Distributed Bayesian Filter using Measurement Dissemination for Multiple UGVs with Dynamically Changing Interaction Topologies

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This paper presents a novel distributed Bayesian filtering (DFB) method using measurement dissemination for multiple unmanned ground vehicles (UGVs) with dynamically changing interaction topologies. Different from statistics dissemination-based algorithms that transmit posterior distributions or likelihood functions, this method relies on a Full-In-and-Full-Out (FIFO) transmission protocol, which significantly reduces the transmission burden between each pair of UGVs. Each UGV only sends a communication buffer (CB) and a track list to its neighbors, in which the former contains a history of sensor measurements from all UGVs, and the latter is used to trim the redundant measurements in the CB to reduce communication overhead. It is proved that by using FIFO each UGV can disseminate its measurements over the whole network within a finite time, and the FIFO-based DBF is able to achieve consistent estimation of the environment state. The effectiveness of this method is validated by comparing with the consensus-based distributed filter and the centralized filter in a multi-target tracking problem.

1 INTRODUCTION

Estimation using a group of networked UGVs has been widely utilized to collectively measure environment status [1], such as intruder detection [2], signal source seeking [3], and pollution field estimation [4], due to its merits on low cost, high efficiency, and good reliability. The commonly adopted estimation approaches include the Kalman Filter, extended Kalman filter, and particle filter [5], and the most generic scheme might be the Bayesian filter because of its applicability for nonlinear systems with arbitrary noise distributions [6, 7]. In fact, a Bayesian filter can be reduced to different methods in certain conditions. For example, under the assumption of linearity and Gaussian noise, a Bayesian filter can be reduced to the Kalman filter [8]; for general nonlinear systems, a Bayesian filter can be numerically implemented as a particle filter [8]. Because of this generality, this study focuses on its networked variant, and use it for tracking targets via local communication between neighboring UGVs.

The interaction topology plays a central role on the design of networked Bayesian filter, of which two types are widely investigated in literature: centralized filters and distributed filters. In the former, local statistics estimated by each agent is transmitted to a single fusion center, where a global posterior distribution is calculated at each filtering cycle [9, 10]. In the latter, each agent individually executes distributed estimation and the agreement of local estimates is achieved by certain consensus strategies [11–13]. In general, the distributed filters are more suitable in practice since they do not require a fusion center with powerful computation capability and are more robust to changes in network topology and link failures. So far, the distributed filters have two mainstream schemes in terms of the transmitted data among agents, i.e., *statistics dissemination* (SD) and *measurement dissemination* (MD). In the SD scheme, each agent exchanges statistics, such as posterior distributions and likelihood functions, within neighboring agents [14]. In the MD scheme, instead of exchanging statistics, each agent sends sensor measurements to neighboring agents.

The statistics dissemination scheme has been widely investigated during the last decade, especially in the field of signal processing, network control, and robotics. Madhavan et al. (2004) presented a distributed extended Kalman filter

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for nonlinear systems [15]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with environmental features. Olfati-Saber (2005) proposed a distributed linear Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [16]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [17]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-linear-average consensus algorithm when local likelihood functions belong to the exponential family of distributions [18]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [6]. Other examples can be found in [7] and [19].

Despite the popularity of statistics dissemination, exchanging statistics can cause high communication burden if the environment to be detected is relatively large in space and complicated in structure. One remedy is to approximate statistics with parametric models, e.g., Gaussian Mixture Model [20], which can reduce communication burden to a certain extent. However, such manipulation increases the computation burden of each agent and sacrifices filtering accuracy due to approximation. The measurement dissemination scheme is an alternative solution to address the issue of exchanging large-scale statistics. An early work on measurement dissemination was done by Coates et al. (2004), who used adaptive encoding of observations to minimize communication overhead [21]. Ribeiro et al. (2006) exchanged quantized observations along with error-variance limits considering more pragmatic signal models [22]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each agent has a complete set of observations of other agents for executing particle filtering [23]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a fixed and complete graph that every pair of distinct agents is constantly connected by a unique edge. In many real applications, the interaction topology can be time-varying due to unreliable links, external disturbances or range limits [24]. In such cases, dynamically changing topologies can cause random packet loss, variable transmission delay, and out-of-sequence measurement (OOSM) issues [25], thus decreasing the performance of distributed estimation. Leung et al. (2010) has explored a decentralized Bayesian filter for dynamic robot networks [26] in order to achieve centralized-equivalent filtering performance. However, it requires the communication of both measurements and statistics, which can still incur large communication overhead.

This paper proposes a distributed Bayesian filtering (DBF) method that only uses measurement dissemination for a group of networked UGVs with dynamically changing interaction topologies. In our previous work [27], we have proposed a Latest-In-and-Full-Out (LIFO) protocol for measurement exchange and developed a corresponding DBF algorithm. However, it only applies to static targets with simple binary sensor model. In this work, we substantially extend the previous work and make the following contributions: (1) We introduce a new protocol called the Full-In-and-Full-Out (FIFO) that allows each UGV to broadcast a history of measurements to its neighbors via single hopping, enabling the tracking of moving targets using general nonlinear sensor models under time-varying topologies. (2) We propose the frequently jointly strongly connectedness condition of the interaction topology and show that, under this condition, FIFO can disseminate UGVs' measurements over the network within a finite time. (3) We develop a FIFO-based distributed Bayesian filter (FIFO-DBF) for each UGV to implement locally, which can avoid the OOSM issue. A track list is designed to reduce the computational complexity of FIFO-DBF and the communication burden. (4) We prove the consistency of FIFO-DBF: each UGV's estimate of target position converges in probability to the true target position asymptotically if the interaction topologies are frequently jointly strongly connected.

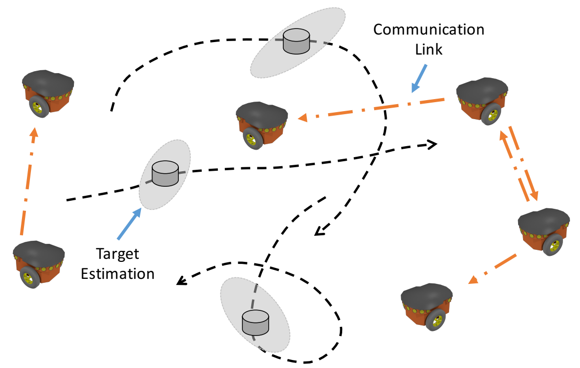


Fig. 1: Target tracking scenario. The interaction topology is dynamically changing and UGVs can only communicate with neighboring UGVs.

The rest of this paper is organized as follows: Section 2 formulates the target tracking problem using multiple UGVs; Section 3 proposes the FIFO protocol for measurement dissemination in dynamically changing interaction topologies; Section 4 introduces the FIFO-DBF algorithm and the track list; Section 5 proves the consistency of FIFO-DBF; Section 6 presents simulation results and Section 7 concludes the paper.

2 Problem Formulation

Consider a network of N UGVs in a bounded two-dimensional space S, as shown in Fig. 1. The interaction topology can be dynamically changing due to limited communication range, team reconfiguration, or intermittently link failure. Each UGV is equipped with a sensor for target detection. Due to the limit of communication range, each UGV can only exchange sensor measurements with its local neighbors. Every UGV locally runs a Bayesian filter to estimate the target position in S utilizing its own measurements and the received measurements from other UGVs.

2.1 Target and Sensor Model

The target motion uses a stochastic discrete-time model:

$$x^g_{k+1} = f(x^g_k, v_k), \tag{1}$$

where the superscript g represents the target and $x^g_k \in S$ is the target position at time k; v_k is the white process noise.

The sensor measurement is described by a stochastic model:

$$z^i_k = h_i(x^g_k, x^i_k) + w^i_k, \tag{2}$$

where the superscript $i \in \{1,\dots,N\}$ represents the index of the UGV; $x^i_k \in S$ is the sensor position and w^i_k is the white measurement noise. The measurement function h_i depends on the type of the sensor.

The design of the Bayesian filter relies on the conditional probability of obtaining a certain measurement z^i_k given the current target and sensor states, which is denoted by $P(z^i_k|x^g_k;x^i_k)$ [5]. The conditional probability $P(z^i_k|x^g_k;x^i_k)$ depends on both h_i and w^i_k in Eq. (2). For example, if w^i_k is a zero-mean Gaussian white noise with covariance Γ^i_k, then $P(z^i_k|x^g_k;x^i_k)$ can be described as $P(z^i_k|x^g_k;x^i_k) = N(h_i(x^g_k,x^i_k),\Gamma^i_k)$. For non-Gaussian noise, such as Poisson noise or Cauchy noise [28], $P(z^i_k|x^g_k;x^i_k)$ can also be similarly defined (for the purpose of simplicity, $P(z^i_k|x^g_k;x^i_k)$ is shorted as $P(z^i_k|x^g_k)$ for the rest of the paper). It should be noted that this work is not confined to any specific distribution of the noise. The measurement function h_i for several typical sensors are listed as follows [29]:

Range-only sensors: h_i is a function of the relative Euclidean distance between the sensor and the target:

$$h_i(x^g_k,x^i_k) = \|x^g_k - x^i_k\|_2,$$

where $\|\cdot\|_2$ is the Euclidean distance in S.

Bearing-only sensors: h_i is a function of the relative bearing between the sensor and the target:

$$h_i(x^g_k,x^i_k) = \measuredangle(x^g_k - x^i_k),$$

where $\measuredangle$ denotes the angle from the sensor to the target.

Range-bearing sensors: h_i includes both the relative distance and bearing:

$$h_i(x^g_k,x^i_k) = x^g_k - x^i_k.$$

2.2 Graphical Model of Interaction Topology

We consider a simple[1] graph $G = (V,E)$ to represent the interaction topology of N networked UGVs, where the vertex set $V = \{1,\dots,N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. For the purpose of narrative simplicity, we use directed graphs to describe our approach in this work. The undirected graphs can actually be treated as bidirectional directed graphs.

The *adjacency matrix* $A = [a_{ij}]$ of the graph G describes the interaction topology:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where a_{ij} is the entry on the i^{th} row and j^{th} column of the adjacency matrix. The notation $a_{ij} = 1$ indicates that the i^{th} UGV can directly communicate to the j^{th} UGV and $a_{ij} = 0$ indicates no direct communication from i to j. A directed graph is *strongly connected* if there is a directed path connecting any two arbitrary vertices in V.

Define the *union* of a set of simple directed graphs as the graph with the vertices in V and the edge set given by the union of each member's edge sets. Such collection is *jointly strongly connected* if the union of its members forms a strongly connected graph[2]. We use G_k to represent the interaction topology of time k and define the *inbound neighbors* and *outbound neighbors* of the i^{th} UGV under G_k as the set $\mathcal{N}_i^{\text{in}}(G_k) = \left\{ j | a^k_{ji} = 1, j \in V \right\}$ and $\mathcal{N}_i^{\text{out}}(G_k) = \left\{ j | a^k_{ij} = 1, j \in V \right\}$. All UGVs in $\mathcal{N}_i^{\text{out}}(G_k)$ can directly receive information from the i^{th} UGV via the single hopping.

3 Full-In-and-Full-Out (FIFO) Protocol

This study proposes a Full-In-and-Full-Out (FIFO) protocol for measurement exchange in dynamically changing interaction topologies. The use of FIFO allows us to apply measurement dissemination-based distributed filters to time-varying networks. Let $Y^i_{\mathcal{K}} = \left\{ \left[x^i_k, z^i_k \right] | k \in \mathcal{K} \right\}$ be the set of state-measurement pairs of the i^{th} UGV, where $\mathcal{K}$ is an index set of time steps. Each UGV contains a communication buffer (CB) and a track list (TL). A CB stores state-measurement pairs of all UGVs:

$$\mathcal{B}^i_k = \left[Y^1_{\mathcal{K}^{i1}_k}, \dots, Y^N_{\mathcal{K}^{iN}_k} \right],$$

where $\mathcal{B}^i_k$ is the CB of i^{th} UGV at time k and $\mathcal{K}^{ij}_k (j \in V)$ is the time index set. $Y^j_{\mathcal{K}^{ij}_k}$ represents the set of j^{th} UGV's state-measurement pairs of time steps in $\mathcal{K}^{ij}_k$ that are stored in i^{th} UGV's CB at time k. A UGV's TL stores the information of this UGV's reception of all UGVs' measurements, and is used for trimming old state-measurement pairs in the CB to reduce the communication burden. The details of TL will be introduced in Section 4.2. Each UGV sends its CB and TL to its outbound neighbors at every time step.

The **FIFO protocol** is stated in Algorithm 1, which consists of the CB and TL parts. For the purpose of clarity, we focus on the CB part in this section and leave the description of the TL part to Section 4.2. Fig. 2 illustrates the FIFO cycles in a network of three UGVs with dynamically changing topologies. The following facts can be observed from Fig. 2: (1) the topologies are jointly strongly connected in

[1] A (directed/undirected) graph $G = (V,E)$ is *simple* if it has no self-loops (i.e., $(i,j) \in E$ only if $i \neq j$) or multiple edges with the same source and target nodes (i.e., E only contains distinct elements).

[2] The counterpart definition for undirected graphs is given in [11].

Algorithm 1 FIFO Protocol

(1) Initialization.

CB: The CB of i^{th} UGV is initialized as an empty set at $k=0$:

$$\mathcal{B}_0^i = \left[Y^1_{\mathcal{K}_0^{i1}}, \ldots, Y^N_{\mathcal{K}_0^{iN}}\right], \text{ where } Y^j_{\mathcal{K}_0^{ij}} = \{[\varnothing, \varnothing]\}.$$

TL: The TL of i^{th} UGV is initialized at $k=0$:

$$\mathbf{q}_1^{ij} = [0, \ldots, 0, 1], \text{ i.e. } q_1^{jl} = 0, \forall j, l \in \{1 \ldots, N\}.$$

(2) At time $k\,(k \geq 1)$ for i^{th} UGV:

(2.1) Receiving Step.

CB: The i^{th} UGV receives all CBs of its inbound neighbors $\mathcal{N}_i^{\text{in}}(G_{k-1})$. The received CB from the l^{th} UGV is $\mathcal{B}_{k-1}^l\,(l \in \mathcal{N}_i^{\text{in}}(G_{k-1}))$.

TL: The i^{th} UGV receives all TLs of its inbound neighbors $\mathcal{N}_i^{\text{in}}(G_{k-1})$. The received TL from the l^{th} UGV is $Q_{k-1}^l\,(l \in \mathcal{N}_i^{\text{in}}(G_{k-1}))$.

(2.2) Observation Step.

CB: The i^{th} UGV updates $Y^i_{\mathcal{K}_k^{ii}}$ by its own state-measurement pair:

$$Y^i_{\mathcal{K}_k^{ii}} = Y^i_{\mathcal{K}_{k-1}^{ii}} \cup \left\{\left[x_k^i, z_k^i\right]\right\}.$$

(2.3) Updating Step.

CB: The i^{th} UGV updates other entries of its own CB, $Y^j_{\mathcal{K}_k^{ij}}\,(j \neq i)$, by merging with all received CBs:

$$Y^j_{\mathcal{K}_k^{ij}} = Y^j_{\mathcal{K}_{k-1}^{ij}} \cup Y^j_{\mathcal{K}_{k-1}^{lj}}, \forall j \neq i, \forall l \in \mathcal{N}_i^{\text{in}}(G_{k-1}).$$

TL: The i^{th} UGV updates its own TL, Q_k^i, using the received TLs (see Algorithm 3). The CB is trimmed based on the updated track list (see Algorithm 4).

(2.4) Sending Step.

CB: The i^{th} UGV sends its updated CB, $\mathcal{B}_k^i$, to all of its outbound neighbors defined in $\mathcal{N}_i^{\text{out}}(G_k)$.

TL: The i^{th} UGV sends its updated track list, Q_k^i, to its outbound neighbors $\mathcal{N}_i^{\text{out}}(G_k)$.

(3) $k \leftarrow k+1$ until stop

the time interval $[0,6)$; (2) each UGV can receive the state-measurement pairs of other UGVs within finite steps. Extending these facts to a network of N UGVs, we have the following properties of FIFO:

Theorem 1. *Consider a network of N UGVs with dynamically changing interaction topologies $\mathbb{G} = \{G_1, G_2, G_3 \ldots, \}$. If $\mathbb{G}$ is frequently jointly strongly connected, i.e.,*

1. *there exists an infinite sequence of time intervals $[k_m, k_{m+1})$, $m = 1, 2, \ldots,$ starting at $k_1 = 0$ and are contiguous, nonempty, and uniformly bounded;*
2. *the union of graphs across each such interval is jointly strongly connected,*

then each pair of UGVs can exchange measurements under FIFO. In addition, it takes no more than NT_u steps for a UGV to communicate to another one, where $T_u = \sup_{m=1,2,\ldots} (k_{m+1} - k_m)T$ is the upper bound of interval lengths.

Proof. Without loss of generality, we consider the transmission of $\mathcal{B}_{t_1}^i$ from the i^{th} UGV to an arbitrary j^{th} UGV $(j \in V \setminus \{i\})$, where $t_1 \in [k_1, k_2)$. Since each UGV will receive inbound neighbors' CBs and send the merged CB to its outbound neighbors at the next time step, the i^{th} UGV can transmit $\mathcal{B}_{t_1}^i$ to j^{th} UGV if and only if a path $[l_1, \ldots, l_n]$ exists, with $l_1 = i$, $l_n = j$, $l_2, \ldots, l_{n-1} \in V \setminus \{i, j\}$, and the edges (l_s, l_{s+1}) appears no later than (l_{s+1}, l_{s+2}), $s = 1, \ldots, n-2$.

As the union of graphs across the time interval $[k_2, k_3)$ is jointly connected, i^{th} UGV can directly send $\mathcal{B}_{t_1}^i$ to at least one another UGV at a time instance, i.e., $\exists l_2 \in V \setminus \{i\}$, $\exists t_2 \in [k_2, k_3)$ s.t. $l_2 \in \mathcal{N}_i^{\text{out}}(G_{t_2})$. If $l_2 = j$, then $\mathcal{B}_{t_1}^i$ has been sent to j. If $l_2 \neq j$, $\mathcal{B}_{t_1}^i$ has been merged into $\mathcal{B}_{t_2+1}^{l_2}$ and will be sent out in the next time step.

Using the similar reasoning for time intervals $[k_m, k_{m+1})$, $, m = 3, 4, \ldots,$ it can be shown that all UGVs can receive the state-measurement pairs in $\mathcal{B}_{t_1}^i$ no later by k_{N+1}. Therefore, the transmission time from an arbitrary UGV to any other UGVs is no greater than NT_u. □

Corollary 1. *For a frequently jointly strongly connected network, each UGV receives the CBs of all other UGVs under FIFO within finite time.*

Proof. According to Theorem 1, each UGV is guaranteed to receive $\mathcal{B}_t^j$ $(\forall t \geq 0, j \in V)$ when $k \geq t + NT_u$. □

Remark 1. *Theorem 1 and Corollary 1 are the consequences of FIFO and the use of the communication buffer (CB). In fact, if we use the traditional methods that each UGV only sends the current sensor measurement to neighboring UGVs without the use of CB, it can happen that two UGVs may never exchange their sensor measurements, even there exists a path connecting them. The condition of frequently jointly strongly connectedness is also crucial for guaranteeing the consistency of the FIFO-based distributed Bayesian filter, as shown in Section 5.*

4 Distributed Bayesian Filter via FIFO Protocol

We first introduce the generic distributed Bayesian filter (DBF). Let $X_k \in S$ be the random variable representing the position of the target at time k. Define $\mathcal{Z}_k^i$ as the set of measurements at time k that are in the i^{th} UGV's CB, i.e., $\mathcal{Z}_k^i = \left\{z_k^j \middle| \left[x_k^j, z_k^j\right] \in \mathcal{B}_k^i, \forall j \in V\right\}$ and let $\mathcal{Z}_{1:k}^i = \bigcup_{t=1}^{k} \mathcal{Z}_t^i$. It is easy to notice that $\mathcal{Z}_{1:k}^i$ is the set of all measurements in $\mathcal{B}_k^i$. We also define $z_{1:k}^i = \left[z_1^i, \ldots, z_k^i\right]$ as the set of the i^{th} UGV's measurements at times 1 through k. The probability density function (PDF) of X_k, called *individual PDF*, of the i^{th} UGV is represented by $P_{pdf}^i(X_k|\mathcal{Z}_{1:k}^i)$. It is the estimation of the target position given all the measurements that the i^{th} UGV has

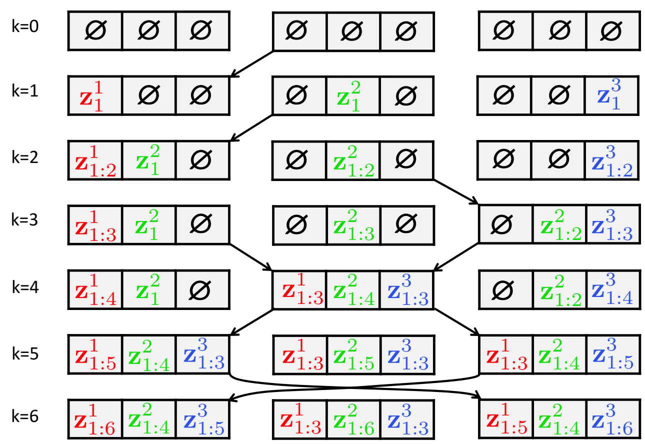


Fig. 2: Example of FIFO with three UGVs under dynamically changing interaction topologies. The arrows represent a directed communication link between two UGVs. $\varnothing$ denotes the empty set. For the purpose of clarity, we only show measurements, not the states, in the CB.

received. The initial individual PDF, $P^i_{pdf}(X_0)$, is constructed given prior information including past experience and environment knowledge. It is necessary to initialize $P^i_{pdf}(X_0)$ such that the probability density of the true target position is nonzero, i.e., $P^i_{pdf}(X_0 = x^g_0) > 0$.

Under the framework of DBF, the individual PDF is recursively estimated by two steps: the prediction step and the updating step.

Prediction. At time k, the prior individual PDF $P^i_{pdf}(X_{k-1}|Z^i_{1:k-1})$ is first predicted forward by using the Chapman-Kolmogorov equation:

$$P^i_{pdf}(X_k|Z^i_{1:k-1}) = \int\limits_{X_{k-1}\in S} P(X_k|X_{k-1})P^i_{pdf}(X_{k-1}|Z^i_{1:k-1})dX_{k-1}, \tag{3}$$

where $P(X_k|X_{k-1})$ represents the state transition probability of the target, based on the Markovian motion model (Eq. (1)).

Updating. The i^{th} individual PDF is then updated by the Bayes' rule using the set of newly received measurements at time k, i.e., Z^i_k:

$$P^i_{pdf}(X_k|Z^i_{1:k}) = K_i P^i_{pdf}(X_k|Z^i_{1:k-1})P(Z^i_k|X_k) \tag{4a}$$

$$= K_i P^i_{pdf}(X_k|Z^i_{1:k-1}) \prod_{z^j_k\in Z^i_k} P(z^j_k|X_k), \tag{4b}$$

where $P(z^j_k|X_k)$ is the sensor model and K_i is a normalization factor, given by:

$$K_i = \left[\int\limits_{X_k\in S} P^i_{pdf}(X_k|Z^i_{1:k-1})P(Z^i_k|X_k)dX_k\right]^{-1}.$$

Here we have utilized the commonly adopted assumption [17, 20, 30] in the distributed filtering literature that the sensor measurement of each UGV at current time is conditionally independent from its own previous measurements and the measurements of other UGVs given the target and the UGV's current position. This assumption allows us to simplify $P(Z^i_k|X_k, Z^i_{1:k-1})$ as $P(Z^i_k|X_k)$ in Eq. (4a) and factorize $P(Z^i_k|X_k)$ as $\prod_{z^j_k\in Z^i_k} P(z^j_k|X_k)$ in Eq. (4b).

Algorithm 2 FIFO-DBF Algorithm

For i^{th} UGV at k^{th} step ($\forall i \in V$):
(1) Initialize a *temporary PDF* by assigning the stored individual PDF to it:

$$P^i_{tmp}(X_t) = P^i_{sto}(X_t),$$

where

$$P^i_{sto}(X_t) = P^i_{pdf}(X_t|z^1_{1:t}, \dots, z^N_{1:t}).$$

(2) For $\xi = t+1$ to k, iteratively repeat two steps of Bayesian filtering:
(2.1) Prediction

$$P^{pre}_{tmp}(X_\xi) = \int_S P(X_\xi|X_{\xi-1})P^i_{tmp}(X_{\xi-1})dX_{\xi-1}.$$

(2.2) Updating

$$P^i_{tmp}(X_\xi) = K_\xi P^{pre}_{tmp}(X_\xi)P(Z^i_\xi|X_\xi),$$

$$K_\xi = \left[\int_S P^{pre}_{tmp}(X_\xi)P(Z^i_\xi|X_\xi)dX_\xi\right]^{-1}.$$

(2.3) If $z^j_\xi \neq \emptyset$ for $\forall j \in V$, update the stored PDF:

$$P^i_{sto}(X_\xi) = P^i_{tmp}(X_\xi).$$

(3) The individual PDF of i^{th} UGV at time k is $P^i_{pdf}(X_k|Z^i_{1:k}) = P^i_{tmp}(X_k)$.

4.1 The FIFO-DBF Algorithm

The generic DBF is not directly applicable to time-varying interaction topologies. This is because changing topologies can cause intermittent and out-of-sequence arrival of measurements from different UGVs, giving rise to the OOSM problem. One possible solution is to ignore all measurements that are out of the temporal order. This is undesirable since this will cause significant information loss. Another possible remedy is to fuse all measurements by running the filtering algorithm from the beginning at each time step. However, this solution causes excessive computational burden. To avoid both OOSM problem and unnecessary computational complexity, we add a new PDF, namely the *stored PDF*, $P^i_{sto}(X_t)$, that is updated from the i^{th} UGV's initial PDF by fusing the state-measurement pairs of *all* UGVs up to a certain time $t \le k$. The choice of t is described in Section 4.2. The individual PDF, $P^i_{pdf}(X_k|Z^i_{1:k})$, is then computed by fusing the measurements from time $t+1$ to k in the CB into $P^i_{sto}(X_t)$, running the Bayesian filter (Eq. (3) and (4)). Note that initially, $P^i_{sto}(X_0) = P^i_{pdf}(X_0)$.

The **FIFO-DBF algorithm** is stated in Algorithm 2. Each UGV runs FIFO-DBF after its CB is updated in the Up-

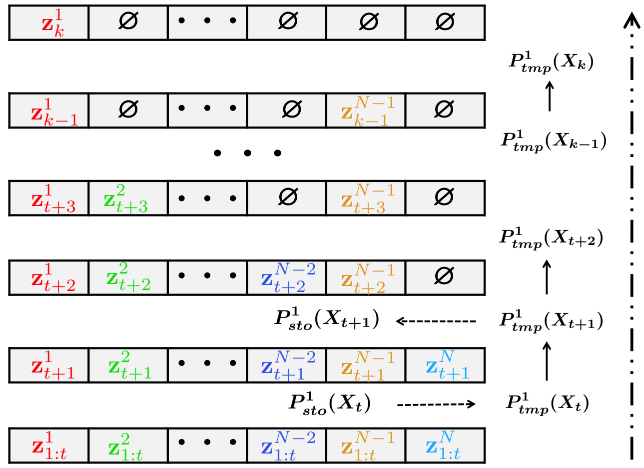


Fig. 3: Example of FIFO-DBF for the 1st UGV at time k. Only the measurement (not the state) is shown in the figure. The UGV first calculates $P^1_{tmp}(X_{t+1})$. Since the UGV has received all UGVs' measurements of $t+1$, the $P^1_{tmp}(X_{t+1})$ is assigned as the new stored PDF. The dashed arrow on the right shows the order to fuse measurements in the CB.

dating Step in Algorithm 1. At the beginning, we assign the stored PDF to a temporary PDF, which will then be updated by sequentially fusing measurements in the CB to obtain the individual PDF. It should be noted that, when the UGV's CB contains all UGVs' state-measurement pairs from t to ξ, the temporary PDF corresponding to time ξ is assigned as the new stored PDF. Fig. 3 illustrates the FIFO-DBF procedure for the 1st UGV as an example. It can be noticed that, the purpose of using the stored PDF is to avoid running the Bayesian filtering from the initial PDF at every time step. Since the stored PDF has incorporated all UGVs' measurements up to time step t, the information loss is prevented. We point out that the time t of each UGV's stored PDF can be different from others. The stored PDF is saved locally by each UGV and not transmitted to others. FIFO-DBF is able to avoid the OOSM issue since all measurements are fused in the correct temporal order.

4.2 Track Lists for Trimming CBs

The size of CBs can keep increasing as measurements cumulate over time. The use of the stored PDF has made it feasible to trim excessive state-measurement pairs from the CBs. To avoid information loss, a state-measurement pair can only be trimmed from a UGV's CB when *all* UGVs have received it. We design the *track list* (TL) for each UGV to keep track of all UGVs' reception of other UGVs' measurements. We first define a binary term $q^{jl}_{k^{ij}}$ $(\forall i,j,l \in V)$: $q^{jl}_{k^{ij}} = 1$ if the i^{th} robot knows that the j^{th} UGV has received the state-measurement pair of the l^{th} UGV of time k^{ij}, $\left[x^l_{k^{ij}}, z^l_{k^{ij}}\right]$; $q^{jl}_{k^{ij}} = 0$ if the i^{th} robot cannot determine whether $\left[x^l_{k^{ij}}, z^l_{k^{ij}}\right]$ has been received by the j^{th} robot. Therefore it can happen that $\left[x^l_{k^{ij}}, z^l_{k^{ij}}\right]$ has been received by the j^{th} UGV but the i^{th} UGV does not know this and thus $q^{jl}_{k^{ij}} = 0$. Now we define the i^{th} UGV's track list as $Q^i_k = \left[\mathbf{q}^{i1}_{k^{i1}}, \dots, \mathbf{q}^{iN}_{k^{iN}}\right]^T$ $(\forall i \in V)$, which is a $N \times (N+1)$ binary matrix with $\mathbf{q}^{ij}_{k^{ij}} = \left[q^{j1}_{k^{ij}}, \dots, q^{jN}_{k^{ij}}, k^{ij}\right]^T$ $(j \in V)$. The last column $\left[k^{i1}, \dots, k^{iN}\right]$ corresponds to measurement times.

Algorithm 3 Updating TLs

Consider updating the i^{th} UGV's TL, Q^i_k, using the received r^{th} UGV's TL, Q^r_{k-1} $(r \in \mathcal{N}^{\text{in}}_i(G_{k-1}))$.

(1) Update the i^{th} row of Q^i_k, using $\mathcal{B}^i_k$:

choose k^{ii} as the minimum integer that satisfies the following conditions: (a) $\exists l \in V$ s.t. $\left[x^l_{k^{ii}}, z^l_{k^{ii}}\right] \notin \mathcal{B}^i_k$ and (b) $k^{ii} \geq t_m - 1$, where t_m is the minimum time of state-measurement pairs in $\mathcal{B}^i_k$.

(2) Update other rows of Q^i_k: $\forall j \in V \setminus \{i\}$

if $k^{ij} > k^{rj}$, keep current $\mathbf{q}^{ij}_{k^{ij}}$;
if $k^{ij} = k^{rj}$, $\mathbf{q}^{ij}_{k^{ij}} = \mathbf{q}^{ij}_{k^{ij}} \vee$ [3] $\mathbf{q}^{rj}_{k^{rj}}$;
if $k^{ij} < k^{rj}$, $\mathbf{q}^{ij}_{k^{ij}} = \mathbf{q}^{rj}_{k^{rj}}$ and $k^{ij} = k^{rj}$.

The exchange and updating of TLs are described in Algorithm 1, with the updating details presented in Algorithm 3. For the i^{th} UGV, it updates the i^{th} row of its TL matrix using the entries of its CB, and updates other rows of the TL using the received TLs from inbound neighbors. The updating rule guarantees that, if the last term of the j^{th} row is k^{ij}, the i^{th} UGV is ensured that every UGV has received all UGVs' state-measurement pairs of times earlier than k^{ij}. Fig. 4 shows the updating of each UGV's TL using Algorithm 3. We can use TLs to trim CBs, which is described in Algorithm 4. In the example of Fig. 4, the 1st and 3rd UGV's CB will be trimmed at $k = 6$ and the trimmed state-measurement pairs corresponds to times $1, 2$, and 3.

The use of TLs can avoid the excessive size of CBs and guarantee that trimming the CBs will not lose any information; the trimmed measurements have been encoded into the stored PDF. The following theorem formalizes this property.

Theorem 2. *Each UGV's estimation result using the trimmed CB is the same as that using the non-trimmed CB.*

Proof. Consider the i^{th} UGV. Let $k^i_m = \min_j k^{ij}$. Trimming $\mathcal{B}^i_k$ happens when all entries in Q^i_k corresponding to time k^i_m equal 1. This indicates that each UGV has received all UGVs' state-measurement pairs of time k^i_m. A UGV has either stored the pairs in its CB or already fused them to obtain the stored PDF. In both cases, such pairs are no longer needed to be transmitted. Therefore, it causes no loss to trim theses measurements. □

The following theorem describes when CBs get trimmed, and it provides an upper bound of the communication burden that FIFO-DBF will incur. A detailed complexity

[3] '$\vee$' is the notation of the logical 'OR' operator.

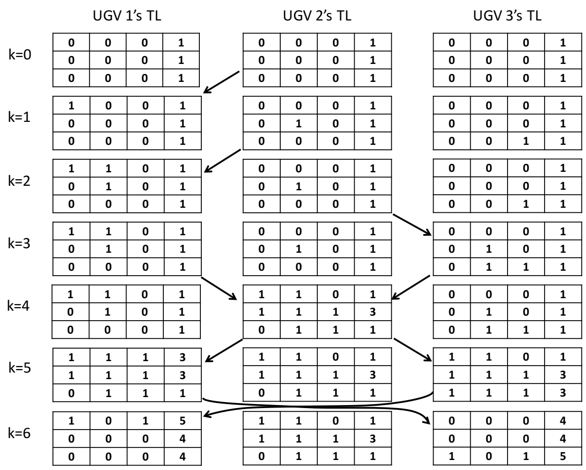


Fig. 4: Example of updating TLs. For the 1st UGV's TL, the j^{th} $(j \in V)$ entry on the i^{th} $(i \in V)$ row represents this UGV's knowledge about whether the i^{th} UGV has received the j^{th} UGV's state-measurement pair of time k^{1i}, where k^{1i} is the last entry of the i^{th} row. TLs are updated using Algorithm 3.

analysis of FIFO-DBF is presented in Section 4.3. Consider trimming all the state-measurement pairs of time t in the i^{th} UGV's CB. Let $k_t^{lj} (> t)$ be the first time that the l^{th} UGV communicates to the j^{th} UGV in the time interval (t, ∞). Define $\tilde{k}_t^j = \max_l k_t^{lj}$, which is the time that the j^{th} UGV receives all other UGVs' measurements of t. Similarly, let $k_t^{ji} (> \tilde{k}_t^j)$ be the first time that the j^{th} UGV communicates to the i^{th} UGV in the time interval $(\tilde{k}_t^j, \infty)$ and define $\tilde{k}_t^i = \max_j k_t^{ji}$. The following theorem gives the time when the i^{th} UGV $(\forall i \in V)$ trims all state-measurement pairs of time t in its own CB.

Theorem 3. *The i^{th} UGV trims $\left\{ \left[x_t^l, z_t^l\right] \ (\forall l \in V) \right\}$ from its CB at the time $\tilde{k}_t^i$.*

Proof. The i^{th} UGV can trim $\left\{ \left[x_t^l, z_t^l\right] \ (\forall l \in V) \right\}$ only when it is sure that all other UGVs have also received these state-measurement pairs. This happens at $\tilde{k}_t^i$ and thus $\tilde{k}_t^i$ is the time when the trim occurs.

□

Corollary 2. *Under the frequently jointly strongly connectedness condition, the size of any UGV's CB is no greater than $2N(N-1)T_u$.*

Proof. We consider an arbitrary i^{th} $(i \in V)$ UGV. According to Theorem 1, a UGV can communicate to any other UGV within NT_u steps. Therefore, $\tilde{k}_t^i \leq 2NT_u$, since it first requires each UGV communicate to all other UGVs and then each UGV communicate to the i^{th} UGV. This implies that, the state-measurement pairs of a certain time of all UGVs will be trimmed from each UGV's CB within $2NT_u$ steps.

The maximum size the of CB occurs when the state-measurement pairs of a certain time from all but one UGV are saved in the i^{th} UGV's CB. Therefore, the size of any UGV's CB is no greater than $2N(N-1)T_u$.

□

Algorithm 4 Trimming CBs using TLs

For the i^{th} UGV: find the smallest time in Q_k^i: $k_m^i = \min\left\{k^{i1}, \ldots, k^{iN}\right\}$.

1. Remove state-measurement pairs in $\mathcal{B}_k^i$ that corresponds to measurement times earlier than k_m^i, i.e., $\mathcal{B}_k^i = \mathcal{B}_k^i \setminus \left\{ \left[x_t^l, z_t^l\right] \right\}, \forall t < k_m^i, \forall l \in V$.
2. If entries associated with time k_m^i in Q_k^i are 1's, then
   (a) set these entries to be 0.
   (b) update the i^{th} row of Q_k^i using the current CB, i.e., $q_{k^{ii}}^{il} = 1$ if $\left[x_{k^{ii}}^l, z_{k^{ii}}^l\right] \in \mathcal{B}_k^i, \forall l \in V$.
   (c) remove all corresponding state-measurement pairs in $\mathcal{B}_k^i$, i.e., $\mathcal{B}_k^i = \mathcal{B}_k^i \setminus \left\{ \left[x_{k_m^i}^l, z_{k_m^i}^l\right] \right\}, \forall l \in V$.
   (d) $k_m^i \leftarrow k_m^i + 1$.

4.3 Complexity Analysis of FIFO-DBF

Compared to statistics dissemination, FIFO is usually more communication-efficient for distributed filtering. To be specific, consider a grid representation of the environment with the size $D \times D$. The transmitted data between each pair of UGVs are the CB and TL of each UGV. The size of the CB is upper bounded by $O(N^2 T_u)$, according to Corollary 2. On the contrary, the communicated data of a statistics dissemination approach that transmits unparameterized posterior distributions or likelihood functions is $O(D^2)$. In applications such as the target localization, D is generally much larger than N. Besides, the consensus filter usually requires multiple rounds to arrive at consensual results. Therefore, when T_u is not comparable to D^2, the FIFO protocol requires much less communication burden.

It is worth noting that each UGV needs to store an individual PDF and a stored PDF, each of which has size $O(D^2)$. In addition, each UGV needs to keep the CB and TL. This is generally larger than that of statistics dissemination-based methods, which only stores the individual PDF. Therefore, the FIFO-DBF sacrifices the local memory for saving the communication resource. This is actually desirable for real applications as local memory of vehicles is usually abundant compared to the limited bandwidth for communication.

Remark 2. *Under certain interaction topologies, CBs can grow to undesirable sizes and cause excessive communication burden if the trim cannot happen frequently. In this case, we can use a time window to constrain the measurements that are saved in CBs. This will cause information loss to the measurements. However, with a decently long time window, FIFO-DBF can still effectively estimate the target position.*

5 Proof of Consistency

This section proves the consistency of the *maximum a posteriori* (MAP) estimator of LIFO-DBF under unbiased sensors (sensors without offset). A state estimator is *consis-*

tent if it converges in probability to the true value of the state [31]. Consistency is an important metric for stochastic filtering approaches [8], which not only implies achieving consensus asymptotically, but also requires the estimated value converge to the true value. We first prove the consistency for static UGVs and then for moving UGVs. Here we assume that S is a finite set (e.g. a finely discretized field) and the target is relatively slow compared to the filtering dynamics. In addition, the target position can be uniquely determined by the multi-UGV network with proper placement (i.e., excluding the special case of ghost targets [32]).

5.1 Static UGVs

The consistency of FIFO-DBF for static UGVs is stated as follows:

Theorem 4. *Assume the UGVs are static and the sensors are unbiased. If the network of N UGVs is frequently jointly strongly connected, then the MAP estimator of target position converges in probability to the true position of the target using FIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X_k^{MAP}=x^g)=1,\ i\in V,$$

where

$$X_k^{MAP}=\arg\max_X P^i_{pdf}(X|\mathcal{Z}^i_{1:k}).$$

Proof. The DBF can be transformed into the batch form by recursively applying Eq. (4) from k to the initial time 1 (back in time):

$$\begin{aligned}
P^i_{pdf}(X|\mathcal{Z}^i_{1:k}) &= K_i P^i_{pdf}(X|\mathcal{Z}^i_{1:k-1}) \prod_{z^j_k\in \mathcal{Z}^i_k} P(z^j_k|X)\\
&= K_i P^i_{pdf}(X|\mathcal{Z}^i_{1:k-2}) \prod_{z^j_{k-1}\in \mathcal{Z}^i_{k-1}} P(z^j_{k-1}|X) \prod_{z^j_k\in \mathcal{Z}^i_k} P(z^j_k|X)\\
&= \dots\\
&= K_i P^i_{pdf}(X) \prod_{z^j_1\in \mathcal{Z}^i_1} P(z^j_1|X)\dots \prod_{z^j_k\in \mathcal{Z}^i_k} P(z^j_k|X)\\
&= K_i P^i_{pdf}(X) \prod_{j=1}^N \prod_{t\in\mathcal{K}^{ij}_k} P(z^j_t|X).
\end{aligned}$$

The last step is obtained by using the relation $\mathcal{B}^i_k = \left[Y^1_{\mathcal{K}^{i1}_k},\dots,Y^N_{\mathcal{K}^{iN}_k}\right]$ and $\mathcal{Z}^i_{1:k}$ is the set of all measurements in $\mathcal{B}^i_k$.

Comparing $P^i_{pdf}(X_k=x|\mathcal{Z}^i_{1:k})$ with $P^i_{pdf}(X_k=x^g|\mathcal{Z}^i_{1:k})$[4] yields

$$\frac{P^i_{pdf}(x|\mathcal{Z}^i_{1:k})}{P^i_{pdf}(x^g|\mathcal{Z}^i_{1:k})} = \frac{P^i_{pdf}(x)\prod_{j=1}^N\prod_{t\in\mathcal{K}^{ij}_k} P(z^j_t|x)}{P^i_{pdf}(x^g)\prod_{j=1}^N\prod_{t\in\mathcal{K}^{ij}_k} P(z^j_t|x^g)}. \tag{5}$$

Take the logarithm of Eq. (5) and average it over k steps:

$$\frac{1}{k}\ln\frac{P^i_{pdf}(x|\mathcal{Z}^i_{1:k})}{P^i_{pdf}(x^g|\mathcal{Z}^i_{1:k})} = \frac{1}{k}\ln\frac{P^i_{pdf}(x)}{P^i_{pdf}(x^g)} + \sum_{j=1}^N \frac{1}{k}\sum_{t\in\mathcal{K}^{ij}_k}\ln\frac{P(z^j_t|x)}{P(z^j_t|x^g)}. \tag{6}$$

[4]For the purpose of simplicity, we use $P^i_{pdf}(x|\mathcal{Z}^i_{1:k})$ to represent $P^i_{pdf}(X_k=x|\mathcal{Z}^i_{1:k})$ in this proof.

Since $P^i_{pdf}(x)$ and $P^i_{pdf}(x^g)$ are bounded and nonzero by the choice of the initial PDF, $\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x)}{P^i_{pdf}(x^g)}=0$. Note that the sensor measurement is a random variable drawn from the underlying distribution associated with the sensor model, i.e., $z^j_t\sim P(\cdot|x^g)$, $j\in V$. Therefore $\ln\frac{P(z^j_t|x)}{P(z^j_t|x^g)}$ is a random variable associated with z^j_t. Due to the finite delay of measurement arrival (Corollary 1), i.e., $k-NT_u\le|\mathcal{K}^{ij}_k|\le k$, where $|\cdot|$ is the set cardinality, we can use the law of large numbers to study the asymptotic behavior of the series in Eq. (6):

$$\begin{aligned}
\frac{1}{k}\sum_{t\in\mathcal{K}^{ij}_k}\ln\frac{P(z^j_t|x)}{P(z^j_t|x^g)} &\xrightarrow{P} \mathbb{E}_{z^j_t}\left[\frac{P(z^j_t|x)}{P(z^j_t|x^g)}\right] &(7a)\\
&= \int_{z^j_t} P(z^j_t|x^g)\frac{P(z^j_t|x)}{P(z^j_t|x^g)}dz^j_t &(7b)\\
&= -D_{KL}\left(P(z^j_t|x)\|P(z^j_t|x^g)\right), &(7c)
\end{aligned}$$

where "$\xrightarrow{P}$" represents "convergence in probability" and $D_{KL}(P_1\|P_2)$ denotes the Kullback-Leibler (KL) divergence between two probability distribution P_1 and P_2. KL divergence has the property that $\forall P_1,P_2$, $D_{KL}(P_1\|P_2)\le 0$, and the equality holds if and only if $P_1=P_2$. Therefore

$$\lim_{k\to\infty}\frac{1}{k}\sum_{t\in\mathcal{K}^{ij}_k}\ln\frac{P(z^j_t|x)}{P(z^j_t|x^g)}<0,\quad x\ne x^g$$

$$\lim_{k\to\infty}\frac{1}{k}\sum_{t\in\mathcal{K}^{ij}_k}\ln\frac{P(z^j_t|x)}{P(z^j_t|x^g)}=0,\quad x= x^g.$$

Considering the limiting case of Eq. (6), we get

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x|\mathcal{Z}^i_{1:k})}{P^i_{pdf}(x^g|\mathcal{Z}^i_{1:k})}<0,\quad x\ne x^g \tag{8}$$

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P^i_{pdf}(x|\mathcal{Z}^i_{1:k})}{P^i_{pdf}(x^g|\mathcal{Z}^i_{1:k})}=0,\quad x= x^g. \tag{9}$$

Eq. (8) and (9) imply that

$$\frac{P^i_{pdf}(x|\mathcal{Z}^i_{1:k})}{P^i_{pdf}(x^g|\mathcal{Z}^i_{1:k})}\xrightarrow{P}\begin{cases}0 & x\ne x^g,\\ 1 & x=x^g.\end{cases}$$

Therefore,

$$\lim_{k\to\infty}P(X_k^{MAP}=x^g)=1.$$

□

5.2 Moving UGVs

The consistency proof for the moving UGVs case is different from the static UGVs case in that each moving UGV makes measurements at multiple different positions. We classify UGV measurement positions into two disjoint sets: *infinite-measurement spots* S_I that contain positions where a UGV keeps revisiting as time tends to infinity, and *finite-measurement spots* S_F that contain positions where the UGV visits finitely many times (i.e., the UGV does not visit again after a finite time period). It is easy to know that each UGV has at least one position where it revisits infinitely many times as k tends to infinity.

Theorem 5. *Assume the UGVs move within a finite set of positions and the sensors are unbiased. If the network of N UGVs is frequently jointly strongly connected, then the MAP estimator of target position converges in probability to the true position of the target using FIFO-DBF, i.e.,*

$$\lim_{k\to\infty} P(X_k^{MAP} = x^g) = 1,\ i \in V,$$

where

$$X_k^{MAP} = \arg\max_X P_{pdf}^i(X|Z_{1:k}^i).$$

Proof. Similar to Eq. (5), comparing $P_{pdf}^i(x|Z_k^i)$ and $P_{pdf}^i(x^g|Z_k^i)$ yields

$$\frac{P_{pdf}^i(x|Z_k^i)}{P_{pdf}^i(x^g|Z_k^i)} = \frac{P_{pdf}^i(x)\prod_{j=1}^{N}\prod_{t\in\mathcal{K}_k^{ij}} P(z_t^j|x;x_t^j)}{P_{pdf}^i(x^g)\prod_{j=1}^{N}\prod_{t\in\mathcal{K}_k^{ij}} P(z_t^j|x^g;x_t^j)}. \tag{10}$$

The only difference from Eq. (5) is that $P(\cdot|x;x_l^j)$ varies as the j^{th} UGV moves, since x_l^j changes over time. Similar to Eq. (6), we obtain

$$\begin{aligned}\frac{1}{k}\ln\frac{P_{pdf}^i(x|Z_{1:k}^i)}{P_{pdf}^i(x^g|Z_{1:k}^i)} &= \frac{1}{k}\ln\frac{P_{pdf}^i(x)}{P_{pdf}^i(x^g)} \\ &+ \sum_{j=1}^{N}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{ij},x_t^j\in S_F}\ln\frac{P(z_t^j|x;x_t^j)}{P(z_t^j|x^g;x_t^j)} \\ &+ \sum_{j=1}^{N}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{ij},x_t^j\in S_I}\ln\frac{P(z_t^j|x;x_t^j)}{P(z_t^j|x^g;x_t^j)},\end{aligned}$$

where the second summand corresponds to the sensor positions that are in the finite-measurement spots set, and the third summand corresponds to the positions in the infinite-measurement spots set. By referring to Eq. (7), it is straightforward to know

$$\sum_{j=1}^{N}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{ij},x_t^j\in S_F}\ln\frac{P(z_t^j|x;x_t^j)}{P(z_t^j|x^g;x_t^j)} \xrightarrow{P} 0,$$

$$\sum_{j=1}^{N}\frac{1}{k}\sum_{t\in\mathcal{K}_k^{ij},x_t^j\in S_I}\ln\frac{P(z_t^j|x;x_t^j)}{P(z_t^j|x^g;x_t^j)} \xrightarrow{P} \mathbb{E}_{z_t^j}\left[\frac{P(z_t^j|x)}{P(z_t^j|x^g)}\right],$$

since only finitely many observations associated with sensor positions in S_F are obtained but infinitely many observations associated with sensor positions in S_I are received. The rest of the proof is similar to that of Theorem 4.

□

6 Simulation

We conduct a simulation that uses a team of six UGVs to localize three moving targets. Every UGV maintains three

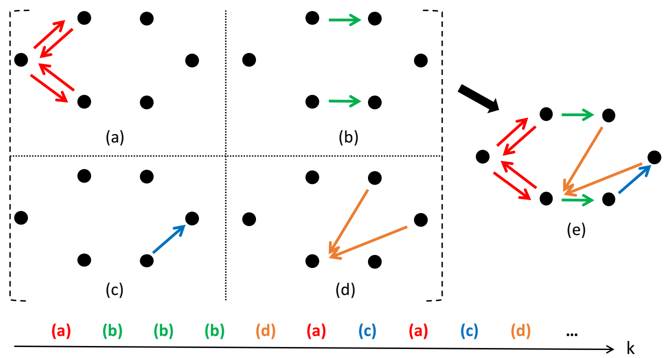
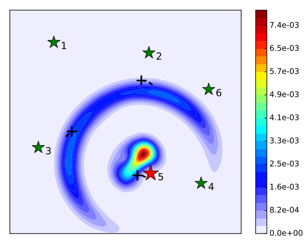
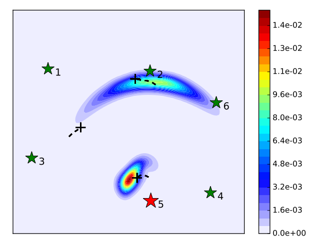


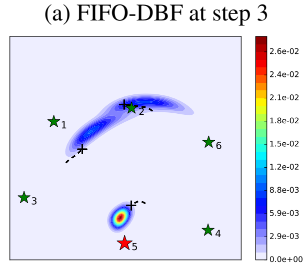
Fig. 5: The dynamically changing interaction topologies used in the simulation: (a)-(d) four types of topologies; (e) the union of these topologies is jointly strongly connected. The bottom axis shows a randomly generated sequence of topologies that satisfy the frequently jointly strongly connectedness condition.



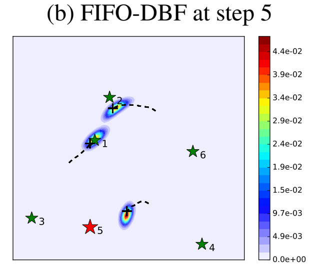
(a) FIFO-DBF at step 3



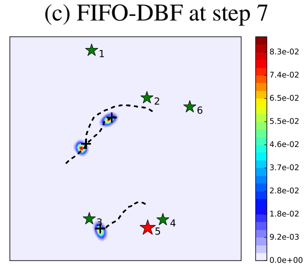
(b) FIFO-DBF at step 5



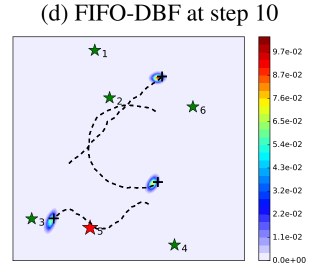
(c) FIFO-DBF at step 7



(d) FIFO-DBF at step 10



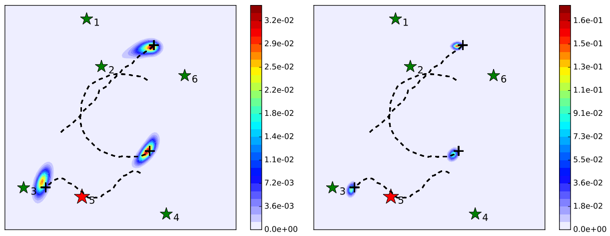
(e) FIFO-DBF at step 20



(f) FIFO-DBF at step 40

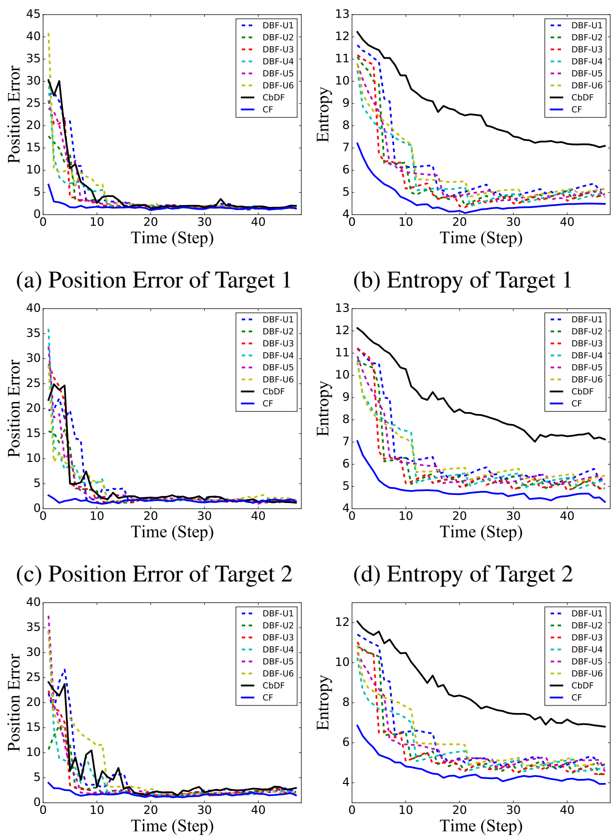
Fig. 6: Evolution of the 1st UGV's target estimation using FIFO-DBF. The colorful background represents the sum of the individual PDF of three targets.

individual PDFs, each corresponding to a target. At each time step, a UGV's sensor can measurement the positions of three targets. We assume that the UGVs know the association between the measurement and the corresponding target. The targets have different motion models, including the linear motion (target 1), sinusoidal motion (target 2), and cir-



(a) CbDF at Step 40 (b) CF at Step 40

Fig. 7: The 1st UGV's target estimation using (a) CbDF and (b) CF. The estimation uncertainty remains large for CbDF.



(a) Position Error of Target 1 (b) Entropy of Target 1

(c) Position Error of Target 2 (d) Entropy of Target 2

(e) Position Error of Target 3 (f) Entropy of Target 3

Fig. 8: The average estimation error (a)(c)(e) and the average entropy (b)(d)(f) of ten trials using different filtering approaches. The dotted lines correspond to the results of FIFO-DBF by the six UGVs ('U1'-'U6' in the legends).

cular motion (target 3). Three of the UGVs have range-only sensors and the other three UGVs have bearing-only sensors. The interaction topology of the UGVs is time-varying and consists of four types, as shown in Fig. 5(a)-(d). A randomly generated sequence of topologies is used (Fig. 5f). It can be noticed that, the interaction topology is frequently jointly strongly connected when all four types appears repeatedly (Fig. 5e). Ten test trials are used, with the randomly generated initial positions of UGVs and targets. There exists different methods to implement a Bayesian filter, including the histogram filter and the particle filter [5]. The histogram filter is easy to implement and can keep track of the probability mass over the whole field, but can be computationally heavy for large fields. The particle filter, on the contrary, is advantageous when the field is very large, but can introduce inaccuracy due to particle deprivation [5]. We use both methods to implement the Bayesian filter in the simulation, and their results are very similar. For the purpose of clarity, we only include the results from the histogram filter here.

We compare FIFO-DBF with two commonly adopted approaches in multi-agent filtering: the consensus-based filter (CbDF) [33] and the centralized filter (CF) [34]. The CbDF requires UGVs to continually exchange their individual PDFs with neighbors, computing the average of its own and the received PDFs. Multiple rounds of communication and averaging are needed at each step to ensure the convergence of UGVs' individual PDFs. The CF assumes a central unit that can constantly receive and fuse all UGVs' latest measurements into a single PDF.

Fig. 6 shows the simulation results of a specific trial. The sum of the 1st UGV's individual PDFs are shown in the figures. Figs. 6a to 6f show that the FIFO-DBF can successfully localize and track moving target's positions and effectively reduce the estimation uncertainty, which is similar to the performance of the CF (Fig. 7b). On the contrary, CbDF is less effective in reducing the estimation uncertainty (Fig. 7a).

We quantitatively compare the three filters in terms of the estimation error and entropy of the uncertainty. The estimation error is defined as the difference between the true target position and the MAP estimate of the individual PDF:

$$\Delta_k = \|X_k^{\text{MAP}} - x_k^g\|_2.$$

The entropy of the uncertainty is

$$H_k = \sum_{X_k \in S} -P_{pdf}(X_k)\log(P_{pdf}(X_k)).$$

The average of the estimation error and entropy of each target across ten trials are shown in Fig. 8. It can be noticed that, the CF achieves the most accurate position estimation and fastest entropy reduction. This is an expected result since the CF utilizes all sensor measurements. The FIFO-DBF achieves similar results as the CF asymptotically. This is a very interesting results, since FIFO-DBF only communicates with neighboring UGVs and have a subset of other UGVs' measurements. The CbDF achieves similar position estimation performance as the CF and FIFO-DBF. However, it fails to effectively reduce the estimation entropy. This is because that, the linear combination of PDFs used in the CbDF does not follow the nonlinear nature of Bayesian filtering, thus information is loss during the combination. The FIFO-DBF, on the other hand, rigorously follows the procedure of Bayesian filtering, and therefore achieves better performance. Besides, CbDF requires multiple rounds of exchanging individual PDFs, which incurs much higher communication burden than FIFO-DBF at each time step. Therefore, FIFO-DBF is more preferable than CbDF.

7 Conclusion

This paper presents a general measurement dissemination-based distributed Bayesian filter (DBF) for a network of multiple unmanned ground vehicles (UGVs) under dynamically changing interaction topologies. The information exchange among UGVs relies on the Full-In-and-Full-Out (FIFO) protocol, under which UGVs exchange the communication buffers and track lists with neighbors. Under the condition that the union of the interaction topologies is frequently jointly strongly connected, FIFO can disseminate measurements over the network within finite time. By using the track list, the CBs can be trimmed to save communication resource without causing information loss. The FIFO-DBF algorithm is developed to estimate individual probability density function for target localization. The FIFO-DBF can significantly reduce the transmission burden between each pair of UGVs compared to the statistics dissemination methods, and can achieve consistent state estimation. Simulations comparing FIFO-DBF with consensus-based distributed filters (CbDF) and the centralized filter (CF) show that FIFO-DBF achieves similar performance as the CF and more superior performance than the CbDF.

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