The type of questions you will practice in this level are

* Quadratic Equations
* Graphs of Quadratic Functions

A quadratic equation is in the form of 𝒂𝒙² + 𝒃𝒙 + 𝒄 = 0. Just as 𝒙² = 1 has two solutions, quadratic equations can also have (and at most have) 2 solutions. Note the word “can” because there may be “repeated roots”.

A quadratic equation may be factorized into the form (𝒙 − 𝒓₁)(𝒙 − 𝒓₂) = 0, and the solutions are 𝒙 = 𝒓₁or 𝒓₂. Several methods may be applied for the factorization, which can be found in this website:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒇𝒂𝒄𝒕𝒐𝒓𝒊𝒏𝒈-𝒒𝒖𝒂𝒅𝒓𝒂𝒕𝒊𝒄𝒔.𝒉𝒕𝒎𝒍

For a more general method of solving quadratic equations, the roots are given by the quadratic formula. See:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒒𝒖𝒂𝒅𝒓𝒂𝒕𝒊𝒄-𝒆𝒒𝒖𝒂𝒕𝒊𝒐𝒏.𝒉𝒕𝒎𝒍

As an exercise, try deriving the sum of roots, 𝒓₁ + 𝒓₂, and product of roots, 𝒓₁𝒓₂, in terms of 𝒂, 𝒃 and 𝒄. You will find that 𝒓₁ + 𝒓₂ = − 𝒃/𝒂 and 𝒓₁𝒓₂ = 𝒄/𝒂.

A quadratic graph has the form 𝒚 = 𝒂𝒙² + 𝒃𝒙 + 𝒄. It is a parabola. There are several properties of the parabola which can be found from the equation, namely the opening direction, the number of x-intercepts, the coordinates of vertices etc. Refer to the following website:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒒𝒖𝒂𝒅𝒓𝒂𝒕𝒊𝒄-𝒆𝒒𝒖𝒂𝒕𝒊𝒐𝒏-𝒈𝒓𝒂𝒑𝒉𝒊𝒏𝒈.𝒉𝒕𝒎𝒍

The “Completing the Square” method may be applied to find the maximum or minimum value of a quadratic formula. It involves writing 𝒂𝒙² + 𝒃𝒙 + 𝒄 into the form 𝒂(𝒙 – 𝒉)^2 + 𝒌. The maximum value is 𝒌, achieved at 𝒙 = 𝒉. For the derivation of the formula of 𝒉 and 𝒌, see the following website:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒄𝒐𝒎𝒑𝒍𝒆𝒕𝒊𝒏𝒈-𝒔𝒒𝒖𝒂𝒓𝒆.𝒉𝒕𝒎𝒍

The following website covers interesting real world applications of quadratic equations in the field of physics. It does not require any knowledge about physics so don’t be afraid to read it:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒒𝒖𝒂𝒅𝒓𝒂𝒕𝒊𝒄-𝒆𝒒𝒖𝒂𝒕𝒊𝒐𝒏-𝒓𝒆𝒂𝒍-𝒘𝒐𝒓𝒍𝒅.𝒉𝒕𝒎𝒍

Have fun mathing!

The type of questions you will practice in this level is

* Arithmetic Sequences

Formula sheet:

General formula: 𝑻(𝒏) = 𝒂 + 𝒅(𝒏 − 1),

where 𝑻(𝒏) is the 𝒏 th term of the sequence

𝒂 is the first term, and

𝒅 is the common difference.

𝑺(𝒏) = ½ 𝒏(𝒂 + 𝒍) = ½ 𝒏(2𝒂 + 𝒅(𝒏 − 1))

where 𝑺(𝒏) is the sum of the first 𝒏 terms of the sequence, and

𝒍 = 𝒂 + 𝒅(𝒏 − 1) is the last term of the section of the sequence.

For a summary of the definition of an AS and the sum, visit the following website:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒔𝒆𝒒𝒖𝒆𝒏𝒄𝒆𝒔-𝒔𝒖𝒎𝒔-𝒂𝒓𝒊𝒕𝒉𝒎𝒆𝒕𝒊𝒄.𝒉𝒕𝒎𝒍

The type of questions you will practice in this level is

* Geometric Sequence
* Polynomials: the factor theorem and the remainder theorem

Note: There will be questions in this game where you have to guess a root in the polynomial, the factorize the whole thing (Don’t blame me ☺). The range of the roots are given as follows:

Integer: from -7 to 7

Fraction: denominators are not greater than 6, the value is not greater than 2.

For a summary of Geometric Sequences, visit

𝒉𝒕𝒕𝒑𝒔://𝒆𝒏.𝒘𝒊𝒌𝒊𝒑𝒆𝒅𝒊𝒂.𝒐𝒓𝒈/𝒘𝒊𝒌𝒊/𝑷𝒐𝒍𝒚𝒂𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒔𝒆𝒒𝒖𝒆𝒏𝒄𝒆𝒔-𝒔𝒖𝒎𝒔-𝒈𝒆𝒐𝒎𝒆𝒕𝒓𝒊𝒄.𝒉𝒕𝒎𝒍

(The formulas cannot be displayed here well, so no formula sheet is given.)

The division algorithm of a polynomial is given by

𝑷(𝒙) = 𝑫(𝒙) 𝑸(𝒙) + 𝑹(𝒙)

where 𝑷 is a polynomial (and the dividend), 𝑫 is the divisor, and 𝑸, 𝑹 are the quotient and remainder respectively. Visit this website for examples:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒂𝒑𝒍𝒖𝒔𝒕𝒐𝒑𝒑𝒆𝒓.𝒄𝒐𝒎/𝒅𝒊𝒗𝒊𝒔𝒊𝒐𝒏-𝒂𝒍𝒈𝒐𝒓𝒊𝒕𝒉𝒎-𝒇𝒐𝒓-𝒑𝒐𝒍𝒚𝒏𝒐𝒎𝒊𝒂𝒍𝒔/

The remainder theorem states that when a polynomial 𝑷(𝒙) is divided by 𝒙 – 𝒂, the remainder is 𝑷(𝒂). An immediate consequence is that when 𝑷(𝒂) is 0, 𝒙 – 𝒂 is a root of 𝑷(𝒙). This is the factor theorem. Please refer to this website:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒂𝒍𝒈𝒆𝒃𝒓𝒂/𝒑𝒐𝒍𝒚𝒏𝒐𝒎𝒊𝒂𝒍𝒔-𝒓𝒆𝒎𝒂𝒊𝒏𝒅𝒆𝒓-𝒇𝒂𝒄𝒕𝒐𝒓.𝒉𝒕𝒎𝒍

Read the following for a definition of the GCF of integers.

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒎𝒂𝒕𝒉𝒔𝒊𝒔𝒇𝒖𝒏.𝒄𝒐𝒎/𝒈𝒓𝒆𝒂𝒕𝒆𝒔𝒕-𝒄𝒐𝒎𝒎𝒐𝒏-𝒇𝒂𝒄𝒕𝒐𝒓.𝒉𝒕𝒎𝒍

This concept applies to polynomials: factorize the polynomials, and find the GCF by comparing these factors.

Quick Maths (Must Read):

The following is an article about the long division of polynomials:

𝒉𝒕𝒕𝒑𝒔://𝒆𝒏.𝒘𝒊𝒌𝒊𝒑𝒆𝒅𝒊𝒂.𝒐𝒓𝒈/𝒘𝒊𝒌𝒊/𝑷𝒐𝒍𝒚𝒏𝒐𝒎𝒊𝒂𝒍\_𝒍𝒐𝒏𝒈\_𝒅𝒊𝒗𝒊𝒔𝒊𝒐𝒏

There is a very quick way of calculating polynomials. The following articles introduces this method:

𝒉𝒕𝒕𝒑𝒔://𝒘𝒘𝒘.𝒑𝒖𝒓𝒑𝒍𝒆𝒎𝒂𝒕𝒉.𝒄𝒐𝒎/𝒎𝒐𝒅𝒖𝒍𝒆𝒔/𝒔𝒚𝒏𝒕𝒉𝒅𝒊𝒗.𝒉𝒕𝒎 (Recommended)

𝒉𝒕𝒕𝒑𝒔://𝒆𝒏.𝒘𝒊𝒌𝒊𝒑𝒆𝒅𝒊𝒂.𝒐𝒓𝒈/𝒘𝒊𝒌𝒊/𝑺𝒚𝒏𝒕𝒉𝒆𝒕𝒊𝒄\_𝒅𝒊𝒗𝒊𝒔𝒊𝒐𝒏