Introducting Relativity

Problem Set 2 (Due 2024/3/28)

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1. (Compton scattering 20%) A photon of wave length λ is scattered by a rest electron of mass m_e . If the scattered photon has wave length λ' with angle θ from the incident direction. Show that

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta)$$

2. (Relativistic energy 20%) Using the relativistic force defined by

$$\vec{F} = \frac{d}{dt}(\gamma m_0 \vec{u}), \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

and the work done by $\vec{F}$ as

$$\frac{dE}{dt} = \vec{F} \cdot \vec{u}$$

to show that

$$E = \frac{m_0 c^2}{\sqrt{1 - u^2/c^2}} + \text{constant}$$

3. (Vector fields 60%) In $\mathbb{R}^2$ a point can be expressed in Cartesian coordinate $(x^a) = (x, y)$ or polar coordinate $(x'^a) = (r, \theta)$.

(1a) Find the transformation matrix $J' = (\partial x'^a / \partial x^b)$ and its inverse $J = (\partial x^a / \partial x'^b)$ in terms of x'^a.

(1b) Let $f(x, y)$ be a function on the circle $x^2 + y^2 = a^2$. Obtain the vector field $X = X^a \partial_a$ such that $df/d\theta = Xf$.

(1c) Using J' to obtain the corresponding X'^a for the vector field in (1b)

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(1) 4-momentum: $p^\mu = (E/c, \vec{p})$ satisfies

$$p^\mu p_\mu = (E/c)^2 - \vec{p}^2 = m_0^2 c^2$$

(2) Energy and momentum for photon

$$E = pc = h\nu, \quad c = \lambda\nu$$