系級............ 組別: A / B 學號 .... 姓名..................

微積分 (I) Quiz #8 (45 minutes) 2023/12/4

除了選擇,填充和簡答題之外,你的答案必須提供完整說明,如果只有答案沒有任何說明得零分!

1. (points) 求以下定積分: (a) $\int_0^{\pi/6} \frac{\sin x}{\cos^2 x}\, dx$ 令 $u = \cos x \Rightarrow du = -\sin x dx$

$\Rightarrow \boxed{\sin x \cdot dx = -du}$

$$= \int_1^{\sqrt{3}/2} \frac{1}{u^2}(-du) = u^{-1}\Big|_1^{\sqrt{3}/2} = \frac{2}{\sqrt{3}} - 1$$

或先算不定積分: $\int \frac{\sin x dx}{\cos^2 x} = \int \frac{1}{u}(-du) = u^{-1} + C = \frac{1}{\cos x} + C$ 再代入積分上下限.

(b) $\int_0^{\pi} (\underbrace{\sin^2 x}_{\frac{1-\cos(2x)}{2}} - \sin^3 x)\, dx$ $= \int_0^{\pi} \frac{1-\cos(2x)}{2} - \int_0^{\pi} \sin^2(x) \sin x\, dx$

$$= \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi} \cos(2x) - \int_0^{\pi} (1-\cos^2 x) \sin x\, dx$$ 令 $u = \cos x$, $du = -\sin x dx$

$$= \frac{\pi}{2} - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \sin(2x)\Big|_0^{\pi} + \int_1^{-1} (1-u^2)\, du$$

$$= \frac{\pi}{2} - \frac{1}{4} \cdot 0 - \int_{-1}^{1} (1-u^2) du = \frac{\pi}{2} - u\Big|_{-1}^{1} + \frac{u^3}{3}\Big|_{-1}^{1}$$

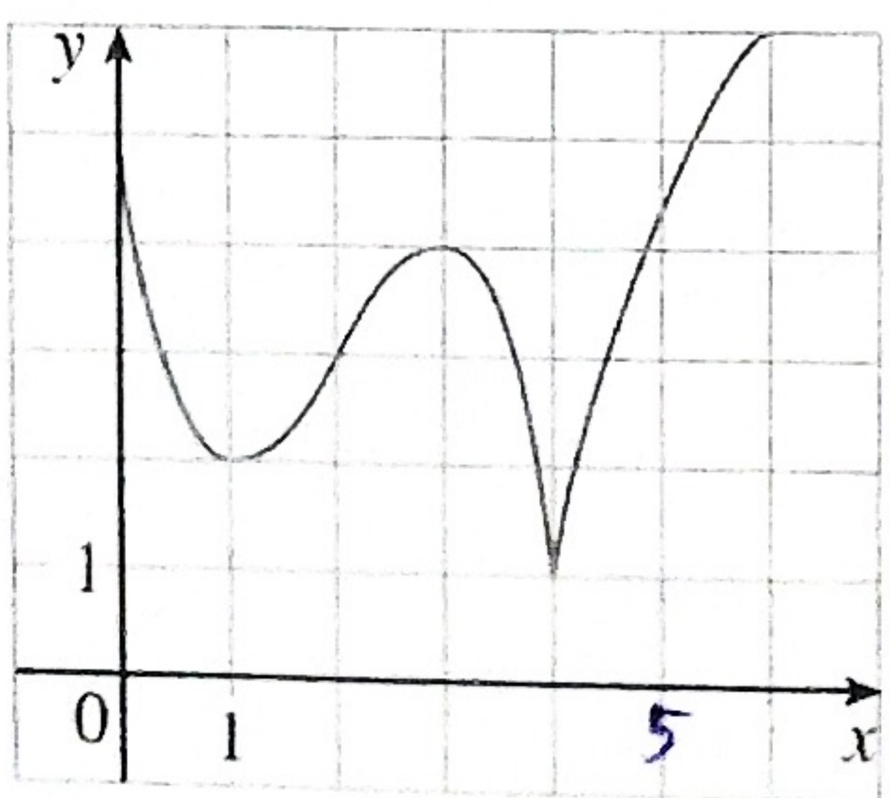
$$= \frac{\pi}{2} - 2 + \frac{2}{3} = \frac{\pi}{2} - \frac{4}{3}$$

(c) $\int_0^{\pi/2} \cos x \cdot \sin(\sin x)\, dx$ 令 $v = \sin x \Rightarrow dv = \cos x \cdot dx$

$$= \int_0^{\pi/2} \sin(\sin x) \cdot [\cos x dx] = \int_0^1 \sin(u) du = -\cos(u)\Big|_0^1$$

$$= \cos(0) - \cos(1) = 1 - \cos(1)$$

2. (10 points) 填充題. 依照以下 y=f(x) 在 [0, 6] 的 圖形回答下列問題:



(a) The open intervals on which f is increasing. (1, 3), (4, 6)

(b) The open intervals on which f is decreasing. (0, 1), (3, 4)

(c) The open intervals on which f is concave upward. (0, 2),

(d) The open intervals on which f is concave downward. (2, 4), (4, 6)

(e) The coordinates of the points of inflection. $x = 2$

(f) absolute maximum / minimum: 絕對極大值: $x = 6$

" 極小值: $x = 4$

3. 計算以下積分：(a) $\int_0^{\pi/2} \sin^2(x)\cos^2(x)\ dx = \frac{1}{4}\int_0^{\pi/2} 4\sin^2 x\cos^2 x\,dx$

$= \frac{1}{4}\int_0^{\pi/2} (2\sin x\cdot\cos x)^2 dx = \frac{1}{4}\int_0^{\pi/2} \sin^2(2x)\,dx$

$= \frac{1}{4}\int_0^{\pi/2} \frac{1-\cos(4x)}{2}\,dx = \frac{1}{8}\int_0^{\pi/2} [1-\cos(4x)]\,dx$

$= \frac{1}{8}\left(\frac{\pi}{2} - \frac{1}{4}\sin(4x)\Big|_0^{\pi/2}\right) = \frac{\pi}{16}$

(b) $\int \tan^2(x)\cos^3(x)\ dx = \int \frac{\sin^2 x}{\cos^2 x}\cdot\cos^3(x)\,dx = \int \sin^2 x\cdot\cos x\,dx$

令 $u = \sin x \Rightarrow du = (\cos x\,dx)$

$= \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3(x) + C$

4. (points) Let $f(x) = e^{2x} + e^{-x}$. $f' = e^{2x}\cdot 2 - e^{-x} = \frac{e^{3x}\cdot 2 - 1}{e^x} = 0$

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) Find the intervals of concavity and the inflection points.

$e^{3x}\cdot 2 - 1 = 0$

$e^{3x} = \frac{1}{2} \Rightarrow 3x = \ln\left(\frac{1}{2}\right)$

$x = \frac{1}{3}(\ln 1 - \ln 2) = \frac{-\ln 2}{3}$

(a) $f'(x) > 0$ if $x > -\frac{\ln 2}{3}$ 或 $\left(-\frac{\ln 2}{3}, \infty\right)$

$f' < 0$ if $x < -\frac{\ln 2}{3}$ 或 $(-\infty, -\ln 2/3)$

(b) $\searrow \mid \nearrow$ at $-\frac{\ln 2}{3}$

在 $x = \frac{-\ln 2}{3}$ 處為 local min，沒有 local max.

(c) $f'' = e^{2x}\cdot 2\cdot 2 - e^{-x}(-1) = 4e^{2x} + e^{-x} > 0\ \forall x \in \mathbb{R}$

f is concave upward $\forall x \in \mathbb{R}$, No inflection points