

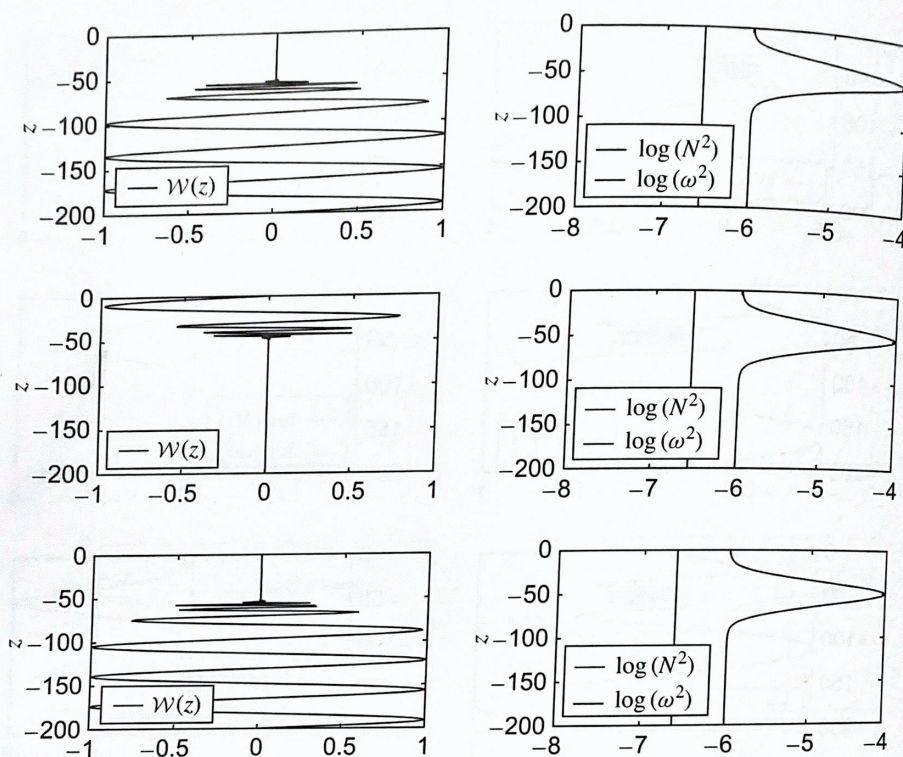
FIGURE 13.8 Modes 10–12 for the same nonuniform stratification as in the previous figure. Again, the left panel shows the vertical structure of the modes, whereas the right panel compares their  $\omega^2$  values to the  $N^2$  values. Note the fine structure in the vicinity of the pycnocline and that the values of  $\omega^2$  fall below the minimum  $N^2$  value.

twice across a peak in  $N^2$ , a pycnocline is accompanied by two turning points, one below and one above.

For higher modes (Fig. 13.8),  $\omega^2$  decreases and approaches the minimum of  $N^2$ . The turning points move away from the pycnocline until they disappear when, for high enough mode numbers, the corresponding  $\omega^2$  values fall below the  $N^2$  minimum. Those higher modes have a structure that is oscillatory everywhere (Fig. 13.8). Surprisingly, amplitudes are now lowest near the pycnocline. This is due to the fact that the frequency difference is maximum ( $\omega^2 - N^2$  largest) near the pycnocline, and resonant behavior is thus stronger away from the pycnocline, where the amplitude is consequently higher.

For even higher modes, the modal frequency  $\omega$  approaches the inertial frequency  $f$ , and a new behavior emerges (Fig. 13.9). The regions above and below the pycnocline start to be decoupled, with one mode being almost entirely confined to one side of the pycnocline, the next mode to the other side, and so alternatively with mode number. The pycnocline appears to act as a barrier. In the limit of an extremely sharp pycnocline (very high  $N^2$  peak), the





**FIGURE 13.9** Modes 26–28 for the same nonuniform stratification as in the previous two figures. Again, the left panel shows the vertical structure of the modes, whereas the right panel compares their  $\omega^2$  values to the  $N^2$  values. These modes, for which the frequencies approach the inertial frequency  $f$ , are almost entirely confined on one side or the other of the pycnocline. In other words, a pycnocline acts as a vertical barrier to near-inertial waves.

stratification effectively becomes a two-layer system, for which waves near the inertial frequency can exist in each layer independently of the other.

### 13.5 LEE WAVES

Internal waves in the atmosphere and ocean can be generated by a myriad of processes, almost wherever a source of energy has some temporal or spatial variability. Oceanic examples include the ocean tide over a sloping bottom, mixing processes in the upper ocean (especially during a hurricane), instabilities of shear flows, and the passage of a submarine. In the atmosphere, one particularly effective mechanism is the generation of internal waves by a wind blowing over an irregular terrain such as a mountain range or hilly countryside. We select the latter example to serve as an illustration of internal-wave theory because it has some meteorological importance and lends itself to a simple mathematical treatment.

To apply the previous linear-wave theory, we naturally restrict our attention to small-amplitude waves and, consequently, to small topographic irregularities.