Distributed Environmental Estiamtion Using A Group of UGVs Under Dynamically Changing Interaction Topologies

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***Abstract*— This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) method for a network of multiple unmanned ground vehicles (UGVs), utilizing the Latest-In-and-Full-Out (LIFO) local exchange protocol of observations. Different from statistics dissemination approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only exchanges with neighboring UGVs a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs from the order of environmental size to that of UGV number. Under the condition of fixed and undirected topology, LIFO can guarantee non-intermittent dissemination of all observations over the network within finite time. Two types of LIFO-based DBF algorithms are then derived to estimate individual posterior density function (PDF) for static and moving target, respectively. For the static target, each UGV locally fuses the newly received observations while for the moving target, a set of historical observations is stored and updated. The consistency of LIFO-based DBF is proved that estimated target position converges in probability to the true target position when the number of observations tends to infinity. The effectiveness of this method is demonstrated by simulations of target localization.**

I. INTRODUCTION

Distributed filtering that focuses on using a group of networked UGVs to collectively infer environment status has been used for various applications, such as intruder detection [1], pedestrian tracking [2] and micro-environmental monitoring [3]. Several techniques have been developed for distributed filtering. For example, Olfati-Saber (2005) proposed a distributed Kalman filter (DKF) for estimating states of linear systems with Gaussian process and measurement noise [4]. Each DKF used additional low-pass and band-pass consensus filters to compute the average of weighted measurements and inverse-covariance matrices. Madhavan et al. (2004) presented a distributed extended Kalman filter for nonlinear systems [5]. This filter was used to generate local terrain maps by using pose estimates to combine elevation gradient and vision-based depth with environmental features. Gu (2007) proposed a distributed particle filter for Markovian target tracking over an undirected sensor network [6]. Gaussian mixture models (GMM) were adopted to approximate the posterior distribution from weighted particles and the parameters of GMM were exchanged via average consensus filter. As a generic filtering scheme for nonlinear systems and arbitrary noise distributions, distributed Bayesian filters (DBF) have received increasing interest during past years [7], [8], which is the focus of this study.

The design of distributed filtering algorithms is closely related to communication topology of multi-UGV network, which can be classified into two types: fusion center (FC)-based and neighborhood (NB)-based. In FC-based approaches, each UGV uses a filter to estimate local statistics of environment status based on its own measurement. The local statistics is then transmitted (possibly via multi-hopping) to a single FC, where a global posterior distribution (or statistical moments in DKF [9]) is calculated at each filtering cycle after receiving all local information [10], [11]. In NB-based approaches, a set of UGVs execute distributed filters to estimate individual posterior distribution. Consensus of individual estimates is achieved by solely communicating statistics and/or observations within local neighbors of these UGVs. The NB-based methods have become popular in recent years since such approaches do not require complex routing protocols or global knowledge of the network and therefore are robust to changes in network topology and to link failures.

So far, most studies on NB-based distributed filtering have mainly focused on the so-called *statistics dissemination* strategy that each UGV actually exchanges statistics, including posterior distributions and likelihood functions, with neighboring UGVs [12]. This strategy can be further categorized into two types: leader-based and consensus-based. In the former, statistics is sequentially passed and updated along a path formed by active UGVs, called leaders. Only leaders perform filtering based on its own measurement and received measurements from local neighbors. For example, Sheng et al. (2005) proposed a multiple leader-based distributed particle filter with Gaussian Mixer for target tracking [13]. Sensors are grouped into multiple uncorrelated cliques, in each of which a leader is assigned to perform particle filtering and the particle information is then exchanged among leaders. In consensus-based distributed filters, every UGV diffuses statistics among neighbors, via which global agreement of the statistics is achieved by using consensus protocols [9], [14], [15]. For example, Hlinka et al. (2012) proposed a distributed method for computing an approximation of the joint (all-sensors) likelihood function by means of weighted-

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linear-average consensus algorithm when local likelihood functions belong to the exponential family of distributions [16]. Saptarshi et al. (2014) presented a Bayesian consensus filter that uses logarithmic opinion pool for fusing posterior distributions of the tracked target [7]. Other examples can be found in [8], [17].

Despite the popularity of statistics dissemination strategy, exchanging statistics can consume high communication resources. Approximating statistics with parametric models, such as Gaussian Mixture Models [13], can significantly reduce communication burden. However, such manipulation increases the computation burden for each UGV and sacrifices accuracy of filtering due to the approximation. One promising remedy is to disseminate measurement instead of statistics among neighbors, which, however, has not been fully exploited. One pioneering work was done by Coates et al. (2004), who used adaptive encoding of observations to minimize communication overhead [18]. Ribeiro et al. (2006) exchanged quantized observations along with error-variance limits considering more pragmatic signal models [19]. A recent work was conducted by Djuric et al. (2011), who proposed to broadcast raw measurements to other agents, and therefore each UGV has a complete set of observations of other UGVs for executing particle filtering [20]. A shortcoming of aforementioned works is that their communication topologies are assumed to be a complete graph that every pair of distinct UGVs is directly connected by a unique edge, which is not always feasible in reality.

This paper extends existing works by introducing a Latest-In-and-Full-Out (LIFO) protocol into distributed Bayesian filters (DBF) for networked UGVs. Each UGV is only allowed to broadcast observations to its neighbors by using single-hopping and then implements individual Bayesian filter locally after receiving transmitted observations. The main benefit of using LIFO is on the reduction of communication burden, with the transmission data volume scaling linearly with the UGV number, while a statistics dissemination-based strategy can suffer from the order of environmental size. The proposed LIFO-based DBF has following properties: (1) For a fixed and undirected network, LIFO guarantees the global dissemination of observations over the network in a non-intermittent manner. (2) The corresponding DBF ensures consistency of estimated target position, which also implies the consensus of target PDFs.

The rest of this paper is organized as follows: The problem of distributed Bayesian filtering is formulated in Section II. The LIFO-based DBF algorithm is described in Section III, followed by the proof of consistency and consensus in Section III-B. Simulation results are presented in Section IV and Section V concludes the paper.

II. Latest-In-and-Full-Out Protocol for Dynamically Changing Topologies

Consider a network of N UGVs in a bounded two-dimensional space S. Each UGV is equipped with a binary sensor for environmental perception. Due to the limit of communication range, each UGV can only exchange observations with its neighbors. The Bayesian filter is run locally on each UGV based on its own and received observations via single-hopping to estimate true position of target.

A. Probabilistic Model of Binary Sensor

The binary sensor only gives two types of observation: 1 if the target is detected, and 0 if no target is detected. The observation of i^{th} sensor at k^{th} time step is denoted as z_k^i. The likelihood function that the target is detected is:

$$P(z_k^i = 1 | x_k^T; x_k^{R,i}) \in [0,1]\,,\ x_k^T \in S \tag{1}$$

where x_k^T denotes the target position; $x_k^{R,i}$ is the UGV position. Correspondingly, the likelihood function that no target is detected is:

$$P(z_k^i = 0 | x_k^T; x_k^{R,i}) = 1 - P(z_k^i = 1 | x_k^T; x_k^{R,i}) \tag{2}$$

The combination of Eq. (1) and Eq. (2) forms a binary sensor model parameterized by x_k^T and $x_k^{R,i}$. For the purpose of simplicity, we will not explicitly write $x_k^{R,i}$ when no confusion may occur. The commonly used likelihood functions for binary sensor include Gaussian function [21] and step function [22].

Remark 1: Given the knowledge of current target and UGV positions, current observation of each UGV is conditionally independent from its own past observations and those of other UGVs.

Remark 2: The proposed LIFO protocol to be described in Section III is applicable for both homogeneous and heterogeneous binary sensors. A homogeneous model can simplify the analysis of completeness, while a heterogeneous model is more close to real sensing characteristics. In addition, it also works for other types of sensors, such as laser scanners and cameras.

B. Graphical Model of Communication Topology

Consider a simple[1], undirected graph $G = (V, E)$ to represent the communication topology of the UGV network, where $V = \{1, \dots, N\}$ represents the index set of UGVs and $E = V \times V$ denotes the edge set. The *adjacency matrix* $M = [m_{ij}]$ of graph G describes the communication topology:

$$m_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases},$$

where m_{ij} denotes the entity of adjacency matrix. The notation $m_{ij} = 1$ indicates that a communication link exists between i^{th} and j^{th} UGV and $m_{ij} = 0$ indicates no communication between them.

The communication topology can be dynamically changing due to limited communication range, varying team formation or link failure. Let $\bar{G} = \{G_1, G_2, \dots, G_L\}$ denote the set of all possible undirected communication graphs defined for the network of UGV. The adjacency matrix associated with graph G_l, $l = \{1, 2, \dots, L\}$ is denoted as $M^l = [m_{ij}^l]$. It is easy to know that $\bar{G}$ has finite elements. Define the *union* of a collection of simple graphs $\{G_{i_1}, G_{i_2}, \dots, G_{i_k}\} \subset \bar{G}$ as

[1]An undirected graph G is *simple* if it has no self-loops or repeated edges, i.e., $(i,j) \in E, only if i \neq j$ and E only contains distinct elements. A graph is *connected* when there is a path between every pair of vertices.

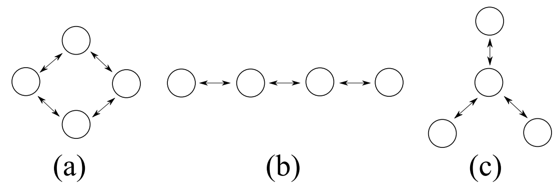


Fig. 1: Three types of topologies: (a) ring topology; (b) line topology; (c) star topology

the undirected graph with nodes in V and edge set given by the union of edge sets of G_{i_j}, $j = 1 \ldots, k$. Such collection is defined to be *jointly connected* if the union of its members forms a connected graph.

We define two concepts of neighborhood in a UGV network. The *direct neighborhood* of i^{th} UGV under topology G_l is defined as $\mathcal{N}_i(G_l) = \left\{j | m_{ij}^l = 1, \forall j \in \{1, \ldots, N\}\right\}$. All the UGVs in $\mathcal{N}_i(G_l)$ can directly exchange information with i^{th} UGV. In addition to direct neighborhood, another set called *available neighborhood* is defined as $\mathcal{Q}_i(G_l)$, which contains indices of UGVs whose observations can be received by the i^{th} UGV given a specific observation exchange protocol and the topology G_l. Note that in general $\mathcal{N}_i(G_l) \subseteq \mathcal{Q}_i(G_l)$. Fig. 1 illustrates three types of typical topologies: ring [23], line [24], and star [25]. All of them are simple and undirected graphs.

C. Latest-In-and-Full-Out (LIFO) Protocol

This study proposes a Latest-In-and-Full-Out (LIFO) protocol for observation exchange. Under LIFO, each UGV contains a communication buffer (CB) to store its latest knowledge of observations of all UGVs:

$$\mathbf{z}_k^{CB,i} = \left[z_{k_1^i}^1, \ldots, z_{k_N^i}^N\right]$$

where $z_{k_j^i}^j$ represents the observation made by j^{th} UGV at time k_j^i. Note that under LIFO, $\mathcal{Q}_i = \{1, \ldots, N\} \setminus \{i\}$, which will be proved in Corollary 1. At time k, $z_{k_j^i}^j$ is received and stored in i^{th} UGV CB, in which k_j^i is the latest observation time of j^{th} UGV available to i^{th} UGV. Due to the communication delay, $k_j^i < k, \forall j \neq i$ and $k_i^i = k$ always holds. The **LIFO protocol** is stated in Algorithm 1. For explanation of DBF in Section III, we define a *new observation set* $\mathbf{z}_k^{new,i}$ for each UGV to denote the set of observations that the i^{th} UGV receives and stores in its CB at k.

Remark 3: Compared to statistics dissemination, LIFO is generally more communication-efficient for distributed filtering. To be specific, consider an $D \times D$ grid environment with a network of N UGVs, the transmitted data of LIFO between each pair of UGVs are only the CB of each UGV and the corresponding UGV positions where observations were made, the length of which is $O(N)$. On the contrary, the length of transmitted data for a statistics dissemination approach that transmits unparameterized posterior distributions or likelihood functions is $O(D^2)$, which is in the order of environmental size. Since D is generally much larger than N in applications such as target localization, LIFO requires much less communication resources.

Algorithm 1 LIFO Protocol

(1) Initialization: The CB of i^{th} UGV is initialized when $k = 0$:

$$z_{k_j^i}^j = \varnothing,\ k_j^i = 0,\ j = 1, \ldots, N$$

(2) At k^{th} step for i^{th} UGV :

(2.1) Receiving Step:

The i^{th} UGV receives all CBs of its direct neighborhood $\mathcal{N}_i$. The received CBs are totally $|\mathcal{N}_i|$ groups, each of which corresponds to the (k-1)-step CB of a UGV in $\mathcal{N}_i$. The received CB from l^{th} ($l \in \mathcal{N}_i$) UGV is denoted as

$$\mathbf{z}_{k-1}^{CB,l} = \left[z_{(k-1)_1^l}^1, \ldots, z_{(k-1)_N^l}^N\right],\ l \in \mathcal{N}_i$$

(2.2) Observation Step:

The i^{th} UGV updates $z_{k_j^i}^j$ $(j = i)$ by its own observation at current step:

$$z_{k_j^i}^j = z_k^i,\ k_j^i = k,\ \text{if } j = i.$$

Add z_k^i to $\mathbf{z}_k^{new,i}$.

(2.3) Comparison Step:

The i^{th} UGV updates other elements of its own CB, i.e., $z_{k_j^i}^j$ $(j \neq i)$, by selecting the latest information among all received CBs from $\mathcal{N}_i$. For all $j \neq i$,

$$l_{\text{latest}} = \underset{l \in \mathcal{N}_i,\, i}{\text{argmax}} \left\{(k-1)_j^i, (k-1)_j^l\right\}$$

$$\text{If} \quad l_{\text{latest}} > z_{(k-1)_j^i}^j, \quad \text{add } z_{(k-1)_j^{l_{\text{latest}}}}^i \text{ to } \mathbf{z}_k^{new,i}.$$

$$z_{k_j^i}^j = z_{(k-1)_j^{l_{\text{latest}}}}^j,\ k_j^i = (k-1)_j^{l_{\text{latest}}}$$

(2.4) Sending Step:

The i^{th} UGV broadcasts its updated CB to all of its neighbors defined in $\mathcal{N}_i$.

(3) $k \leftarrow k + 1$ until stop

(**TODO:** change the figure to be a switching topology. change the contents here accordingly) Fig. 2 illustrates the LIFO cycles with 3 UGVs using a connected line topology. Two facts can be noticed in Fig. 2: (1) all UGV CBs are filled within 3 steps, which means under LIFO each UGV has a maximum delay of 2 steps for receiving observations from other UGVs; (2) after filled, CBs are updated non-intermittently, which means each UGV continuously receives new observations of other UGVs. Extending the two facts to a network of N UGVs, we have the following proposition:

Proposition 1: Consider an undirected network of N UGVs. Let $G[k] \in \bar{G}$ be the communication topology at time $t = kT$. If there exists an infinite sequence of contiguous, nonempty and uniformly bounded time intervals $[k_m, k_{m+1})$, $m = 1, 2, \ldots,$ starting at $k_1 = 0$, with the property that the union of graphs across each such interval is jointly connected, then any pair of UGVs can exchange

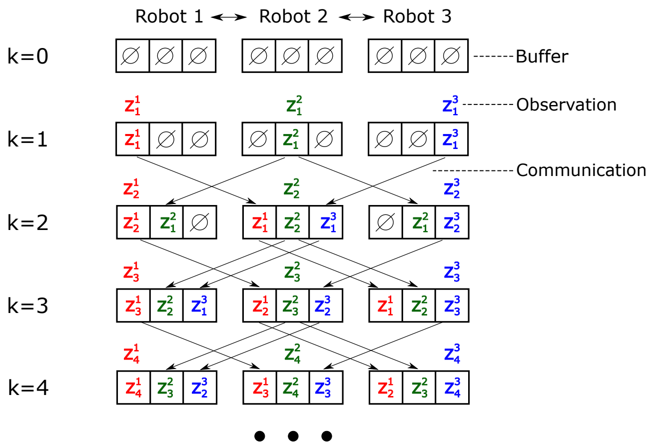


Fig. 2: Example of LIFO with three UGVs using line communication topology

observations under LIFO. In addition, the delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$, where $T_u = \sup_{m=1,2,\dots} (k_{m+1} - k_m)T$ is the upper bound of intervals.

Proof: Consider the data transmission between i^{th} and j^{th} UGV. Since the union of graphs across time interval $[k_1, k_2)$ is jointly connected, i^{th} UGV is connected to at least one another UGV at some time step, i.e., $\exists l_1 \in V$, $t_1 \in [k_1, k_2)$ such that $i \in \mathcal{N}_{l_1}(G_{t_1})$. This implies that observation $z^i_{t_1}$ is received and stored in the CB of l_1^{th} UGV at $t_1 + 1$ under LIFO. Therefore, we have shown that at least one UGV have received observation from i^{th} UGV by k_2. If $l_1 = j$, then we have proved the exchange of observations between i and j. If $l_1 \neq j$, we consider time interval $[k_2, k_3)$. By using similar reasoning as before, it is easy to understand that $\exists l_2 \in V$, $t_2 \in [k_2, k_3)$ such that $i \in \mathcal{N}_{l_2}(G_{t_2})$ or $l_1 \in \mathcal{N}_{l_2}(G_{t_2})$. For the former case, $z^i_{t_2}$ is received and stored in the CB of l_2^{th} UGV at $t_2 + 1$ under LIFO; For the latter case, $z^i_{t_1}$ is received by l_2^{th} UGV at $t_2 + 1$ but may not be stored in its CB. This occurs if l_2^{th} UGV has received a newer observation $z^i_{t'_2}$, $t'_2 > t_1$ from UGVs other than l_1. In both cases, it is shown that at least two UGVs have received and stored an observation from i^{th} UGV by k_3. Using similar derivation, it can be shown that $N-1$ UGVs, except i^{th} UGV, will receive and store an observation from i no later by k_N. Therefore, the transmission delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$. ■

Corollary 1: For the same network condition in Proposition 1, all elements in $\mathbf{z}^{CB,i}_k$ under LIFO become filled when $k \geq (N-1)T_u$, i.e., $\mathcal{Q}_i = \{1, \dots, N\} \setminus \{i\}$. Additionally, each element keeps getting updated after finite intervals.

Proof: Based on Proposition 1, the transmission delay between an arbitrary pair of UGVs is no greater than $(N-1)T_u$. Therefore, CBs of all UGV will be filled when $k \geq (N-1)T_u$. In addition, each element in CBs gets updated after finite interval no greater than $(N-1)T_u$. ■

III. Distributed Bayesian Filter via Latest-In-and-Full-Out Protocol

A. Distributed Bayesian Filter for Multiple UGVs

The distributed Bayesian filter (DBF) using LIFO protocol is introduced in this section. Each UGV has its individual estimation of posterior density function (PDF) of target position, called *individual PDF*. The individual PDF of i^{th} UGV at time k is defined as $P^i_{pdf}(x^T_k|\mathbf{z}^{new,i}_{1:k})$, where $\mathbf{z}^{new,i}_{1:k}$ denotes the set of new observation set by i^{th} UGV from time 1 to k. The individual PDF is initialized as $P^i_{pdf}(x^T_0|\mathbf{z}^{new,i}_0) = P(x^T_0)$, given all available prior information including past experience and environmental knowledge. Under the framework of DBF, the individual PDF is recursively estimated by two steps, i.e., prediction step and updating step, based on observations of i^{th} UGV and UGVs in $\mathcal{Q}_i$.

1) Prediction: At time k, the prior individual PDF $P^i_{pdf}(x^T_{k-1}|\mathbf{z}^{new,i}_{1:k-1})$ is first predicted forward by using the Chapman-Kolmogorov equation:

$$P^i_{pdf}(x^T_k|\mathbf{z}^{new,i}_{1:k-1}) = \int P(x^T_k|x^T_{k-1}) P^i_{pdf}(x^T_{k-1}|\mathbf{z}^{new,i}_{1:k-1}) dx^T_{k-1} \tag{3}$$

where $P(x^T_k|x^T_{k-1})$ is a Markov motion model of the target, independent of UGV states. This model describes the state transition probability of the target from a prior state x^T_{k-1} to posterior state x^T_k. Note that the target is static in many search applications, such as the indoor search for stationary objects [26]. For a static target, its Markov motion model is simplified to be

$$P(x^T_k|x^T_{k-1}) = \begin{cases} 1 & \text{if } x^T_k = x^T_{k-1} \\ 0 & \text{if } x^T_k \neq x^T_{k-1} \end{cases}.$$

This work focuses on the localization of static target. Therefore, we remove the subscript of x^T_k for the rest of the paper.

2) Updating: The i^{th} individual PDF is then updated by Bayes' formula using the set of newly received observations at time k, $\mathbf{z}^{new,i}_k$:

$$\begin{aligned} P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k}) &= K_i P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k-1}) P(\mathbf{z}^{new,i}_k|x^T) \\ &= K_i P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k-1}) \prod_{z^j_{k^i_j} \in \mathbf{z^{new,i}_k}} P(z^j_{k^i_j}|x^T) dx^T \end{aligned} \tag{4}$$

where K_i is a normalization factor, given by:

$$K_i = 1 / \int P^i_{pdf}(x^T|\mathbf{z}^{new,i}_{1:k-1}) \prod_{z^j_{k^i_j} \in \mathbf{z^{new,i}_k}} P(z^j_{k^i_j}|x^T) dx^T$$

and $P^i_{pdf}(x^T_k|\mathbf{z}^{new,i}_{1:k})$ is called posterior individual PDF; $P(z^j_{k^i_j}|x^T_k)$ is the likelihood function of observation $z^j_{k^i_j}$, described in Eq. (1) and Eq. (2).

B. Proof of Consistency and Consensus

This section proves consistency and consensus of LIFO-DBF. Considering S is finite and x^{T^*} is the true location of target, the consistency of LIFO-DBF for static UGVs is stated as follows:

Theorem 1: When UGVs are static, the estimated target position converges to the true position of target in probability using LIFO-DBF, i.e.,

$$\lim_{k\to\infty} P(x^T = x^{T^*}|\mathbf{z}_{1:k}^{new,i}) = 1,\ i = 1,\dots,N.$$

Proof: For the purpose of clarity, define a time set $\mathscr{K}_j^i$, $j \in \{1,\dots,N\}$ for i^{th} UGV that contains time steps when associated observations by j^{th} UGV are in $\mathbf{z}_{1:k}^{new,i}$. Considering the conditional independence of observations given $x^T \in S$, the batch form of DBF at k^{th} step is:

$$P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) = \frac{P_{pdf}^i(x^T)\prod_{j=1}^{N}\prod_{l\in\mathscr{K}_j^i} P(z_l^j|x^T)}{\sum_{x^T\in S} P_{pdf}^i(x^T)\prod_{j=1}^{N}\prod_{l\in\mathscr{K}_j^i} P(z_l^j|x^T)}, \tag{5}$$

where P_{pdf}^i is i^{th} initial individual PDF. It is known from Corollary 1 that the cardinality of $\mathscr{K}_j^i$ has following property: $k-(N-1)T_u < |\mathscr{K}_j^i| \le k$.

Comparing $P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})$ with $P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})$ yields

$$\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})} = \frac{P_{pdf}^i(x^T)\prod_{j=1}^{N}\prod_{l\in\mathscr{K}_j^i} P(z_l^j|x^T)}{P_{pdf}^i(x^{T^*})\prod_{j=1}^{N}\prod_{l\in\mathscr{K}_j^i} P(z_l^j|x^{T^*})} \tag{6}$$

Take the logarithm of Eq. (6) and average it over k steps:

$$\frac{1}{k}\ln\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})} = \frac{1}{k}\ln\frac{P_{pdf}^i(x^T)}{P_{pdf}^i(x^{T^*})} + \sum_{j=1}^{N}\frac{1}{k}\sum_{l\in\mathscr{K}_j^i}\ln\frac{P(z_l^j|x^T)}{P(z_l^j|x^{T^*})}. \tag{7}$$

Since $P_{pdf}^i(x^T)$ and $P_{pdf}^i(x^{T^*})$ are bounded, then

$$\lim_{k\to\infty}\frac{1}{k}\ln\frac{P_{pdf}^i(x^T)}{P_{pdf}^i(x^{T^*})} = 0. \tag{8}$$

The binary observations subject to Bernoulli distribution $B(1,p_j)$, yielding

$$P(z_l^j|x^T) = p_j^{z_l^j}(1-p_j)^{1-z_l^j}$$

where $p_j = P(z_l^j = 1|x^T)$. Utilizing the facts: (1) z_l^j are conditionally independent samples from $B(1,p_j^*)$ and (2) $k-(N-1)T_u < |\mathscr{K}_j^i| \le k$, the law of large numbers yields

$$\frac{1}{k}\sum_{l\in\mathscr{K}_j^i} z_l^j \xrightarrow{P} p_j^*, \quad \frac{1}{k}(|\mathscr{K}_j^i| - \sum_{l\in\mathscr{K}_j^i} z_l^j) \xrightarrow{P} 1-p_j^*$$

where $p_j^* = P(z_l^j = 1|x^{T^*})$ and "$\xrightarrow{P}$" denotes "convergence in probability". Then,

$$\frac{1}{k}\sum_{l\in\mathscr{K}_j^i}\ln\frac{P(z_l^j|x^T)}{P(z_l^j|x^{T^*})} \xrightarrow{P} p_j^*\ln\frac{p_j}{p_j^*} + (1-p_j^*)\ln\frac{1-p_j}{1-p_j^*} \tag{9}$$

Note that the right-hand side of Eq. (9) achieves maximum value 0 if and only if $p_j = p_j^*$. Define

$$c(x^T) = \sum_{j=1}^{N} p_j^*\ln\frac{p_j}{p_j^*} + (1-p_j^*)\ln\frac{1-p_j}{1-p_j^*}.$$

Considering Eq. (8) and Eq. (9), the limit of Eq. (7) is

$$\frac{1}{k}\ln\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})} \xrightarrow{P} c(x^T) \tag{10}$$

It follows from Eq. (10) that

$$\frac{P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})e^{c(x^T)k}} \xrightarrow{P} 1 \tag{11}$$

Define the set $\bar{X}^T = S \setminus \{x^{T^*}\}$ and $c_M = \max_{x^T\in\bar{X}^T} c(x^T)$. Then $c_M < 0$. Summing Eq. (11) over $\bar{X}^T$ yields

$$\frac{\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})e^{[c_M - c(x^T)]k}}{P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})e^{c_M k}} \xrightarrow{P} |\bar{X}^T| \tag{12}$$

where $|\bar{X}^T|$ denotes the cardinality of $\bar{X}^T$.

Since $c_M < 0$, $P_{pdf}^i(x^{T^*}|\mathbf{z}_{1:k}^{new,i})e^{c_M k} \longrightarrow 0$, Eq. (12) implies

$$\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})e^{[c_M - c(x^T)]k} \xrightarrow{P} 0 \tag{13}$$

Utilizing the relation

$$0 \le P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) \le P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i})e^{[c_M - c(x^T)]k},$$

it can be derived from Eq. (13) that

$$\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) \xrightarrow{P} 0$$

Therefore,

$$\lim_{k\to\infty} P(x^T = x^{T^*}|\mathbf{z}_{1:k}^{new,i}) = 1 - \lim_{k\to\infty}\sum_{x^T\in\bar{X}^T} P_{pdf}^i(x^T|\mathbf{z}_{1:k}^{new,i}) = 1$$

■

Remark 4: Consistency implies that all individual PDFs converge to the same distribution (true target PDF), thus the consensus is also guaranteed. It must be noted that traditional statistics dissemination-based methods only ensure consensus of individual PDFs [7], [8]. To the best knowledge of authors, there is no proof of consistency on estimated target position.

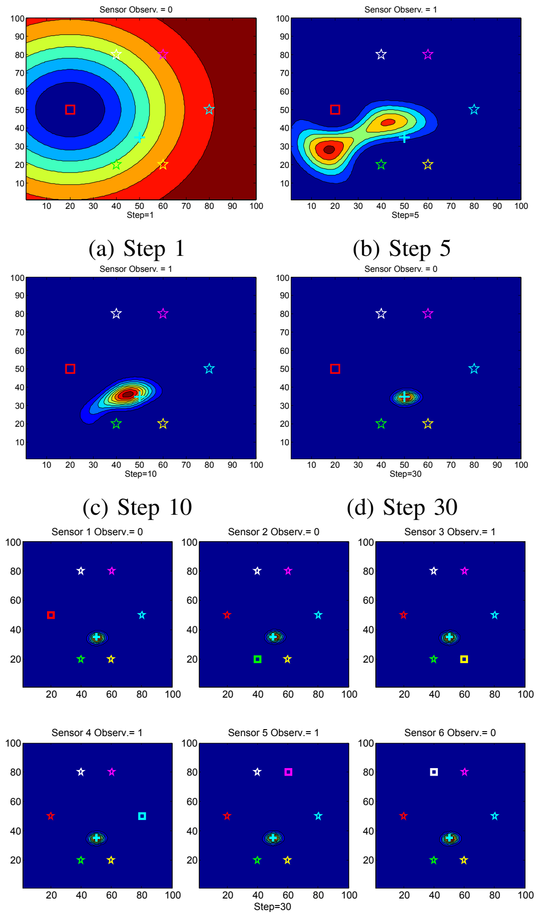
IV. Simulation

(**TODO:** change simulation plots) This section simulates two scenarios of target localization to demonstrate the effectiveness of LIFO-BDF. In all scenarios, six UGVs are utilized and each UGV is equipped with a binary sensor. All sensors are modeled with identical Gaussian functions [21]:

$$P(z_k^i = 1|x_k^T; x_k^{R,i}) = e^{-\frac{1}{2}(x_k^T - x_k^{R,i})^T\Sigma^{-1}(x_k^T - x_k^{R,i})} \tag{14a}$$

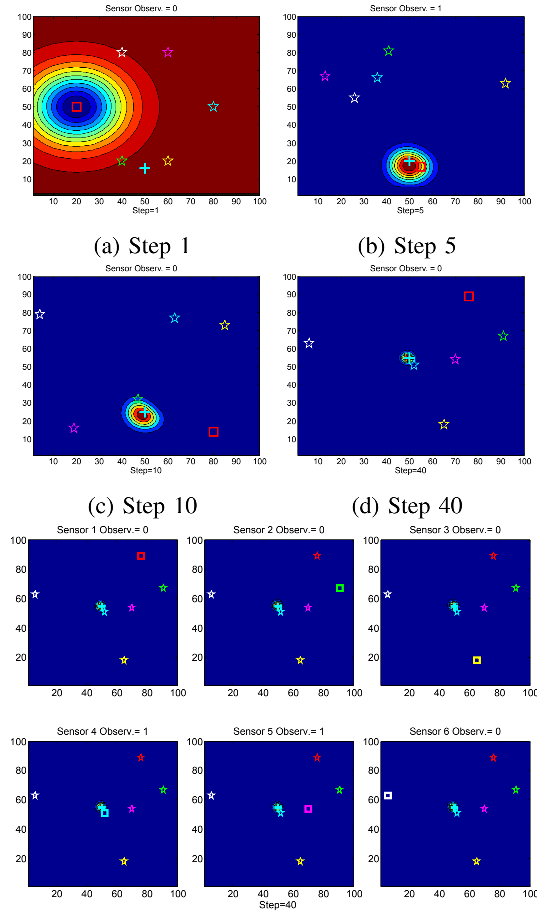
$$P(z_k^i = 0|x_k^T; x_k^{R,i}) = 1 - P(z_k^i = 1|x_k^T; x_k^{R,i}). \tag{14b}$$

The first scenario consists of six static UGVs and a single static target. The second scenario subsequently deals with six moving UGVs for localizing a moving target. UGV positions are randomly generated at each time step. A single-integrator dynamics is used as the target motion model.



(a) Step 1 (b) Step 5 (c) Step 10 (d) Step 30 (e) Step 30

Fig. 3: (a)-(d) The 1st UGV's individual PDFs at different times; (e) All UGVs' individual PDFs at time 30. The square denotes the current UGV and stars represent other UGVs. The cross stands for the target.



(a) Step 1 (b) Step 5 (c) Step 10 (d) Step 40 (e) Step 40

Fig. 4: (a)-(d) The 1st UGV's individual PDFs at different times; (e) All UGVs' individual PDFs at time 40.

A. Static UGVs, Static Target

The positions of six static UGVs are shown as stars and square in Fig. 3. The LIFO-DBF for static target is implemented on each UGV for target localization. The networked UGVs use a ring communication topology that each UGV can communicate with two fixed neighbors. Fig. 3 shows the estimation results of the static target. After the initial observation, each UGV forms a circular individual PDF, centered at its own position. The circular PDF happens because the Gaussian sensor model (Eq. (14)) only depends on the distance between UGV and target. As more observations are used, the posterior individual PDF concentrates to the true location of the target (Fig. 3d), which accords with the consistency of LIFO-DBF.

V. CONCLUSION

This paper presents a measurement dissemination-based distributed Bayesian filtering (DBF) method for a multi-UGV network, utilizing the Latest-In-and-Full-Out (LIFO) protocol for observation exchange. Different from statistics dissemination approaches that transmit posterior distributions or likelihood functions, each UGV under LIFO only exchanges with neighboring UGVs a full communication buffer consisting of latest available measurements, which significantly reduces the transmission burden between each pair of UGVs from the order of environmental size to that of UGV number. Under the condition of fixed and undirected topology, LIFO can guarantee non-intermittent dissemination of all observations over the network within finite time via local communication among direct neighborhood. It is worth noting that LIFO is a general measurement exchange protocol and applicable to various sorts of sensors. Two types of LIFO-based DBF algorithms are proposed to estimate individual PDF for static and moving target, respectively. For the static target, each UGV locally fuses the newly received observations while for the moving target, a record set of historical observations is stored and updated. The consistency of LIFO-based DBF is proved by utilizing the law of large numbers, which ensures that estimated target position converges in probability to the true target position when the number of observations tends to infinity.

Future work includes how to handle other types of sensors and switching topology. Other types of sensors may have biased observations and subject to non-Bernoulli distribution, which complicates the design and analysis of LIFO-based Bayesian filters. The switching topology, including package loss, can lead to unpredictable delay and intermittent transmission, which may affect the consistency and consensus of LIFO-DBF.

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