Homework 10

1 Graded Problems

1. True/False Questions (9pt)
   State True or False for the following sentences and give a brief explanation.

   (a) If someone proves P=NP, then it would imply that every decision problem can be solved in polynomial time.

   (b) Assume A is a decision problem, If $A \leq_p B$ and $B \in$ NP, then $A \in$ NP.

   (c) Assume $P \notin NP$. Let A and B be decision problems. If $A \in NPC$ and $A \leq_p B$, then $B \in P$.

2. Show that vertex cover remains NP-Complete even if the instances are restricted to graphs with only even degree vertices. (15pts)

3. Consider the partial satisfiability problem, denoted as 3-Sat(α). We are given a collection of k clauses, each of which contains exactly three literals, and we are asked to determine whether there is an assignment of true/false values to the literals such that at least αk clauses will be true. Note that 3-Sat(1) is exactly the 3-SAT problem from lecture.
   Prove that 3-Sat(15/16)is NP-complete. (20 points)
   Hint: If x, y, and z are literals, there are eight possible clauses containing them:

   $(x \vee y \vee z), (!x \vee y \vee z), (x \vee !y \vee z), (x \vee y \vee !z), (!x \vee !y \vee z), (!x \vee y \vee !z), (x \vee !y \vee !z), (!x \vee !y \vee !z)$

4. Given a graph $G = (V, E)$ and two integers k, m, the *Dense Subgraph* Problem is to find a subset V' of V, whose size is at most k and are connected by at least m edges. Prove that the *Dense Subgraph* Problem is NP-Complete.(20 pts)

2 Ungraded Problems

1. There are N cities, and there are some undirected roads connecting them, so they form an undirected graph $G(V, E)$. You want to know, given K and M, if there exists a subset of cities of size K, and the total number of roads between these cities is larger or equal to M. Prove that the problem is NP-Complete.

2. Suppose we have a variation on the 3-SAT problem called Min-3-SAT, where the literals are never negated. Of course, in this case it it possible to satisfy all clauses by simply setting all literals to true. But, we are additionally given a number k, and are asked to determine whether we can satisfy all clauses while setting at most k literals to be true. Prove that Min-3-SAT is NP-Complete.