Distributed Bayesian Filter in Multi-Robot Network by using Local-Exchange-of-Observation Strategy

Albert Author[1] and Bernard D. Researcher[2]

Abstract— **This electronic document is a live template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document.**

I. INTRODUCTION

Distributed filtering in multi-robot network that focuses on utilizing a group of networked robots to collectively infer the state of an environment has been used for various applications, such as object detection (Chamberland), target tracking (Beaudeau) and environmental monitoring etc. [need to mention target search somewhere]

The communication topology of multi-robot network plays a vital role in distributed filtering algorithms. There have been a number of works on distributed filtering using fusion center-based topology that local information is transmitted (via multihopping) to the fusion center for forming global estimation. For example, in (L. Zuo) each robot sensor transmits local estimation of environment state to the fusion center, where the Best Linear Unbiased Estimator is utilized forming the global estimation of the environment. Ribero et.al. (A) studied deterministic mean-location parameter estimation using quantized sensor observations. At the fusion center, maximum-likelihood estimators were applied to the quantized observations transmitted from sensors for achieving a parameter estimation with variance close to that of the clairvoyant sample mean estimator.

More recently, many works focus on the distributed filtering using the neighborhood-based communication topology that each robot can only exchange information with neighboring robots. For example, Sheng et. al. (S.) proposed distributed particle filters with Gaussian Mixer to localize and track multiple moving targets in a wireless sensor network. The distributed particle filters run on a set of uncorrelated sensor cliques and information among cliques is exchanged for global estimation of targets. In (Ram.S), a circular topology that each sensor can only communicate with a fixed neighboring sensor is deployed for parameter estimation of a spatial field, using the incremental Robbins-Monro gradient algorithm locally at each sensor.

Consensus algorithms, as proposed in (Olfat-Saber) (Ren.) (Jadbabie.) have become a popular distributed filtering approach for robot teams with neighborhood-based communication topology. As far, the commonly used consensus strategy is based on the statistics dissemination (Hlinka), which actually exchanges posterior distribution or likelihood functions to neighboring robots for distributed estimation. For example, Saptarshi et. al. (Bandyopadhyay) presented a Bayesian consensus filter (BCF) that uses logarithmic opinion pool for fusing posterior distributions among neighboring robots. The proposed BCF can incorporate non-Gaussian uncertainties and nonlinearity in target dynamic models and measurement models. A consensus-based distributed particle filter (DPF) is proposed by Julian et. al. (Julian) for estimating environmental state by exchanging and fusing posterior functions of the state among neighbors. The DPF can work even when the network diameter, the maximum in/out degree, and the number of robots are unknown.

Despite the popularity of these distributed filtering approaches, exchanging posterior distribution or likelihood functions can consume high communication resources, which may be infeasible in many applications in vast area or complex environment, such as marine search, seismological rescue, etc. Different from the statistics dissemination-based approaches, this study focuses on the strategy of exchanging observation in the neighborhood of each robot for the purpose of achieving a consensus of local target PDFs. The main benefit of this strategy is the reduction of communication burden, the data volume of which scales linearly with the robot number while statistics dissemination-based strategies suffer from the data volume on the order of the environmental size.

Several works have focused on the dissemination of observation for distributed filtering, in which raw or quantized observations are exchanged among robots for environmental state estimation. For example, Coates et. al. (Coates) proposed a distributed particle filter that uses adaptive encoding of the observations to minimize communication overhead. In (Djuric), each robot communicate the observation of the tracked targets to all the remaining agents and apply local particle filter for tracking targets. In both works, communication topology is assumed fully connected that each robot broadcasts observations to all robots in single transmission step.

In this work, we propose a local-exchange-of-observation (LEO) strategy for distributed Bayesian filters (DBF). Each robot broadcasts its own observation and previously received neighboring robots? observations to all neighbors. After receiving observation from neighbors, each robot runs Bayesian filter locally for environmental state estimation.

*This work was not supported by any organization

[1]Albert Author is with Faculty of Electrical Engineering, Mathematics and Computer Science, University of Twente, 7500 AE Enschede, The Netherlands albert.author@papercept.net

[2]Bernard D. Researcheris with the Department of Electrical Engineering, Wright State University, Dayton, OH 45435, USA b.d.researcher@ieee.org

The proposed LEO-DBF has following properties: (1) LEO-DBF reduces the communication burden for multi-robot network compared to statistics dissemination-based distributed filtering approaches (2) LEO-DBF can ensure consistency of the state estimate, thus guaranteeing consensus.

The consistency property, referring to the agreement between the state estimates and the true state, differentiates LEO-DBF from the statistics dissemination-based filtering approaches. In fact, the statistics dissemination-based methods can ensure the convergence of the state estimate among robots. However, there?s no guarantee whether the agreed quantity is close to the true value. In this study, a formal proof of the consistency of the proposed LEO-based DBF is provided, ensuring the agreement between robots? state estimate and true environmental state.

II. PROBLEM FORMULATION

Consider a network of N robots in bounded two-dimensional space S. Each robot is equipped with a binary sensor for observing the environment state. Due to the limit of communication range and bandwidth, each robot can only exchange observations with its neighboring robots using the local-exchange-of-observation strategy. Distributed Bayesian filter is run locally on each robot for environmental state estimation based on own and received observations.

A. Probabilistic Model of Binary Sensor

The sensor model is subject to a Gaussian distribution.

B. Bayesian Filtering for Multiple Sensors

The binary sensor only gives two types of observation: 1 if the target is detected, and 0 if no target is detected. For the sensor of i-th robot, the observation at k-th time step is denoted as . The observation likelihood function is used to describe the probability that the target is detected:

(1) where is the target position. Correspondingly, the probability that no target is detected is

(2) The commonly used observation likelihood functions for binary sensor are Gaussian function (Bonnie) (Liu) and step function (Djuric P.). In addition, it should be noted that, in spite of the usage of binary sensors, the observation exchange strategy proposed in this study also works for other types of sensors, including range sensors, such as Sonars (Coraluppi), laser scanners (D) and cameras (J.).

Remark (1): The current observation of each robot is conditionally independent from both its own past observations and those of other robots, given the current state of the target. Remark (2): In this study, both homogenous and heterogeneous models of binary sensor are applicable. The definition of heterogeneity is in the sense of observation likelihood function, which means that the observation spaces are the same, but the observation probability can differ. The homogenous assumption can simplify the analysis of completeness, while the heterogeneous models is more close to real sensing characteristics.

1) Graphical model of communication topology: Consider an undirected and connected graph , where is the set of nodes (i.e., robots) and is the edge set of nodes. The adjacency matrix is used to describe the communication topology of the robot network: , where denotes the entity of adjacency matrix. The notation indicates there is a communication link between i-th and j-th robot and indicates no communication. The direct neighborhood of i-th robot is defined as . All the robots in can directly exchange information with the i-th robot. In addition to the direct neighborhood, another neighborhood, called the available neighborhood, is defined as , which contains robots whose observations is received by the i-th robot through single-hopping and multi-hopping. Note that if only single-hopping is allowed, then is equal to . Figure 1 illustrates three types of common communication topologies: circular, serial, and star. Remarks (3): The communication topology in this study is connected, which means that there exists a communication path between any two robots.

C. Distributed Bayesian Filter for Multiple Robots

The generic Distributed Bayesian Filter (DBF) is introduced in this section, which was also stated in (O, Distributed Sequential Estimation in Asynchronous Wireless Sensor Networks) and (T). Each robot has its individual estimation of the target PDF, called individual PDF. The i-th individual PDF at time k is defined as . Before starting DBF, the individual PDF is initialized by the prior function , given all available prior information including past experience and domain knowledge. Then the individual PDF is recursively estimated by two steps, i.e., prediction step and updating step, based on the observations of the robots in .

1) Prediction: The i-th individual PDF at time k-1 is known, denoted as . At time k, the prior individual PDF is first predicted forward by using the Chapman-Kolmogorov equation:

(3) where is a Markov motion model of the target, independent of robot states. This model describes the state transition probability of the target from the prior state to the posterior state . Note that the target can be static in many searching applications, for example, [] and []. For a static target, its motion model is simplified to be

and Eq. (3) can be reduced to

2) Updating: At time step k, the i-th individual PDF is then updated by Bayes? formula considering the latest available observation at time k:

(4) where is a normalization factor, given by: . where is called posterior individual PDF, is the observation likelihood of for a given target , expressed in Eq. (1) and (2).

III. DISTRIBUTED BAYESIAN FILTER VIA LOCAL EXCHANGE OF OBSERVATION

This study proposes a Distributed Bayesian Filter (DBF) algorithm based on local exchange of observation (LEO) strategy. The LEO still uses the local communication within neighborhood of each robot, but allows the broadcast of the observation of each robot to any other nodes by multi-hopping along the shortest path in the communication network. The theoretical analysis show that LEO-based DBF can ensure the consistency of individual estimation (consistency also implies consensus) while requiring much less communication burden than the statistics-dissemination based DBF.

3) Algorithm for Local Exchange of observations (LEO): Under the LEO, each robot contains a communication buffer (CB) to store its latest knowledge of the observations of all robots:

where represents the observation made by j-th robot at time . At time k, is received and stored in i-th robot CB, in which is the latest available step of j-th robot known by the i-th robot. Due to the communication delay of multi-hopping, always holds in practice.

The LEO algorithm is stated as below:

(1) Initialization: The buffer of i-th robot is initialized when k=0:

(2) Repeat the following steps for i-th robot until stop (2.1) Receiving Step: The i-th robot receives all CBs of its neighboring robots. The received CBs are totally groups, each of which is actually the (k-1)-step CB of a robot in . The received CB from l-th ( ) robot is denoted as

(2.2) Observation Step: The i-th robot updates by its personal observation at current step: . (2.3) Comparison Step: The i-th robot updates other elements of its own CB, i.e., , by selecting the latest information among all received CBs from . For all ,

(5) (2.4) Sending Step: The i-th robot broadcasts its updated CB to all of its neighbors defined in . (End of LEO)

Figure 2 illustrates the LEO algorithm with 3 robots using linear topology. Two facts can be observed in Figure 2: (1) the CBs are filled within 3 steps, which means LEO has a maximum delay of 2 steps; (2) after filled, the updating of CBs are non-intermittent, which means each robot continuously receives newer observations of other robots. Extending the two facts to a network of N robots, we have the following proposition:

Proposition 1: For an undirected and connected network of N robots with fixed communication topology, the latest observation of an arbitrary robot i can be received and stored in another robot j?s CB under LEO only via the shortest paths between i and j, with the delay equivalent to the length of shortest paths.

Proof: Let denote a loopless path between two arbitrary robots, the i-th and j-th robot, with and all ?s being unique. The path length is n-1. For the purpose of simplicity, we assume there?s only one shortest path between i and j and let be the shortest path, with . Define the distance between two robots, , as the length of the shortest path between the two robots. Then .

Next, we will prove the following shortest-path transmission statement by the mathematical induction:

Let be the j-th robot?s observation at time k. Then i-th robot can only receive and store in its CB via the shortest path at time .

Step (1): For , this means and j can directly send to i. Since is the latest observation that i-th robot can receive, is stored in i-th CB at time k+1. transmitted via other paths with length greater than 1 arrives at i later than k+1, thus getting discarded by i-th robot, since it has already received at k+1. The delay . Since i and j are arbitrary robots with distance equal to 1, the shortest-path transmission statement holds for all pairs of robots with distance equal to 1 in the network.

Step (2): Suppose the shortest-path transmission statement holds for . Then for , by the Bellman?s principle of optimality, the path is the shortest path between j and l, where l is the ( )-th robot on the path and i is a neighboring robot of l. The assumption that the shortest-path transmission statement holds for implies that is received and stored in l-th robot?s CB at time . Since , i-th robot receives at . Therefore . For any other path with , cannot be received by i-th robot at . This proves the shortest-path transmission statement for .

Based on the principle of mathematical induction, the shortest-path transmission statement holds. (End of proof)

Corollary 1: For the same topology assumption in Proposition 1, all elements of under LEO become filled when .

Proof: Based on proposition 1, the delay between any pair of two robots is the length of the shortest path connecting these two robots. In a network of N robots, the maximal length of shortest paths is no greater than N-1 []. Thus and all elements of becomes filled when . (End of proof)

Corollary 2: For the same topology in Proposition 1, once is filled, the updating of each element in is non-intermittent, i.e., is a constant.

Proof: For a connected network with fixed topology, the shortest path between any nodes is fixed. Based on Proposition 1, is constant. (End of proof)

Remarks (4): Compared to statistics dissemination, LEO is a more transmission-efficient approach for the DBF consensus. To be specific, consider a grid environment with a network of robots, the transmitted data of LEO between each pair of robots are only the CB of each robot, the length of which is . On the contrary, the length of transmitted data for a statistics-dissemination approach is . Since is generally much larger than in robot search community, LEO requires much less transmission sources than the statistics-dissemination approaches.

Remarks (5): This study only discusses a network of fixed topology, which has been commonly used in multi-robot search, e.g., (T) and (Bandyopadhyay). The switching topology can leads to unpredictable delay and intermittent transmission, thus significantly complicating the analysis of the consensus of individual PDFs. Intuitively, a consensus is till achievable in practice if enough observations are transmitted between any two topological switches for slowly changing topology, or if a certain percentage of observations in probability from other robots can be always received for fast changing topology.

A. Algorithm for LEO-based DBF for a Static Target

This section derives the DBF for a static target. The robot store last step individual PDF, i.e., (k-1)-th step. The assumption of static target can simplify the Bayesian filter, in which the prediction step is unnecessary. Therefore, the i-th individual PDF is only updated by

B. Algorithm for LEO-DBF for a Moving target

This section derives the DBF for a moving target. Instead of storing last step PDF , the robot maintains (k-N)-th individual PDF, and a triangular matrix of history observations from (k-N+1)-th to k-th steps. The i-th individual PDF is alternatively predicted and updated by using aforementioned Bayesian filter (Eq. (3) and (4)) from (k-N)-th individual PDF to k-th step.

Without loss of generality, assume and let . For the i-th robot at k-th step: 1) The stored individual PDF for (k-N)-th step is 2) Initialize a virtual PDF by assigning the assigned PDF to it:

3) From to N, repeat two steps of Bayesian filtering (a) Prediction

(b) Updating

4) Store (k-N+1)-th individual PDF to be first-step virtual PDF

The target PDF estimation of i-th robot at current step is .

Remarks (6): For the static target, each robot only needs current step CB to update individual PDFs. Except storing individual PDFs, all historic CBs can be discarded and only current step CB is stored in robot memory, whose length is . On the contrary, for the moving target, each robot needs to store a triangular matrix of history observation (except current step CB) with N-by-N dimension and a M-by-M virtual PDF, which means that the length of occupied memory in each robot is $O(M^2 + N^2)$.

IV. Proof of Consistency and Consensus

This section presents a consistency and consensus proof of LEO-based DBF. Only the scenarios for the static target are presented, including both static robot and moving robot. The proof for LEO-based DBF for moving target is similar to that of static target, if we can assume that the target positions are finite (such in a discretized bounded field) and the dynamic model of the target is accurately known.

A. Proof for static robots

Theorem 1 (consistency of LEO-DBF for static robots) Under the condition of a binary sensor model that is parameterized by a finite set of target positions, the point estimate of the target position obtained by LEO-DBF converges to the true target position value (in what sense ?) as the number of observations tends to infinity, i.e.

where denotes the true location of the target and .

Proof: Define as the set of possible target positions. Thus . Considering the conditional independence of observations for given , the batch form of DBF at k-th step is:

(6) where is initial i-th local PDF. It is known from the proposition 1:

(7) The binary observations subject to Bernoulli distribution, yielding

(8) where

Take the logarithm of (6) and average it over the k steps:

(9) where

. Utilizing the fact that (1) are conditionally independent, and (2) , the law of large numbers yields

where . Then, the first term of Eq. (9) has the following limit

(10) Note that the r.h.s of (10) achieves maximum iff . Considering the equality

The third term of Eq. (9) is simplified to

Further, considering the equality

Considering Eq. (10), we have in the condition of when . Then,

Therefore, the limit of Eq. (9) becomes

(11)

It is known from Eq. (11): (1) When , and ; (2) When , and . (End of proof)

B. Proof for moving sensors

Lemma 1: For a finite number of robots within a finite number of possible positions, there exists at least one position for each robot that it visits for infinite times as k goes to infinity.

Proof: Let denote the times that i-th robot visits j-th position up to time k. Then . It is straightforward to see that .

Theorem 2 (consistency of LEO-DBF for moving robots) Consider a finite set of target positions, . Under the condition of binary sensors, the individual PDF given by LEO-DBF will concentrates on the true location of the target after infinitely many observations, i.e.

where denotes the true location of the target and . Proof: Similar to Theorem 1, the batch form of DBF at k-th step

(12) where is initial i-th local PDF. By converting [change this part, may define to show all the (time, position) pairs]

(13) The only difference is that Eq. (7) does not hold, but for each sensor, at least there is one position has infinite observation as , according to lemma 1. We can classify all the positions into finite-observation spots and infinite-observation spots. For the former, it is easy to know that

The corresponding item in has zero-limit. Therefore, the proof of Eq. (13) can be reduced to infinite-observation spot, which is similar to Theorem 1. (End of Proof)

Remarks (7): For single target, the true location is unique. Since all individual PDFs concentrate on the same location, the consensus of individual PDFs is achieved.

V. Simulation

This section presents three scenarios in order to demonstrate the use of the LEO strategy for recursive Bayesian filtering in autonomous target search. In all scenarios, six sensors are utilized for target search. For the purpose of simplicity, a Gaussian binary sensor model is used:

where denotes the sensor position where the observation is made. Figure 1 shows the 1-D illustration of the sensor model.

VI. Simulation

This section presents three scenarios in order to demonstrate the use of the LEO strategy for recursive Bayesian filtering in autonomous target search. In all scenarios, six sensors are utilized for target search, each receiving binary observation following the sensor model in xxx.

The first scenario consists of six static sensors and single static target, which acts as a proof of concept of the LEO strategy for static cases. The second scenario subsequently deals with the six moving sensors for searching the single static target. Finally, a general scenario is presented that contains six moving sensors and one moving target.

The first scenario consists of six static sensors and single static target, which acts as a proof of concept of the LEO strategy for static target case. The second scenario subsequently deals with the six moving sensors for searching the single static target. Finally, a general scenario is presented that contains six moving sensors and one moving target.

Add entropy reduction plot to show the convergence rate.

A. Static Sensors, Static Target

The six static sensors are places at. Each sensor constantly receives binary observations from the target, using the sensor model in xxx. Sensors use LEO strategy to communicate their current and saved observations with their communication neighbors. Recursive Bayesian filtering is conducted for target position estimation.

Figure 1 shows the result of the estimation. After the initial observation, each sensor forms a semicircle of the probability map, centered at the corresponding sensor position. As more observations are received, the posterior probability concentrates on the true location of the target. This demonstrates the effectiveness of the LEO strategy for achieving the consensus on the estimate of the target position.

B. Moving Sensors, Static Target

The six sensors start moving from the positions respectively to estimate the target position. The motion planning of sensors for effective target search has received much attention in the past decade. In this work, the sensor positions are randomly generated at each time in order to demonstrate the effectiveness of the LEO strategy. Readers interested in sensor motion planning can refer to xxx.

Figure 2 shows the result of the estimation. Similar to sec. 5.1, the posterior probability concentrates to the true target location. Figure 2(b) gives the decrease of the entropy of the posterior distribution, showing the reduction of uncertainty in the estimated target position.

C. Moving Sensors, Moving Target

The target in the scenario moves on a horizontal plane and the model is given by. The sensor positions are randomly generated at each time. LEO given in section 4 is utilized for distributed recursive Bayesian filtering. Figure 3 shows the result of the estimation. As expected, the posterior probability concentrates to the true target location.

VII. Conclusion

In this study, we proposed the local-exchange-of-observation (LEO) strategy for distributed Bayesian filter (LEO-DBF) in a multi-robot network. With fixed communication topology, each robot can receive all robots? observations non-intermittently under LEO strategy. Two LEO-DBFs are proposed for estimating the position of the static and the moving target, respectively. For the static target, each robot fuses the latest knowledge of all robots? observations by only considering the updating step of the Bayesian filter. The received observations are then fused locally by each robot using Bayesian filters for state estimation. For the moving target, a triangle matrix of history observations are maintained by each robot. Upon obtaining the latest available observations of all robots, an iterative Bayesian filtering procedure is applied that alternates between prediction and updating steps. The consistency of LEO-DBF is proved, ensuring the agreement between robots? state estimate and true environmental state. Simulations have demonstrated the effectiveness of the LEO-DBF for position estimation of both static and moving target.

TABLE I

An Example of a Table

One	Two
Three	Four

We suggest that you use a text box to insert a graphic (which is ideally a 300 dpi TIFF or EPS file, with all fonts embedded) because, in an document, this method is somewhat more stable than directly inserting a picture.

Fig. 1. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

Appendix

Appendixes should appear before the acknowledgment.

Acknowledgment

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

REFERENCES

[1] G. O. Young, Synthetic structure of industrial plastics (Book style with paper title and editor), in Plastics, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 1564.

[2] W.-K. Chen, Linear Networks and Systems (Book style). Belmont, CA: Wadsworth, 1993, pp. 123135.

[3] H. Poor, An Introduction to Signal Detection and Estimation. New York: Springer-Verlag, 1985, ch. 4.

[4] B. Smith, An approach to graphs of linear forms (Unpublished work style), unpublished.

[5] E. H. Miller, A note on reflector arrays (Periodical styleAccepted for publication), IEEE Trans. Antennas Propagat., to be publised.

[6] J. Wang, Fundamentals of erbium-doped fiber amplifiers arrays (Periodical styleSubmitted for publication), IEEE J. Quantum Electron., submitted for publication.

[7] C. J. Kaufman, Rocky Mountain Research Lab., Boulder, CO, private communication, May 1995.

[8] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, Electron spectroscopy studies on magneto-optical media and plastic substrate interfaces(Translation Journals style), IEEE Transl. J. Magn.Jpn., vol. 2, Aug. 1987, pp. 740741 [Dig. 9th Annu. Conf. Magnetics Japan, 1982, p. 301].

[9] M. Young, The Techincal Writers Handbook. Mill Valley, CA: University Science, 1989.

[10] J. U. Duncombe, Infrared navigationPart I: An assessment of feasibility (Periodical style), IEEE Trans. Electron Devices, vol. ED-11, pp. 3439, Jan. 1959.

[11] S. Chen, B. Mulgrew, and P. M. Grant, A clustering technique for digital communications channel equalization using radial basis function networks, IEEE Trans. Neural Networks, vol. 4, pp. 570578, July 1993.

[12] R. W. Lucky, Automatic equalization for digital communication, Bell Syst. Tech. J., vol. 44, no. 4, pp. 547588, Apr. 1965.

[13] S. P. Bingulac, On the compatibility of adaptive controllers (Published Conference Proceedings style), in Proc. 4th Annu. Allerton Conf. Circuits and Systems Theory, New York, 1994, pp. 816.

[14] G. R. Faulhaber, Design of service systems with priority reservation, in Conf. Rec. 1995 IEEE Int. Conf. Communications, pp. 38.

[15] W. D. Doyle, Magnetization reversal in films with biaxial anisotropy, in 1987 Proc. INTERMAG Conf., pp. 2.2-12.2-6.

[16] G. W. Juette and L. E. Zeffanella, Radio noise currents n short sections on bundle conductors (Presented Conference Paper style), presented at the IEEE Summer power Meeting, Dallas, TX, June 2227, 1990, Paper 90 SM 690-0 PWRS.

[17] J. G. Kreifeldt, An analysis of surface-detected EMG as an amplitude-modulated noise, presented at the 1989 Int. Conf. Medicine and Biological Engineering, Chicago, IL.

[18] J. Williams, Narrow-band analyzer (Thesis or Dissertation style), Ph.D. dissertation, Dept. Elect. Eng., Harvard Univ., Cambridge, MA, 1993.

[19] N. Kawasaki, Parametric study of thermal and chemical nonequilibrium nozzle flow, M.S. thesis, Dept. Electron. Eng., Osaka Univ., Osaka, Japan, 1993.

[20] J. P. Wilkinson, Nonlinear resonant circuit devices (Patent style), U.S. Patent 3 624 12, July 16, 1990.