Homework 2

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Part 1

Task 1.5

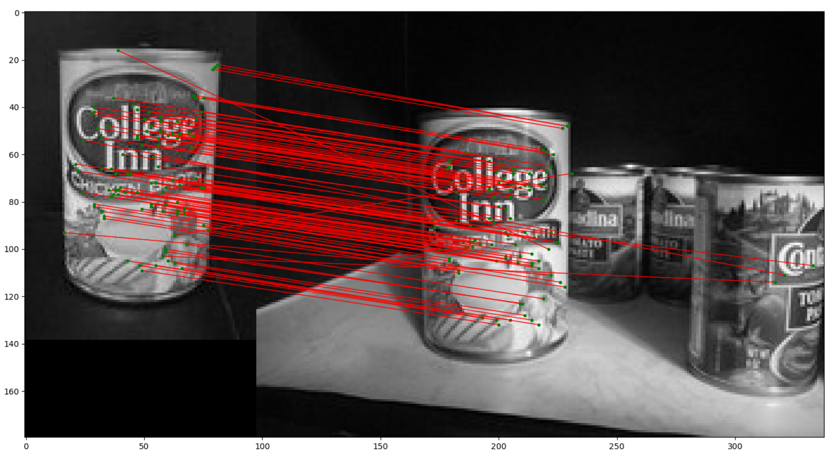
Image (model_chickenbroth.jpg) with the detected keypoints:

Part 2

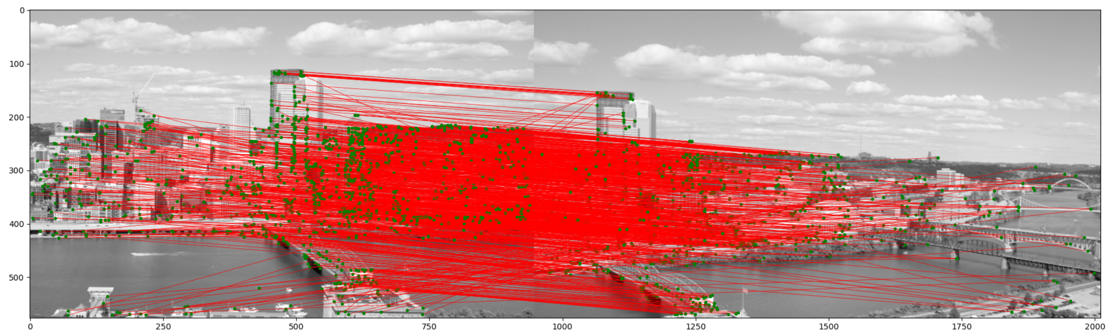
Task 2.4

Results of BRIEF descriptor matching for 3 sets of images:

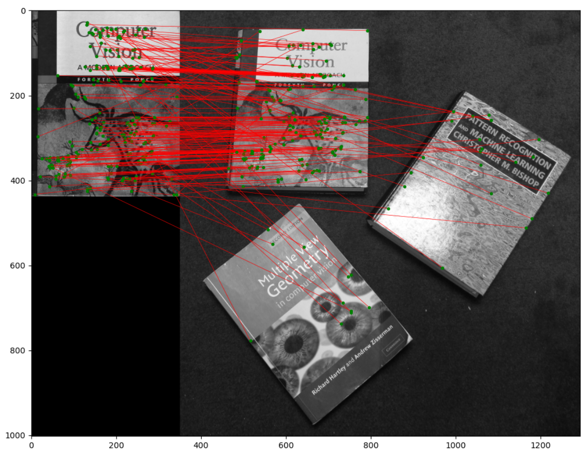
1. Left: `model_chickenbroth.jpg`, right: `chickenbroth_01.jpg`



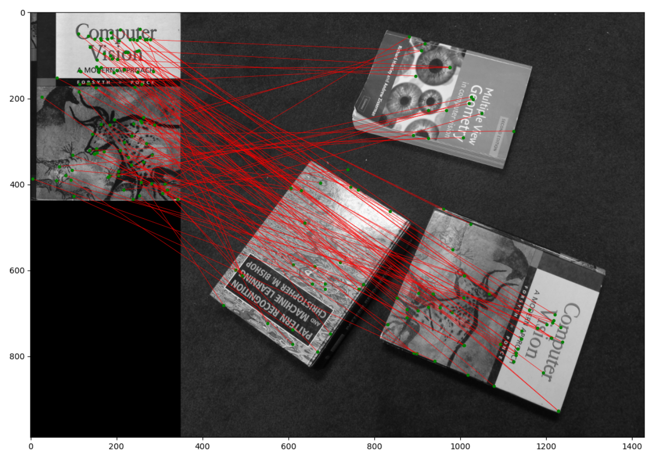
2. Left: `incline_L.png`, right: `incline_R.png`



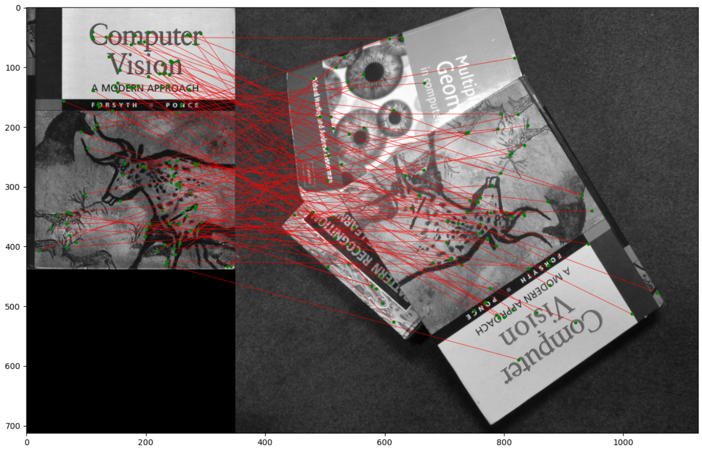
3. Left: `pf_scan_scaled.jpg`, right: `pf_floor.jpg`



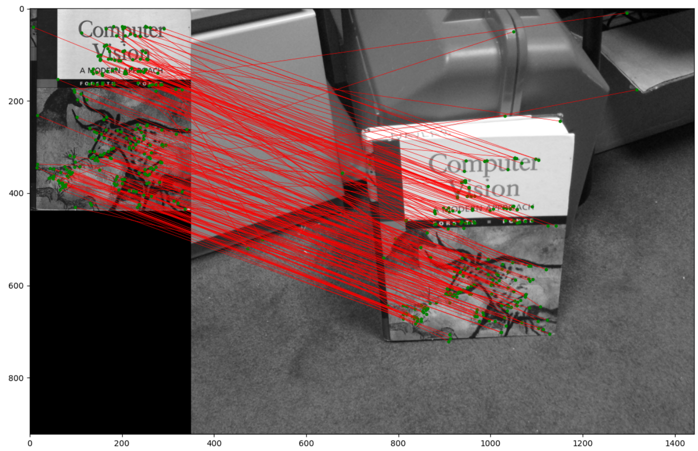
4. Left: `pf_scan_scaled.jpg`, right: `pf_floor_rot.jpg`



5. Left: `pf_scan_scaled.jpg`, right: `pf_pile.jpg`



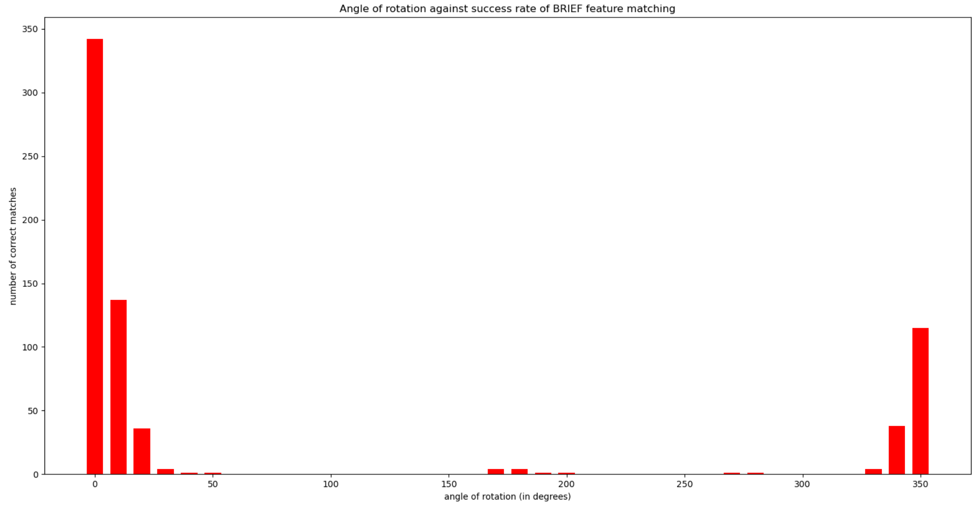
6. Left: `pf_scan_scaled.jpg`, right: `pf_stand.jpg`



Discussion: The `pf_pile.png` and `pf_floor_rot.png` image have the worst performance, with over half of the features wrongly mapped to other books and having much less matches than other images. The `pf_scan_scaled.jpg` also has incorrectly matched features. In contrast, the other images perform rather well, with most keypoints correctly mapped.

Task 2.5

Required bar graph:



The algorithm has relative high accuracy when the image is only rotated slightly, and completely fails at rotation angles greater than 20 degrees.

I believe this is due to the way the test point pairs are selected in BRIEF. Since the tests will compare the intensities between corresponding pixels in a patch around a keypoint, when the image is rotated by a large angle, the pixels being compared does not have a correspondence relationship. Slight rotations are counteracted by gaussian blur which merges rotated pixels, but large rotation will completely move the pixel away. That is why BRIEF descriptors are not rotation-invariant.

Part 3

Part (a)

For a point w on the common plane, $\tilde{u}$ and $\tilde{x}$ are the 2D projected point on the first and second camera respectively. The two are related by a relationship

$$\lambda \tilde{x} = H \tilde{u}$$

For N pairs of such points, we have

$$\lambda_n \widetilde{x_n} = H \widetilde{u_n}$$
$$\lambda_n \begin{bmatrix} \widetilde{x_{xn}} \\ x_{yn} \\ 1 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} \widetilde{u_{xn}} \\ \widetilde{u_{yn}} \\ 1 \end{bmatrix} = \begin{bmatrix} -h_1^T- \\ -h_2^T- \\ -h_3^T- \end{bmatrix} \widetilde{u_n}$$

We let $h = \begin{bmatrix} H_{11} & H_{12} & \dots & H_{33} \end{bmatrix}^T$. Then for one set of points,

$$\lambda_n \widetilde{x_{xn}} = h_1^T \widetilde{u_n}$$
$$\lambda_n \widetilde{x_{yn}} = h_2^T \widetilde{u_n}$$
$$\lambda_n = h_3^T \widetilde{u_n}$$

which can be reduced to

$$h_1^T \widetilde{u_n} - \left(h_3^T \widetilde{u_n}\right) \widetilde{x_{xn}} = 0$$
$$h_2^T \widetilde{u_n} - \left(h_3^T \widetilde{u_n}\right) \widetilde{x_{yn}} = 0$$

In matrix form,

$$A_n = \begin{bmatrix} \widetilde{u_n}^T & 0 & -\widetilde{x_{xn}}\widetilde{u_n}^T \\ 0 & \widetilde{u_n}^T & -\widetilde{x_{yn}}\widetilde{u_n}^T \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

Repeat this N times to get

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix}$$

Part (b)

There are 9 elements in h.

Part (c)

H has 9 elements, but is only defined up to scale (because it relates homogenous coordinates). Hence, it only has $9 - 1 = 8$degrees of freedom.

Each point correspondence gives two linear equations (derived above).

Therefore, we need $8/2 = 4$ point correspondences to solve the system.

Part (d)

1. Compute the SVD of A to get $U\Sigma V^T$.

2. The solutions for h is given by the rightmost column of the matrix V.

Part 6

Task 6.3

This is the final unclipped panorama:

Part 7

The code for this section can be found in `augmentedReality.py`.

Task 7.2

Here is the final image: