BIEN/CENG 2310

MODELING FOR CHEMICAL AND BIOLOGICAL ENGINEERING

HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY, FALL 2022

HOMEWORK #3 (DUE OCT. 4, 2022)

1. The Lotka-Volterra model for an ecosystem consisting of a predator and a prey is as follows:

$$\begin{cases} \dfrac{dR}{dt} = \alpha R - \beta RF \quad ; \quad R(t=0) = R_0 \\ \dfrac{dF}{dt} = \gamma RF - \delta F \quad ; \quad F(t=0) = F_0 \end{cases}$$

(a) Using MATLAB's `ode45` function, simulate this system of ODEs and plot the two populations R and F, versus time t from $t = 0$ to $t = t_f$. Allow the users to specify the parameters, the initial populations, and the final time point t_f.

(b) Plot the phase space diagram F vs. R. Keeping the rate constants the same, vary the initial populations and overlay the plots for different initial populations. Can you find a combination of initial populations for which the populations will stay the same forever?

(c) In the basic Lotka-Volterra model above, it was assumed that the prey is not subject to resource constraints and follow a Malthusian growth model. Replace the birth term for the prey to relax this assumption, and use the logistic growth model instead. Modify the provided `lotkaVolterra.m` to simulate this new model. Your function definition should be:

```
Nf = lotkaVolterraLogistic_<LastName>_<FirstName>(param, tf)
```

where `param` is a vector containing the parameters $\alpha, \beta, \gamma, \delta, K, R_0, F_0$ in that order. Here, K is the carrying capacity of the prey, and α is the net birth rate of the prey at the limit of $R = 0$.

(d) Experiment with different parameter settings and observe the populations over long time. How does the general behavior differ from the basic Lotka-Volterra model? Locate any fixed point(s) for this system in your simulation, and verify that they are exactly as predicted by setting $dR/dt = dF/dt = 0$.

(e) Modify your program from Part (c) to add an event trigger to stop the simulation if/when the system reaches steady state, or at t_f, whichever is earlier. The program should still return the populations at t_f.

DELIVERABLES:

No need to submit anything for Parts (a) and (b). They will be done in class.

Modify the provided `lotkaVolterra.m` and submit the modified MATLAB program for Part (c), naming it `lotkaVolterraLogistic_<LastName>_<FirstName>.m`. Also submit two plots for this model, with your choice of parameter setting: (i) populations vs. time and (ii) the phase space diagram. Insert your name in the title of the plots, and the parameters you used in the subtitle.

In a type-written or scanned hand-written document, provide your answers for Part (d) in a write-up.

2. Two bacteria species A and B compete for nutrients in the same environment, leading to the following model for their populations, A and B, as functions of time, t:

$$\frac{dA}{dt} = r_A\left(1 - \frac{A + \beta B}{K}\right)A \qquad ; \qquad A(t = 0) = A_0$$

$$\frac{dB}{dt} = r_B\left(1 - \frac{\alpha A + B}{K}\right)B \qquad ; \qquad B(t = 0) = B_0$$

(a) Without solving the ODEs, find all the fixed point(s) of this system. Discuss the parameter settings at which the fixed point(s) are feasible. You can assume that all the parameters r_A, r_B, α, β and K are positive numbers.

(b) Write a MATLAB program to solve this system of ODEs and plot the populations over time $A(t)$ and $B(t)$ on the same graph, allowing the user to specify $r_A, r_B, \alpha, \beta, K, A_0, B_0$. The program should stop when it reaches steady state, or until $t = 10^4$, whichever is earlier, and return the final values for A and B. Your function definition should be:

```
[Af,Bf] = bacteriaCompetition_<LastName>_<FirstName>(param)
```

where `param` contains the parameters $r_A, r_B, \alpha, \beta, K, A_0, B_0$, in that order.

(c) Based on your results from Part (a), find a parameter setting for which both species will co-exist indefinitely, plot the populations versus time graphs using your program from Part (b), and verify that the observed steady-state populations A^* and B^* are indeed the fixed point(s) you predicted.

DELIVERABLES:

Submit the MATLAB program for Part (b). Remember to document your code, and name it "`bacterialCompetition_<LastName>_<FirstName>.m`"

In a typewritten or scanned hand-written document, provide your answers for Parts (a) and (c). For Part (c), submit the plot showing the "non-trivial" steady state when both species co-exist indefinitely. Insert your name in the title of the plot, and the parameters you used in the subtitle.