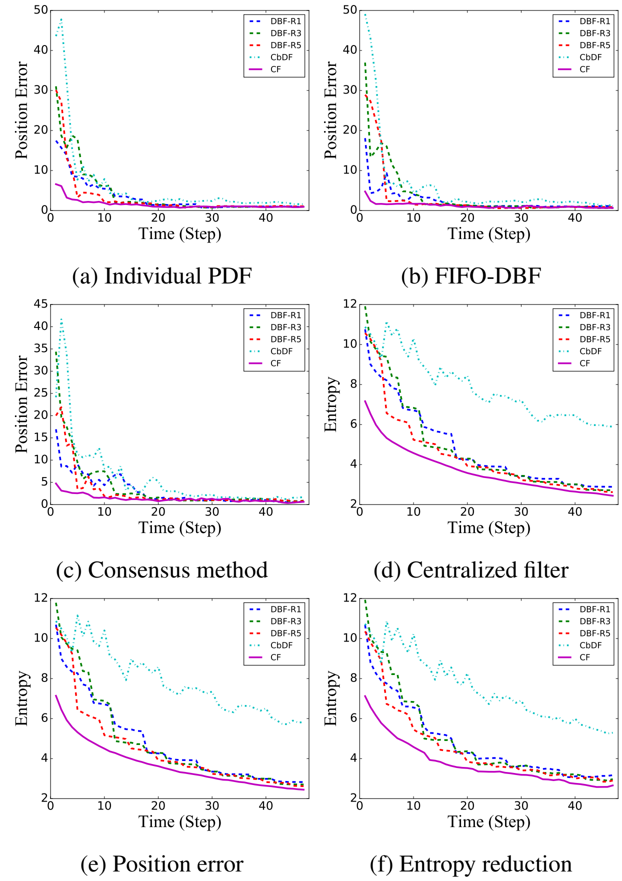
Fig. 3: First scenario: (a) two types of topologies; (b) individual PDF of the 3rd UGV after initial observation; (c)-(e) PDFs at the end of simulation using different filters; (f) average position estimation errors; (g) average entropy of PDF. In last two figures, metrics are based on the PDFs of the 1st, 3rd and 5th UGV using FIFO-DBF, the common PDF using CbDF and using CF.

6 Simulation

An example result is shown in Figure 3. Will generate better simulation later.

7 Conclusion

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) method for a network of multiple unmanned ground vehicles (UGVs) under dynamically changing interaction topologies. The information exchange among UGVs relies on the Full-In-and-Full-Out (FIFO) protocol, under which UGVs exchange the communication buffers and track lists with neighbors. The FIFO can significantly reduce the transmission burden between each pair of UGVs compared to the statistics dissemination methods. Under the condition that the union of the switching topologies is frequently jointly strongly connected, FIFO can disseminate measurements over the network within finite time. By using the track list, the CBs can be trimmed without causing information loss. The FIFO-based DBF algorithm is then derived to estimate individual probability density function for target localization. The consistency of this algorithm is proved by utilizing the law of large numbers, ensuring that each individual estimate of target position converges in probability to the true value. Simulations comparing FIFO-DBF with consensus-based distributed filters (CbDF) and the centralized filter (CF) show that FIFO-DBF achieves similar performance as the CF and superior performance over the CbDF while requiring less communication resource.



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