Proof of consistency for arbitrary sensors:

## Probabilistic Model of Sensor

This study assumes each robot is equipped with a sensor. For the sensor of i-th robot, the observation at k-th time step is denoted as . The observation likelihood function is used to describe the probability that the  is obtained given the position of the target:

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where  is the target position, and  are set of possible observation results and target positions, respectively. The sensor model can be regarded as being parameterized by the target position. The goal of Bayesian filter is to identify the true target position .

Remark (1): Besides the target position, the sensor model also depends on the robot states, such as the position, sensor heading and other property of the sensor. Since these states are usually known in practice, we do not explicitly write them in the sensor model for the purpose of simplicity.

## Proof for static robots

For the purpose of simplicity, we assume  is finite, such as in a discretized field. Define an *equi-parameter* set  such that



Since  is finite, is also finite. Let denote all *equi-parameter* sets. By definition, the following properties hold:

(1) 

(2) 

Without loss of generality, assume , where denotes the true location of the target.

Remark (2): the *equi-parameter* sets depend on the property of the sensor. For example, for a range-only sensor, all locations with equivalent distance to the sensor belong to the same *equi-parameter* set. For a laser scanner, it is likely that every *equi-parameter* set contains only one element. In addition, when multiple robots are deployed, each *equi-parameter* set usually contains only one element.

*Theorem 1* (consistency of LEO-DBF for static robots) Under the condition of static robots, the estimation of target position obtained by LEO-DBF converges to the set of positions in when the number of observations tends to infinity, i.e.

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where ,  are the timestamps of the i-th robot’s latest knowledge of all robots’ observations.

*Proof*: Considering the conditional independence of observations  for given , the batch form of DBF at k-th step is:

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where  is *i*-th robot’s initial local PDF. It is known from the Corollary 2:

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Compare  with **:**

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Take the logarithm of (5) and average it over the k steps:

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Since, and  are bounded,

Utilizing the fact that (1)are conditionally independent and (2) , the law of large numbers yields



where denotes the Kullback–Leibler divergence. It has the property thatand the equality holds iff .

Therefore,



This implies



Therefore:



Moreover, each element in have equivalent probability mass.

(End of proof)

## Proof for moving robots

*Lemma 1*: For a set of robots moving among a set of finite possible states , there exists at least one state for each robot that it visits infinite times as k tends to infinity.

*Proof:* Let  denote the times that i-th robot is at the j-th state up to time k. Then . It is straightforward to see that .

*Theorem* *2* (consistency of LEO-DBF for moving robots) Under the condition of moving robots, the estimation of target position obtained by LEO-DBF converges to the set of positions in when the number of observations tends to infinity, i.e.

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where ,  are the timestamps of the i-th robot’s latest knowledge of all robots’ observations.

*Proof:*

Let  denote the set of (time, state) pairs to indicate j-th robot’s state at the corresponding time.

Similar to Theorem 1, the batch form of DBF at k-th step is:

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The only difference is that  needs to be grouped according to the robot state. For each robot, there exists at least one state from which the robot obtains infinite observations as , according to *lemma 1*. We can classify all the states into the finite-observation states and the infinite-observation states. For the former, it is straightforward to know that their contribution to (6) tends to zeros as .

Therefore, the proof of (7) can be reduced to infinite-observation states, which is similar to Theorem 1. (End of Proof)

Remarks (3): Since the true location of a target is unique, consistency of LEO-DBF ensures the consensus of individual PDFs.