Distributed Bayesian Filters for Multi-Vehicle Network by Using Latest-In-and-Full-Out Exchange Protocol of Observations

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Abstract— **This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a network of unmanned ground vehicles (UGVs). The DBF utilizes the Latest-In-and-Full-Out (LIFO) local exchange protocol of sensor measurements for data communication within the network. Different from existing statistics dissemination-based approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only exchanges with neighboring UGVs a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs to scale linearly with the size of the network. Under the condition of fixed and undirected topology, LIFO can guarantee non-intermittent dissemination of all observations over the network within finite time. Two types of LIFO-based DBF algorithms are then derived to estimate individual probability density function (PDF) for a static target and for a moving target, respectively. For the static target, each UGV locally fuses the newly received observations while for the moving target, a set of historical observations is stored and sequentially fused. The consistency of LIFO-based DBF is proved that the estimated target position converges in probability to the true target position. The effectiveness of this method is demonstrated by comparing with consensus-based distributed filters and a centralized filter in simulations of target localization.**

I. INTRODUCTION

Distributed filtering that focuses on using a group of networked UGVs to collectively infer environment status has been used for various applications, such as intruder detection [?], pedestrian tracking [?] and micro-environmental monitoring [?]. Several techniques have been developed for distributed filtering. For example, Olfati-Saber (2005) proposed a distributed Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [?]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Madhavan et al. (2004) presented a distributed extended Kalman filter for nonlinear systems [?]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with environmental features. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [?]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. As a generic filtering scheme for nonlinear systems and arbitrary noise distributions, distributed Bayesian filters (DBF) have received increasing interest during past years [?], [?], which is the focus of this study.

The design of distributed filtering algorithms depends on the communication topology of multi-UGV network, which can be classified into two types: fusion center (FC)-based and neighborhood (NB)-based. In FC-based approaches, each UGV uses a filter to estimate local statistics of environment status based on its own measurement. The local statistics is then transmitted (possibly via multi-hopping) to a single FC, where a global posterior distribution (or statistical moments in DKF [?]) is calculated at each filtering cycle after receiving all local information [?]. In NB-based approaches, a set of UGVs execute distributed filters to estimate individual posterior distribution. Consensus of individual estimates is achieved by solely communicating statistics and/or observations within local neighbors of these UGVs. The NB-based methods have become popular in recent years since such approaches do not require complex routing protocols or global knowledge of the network and therefore are robust to changes in network topology and to link failures.

So far, most studies on NB-based distributed filtering have mainly focused on the so-called *statistics dissemination* strategy that each UGV actually exchanges statistics, including posterior distributions and likelihood functions, with neighboring UGVs [?]. This strategy can be further categorized into two types: leader-based and consensus-based. In the former, statistics is sequentially passed and updated along a path formed by active UGVs, called leaders. Only leaders perform filtering based on its own measurement and received measurements from local neighbors. For example, Sheng et al. (2005) proposed a multiple leader-based distributed particle filter with Gaussian Mixer for target tracking [1]. Sensors are grouped into multiple uncorrelated cliques, in each of which a leader is assigned to perform particle filtering and the particle information is then exchanged among leaders. In consensus-based distributed filters, every UGV diffuses statistics among neighbors, via which global agreement of the statistics is achieved by using consensus protocols [?], [?], [?]. For example, Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-

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linear-average consensus algorithm when local likelihood functions belong to the exponential family of distributions [?]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [?]. Other examples can be found in [?], [?].

Despite the popularity of statistics dissemination strategy, exchanging statistics can consume high communication resources. One promising remedy is to disseminate measurement instead of statistics among neighbors, which, however, has not been fully exploited. One pioneering work was done by Coates et al. (2004), who used adaptive encoding of observations to minimize communication overhead [?]. Ribeiro et al. (2006) exchanged quantized observations along with error-variance limits considering more pragmatic signal models [?]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each UGV has a complete set of observations of other UGVs for executing particle filtering [?]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a complete graph that every pair of distinct UGVs is directly connected by a unique edge, which is not always feasible in reality.

This paper extends existing works by introducing a Latest-In-and-Full-Out (LIFO) protocol into distributed Bayesian filters (DBF) for networked UGVs. Each UGV is only allowed to broadcast observations to its neighbors by using single-hopping and then implements individual Bayesian filter locally after receiving transmitted observations. The main benefit of using LIFO is on the reduction of communication burden, with the transmission data volume scaling linearly with the UGV number, while a statistics dissemination-based strategy can suffer from the order of environmental size. The proposed LIFO-based DBF has following properties: (1) For a fixed and undirected network, LIFO guarantees the global dissemination of observations over the network in a non-intermittent manner. (2) The corresponding DBF ensures the consistency of estimated target position, i.e., the estimated position converges in probability to the true target position when the number of observations tends to infinity.

The rest of this paper is organized as follows: The problem of distributed Bayesian filtering is formulated in Section II. The LIFO-based DBF algorithm is described in **??**, followed by the proof of consistency in **??**. Simulation results are presented in **??** and Section V concludes the paper.

II. PROBLEM FORMULATION

Let $S = \left\{x | x \in \mathbb{R}^2\right\}$ denote a two-dimensional planar space containing a target to be localized. An autonomous ground robot equipped with a camera sensor is tasked with localizing the target.

A. Robot and Target Motion Model

We consider a discrete-time unicycle motion model for the robot:

$$x^R_{k+1} = f(x^R_k, u^R_k), \tag{1}$$

where

$$f(x^R_k, u^R_k) = x^R_k + \begin{bmatrix} \cos\theta^R_k \Delta t & 0 \\ \sin\theta^R_k \Delta t & 0 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} u^R_k.$$

The robot state $x^R_k = [x^R_{1,k}, x^R_{2,k}, \theta^R_k, v^R_k]$ consists of robot position, heading angle and speed at time k. The control input $u^R_k = [w^R_k, a^R_k]$ includes angular velocity and linear acceleration.

We assume a stochastic single integrator model for the target:

$$x^t_{k+1} = Ax^t_k + w_k,\ w_t \sim \mathcal{N}(0, Q) \tag{2}$$

with a zero-mean Gaussian noise. The target state $x^t_k = [x^t_{1,k}, x^t_{2,k}]$ include its position. Q is the covariance matrix for Gaussian distribution.

B. Sensor Model

Due to the fact that the target position is unknown a-priori and thus the target may not be within the FOV at the beginning of the search process, the sensor model needs to account for the cases that the target is within and out of FOV. We adopt the linear measurement model from Schenato et. al. that considers the missing measurements:

$$y_{k+1} = Cx^t_k + v_t,\ v_t \sim \begin{cases} \mathcal{N}(0, R) & \text{if } x^t_k \in \mathcal{F}_k \\ \mathcal{N}(0, \sigma^2 I) & \text{if } x^t_k \notin \mathcal{F}_k \end{cases}, \tag{3}$$

where a Gaussian white noise with covariance R is added when the target is within the FOV $\mathcal{F}_k$ of the sensor. Once the target is outside of FOV, the measurement is equivalent to receiving a measurement containing a white noise of infinite covariant, i.e. $\sigma \to \infty$.

III. MPC-BASED INFORMATIVE PATH PLANNING

A. kalman Filter with Limited FOV

According to Schenato paper, a discrete-time kalman filter with intermittent measurement can be formulated as:

$$\hat{x}^t_{k+1|k} = A\hat{x}^t_{k|k} \tag{4a}$$

$$P_{k+1|k} = AP_{k|k}A' + Q \tag{4b}$$

$$K_{k+1} = P_{k+1|k}C(CP_{k+1|k}C' + R)^{-1} \tag{4c}$$

$$\hat{x}^t_{k+1|k+1} = \hat{x}^t_{k+1|k} + \gamma_{k+1}K_{k+1}(y_{k+1} - C\hat{x}^t_{k+1|k}) \tag{4d}$$

$$P_{k+1|k+1} = P_{k+1|k} - \gamma_{k+1}K_{k+1}CP_{k+1|k} \tag{4e}$$

where γ_{k+1} takes a binary value (1 or 0), corresponding to situation that a measurement is obtained or not obtained. Such Kalman filter provides a unifying a framework for handling different measurement results. However, γ_{k+1} is a discontinuous function of the robot and target states, which is inconvenient for formulating an optimization problem. Here we use a sigmoid function to approximate γ_{k+1}. To be specific, we assume that the sensor can always obtain a measurement when the target is inside its FOV and no measurement if outside of FOV. Therefore, $\gamma_{k+1} = \mathbb{1}_{x^t_{k+1} \in \mathcal{F}_{k+1}}$, which is an indicator function specifying whether the target is inside the sensor's FOV.

A typical camera sensor FOV can be modeled as a sector (**??**). We use a sigmoid function to approximate its boundary:

$$\gamma_k \approx \frac{d(x_k^t, e_i)}{1 + d(x_k^t, e_i)^2}, \tag{5}$$

where d Since the sigmoid function is differentiable, it can be utilized in the MPC framework.

B. Informative Path Planning

A model predictive control problem with planning horizon N can be formulated as:

$$\begin{aligned}
\min_{u_{1:N}} \; & J(b_{1:N+1}) \\
\text{s.t.} \; & x_{k+1}^R = f(x_k^R, u_k), \\
& b_{k+1} = g(b_k, u_k), \\
& x_{k+1}^R \in \mathcal{X},\, u_{k+1} \in \mathcal{U}, \\
& k = 1, \ldots, N,
\end{aligned}$$

where $\mathcal{X}$ and $\mathcal{U}$ are feasible set of robot state and control input, respectively.

To drive the robot to configurations in which the sensor can obtain information about the target, we want to maximize the entropy reduction, $H(\hat{x}_1^t) - H(\hat{x}_{k+1}^t)$, for the planning horizon. Since $H(\hat{x}_1^t)$ depends only on the target estimation at the current time and is not affected by control input of the robot, it is equivalently to minimizing $H(\hat{x}_{k+1}^t)$. For a multivariate normal distribution, such entropy is equivalent to $\frac{k}{2}(1+\ln(2\pi)) + \frac{1}{2}\ln|P_{N+1|N+1}|$. Minimizing a determinant can be troublesome for optimization problem. Utilizing the relation that $det(A)^{\frac{1}{n}} \leq \frac{1}{n}tr(A)$ for a positive definite matrix A, we define the objective function of the MPC problem as:

$$J(b_{1:N+1}) = tr(P_{N+1|N+1}).$$

IV. Simulation

A. Static Target

B. Moving Target

V. Conclusion

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) approach for a multi-UGV network, utilizing the Latest-In-and-Full-Out (LIFO) protocol for measurement exchange. By exchanging full communication buffers among neighboring UGVs, LIFO significantly reduces the transmission burden between each pair of UGVs to scale linearly with the network size. It should be noted that LIFO is a general measurement exchange protocol and thus applicable to various sorts of sensors. Two types of LIFO-based DBF algorithms are proposed to estimate individual PDFs for a static target and a moving target, respectively. The consistency of LIFO-based DBF is proved by utilizing the law of large numbers, which ensures that the estimated target position converges in probability to the true target position.

Future work includes considering other types of sensors and imperfect communication between UGVs. Other types of sensors may have biased measurement , which complicates the design and analysis of LIFO-DBF. Imperfect communication, including package loss and transmission delay, requires extensions of current LIFO-DBF approach.

References

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