1. (10 points) 若 $\displaystyle\int_0^\infty x^k e^{-x}\ dx\ =\ k!$ (k 為正整數), 試證明 $\displaystyle\int_0^\infty x^{k+1} e^{-x}\ dx\ =\ (k+1)!$.

2. (10 points) 判斷數字 a 必須有多大才能滿足不等式:

$$\int_a^\infty \frac{1}{1+x^2}\ dx\ <\ 0.001$$

3. (10+10=20 points) 計算以下積分: (a) $\displaystyle\int_0^\infty \sin\theta \cdot e^{\cos\theta} \, d\theta$

(b) $\displaystyle\int_1^\infty \frac{\ln x}{x^2} \, dx$