Introducting Relativity

Problem Set 1 (Due 2024/3/12)

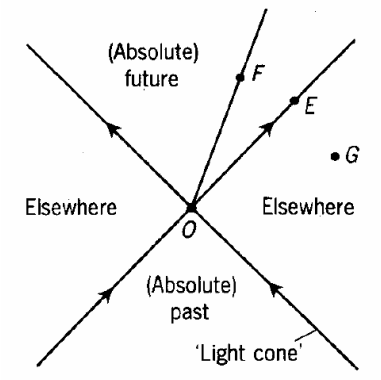

Figure 1: Light cone and causality of the frame S

1. (Causality)

   Let E be the event on the light-cone and G be an event outside light-cone in the inertia frame S.

   (a) Draw the axis of the frame S' in S so that G occurs earlier than O.

   (b) Draw the axis of the frame S'' in S so that G occurs later than O.

2. (Lorentz transformation) Given the Lorentz transformation in x-directin

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \equiv L(v) \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

   Show that from the combination of two Lorentz transformations $L(v_2)L(v_1) = L(v)$ one has

$$v = \frac{v_1 + v_2}{1 + v_1 v_2/c^2}$$

3. (Length contraction) Let S and S' are inertial frames relative with $v = \alpha c$ where $0 < \alpha < 1$. If a rod at rest in S' makes an angle of $\pi/6$ with Ox' in S' and $\pi/4$ with Ox in S. Find the parameter α.

4. (Aberration) A light ray from a star to a telescope observer has an inclination θ' to the horizontal in S' and θ in S, where S and S' are related by speed v. Show that

$$\tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta + v/c)}$$

   where $\gamma = 1/\sqrt{1 - v^2/c^2}$.