Note: Vector Calculus

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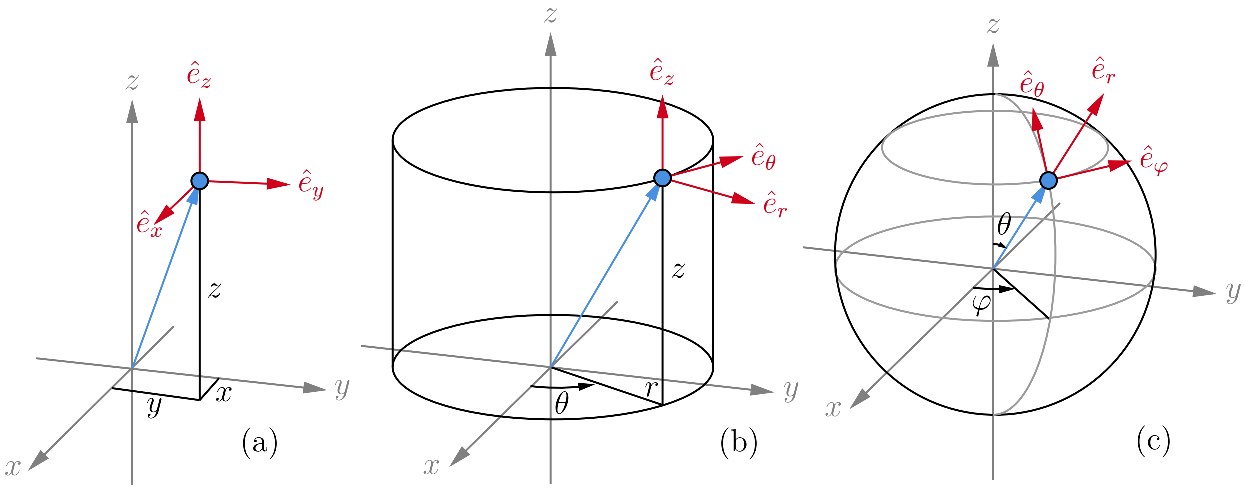
0.1 coordinate convention



0.2 unit vector(basis) transformation

Table 1: Basis Transformation

	Cartesian	Cylindrical	Spherical
Cartesian	$\hat{e}_x = \hat{e}_x$ $\hat{e}_y = \hat{e}_y$ $\hat{e}_z = \hat{e}_z$	$\hat{e}_x = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta$ $\hat{e}_y = \sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta$ $\hat{e}_z = \hat{e}_z$	$\hat{e}_x = \sin\theta\cos\varphi\,\hat{e}_r + \cos\theta\cos\varphi\,\hat{e}_\theta - \sin\varphi\,\hat{e}_\phi$ $\hat{e}_y = \sin\theta\sin\varphi\,\hat{e}_r + \cos\theta\sin\varphi\,\hat{e}_\theta + \cos\varphi\,\hat{e}_\phi$ $\hat{e}_z = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta$
Cylindrical	$\hat{e}_r = \dfrac{x\hat{e}_x + y\hat{e}_y}{\sqrt{x^2+y^2}}$ $\hat{e}_\theta = \dfrac{-y\hat{e}_x + x\hat{e}_y}{\sqrt{x^2+y^2}}$ $\hat{e}_z = \hat{e}_z$	$\hat{e}_r = \hat{e}_r$ $\hat{e}_\theta = \hat{e}_\theta$ $\hat{e}_z = \hat{e}_z$	$\hat{e}_r = \sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta$ $\hat{e}_\theta = \hat{e}_\varphi$ $\hat{e}_z = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta$
Spherical	$\hat{e}_r = \dfrac{x\hat{e}_x + y\hat{e}_y + z\hat{e}_z}{\sqrt{x^2+y^2+z^2}}$ $\hat{e}_\theta = \dfrac{z\left(x\hat{e}_x + y\hat{e}_y\right) - \left(x^2+y^2\right)\hat{e}_z}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}$ $\hat{e}_\varphi = \dfrac{-y\hat{e}_x + x\hat{e}_y}{\sqrt{x^2+y^2}}$	$\hat{e}_r = \dfrac{r\hat{e}_r + z\hat{e}_z}{\sqrt{r^2+z^2}}$ $\hat{e}_\theta = \dfrac{z\hat{e}_r - r\hat{e}_z}{\sqrt{r^2+z^2}}$ $\hat{e}_\varphi = \hat{e}_\varphi$	$\hat{e}_r = \hat{e}_r$ $\hat{e}_\theta = \hat{e}_\theta$ $\hat{e}_\varphi = \hat{e}_\varphi$