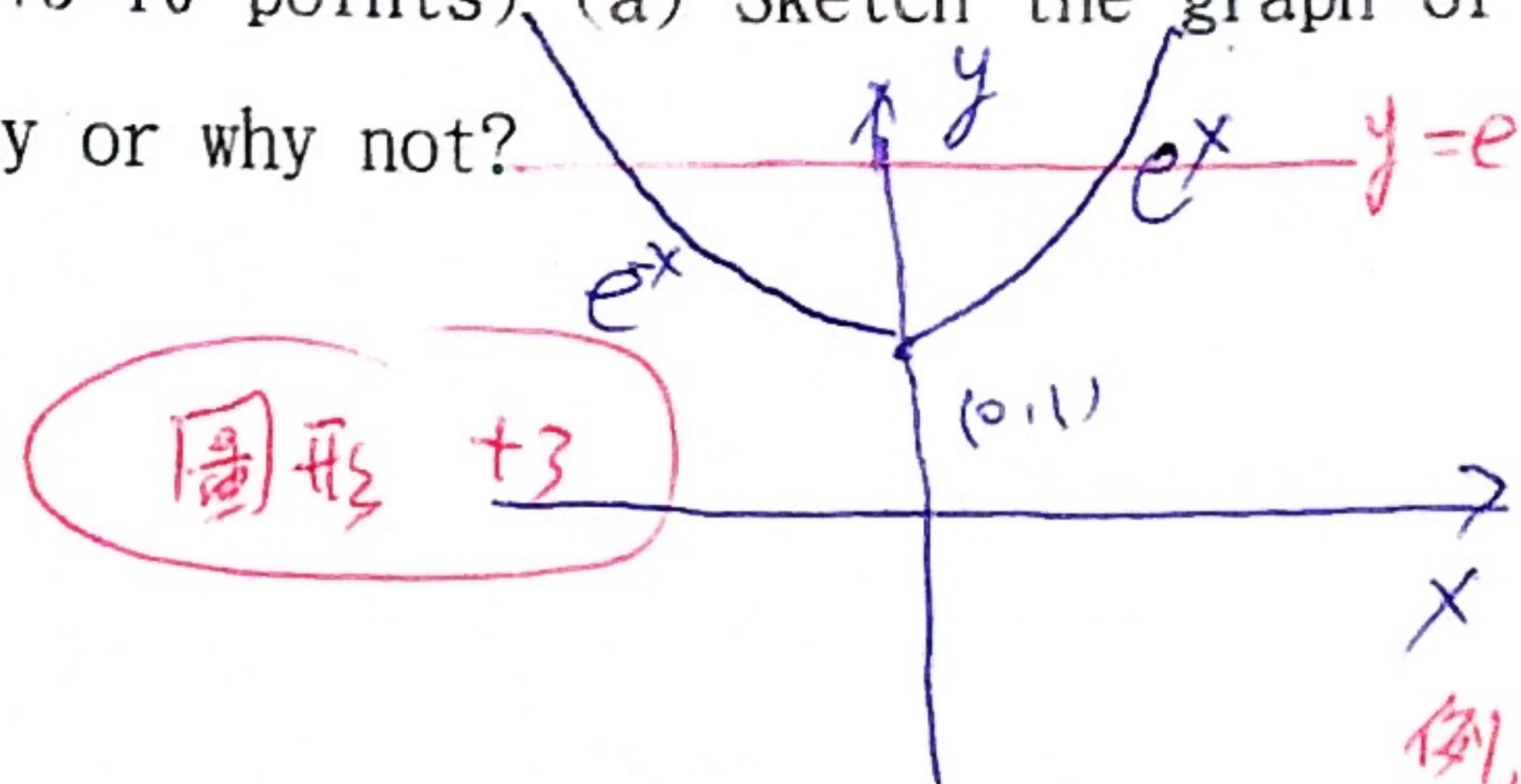


除了選擇, 填充和簡答題之外, 你的答案必須提供完整說明, 如果只有答案沒有任何說明得零分!

1. (5+5=10 points) (a) Sketch the graph of $f(x) = e^{|x|}$. Is it a one-to-one function?

Why or why not?



$$\begin{cases} e^x, & x \geq 0 \\ e^{-x}, & x < 0 \end{cases}$$

$$x = \pm 1 \text{ 時, } y = e$$

無法通過水平線檢測
→ 不是 1-1

- (b) Find the inverse function $g^{-1}(x)$ of $g(x) = \ln(x+3)$.

$$\text{令 } y = \ln(x+3)$$

$$\Rightarrow e^y = x+3$$

$$\Rightarrow x = e^y - 3$$

$$\therefore g^{-1}(x) = e^x - 3$$

2. (5+5=10 points) Solve for x : (a) $e^{2x} - 3e^x + 2 = 0$

- (b) $1 - 2\ln x < 3$

Note: $e^{2x} = (e^x)^2$

$$(e^x)^2 - 3e^x + 2 = 0$$

$$(e^x - 1)(e^x - 2) = 0$$

$$\therefore \begin{cases} e^x - 1 = 0 \Rightarrow e^x = 1 \\ \text{or } e^x - 2 = 0 \Rightarrow e^x = 2 \end{cases}$$

$$\Rightarrow x = \ln(1) = 0 \text{ or } \ln 2$$

$$2\ln x > 1 - 3 = -2$$

$$\ln x > -1$$

$$\Rightarrow x > e^{-1}$$

3. (4+3+3=10 points) 簡答題.

The graph of f is given.

$$f^{-1}: \text{定義域: } [-1, 3] \quad (+2)$$

$$f^{-1}: \text{值域: } [-3, 3] \quad (+2)$$

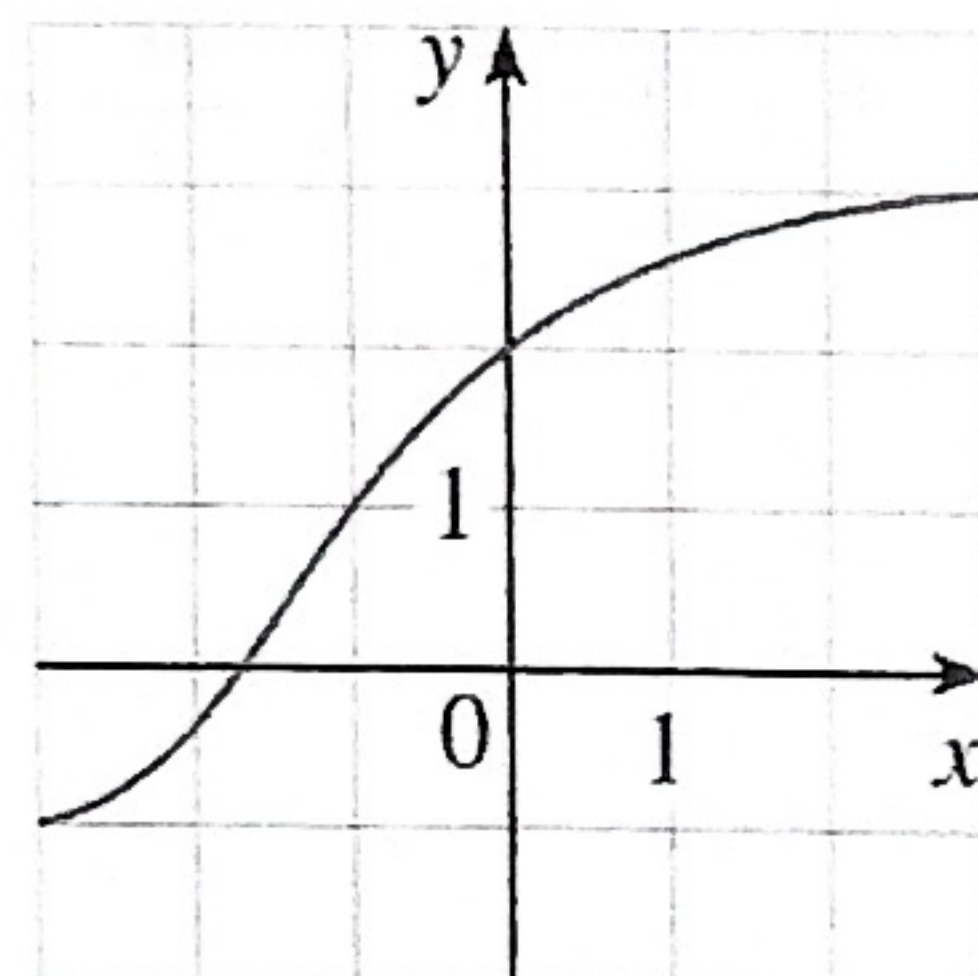
- (a) What are the domain and range of f^{-1} ?

- (b) What is the value of $f^{-1}(2)$?

$$\therefore f(0) = 2 \therefore f^{-1}(2) = 0 \quad (+3)$$

- (c) What is the value of $f^{-1}(-1)$?

$$\therefore f(3) = -1, \therefore f^{-1}(-1) = 3 \quad (+3)$$



4. (5+5=10 points) If $f(x) = e^x \sin x$, find $f'(x)$ and $f''(x)$.

$$f' = (e^x)' \sin x + e^x \cdot (\sin x)' = e^x \sin x + e^x \cos x = e^x (\sin x + \cos x) \quad (+5)$$

$$\begin{aligned} f'' &= \left(\frac{d}{dx}(e^x) \right) \cdot (\sin x + \cos x) + e^x \cdot \frac{d}{dx}(\sin x + \cos x) \\ &= e^x (\sin x + \cos x) + e^x (\cos x - \sin x) \\ &= 2e^x \cos x \end{aligned} \quad (+5)$$

5. (10 points) Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$.

$$y' = \frac{d}{dx}(x^{\frac{3}{2}}) = \frac{3}{2}x^{\frac{1}{2}} \quad (+2)$$

在 $x=a$ 處的斜率 $= 3$, $\Rightarrow \frac{3}{2}\sqrt{a} = 3$ (+4)

$$\therefore \sqrt{a} = 2, \Rightarrow a = 4$$

此處 y 座標 $= a\sqrt{a} = 4\sqrt{4} = 8$; $(4, 8)$ (+2)

\therefore 切線方程式: $y - 8 = 3(x - 4)$ 或 $y = 3x - 4$ (+2)

6. (5+5=10 points) Find the derivatives:

(a) $y = (x + 2\sqrt{x})e^x \Rightarrow y' = (1 + 2 \cdot \frac{1}{2\sqrt{x}})e^x + (x + 2\sqrt{x})e^x$ (+5)

$$= (1 + \frac{1}{\sqrt{x}} + x + 2\sqrt{x})e^x$$

(b) $f(t) = \frac{5t}{t^3 - t - 1}$

$$f' = \frac{5(t^3 - t - 1) - 5t(3t^2 - 1)}{(t^3 - t - 1)^2} \quad (+5)$$

$$\text{或} \frac{5t^3 - 5t - 15t^3 + 5t}{(t^3 - t - 1)^2} = \frac{-10t^3 - 5}{(t^3 - t - 1)^2}$$