Formulation of the problem

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I. PROBLEM FORMULATION

A. Robot and Target Motion Model

Unicycle motion model for the mobile robot:

$$z_{k+1} = h(z_k, u^r_k), \tag{1}$$

where

$$f(z_k, u^r_k) = z_k + \begin{bmatrix} \cos\theta^r_k \Delta t & 0 \\ \sin\theta^r_k \Delta t & 0 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} u^r_k.$$

Motion model of the target:

$$x_{k+1} = f(x_k) + w_k,\ w_t \sim \mathcal{N}(0, Q) \tag{2}$$

$$\tag{3}$$

B. Modeling Sensing Domain

Sensor sensing domain is represented as $\mathcal{F}_k = \left\{[x_{1,k}, x_{2,k}] \in \mathbb{R}^2 | \ \|v\|_2 \leq r, \angle v \in [\theta_1, \theta_2]\right\}$, where $v = [x_{1,k} - z_{1,k}, x_{2,k} - z_{2,k}]$.

C. Sensor Measurement Model

Measurement model:

$$y_k = g(x_k) + v_k,\ v_k \sim \begin{cases} \mathcal{N}(0, R) & \text{if } \gamma_k = 1 \\ \mathcal{N}(0, \sigma^2 I) & \text{if } \gamma_k = 0 \end{cases}, \tag{4}$$

$$\gamma_k = \mathbb{1}_{\{x_k \in \mathcal{F}_k\}} \tag{5}$$

II. MPC-BASED PATH PLANNING

A. EKF with Limited Sensing Domain

$$\hat{x}^t_{k+1|k} = f(\hat{x}^t_{k|k}) \tag{6a}$$

$$P_{k+1|k} = A^i_k P^i_{k|k} A^{i'}_k + Q \tag{6b}$$

$$K^i_{k+1} = P^i_{k+1|k} C^{i'}_{k+1} (C^i_{k+1} P^{i'}_{k+1|k} C^{i'}_{k+1} + R)^{-1} \tag{6c}$$

$$\hat{x}^i_{k+1|k+1} = \hat{x}^i_{k+1|k} + \gamma_{k+1} K^i_{k+1} (y_{k+1} - h(\hat{x}^t_{k+1|k})) \tag{6d}$$

$$P^i_{k+1|k+1} = P^i_{k+1|k} - \gamma_{k+1} K^i_{k+1} C^i_{k+1} P^i_{k+1|k}, \tag{6e}$$

where $A^i_k = \frac{\partial f}{\partial x}|_{x=\hat{x}^i_{k|k}}$ and $C^i_{k+1} = \frac{\partial g}{\partial x}|_{x=\hat{x}^i_{k+1|k}}$. The $\hat{x}^t_{k|k}$ and $P_{k|k}$ represent the estimated target position and covariance matrix. For notational simplicity, we define $b_k = [\hat{x}^t_k, P_{k|k}]$ and let $b_{k+1} = g(b_k, u^r_k)$ represent the Kalman filter defined in Eq. (6).

γ is approximated by

$$\gamma_k \approx \frac{1}{1 + \alpha_1 \|[x_{1,k}, x_{2,k}] - [z_{1,k}, z_{2,k}]\|^2_2} \times \frac{1}{1 + \exp\left\{-\alpha_2 (\cos(\theta^r_k - \tilde{\theta}_k) - \cos(\theta_0))\right\}}, \tag{7}$$

where $\tilde{\theta}_k = \angle([x_{1,k}, x_{2,k}] - [z_{1,k}, z_{2,k}])$ is the direction angle from the sensor position to target position; $\theta_0 = \frac{\theta_2 - \theta_1}{2}$ is half of the sensing angle; α_1 and α_2 are tunning parameters that controls the shape of the function. Eq. (7) can be interpreted as follows: when the robot is close to the target, it is more likely that the target can be detected; besides, the closer the target direction aligns with the center direction of the sensor, the higher possibility that the target will get detected.

B. Path Planning for Target Search and Tracking

The MPC-based path planner with planning horizon N can be formulated as:

$$\min_{u_{1:N}} J(b_{1:N+1}, u_{1:N}) \tag{8a}$$

$$\text{s.t. } z_{k+1} = f(z_k, u^r_k), \tag{8b}$$

$$b_{k+1} = g(b_k, u^r_k), \tag{8c}$$

$$z_{k+1} \in \mathcal{X},\ u^r_{k+1} \in \mathcal{U}, \tag{8d}$$

$$k = 1, \ldots, N, \tag{8e}$$

The objective function is

$$J(b_{1:N+1}) = \sum_{k=1}^{N+1} H(b_k) \tag{9}$$

$$\approx -\sum_{k=1}^{N+1} \sum_{i=1}^{L} w_i \log b^i_k. \tag{10}$$

The approximation is the 0-order approximation of entropy.

C. Possible linearization in the iterative planning process

1) target motion matrix A
2) sensor measurement matrix C
3) initial solution
4) approximate the sensor boundary
5) linearize robot motion model

III. POSSIBLE EXTENSIONS

1) GSF with good weight update law
2) efficient computation of the objective function
3) how to represent and compute γ (incorporating b_t or just using a point estimate (e.g. MAP))
4) incorporate negative info in a better way than γ.
5) the way to do iterative planing: updating γ in SQP or outside SQP; whether updating w. if not updating, can I obtain an upper bound of the error?
6) control of nonholonomic vehicle
7) make the problem a cvx optimization or some other form (e.g., proximal gradient) to better utilize the form of the problem.

1,3,4 are the possible main contributions of the work.