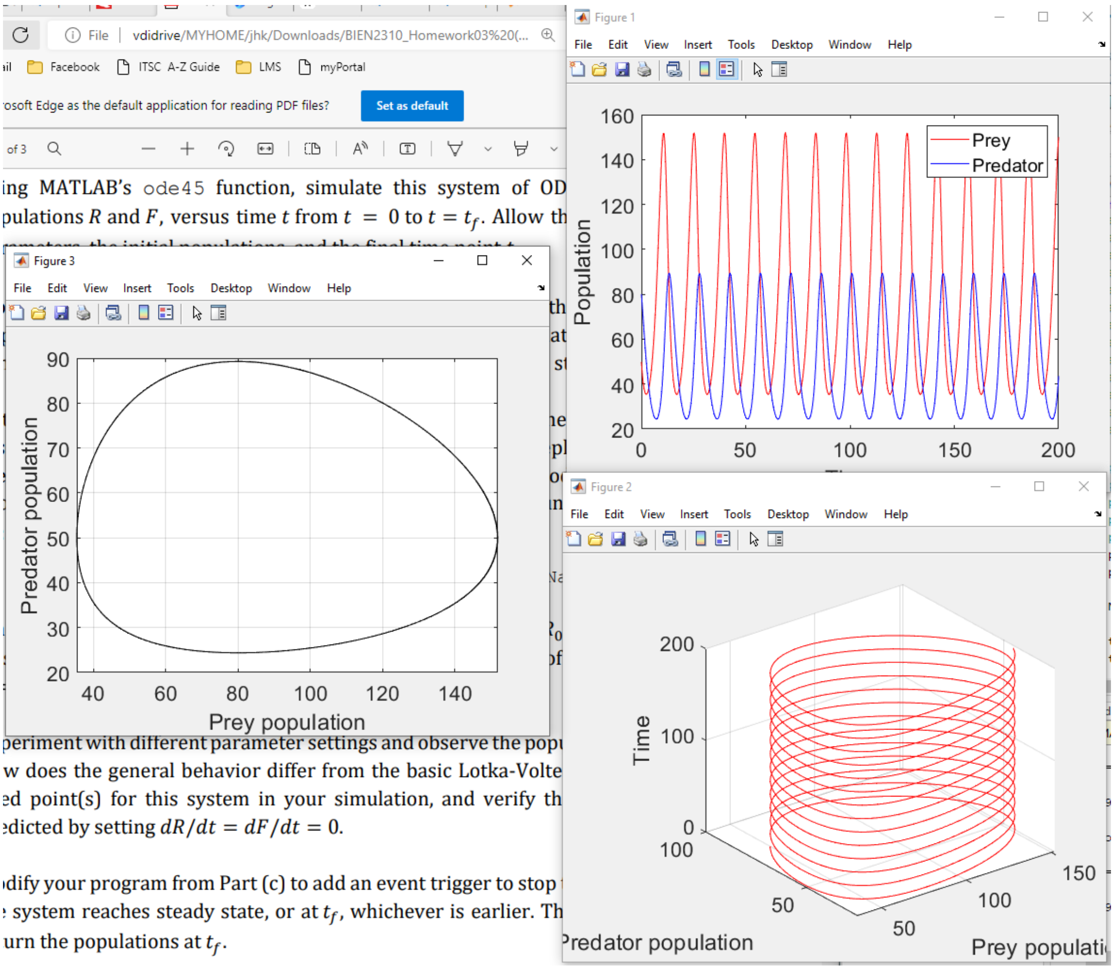
Question 1

Part (d)

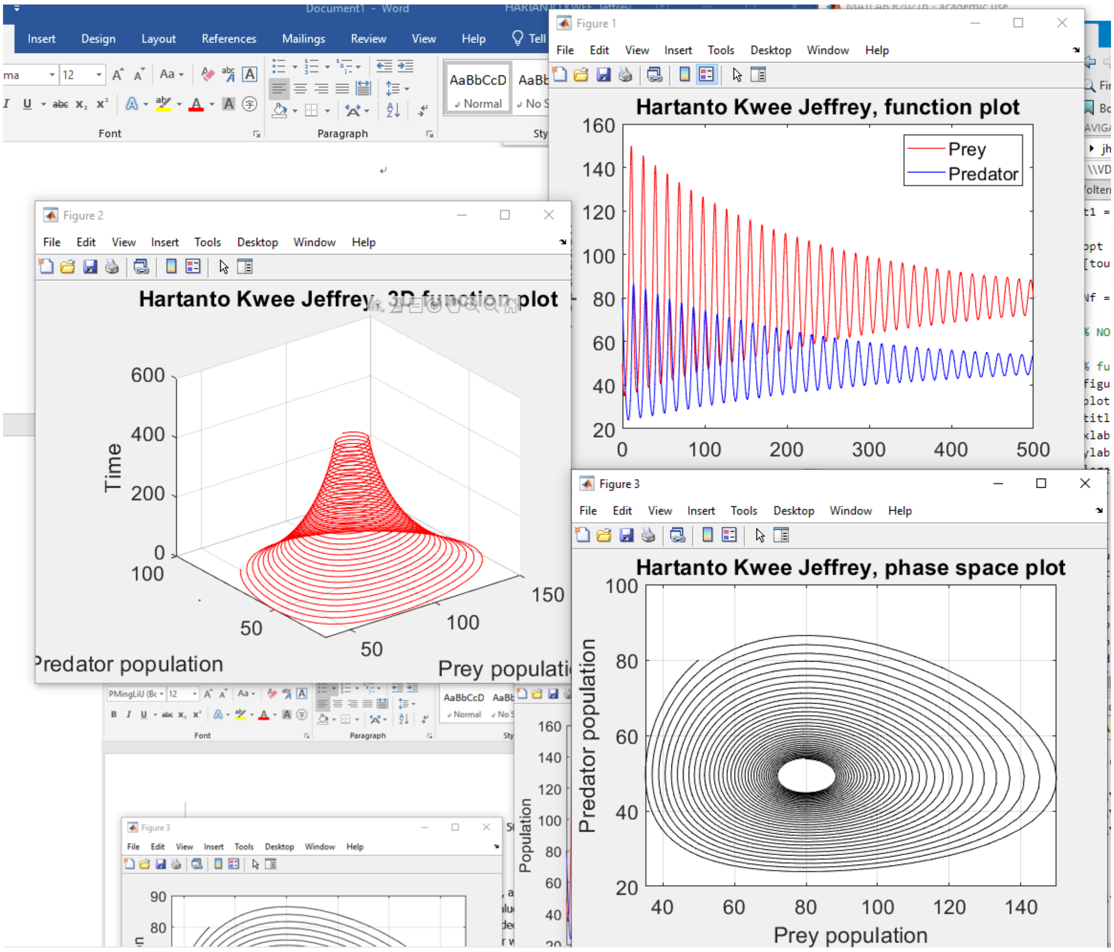
- The oscillatory behavior of the population dies down over time, and the populations of the prey and predator will approach a certain value.
- The prey carrying capacity acts as a parameter for the rate of decay. The smaller the carrying capacity, the faster the oscillatory behavior will decay. A large carrying capacity gives the limiting behavior of the usual Lotka-Volterra model.

The following is an example showing the differences between the two models:

```
lotkaVolterra([0.5, 0.01, 0.005, 0.4, 50, 80], 200)
```



```
lotkaVolterraLogistic_HartantoKwee_Jeffrey([0.5, 0.01, 0.005, 0.4, 5000, 50, 80], 500)
```



For the fixed point, let's consider the following parameters:

params = [0.5, 0.01, 0.005, 0.4, 5000, _, _]

$$\begin{cases} \dfrac{dR}{dt} = (0.5)R\left(1 - \dfrac{R}{5000}\right) - (0.01)RF \\ \dfrac{dF}{dt} = (0.005)RF - 0.4F \end{cases}$$

Solving $dR/dt = 0$ and $dF/dt = 0$,

$\dfrac{dR}{dt} = 0$ $(0.5)R\left(1 - \dfrac{R}{5000}\right) - (0.01)RF = 0$ $R = 0$ or $0.5\left(1 - \dfrac{R}{5000}\right) - 0.01F$	$\dfrac{dF}{dt} = 0$ $(0.005)RF - 0.4F = 0$ $F = 0$ or $R = \dfrac{0.4}{0.005} = 80$

Consider the second solution of $dR/dt = 0$, with F as the subject,

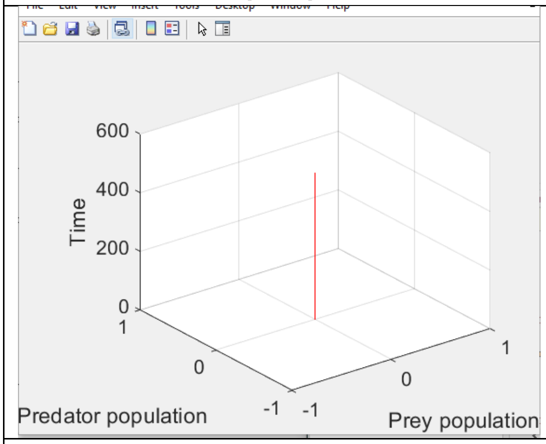
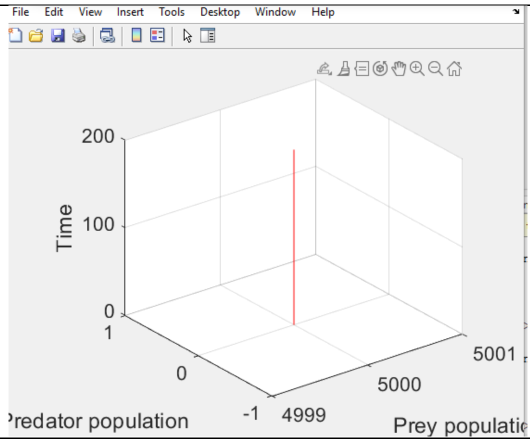
$$F = 50\left(1 - \frac{R}{5000}\right)$$

And with R as the subject,

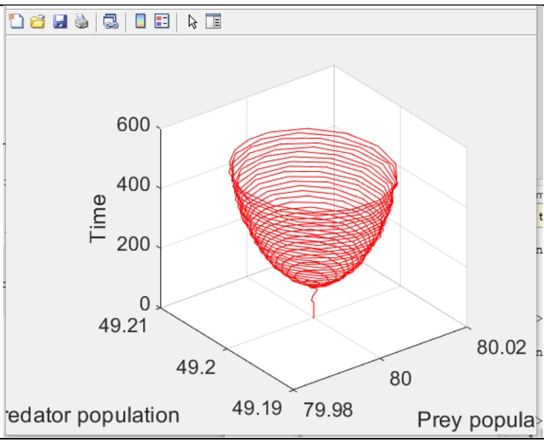
$$R = 5000\left(1 - \frac{F}{50}\right)$$

So, we have found the fixed points

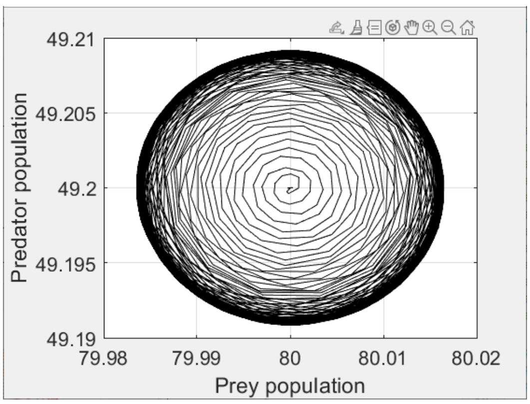
$$(R, F) = (0,0), (5000,0), (80,49.2)$$

(0, 0)	(5000, 0)
	

(80, 49.2)



(80, 49.2) is a fixed point, but the instability of numeric methods (e.g. floating-point errors) has caused the solution to deviate from the fixed point and oscillate around the fixed point). If we observe the phase diagram when we simulate for a very long time, we see that the oscillation stabilizes in a fixed radius, which is probably the precision limit of the ode solver.



Question 2
Part (a)

$$\frac{dA}{dt} = r_A\left(1 - \frac{A + \beta B}{K}\right)A \qquad ; \qquad A(t=0) = A_0$$

$$\frac{dB}{dt} = r_B\left(1 - \frac{\alpha A + B}{K}\right)B \qquad ; \qquad B(t=0) = B_0$$

Solving $dA/dt = 0$ and $dB/dt = 0$, we have

$\frac{dA}{dt} = 0$ $A = 0$ or $1 - \frac{A + \beta B}{K} = 0$ Second solution: $A + \beta B = K$	$\frac{dB}{dt} = 0$ $B = 0$ or $1 - \frac{\alpha A + B}{K} = 0$ Second solution: $\alpha A + B = K$

If $A + \beta B = K$ and $\alpha A + B = K$ simultaneously, then

$$\begin{vmatrix} 1 & \beta \\ \alpha & 1 \end{vmatrix} = 1 - \alpha\beta$$

$$A = \frac{\begin{vmatrix} K & \beta \\ K & 1 \end{vmatrix}}{1 - \alpha\beta} = \frac{K(1-\beta)}{1 - \alpha\beta}$$

$$B = \frac{\begin{vmatrix} 1 & K \\ \alpha & K \end{vmatrix}}{1 - \alpha\beta} = \frac{K(1-\alpha)}{1 - \alpha\beta}$$

Hence, the fixed points are

$$(A, B) = (0,0), (0, K), (0, K), \left(\frac{K(1-\beta)}{1-\alpha\beta}, \frac{K(1-\alpha)}{1-\alpha\beta}\right)$$

Noting that A and B are populations and are greater than zero, the final fixed point implies that $\alpha < 1$ and $\beta < 1$ such that $\alpha\beta < 1$, or $\alpha > 1$ and $\beta > 1$ such that $\alpha\beta > 1$.

Part (c)

The params used are

```
[0.5, 0.2, 0.2, 0.6, 1000, 20, 20]
```

giving us

```
Af = 454.5803
Bf = 909.0662
```

The non-trivial fixed point is

$$\left(\frac{1000(1-0.6)}{1-0.2\times 0.6}, \frac{1000(1-0.2)}{1-0.2\times 0.6}\right) \approx (454.545, 909.091)$$

which more or less matches with the steady-state populations.

The following is the plot: