Question 1
Part (c)

$$\frac{d^2y}{dt^2} = -\frac{gR^2}{(y+R)^2} - \frac{D}{m}\left(\frac{dy}{dt}\right)^2 sgn\left(\frac{dy}{dt}\right)$$

Boundary Conditions: $y(t=0) = y(t = t_{ground}) = 0$

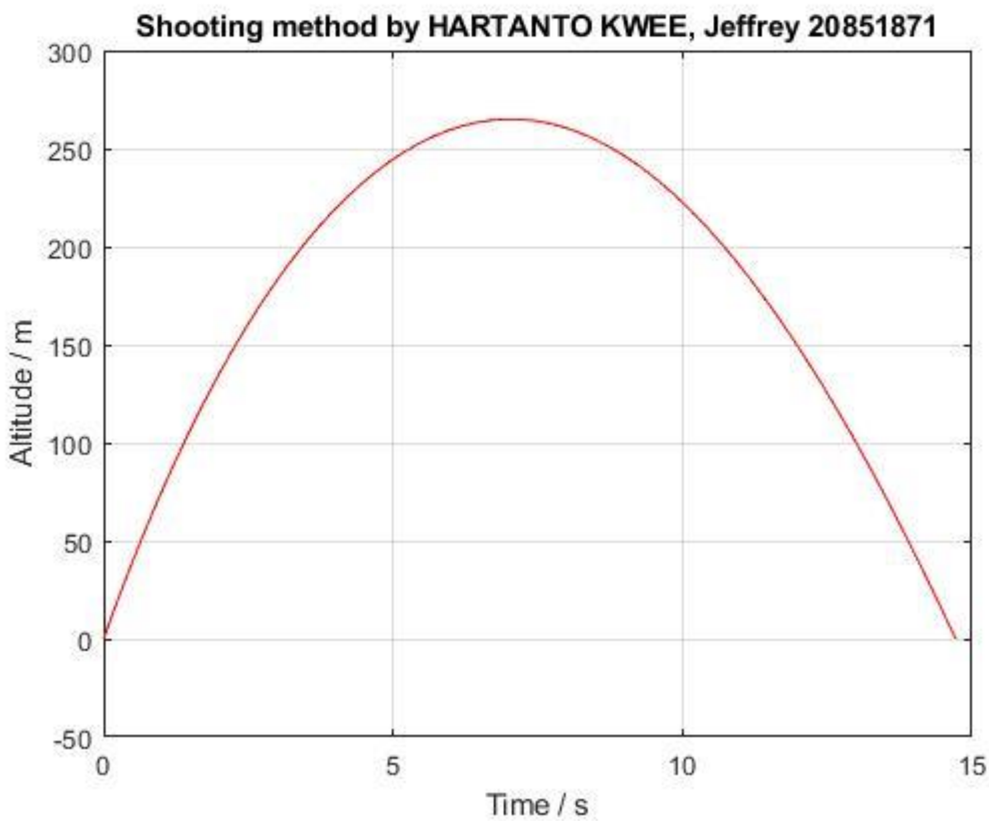
$$\left.\frac{dy}{dt}\right|_{t=t_i} = \frac{y_{i+1} - y_{i-1}}{2h}, \left.\frac{d^2y}{dt^2}\right|_{t=t_i} = \frac{\frac{y_{i+1} - y_i}{h} - \frac{y_i - y_{i-1}}{h}}{h} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

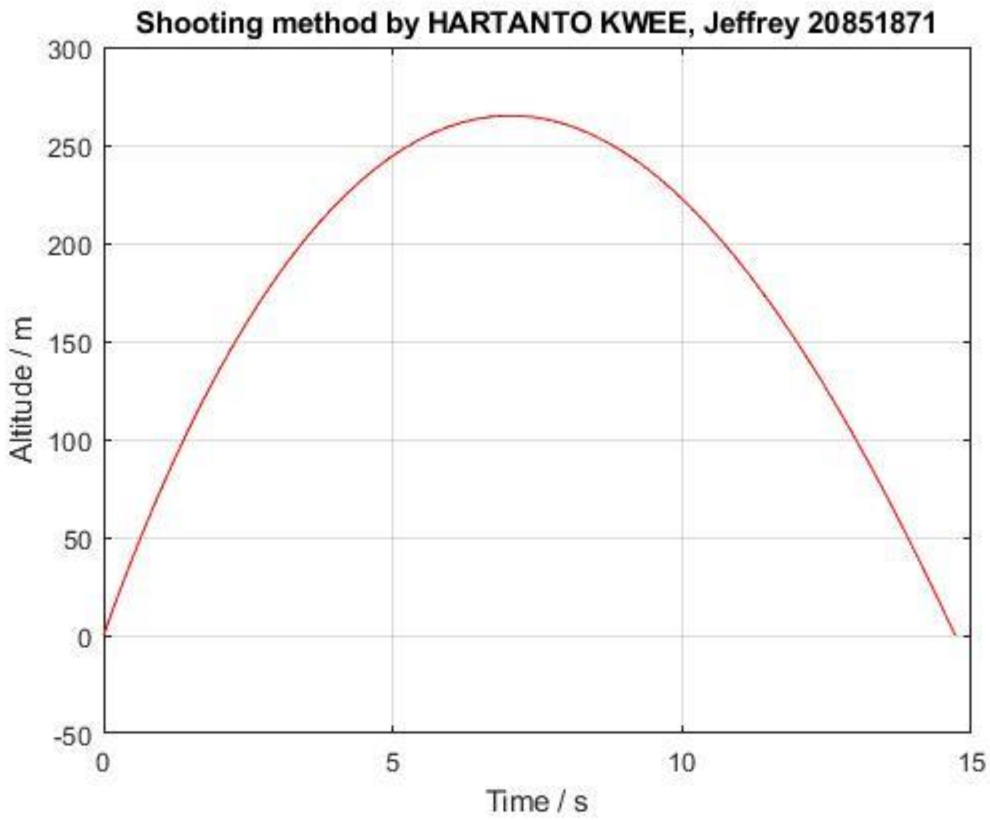
Substituting, we have

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = -\frac{gR^2}{(y_i+R)^2} - \frac{D}{m}\left(\frac{y_{i+1} - y_{i-1}}{2h}\right)\left|\frac{y_{i+1} - y_{i-1}}{2h}\right|$$

For $i = 1,2,3,\ldots,n-1$, where $t_{n-1} = t_{ground} - h$.

In total, we have $n+1$ unknowns and $n+1$ equations.




Shooting method by HARTANTO KWEE, Jeffrey 20851871
Altitude / m
Time / s
300
250
200
150
100
50
0
-50
0
5
10
15

Question 2

This is a general derivation. For part (a), $n = 2$.

$$\vec{x}_k = (x_k, y_k), \vec{v}_k = \frac{d\vec{x}_k}{dt}, \vec{a}_k = \frac{d^2\vec{x}_k}{dt^2}$$

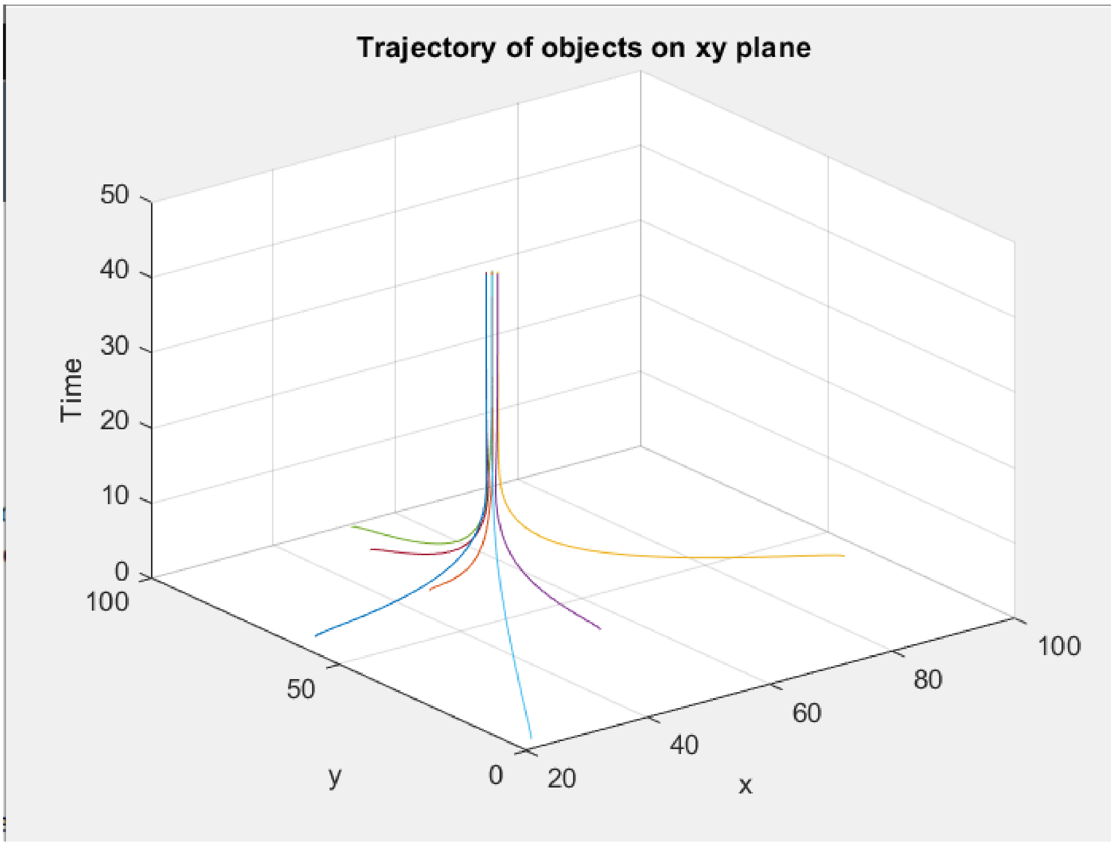
The ODE is given by Newton's second law:

$$m\vec{a}_k = \sum_{i\in[1,n]\backslash k} \vec{F}_{e,i} + \vec{F}_{s,i} + \vec{F}_{d,i}$$

$$m\vec{a}_k = \sum_{i\in[1,n]\backslash k} \left\{-\frac{q}{r^3}(\vec{x}_k - \vec{x}_i) + \alpha(\vec{x}_k - \vec{x}_i) - \sigma\vec{v}_k\right\}$$

$$\vec{a}_k = \frac{1}{m}\sum_{i\in[1,n]\backslash k} \left\{\left(-\frac{q}{|\vec{x}_k - \vec{x}_i|^3} + \alpha\right)(\vec{x}_k - \vec{x}_i) - \sigma\vec{v}_k\right\}$$

We are given the initial positions of these objects, $\vec{x}_{k,0}$, and the initial velocities of these objects are 0, $\vec{v}_{k,0} = 0$.



Question 3

C_0 concentration of dye at source
D diffusion coefficient of dye in water
v speed of the river flow
k rate constant of bacterial degradation

$$D\left(\frac{d^2C}{dx^2}\right) - v\left(\frac{dC}{dx}\right) - kC = 0$$

$$\frac{d^2C}{dx^2} = \frac{1}{D}\left(v\frac{dC}{dx} + kC\right)$$

$$C(x = 0) = C_0$$

$$\left.\frac{\partial C}{\partial x}\right|_{x=L} = 0$$

$$L = 10\,\max\left(\sqrt{\frac{d}{k}}, \frac{v}{k}\right)$$