Distributed Bayesian Filters for Multi-UGV Network by Using Latest-In-and-Full-Out Exchange Protocol of Observations

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Abstract— **This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a network of unmanned ground vehicles (UGVs). The DBF utilizes the Latest-In-and-Full-Out (LIFO) local exchange protocol of sensor measurements for data communication within the network. Different from statistics dissemination-based approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only exchanges with neighboring UGVs a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs to scale linearly with the size of the network. Under the condition of fixed and undirected topology, LIFO can guarantee non-intermittent dissemination of all observations over the network within finite time. Two types of LIFO-based DBF algorithms are then derived to estimate individual probability density function (PDF) for a static target and for a moving target, respectively. For the static target, each UGV locally fuses the newly received observations while for the moving target, a set of historical observations is stored and updated. The consistency of LIFO-based DBF is proved that the estimated target position converges in probability to the true target position. The effectiveness of this method is demonstrated by comparing with consensus-based distributed filters and a centralized filter in simulations of target localization.**

I. INTRODUCTION

Distributed filtering that focuses on using a group of networked UGVs to collectively infer environment status has been used for various applications, such as intruder detection [1], pedestrian tracking [2] and micro-environmental monitoring [3]. Several techniques have been developed for distributed filtering. For example, Olfati-Saber (2005) proposed a distributed Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [4]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Madhavan et al. (2004) presented a distributed extended Kalman filter for nonlinear systems [5]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with environmental features. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [6]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. As a generic filtering scheme for nonlinear systems and arbitrary noise distributions, distributed Bayesian filters (DBF) have received increasing interest during past years [7], [8], which is the focus of this study.

The design of distributed filtering algorithms depends on the communication topology of multi-UGV network, which can be classified into two types: fusion center (FC)-based and neighborhood (NB)-based. In FC-based approaches, each UGV uses a filter to estimate local statistics of environment status based on its own measurement. The local statistics is then transmitted (possibly via multi-hopping) to a single FC, where a global posterior distribution (or statistical moments in DKF [9]) is calculated at each filtering cycle after receiving all local information [10], [11]. In NB-based approaches, a set of UGVs execute distributed filters to estimate individual posterior distribution. Consensus of individual estimates is achieved by solely communicating statistics and/or observations within local neighbors of these UGVs. The NB-based methods have become popular in recent years since such approaches do not require complex routing protocols or global knowledge of the network and therefore are robust to changes in network topology and to link failures.

So far, most studies on NB-based distributed filtering have mainly focused on the so-called *statistics dissemination* strategy that each UGV actually exchanges statistics, including posterior distributions and likelihood functions, with neighboring UGVs [12]. This strategy can be further categorized into two types: leader-based and consensus-based. In the former, statistics is sequentially passed and updated along a path formed by active UGVs, called leaders. Only leaders perform filtering based on its own measurement and received measurements from local neighbors. For example, Sheng et al. (2005) proposed a multiple leader-based distributed particle filter with Gaussian Mixer for target tracking [13]. Sensors are grouped into multiple uncorrelated cliques, in each of which a leader is assigned to perform particle filtering and the particle information is then exchanged among leaders. In consensus-based distributed filters, every UGV diffuses statistics among neighbors, via which global agreement of the statistics is achieved by using consensus protocols [9], [14], [15]. For example, Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-

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linear-average consensus algorithm when local likelihood functions belong to the exponential family of distributions [16]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [7]. Other examples can be found in [8], [17].

Despite the popularity of statistics dissemination strategy, exchanging statistics can consume high communication resources. Approximating statistics with parametric models, such as Gaussian Mixture Models [13], can significantly reduce communication burden. However, such manipulation increases the computation burden for each UGV and sacrifices accuracy of filtering due to the approximation. One promising remedy is to disseminate measurement instead of statistics among neighbors, which, however, has not been fully exploited. One pioneering work was done by Coates et al. (2004), who used adaptive encoding of observations to minimize communication overhead [18]. Ribeiro et al. (2006) exchanged quantized observations along with error-variance limits considering more pragmatic signal models [19]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each UGV has a complete set of observations of other UGVs for executing particle filtering [20]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a complete graph that every pair of distinct UGVs is directly connected by a unique edge, which is not always feasible in reality.

This paper extends existing works by introducing a Latest-In-and-Full-Out (LIFO) protocol into distributed Bayesian filters (DBF) for networked UGVs. Each UGV is only allowed to broadcast observations to its neighbors by using single-hopping and then implements individual Bayesian filter locally after receiving transmitted observations. The main benefit of using LIFO is on the reduction of communication burden, with the transmission data volume scaling linearly with the UGV number, while a statistics dissemination-based strategy can suffer from the order of environmental size. The proposed LIFO-based DBF has following properties: (1) For a fixed and undirected network, LIFO guarantees the global dissemination of observations over the network in a non-intermittent manner. (2) The corresponding DBF ensures the consistency of estimated target position, i.e., the estimated position converges in probability to the true target position when the number of observations tends to infinity.

The rest of this paper is organized as follows: The problem of distributed Bayesian filtering is formulated in Section II. The LIFO-based DBF algorithm is described in Section III, followed by the proof of consistency in Section IV. Simulation results are presented in Section V and Section VI concludes the paper.

II. PROBLEM FORMULATION

Consider a network of N UGVs in a bounded two-dimensional space S. Each UGV is equipped with a binary sensor for environmental perception. The aim of UGVs is to efficiently localize a target in S. Due to the limit of communication range, each UGV can only exchange observations with its neighbors. The Bayesian filter is run locally on each UGV based on its own and received observations via single-hopping to estimate true position of target.

A. Probabilistic Model of Binary Sensor

The target motion takes a discrete-time linear model that can described by

$$x^g_{k+1} = A_k x^g_k + B_k u_k, \tag{1}$$

where $x^g_k = \begin{bmatrix} x^g_k(1) \\ x^g_k(2) \end{bmatrix}$ denotes the target position at time k; $u_k \in \mathbb{R}^{2\times 1}$ represents the control input of the target; $A_k \in \mathbb{R}^{2\times 2},\ B_k \in \mathbb{R}^{2\times 2}$.

The binary sensor constantly measures the target state and The measurement model of i^{th} UGV's sensor can be modeled as

$$y^i_k = h^i(x^g_k),\ x^g_k \in S,$$

where h^i is a sensor property that characterize the target state. For example, h^i can be the power received by an ultrasonic sensor. A binary sensor does not provide y^i_k as its output, instead, it only gives one of two possible values: when the signal y^i_k is greater than a threshold γ, the sensor gives 1, indicating the target is detected in the sensor field of view; otherwise, 0 is given by the sensor. Such model is shown below:

$$z^i_k = \begin{cases} 1 & y^i_k \geq \gamma \\ 0 & y^i_k < \gamma, \end{cases}$$

where z^i_k denotes the observation of i^{th} sensor at time k and its value follows a Bernoulli distribution.

We can use a likelihood function to represent the probability of the target being detected by the binary sensor as a function of the target and sensor positions:

$$P(z^i_k = 1 | x^g_k; x^{R,i}_k) \in [0,1]\,,\ x^g_k \in S \tag{2}$$

where $x^{R,i}_k$ is the i^{th} sensor's position. Correspondingly, the likelihood function that no target is detected is:

$$P(z^i_k = 0 | x^g_k; x^{R,i}_k) = 1 - P(z^i_k = 1 | x^g_k; x^{R,i}_k) \tag{3}$$

The combination of Eq. (2) and Eq. (3) forms the probabilistic model for a binary sensor. For the purpose of simplicity, we will not explicitly write $x^{R,i}_k$ when no confusion may occur. The commonly used likelihood functions for binary sensors include Gaussian function [21], [22] and step function [23].

Remark 1: Given the knowledge of current target and UGV positions, current observation of each UGV is conditionally independent from its own past observations and those of other UGVs.

Remark 2: The proposed LIFO protocol to be described in Section III is applicable for both homogeneous and heterogeneous binary sensors. A homogeneous model can simplify the analysis of completeness, while a heterogeneous model is closer to real sensing characteristics. In addition, it also works for other types of sensors, such as laser scanners and cameras.

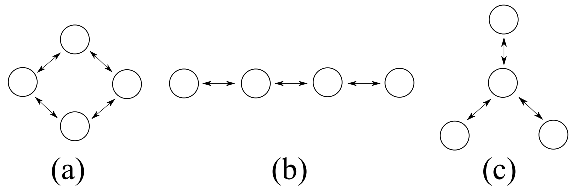


Fig. 1: Three types of topologies: (a) ring topology; (b) line topology; (c) star topology

B. Graphical Model of Communication Topology

The UGV network is always assumed to be connected, i.e., there exists a path, either direct or indirect, between every pair of UGVs. Under this assumption, consider an undirected and fixed graph $G = (V, E)$, where $V = \{1, \dots, N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. The adjacency matrix $M = [m_{ij}]$ describes the communication topology of G:

$$m_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where m_{ij} denotes the entity of adjacency matrix. The notation $m_{ij} = 1$ indicates that a communication link exists between i^{th} and j^{th} UGV and $m_{ij} = 0$ indicates no communication between them.

The *direct neighborhood* of i^{th} UGV is defined as $\mathcal{N}_i = \{j | m_{ij} = 1, \forall j \in \{1, \dots, N\}\}$. All the UGVs in $\mathcal{N}_i$ can directly exchange information with i^{th} UGV. In addition to direct neighborhood, another set called *available neighborhood* is defined as $\mathcal{Q}_i$, which contains indices of UGVs whose observations can be received by the i^{th} UGV given a specific observation exchange protocol. Note that in general $\mathcal{N}_i \subseteq \mathcal{Q}_i$. Fig. 1 illustrates three types of typical topologies: ring [24], line [25], and star [26]. All of them are undirected and connected topologies.

C. Distributed Bayesian Filter for Multiple UGVs

The generic distributed Bayesian filter (DBF) is introduced in this section. Each UGV has its individual estimation of probability density function (PDF) of target position, called *individual PDF*. The individual PDF of i^{th} UGV at time k is defined as $P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k})$, where $\mathbf{z}^i_{1:k}$ denotes the set of observations by i^{th} UGV and by UGVs in $\mathcal{Q}_i$ that are transmitted to i^{th} UGV by time k. The individual PDF is initialized as $P^i_{pdf}(x^g_0 | \mathbf{z}^i_0) = P(x^g_0)$, given all available prior information including past experience and environmental knowledge. Under the framework of DBF, the individual PDF is recursively estimated by two steps, i.e., prediction step and updating step, based on observations of i^{th} UGV and UGVs in $\mathcal{Q}_i$.

1) Prediction: At time k, the prior individual PDF $P^i_{pdf}(x^g_{k-1} | \mathbf{z}^i_{1:k-1})$ is first predicted forward by using the Chapman-Kolmogorov equation:

$$P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k-1}) = \int P(x^g_k | x^g_{k-1}) P^i_{pdf}(x^g_{k-1} | \mathbf{z}^i_{1:k-1}) dx^g_{k-1} \quad (4)$$

where $P(x^g_k | x^g_{k-1})$ is a Markov motion model of the target, independent of UGV states. This model describes the state transition probability of the target from a prior state x^g_{k-1} to posterior state x^g_k. Note that the target is static in many search applications, such as the indoor search for stationary objects [27]. For a static target, its Markov motion model is simplified to be

$$P(x^g_k | x^g_{k-1}) = \begin{cases} 1 & \text{if } x^g_k = x^g_{k-1} \\ 0 & \text{if } x^g_k \neq x^g_{k-1} \end{cases}.$$

2) Updating: The i^{th} individual PDF is then updated by Bayes' theorem using the set of newly received observations at time k, $\mathbf{z}^i_k$:

$$P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k}) = K_i P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k-1}) P(\mathbf{z}^i_k | x^g_k) \quad (5)$$

where K_i is a normalization factor, given by:

$$K_i = 1 / \int P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k-1}) P(\mathbf{z}^i_k | x^g_k) dx^g_k$$

and $P^i_{pdf}(x^g_k | \mathbf{z}^i_{1:k})$ is called posterior individual PDF; $P(z^i_k | x^g_k)$ is the likelihood function of observation z^i_k, described in Eq. (2) and Eq. (3).

III. Distributed Bayesian Filter via Latest-In-and-Full-Out Protocol

This study proposes a Latest-In-and-Full-Out (LIFO) protocol for observation exchange and derives two corresponding distributed Bayesian filtering (DBF) algorithms, shorted as LIFO-DBF. The data communication in LIFO is synchronized with the execution of DBF. In each step, LIFO only allows single-hopping communication within the direct neighborhood, but is able to broadcast observations of each UGV to any other agent after a finite number of steps. The individual PDF is forward predicted and updated in DBF after each LIFO cycle. The theoretical analysis show that LIFO-DBF can ensure the consistency and consensus of distributed estimation while requiring much less communication burden than statistics dissemination-based methods.

A. Latest-In-and-Full-Out (LIFO) Protocol

Under LIFO, each UGV contains a communication buffer (CB) to store its latest knowledge of observations of all UGVs:

$$\mathbf{z}^{CB,i}_k = \left[z^1_{k^i_1}, \dots, z^N_{k^i_N} \right]$$

where $z^j_{k^i_j}$ represents the observation made by j^{th} UGV at time k^i_j. Note that under LIFO, $\mathcal{Q}_i = \{1, \dots, N\} \setminus \{i\}$, which will be proved in Corollary 1. At time k, $z^j_{k^i_j}$ is received and stored in i^{th} UGV CB, in which k^i_j is the latest observation time of j^{th} UGV available to i^{th} UGV. Due to the communication delay, $k^i_j < k, \forall j \neq i$ and $k^i_i = k$ always holds. The **LIFO protocol** is stated in Algorithm 1.

Fig. 2 illustrates the LIFO cycles with 3 UGVs using a line topology. Two facts can be noticed in Fig. 2: (1) all UGV CBs are filled within 3 steps, which means under LIFO each UGV has a maximum delay of 2 steps for receiving observations from other UGVs; (2) after filled, CBs are updated non-intermittently, which means each UGV continuously receives new observations of other UGVs. Extending the two facts to a network of N UGVs, we have the following proposition:

Algorithm 1 LIFO Protocol

(1) Initialization: The CB of i^{th} UGV is initialized when $k = 0$:

$$z^j_{k^i_j} = \varnothing,\ k^i_j = 0,\ j = 1, \dots, N$$

(2) At k^{th} step for i^{th} UGV :

(2.1) Receiving Step:

The i^{th} UGV receives all CBs of its direct neighborhood $\mathcal{N}_i$, each of which corresponds to the (k-1)-step CB of a UGV in $\mathcal{N}_i$. The received CB from l^{th} ($l \in \mathcal{N}_i$) UGV is denoted as

$$\mathbf{z}^{CB,l}_{k-1} = \left[z^1_{(k-1)^l_1}, \dots, z^N_{(k-1)^l_N} \right],\ l \in \mathcal{N}_i$$

(2.2) Observation Step:

The i^{th} UGV updates $z^j_{k^i_j}$ ($j = i$) by its own observation at current step:

$$z^j_{k^i_j} = z^i_k,\ k^i_j = k, \text{ if } j = i.$$

(2.3) Comparison Step:

The i^{th} UGV updates other elements of its own CB, i.e., $z^j_{k^i_j}$ ($j \neq i$), by selecting the latest information among all received CBs from $\mathcal{N}_i$. For all $j \neq i$,

$$l_{\text{latest}} = \underset{l \in \mathcal{N}_i,\ i}{\operatorname{argmax}} \left\{ (k-1)^i_j, (k-1)^l_j \right\}$$

$$z^j_{k^i_j} = z^j_{(k-1)^{l_{\text{latest}}}_j},\ k^i_j = (k-1)^{l_{\text{latest}}}_j$$

(2.4) Sending Step:

The i^{th} UGV broadcasts its updated CB to all of its neighbors defined in $\mathcal{N}_i$.

(3) $k \leftarrow k + 1$ until stop

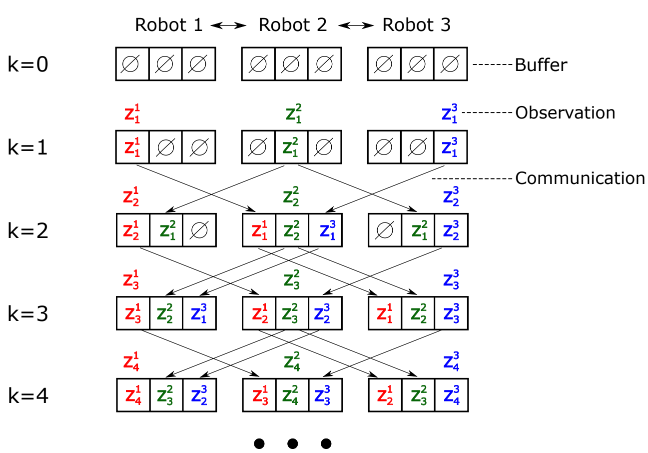


Fig. 2: Example of LIFO with three UGVs using line communication topology

Proposition 1: For a fixed and undirected network of N UGVs, LIFO uses the shortest path(s) between i^{th} and j^{th} UGV to exchange observation, the length of which is the delay $\tau_{i,j}$ between these two UGVs.

Proof: Without loss of generality, assume that there is a unique shortest path between i and j, denoted by $T^{j,i}_{n^*} = (v_1, \dots, v_{n^*})$, with $v_1 = j, v_{n^*} = i, v_{m+1} \in \mathcal{N}_{v_m}$. Then, the distance between i and j is $d(j,i) = n^* - 1$. The following mathematical induction will prove Proposition 1.

Step (1): When $d(j,i) = 1$, $j \in \mathcal{N}_i$ and j can directly send z^j_k to i. Then z^j_k is stored in i^{th} CB at time $k+1$, i.e. $\tau_{i,j} = 1$. Proposition 1 holds for $d(j,i) = 1$, $\forall i, j \in \{1, \dots, N\}$.

Step (2): Suppose that Proposition 1 holds for $d(j,i) = s$, $s \geq 2$, $\forall i, j \in \{1, \dots, N\}$. Then for $d(j,i) = s + 1$, i.e., $n^* = s + 2$, by the Bellman's principle of optimality, the path $T^{j,l}_{n^*-1} = (v_1, \dots, v_{n^*-1})$ is a shortest path between j and l, where $v_{n^*-1} = l$ and $i \in \mathcal{N}_l$. The assumption that Proposition 1 holds for $d(j,i) = s$ implies that z^j_k is received and stored in l^{th} UGV's CB at time $k + s$. Since $i \in \mathcal{N}_l$, i^{th} UGV receives z^j_k at $k + s + 1$. For any other path $T^{j,i}_n = (v_1, \dots, v_n)$ with $n > n^*$, z^j_k cannot be received by i earlier than $k + s + 1$. Therefore $\tau_{i,j} = s + 1$. This proves the Proposition 1 for $d(j,i) = s + 1$. ■

Corollary 1: For the same topology in Proposition 1, all elements in $\mathbf{z}^{CB,i}_k$ under LIFO become filled when $k \geq N$, i.e., $\mathcal{Q}_i = \{1, \dots, N\} \setminus \{i\}$.

Proof: In a network of N UGVs, the maximal length of shortest paths is no greater than $N - 1$. Based on Proposition 1, $\tau_{i,j} \leq N - 1$ and thus all elements of $\mathbf{z}^{CB,i}_k$ become filled when $k \geq N$. ■

Corollary 2: For the same topology in Proposition 1, once all elements in $\mathbf{z}^{CB,i}_k$ are filled, the updating of each element is non-intermittent.

Proof: For a network with fixed topology, shortest path(s) between any pair of nodes are fixed. Therefore, based on Proposition 1, $\tau_{i,j}$ is constant and the updating of each element in $\mathbf{z}^{CB,i}_k$ is non-intermittent. ■

Compared to statistics dissemination, LIFO is generally more communication-efficient for distributed filtering. To be specific, consider an $M \times M$ grid environment with a network of N UGVs, the transmitted data of LIFO between each pair of UGVs are only the CB of each UGV and the corresponding UGV positions where observations were made, the size of which is $O(N)$, scaling linearly with UGV number. On the contrary, the length of transmitted data for a statistics dissemination approach that transmits unparameterized posterior distributions or likelihood functions is $O(M^2)$, which is in the order of environmental size. Since M is generally much larger than N in applications such as target localization, LIFO requires much less communication resources.

B. Algorithm of LIFO-DBF for Static Target

This section derives the LIFO-DBF algorithm for localizing a static target. Each UGV stores last-step individual PDF, i.e., $P^i_{pdf}(x^g|\mathbf{z}^i_{1:k-1})$. According to Corollary 2, $\mathbf{z}^i_k = \mathbf{z}^{CB,i}_k$

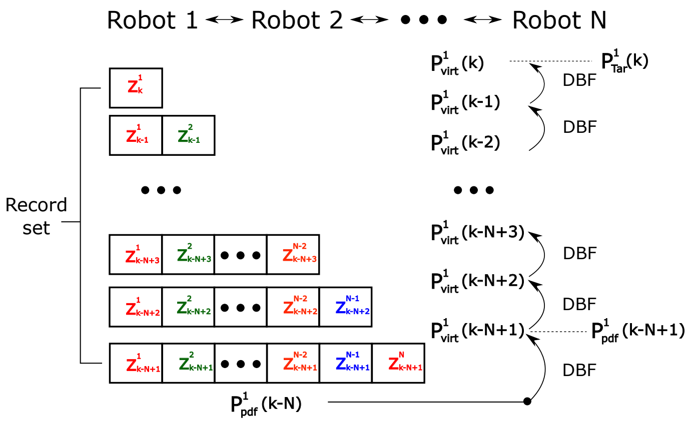


Fig. 3: Example of LIFO-DBF for 1st UGV at time k. Networked UGVs take a line topology, shown in the top. The stored individual PDF is $P^1_{pdf}(k-N)$. The UGV first calculates $P^1_{virt}(k-N+1)$ using DBF and stores it as $P^1_{pdf}(k-N+1)$. Repeating DBF until obtaining $P^1_{pdf}(k)$, which is then used as the target PDF estimation of 1st UGV at time k. In this example, $\Omega^1_\xi = \{1, 2, \ldots, N+1-\xi\}$, $\xi = 1, \ldots, N$.

and $\mathbf{z}^i_{1:k} = \mathbf{z}^{CB,i}_{1:k} = \left[z^1_{1:k^i_1}, \ldots, z^N_{1:k^i_N}\right]$. The assumption of static target can simplify the Bayesian filter as the prediction step becomes unnecessary. Therefore, the i^{th} individual PDF is only updated by

$$\begin{aligned} P^i_{pdf}(x^g|\mathbf{z}^i_{1:k}) &= K_i P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k-1}) P(\mathbf{z}^i_k|x^g_k) \\ &= K_i P^i_{pdf}(x^g|\mathbf{z}^i_{1:k-1}) \prod_{j=1}^{N} P(z^j_{k^i_j}|x^g) \end{aligned} \tag{6}$$

where

$$K_i = 1/\int P^i_{pdf}(x^g|\mathbf{z}^i_{1:k-1}) \prod_{j=1}^{N} P(z^j_{k^i_j}|x^g) dx^g$$

C. Algorithm of LIFO-DBF for Moving Target

This section derives the LIFO-DBF for localizing a moving target. Instead of storing last-step PDF, at time k each UGV maintains an individual PDF of time $(k-N)$ and a collection of historical observations, called the *record set*, from time $(k-N+1)$ to k. The i^{th} individual PDF is then alternatively predicted and updated by using the aforementioned Bayesian filter (Eq. (4) and Eq. (5)) from $(k-N)$ to k. Fig. 3 illustrates the LIFO-DBF procedure for the 1st UGV as an example. With a line topology, the record set of 1st UGV is shown as a triangle.

Let Ω^i_ξ $(\xi = 1, \ldots, N)$ denote the index set of UGVs whose observation at time $(k-N+\xi)$ is stored in i^{th} UGV's record set. The **LIFO-DBF algorithm** for moving target is then stated in Algorithm 2.

Remark 3: For the static target, each UGV only needs current step CB to update individual PDFs. Therefore, besides storing individual PDFs, only current-step CB is stored in UGV memory and all historical CBs can be discarded, which means that the size of occupied memory is $O(N)$. On the contrary, for the moving target, each UGV needs to store a triangular matrix of historical observation with size of $O(N^2)$ and an individual PDF with size $O(M^2)$, which means that the size of occupied memory in each UGV is $O(M^2+N^2)$. This is generally larger that statistics dissemination, the storage size of which is $O(M^2)$. Therefore, LIFO-DBF sacrifices storage size for reducing communication burden. This is desirable for contemporary usage as local memory of vehicles are abundant compared to the limited bandwidth for communication.

IV. Proof of Consistency

This section proves the consistency of LIFO-DBF. Only proofs for localizing a static target are presented, including static UGVs and moving UGVs. The proof of LIFO-DBF for localizing a moving target is similar but requires extra algebraic manipulation, which is omitted due to space limit.

Algorithm 2 LIFO-DBF Algorithm

For i^{th} UGV at k^{th} step:
After updating CB by Algorithm 1,
(1) The stored individual PDF for time $(k-N)$ is:

$$P^i_{pdf}(x^g_{k-N}|z^1_{1:k-N}, \ldots, z^N_{1:k-N})$$

(2) Initialize a virtual PDF by assigning the individual PDF to it:

$$P^i_{virt}(x^g_{k-N}) = P^i_{pdf}(x^g_{k-N}|z^1_{1:k-N}, \ldots, z^N_{1:k-N})$$

(3) From $\xi = 1$ to N, iteratively repeat two steps of Bayesian filtering:
(3.1) Prediction

$$\begin{aligned} &P^{pre}_{virt}(x^g_{k-N+\xi}) \\ &= \int P(x^g_{k-N+\xi}|x^g_{k-N+\xi-1}) P^i_{virt}(x^g_{k-N+\xi-1}) dx^g_{k-N+\xi-1} \end{aligned}$$

(3.2) Updating

$$P^i_{virt}(x^g_{k-N+\xi}) = K_\xi P^{pre}_{virt}(x^g_{k-N+\xi}) \prod_{j \in \Omega^i_\xi} P(z^j_{k-N+\xi}|x^g_{k-N+\xi})$$

$$K_\xi = 1/\int P^{pre}_{virt}(x^g_{k-N+\xi}) \prod_{j \in \Omega^i_\xi} P(z^j_{k-N+\xi}|x^g_{k-N+\xi}) dx^g_{k-N+\xi}$$

(3.3) When $\xi = 1$, store the virtual PDF as the individual PDF for time $(k-N+1)$

$$P^i_{pdf}(x^g_{k-N+1}|z^1_{1:k-N+1}, \ldots, z^N_{1:k-N+1}) = P^i_{virt}(x^g_{k-N+1}).$$

(4) Individual PDF of i^{th} UGV at time k is $P^i_{pdf}(x^g_k|\mathbf{z}^i_{1:k}) = P^i_{virt}(x^g_k)$.

A. Proof for static UGVs

Considering S is finite and x^{T^*} is the true location of target, the consistency of LIFO-DBF for static UGVs is stated as follows:

Theorem 1: When UGVs are static, the estimated target position converges to the true position of target in probability using LIFO-DBF, i.e.,

$$\lim_{k \to \infty} P(x^g = x^{T^*}|\mathbf{z}^i_{1:k}) = 1, \; i = 1, \ldots, N.$$

Proof: Considering the conditional independence of observations given $x^g \in S$, the batch form of DBF at k^{th} step is:

$$P^i_{pdf}(x^g|\mathbf{z}^i_{1:k}) = P^i_{pdf}(x^g|z^1_{1:k^i_1}, \dots, z^N_{1:k^i_N})$$
$$= \frac{P^i_{pdf}(x^g)\prod_{j=1}^{N}\prod_{l=1}^{k^i_j} P(z^j_l|x^g)}{\sum_{x^g\in S} P^i_{pdf}(x^g)\prod_{j=1}^{N}\prod_{l=1}^{k^i_j} P(z^j_l|x^g)},$$

where P^i_{pdf} is i^{th} initial individual PDF. It is known from Corollary 1 and Corollary 2 that $k - N < k^i_j \le k$.

Comparing $P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})$ with $P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})$ yields

$$\frac{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})} = \frac{P^i_{pdf}(x^g)\prod_{j=1}^{N}\prod_{l=1}^{k^i_j} P(z^j_l|x^g)}{P^i_{pdf}(x^{T^*})\prod_{j=1}^{N}\prod_{l=1}^{k^i_j} P(z^j_l|x^{T^*})} \tag{7}$$

Take the logarithm of Eq. (7) and average it over k steps:

$$\frac{1}{k}\ln\frac{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})} = \frac{1}{k}\ln\frac{P^i_{pdf}(x^g)}{P^i_{pdf}(x^{T^*})} + \sum_{j=1}^{N}\frac{1}{k}\sum_{l=1}^{k^i_j}\ln\frac{P(z^j_l|x^g)}{P(z^j_l|x^{T^*})}. \tag{8}$$

Since $P^i_{pdf}(x^g)$ and $P^i_{pdf}(x^{T^*})$ are bounded, then

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x^g)}{P^i_{pdf}(x^{T^*})} = 0. \tag{9}$$

The binary observations subject to Bernoulli distribution $B(1, p_j)$, yielding

$$P(z^j_l|x^g) = p_j^{z^j_l}(1-p_j)^{1-z^j_l}$$

where $p_j = P(z^j_l = 1|x^g)$. Utilizing the facts: (1) z^j_l are conditionally independent samples from $B(1, p^*_j)$ and (2) $k - N < k^i_j \le k$, the law of large numbers yields

$$\frac{1}{k}\sum_{l=1}^{k^i_j} z^j_l \xrightarrow{P} p^*_j, \quad \frac{1}{k}(k^i_j - \sum_{l=1}^{k^i_j} z^j_l) \xrightarrow{P} 1 - p^*_j$$

where $p^*_j = P(z^j_l = 1|x^{T^*})$ and "$\xrightarrow{P}$" denotes "convergence in probability". Then,

$$\frac{1}{k}\sum_{l=1}^{k_j}\ln\frac{P(z^j_l|x^g)}{P(z^j_l|x^{T^*})} \xrightarrow{P} p^*_j\ln\frac{p_j}{p^*_j} + (1-p^*_j)\ln\frac{1-p_j}{1-p^*_j} \tag{10}$$

Note that the right-hand side of Eq. (10) achieves maximum value 0 if and only if $p_j = p^*_j$. Define

$$c(x^g) = \sum_{j=1}^{N} p^*_j\ln\frac{p_j}{p^*_j} + (1-p^*_j)\ln\frac{1-p_j}{1-p^*_j}.$$

Considering Eq. (9) and Eq. (10), the limit of Eq. (8) is

$$\frac{1}{k}\ln\frac{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})} \xrightarrow{P} c(x^g) \tag{11}$$

It follows from Eq. (11) that

$$\frac{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})e^{c(x^g)k}} \xrightarrow{P} 1 \tag{12}$$

Define the set $\bar{X}^T = S \setminus \{x^{T^*}\}$ and $c_M = \max_{x^g\in\bar{X}^T} c(x^g)$. Then $c_M < 0$. Summing Eq. (12) over $\bar{X}^T$ yields

$$\frac{\sum_{x^g\in\bar{X}^T} P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})e^{[c_M - c(x^g)]k}}{P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})e^{c_M k}} \xrightarrow{P} |\bar{X}^T| \tag{13}$$

where $|\bar{X}^T|$ denotes the cardinality of $\bar{X}^T$.

Since $c_M < 0$, $P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})e^{c_M k} \longrightarrow 0$, Eq. (13) implies

$$\sum_{x^g\in\bar{X}^T} P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})e^{[c_M - c(x^g)]k} \xrightarrow{P} 0 \tag{14}$$

Utilizing the relation

$$0 \le P^i_{pdf}(x^g|\mathbf{z}^i_{1:k}) \le P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})e^{[c_M - c(x^g)]k},$$

it can be derived from Eq. (14) that

$$\sum_{x^g\in\bar{X}^T} P^i_{pdf}(x^g|\mathbf{z}^i_{1:k}) \xrightarrow{P} 0$$

Therefore,

$$\lim_{k\to\infty} P(x^g = x^{T^*}|\mathbf{z}^i_{1:k}) = 1 - \lim_{k\to\infty}\sum_{x^g\in\bar{X}^T} P^i_{pdf}(x^g|\mathbf{z}^i_{1:k}) = 1$$

■

B. Proof for moving UGVs

The difficulty of consistency proof for moving UGVs lies in the fact that each UGV makes observations at multiple positions. Here, the main idea is to classify UGV observation positions into two disjoint sets: *infinite-observation spots* that contain positions where a UGV makes infinite observations as time tends to infinity, and *finite-observation spots* that contain positions where the UGV makes finite observations. Before stating the main theorem, the following lemma is introduced.

Lemma 1: For a set of UGVs moving within a collection of finite positions, each UGV has at least one position where infinite observations are made as k tends to infinity.

Proof: Let $n^{i,k}_j$ denote the times that i^{th} UGV visits j^{th} position up to time k. Then, $\sum_j n^{i,k}_j = k$. It is straightforward to see that $\exists n^{i,k}_j$, such that $n^{i,k}_j \to \infty$, as $k \to \infty$. ■

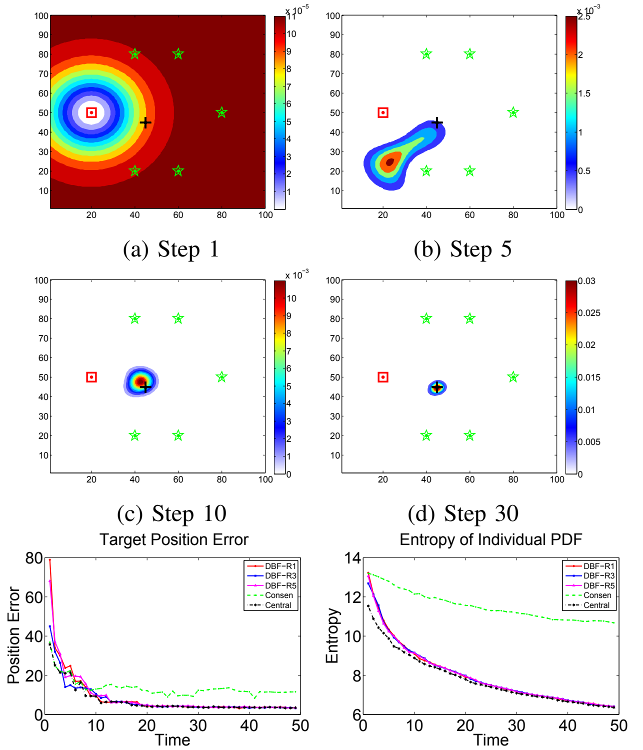
Theorem 2: If UGVs move within a collection of finite positions, the estimated target position converges to the true position of target in probability using LIFO-DBF, i.e.,

$$\lim_{k\to\infty} P(x^g = x^{T^*}|\mathbf{z}^i_{1:k}) = 1,\ i = 1, \dots, N$$

Proof: Similar to Theorem 1, the batch form of DBF at k^{th} step is

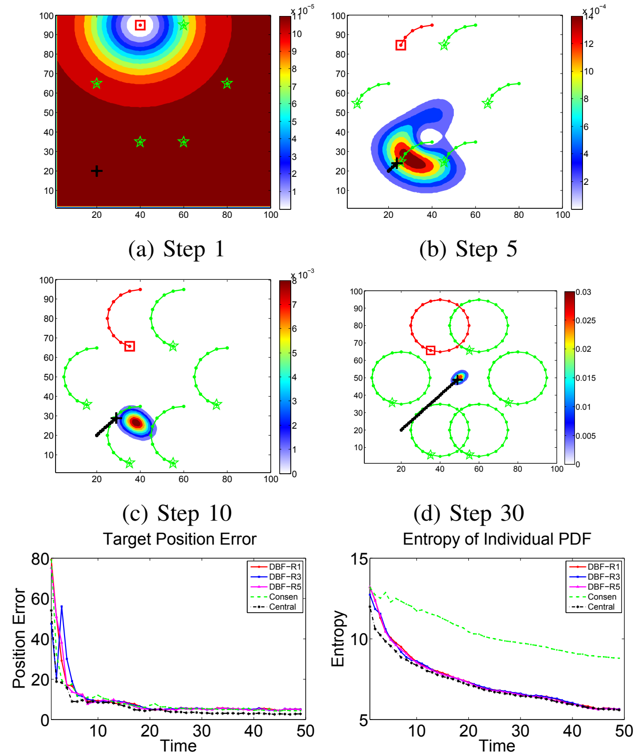
$$\frac{P^i_{pdf}(x^g|\mathbf{z}^i_{1:k})}{P^i_{pdf}(x^{T^*}|\mathbf{z}^i_{1:k})} = \frac{P^i_{pdf}(x^g)\prod_{j=1}^{N}\prod_{l=1}^{k^i_j} P(z^j_l|x^g; x^R_l)}{P^i_{pdf}(x^{T^*})\prod_{j=1}^{N}\prod_{l=1}^{k^i_j} P(z^j_l|x^{T^*}; x^R_l)} \tag{15}$$

The only difference from Eq. (7) is that $P(z^j_l|x^g; x^R_l)$ in Eq. (15) varies as the UGV moves. For each UGV, there exists at least one position where infinite observations are made as $k \to \infty$, according to Lemma 1. All positions



(a) Step 1 (b) Step 5 (c) Step 10 (d) Step 30 (e) Position Error (f) Entropy of Individual PDF

Fig. 4: (a)-(d) The 1st UGV's individual PDF at different times. The square denotes the current UGV and stars represent other UGVs. The cross stands for the target. (e) Average position estimation errors of 1st, 3rd and 5th UGV's LIFO-DBF, CbDF and CF. (f) Average entropy of individual PDFs.



(a) Step 1 (b) Step 5 (c) Step 10 (d) Step 30 (e) Position Error (f) Entropy of Individual PDF

Fig. 5: (a)-(d) The 6th UGV's individual PDFs at different times. (e) Average position estimation errors. (f) Average entropy of individual PDFs.

can be classified into finite-observation spots and infinite-observation spots. For the former, by referring to Eq. (11) in proof of Theorem 1, it is easy to know that their contribution to Eq. (15) is zero when $k \to \infty$. Therefore, Eq. (15) can be reduced to only consider infinite-observation spots, which is similar to proof of Theorem 1. ■

V. Simulation

This section simulates two scenarios of target localization to demonstrate the effectiveness of LIFO-DBF. The networked UGVs take a ring communication topology that each UGV can communicate with two fixed nearest neighbors. The probabilistic sensor model, eq. (2) and (3), takes the form of Gaussian functions [21]:

$$P(z_k^i = 1 | x_k^g; x_k^{R,i}) = e^{-\frac{1}{2}(x_k^g - x_k^{R,i})^T \Sigma^{-1} (x_k^g - x_k^{R,i})} \tag{16a}$$

$$P(z_k^i = 0 | x_k^g; x_k^{R,i}) = 1 - P(z_k^i = 1 | x_k^g; x_k^{R,i}), \tag{16b}$$

which reflects the fact that target detection becomes more probable as the UGV approaches the target position.

The first scenario consists of six static UGVs and a single static target. The second scenario subsequently deals with six moving UGVs for localizing a moving target. In both scenarios, LIFO-DBF is compared with two commonly adopted approaches in multi-agent filtering: the consensus-based distributed filtering (CbDF) method and the centralized filtering (CF) method. The CbDF requires UGVs to continually exchange their individual PDFs with direct neighbors, computing the average of all received and its own target PDFs. Multiple rounds of communication and averaging are conducted at each time step to ensure the convergence of each UGV's individual PDFs. The CF assumes a central unit that can constantly receive and fuse all UGVs' latest observations into a single PDF. 10 test trials with randomly generated initial target positions are run and each trial is terminated after 50 time steps. The average error between the estimated and true target position and the average entropy of individual PDFs of all 10 trials are compared among these three approaches.

A. Static UGVs, Static Target

Six static UGVs, as represented by green starts and red square in Fig. 4a, are placed in the field. The target is also assumed to be static, with A being an identity matrix and B_k being a zero matrix in eq. (1). The individual PDF of the first UGV at different time steps are shown in Figs. 4a to 4d. It can be noticed that, as more measurements are fused, the individual PDF asymptotically concentrates to the true location of the target (Fig. 4d), which accords with the consistency of LIFO-DBF.

LIFO-DBF is compared with CbDF and CF, results of which are presented in Figs. 4e and 4f. Unsurprisingly, the CF achieves the best performance in terms of both small position estimation error and fast reduction of entropy. This happens because the central unit has access to the latest observations of all UGVs, thus making most use of all available information. It is worth noting that, LIFO-DBF achieves similar asymptotic performance as the CF, both in position estimation error and entropy reduction; this is achieved even though each UGV only communicates with its two neighboring UGVs, which requires less communication burden than the CF. The CbDF has the worst performance among these three filtering approaches. It results in slow reduction of entropy and the position error remains large.

B. Moving UGVs, Moving Target

In this scenario, each UGV follows a pre-defined circular trajectory. The target motion is modeled as a single-integrator, where A_k is an identity matrix and $B_k = \begin{bmatrix} \Delta T & 0 \\ 0 & \Delta T \end{bmatrix}$, where ΔT is the sampling time. Figs. 5a to 5d illustrates the test trial, in which the LIFO-DBF described in Section III-C is utilized for target localization. It can be noticed that the individual PDF concentrates to the true target location when the target constantly moves.

Figs. 5e and 5f compares LIFO-DBF with CbDF and CF. Similar to results in Section V-A, CF achieves best performance with smallest position estimation error and fastest entropy reduction; LIFO-DBF shows similar asymptotic performance as the CF; and CbDF has the slowest entropy reduction among all three filtering approaches. However, CbDF achieves comparable position estimation error as the CF, which are marginally better than LIFO-DBF. This is not an unexpected result: due to the consensus procedure, CbDF is able to fuse the latest individual PDFs of each UGV via the averaging process. This is especially useful when the environment is dynamically changing. In contrast, LIFO-DBF for each UGV only uses delayed information from neighboring UGVs, thus gaining less knowledge about the target's current position. However, it is worth noting that, CbDF requires multiple rounds of exchanging individual PDFs, which incurs much higher communication burden than LIFO-DBF's approach of transmitting the communication buffer once at each time step. Considering the small difference in position estimation error and significantly faster entropy reduction, LIFO-DBF is still preferable over CbDF for moving target scenario.

VI. CONCLUSION

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a multi-UGV network, utilizing the Latest-In-and-Full-Out (LIFO) protocol for observation exchange. By exchanging full communication buffers among neighboring UGVs, LIFO significantly reduces the transmission burden between each pair of UGVs to scale linearly with the network size. It should be noted that LIFO is a general measurement exchange protocol and thus applicable to various sorts of sensors. Two types of LIFO-based DBF algorithms are proposed to estimate individual PDFs for a static target and a moving target, respectively. The consistency of LIFO-based DBF is proved by utilizing the law of large numbers, which ensures that the estimated target position converges in probability to the true target position when the number of observations tends to infinity.

Future work includes considering other types of sensors and imperfect communication between UGVs. Other types of sensors may have biased observations and subject to non-Bernoulli distribution, which complicates the design and analysis of LIFO-based Bayesian filters. Imperfect communication, including package loss, out-of-order measurement and transmission delay, requires extension to LIFO-DBF and may affect its consistency properties.

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