Distributed Bayesian Filter for Multi-Robot Network by Using Local-Exchange-of-Observation Strategy

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***Abstract*— This electronic document is a live template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document.**

I. INTRODUCTION

Distributed filtering in multi-robot network that focuses on using a group of networked robots to collectively infer the state of an environment has been used for various applications, such as object detection (Chamberland), target tracking (Beaudeau) and environmental monitoring etc (give 2nd-level details of the examples). This study focuses on the distributed estimation of a target position in the environment for target search and tracking applications. History, main stream methods (Kalman filter, particle filter, Bayesian filter)? ???Bayesian filter, generalization.

The communication topology of multi-robot network plays a vital role in distributed filtering algorithms. Fusion center-based DBF has been the most common structure for distributed filtering, in which local information of robots is transmitted (possibly via multi-hopping) to the fusion center for forming global estimation (L. Zuo) (A). Pros and cos. The other is ****. , many works focus on the neighborhood-based DBF that each robot can only exchange information with neighboring robots and each robot forms local estimation of the environment state. Pros and cons. Definition,

Statistics dissemination based ****. For example, Sheng et. al. (S.) proposed a leader agent based distributed particle filters with Gaussian Mixer to localize and track multiple moving targets. The distributed particle filters ran on a set of uncorrelated sensor cliques and information among cliques was exchanged for global estimation of targets. An improvement is conducted by ****. *****

Another popular strategy for statistics dissemination is to use ***** consensus. ***

Consensus algorithms, as proposed in (Olfat-Saber) (Ren.) (Jadbabie.), have become a popular distributed filtering approach for neighborhood-based communication topology. As far, the commonly used consensus strategy is based on the statistics dissemination (Hlinka), which actually exchanges posterior distribution or likelihood functions to neighboring robots for distributed estimation. For example, Saptarshi et. al. (Bandyopadhyay) presented a Bayesian consensus filter (BCF) that uses logarithmic opinion pool for fusing posterior distributions of the tracked target among neighboring robots. The proposed BCF can incorporate non-Gaussian uncertainties and nonlinearity in target dynamic models and measurement models. A consensus-based distributed particle filter (DPF) is proposed by Julian et. al. (Julian) for estimating environmental state by exchanging and fusing posterior functions of the state among neighbors. The DPF can work even when the network diameter, the maximum in/out degree, and the number of robots are unknown.

There are other types of variants, for example, In (Ram.S), a circular topology that each sensor can only communicate with a fixed neighboring sensor is deployed for parameter estimation of a spatial field. Each sensor generates state estimate based on the estimate of previous sensor and its own observation and sequentially passes the estimate to its neighbor, who using the incremental Robbins-Monro gradient algorithm locally at each sensor.

Despite the popularity of statistics dissemination approaches, exchanging posterior distribution or likelihood functions can consume high communication resources, which may be infeasible for applications in vast area or complex environment, such as marine search, seismological rescue, etc. This study focuses on the strategy of exchanging observation in the neighborhood of each robot for the purpose of achieving a consensus of local target PDFs. Some pioneering studies have been done on measurement dissemination strategies, in which raw or quantized observations are exchanged among robots. For example, Coates et. al. (Coates) used adaptive encoding of observations to minimize communication overhead. In (Djuric), each robot communicates the observation of the targets to all the remaining agents and apply local particle filter for target tracking (add more details about how the observations are exchanged). Another example can be found in ***, in which both observation and statistics are exchanged. In aforementioned works, communication topology is assumed fully connected that each robot broadcasts observations to all robots in single transmission step.

This paper proposes a local-exchange-of-observation (LEO) strategy for distributed Bayesian filters (DBF) for undirected local topology. Each robot only broadcasts its observation to its local neighbors and ****. introduce DBF, two steps. The main benefit of this strategy is the reduction of communication burden, with the transmission data volume scaling linearly with the robot number, while statistics dissemination-based strategies suffer from the data volume on the order of environmental size. After receiving

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observation from neighbors, each robot runs Bayesian filter locally for environment state estimation. The proposed DBF has following properties: (1) For *** topology, LEO can guarantee , all elements in under LEO become filled when . (mainly about proposition 1, corollary 1,2) (2) LEO-DBF can ensure consistency of the state estimate, thus guaranteeing consensus. (mainly about theorem 1 and 2)

(merge this with property (2))The consistency property refers to the agreement between the robot?s state estimate and the true state. In this study, a formal proof of the consistency of LEO-DBF is provided. This property differentiates LEO-DBF from the statistics dissemination-based filtering approaches. In fact, the statistics dissemination-based methods can ensure the convergence of the state estimate among robots. However, there?s no guarantee whether the agreed estimate is close to the true state.

The rest of this paper is organized as follows: Section 1 ***; section 2 ***.

II. PROBLEM FORMULATION

Consider a network of N robots in bounded two-dimensional space S. Each robot is equipped with a binary sensor for environmental perception. Due to the limit of communication range, each robot can only exchange observations with its neighbors. Bayesian filter is run locally on each robot for environment state estimation based on its own and received observations.

A. Probabilistic Model of Binary Sensor

The binary sensor only gives two types of observation: 1 if the target is detected, and 0 if no target is detected. The observation of i-th sensor at k-th time step is denoted as . The likelihood function that the target is detected is:

(1) where is the target position, is the set of all possible target positions. Correspondingly, the likelihood function that no target is detected is

(2) The combination of Eq. (1) and (2) is actually binary sensor model, parameterized by . The goal of Bayesian filter is to estimate the actual target position, i.e., , with which all observations are generated.

The commonly used likelihood functions for binary sensor are Gaussian function (Bonnie) (Liu) and step function (Djuric P.). In addition, LEO also works for other types of sensors, including imaging sensors (need to think about this wording. Is it possible to find something opposite to binary sensor) such as laser scanners (D) and cameras (J.).

Remark (1): The current observation of each robot is conditionally independent from both its own past observations and those of other robots, given the information of target position.

Remark (3): This study is applicable for both homogenous and heterogeneous binary sensors A homogeneous model can simplify the analysis of completeness, while the heterogeneous model is more close to real sensing characteristics.

B. Bayesian Filtering for Multiple Sensors

The binary sensor only gives two types of observation: 1 if the target is detected, and 0 if no target is detected. For the sensor of i-th robot, the observation at k-th time step is denoted as . The observation likelihood function is used to describe the probability that the target is detected:

(1) where is the target position. Correspondingly, the probability that no target is detected is

(2) The commonly used observation likelihood functions for binary sensor are Gaussian function (Bonnie) (Liu) and step function (Djuric P.). In addition, it should be noted that, in spite of the usage of binary sensors, the observation exchange strategy proposed in this study also works for other types of sensors, including range sensors, such as Sonars (Coraluppi), laser scanners (D) and cameras (J.).

Remark (1): The current observation of each robot is conditionally independent from both its own past observations and those of other robots, given the current state of the target. Remark (2): In this study, both homogenous and heterogeneous models of binary sensor are applicable. The definition of heterogeneity is in the sense of observation likelihood function, which means that the observation spaces are the same, but the observation probability can differ. The homogenous assumption can simplify the analysis of completeness, while the heterogeneous models is more close to real sensing characteristics.

1) Graphical model of communication topology: Consider an undirected and connected graph , where is the set of robots and denotes the edge set. The adjacency matrix describes the communication topology of G: , where denotes the entity of adjacency matrix. The notation indicates that a communication link exists between i-th and j-th robot and indicates no communication. The direct neighborhood of i-th robot is defined as . All the robots in can directly exchange information with the i-th robot. In addition to direct neighborhood, another set, called available neighborhood, is defined as , which contains robots whose observation is received by the i-th robot given a specific observation exchange algorithm. Note that in general , but if only single-hopping is allowed, is equal to . Figure 1 illustrates three types of typical topologies: circular (Ram.S), serial (Bahceci), and star (S.). All of them are undirected and connected topologies.

C. Distributed Bayesian Filter for Multiple Robots

The generic Distributed Bayesian Filter (DBF) is introduced in this section, which was also stated in (O, Distributed Sequential Estimation in Asynchronous Wireless Sensor Networks) and (T). Each robot has its individual estimation of target PDF, called individual PDF. The i-th individual PDF at time k is defined as . Before starting DBF, the individual PDF is initialized by the prior function , given all available prior information including past experience and environmental knowledge. Then the individual PDF is recursively estimated by two steps, i.e., prediction step and updating step, based on the observations of the robots in .

1) Prediction: The i-th individual PDF at time k-1 is known, denoted as . At time k, the prior individual PDF is first predicted forward by using the Chapman-Kolmogorov equation:

(3) where is a Markov motion model of the target, independent of robot states. This model describes the state transition probability of the target from a prior state to posterior state . Note that the target can be static in many searching applications, for example, ****** (Kulich). For a static target, its Markov motion model is simplified to be

and Eq. (3) can be reduced to .

2) Updating: At time step k, the i-th individual PDF is then updated by Bayes? formula using the latest available observations at time k:

(4) where is a normalization factor, given by: . where is called posterior individual PDF; is the likelihood function of observation , described in Eq. (1) and (2).

III. Distributed Bayesian Filter via Local Exchange of Observation

This study proposes a Distributed Bayesian Filtering (DBF) algorithm based on LEO strategy, shorted as LEO-DBF. The LEO only uses the local communication within the neighborhood of each robot, but allows to broadcast observations of each robot to any other nodes by multi-hopping along the shortest path in the undirected and connected network. The theoretical analysis show that LEO-DBF can ensure the consistency and consensus of individual PDF while requiring much less communication burden than any statistics dissemination-based DBFs.

A. Algorithm for Local Exchange of observations (LEO)

Under the LEO, each robot contains a communication buffer (CB) to store its latest knowledge of the observations of all robots:

$$\mathbf{z}_k^i = \left[z^1_{k^i_1}, \ldots, z^N_{k^i_N}\right] \tag{1}$$

where $z^j_{k^i_j}$ represents the observation made by j^{th} robot at time k^i_j. Note that under LEO, $\mathcal{Q}_i = \{1, \ldots, N\}$. At time k, $z^j_{k^i_j}$ is received and stored in i^{th} robot CB, in which k^i_j is the latest available observation time of j^{th} robot known by i^{th} robot. Due to the communication delay of multi-hopping, $k^i_j < k$ always holds in practice.

The LEO algorithm is stated as follows:

(1) Initialization: The buffer of i-th robot is initialized when k=0:

(2) Repeat the following steps for i-th robot until stop (2.1) Receiving Step: The i-th robot receives all CBs of its neighboring robots. The received CBs are totally groups, each of which corresponding to the (k-1)-step CB of a robot in . The received CB from l-th ( ) robot is denoted as

(2.2) Observation Step: The i-th robot updates by its own observation at current step: . (2.3) Comparison Step: The i-th robot updates other elements of its own CB, i.e., , by selecting the latest information among all received CBs from . For all ,

(5) (2.4) Sending Step: The i-th robot broadcasts its updated CB to all of its neighbors defined in . (End of LEO)

Figure 2 illustrates the LEO algorithm with 3 robots using a serial topology. Two facts can be observed in Figure 2: (1) all robot CBs are filled within 3 steps, which means under LEO each robot has a maximum delay of 2 steps when receiving observations from other robots; (2) after filled, the updating of CBs are non-intermittent, which means each robot continuously receives newer observations of other robots. Extending the two facts to a network of N robots, we have the following proposition:

Proposition 1: For an undirected and connected network of N robots with fixed topology, LEO uses the shortest path(s) between i and j to exchange observation, i.e., the delay between i and j is equivalent to the length of their shortest path(s).

Proof: Without loss of generality, assume that there is a unique shortest path between i and j, denoted by , with . Then, the distance between i and j is . The following mathematical induction will prove Proposition 1.

Step (1): For , this means and j can directly send to i. Then, is stored in i-th CB at time k+1, i.e. . Proposition 1 holds for . Step (2): Suppose that Proposition 1 holds for . Then for , i.e., , by the Bellman?s principle of optimality, the path is the shortest path between j and l, where and . The assumption that Proposition 1 holds for implies that is received and stored in l-th robot?s CB at time . Since , i-th robot receives at . Therefore, . For any other path with , cannot be received by i-th robot at . This proves the Proposition 1 for . (End of proof)

Corollary 1: For the same topology assumption in Proposition 1, all elements in under LEO become filled when .

Proof: In a network of N robots, the maximal length of shortest paths is no greater than N-1. Based on proposition 1, and thus all elements of become filled when . (End of proof)

Corollary 2: For the same topology in Proposition 1, once is filled, the updating of each element in is non-intermittent.

Proof: For a network with fixed topology, the shortest path between any nodes is fixed. Based on Proposition 1, is constant, and then the updating of each element in is non-intermittent. (End of proof)

Remarks (5): Compared to statistics dissemination, LEO is a more communication-efficient approach for distributed filtering. To be specific, consider an grid environment with a network of robots, the transmitted data of LEO between each pair of robots are only the CB of each robot, the length of which is . On the contrary, the length of transmitted data for a statistics-dissemination approach is , which is the size of environment grid. Since is generally much larger than in target search and tracking community, LEO requires much less communication resources than the statistics-dissemination approaches.

B. Algorithm for LEO-based DBF for a Static Target

This section gives the LEO-DBF for a static target. Each robot stores last-step individual PDF, i.e., (k-1)-th step. The assumption of static target can simplify the Bayesian filter, in which the prediction step is unnecessary. Therefore, the i-th individual PDF is only updated by

where

C. Algorithm for LEO-DBF for a Moving target

This section gives the LEO-DBF for a moving target. Instead of storing last-step PDF , each robot maintains (k-N)-th individual PDF, and a triangular matrix of history observations from (k-N+1)-th to k-th steps. The i-th individual PDF is alternatively predicted and updated by using aforementioned Bayesian filter (Eq. (3) and (4)) from (k-N)-th individual PDF to k-th step. Figure 3 illustrates how to *****.

For the i-th robot at k-th step: 1) The stored individual PDF for (k-N)-th step is 2) Initialize a virtual PDF by assigning the individual PDF to it:

3) From to N, repeat two steps of Bayesian filtering (a) Prediction

(b) Updating

4) Store the first-step virtual PDF as the (k-N+1)-th individual PDF:

where denotes the index set of robots whose observation at time is stored in i-th robot?s CB. The target PDF estimation of i-th robot at current step is . *** (??????????????).

Remarks (7): For the static target, each robot only needs current step CB to update individual PDFs. Therefore, except storing individual PDFs, all historical CBs can be discarded and only current-step CB is stored in robot memory, the length of which is . On the contrary, for the moving target, each robot needs to store a triangular matrix of history observation (except current step CB) with N-by-N dimension and an M-by-M individual PDF, which means that the length of occupied memory in each robot is $O(M^2 + N^2)$.

IV. Proof of Consistency and Consensus

This section presents a consistency and consensus proof of LEO- DBF. Only the scenarios for the static target are presented, including both static robots and moving robots. The proof for LEO- DBF for moving target is similar to that of static target by assuming that the dynamic model of the target is accurately known, but with more complicated algebraic manipulation.

Assume that is finite. Define an equi-parameter set such that

Since is finite, is also finite. Let denote all equi-parameter sets. By definition, the following properties hold: (1) (2) Without loss of generality, assume , where denotes the true location of the target.

Remarks (8): the equi-parameter sets depend on the property of the sensor. For example, for a range-only sensor, all locations with equivalent distance to the sensor belong to the same equi-parameter set. For a laser scanner with perfect sensing capability, every equi-parameter set contains only one element.

A. Proof for static robots

The consistency of LEO-DBF for static robots is stated as follows: Theorem 1 For static robots, each individual PDF by LEO-DBF converges to when the number of observations goes to infinity, i.e.

(6) where , are the timestamps of the i-th robot?s latest knowledge of all robots? observations.

Proof: Considering the conditional independence of observations for given , the batch form of DBF at k-th step is:

(7) where is initial i-th local PDF. It is known from the Corollary 2 that

(8) Compare with :

(9) Take the logarithm of (9) and average it over k steps:

(10) Since and are bounded,

(11) The binary observations subject to Bernoulli distribution , yielding

(12) where . Utilizing the fact that (1) are conditionally independent samples from , and (2) , the law of large numbers yields

where . Then,

(13) Note that the r.h.s of (13) achieves maximum iff . Considering (11) and (13), the limit of (10) can be obtained:

(14) It is known from Eq. (14) that (1) When , i.e., , and ; (2) When , i.e., , and . (End of proof)

B. Proof for moving sensors

Lemma 1: For a finite number of robots within a finite number of possible positions, there exists at least one position for each robot that it visits for infinite times as k goes to infinity.

Proof: Let denote the times that i-th robot visits j-th position up to time k. Then . It is straightforward to see that .

Theorem 2 (consistency of LEO-DBF for moving robots) Consider a finite set of target positions, . Under the condition of binary sensors, the individual PDF given by LEO-DBF will concentrates on the true location of the target after infinitely many observations, i.e.

where denotes the true location of the target and . Proof: Similar to Theorem 1, the batch form of DBF at k-th step

(12) where is initial i-th local PDF. By converting [change this part, may define to show all the (time, position) pairs]

(13) The only difference is that Eq. (7) does not hold, but for each sensor, at least there is one position has infinite observation as , according to lemma 1. We can classify all the positions into finite-observation spots and infinite-observation spots. For the former, it is easy to know that

The corresponding item in has zero-limit. Therefore, the proof of Eq. (13) can be reduced to infinite-observation spot, which is similar to Theorem 1. (End of Proof)

Remarks (7): For single target, the true location is unique. Since all individual PDFs concentrate on the same location, the consensus of individual PDFs is achieved.

V. Simulation

This section simulates three searching scenarios in order to demonstrate the effectiveness of LEO-BDF. In all scenarios, six robots are utilized and each robot is equipped with a binary sensor. All sensors are modeled with identical Gaussian functions:

where denotes the robot position where current observation is executed (find a proper word??). Figure 4 shows the 1-D illustration of Gaussian binary sensor model.

The first scenario consists of six static robots and single static target, which acts as a proof of concept of the LEO strategy for static target case. The second scenario subsequently deals with six moving robots for searching the single static target. The third scenario is presented that contains six moving sensors and one moving target. moving robots*****; moving target *****.

A. Static Robots, Static Target

The positions of 6 static sensors are shown as stars in Figure 5. Each sensor constantly receives binary observations of the target. Distributed Bayesian filters described Eq. (**), Eq. () in section 3.2 are implemented on each robot for target position estimation.

Figure 5 shows the estimation results of the static target. After the initial observation, each sensor forms a circular individual PDF, centered at the corresponding sensor position. As more observations are received, the posterior individual PDF concentrates on the true location of the target. Figure 6 shows the decrease of the entropy of the posterior distribution (add a ref), indicating the uncertainty reduction of the estimated target position.

B. Moving Robots, Static Target

The 6 robots move within the field to estimate the target position. The motion planning of robots for effective target search has received much attention in the past years. In this work, the robot positions are randomly generated at each time in order to demonstrate the effectiveness of the LEO-DBF approach. Readers interested in robot motion planning can refer to (J.) (Furukawa).

Figure 7 shows the estimation results over time. Similar to the results in sec. 5.1, the posterior individual PDF concentrates to the true target location. Figure 8 shows the decrease of the entropy of the posterior distribution.

C. Moving Robots, Moving Target

The target in this scenario moves on the horizontal plane and the dynamics is given by a single-integrator model:

where is the constant velocity of the target.

The LEO-DBF described in section 3.3 is used as the distributed Bayesian filters. Figure 9 shows the estimation results of the moving target. It can be noticed that similar to the case of static target, the posterior individual PDFs concentrate to the true target location at each time, even when the target constantly moves.

VI. Conclusion

In this study, we proposed the local-exchange-of-observation (LEO) strategy for distributed Bayesian filters (LEO-DBF) in a multi-robot network with the application of distributed estimation of target position in the environment. With fixed communication topology, each robot can receive all robots? observations non-intermittently under the LEO strategy. Two LEO-DBFs are proposed for estimating the position of the static and the moving target, respectively. (in the above two sentences, explain the two properties of LEO). For the static target, each robot locally fuses the latest knowledge of all robots? observations by only considering the updating step of the Bayesian filter. For the moving target, a triangle matrix of historical observations is maintained by each robot. Upon obtaining the latest available observations of all robots, an iterative Bayesian filtering procedure is applied that alternates between prediction and updating steps. The consistency of LEO-DBF is proved, ensuring the agreement between robots? state estimate and the true environment state (explain briefly how the consistency is proved, such as via what principle etc.). Simulations demonstrated the effectiveness of LEO-DBF for both static and moving target.

Future work may include several extensions to the proposed LEO-DBF. LEO under switching topology can leads to unpredictable delay and intermittent transmission. Therefore consensus analysis of individual PDFs requires further analysis. In addition, combining LEO-DBF with robot motion planning is promising for more effective estimation of target state in the environment.

TABLE I

An Example of a Table

One	Two
Three	Four

We suggest that you use a text box to insert a graphic (which is ideally a 300 dpi TIFF or EPS file, with all fonts embedded) because, in an document, this method is somewhat more stable than directly inserting a picture.

Fig. 1. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

Appendix

Appendixes should appear before the acknowledgment.

Acknowledgment

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

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