Model Predictive Control-based Target Search and Tracking Using Autonomous Mobile Robot with Limited Sensing Domain

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***Abstract*— Target search and tracking using autonomous robots is important for both civil and military applications. In this work, we propose a model predictive control (MPC)-based path planning approach for a ground mobile robot to autonomously search and track a moving target. The robot is equipped with a sensor with limited sensing domain for target detection. Both target motion and sensor measurement use linear time-invariant models. Due to the limited sensing domain, we utilize a modified Kalman filter to handle the intermittent measurements. Under the MPC framework, the sensing domain is approximated with a bell-shaped differentiable function and is explicitly considered in the optimization problem. To reduce the computation burden of solving MPC, we propose a two-step procedure: it first considers the limited sensing range and computes a reference trajectory, which is then used for solving the original MPC that considers both limited sensing range and angle. The effectiveness of the proposed method is demonstrated by numerical simulations.**

I. INTRODUCTION

Using autonomous robots to search and track targets has attracted wide interest in recent years. In such application, an autonomous robot first needs to explore the environment and search for the targets of interest. After detecting a target, the robot will then switch mode and keep tracking the target. Some applications include indoor search [1], marine search-and-rescue [2] and object search-and-identify in the wilderness [3].

Studies on search and tracking were initiated during World War II. However, later research efforts have treated these two modes rather separately. On one hand, much work has been focused on tracking moving targets [4]–[6], which assumes that targets are within the sensor's sensing domain from the beginning. The sensor is then controlled to maintain the target's visibility over time. On the other hand, intense efforts have been put into search problems, which started with simple environment coverage strategies and gradually evolved to more advanced approaches for handling complex, dynamic and stochastic environments [7], [8]. Probabilistic measures, such as probability of detection, were used as metrics for designing search path.

A pioneering work that unifies the target search and tracking under the same framework was conducted by Bourgault et al. [9], who designed a unified objective function for search and tracking that represents the cumulative probability of detection. It utilized a Bayesian filter for updating the target's state based on sensor measurement. Due to the numerical complexity of evaluating the objective function, the work used a one-step look-ahead scheme for path planning. The idea of using such greedy policy for path planning has also been utilized in [10], where robots follow the gradient of mutual information to estimate environment events and hazards online. Research has also been focused on distributed information gathering and target search using the greedy path planning policy [11], [12].

Approaches for non-myopic target search and tracking have also been developed. For example, Tisdale et al. [13] utilized a receding horizon path planning approach for a team of aerial vehicles to cooperatively search for targets, with the probability of detection being maximized over a finite horizon. Ryan et al. [14] used an information-theoretic objective function and developed a control-based formulation to minimize the entropy of an estimated distribution for moving targets over a multiple-step horizon. The prediction of conditional entropy is computed by sequential Monte Carlo method in the context of particle filtering. However, due to the complexity of Monte Carlo method, such approach could not be implemented in real time. Lanillos et al. [15] used the cross entropy approach to solve an optimization problem with a discounted probability of detection as the objective function, which leads to minimum time search of the target. However, these works did not explicitly consider the limited sensing domain of sensors in the planning process.

Sampling-based path planning approaches have also been adopted for information gathering. For example, Hollinger et al. [16] has proposed a rapidly exploring information gathering graph approach to effectively optimize a modular or submodular objective functions for information gathering. Levine [17] proposed a variant of RRT [18] algorithm to search for targets in the environment, considering the limited sensing domain and occlusion caused by obstacles. However, sampling-based methods currently suffer from poor scalability to higher dimensional space and thus is of limited use.

In this work, we use the model predictive control framework for non-myopic path planning, which is formulated as an optimization problem that explicitly considers the limited sensing domain of the sensor. Some previous works have considered the similar problem [19], [20]. However, their assumption of certain sensor type (binary sensors) limits the applicability to general situations. A particularly relevant work was done by Patil et al. [21], who considered the path planning in Gaussian belief space and proposed a sequential

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planning algorithm in MPC framework to deal with the discontinuity of sensing domains. Our work is different from this work in the way we handle the limited sensing domain and the strategy for solving the MPC problem. To be specific, we use the geometric representation of sensing domain and use sigmoid function to approximate its boundary. In addition, we propose a two-step optimization structure that successively improves the planned path to reduce the uncertainty of the target. Such approach can alleviate the computational burden of solving the MPC problem.

The remainder of this paper is organized as follows: the target search and tracking problem is formulated in Section II. The MPC-based path planning method that incorporates limited sensing domain is described in Section III. Simulation results are presented in Section IV.

II. PROBLEM FORMULATION

We consider a two-dimensional planar space, as shown in Figure 1, that contains a moving target (blue toy car) and an autonomous mobile robot (red wheeled robot). The robot is equipped with a sensor that has limited sensing domain to measure the target position. We let the sensor share the same state as the robot (i.e. position and heading angle). The target position is unknown a priori and the robot needs to autonomously search for the target. Once having detected the target, the robot will need to continue tracking it by keeping the target in its sensor's sensing domain. As shown in the figure, the target position comes with large initial uncertainty due to the lack of prior information. However, when the target is within the sensing domain, the uncertainty is significantly reduced and the robot will then keep tracking the target.

A. Robot and Target Motion Model

We use a discrete-time unicycle motion model for the mobile robot:

$$x^r_{k+1} = f(x^r_k, u^r_k), \tag{1}$$

where

$$f(x^r_k, u^r_k) = x^r_k + \begin{bmatrix} \cos\theta^r_k \Delta t & 0 \\ \sin\theta^r_k \Delta t & 0 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} u^r_k.$$

The robot state $x^r_k = [x^r_{1,k}, x^r_{2,k}, \theta^r_k, v^r_k] \in \mathbb{R}^4$ consists of its position, steering wheel angle and speed at time k. The control input $u^r_k = [w^r_k, a^r_k] \in \mathbb{R}^2$ includes angular velocity and linear acceleration. Δt represents the sampling time.

We assume a stochastic linear time-invariant model for the target:

$$x^t_{k+1} = Ax^t_k + Bu^t_k + w_k,\ w_t \sim \mathcal{N}(0, Q), \tag{2}$$

where $A, B \in \mathbb{R}^{2\times 2}$ are system matrices; $w_t \in \mathbb{R}^2$ is a zero-mean Gaussian noise with $Q \succeq 0$ being the covariance matrix. The target state $x^t_k = [x^t_{1,k}, x^t_{2,k}] \in \mathbb{R}^2$ represents its position.

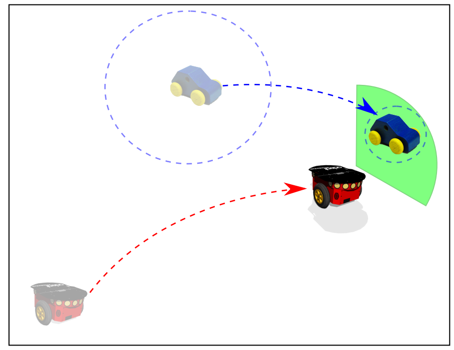

Fig. 1: Scenario of the target search and tracking mission. Initial positions of the robot and target are represented by translucent red wheeled robot and blue toy car. Their corresponding trajectories are shown as the red and blue dashed lines. The blue dashed circles corresponds to the uncertainty of target position. The green sector shows the sensor's limited sensing domain.

B. Modeling Sensing Domain

Common sensors, such as cameras and radars, have limited sensing domain that are bounded in both sensing range (distance) and angle (angle of view) [25]. We approximate the sensing domain as a sector (Figure 2), which can be represented as $\mathcal{F}_k = \left\{v = [x_1, x_2] \in \mathbb{R}^2 |\ \|v\|_2 \le r, \angle v \in [\theta_1, \theta_2]\right\}$. It should be noted that, though we use a sector sensing domain model in this work, the presented path planning method can also apply to other geometries of sensor's sensing domain, such as the lobe or cone shape.

C. Sensor Measurement Model

We assume a linear measurement model. Since the target position is unknown a-priori, the target can be outside of the sensing domain and results in missing measurement. To tackle this problem, we adopt the measurement model from Sinopoli et al. [22], which provides a unified model for handling intermittent measurements:

$$y_k = Cx^t_k + v_k,\ v_k \sim \begin{cases} \mathcal{N}(0, R) & \text{if } \gamma_k = 1 \\ \mathcal{N}(0, \sigma^2 I) & \text{if } \gamma_k = 0 \end{cases}, \tag{3}$$

where $C \in \mathbb{R}^{2\times 2}$ is the measurement matrix and $v_k \in \mathbb{R}^2$ is a zero-mean Gaussian noise. γ_k is a binary random variable denoting whether a measurement is received ($\gamma_k = 1$) or not ($\gamma_k = 0$) at time step k. Different from the measurement noise in traditional Kalman filter, covariance of v_k now depends on whether a measurement is obtained. In practice, the receiving of a measurement corresponds to a finite covariance $R \succeq 0$ and the absence of measurement corresponds to the limiting case of $\sigma \to \infty$.

To adapt Eq. (3) for handling limited sensing domain, we define

$$\gamma_k = \mathbb{1}_{\{x^t_k \in \mathcal{F}_k\}} \tag{4}$$

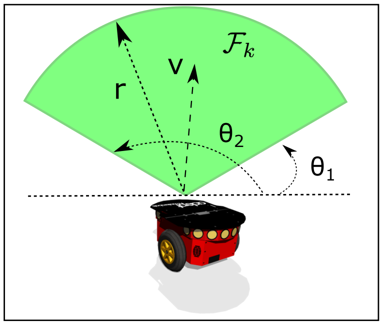

Fig. 2: Illustration of sensor's sensing domain $\mathcal{F}_k$. The sensing range is r and the sensing angle is $[\theta_1, \theta_2]$. v represents the vector from the sensor origin to the target position.

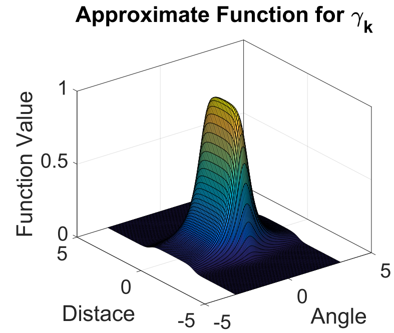

Fig. 3: Illustration of the bell-shaped function for approximating γ_k. In this example, parameters take the following values: $\tilde{\theta}_k = 0,\ \theta_0 = \pi/3,\ \alpha_1 = 1,\ \alpha_2 = 10$.

to be an indicator function that reflects whether the target is within sensing domain (a measurement is thus received) and the case that the target is outside of sensing domain (therefore the measurement is missing). This formulation will facilitate the development of a unified MPC that explicitly handles intermittent measurements caused by limited sensing domain, which is to be described in following sections.

III. MPC-based Path Planning

A. Kalman Filter with Limited Sensing Domain

Since limited sensing domain can cause intermittent measurements, we utilize the discrete-time Kalman filter from [22] to consider this effect, which is defined as

$$\begin{aligned}
\hat{x}^t_{k+1|k} &= A\hat{x}^t_{k|k} + Bu^t_k && \text{(5a)}\\
P_{k+1|k} &= AP_{k|k}A' + Q && \text{(5b)}\\
K_{k+1} &= P_{k+1|k}C(CP_{k+1|k}C' + R)^{-1} && \text{(5c)}\\
\hat{x}^t_{k+1|k+1} &= \hat{x}^t_{k+1|k} + \gamma_{k+1}K_{k+1}(y_{k+1} - C\hat{x}^t_{k+1|k}) && \text{(5d)}\\
P_{k+1|k+1} &= P_{k+1|k} - \gamma_{k+1}K_{k+1}CP_{k+1|k}, && \text{(5e)}
\end{aligned}$$

where $\hat{x}^t_{k|k}$ and $P_{k|k}$ represent the estimated target position and covariance matrix. For notational simplicity, we define $b_k = [\hat{x}^t_k, P_{k|k}]$ and let $b_{k+1} = g(b_k, u^r_k)$ represent the Kalman filter defined in Eq. (5). Note that $\hat{x}^t_{k+1|k+1}$ and $P_{k+1|k+1}$ are a function of γ_{k+1} now and thus depend on the state of both the robot and target.

B. Path Planning for Target Search and Tracking

We formulate the path planning problem using the model predictive control framework. MPC is a suitable method for this problem. This is because the target position constantly gets updated as new sensor measurement is obtained and MPC can utilize the updated target information to re-generate a path at each time instance. The MPC-based path planner with planning horizon N can be formulated as:

$$\begin{aligned}
&\min_{u_{1:N}} J(b_{1:N+1}, u_{1:N}) && \text{(6a)}\\
&\text{s.t. } x^r_{k+1} = f(x^r_k, u^r_k), && \text{(6b)}\\
&\quad b_{k+1} = g(b_k, u^r_k), && \text{(6c)}\\
&\quad x^r_{k+1} \in \mathcal{X},\ u^r_{k+1} \in \mathcal{U}, && \text{(6d)}\\
&\quad k = 1, \ldots, N, && \text{(6e)}
\end{aligned}$$

where J is the objective function; $\mathcal{X}$ and $\mathcal{U}$ represent the feasible sets of robot state and control input, consisting of constraints on robot speed, steering rate and linear acceleration.

Information-theoretic measures have been proven to be effective metrics for path planning in information gathering [9], [11], [14], [23]. To drive the robot to configurations in which the sensor can effectively obtain target information, we choose the cumulative entropy and the distance between the robot and predicted target position as the objective function. Because the linear Gaussian noise assumption in the target model and measurement model, target uncertainty follows a Gaussian distribution. Therefore, the entropy at k is $H(b_k) = \frac{k}{2}(1+\ln(2\pi)) + \frac{1}{2}\ln|P_{k|k}|$ ([24]). Minimizing a determinant can incur large computation burden for an optimization problem. Therefore, we utilize the relation $|A|^{\frac{1}{n}} \leq \frac{1}{n}tr(A)$ for a positive definite matrix A and define the objective function of the MPC problem as:

$$J(b_{1:N+1}) = \sum_{i=1}^{N} w_1 tr(P_{i+1|i+1}) + w_2\|x^t_{i+1} - x^r_{i+1}\|^2,$$

where $tr(P_{k|k})$ is the trace of the matrix $P_{k|k}$.

The binary variable γ_k (Eq. (4)) is a discontinuous function of the robot and target states, which is inconvenient for an optimization problem that usually requires differentiability of functions in it. Therefore we approximate γ_k as a product of two bell-shaped differentiable functions, corresponding to bounds on sensing range and angle:

$$\gamma_k \approx \frac{1}{1+\alpha_1\|[x^t_{1,k}, x^t_{2,k}] - [x^r_{1,k}, x^r_{2,k}]\|_2^2} \times \frac{1}{1+\exp\left\{-\alpha_2(\cos(\theta^r_k - \tilde{\theta}_k) - \cos(\theta_0))\right\}}, \quad (7)$$

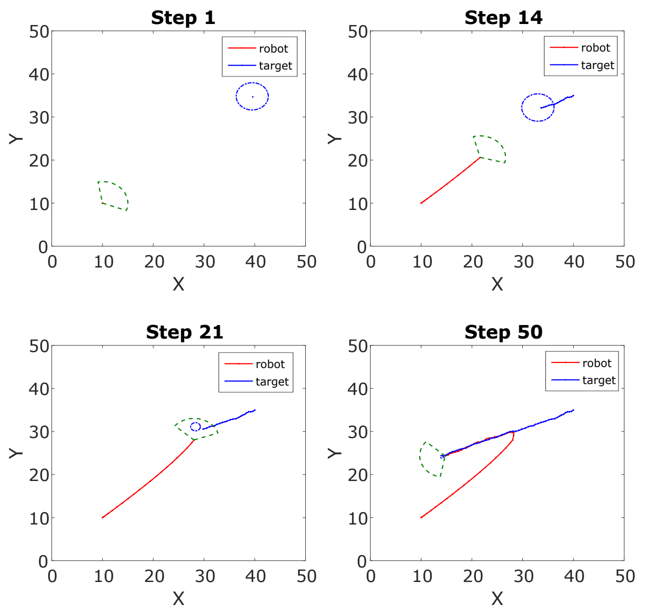

Fig. 4: Intermediate steps of search and tracking. Green dashed sector represents the sensor's sensing domain and the blue dashed circle show the uncertainty of the target position (based on diagonal elements of $\sqrt{P_{k|k}}$). Red and blue lines corresponds to the trajectories of the robot and target.

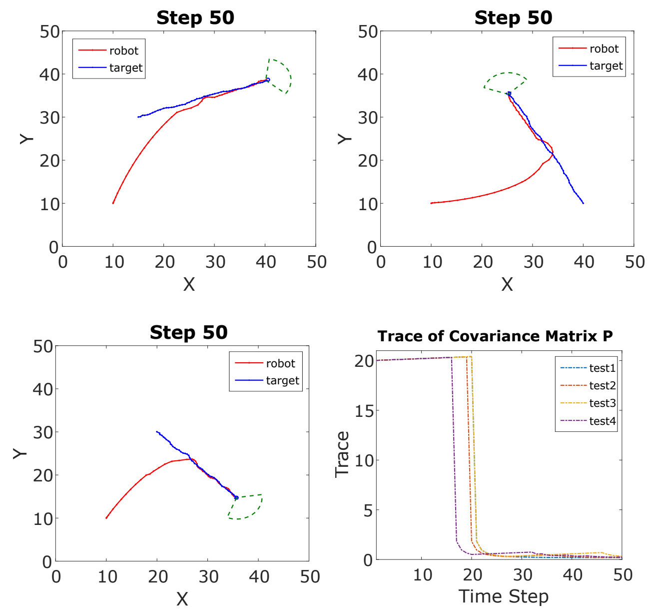

Fig. 5: First three figures show the final steps of search and tracking mission in three different scenarios. The last figure shows the trace of $P_{k|k}$ at each time step.

where $\tilde{\theta}_k = \angle([x^t_{1,k}, x^t_{2,k}] - [x^r_{1,k}, x^r_{2,k}])$ is the direction angle from the sensor position to target position; $\theta_0 = \frac{\theta_2 - \theta_1}{2}$ is half of the sensing angle; α_1 and α_2 are tunning parameters that controls the shape of the function. Eq. (7) can be interpreted as follows: when the robot is close to the target, it is more likely that the target can be detected; besides, the closer the target direction aligns with the center direction of the sensor, the higher possibility that the target will get detected. Figure 3 shows the shape of γ_k.

C. *Sequential Planning*

Directly solving the MPC problem in Eq. (6) is time consuming, mainly due to the coupling of robot and target state in γ_k, as defined in Eq. (7). To reduce the computation burden for real-time applications, we make several relaxation to the original MPC problem. To be specific, in the predictive horizon, the target position is assumed to be propagated without noise from the estimated target position at the beginning of the prediction. This is equivalent to predicting target position using Eq. (5a) but without updating step (Eq. (5d)). This is a reasonable assumption for MPC framework. In fact, because of the stochasticity of target motion and sensor measurement process, it is difficult to accurately predict the evolution of the environment state for a long horizon. The advantage of MPC is that it will takes into account the newly received information about the target and then utilize it to update the path at each time step. This can compensate for inaccuracy caused by using a simple model to predict target position in the predictive horizon.

We also devise a two-step process to further reduce the computation burden for the MPC problem. In the first step, we only consider the limited sensing range of the sensor, assuming a full sensing angle. In this step, γ_k is equivalent to the first multiplier in Eq. (7). With this simplified γ_k, the MPC computes a reference trajectory of the robot. Utilizing the generated reference trajectory, the heading angle of sensor $\hat{\theta}_k$ at each predicted time can be computed. In the second step, MPC is solved again but with the full expression of γ_k, in which $\hat{\theta}_k$ takes the value from the first step. The benefits of conducting this sequential two-step planning process lies in that computing the reference trajectory in the first step generates the reference heading angle of the robot, which reduces the computation burden when later a full version of MPC is solved.

IV. Simulation

We have tested the algorithm on several scenarios. Each scenario consists of a mobile robot equipped with a camera and a moving target in a $50m \times 50m$ field. The initial position of the target is different in each scenario while the robot starts from the same position. Specifications for simulation is described in Appendix.

Figure 4 shows the intermediate search and tracking steps of one test scenario. At the beginning, the robot starts from the lower left corner and the target is at the upper right corner. The estimated target position is initialized as the target's true position but with a large uncertainty. The robot then moves towards the target by using the control input computed from the MPC problem (Eq. (6)). At step 21, the robot successfully detects the target and thus the uncertainty significantly shrinks. The robot then keeps tracking the target and the uncertainty is further reduced, as the figure of Step 50 shows.

Robot and target trajectories in some other test scenarios are shown in Figure 5. The lower right figure in Figure 5 shows the trace of the covariance matrix of the target position. As expected, uncertainty keeps increasing at the beginning since the target is not found and thus no uncertainty reduction can be conducted by Kalman filter. Once the target is within the sensor's sensing domain, uncertainty drops significantly since Kalman filter is able to use measurements to improve the estimation of the target position. The robot then keeps measuring the target position and the uncertainty continues being reduced. Fluctuation of the trace can also be observed in later part of the search and tracking. This is mainly due to the fact that the target moves in a stochastic manner and therefore there exists some cases that the target falls out of the sensing domain. However, the robot is able to adjust its sensor and keep the target in its sensing domain again. These figures shows that the robot successfully detects the target and then keeps tracking it afterwards in all cases using the presented approach.

V. CONCLUSION

In this work, we propose a model predictive control (MPC)-based path planning approach for a ground mobile robot to autonomously search and track a moving target. We consider the effects of sensor's limited sensing domain in the predictive horizon by utilizing the modified Kalman filter for handling intermittent measurements. To deal with the discontinuity caused by limited sensing domain, we propose a bell-shaped function to approximate the sensing domain's boundary. A two-step sequential planning approach is then developed for solving the MPC problem. Simulation results have demonstrated the effectiveness of the proposed path planning algorithm in searching and tracking the target.

Though this work achieves good results, we realize some limitations that should be addressed in our future work. First, the developed approach relies on the assumption of Gaussian distribution of target uncertainty and is therefore restricted to linear target model and sensor measurement model. For general nonlinear dynamic systems, this method may not be directly applied. [21] has used Extended Kalman filter to deal with this issue. However, the Gaussian distribution assumption in that work also limits its generalizability. Second, when measurement arrival rate is below a certain threshold, as [22] indicated, Kalman filter is not able to converge. Some other general nonlinear filters, such as Bayes filter and Particle filter, can therefore be used to improve the robustness of the filtering results. Lastly, when there exist obstacles in the environment, occlusion may happen that affects the sensor measurement process. Taking into account the occlusion in the planning process is our ongoing work.

APPENDIX

PARAMETERS FOR SIMULATION

- Feasible sets: $\mathcal{X} = \{x_k^r | v_k^r \in [0,3]\}$, $\mathcal{U} = \{u_k^r | a_k^r \in [-3,1],\ w_k^r \in [-\frac{pi}{4}, \frac{pi}{4}]\}$.
- Sensor sensing domain: $r = 5,\ \theta_0 = 60°$.
- $\Delta t = 0.5$.
- System matrices: $A, C, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$.

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