系級............ 組別： A / B 學號 .... 姓名.....................

微積分 (I) Quiz #9 (45 minutes) 2023/12/11

除了選擇，填充和簡答題之外，你的答案必須提供完整說明，如果只有答案沒有任何說明得零分！

1. (10+10=20 points) 求以下積分：(a) $\int \frac{\sin x}{\cos^2 x\sqrt{1+\sec x}}\,dx$

$\frac{\sin x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)$

$= \int \frac{\tan(x)\sec(x)}{\sqrt{1+\sec(x)}}\,dx$ (令 $u = \sec(x)$, $\Rightarrow du = \sec(x)\tan(x)\,dx$)

$= \int u^{-1/2}\,du = 2u^{\frac{1}{2}}+C = 2\sqrt{1+\sec(x)}+C$

(b) $\int (\tan^2 x + \tan^4 x)\,dx = \int \tan^2(x)\,(1+\tan^2(x))\,dx$

$= \int \tan^2(x)\sec^2(x)\,dx$ (令 $u = \tan x \Rightarrow du = \sec^2(x)\,dx$)

$= \int u^2\,du = \frac{u^3}{3}+C = \frac{\tan^3(x)}{3}+C$

2. (10 points) 描繪滿足以下條件的函數圖形($y = f(x)$)，標示局部極值與反曲點座標(如果存在)。

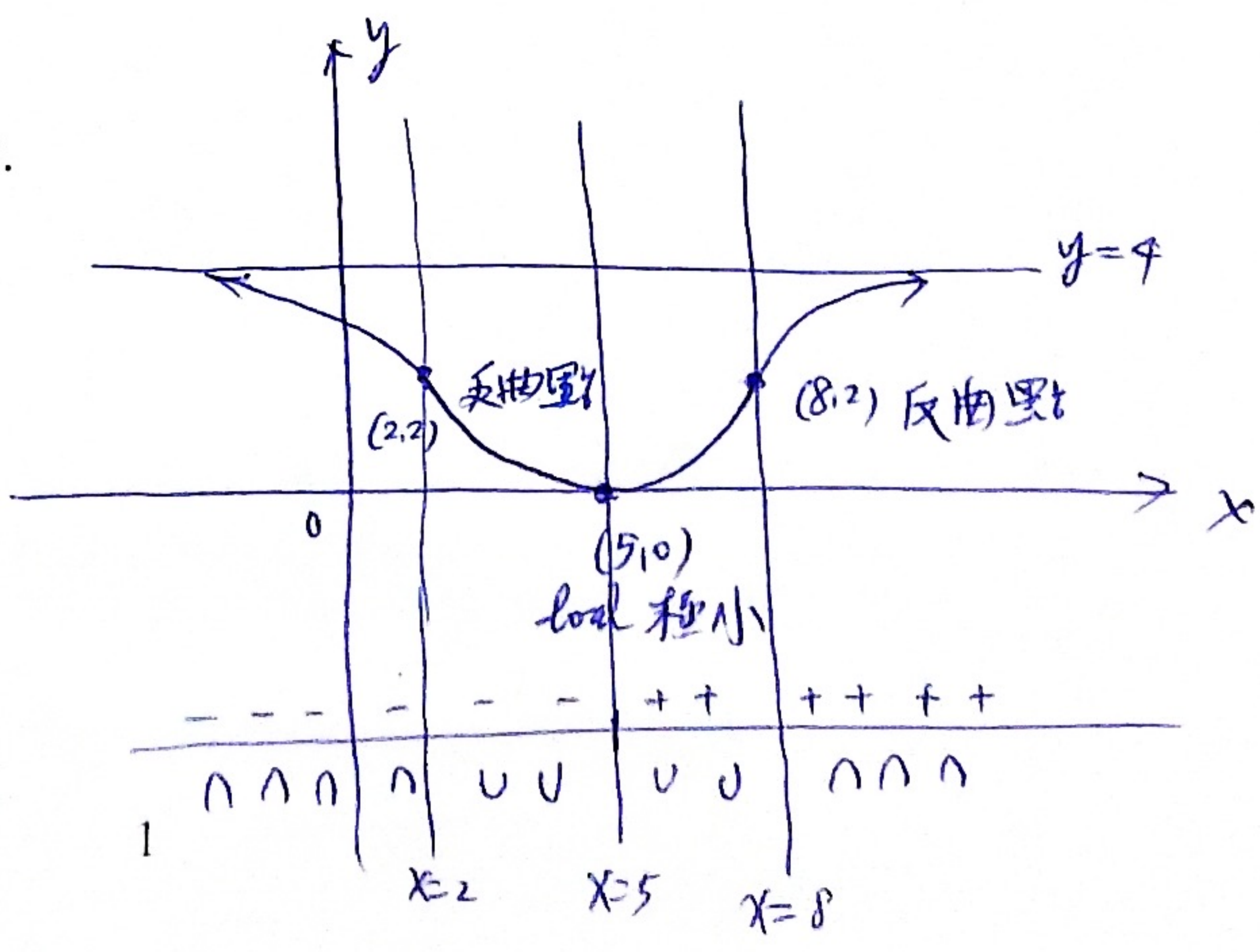
(a) $f'(5) = 0$, $f'(x) < 0$ when $x < 5$, $f'(x) > 0$ when $x > 5$,

(b) $f''(2) = 0$, $f''(8) = 0$, $f''(x) < 0$ when $x < 2$ or $x > 8$,

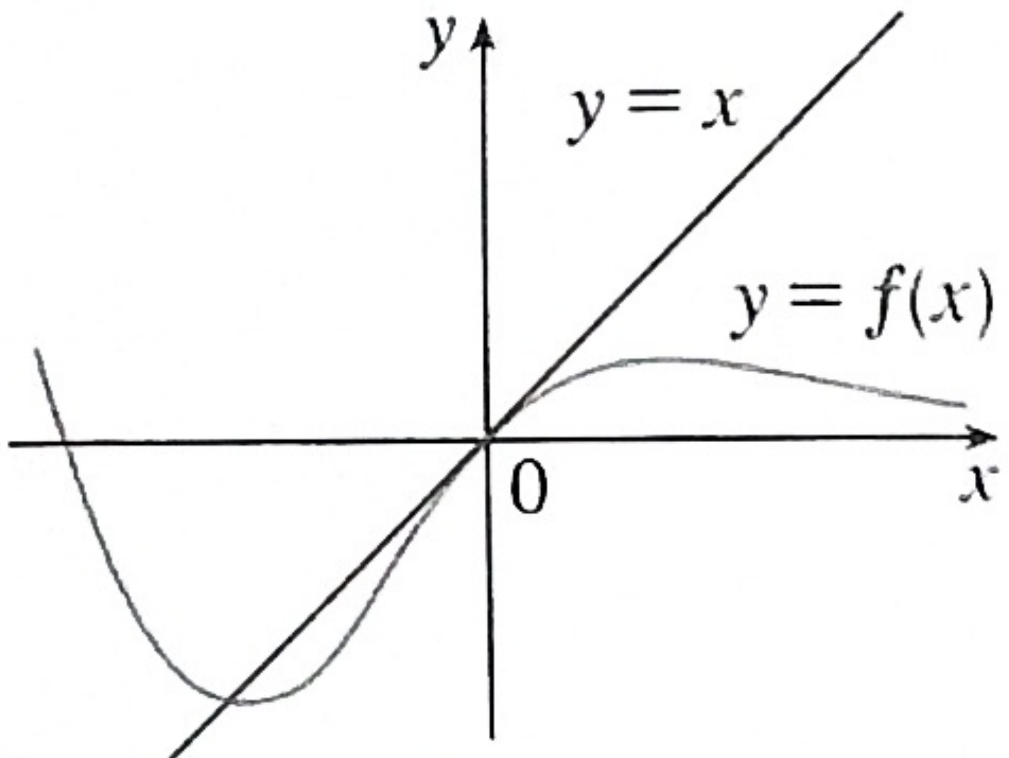
(c) $f''(x) > 0$ for $2 < x < 8$,

(d) $\lim_{x\to\infty} f(x) = 4$, $\lim_{x\to-\infty} f(x) = 4$

(e) $f(5) = 0$, $f(0) = 3$, $f(2) = f(8) = 2$.



3. (7 points) 根據右圖的函數$f(x)$計算極限: $\lim\limits_{x\to 0} \dfrac{f(x)}{\tan^{-1}(4x)}$ $\left(\frac{0}{0}\right)$



函數 f 與其在$x=0$的切線 $y=x$ $\Rightarrow f'(0)=1$

$$= \lim_{x\to 0} \frac{f'(x)}{\frac{1}{1+(4x)^2}\cdot 4} = \frac{1}{\frac{1}{1+0^2}\cdot 4} = \frac{1}{4} \quad \#$$

4. (7+8+8=23 points) 計算極限: (a) $\lim\limits_{x\to 0} \dfrac{1}{h}\displaystyle\int_2^{2+\frac{h}{2}} \sqrt{1+t^3}\, dt$

$$= \lim_{h\to 0} \frac{\int_2^{2+h/2} \sqrt{1+t^3}\, dt}{h} \left(\frac{0}{0}\right) = \lim_{h\to 0} \frac{\sqrt{1+(2+h/2)^3}\cdot \frac{d}{dh}(2+h/2)}{1}$$

$$= \lim_{h\to 0} \sqrt{1+(2+h/2)^3}\cdot\left(\frac{1}{2}\right) = \sqrt{1+8}\cdot\frac{1}{2} = \frac{3}{2}$$

(b) $\lim\limits_{x\to 0+} \left(\dfrac{1}{x} - \dfrac{1}{\tan x}\right) = \lim\limits_{x\to 0^+} \left(\dfrac{\tan x - x}{x\tan x}\right) \left(\frac{0}{0}\right)$

$(\infty-\infty)$不定式

$$= \lim_{x\to 0^+} \frac{\sec^2 x - 1}{\tan x + x\cdot \sec^2 x} \left(\frac{0}{0}\right) = \lim_{x\to 0^+} \frac{2\sec(x)\cdot\sec(x)\tan(x)}{\sec^2(x) + \sec^2 x + x(2\sec x\cdot \sec x \tan x)}$$

$$= \lim_{x\to 0^+} \frac{0}{1+1+0} = 0$$

(c) $\lim\limits_{x\to\infty} x^{3/2}\sin(1/x)$ $(\infty\cdot 0$不定式$)$

$$= \lim_{x\to\infty} \frac{\sin(1/x)}{x^{-3/2}} \left(\frac{0}{0}\right) = \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot(-1)x^{-2}}{\left(-\frac{3}{2}\right)x^{-\frac{5}{2}}} = \lim_{x\to\infty} \cos\left(\frac{1}{x}\right)\left(\frac{2}{3}\right)\cdot x^{-2}\cdot x^{5/2}$$

$$= \frac{2}{3}\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)\cdot x^{\frac{1}{2}} = \frac{2}{3}\cdot 1\cdot\infty = \infty$$

(另解) $\lim\limits_{x\to\infty} x^{3/2}\cdot\sin\left(\frac{1}{x}\right) = \lim\limits_{x\to\infty} x^{\frac{1}{2}}\cdot x\sin\left(\frac{1}{x}\right)$

$$= \lim_{x\to\infty} x^{\frac{1}{2}}\cdot\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \infty\cdot 1 = \infty$$

$\boxed{利用 \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1}$