

# Towards multi-drug adaptive therapy

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CATMo2020

# Towards multi-drug adaptive therapy



1.

CANCER RESEARCH

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Research Article

Turnover modulates the need for a cost of resistance in adaptive therapy

Maximilian A.R. Strobl, Jeffrey West, Yannick Viossat, Mehdi Damaghi, Mark Robertson-Tessi, Joel S Brown, Robert A Gatenby, Philip K Maini, and Alexander R.A. Anderson

DOI: 10.1158/0008-5472.CAN-20-0806 Check for updates

2.

## Spatial structure impacts adaptive therapy by shaping intra-tumoral competition

Maximilian A. R. Strobl, Jill Gallaher, Jeffrey West, Mark Robertson-Tessi, Philip K. Maini, Alexander R.A. Anderson

doi: <https://doi.org/10.1101/2020.11.03.365163>

This article is a preprint and has not been certified by peer review [what does this mean?].

3.

Translational Cancer Mechanisms and Therapy

Clinical Cancer Research

Multidrug Cancer Therapy in Metastatic Castrate-Resistant Prostate Cancer: An Evolution-Based Strategy

Jeffrey B. West<sup>1</sup>, Mina N. Dinh<sup>1,2</sup>, Joel S. Brown<sup>1</sup>, Jingsong Zhang<sup>3</sup>, Alexander R. Anderson<sup>1</sup>, and Robert A. Gatenby<sup>1</sup>

Check for updates

4.

## CANCER RESEARCH | CONVERGENCE AND TECHNOLOGIES

### Towards Multidrug Adaptive Therapy

Jeffrey West<sup>1</sup>, Li You<sup>2</sup>, Jingsong Zhang<sup>3</sup>, Robert A. Gatenby<sup>1</sup>, Joel S. Brown<sup>1,4</sup>, Paul K. Newton<sup>5</sup>, and Alexander R.A. Anderson<sup>1</sup>



Clinically feasible today



1

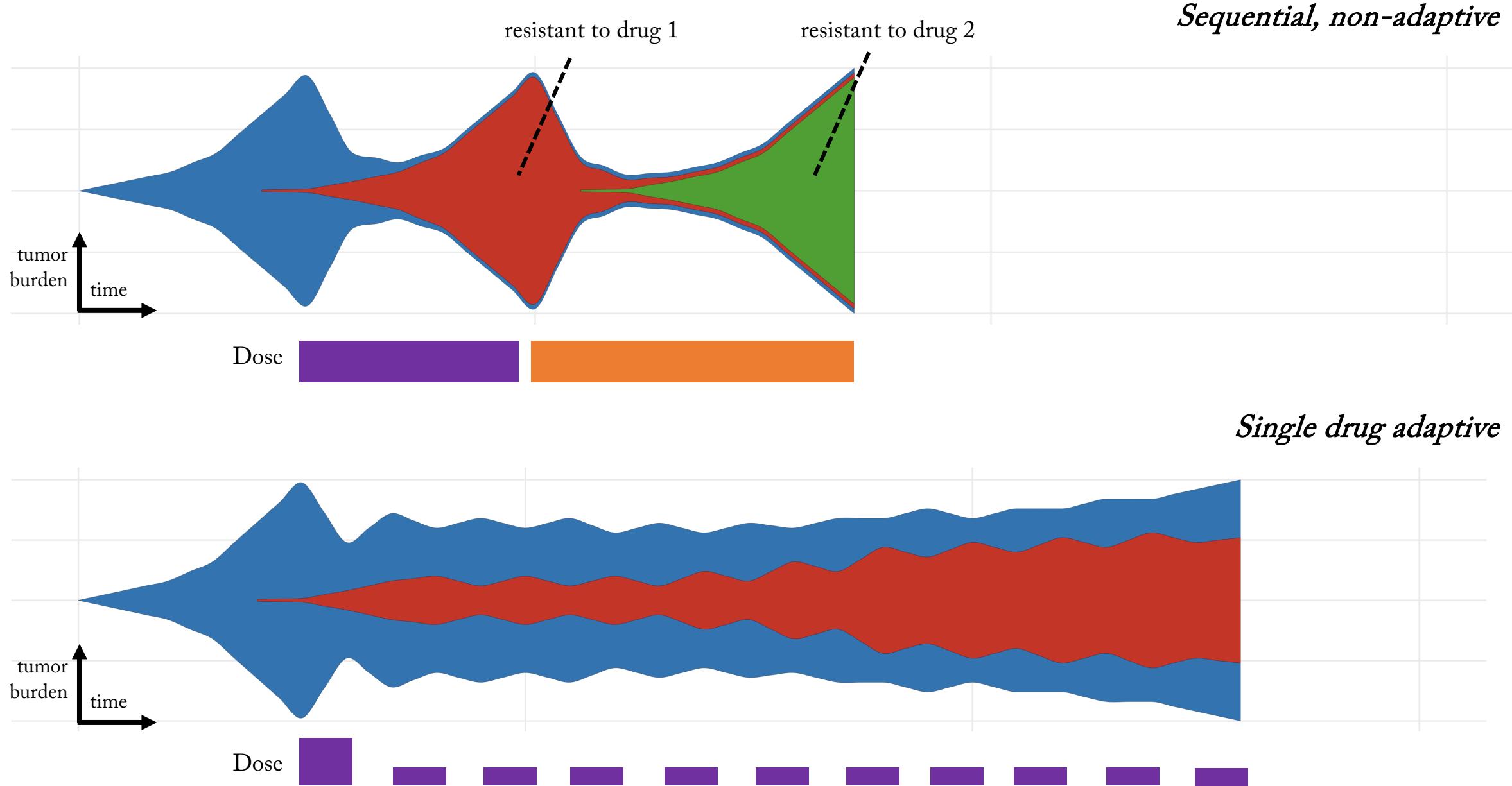
2

3

4

Conceptual ideas for tomorrow

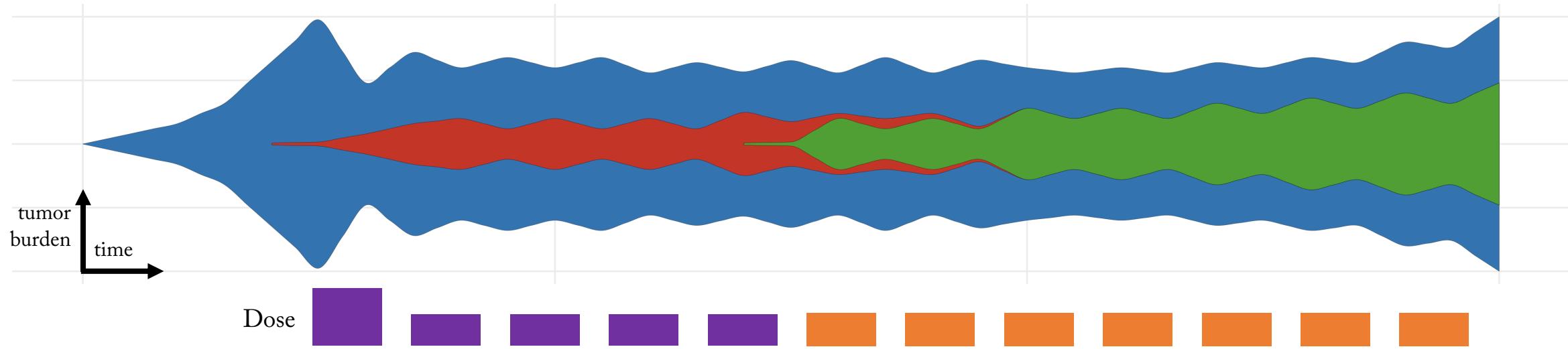
# Sequential and Adaptive Therapy



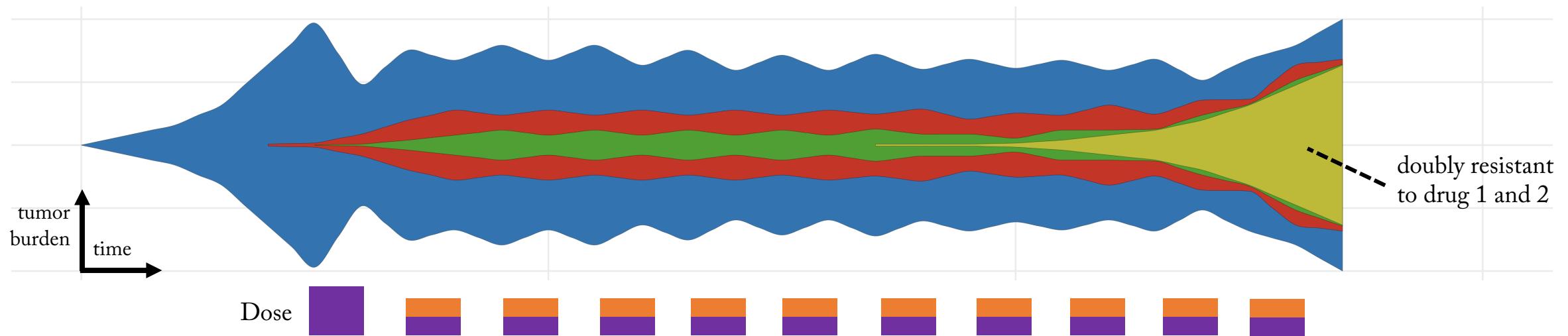
# Designing Adaptive Therapy schedules w/ multiple drugs



*Two drug combination adaptive*



*Two drug sequential adaptive*



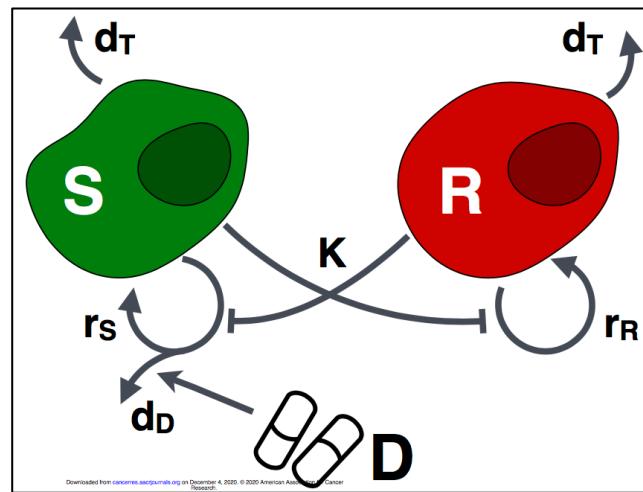
doubly resistant  
to drug 1 and 2

# What is the purpose of adding a second drug?



## The model:

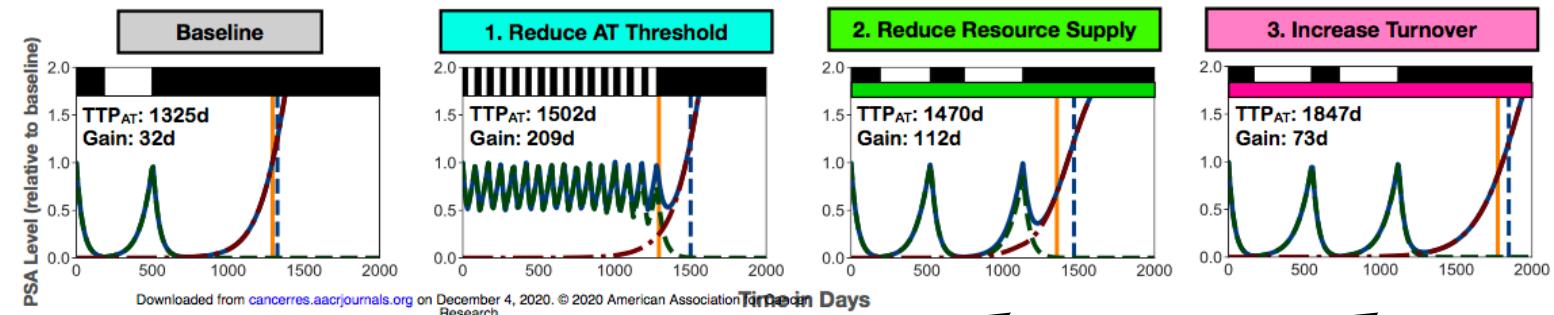
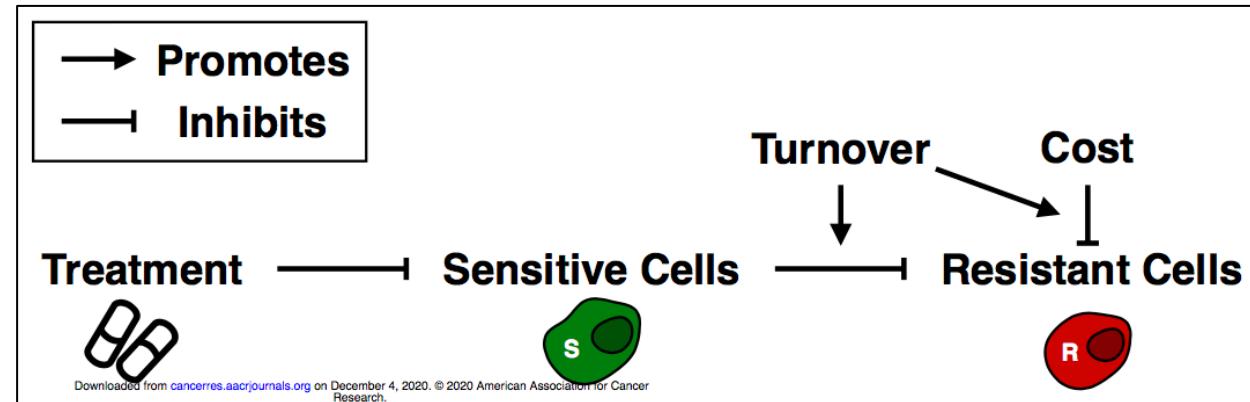
- Lotka-Volterra competition
- Norton-Simon drug effect



$$\frac{dS}{dt} = r_S \left(1 - \frac{S+R}{K}\right) \left(1 - \frac{2d_D}{D_{Max}} D(t)\right) S - d_TS,$$

$$\frac{dR}{dt} = r_R \left(1 - \frac{R+S}{K}\right) R - dTR,$$

(Strobl, et. al. Cancer Research 2020)



## Idea:

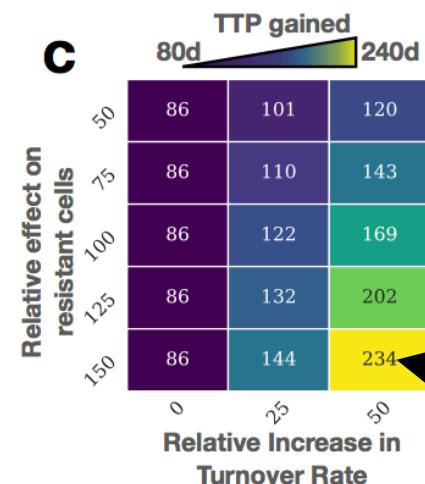
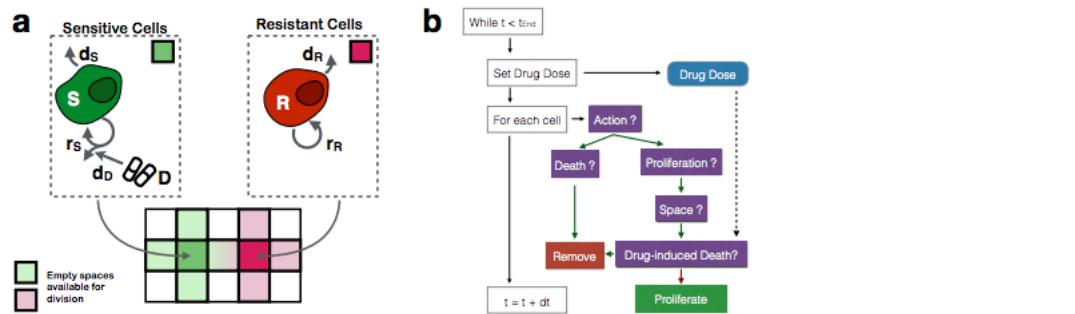
- Targeting shared resources or increasing turnover increases benefit of single drug Adaptive Therapy

# Pro-turnover treatment (off treatment cycle)

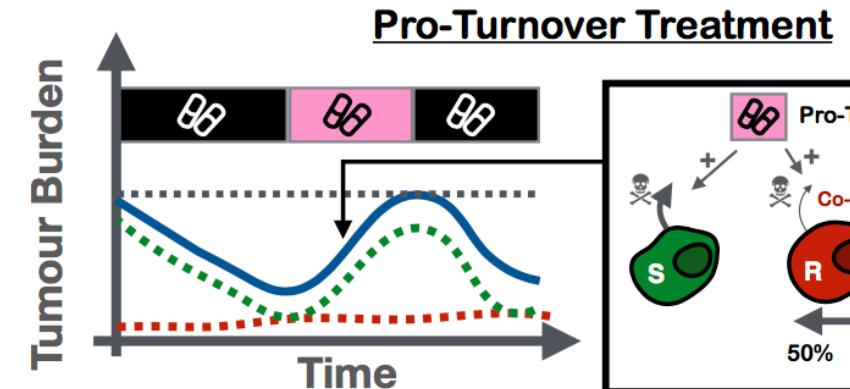


## The model:

- Agent-based 2-d analog of Lotka-Volterra model



**a**



## Idea:

- Turnover increases competition – can we increase competition during off treatment periods?

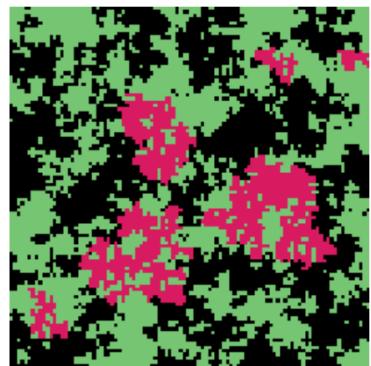
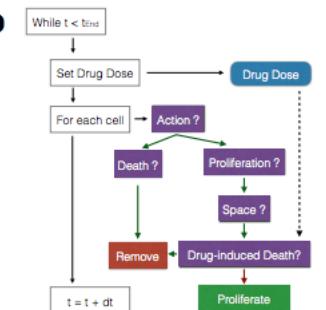
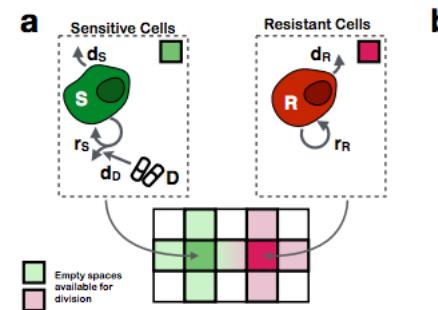
Increasing turnover, targeted to resistant cells

# Pro-proliferation treatment (off treatment cycle)

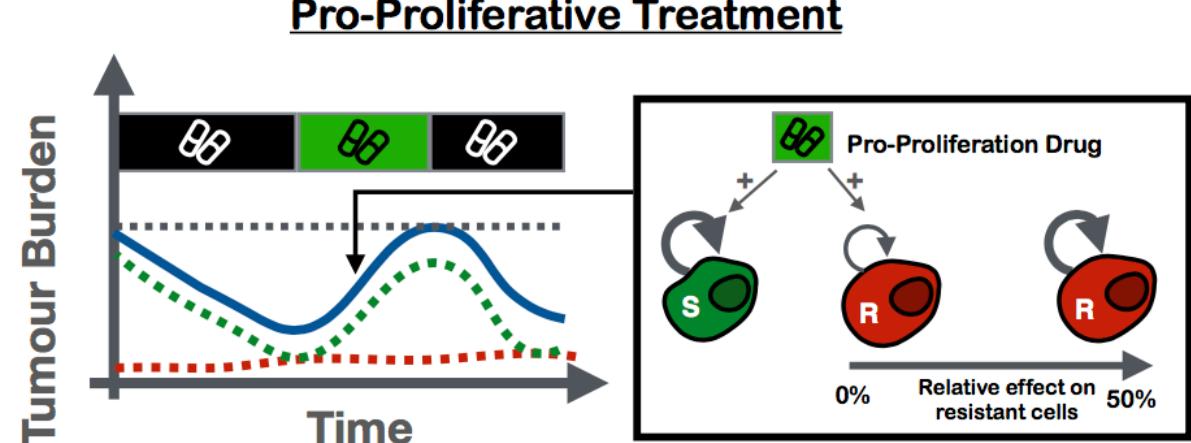
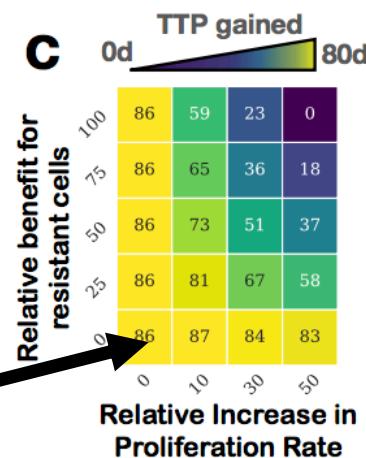


## The model:

- Agent-based 2-d analog of Lotka-Volterra model



Optimal is to leave well-enough alone!



## Idea:

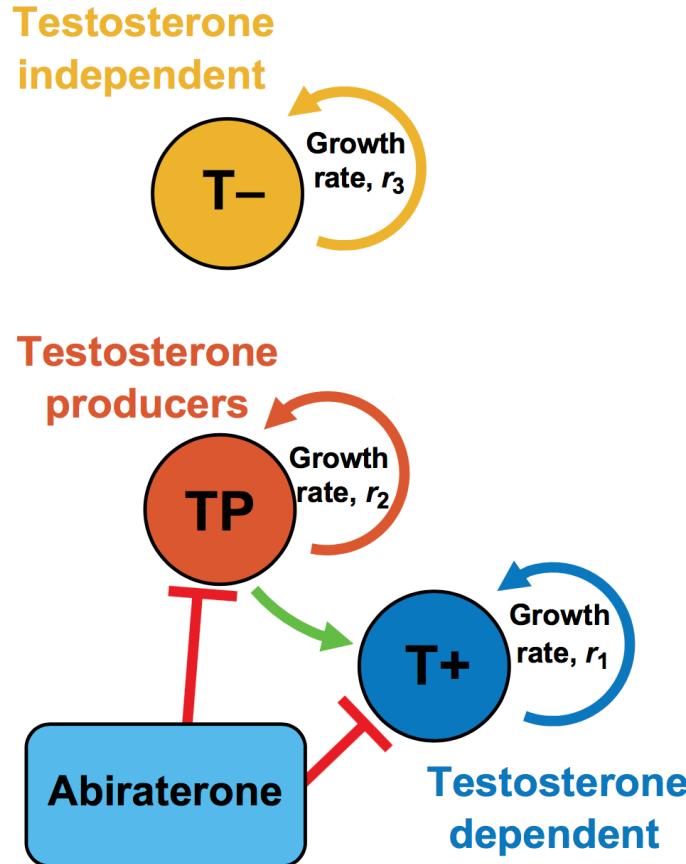
- Sensitive cells suppress Resistant -- can we boost competition by increasing proliferation off treatment?

# **1. Primary-secondary adaptive therapy**

**"What's the difference between theory and practice? Small in theory; large in practice."**

- Author attribution unknown -

# Primary-secondary therapy

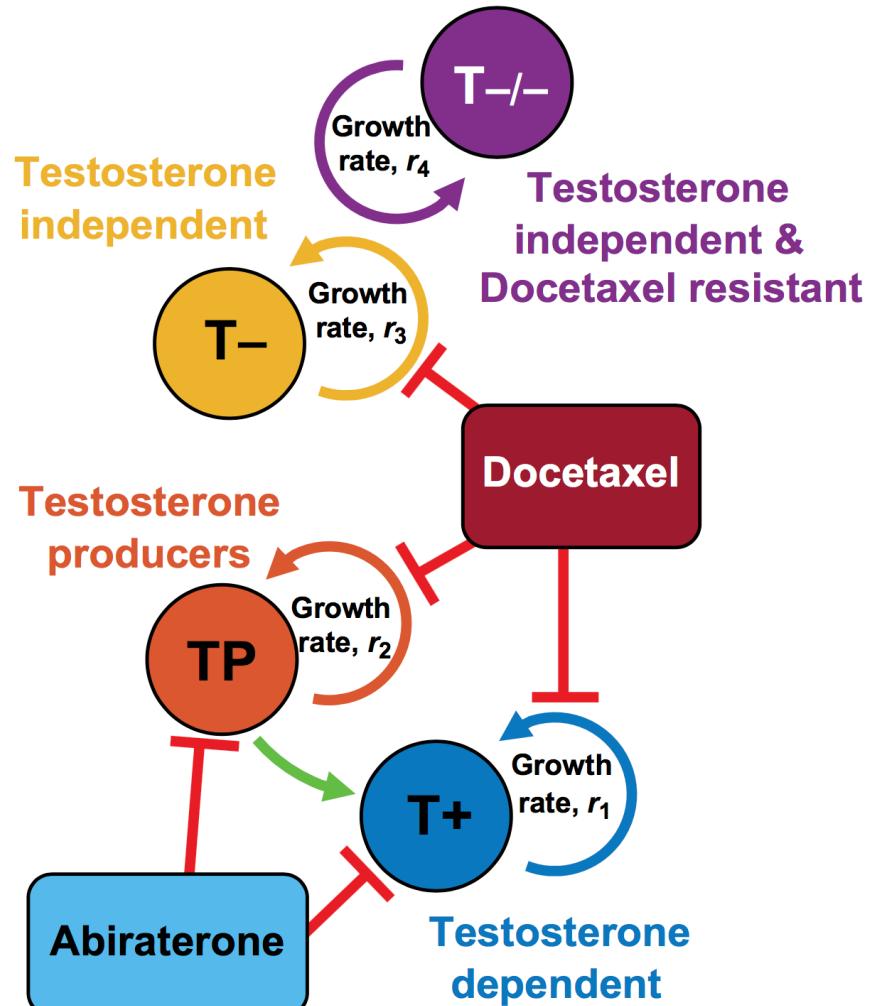


## Cell Populations:

- Testosterone **producers** (TP)
- Testosterone **dependent** (T+)
- Testosterone **independent**

$$\dot{y}_i = r_i y_i \left( 1 - \frac{\sum_{j=1}^3 a_{ij} y_j}{K_i} \right)$$

# Primary-secondary therapy



## Cell Populations:

- Testosterone **producers** (TP)
  - Testosterone **dependent** (T+)
  - Testosterone **independent**
    - **Susceptible** to Docetaxel
    - **Resistant** to Docetaxel
- Targeted by  
“Primary” drug
- Targeted by  
“Secondary” drug

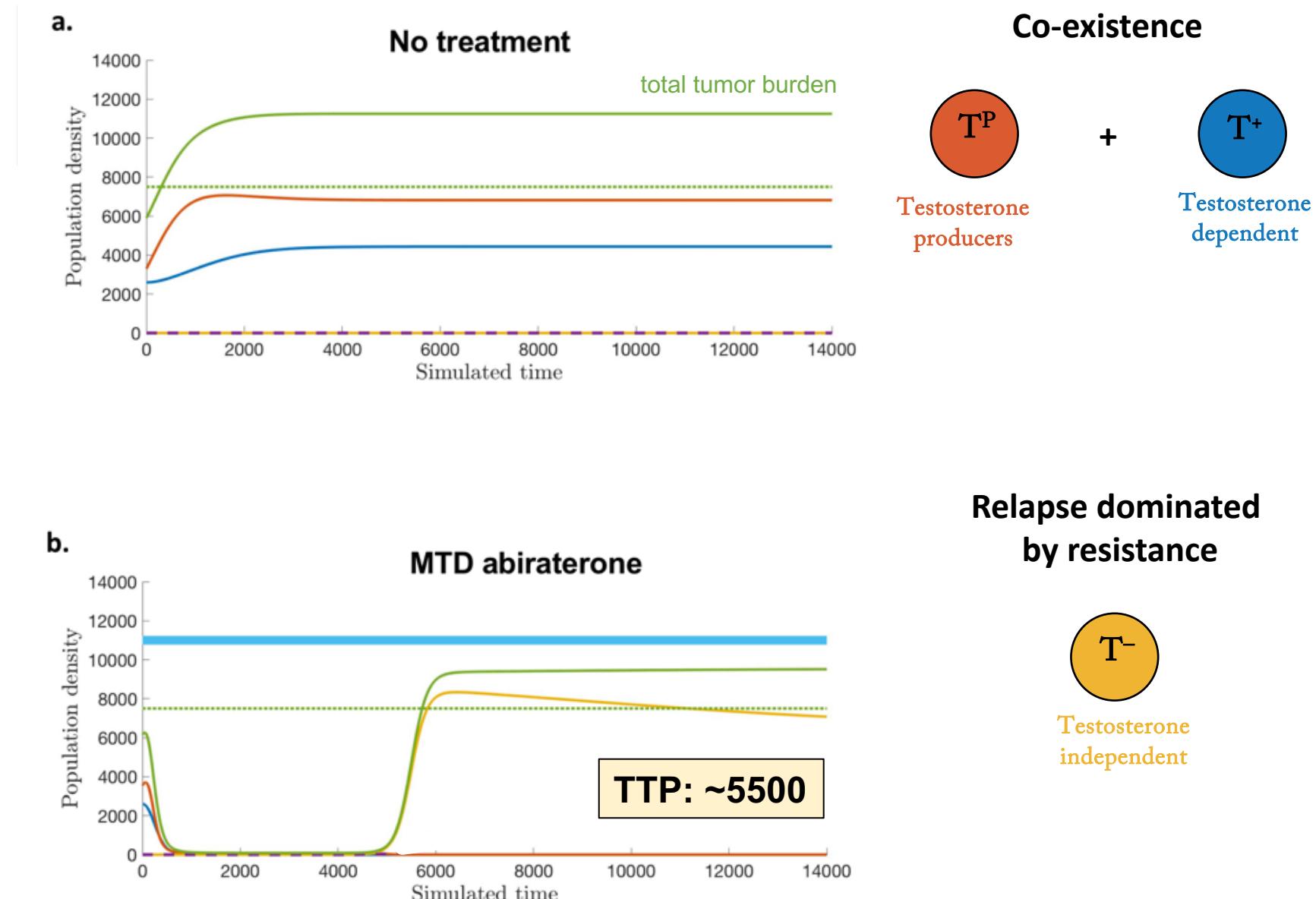
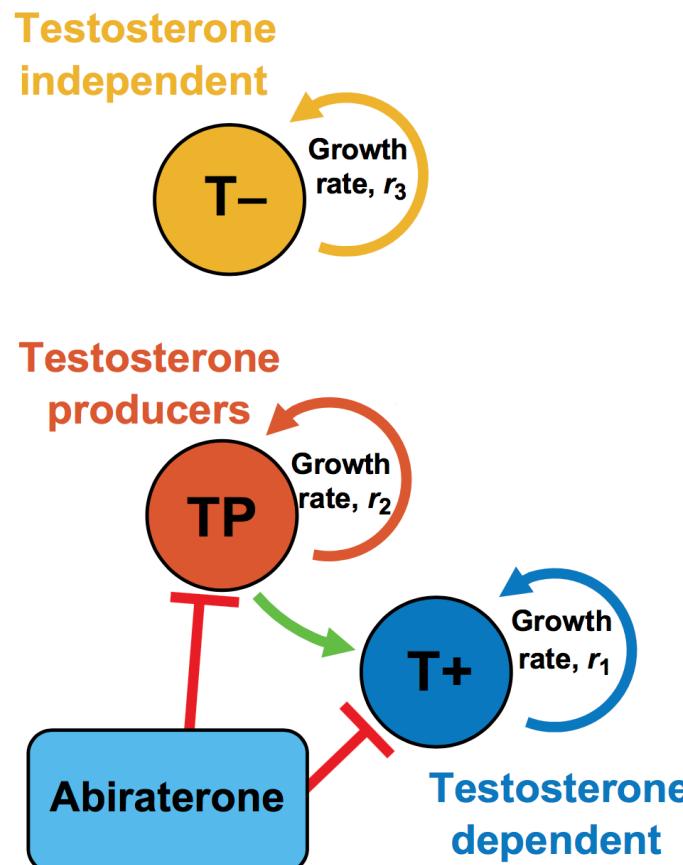
## Primary drug

- Greatest efficacy and/or lower toxicity

## Secondary drug

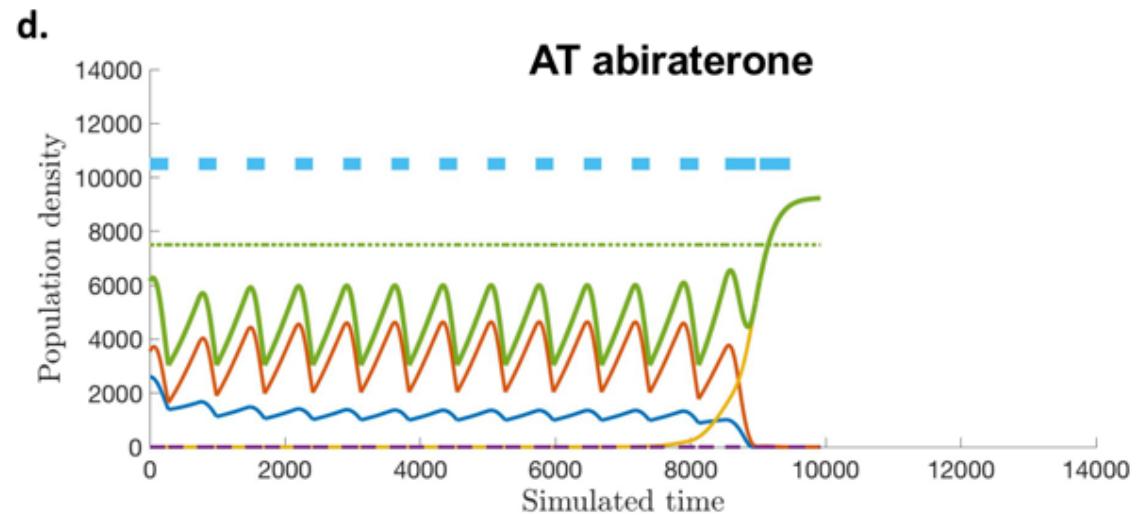
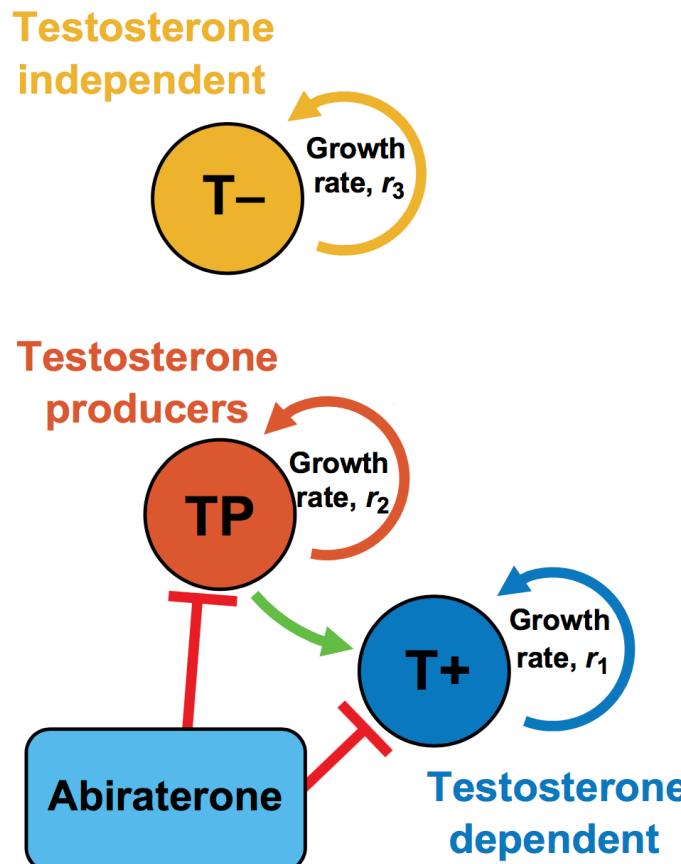
- Targets cell population which is resistant to primary drug

# Primary mono-therapy

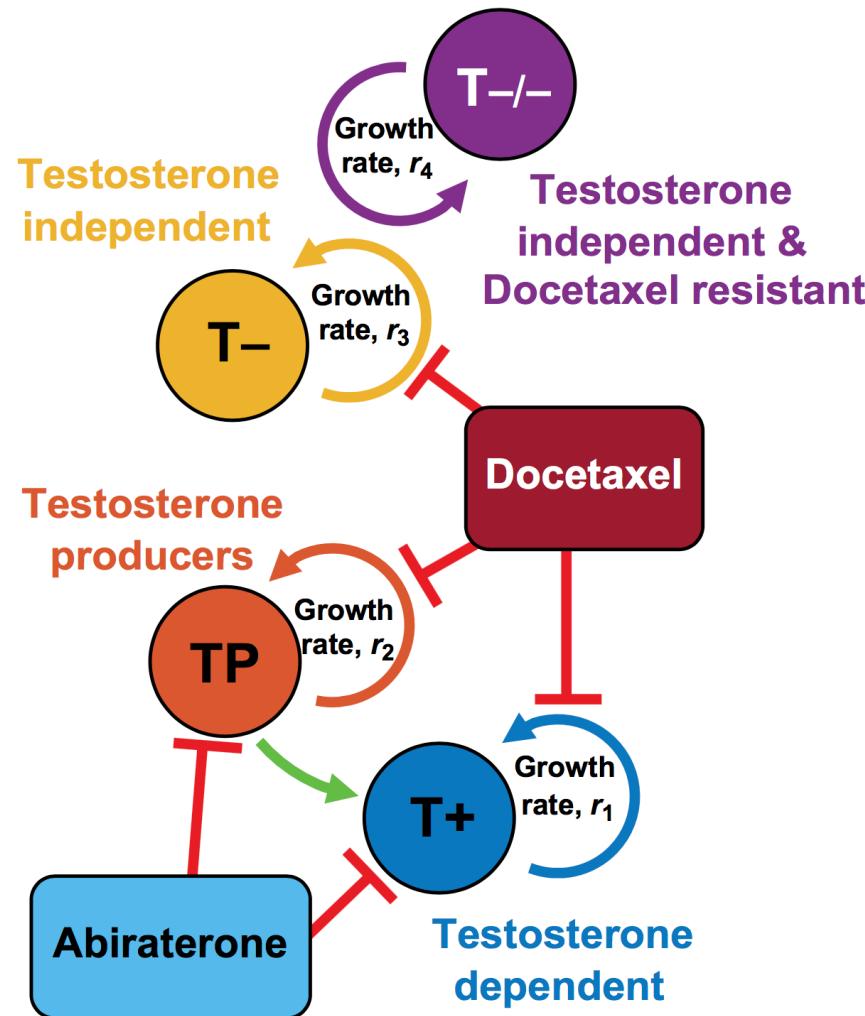


(Zhang. et. al. Nature Comm. 2017)

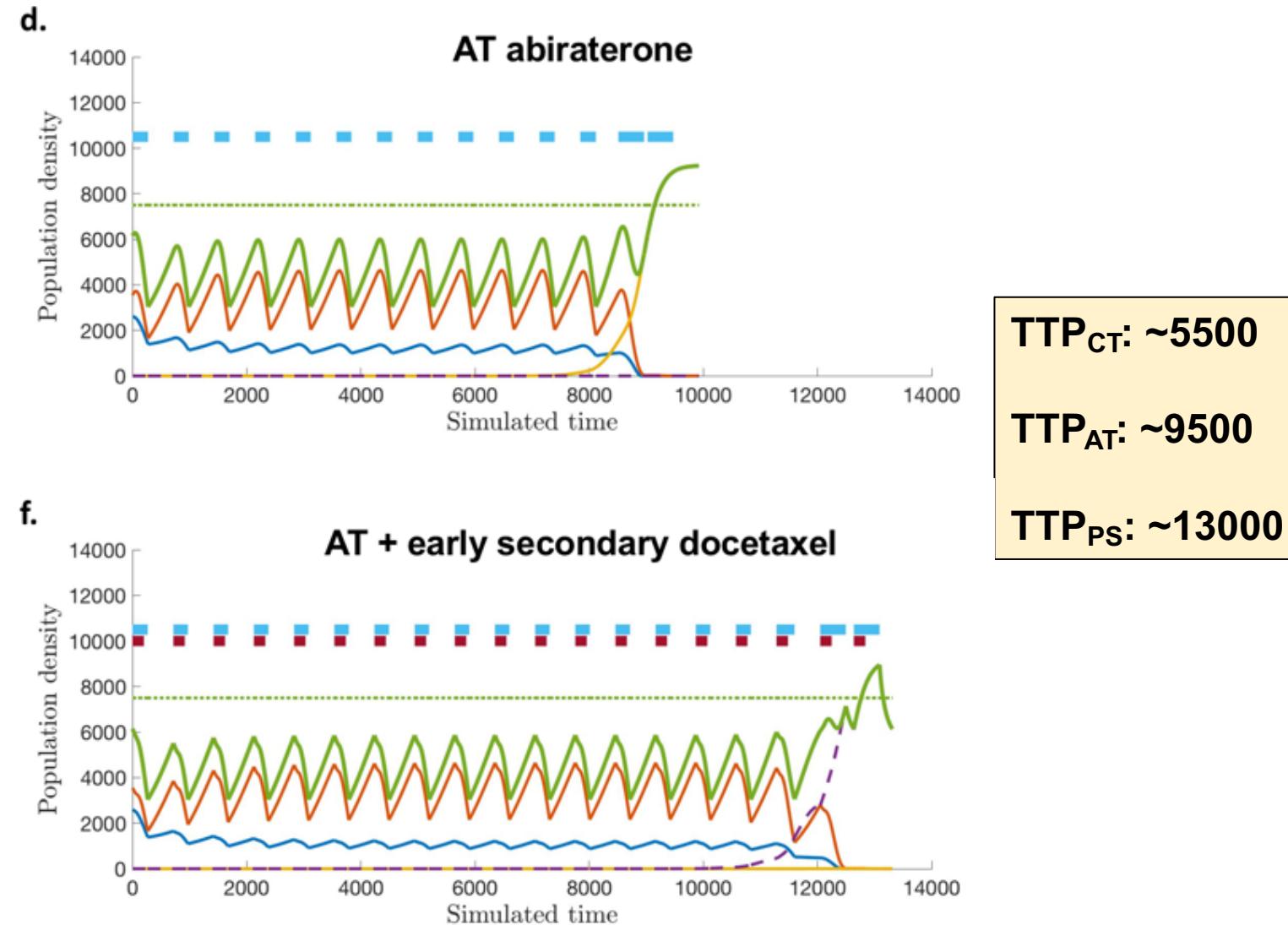
# Primary mono-therapy (adaptive)



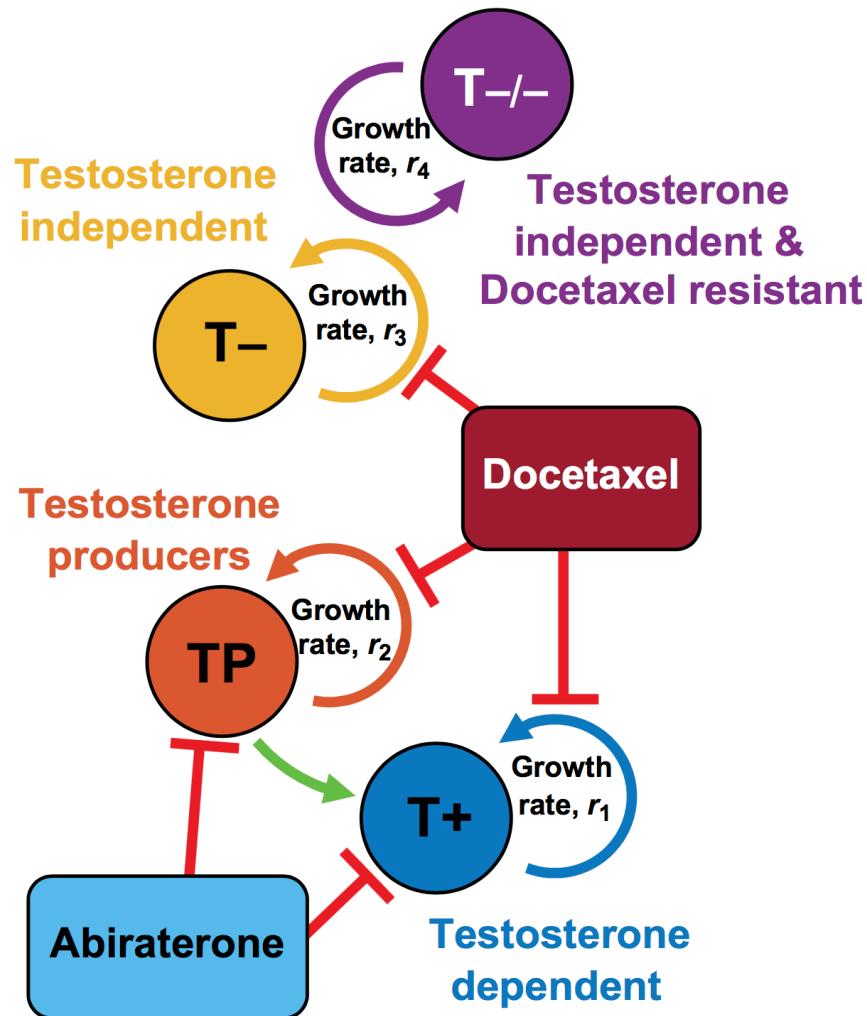
# Can docetaxel be used as “Secondary” drug?



(Zhang. et. al. Nature Comm. 2017)



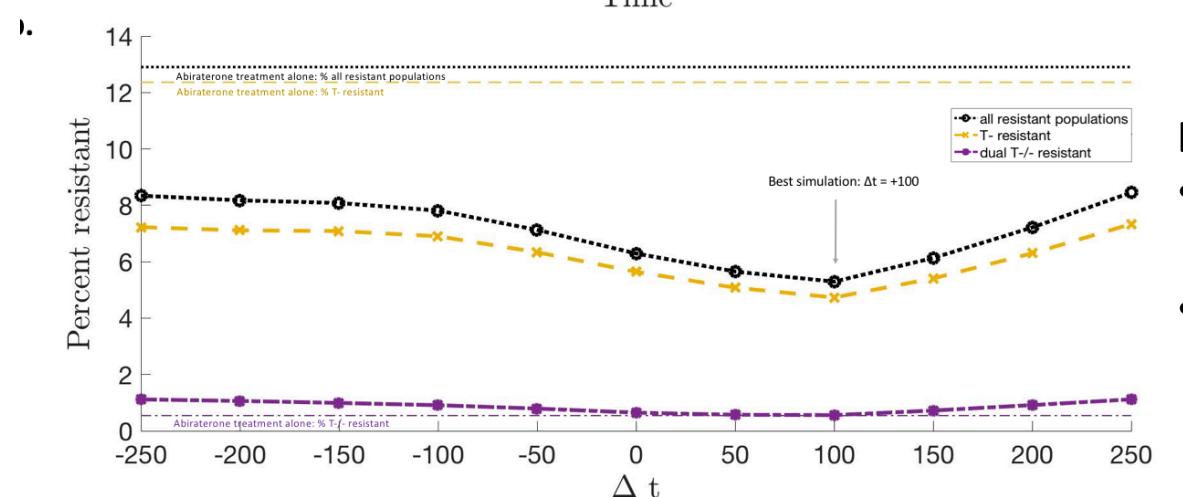
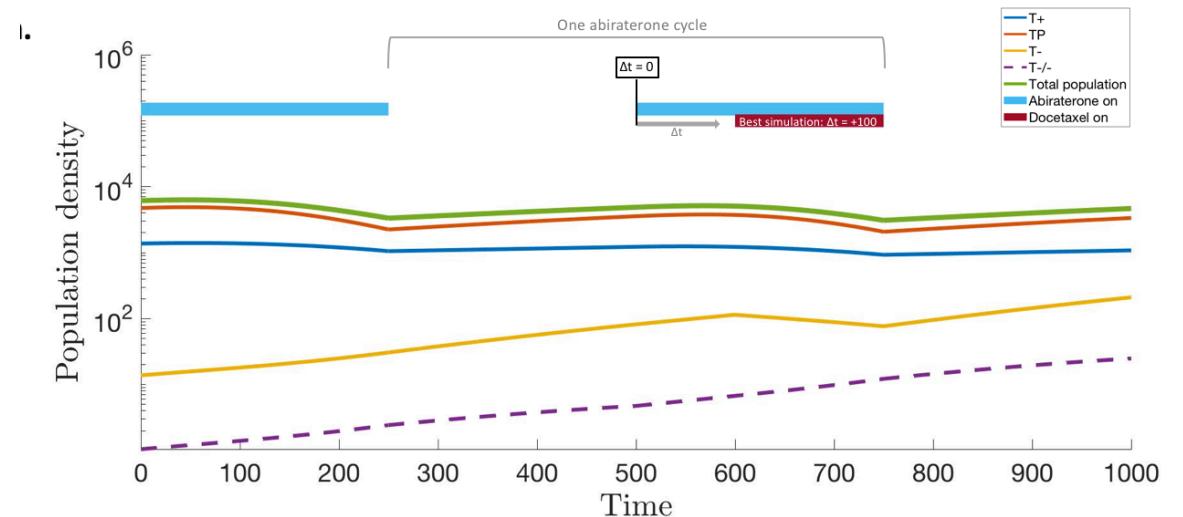
# Can docetaxel be used as “Secondary” drug?



(Zhang. et. al. Nature Comm. 2017)

## Problem with our secondary drug:

- Docetaxel is indiscriminately targeting 3 cell types



- Delayed docetaxel:**
- resistant cell type ( $T-$ ) is growing
  - sensitive cell types (TP, T+) are decaying

## **2. Evolutionary cycles**

**" Much is known but unfortunately in different heads."**

- Werner Kollath -

## Population Dynamics

Lotka-Volterra

$$\dot{y}_i = r_i y_i \left( 1 - \frac{\sum_{j=1}^3 a_{ij} y_j}{K_i} \right)$$

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

## Frequency Dynamics

Replicator Dynamics

$$\dot{x}_i = (f_i - \phi) x_i$$

$$f_i = w_i (\vec{Ax})_i$$

$$w_i = K_i / K_{\max}$$

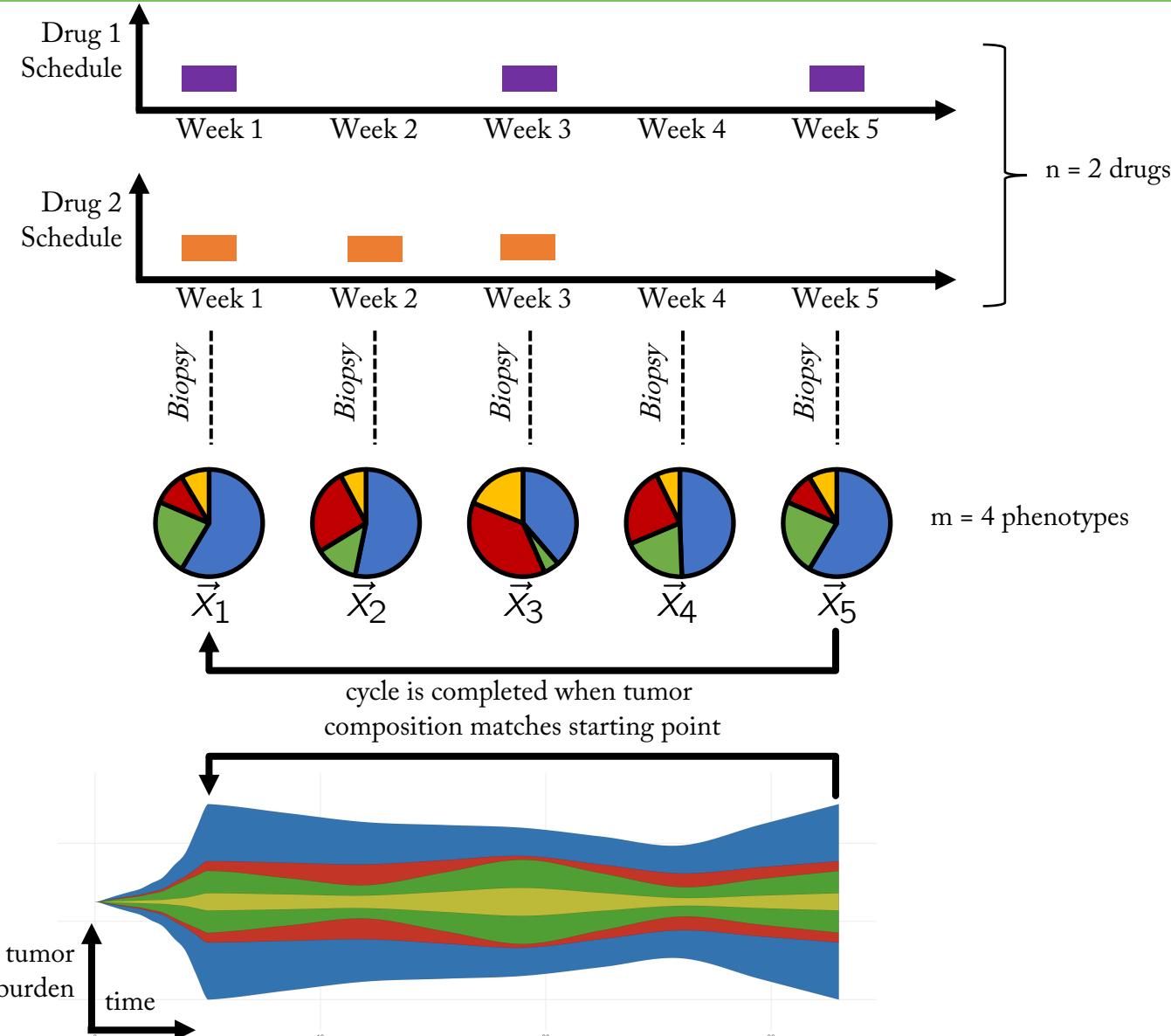
$$A = [1 - a_{ij}] = \begin{bmatrix} 1 - a_{11} & 1 - a_{12} & 1 - a_{13} \\ 1 - a_{21} & 1 - a_{22} & 1 - a_{23} \\ 1 - a_{31} & 1 - a_{32} & 1 - a_{33} \end{bmatrix}$$

# Frequency-dependent Evolutionary Cycles



## *Key assumption:*

- since the goal of an adaptive therapy is to maintain a stable volume
- we can study frequency-dependent dynamics



# What defines a treatment?



Combinations of  
 $n$  drugs:

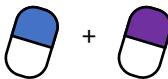
Treatment 1



Treatment 2



Treatment 3



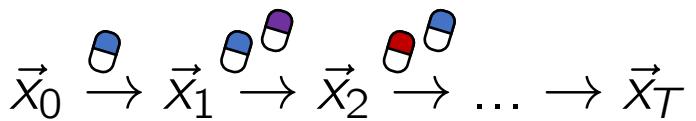
... and so on.

$2^n$  possible  
combinations  
(including no  
treatment)

Controlling  $m$  cell types  
(*genotype or phenotype*)

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Frequency-dependent cycles  
of tumor evolution



A sequence of treatments gives rise  
to an evolutionary “cycle” if:

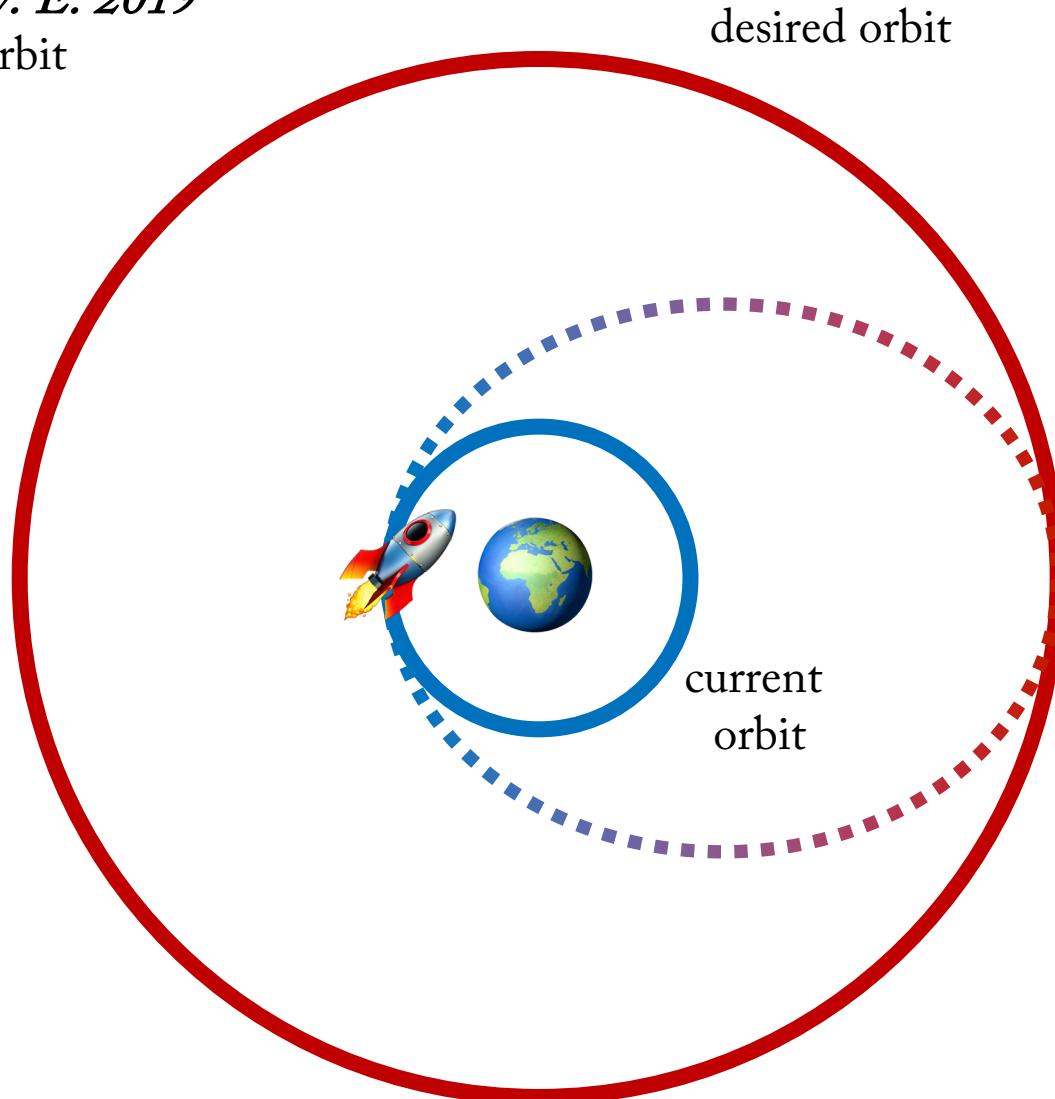
$$\vec{x}_T \approx \vec{x}_0$$

for some time period  $T > 0$ .

# Drawing inspiration from orbital mechanics

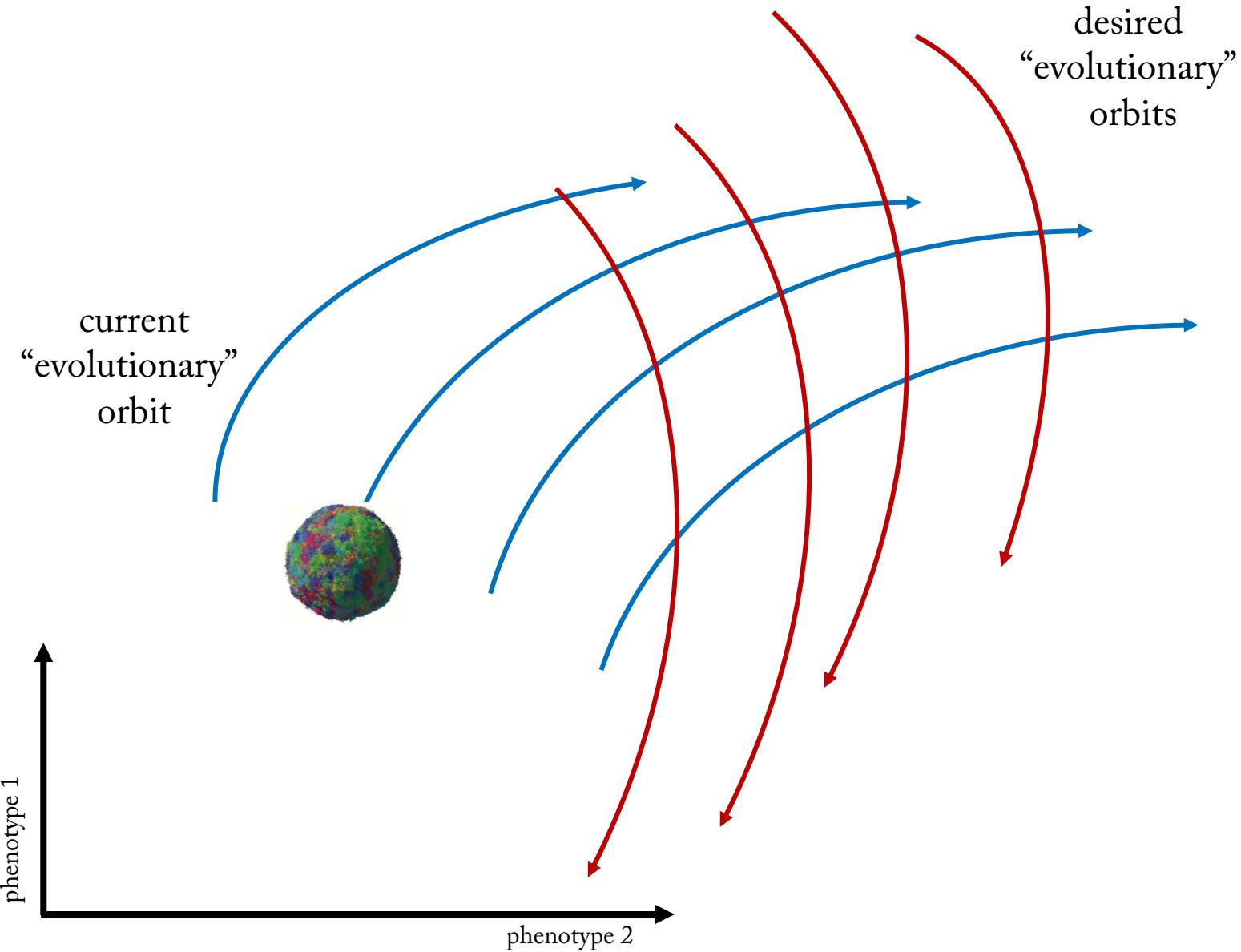
*Newton et. al. Phys. Rev. E. 2019*

- Hohmann Transfer Orbit



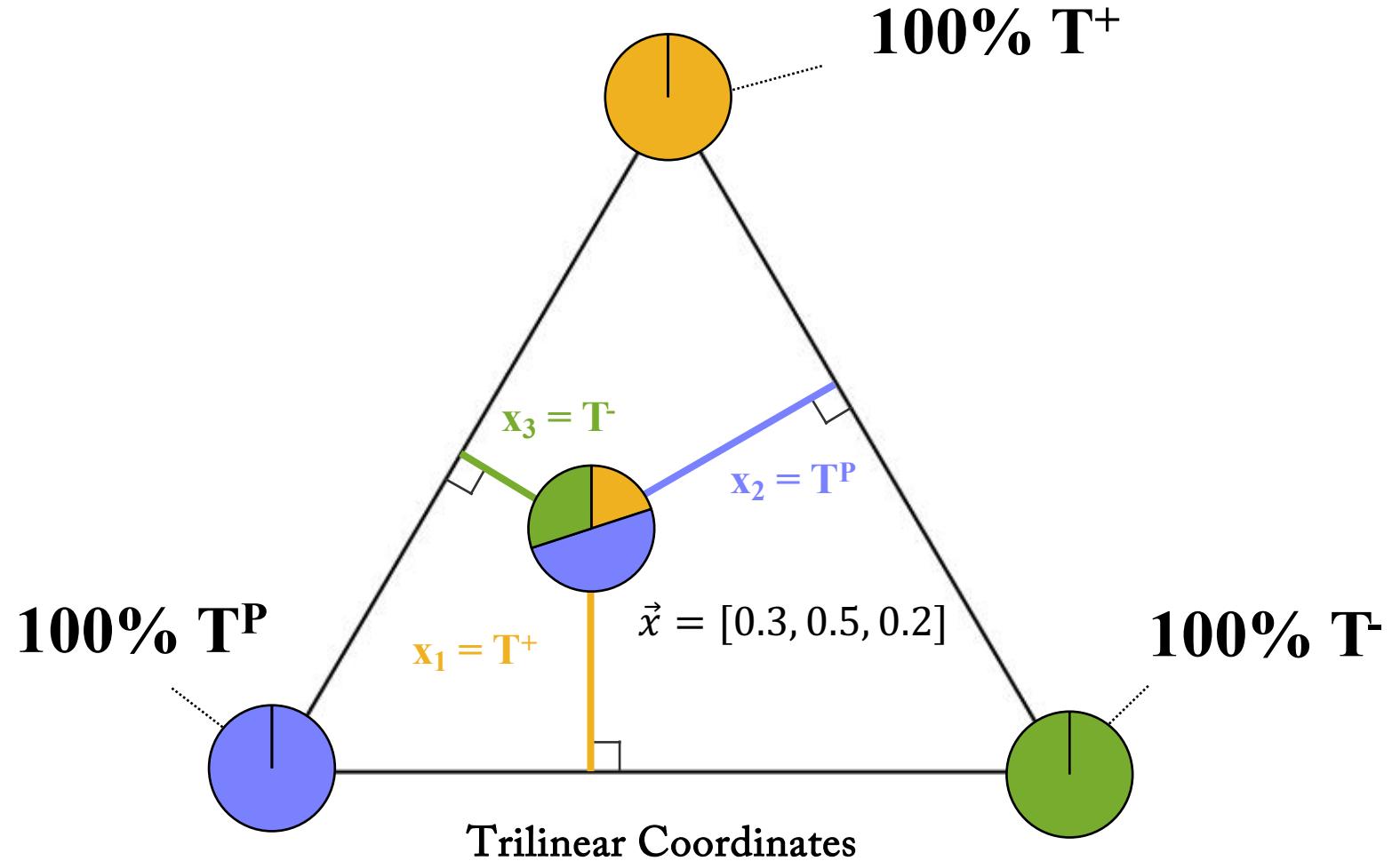
1. Current orbit is fixed & known
2. Desired orbit is fixed & known
3. The connecting orbit is tangent to current orbit and intersects desired orbit

# Drawing inspiration from orbital mechanics



1. Current orbit is fixed & known
2. Desired orbit is fixed & known
3. The connecting orbit is tangent to current orbit and intersects desired orbit

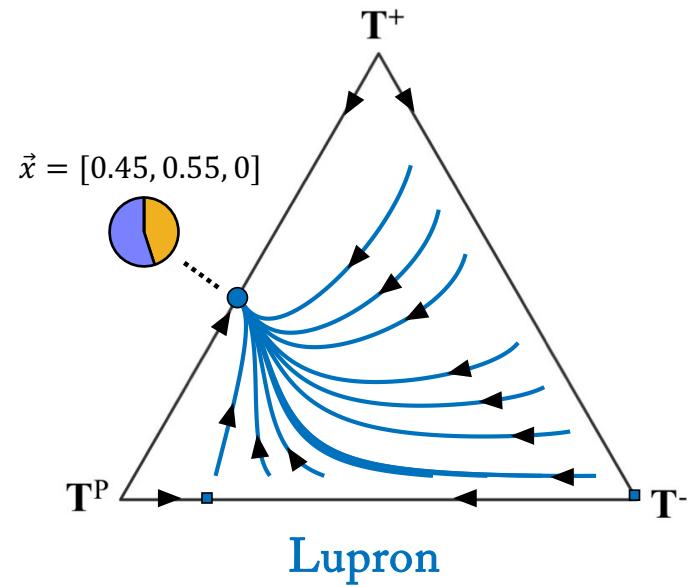
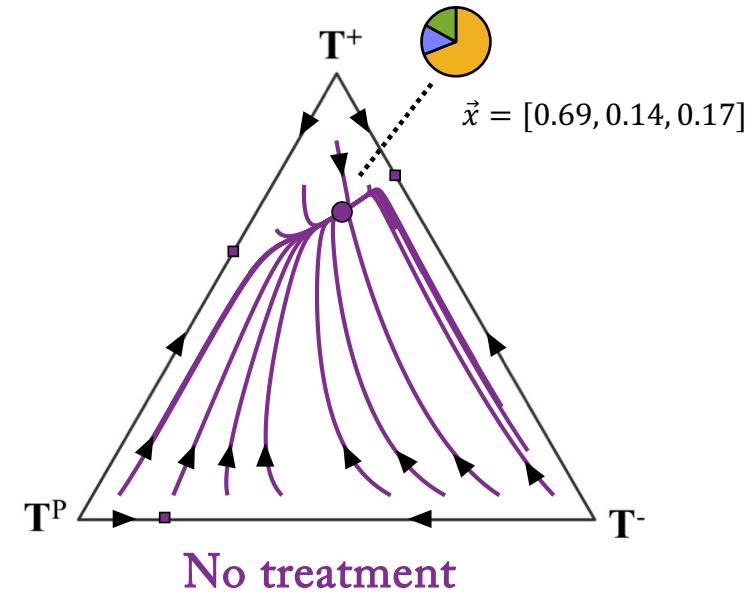
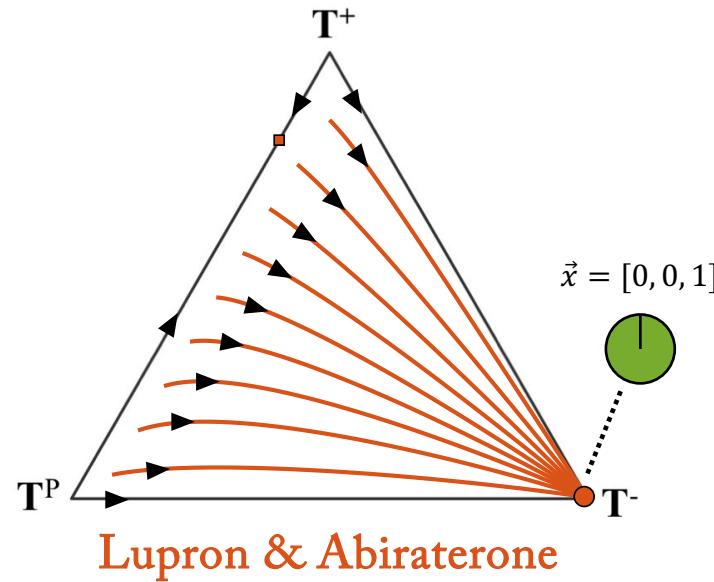
# Triangular simplex



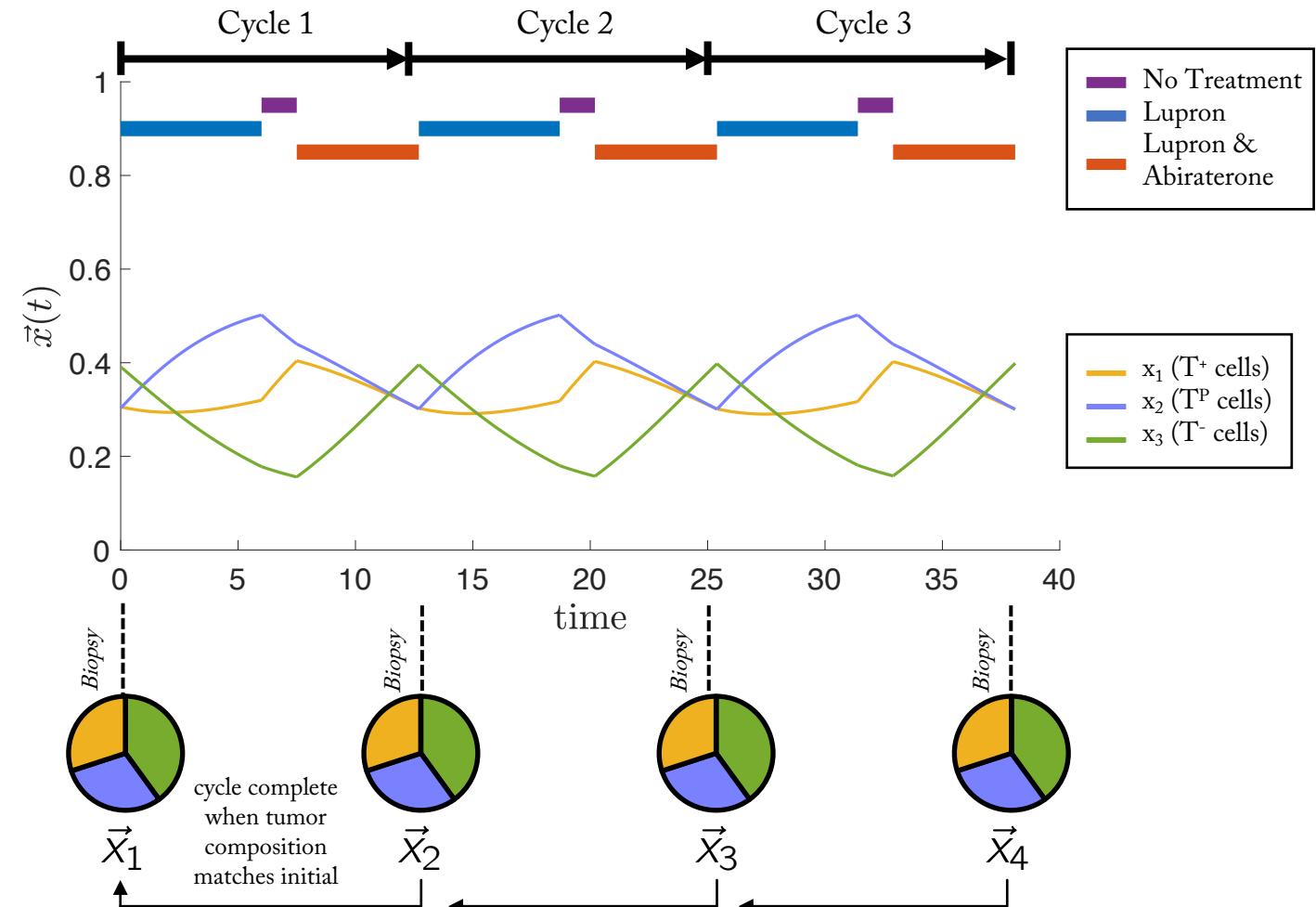
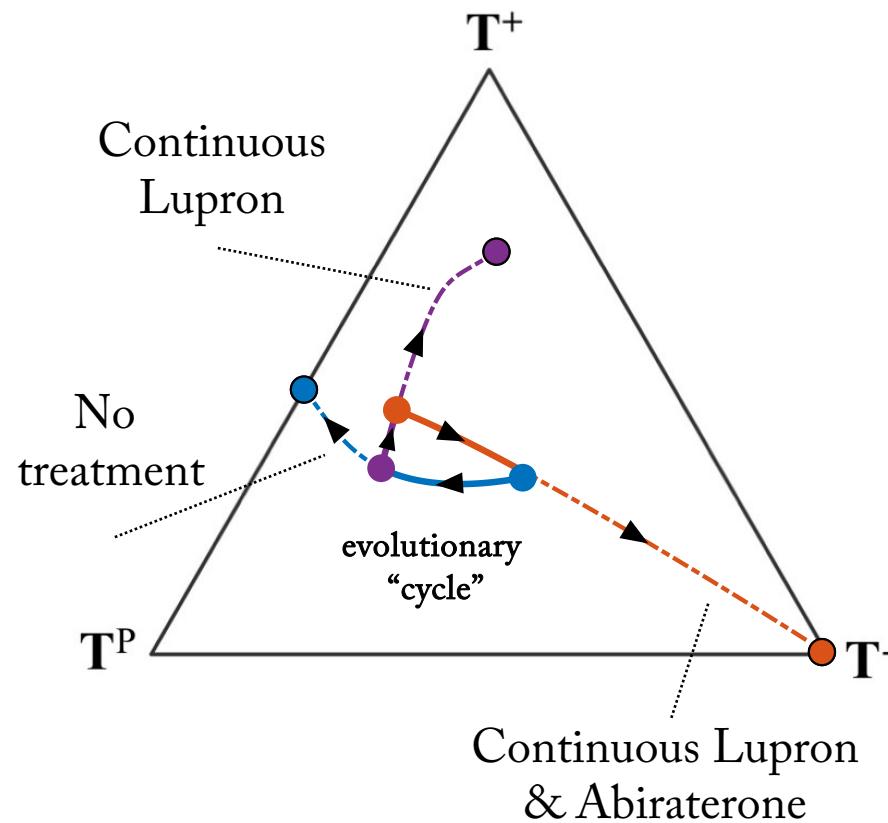
## Trilinear Simplex

- The state space of all possible states of three populations
- $x_1 + x_2 + x_3 = 1$

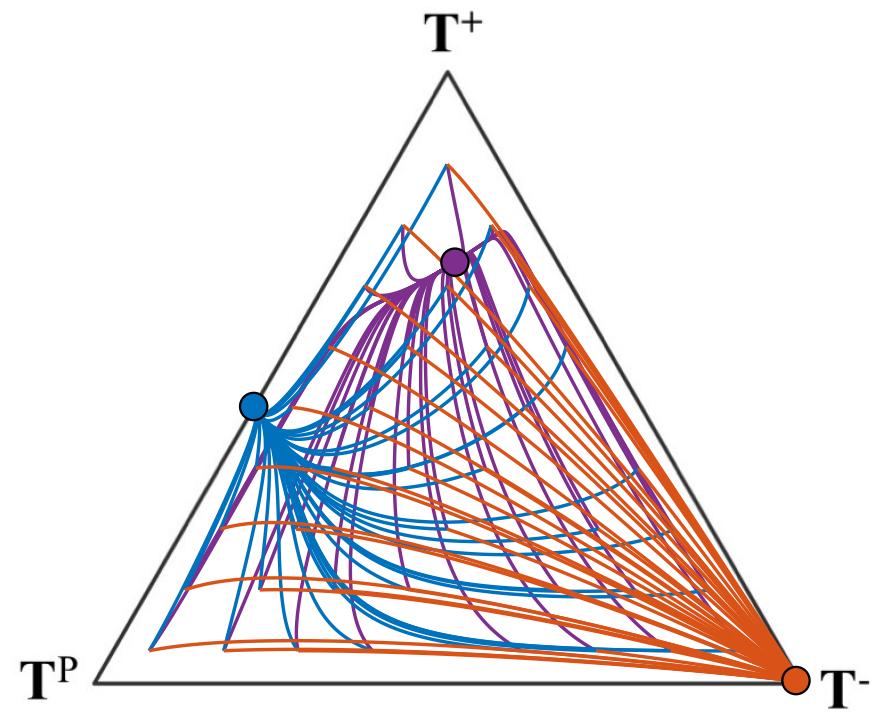
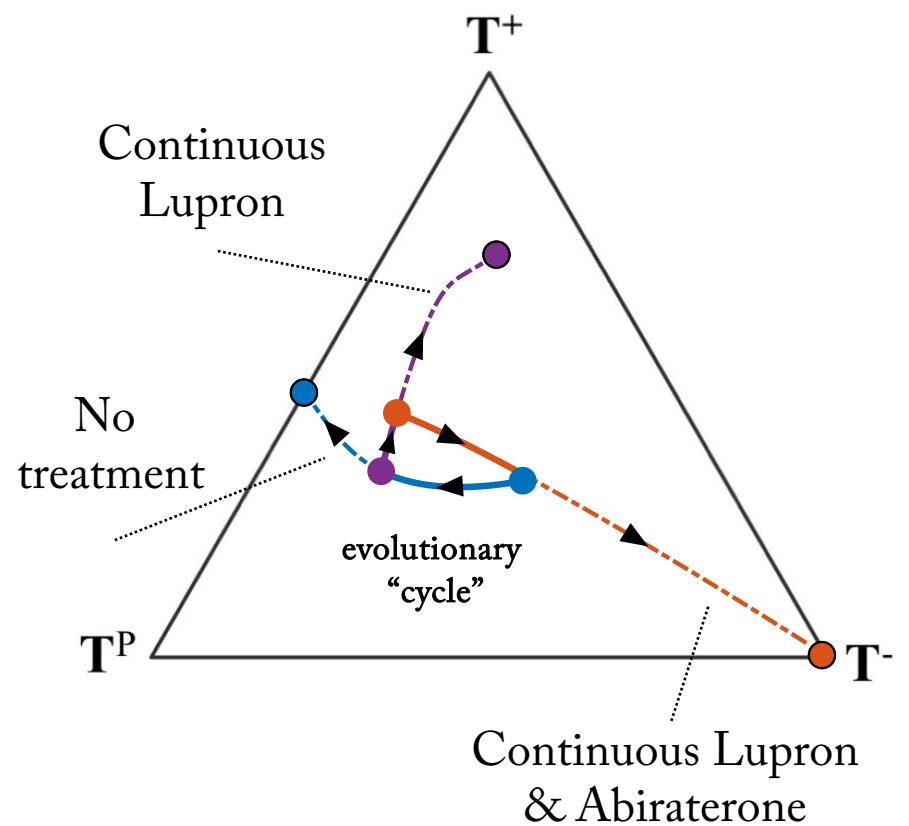
# Frequency-dependent treatment dynamics



# Frequency-dependent treatment dynamics



# Frequency-dependent treatment dynamics



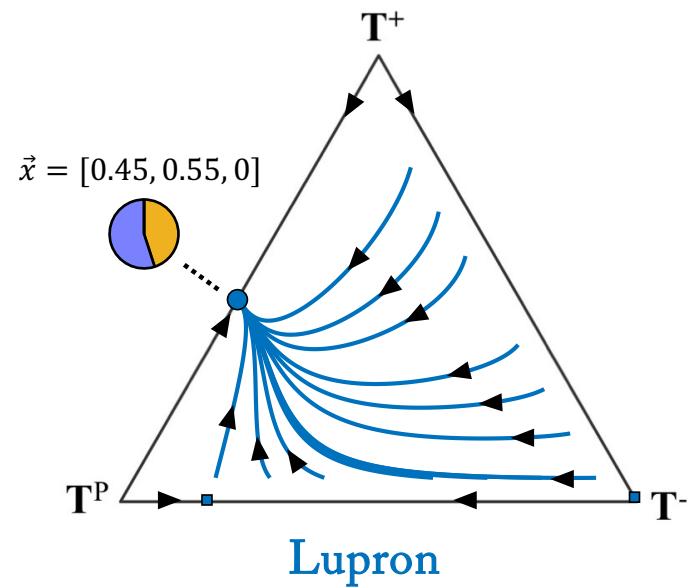
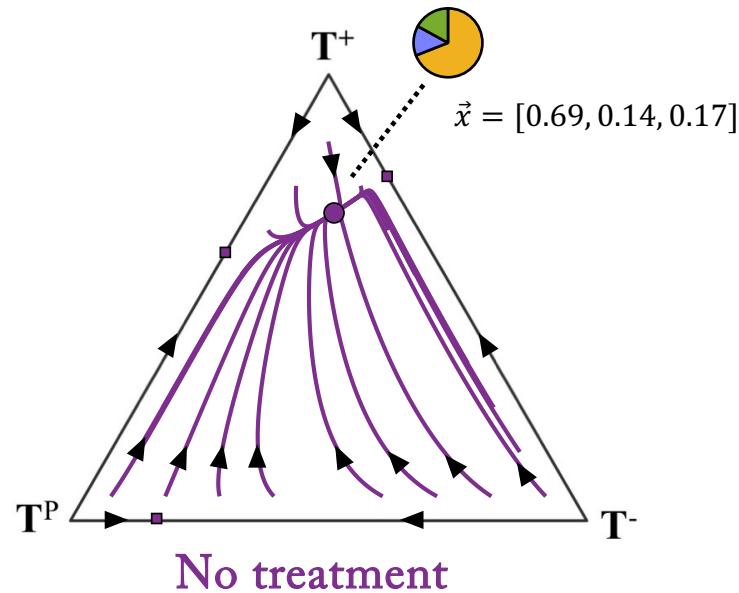
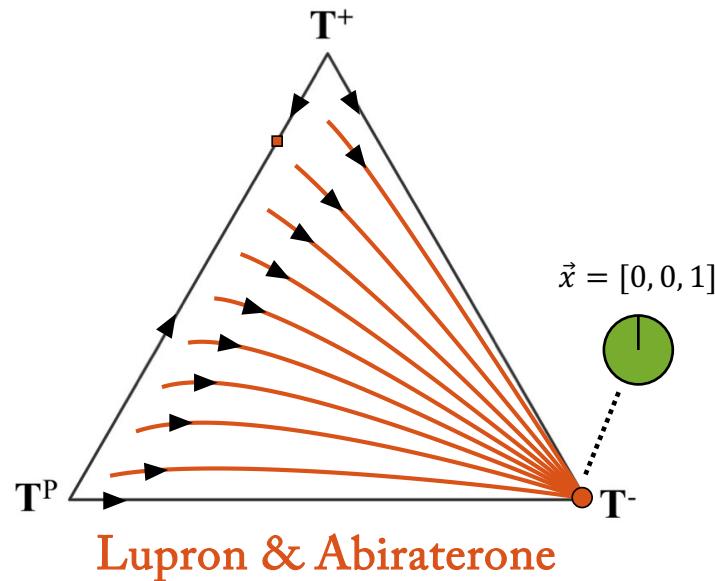
### **3. Evolutionary search space**

**" Most discoveries even today are a combination of serendipity and of searching."**

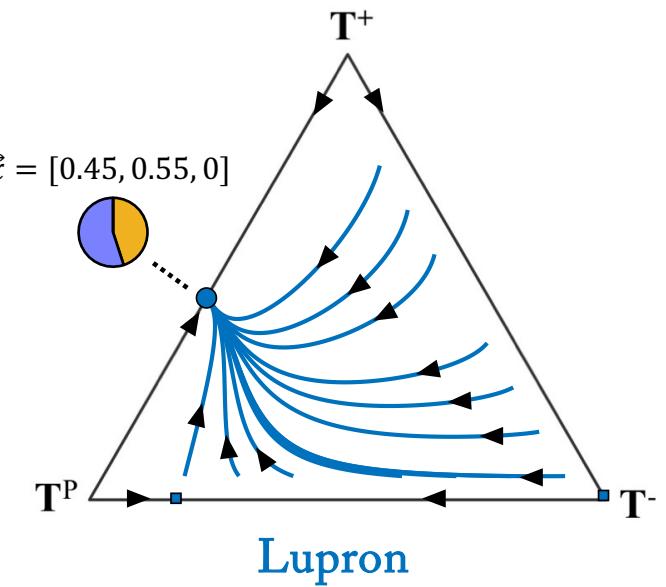
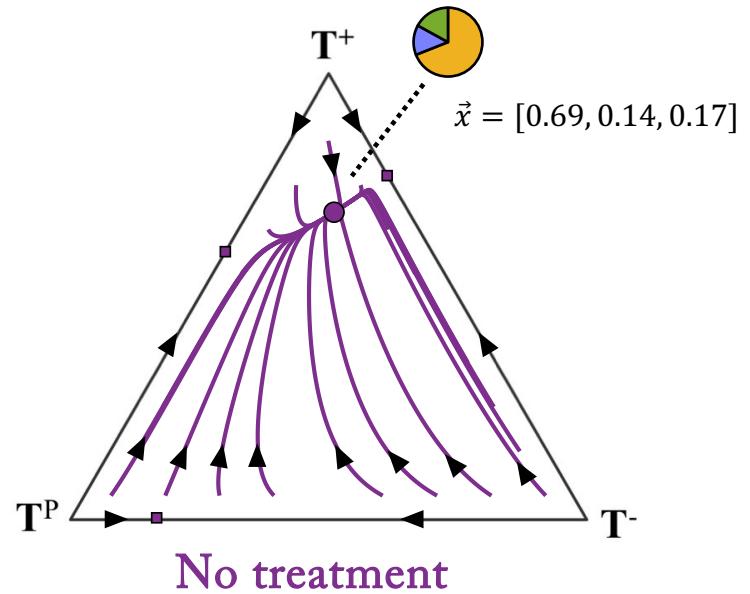
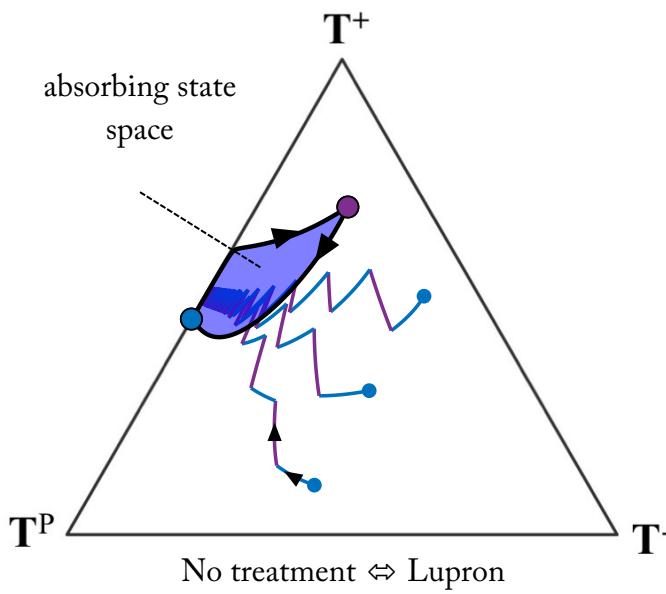
- Siddhartha Mukherjee -

# Evolutionary Search Space

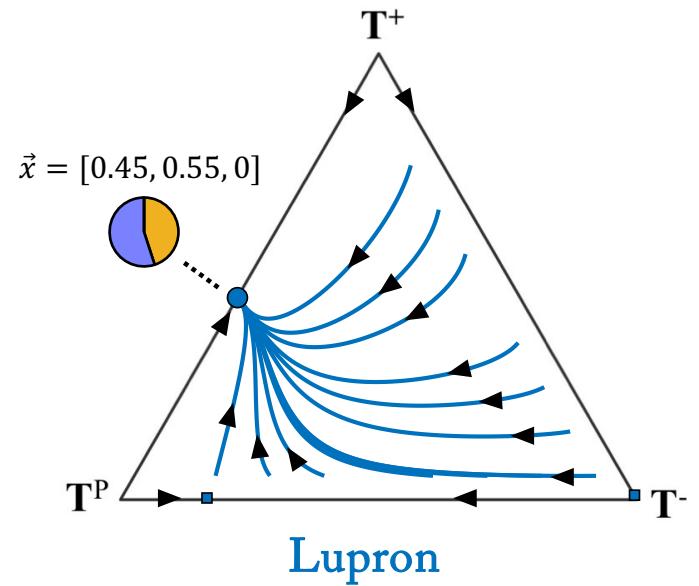
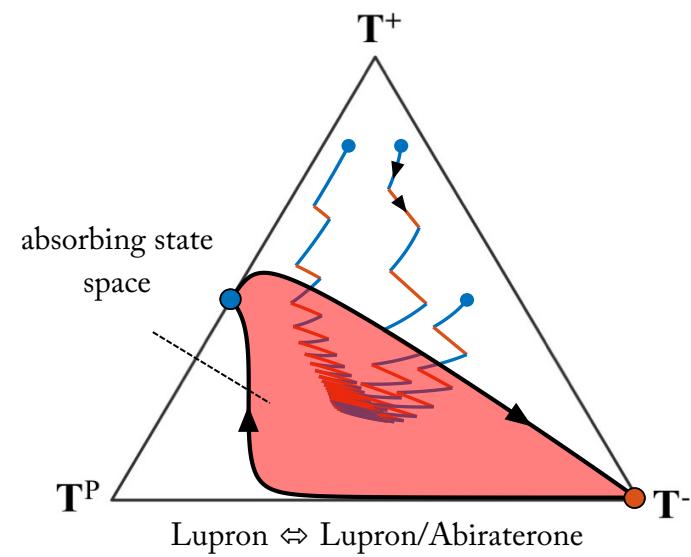
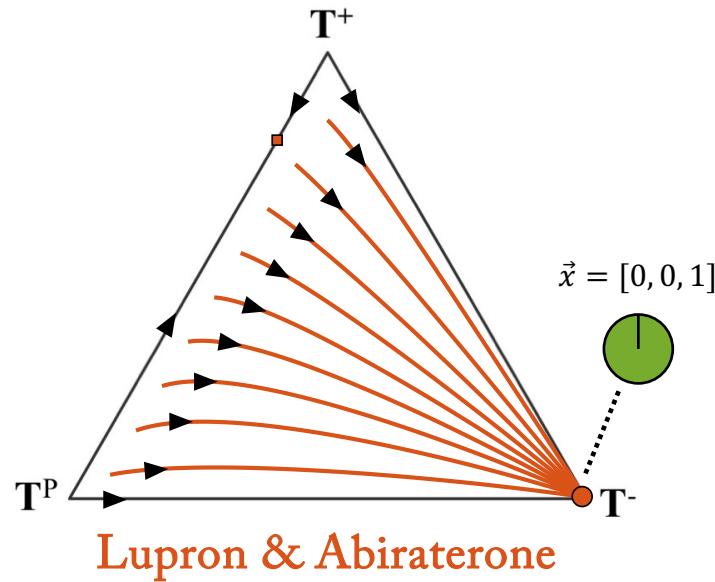
## Pairwise treatment combinations



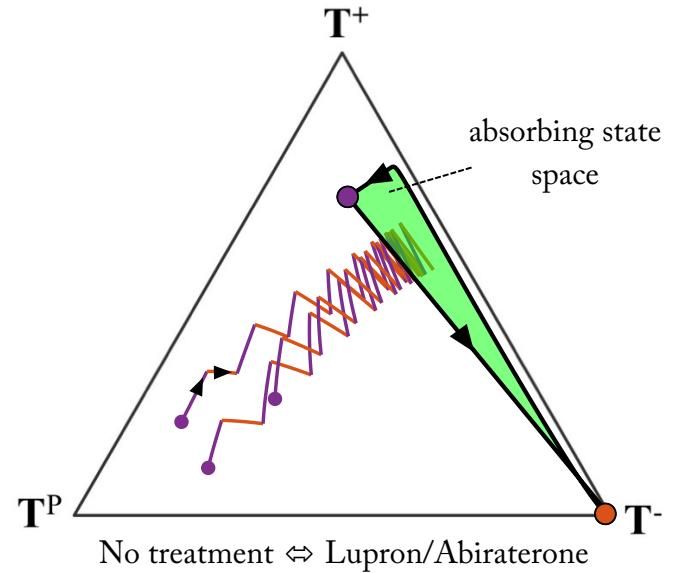
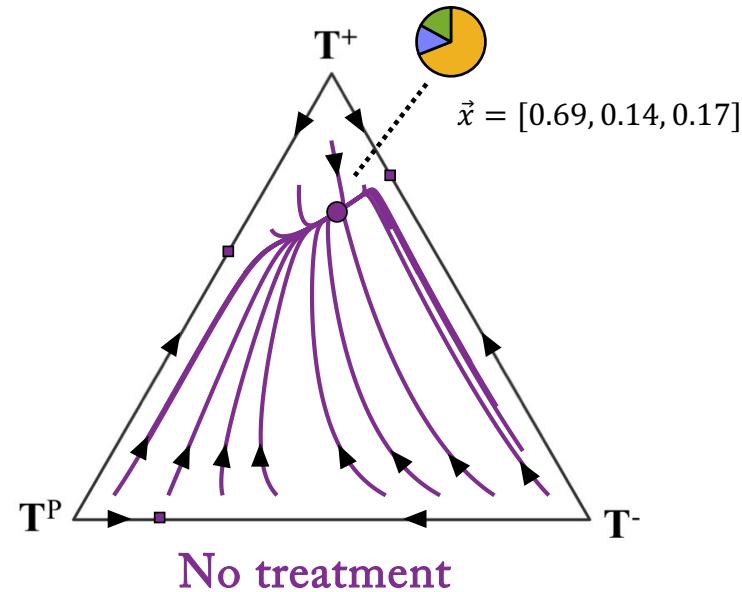
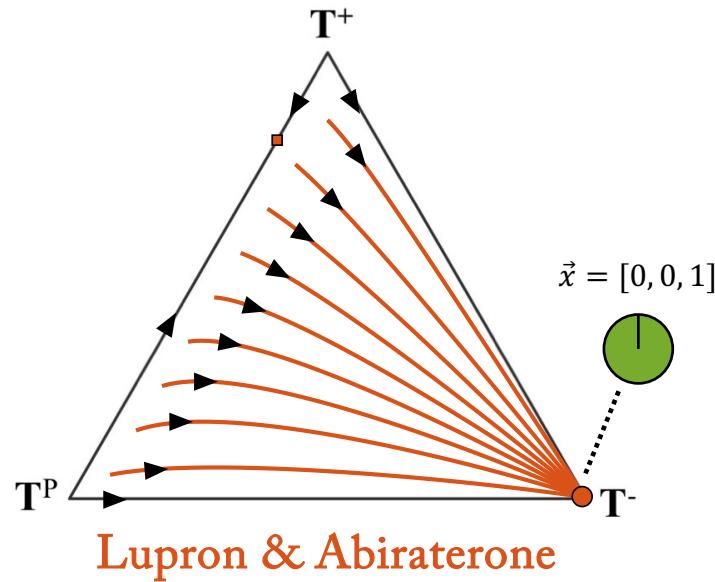
# Evolutionary Search Space



# Evolutionary Search Space



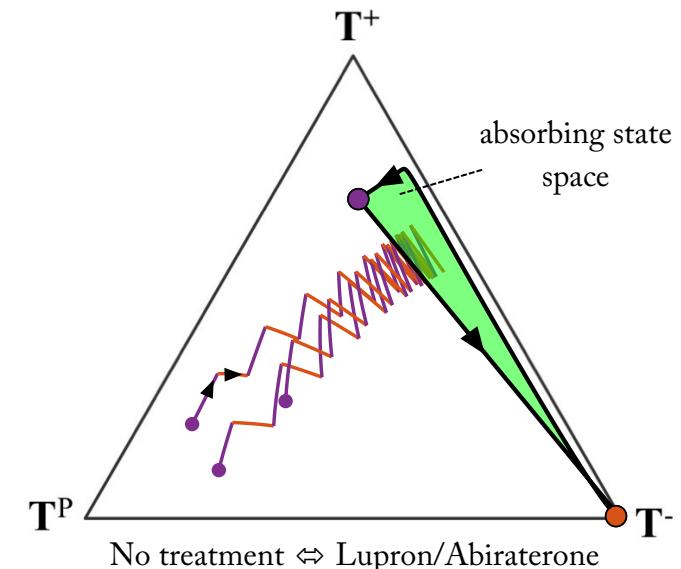
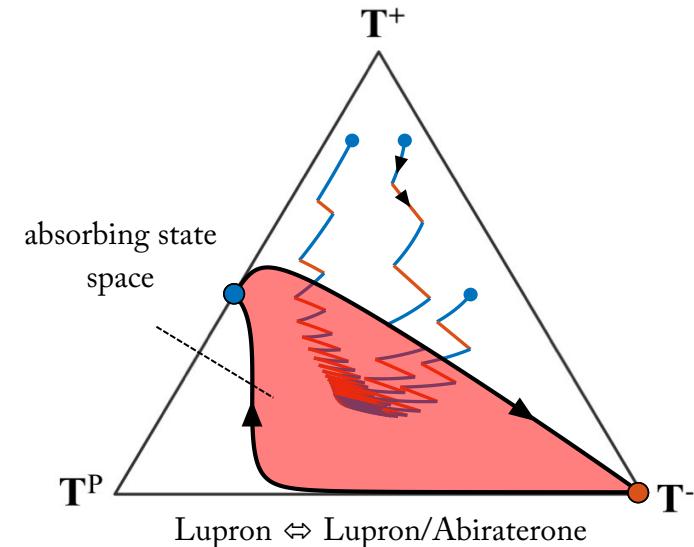
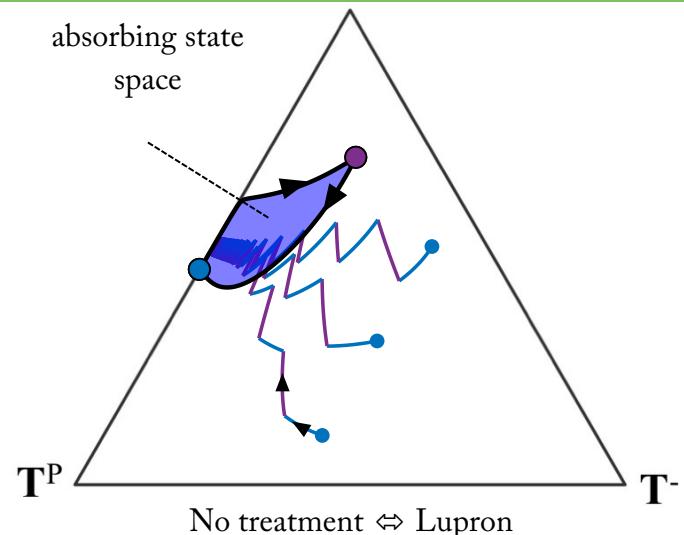
# Evolutionary Search Space



# Evolutionary Search Space

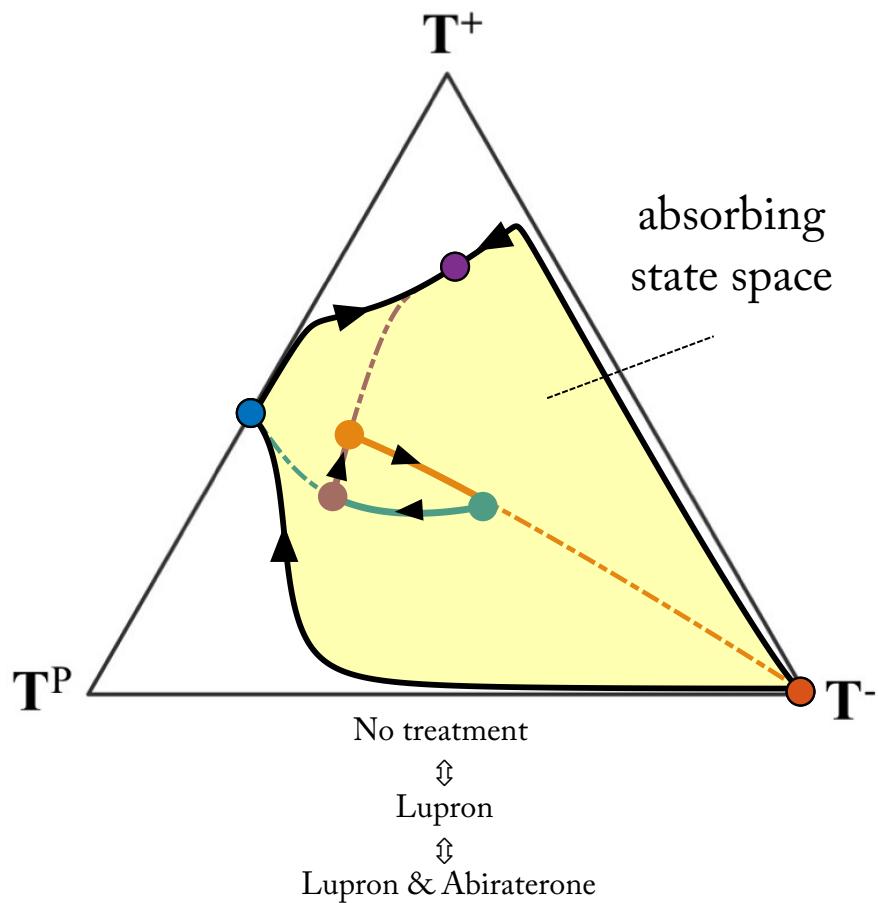
## *Key observations:*

- All possible drug sequences tend toward absorbing area
- Various drug combinations have varied size of absorbing space
- The longer time cycled, closer to absorbing space
- No treatment is an evolutionary process



## *3 treatments*

- Vastly expands absorbing state space
- “Orthogonal” drugs are desirable



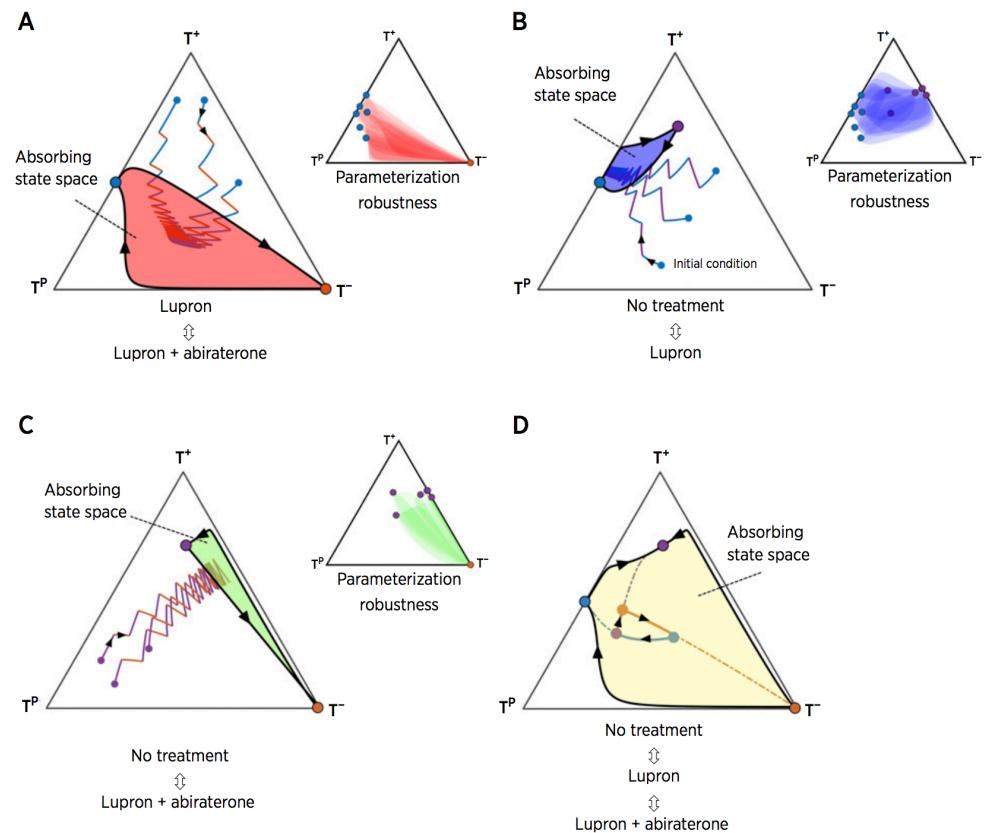
# Payoff assumptions

## No treatment

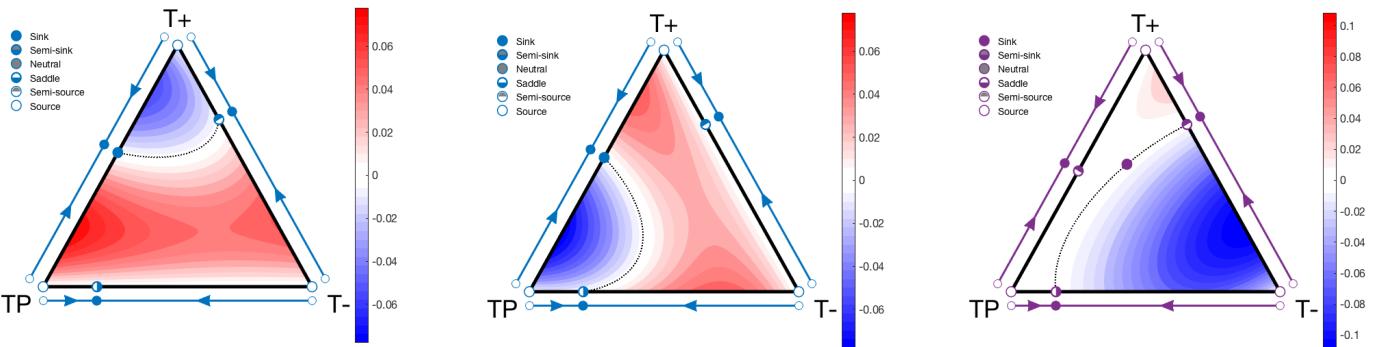
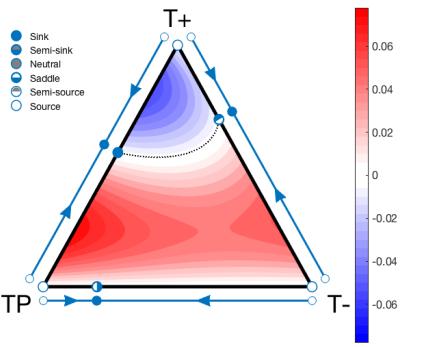
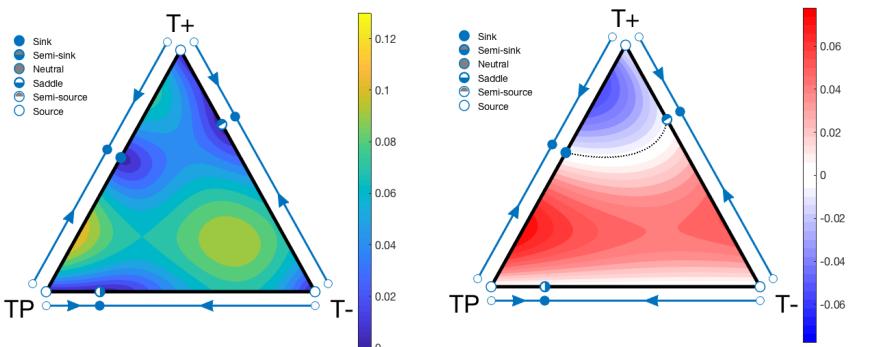
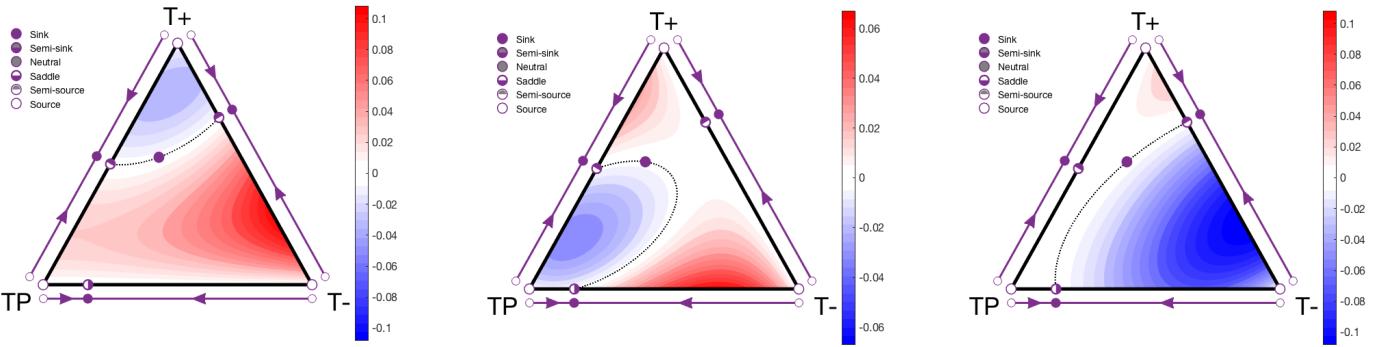
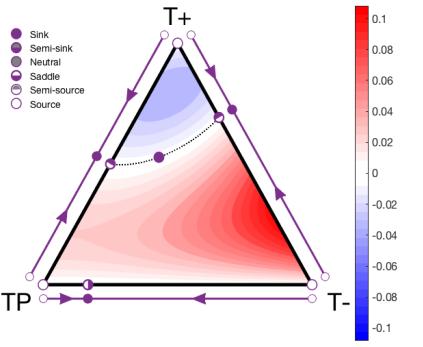
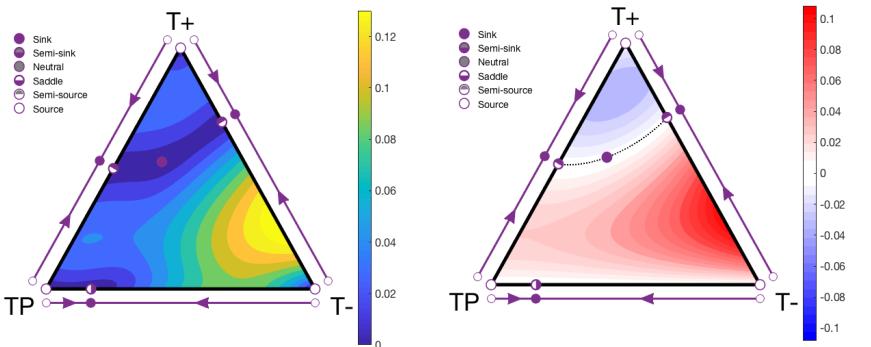
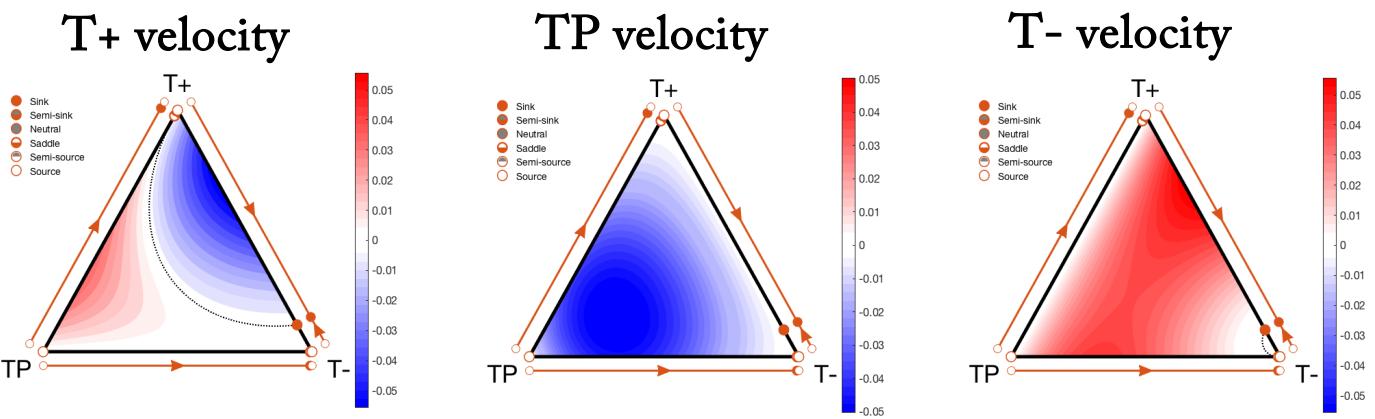
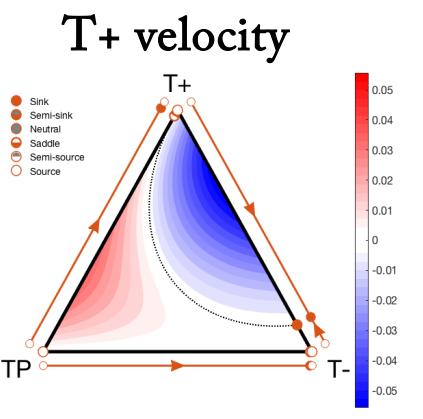
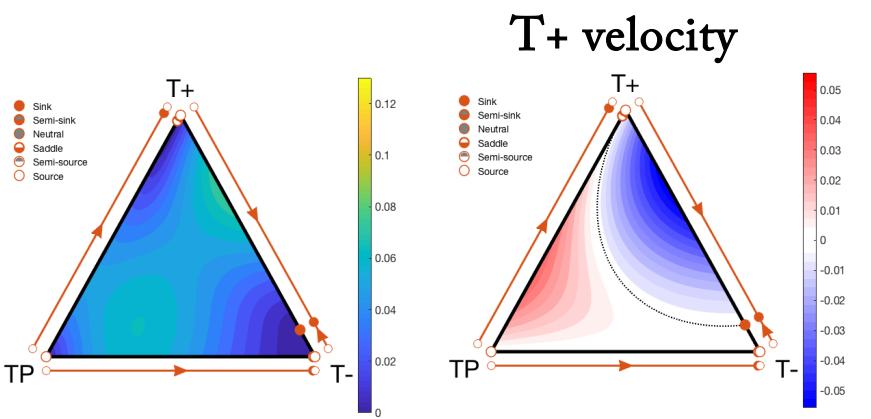
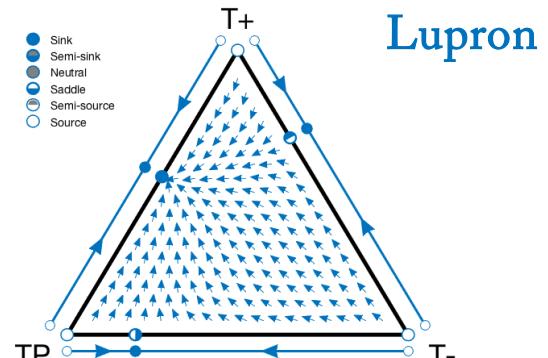
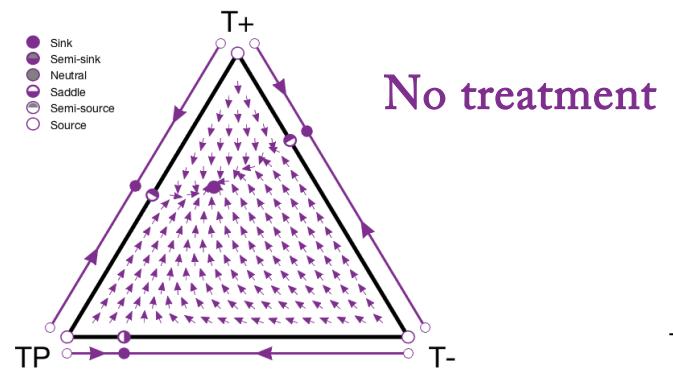
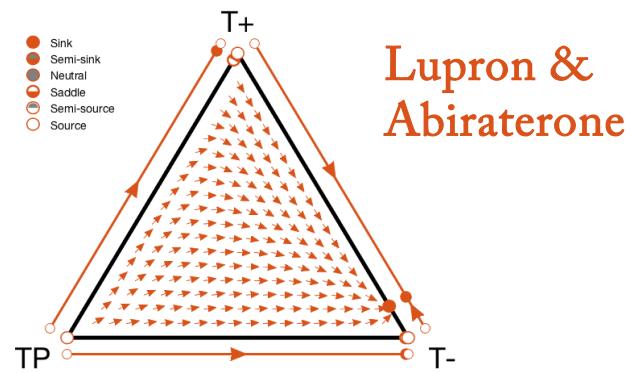
$c < e$	$T^-$ cells have a higher fitness than $T^P$ cells when interacting with many $T^+$ , especially in low vasculature regions. Testosterone production by $T^P$ production comes at some cost to provide public good to both self ( $T^P$ ) and neighbor ( $T^+$ ). Both $c$ and $e$ should decrease in the pre-treatment condition
$a > f$	$T^+$ cells have a higher fitness than $T^-$ cells when interacting with many $T^P$ , receiving advantage from the public good. The parameter $a$ should increase in the pre-treatment condition $f$ slightly decrease
$b > d$	$T^+$ cells have a higher fitness than $T^P$ cells when interacting with many $T^-$ because there is lack of spatial competition near vasculature for $T^+$ cells as testosterone is not being used
$a < b$	$T^+$ cells have a higher fitness competing with $T^-$ over competition with $T^P$
$c < d$	Similarly, $T^P$ cells have a higher fitness competing with $T^-$ over competition with $T^+$
$e > f$	$T^-$ cells have less competition for space in a tumor with mostly $T^+$ than with mostly $T^P$ . The parameter $f$ should decrease in the pre-treatment condition slightly

## Lupron & Abiraterone

$c > e$	$T^P$ cells have a higher fitness than $T^-$ cells when interacting with few $T^+$ (absence of competition)
$a > f$	Interacting with mostly $T^P$ cells, $T^+$ gains from the public good and from the new available space in low vasculature regions
$b < d$	Interacting with mostly $T^-$ cells, $T^P$ cells see little competition near vasculature. Payoffs to $T^+$ cells, $b$ , may be small or zero
$a > b = 0$	$T^+$ cells need the $T^P$ cells to succeed in the absence of systemic testosterone
$c > d$	$c$ is likely the largest parameter as $T^P$ cells have the highest fitness in a mostly $T^-$ tumor without systemic testosterone
$e > f$	Again, $T^P$ cells outcompete $T^-$ cells in the absence of systemic testosterone



# Velocity of frequency-dependent dynamics



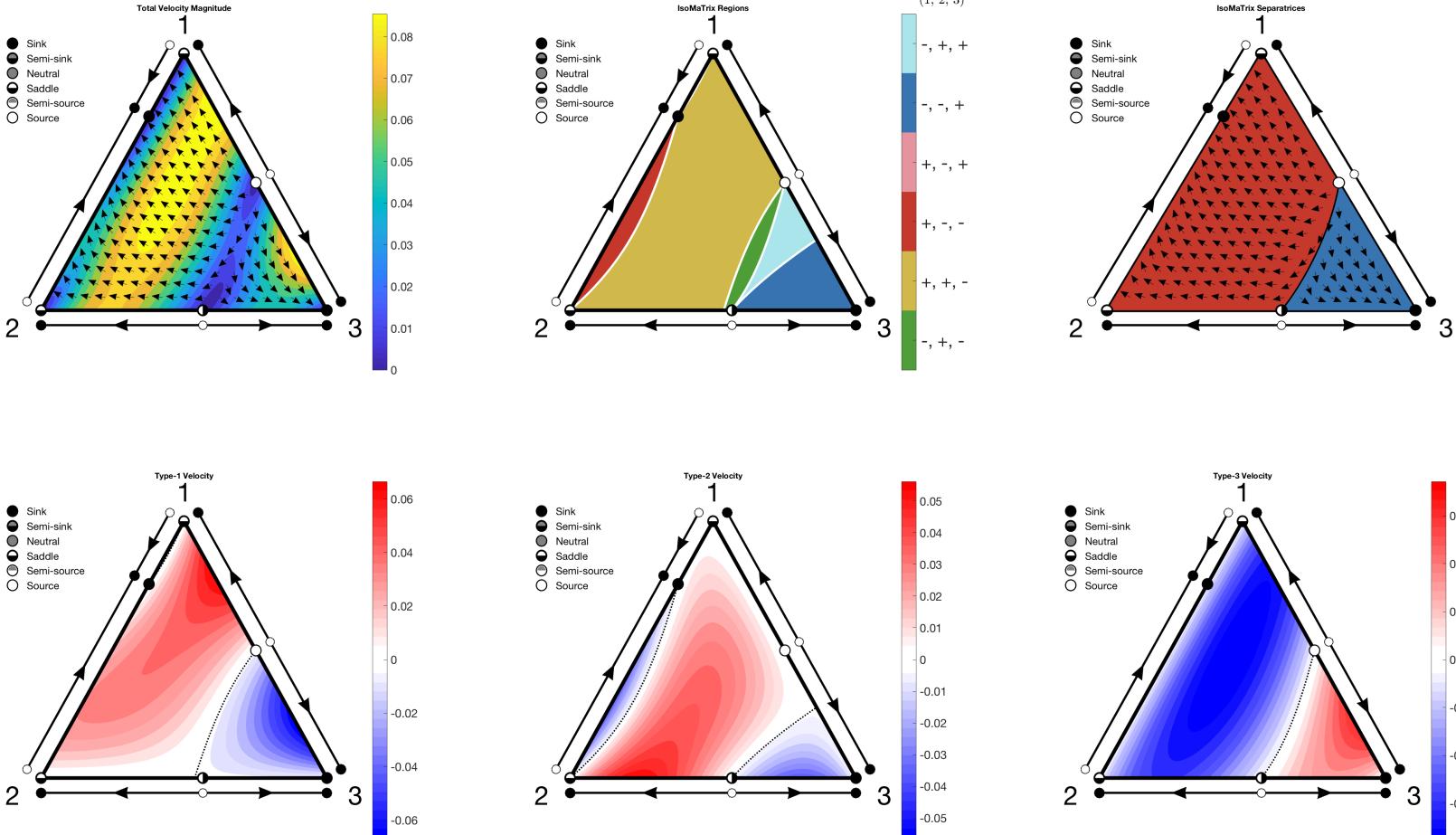
## **4. IsoMaTrix**

### **A tool for visualizing matrix games**

"The best investment is in the tools of own's own trade."

- Benjamin Franklin -

# IsoMaTrix: visualizing Isoclines of Matrix games



**Download at:**

- [github.com/MathOnco/isomatrix](https://github.com/MathOnco/isomatrix)

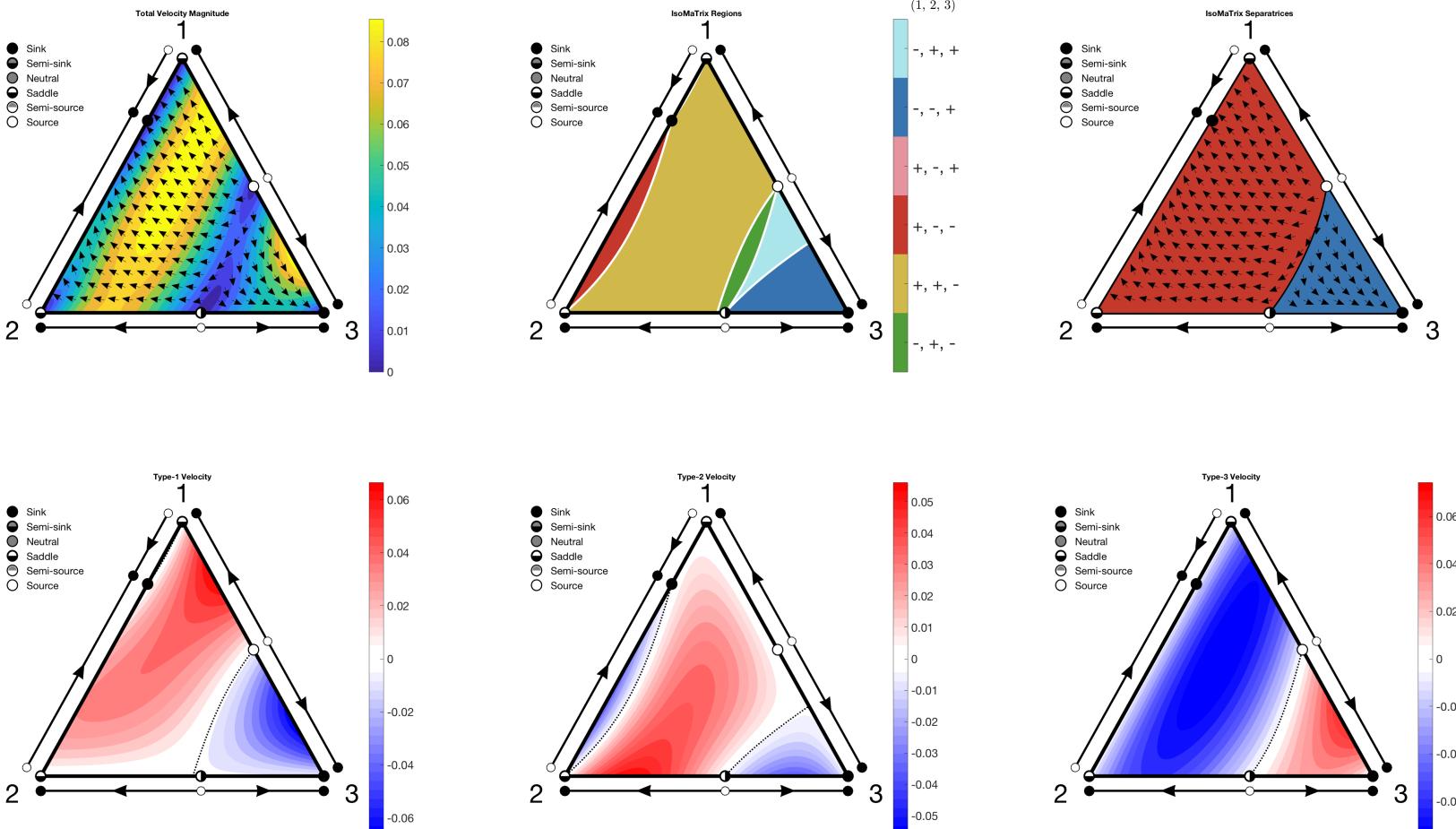
**Joint work with:**

- Yongqian Ma (Univ. Southern California)
- Artem Kaznatcheev (Oxford Univ)
- Alexander Anderson (Moffitt)

**How to use:**

- 6 graphs with one command: `isomatrix(A)`

# IsoMaTrix: visualizing Isoclines of Matrix games



- 1 IsoMaTrix (MATLAB)**
  - 1.1 `isomatrix(A)` . . . . .
  - 1.2 `isomatrix_fixedpoint(A,index)` . . . . .
  - 1.3 `isomatrix_quiver(A)` . . . . .
  - 1.4 `isomatrix_isocline(A,id)` . . . . .
  - 1.5 `isomatrix_trajectory(A,x0,tF)` . . . . .
  - 1.6 `isomatrix_velocity(A,id)` . . . . .
  - 1.7 `isomatrix_fitness(A,id)` . . . . .
  - 1.8 `isomatrix_region(A)` . . . . .
  - 1.9 `isomatrix_surface(A,id)` . . . . .
  - 1.10 `isomatrix_separatrix(A)` . . . . .
- 2 IsoMaTrix Helper Functions (MATLAB)**
  - 2.1 Coordinate transformations . . . . .
  - 2.2 `replicator(t,x,A)` . . . . .
  - 2.3 `line_plot(A,x0,tF)` . . . . .
  - 2.4 `add_labels(string_array)` . . . . .
  - 2.5 `add_gridlines(gridlines)` . . . . .
  - 2.6 `pairwise_fixedpoint(A)` . . . . .
  - 2.7 `hessian(x,A)` . . . . .
  - 2.8 `A_subset(A,types)` . . . . .
  - 2.9 `Ohtsuki_Nowak_transform(A,k,rule)` . . . . .
- 3 HAL integration with IsoMaTrix (Java)**
  - 3.1 Setting up Integrated Development Environment . . . . .
  - 3.2 HALMatrixGame2D and HALMatrixGame3D . . . . .
  - 3.3 Fitness Neighborhood . . . . .
  - 3.4 Deterministic or Stochastic Updating . . . . .
  - 3.5 Population Update Fraction . . . . .
  - 3.6 SingleSimulation(int timesteps) . . . . .
  - 3.7 MeshGrid(int timesteps, int nSims) . . . . .
- 4 Visualizing HALMatrixGames using IsoMaTrix**
  - 4.1 `HAL_isomatrix()` . . . . .
  - 4.2 `HAL_isomatrix_trajectory(color)` . . . . .
  - 4.3 `HAL_isomatrix_quiver(uncertainty_boolean)` . . . . .
  - 4.4 `HAL_isomatrix_velocity(id)` . . . . .
  - 4.5 `HAL_isomatrix_region()` . . . . .
  - 4.6 `HAL_isomatrix_uncertainty(id)` . . . . .



# Integrated Mathematical Oncology

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