# Homework 4

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**What exactly is a Rodrigues vector?**

**Conventions**

We will define the fundamental matrix as

where

and the epipolar lines are nd respectively.

**Our convention**: If we define , then

**Alternative conventions**: defining would give

Hence, we should be extra cautious about the definition.

## Part 1

### Q1.1

The epipolar constraint suggests that

Since and satisfy this equation, we have

### Q1.2

Note that is the essential matrix. With a pure translation in the x-axis, we have , and . Hence , and the epipolar line for the first camera is

is a horizontal line since any point on it satisfies , and since , we have

which is the equation of a horizontal line, i.e. parallel to the -axis. Similar calculations show that is also a horizontal line.

### Q1.3

Suppose the next timestamp following is . The relationship between camera and world coordinates in both frames are and . We invert the latter to get , which we substitute into the former to get

Inverting gives

Remember that our convention is

Hence, and , and the fundamental matrix is

### Q1.4

Denote and to be the real and imaginary image of the object as viewed by our camera. Set the world coordinate system at the mirror plane. Then, the world coordinates of the objects can be let as and , where is a vector lying on the mirror plane, and is a vector perpendicular to the mirror plane. Note that is variable and is constant as the object is “flat”. If the camera is offset by rotation and translation , then the camera coordinates of the objects are and and the image coordinates are and . Hence, this set up can also be viewed as having two cameras with the same rotation matrix and intrinsic matrix but different translations and viewing the same object. Applying this to the result of Q1.3, we have and . The fundamental matrix of this setup is

Noting that is skew symmetric since , we see that

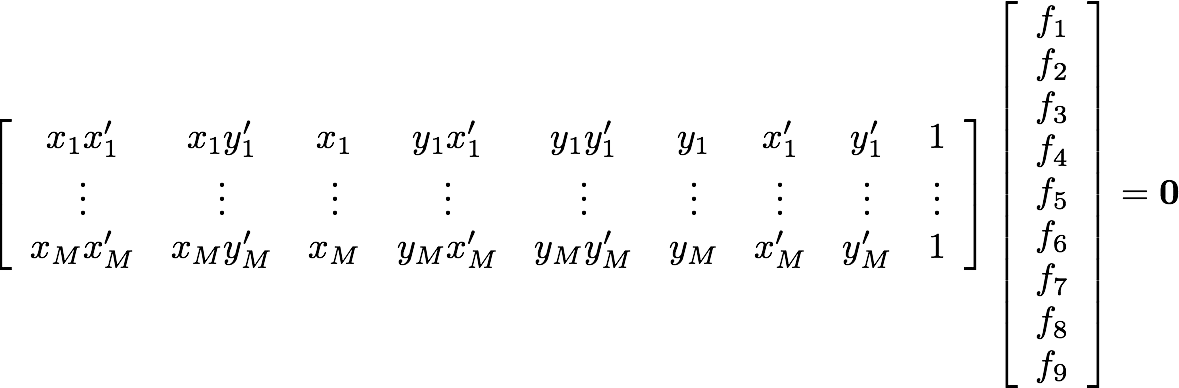
Hence, the fundamental matrix is skew-symmetric.

## Part II

### Q2.1

Carefully following our conventions, is given by

In this case, and . Referring to the lecture notes, our system is given by



[[-8.33149234e-09 1.29538462e-07 -1.17187851e-03]

[ 6.51358336e-08 5.70670059e-09 -4.13435037e-05]

[ 1.13078765e-03 1.91823637e-05 4.16862079e-03]]

### Q3.1

By our conventions,

Hence

### Q3.2

Note: we omit the index in the following derivation.

The relationship between the camera matrix , 3D object point and 2D image point is

Then, we have

where is a column vector containing the elements of the th row of . We obtain three linear equations from this

Substituting the third equality into the first two, we have

In matrix form,

We can get a similar matrix from the second camera:

Therefore, the homogenous system we want to find is

How are babies made, you ask:

[[ 0.03519757 0.03519757 -0.07009456 -0.07009456]

[ 0.96623389 0.96623389 0.99394507 0.99394507]

[-0.25525126 -0.25525126 0.08461648 0.08461648]]

[[ 0.03519757 0.03519757 -0.07009456 -0.07009456]

[ 0.96623389 0.96623389 0.99394507 0.99394507]

[-0.25525126 -0.25525126 0.08461648 0.08461648]]

Recall that , where is the essential matrix. With a pure translation in the x-axis, we have , and . Note that the intrinsic matrices are affine, and their inverses are also affine. The fundamental matrix will take the form:

Epipolar lines are given by and . Hence,

5.2

The Rodrigues vector represents a rotation using the axis-angle representation:

where is a unit vector for the rotation, and is the angle of rotation. We strongly emphasize here that this is different from the axis-angle representation where . A rotation is uniquely represented by the direction (up/down) of the rotation vector and the angle which can take up values within . These properties account for the full rotation around a rotation vector. The zero-rotation can be uniquely represented by . However, note that rotations at an angle are not defined, and we represent all rotations at with the infinity vector in our implementation.

References which specifically mention the term “Rodrigues’ vector”:

* <https://www.ctcms.nist.gov/~langer/oof2man/RegisteredClass-Rodrigues.html>
* <https://www.researchgate.net/publication/231052227_Rotations_with_Rodrigues'_vector>

The original definition of the Rodrigues’ vector comes from Rodrigues’ own publications in French, so we will mainly refer to the second reference above for our implementation.

A rotational matrix can be written as

where .

To obtain from , simply normalize , find and construct from the given formula.

To obtain from , we refer to the following four references:

<https://handwiki.org/wiki/Rodrigues%27_rotation_formula>

<https://en.wikipedia.org/wiki/Rotation_matrix>

<https://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.110.5134>

<https://courses.cs.duke.edu/fall13/compsci527/notes/rodrigues.pdf>