Complimentary Material to "A Guaranteed Cost Approach to Robust Model Predictive Control of Uncertain Linear Systems"

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Abstract

This complimentary material discusses the independence of K_k and $\widetilde{K}$ controllers used at Equation (45) of the paper and presents the proof of Lemma 3.

1 Independence of controllers

Consider the system

$$\bar{x}_{k+1} = F\bar{x}_k + Gu_k + Hw_k, \tag{1}$$

and its deterministic ($w_k \equiv 0$) counterpart

$$x_{k+1} = Fx_k + Gu_k. \tag{2}$$

Consider a nil-potent controller $\widetilde{K}$ and let $u_k = -\widetilde{K}\bar{x}_k + p_k$, then the relation between the systems can be expressed as

$$\bar{x}_k = x_k + \sum_{i=0}^{k-1} (F - G\widetilde{K})^{k-i-1} Hw_i \tag{3}$$

and the closed loop deterministic system is

$$x_{k+1} = (F - G\widetilde{K})x_k + Gp_k. \tag{4}$$

Note that the control law $\widetilde{K}$ is used w.r.t. the uncertain state $\bar{x}_k$ in Equation (3).

Consider now the Guaranteed Cost Controller K_k at timestep k and let $p_k = -(K_k - \widetilde{K})x_k + v_k$, then the closed loop deterministic system becomes

$$\begin{aligned} x_{k+1} &= (F - G\widetilde{K})x_k - G(K_k - \widetilde{K})x_k + Gv_k \\ &= (F - GK_k)x_k + Gv_k. \end{aligned} \tag{5}$$

It is possible to observe that the independence of controller guaranteed cost controller K_k and nil-potent controller $\widetilde{K}$ comes from the fact that the nil-potent controller is used to stabilize the uncertain system dynamics while the guaranteed cost controller is used to stabilize the deterministic system.

2 Proof of Lemma 3

To be added.