系級........... 組別: A / B 學號 .... 姓名....................

微積分 (I) Quiz #2 (45 minutes) 2023/10/02

除了選擇,填充和簡答題之外，你的答案必須提供完整說明，如果只有答案沒有任何說明得零分!

1. (5+5=10 points) Find the limits: (a) $\lim_{x\to 0}\frac{\sin(x^2)}{x}$ (b) $\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}$

(a) $= \lim_{x\to 0}\frac{\sin(x^2)}{(x^2)}\cdot x = 1\cdot 0 = 0$

$(u = x^2)$

(b) $= \lim_{x\to 1}\frac{\sin(x-1)}{(x+2)(x-1)} = \lim_{x\to 0}\frac{\sin(x-1)}{(x-1)}\cdot\frac{1}{(x+2)}$, 令 $u=(x-1)$, $x=u+1$, $x\to 1 \Leftrightarrow u\to 0$

$= \lim_{u\to 0}\frac{\sin(u)}{u}\cdot\left(\frac{1}{u+3}\right) = 1\cdot\left(\frac{1}{0+3}\right) = \frac{1}{3}$

2. (3+3+4=10 points) 簡答題. Find the derivatives: (a) $y = 5^{\sqrt{x}}$

(a) $y' = 5^{\sqrt{x}}\cdot \ln 5\cdot \frac{d}{dx}\sqrt{x} = \underline{(\ln 5)\, 5^{\sqrt{x}}\cdot\frac{1}{2}\frac{1}{\sqrt{x}}}$

(b) $f(t) = \csc(\pi t)$

$f'(t) = -\csc(\pi t)\cdot\cot(\pi t)\cdot\frac{d}{dt}(\pi t) = \underline{-\pi\csc(\pi t)\cdot\cot(\pi t)}$

(c) $y = e^{\tan x}$. $y' = e^{\tan(x)}\cdot\frac{d}{dx}(\tan x) = \underline{e^{\tan(x)}\cdot\sec^2(x)}$

3. (5+5=10 points) 簡答題. Find the derivatives: (a) $F(t) = \left(\frac{1}{2t+1}\right)^4 = (2t+1)^{-4}$

(b) $y = e^{z/(z-1)}$

$\underline{F'(t) = (-4)\cdot(2t+1)^{-5}\cdot 2}$

$y' = e^{\frac{z}{(z-1)}}\cdot\frac{d}{dz}\left(\frac{z}{z-1}\right) = e^{z/(z-1)}\cdot\frac{(z-1)-z}{(z-1)^2} = \underline{\frac{e^{z/(z-1)}\cdot(-1)}{(z-1)^2}}$

4. (10 points) Find an equation of the tangent line to the curve $y = \dfrac{\cot 2t}{e^t}$ at the point $(\pi/4, 0)$.

$= e^{-t}\cot(2t)$

$y' = -e^{-t}\cot(2t) + e^{-t}\cdot(-\csc^2 2t)\cdot 2$

$= -e^{-t}(\cot 2t + 2\csc^2 2t)$

$y'(\pi/4) = -e^{-\pi/4}\left(\underset{0}{\cot\frac{\pi}{2}} + 2\underset{2}{\csc^2\frac{\pi}{2}}\right) = \dfrac{-2}{e^{\pi/4}}$

$y - 0 = \dfrac{-2}{e^{\pi/4}}(x - \pi/4)$

5. (10 points) Find the derivative $f(t) = \tan\sqrt{1+t^2}$ and compute $f'(1)$.

$= \tan(1+t^2)^{\frac{1}{2}}$

$f' = \sec^2\sqrt{1+t^2}\cdot\frac{d}{dt}\sqrt{1+t^2}$

$= \sec^2\sqrt{1+t^2}\cdot\frac{1}{2}\dfrac{2t}{\sqrt{1+t^2}} = \sec^2(\sqrt{1+t^2})\cdot\dfrac{t}{\sqrt{1+t^2}}$

$f'(1) = \sec^2(\sqrt{2})\cdot\dfrac{1}{\sqrt{2}}$

6. (10 points) Find the derivative $f(x) = (1+\cos^2 x)^3$ and compute $f'(\pi/4)$.

$f'(x) = 3(1+\cos^2 x)^2\cdot\frac{d}{dx}(1+\cos^2 x)$

$= 3(1+\cos^2 x)^2\cdot 2\cos x\cdot\frac{d}{dx}(\cos x)$

$= 6(1+\cos^2 x)^2\cdot\cos x\cdot(-\sin x)$

$f'(\pi/4) = 6\left(1+\left(\frac{1}{\sqrt{2}}\right)^2\right)^2\cdot\frac{1}{\sqrt{2}}\cdot\frac{-1}{\sqrt{2}}$

$= 6\cdot\left(1+\frac{1}{2}\right)^2\cdot\left(\frac{-1}{2}\right) = 6\left(\frac{3}{2}\right)^2\left(\frac{-1}{2}\right)$

$= \dfrac{-27}{4}$