BIEN/CENG 2310

MODELING FOR CHEMICAL AND BIOLOGICAL ENGINEERING

HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY, FALL 2022

HOMEWORK #1 (DUE SEPT. 17, 2022)

1. A well-stirred tank, with an inlet and an outlet, is initially filled with water. The inlet stream contains a chemical dissolved in water at a fixed concentration.

   (a) Assuming the volume of the solution in the tank, V, is constant throughout the process, write down an ODE for the concentration of the chemical in the tank, $C(t)$. List any assumptions and define any parameters.

   (b) Solve the ODE in Part (b) analytically, and also simulate it using the Euler Method in Excel. Overlay the numerical solution over the analytical one.

   (c) Generalize your model for $C(t)$ to the case where V is not constant.

   (d) Solve the ODE in part (c) analytically, and also simulate it using the Euler Method in Excel. Overlay the numerical solution over the analytical one.

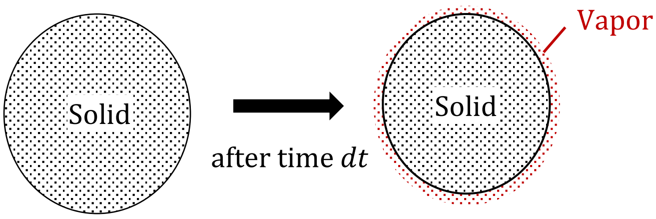
DELIVERABLES:

(i) Type-written or hand-written solution for Part (c) and the analytical solution of Part (d). Use a scanner app to take photos of your hand-written work for better image quality.

(ii) Modify the provided Excel file – which is to be done in class for Part (b) – to simulate the more general model for Part (d). Rename your Excel file “HW1_Q1_<8-digit-student-id>.xlsx”.

Note: No need to submit Parts (a) and (b), which is to be done in class in the demo.

2. One convenient way to prevent moth larvae from ruining our clothing is the use of mothballs, which slowly release a chemical vapor (typically 1,2-dicholorbenzene) in our wardrobes to repel the pests. The mechanism is that under ambient conditions, the solid repellent of the mothball sublimes to produce the vapor, and as time goes on, the mothball becomes smaller and smaller. The rate of sublimation (in g/month) is proportional to the surface area of the mothball in contact with air.



(a) Assuming the mothball is a perfect sphere throughout the process, model the radius of the mothball as a function of time, $R(t)$, as an ODE with an appropriate initial condition. Be explicit about your assumptions, and list and define all your parameters.

(b) Solve the ODE in Part (a) analytically, and provide an expression for $s(t)$, the rate of release of the vapor from one mothball.

(c) Suppose we put n mothball in a wardrobe of volume V_w, and we are interested in the concentration of the chemical vapor $L(t)$ in the wardrobe over a long period of many months. The wardrobe is not air-tight, and the air inside is continuously exchanged with fresh air outside. This can be modeled by an air stream going through the wardrobe with some small constant flow rate F (in L/month). Create a model in Excel to predict $L(t)$, allowing the user to specify all the relevant parameters. *(No need to solve this equation analytically.)*

DELIVERABLES:

(i) Type-written or hand-written solution for Parts (b) and (c), including defining systems and symbols, stating assumptions, setting up the ODEs, etc. Use a scanner app to take photos of your hand-written work for better image quality.

(ii) An Excel file with your model and the plots for Part (c), similar to what was demonstrated in class in Q1. Label everything as clearly as possible. The TA will experiment with your model as part of the grading. Do not hard-code any of the parameters, and make it easy for the TA to change them. Name your Excel file “HW1_Q2_<8-digit-student-id>.xlsx”.

Note: No need to submit Part (a), which is to be done in class in the demo.