Distributed Environmental Estimation Using A Group of UGVs Under Dynamically Changing Interaction Topologies

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***Abstract*— This paper presents a distributed Bayesian filtering (DBF) method for a network of multiple unmanned ground vehicles (UGVs) under dynamically changing interaction topologies. The information exchange among UGVs relies on a measurement dissemination scheme, called Latest-In-and-Full-Out (LIFO) protocol. Different from statistics dissemination approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only sends a buffer that contains latest available measurements to neighboring nodes, which reduces the transmission burden from the order of environmental size to that of UGV number. Under the condition that the union of undirected switching topologies is connected frequently enough, LIFO can disseminate observations over the network within finite time. The LIFO-based DBF algorithm is then derived to estimate individual posterior density function (PDF) for target localization in a static environment. The consistency of this algorithm is proved that each individual estimate of target position converges in probability to the true value when the number of observations tends to infinity. The effectiveness of this method is demonstrated by a series of simulations.**

***Index Terms*— Multiple vehicle system, environment estimation, distributed filtering, switching interaction topology**

I. INTRODUCTION

Unmanned ground vehicles (UGV) that operate without on-board operators have been used for many applications that are inconvenient, dangerous, or impossible to human. Distributed estimation using a group of networked UGVs has been applied to collectively infer status of complex environment, such as intruder detection [1], and object tracking [2]. Several techniques have been developed for distributed estimation, including distributed linear Kalman filters (DKF) [3], distributed extended Kalman filters [4] and distributed particle filters [5], etc. The most generic filtering scheme is distributed Bayesian filters (DBF), which can be applied for nonlinear systems with arbitrary noise distributions [6], [7]. This paper focuses on a communication-efficient DBF for networked UGVs.

The interaction topology plays a central role on the design of DBF, of which two types are widely investigated in literature: fusion center (FC) and neighborhood (NB). In the former, local statistics estimated by each agent is transmitted to a single FC, where a global posterior distribution is calculated at each filtering cycle [8], [9]. In the latter, each agent individually executes distributed estimation and the agreement of local posterior distributions is achieved by a certain consensus strategy [10]–[12]. In general, the NB-based distributed filters are more suitable in practice since they do not require a fusion center with powerful computation capability and are more robust to changes in network topology and link failures. So far, the NB-based approaches have two mainstream schemes according to the transmitted data among agents, i.e., *statistics dissemination* (SD) and *measurement dissemination* (MD). In the SD scheme, each agent exchanges such statistics as posterior distributions and likelihood functions within neighboring nodes [13]. In the MD scheme, instead of exchanging statistics, each agent sends its observations to neighboring nodes.

Statistics dissemination scheme has gained increasing interest and been widely investigated during last decade. Madhavan et al. (2004) presented a distributed extended Kalman filter for nonlinear systems [4]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with environmental features. Olfati-Saber (2005) proposed a distributed linear Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [3]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [5]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-linear-average consensus algorithm when local likelihood functions belong to the exponential family of distributions [14]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [6]. Other examples can be found in [7] and [15].

Despite the popularity of statistics dissemination, exchanging statistics can consume high communication resources. One remedy is to approximate statistics with parametric models, e.g., Gaussian Mixture Model [16], which can reduce

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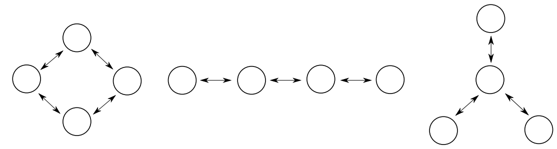
communication burden to a certain extent. However, such a manipulation increases the computation burden of each agent and sacrifices filtering accuracy due to approximation. The measurement dissemination scheme is an alternative solution to address the issue of exchanging statistics. An early work on measurement dissemination was done by Coates et al. (2004), who used adaptive encoding of observations to minimize communication overhead [17]. Ribeiro et al. (2006) exchanged quantized observations along with error-variance limits considering more pragmatic signal models [18]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each agent has a complete set of observations of other UGVs for executing particle filtering [19]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a fixed and complete graph that every pair of distinct UGVs is constantly connected by a unique edge. In many real applications,the interaction topology may change dynamically due to unreliable links, external disturbances and/or range limits [20]. In such cases, dynamically changing topologies can cause random packet loss and variable transmission delay, thus decreasing the performance of distributed estimation, and even leading to inconsistency and non-consensus.

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) method for a group of networked UGVs with dynamically changing interaction topologies. The measurement dissemination scheme uses the so-called Latest-In-and-Full-Out (LIFO) protocol, in which each UGV is only allowed to broadcast observations to its neighbors by using single-hopping. Individual Bayesian filter is implemented locally for each UGV after exchanging observations using LIFO. Under the condition that the union of undirected switching topologies is connected frequently enough, two properties are achieved: (1) LIFO can disseminate observations over the network within finite time; (2) LIFO-based DBF guarantees the consistency of estimation that each individual estimate of target position converges in probability to the true value as the number of observations tends to infinity. The main benefit of using LIFO is on the reduction of communication burden, with the transmission data volume scaling linearly with the UGV number.

The rest of this paper is organized as follows: The LIFO protocol for dynamically changing interaction topologies is formulated in Section II. The LIFO-based DBF algorithm is described in Section III, where the consistency and consensus of estimation are proved. Simulation results are presented in Section IV and Section V concludes the paper.

II. LIFO Protocol for Dynamically Changing Interaction Topologies

Consider a network of N UGVs in a bounded two-dimensional space S. The interaction topology can be dynamically changing due to limited communication range, varying team formation or link failure. Each UGV is equipped with a sensor for environmental perception. Due to the limit of communication range, each UGV can only exchange sensor observations with its neighbors. The Bayesian filter is run locally on each UGV based on its own and received observations via single-hopping to estimate the target position.



(a) (b) (c)

Fig. 1: Three types of topologies: (a) ring topology; (b) line topology; (c) star topology

A. Graphical Model of Interaction Topology

Consider a simple[1], undirected graph $G = (V, E)$ to represent the interaction topology of N networked UGVs, where $V = \{1, \dots, N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. The *adjacency matrix* $M = [m_{ij}]$ of graph G describes the interaction topology:

$$m_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where m_{ij} denotes the entity of adjacency matrix. The notation $m_{ij} = 1$ indicates that a communication link exists between i^{th} and j^{th} UGV and $m_{ij} = 0$ indicates no communication between them. Fig. 1 illustrates three types of typical topologies: ring [21], line [22], and star [23]. All of them are represented by simple and undirected graphs.

Let $\bar{G}$ denote the set of all possible simple and undirected graphs defined for the network of UGVs. It is easy to know that $\bar{G}$ has finite elements. The adjacency matrix associated with a graph $G_l \in \bar{G}$ is denoted as $M^l = [m^l_{ij}]$. Define the *union* of a collection of graphs $\{G_{i_1}, G_{i_2}, \dots, G_{i_l}\} \subset \bar{G}$ as the undirected graph with nodes in V and edge set given by the union of edge sets of G_{i_j}, $j = 1 \dots, l$. Such collection is defined to be *jointly connected* if the union of its members forms a connected graph.

We define two concepts of neighborhood in a UGV network. The *direct neighborhood* of i^{th} UGV under topology G_l is defined as $\mathcal{N}_i(G_l) = \{j | m^l_{ij} = 1, j \in \{1, \dots, N\}\}$. All UGVs in $\mathcal{N}_i(G_l)$ can directly exchange information with i^{th} UGV via single-hopping. In addition to direct neighborhood, another set called *available neighborhood* is defined as $\mathcal{Q}_i(G_l)$, which contains indices of UGVs whose observations can be received by the i^{th} UGV given a specific observation exchange protocol and the interaction topology G_l. Note that in general $\mathcal{N}_i(G_l) \subseteq \mathcal{Q}_i(G_l)$.

B. Latest-In-and-Full-Out (LIFO) Protocol

This study proposes a Latest-In-and-Full-Out (LIFO) protocol for observation exchange. Under LIFO, each UGV

[1]An undirected graph $G = (V, E)$ is *simple* if it has no self-loops or repeated edges, i.e., $(i, j) \in E$, only if $i \neq j$ and E only contains distinct elements. A graph is *connected* when there is a path between every pair of vertices in V.

contains a communication buffer (CB) to store its latest knowledge of observations of all UGVs:

$$\mathbf{z}_k^{CB,i} = \left[z^1_{k^i_1}, \ldots, z^N_{k^i_N} \right]$$

where $z^j_{k^i_j}$ represents the observation made by j^{th} UGV at time k^i_j. Note that under LIFO and certain conditions of interaction topologies, $\mathcal{Q}_i = \{1, \ldots, N\} \setminus \{i\}$, which will be proved in Corollary 1. $z^j_{k^i_j}$ is stored in the CB of i^{th} UGV, where k^i_j is the latest observation time of j^{th} UGV available to i^{th} UGV by time k. Due to the communication delay, $k^i_j < k, \forall j \neq i$ and $k^i_i = k$ always holds. Let $G[k] \in \bar{G}$ represent the interaction topology at time k. The **LIFO protocol** is stated in Algorithm 1. For the clarity of explanation of DBF in Section III, we define a *new observation set* $\mathbf{z}_k^{new,i}$ for i^{th} UGV to denote the set of observations that the i^{th} UGV receives and stores in its CB at k.

Algorithm 1 LIFO Protocol

(1) Initialization: The CB of i^{th} UGV is initialized when $k = 0$,

$$z^j_{k^i_j} = \varnothing,\ k^i_j = 0,\ j = 1, \ldots, N$$

(2) At k^{th} step for i^{th} UGV:

(2.1) Receiving Step:

The i^{th} UGV receives all CBs of its direct neighborhood $\mathcal{N}_i(G[k-1])$. The received CBs are totally $|\mathcal{N}_i(G[k-1])|$ groups, each of which corresponds to the $(k-1)^{\text{th}}$ step CB of a UGV in $\mathcal{N}_i(G[k-1])$. The received CB from l^{th} ($l \in \mathcal{N}_i(G[k-1])$) UGV is denoted as

$$\mathbf{z}_{k-1}^{CB,l} = \left[z^1_{(k-1)^l_1}, \ldots, z^N_{(k-1)^l_N} \right],\ l \in \mathcal{N}_i(G[k-1])$$

(2.2) Observation Step:

The i^{th} UGV updates $z^j_{k^i_j}$ ($j = i$) by its own observation at current step.

$$\text{add } z^i_k \text{ to } \mathbf{z}_k^{new,i},$$
$$z^j_{k^i_j} = z^i_k,\ k^i_j = k,\ \text{if } j = i.$$

(2.3) Comparison Step:

The i^{th} UGV updates other elements of its own CB, i.e., $z^j_{k^i_j}$ ($j \neq i$), by selecting the latest information among all received CBs from $\mathcal{N}_i(G[k-1])$. For all $j \neq i$,

$$l_{\text{latest}} = \underset{l \in \mathcal{N}_i,\, i}{\operatorname{argmax}} \left\{ (k-1)^i_j, (k-1)^l_j \right\}$$

If $l_{\text{latest}} > (k-1)^i_j$, add $z^i_{(k-1)^{l_{\text{latest}}}_j}$ to $\mathbf{z}_k^{new,i}$.

$$z^j_{k^i_j} = z^j_{(k-1)^{l_{\text{latest}}}_j},\ k^i_j = (k-1)^{l_{\text{latest}}}_j.$$

(2.4) Sending Step:

The i^{th} UGV broadcasts its updated CB to all of its neighbors defined in $\mathcal{N}_i(G[k])$.

(3) $k \leftarrow k + 1$ until stop

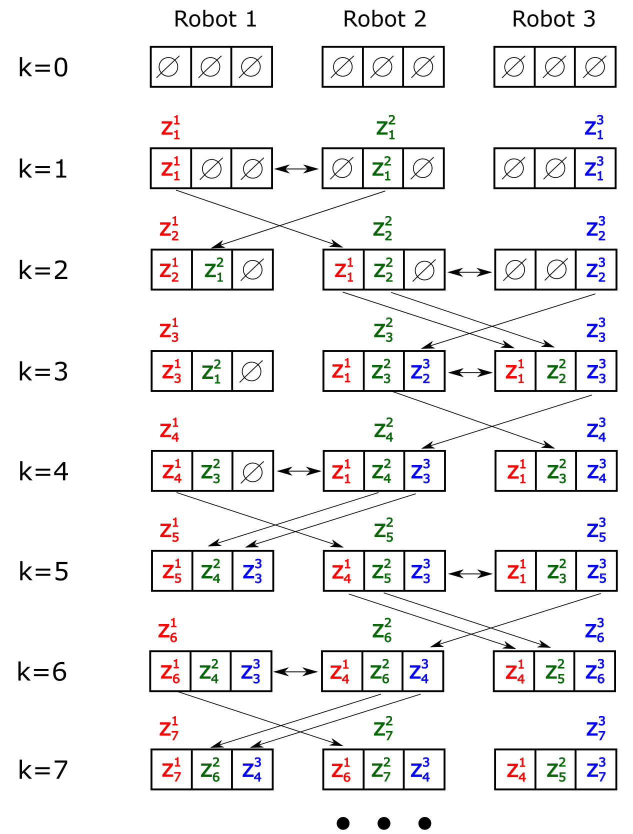


Fig. 2: Example of LIFO with three UGVs using switching line interaction topologies. The hollow arrow represents a communication link between two UGVs.(**TODO:** change 'robot' to UGV; add topology for k=0 and 7.)

Remark 1: Compared to statistics dissemination, LIFO is generally more communication-efficient for distributed filtering. To be specific, consider an $D \times D$ grid environment with a network of N UGVs, the transmitted data of LIFO between each pair of UGVs are only the CB of each UGV and the corresponding UGV positions where observations were made, the length of which is $O(N)$. On the contrary, the length of transmitted data for a statistics dissemination approach that transmits unparameterized posterior distributions or likelihood functions is $O(D^2)$, which is in the order of environmental size. Since D is generally much larger than N in applications such as target localization, LIFO requires much less communication resources.

Fig. 2 illustrates the LIFO cycles of a network of 3 UGVs with switching line topologies. There are two types of topologies, the first only UGV 1 and UGV 2 or UGV 2 and UGV 3 can communicate with each other. Several facts can be noticed in Fig. 2: (1) the two topologies are jointly connected within each time intervals $[0, 3)$, $[3, 5)$, $[5, 7)$. (2) CBs of all UGVs are filled within 5 steps. (3) after being filled, each CB keeps updated every finite time steps, which means each UGV receives new observations of other UGVs with finite delay. Extending the these facts to a network of N UGVs, we have the following proposition:

Proposition 1: Consider a network of N UGVs with undirected switching interaction topologies. If the following two conditions are satisfied: (1) there exists an infinite sequence of time intervals $[k_m, k_{m+1})$, $m = 1, 2, \ldots$, starting at $k_1 = 0$ and are contiguous, nonempty and uniformly bounded; (2) the union of graphs across each such interval is jointly connected, then arbitrary pair of UGVs can exchange observations under LIFO. In addition, the delay between the pair of UGVs is no greater than $(N-1)T_u$, where $T_u = \sup_{m=1,2,\ldots} (k_{m+1} - k_m)T$ is the upper bound of interval lengths.

Proof: Consider the transmission between two arbitrary UGVs, i and j. Since the union of graphs across time interval $[k_1, k_2)$ is jointly connected, i^{th} UGV can directly communicate with at least one another UGV at a time instance, i.e., $\exists l_1 \in V$, $t_1 \in [k_1, k_2)$ such that $i \in \mathcal{N}_{l_1}(G[t_1])$. This implies that observation $z^i_{t_1}$ is received and stored in the CB of l_1^{th} UGV at t_1+1 under LIFO. Therefore, at least one UGV other than i^{th} UGV has received and received observation from i^{th} UGV by k_2. If $l_1 = j$, then we have proved the exchange of observations between i and j. If $l_1 \neq j$, we consider time interval $[k_2, k_3)$. By using similar derivation as before, it is easy to understand that $\exists l_2 \in V$, $t_2 \in [k_2, k_3)$ such that $i \in \mathcal{N}_{l_2}(G[t_2])$ or $l_1 \in \mathcal{N}_{l_2}(G[t_2])$. For the former case, $z^i_{t_2}$ is received and stored in the CB of l_2^{th} UGV at $t_2 + 1$ under LIFO; for the latter case, $z^i_{t_1}$ is received by l_2^{th} UGV at t_2+1 but may not be stored in its CB. This happens if l_2^{th} UGV has received a newer observation $z^i_{t'_2}$, $t_1 < t'_2 < t_2$, from UGVs other than l_1. In both cases, at least two UGVs have received and stored an observation from i^{th} UGV by k_3. Using similar derivation, it can be shown that all $N-1$ UGVs, except i^{th} UGV, will receive and store an observation from i no later by k_N. Therefore, the transmission delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$. ■

Corollary 1: With the same network condition in Proposition 1, all elements in $\mathbf{z}^{CB,i}_k$ under LIFO become filled within finite time, i.e., $\mathcal{Q}_i = \{1, \ldots, N\} \setminus \{i\}$. Additionally, each element keeps updating every finite period of time.

Proof: According to Proposition 1, the transmission delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$. Therefore, CBs of all UGVs becomes filled when $k \geq (N-1)T_u$. In addition, each element in CBs gets updated every finite period of time that is no greater than $(N-1)T_u$. ■

III. Distributed Bayesian Filter via Latest-In-and-Full-Out Protocol

A. Probabilistic Model of Binary Sensor

In this study, each UGV is equipped with a binary sensor, which only gives two types of observation: 1 if the target is detected, and 0 if no target is detected. The observation of i^{th} sensor at k^{th} time step is denoted as z^i_k. The likelihood function that the target is detected is

$$P(z^i_k = 1|x^T; x^{R,i}) \in [0,1]\,,\ x^T \in S, \tag{1}$$

where x^T denotes the target position; $x^{R,i}$ is the UGV position. Correspondingly, the likelihood function that no target is detected is

$$P(z^i_k = 0|x^T; x^{R,i}) = 1 - P(z^i_k = 1|x^T; x^{R,i}). \tag{2}$$

The combination of Eq. (1) and Eq. (2) forms a binary sensor model parameterized by x^T and $x^{R,i}$. For the purpose of simplicity, we will not explicitly write $x^{R,i}$ when no confusion may occur. The commonly used likelihood functions for binary sensor include Gaussian function [24] and step function [25].

Remark 2: Given the knowledge of current target and UGV positions, current observation of each UGV is conditionally independent from its own past observations and those of other UGVs.

B. Distributed Bayesian Filter for Multiple UGVs

The distributed Bayesian filter (DBF) using LIFO protocol is introduced in this section. Each UGV has its individual estimation of posterior density function (PDF) of target position, called *individual PDF*. The individual PDF of i^{th} UGV at time k is defined as $P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k})$, where $\mathbf{z}^{new,i}_{1:k}$ denotes the collection of new observation set by i^{th} UGV from time 1 to k. The individual PDF is initialized as $P^i_{pdf}(x^T|\mathbf{z}^{new,i}_0) = P(x^T)$, given all available prior information including past experience and environmental knowledge. Under the framework of DBF, the individual PDF is recursively estimated using Bayes' formula, based on observations of i^{th} UGV and that of UGVs in $\mathcal{Q}_i$.

To be specific, at time k, the i^{th} individual PDF is updated using the set of newly received observations $\mathbf{z}^{new,i}_k$:

$$\begin{aligned} P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k}) &= K_i P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k-1}) P(\mathbf{z}^{new,i}_k|x^T) \\ &= K_i P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k-1}) \prod_{z^j_{k^i_j} \in \mathbf{z}^{new,i}_k} P(z^j_{k^i_j}|x^T) dx^T. \end{aligned} \tag{3}$$

where K_i is a normalization factor, given by

$$K_i = 1 \Big/ \int P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k-1}) \prod_{z^j_{k^i_j} \in \mathbf{z}^{new,i}_k} P(z^j_{k^i_j}|x^T) dx^T,$$

and $P^i_{pdf}(x^T_k|\mathbf{z}^{new,i}_{1:k})$ is called posterior individual PDF; $P(z^j_{k^i_j}|x^T_k)$ is the likelihood function of observation $z^j_{k^i_j}$, described in Eq. (1) and Eq. (2). Note that the factorization of $P(\mathbf{z}^{new,i}_k|x^T)$ in Eq. (3) results from the conditional independence of observations by different UGVs given the position of the target.

C. Proof of Consistency and Consensus

This section presents the main result of this study that LIFO-DBF achieves consistent and consensual estimation of target position provided that the union of interaction topologies across some time intervals are jointly connected frequently enough as the system evolves. To be specific, considering S is finite and x^{T^*} is the true position of target, the consistency of LIFO-DBF for static UGVs is stated as follows:

Theorem 1: Considering a network of N static UGVs with the interaction topology condition in proposition 1, the estimated target position converges to the true position of target in probability using LIFO-DBF, i.e.,

$$\lim_{k\to\infty} P(x^T = x^{T^*}|\mathbf{z}_{1:k}^{new,i}) = 1,\ i = 1,\dots,N.$$

Proof: For the purpose of clarity, define time sets of i^{th} UGV, $\mathscr{K}_{j,k}^i$, $j \in \{1,\dots,N\}$, that contain time steps of observations by j^{th} UGV that are contained in $\mathbf{z}_{1:k}^{new,i}$. It is known from Corollary 1 that the cardinality of $\mathscr{K}_{j,k}^i$ has following property: $k-(N-1)T_u < |\mathscr{K}_{j,k}^i| \le k$. Considering the conditional independence of observations given $x^T \in S$, the batch form of DBF at k^{th} step is

$$P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) = \frac{P_{pdf}^i(x^T)\prod_{j=1}^N \prod_{l\in\mathscr{K}_{j,k}^i} P(z_l^j|x^T)}{\sum_{x^T\in S} P_{pdf}^i(x^T)\prod_{j=1}^N \prod_{l\in\mathscr{K}_{j,k}^i} P(z_l^j|x^T)}, \quad (4)$$

where P_{pdf}^i is the initial individual PDF of i^{th} UGV. Comparing $P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})$ with $P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})$ yields

$$\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})} = \frac{P_{pdf}^i(x^T)\prod_{j=1}^N \prod_{l\in\mathscr{K}_{j,k}^i} P(z_l^j|x^T)}{P_{pdf}^i(x^{T^*})\prod_{j=1}^N \prod_{l\in\mathscr{K}_{j,k}^i} P(z_l^j|x^{T^*})}. \quad (5)$$

Take the logarithm of Eq. (5) and average it over k steps:

$$\frac{1}{k}\ln\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})} = \frac{1}{k}\ln\frac{P_{pdf}^i(x^T)}{P_{pdf}^i(x^{T^*})} + \sum_{j=1}^N \frac{1}{k}\sum_{l\in\mathscr{K}_{j,k}^i} \ln\frac{P(z_l^j|x^T)}{P(z_l^j|x^{T^*})}. \quad (6)$$

Since $P_{pdf}^i(x^T)$ and $P_{pdf}^i(x^{T^*})$ are bounded, then

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P_{pdf}^i(x^T)}{P_{pdf}^i(x^{T^*})} = 0. \quad (7)$$

The binary observations subject to Bernoulli distribution $B(1,p_j)$, yielding

$$P(z_l^j|x^T) = p_j^{z_l^j}(1-p_j)^{1-z_l^j},$$

where $p_j = P(z_l^j = 1|x^T)$. Utilizing the facts: (1) z_l^j are conditionally independent samples from $B(1,p_j^*)$ and (2) $k-(N-1)T_u < |\mathscr{K}_{j,k}^i| \le k$, the law of large numbers yields

$$\frac{1}{k}\sum_{l\in\mathscr{K}_{j,k}^i} z_l^j \xrightarrow{P} p_j^*, \quad \frac{1}{k}(|\mathscr{K}_{j,k}^i| - \sum_{l\in\mathscr{K}_{j,k}^i} z_l^j) \xrightarrow{P} 1-p_j^*,$$

where $p_j^* = P(z_l^j = 1|x^{T^*})$ and "$\xrightarrow{P}$" denotes "convergence in probability". Then,

$$\frac{1}{k}\sum_{l\in\mathscr{K}_{j,k}^i}\ln\frac{P(z_l^j|x^T)}{P(z_l^j|x^{T^*})} \xrightarrow{P} p_j^*\ln\frac{p_j}{p_j^*} + (1-p_j^*)\ln\frac{1-p_j}{1-p_j^*}. \quad (8)$$

Note that the right-hand side of Eq. (8) achieves maximum value 0 if and only if $p_j = p_j^*$. Define

$$c(x^T) = \sum_{j=1}^N p_j^*\ln\frac{p_j}{p_j^*} + (1-p_j^*)\ln\frac{1-p_j}{1-p_j^*}.$$

Considering Eq. (7) and Eq. (8), the limit of Eq. (6) is

$$\frac{1}{k}\ln\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})} \xrightarrow{P} c(x^T). \quad (9)$$

It follows from Eq. (9) that

$$\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})e^{c(x^T)k}} \xrightarrow{P} 1. \quad (10)$$

Define the set $\bar{X}^T = S\setminus\{x^{T^*}\}$ and $c_M = \max_{x^T\in\bar{X}^T} c(x^T)$. Then $c_M < 0$. Summing Eq. (10) over $\bar{X}^T$ yields

$$\frac{\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})e^{[c_M - c(x^T)]k}}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})e^{c_M k}} \xrightarrow{P} |\bar{X}^T|, \quad (11)$$

where $|\bar{X}^T|$ denotes the cardinality of $\bar{X}^T$. Since $c_M < 0$, $P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})e^{c_M k} \longrightarrow 0$, Eq. (11) implies

$$\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})e^{[c_M - c(x^T)]k} \xrightarrow{P} 0. \quad (12)$$

Utilizing the relation

$$0 \le P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) \le P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})e^{[c_M - c(x^T)]k},$$

it can be derived from Eq. (12) that

$$\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) \xrightarrow{P} 0.$$

Therefore,

$$\lim_{k\to\infty} P(x^T = x^{T^*}|\mathbf{z}_{1:k}^{new,i}) = 1 - \lim_{k\to\infty}\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) = 1.$$

■

Remark 3: This paper can guarantee both consistency and consensus of individual PDF. The reason is that all individual PDFs converge to the same distribution, thus the consensus is also achieved. Different from this study, traditional statistics dissemination-based methods only ensure consensus of individual PDFs [6], [7]. To the best knowledge of authors, there is no proof of consistency on estimated target position.

IV. Simulation

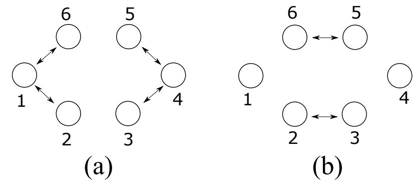
This section simulates two sets of dynamically changing interaction topologies to demonstrate the effectiveness of LIFO-DBF. In both cases, six static UGVs, represented as one square and five stars in Fig. 3 and Fig. 4, are utilized and each UGV is equipped with a binary sensor. The square represents the UGV, whose individual PDF is shown in the figure. All sensors are modeled with identical Gaussian functions [24]:

$$P(z_k^i = 1|x_k^T; x_k^{R,i}) = e^{-\frac{1}{2}(x_k^T - x_k^{R,i})^T\Sigma^{-1}(x_k^T - x_k^{R,i})}, \quad (13a)$$

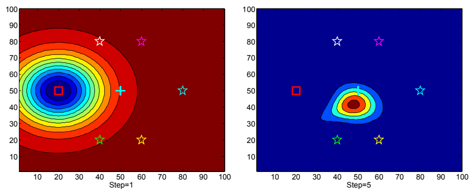
$$P(z_k^i = 0|x_k^T; x_k^{R,i}) = 1 - P(z_k^i = 1|x_k^T; x_k^{R,i}). \quad (13b)$$

Fig. 3a and Fig. 4a illustrates the collections of changing interaction topologies for the first and second scenario, respectively, one with two topologies and the other with three topologies. The union of both collections are designed to be jointly connected. In both scenarios, topologies appear alternatively such that they can be said to be connected frequently enough.

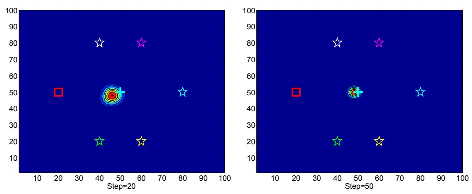
The LIFO-DBF is implemented on each UGV for target localization. After the initial observation, each UGV forms a circular individual PDF, centered at its own position. The circular PDF happens because the Gaussian sensor model



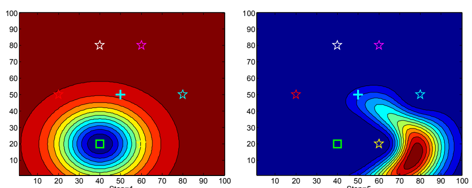
(a) Collection of changing topologies



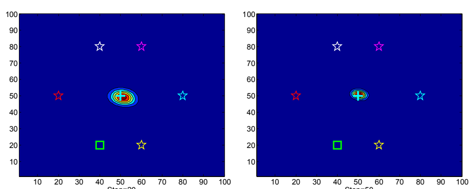
(b) Step 1 (c) Step 5



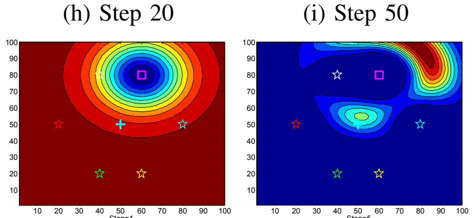
(d) Step 20 (e) Step 50



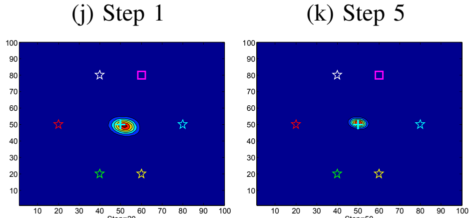
(f) Step 1 (g) Step 5



(h) Step 20 (i) Step 50

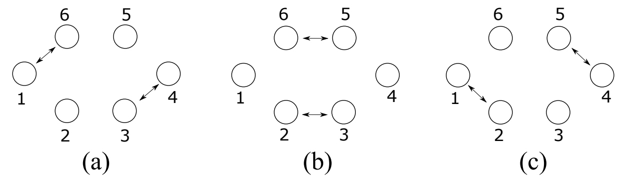


(j) Step 1 (k) Step 5

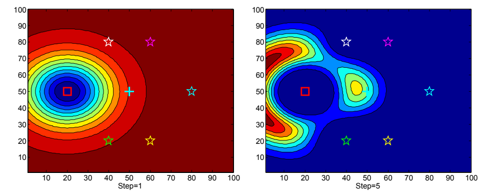


(l) Step 20 (m) Step 50

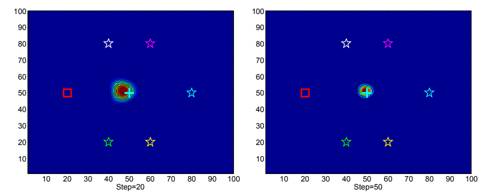
Fig. 3: Individual PDFs for two changing interaction topologies: (b)-(e) The 1st UGV's individual PDFs at different times; (f)-(i) The 2nd UGV's individual PDFs; (j)-(m) The 5nd UGV's individual PDFs. The cross stands for the target.(**TODO:** four plots combine into one single plot. PDF evolution)



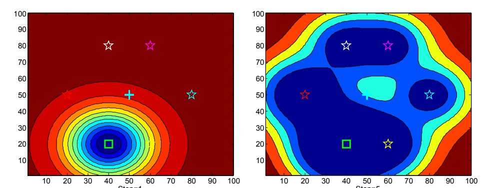
(a) Collection of changing topologies



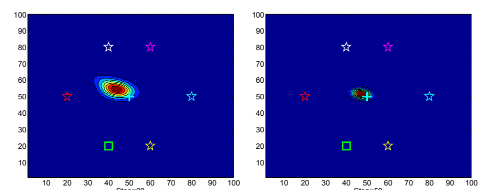
(b) Step 1 (c) Step 5



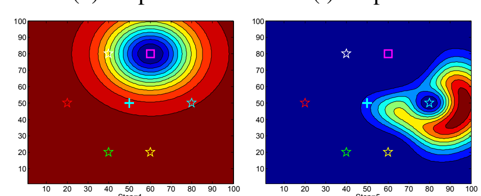
(d) Step 20 (e) Step 50



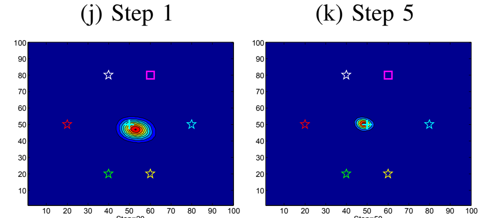
(f) Step 1 (g) Step 5



(h) Step 20 (i) Step 50



(j) Step 1 (k) Step 5



(l) Step 20 (m) Step 50

Fig. 4: Individual PDFs for three switching interaction topologies: (b)-(e) The 1st UGV's individual PDFs; (f)-(i) The 2nd UGV's individual PDFs; (j)-(m) The 5nd UGV's individual PDFs.

(Eq. (13)) only depends on the distance between UGV and target. As more observations are received by each UGV, the posterior individual PDF concentrates to the true location of the target, which accords with the consistency of LIFO-DBF.

V. Conclusion

This paper presents a distributed Bayesian filtering (DBF) method for a network of multiple unmanned ground vehicles (UGVs) under dynamically changing interaction topologies. The information exchange among UGVs relies on a measurement dissemination scheme, called Latest-In-and-Full-Out (LIFO) protocol. Different from statistics dissemination approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only sends a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs from the order of environmental size to that of UGV number. Under the condition that the union of undirected switching topologies is connected frequently enough, LIFO can disseminate observations over the network within finite time. The LIFO-based DBF algorithm is then derived to estimate individual posterior density function (PDF) for target localization in a static environment. The consistency of this algorithm is proved by utilizing the law of large numbers, ensuring that each individual estimate of target position converges in probability to the true value when the number of observations tends to infinity.

Future work includes how to handle other types of sensors and directed interaction topologies. Other types of sensors may have biased observations and subject to non-Bernoulli distribution, which complicates the design and analysis of LIFO-based Bayesian filters. The directed interaction topologies, due to the constraint of unidirectional communication, may affect condition for the consistency and consensus of LIFO-DBF.

Acknowledgment

The authors gratefully acknowledges the Office of Naval Research for supporting the research described in this paper. They would also like to thank Yuting Wei in the Department of Statistics, UC Berkeley for her sincere help and fruitful discussion on the consistency proof.

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