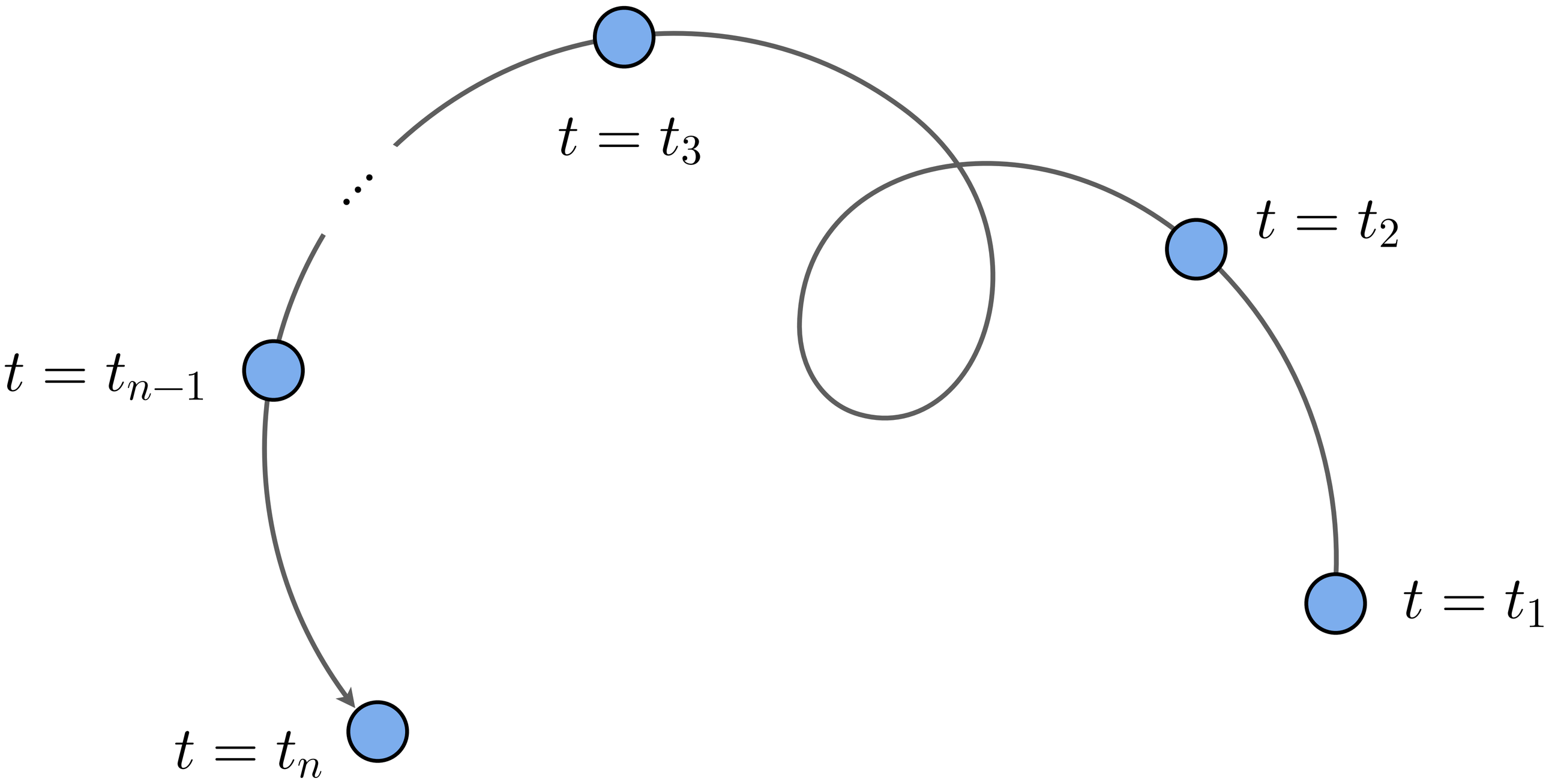
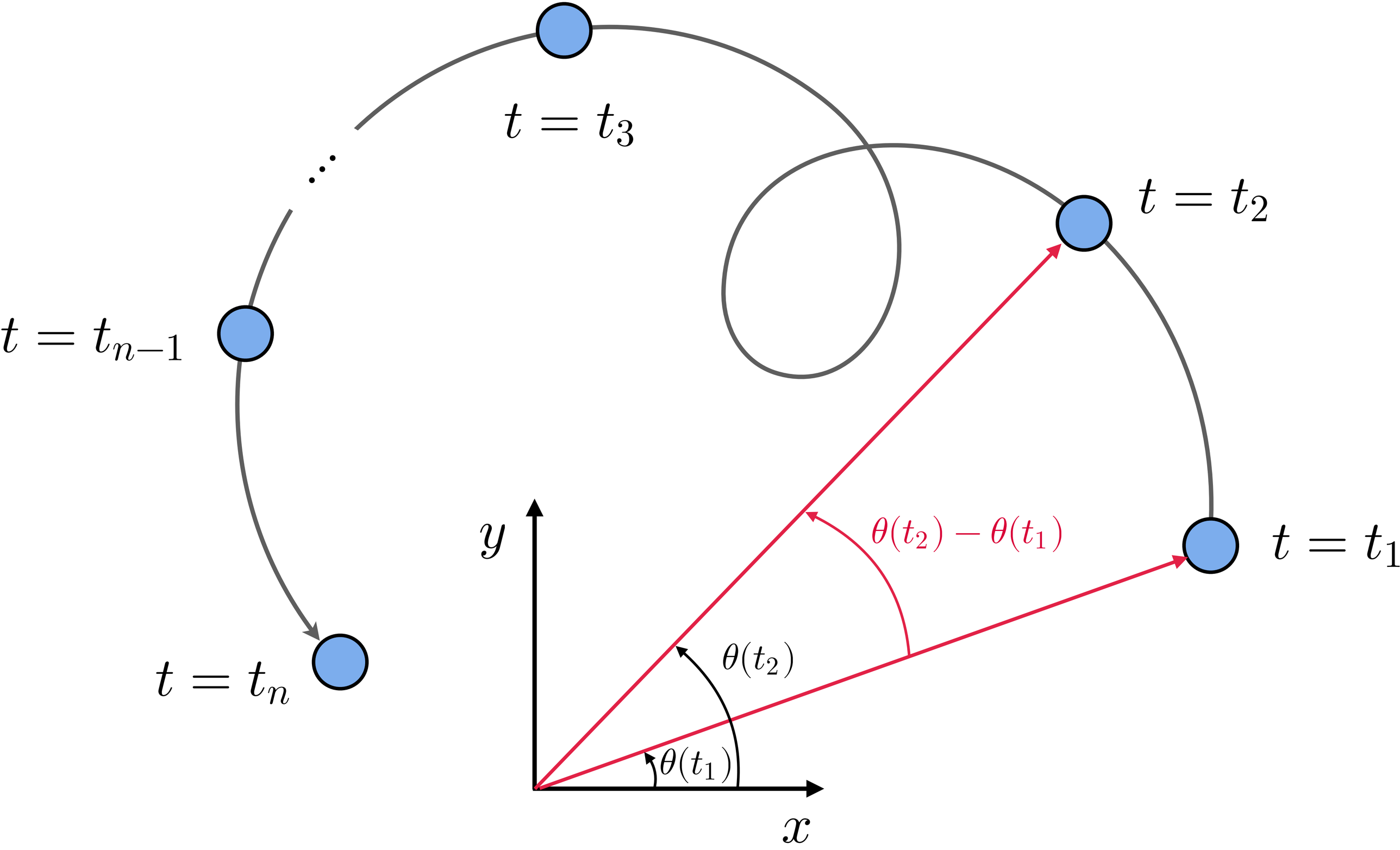
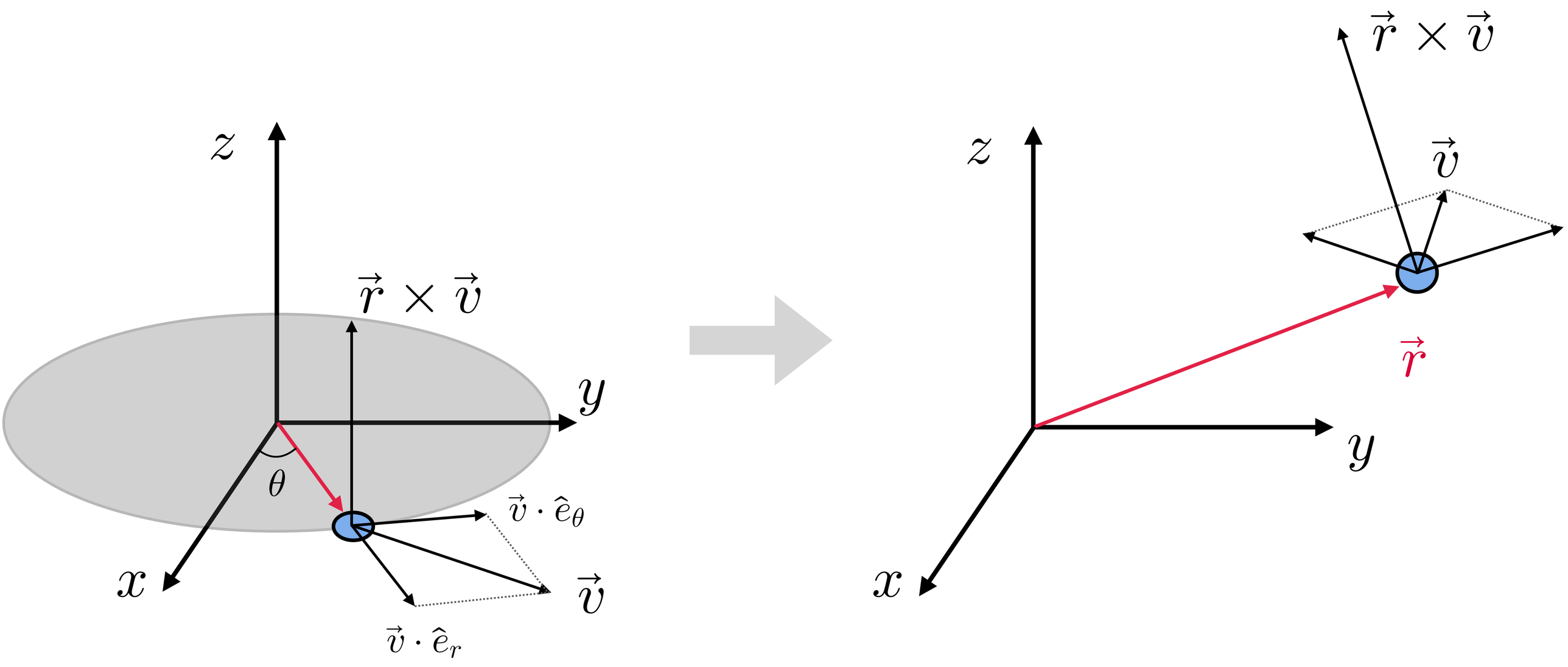
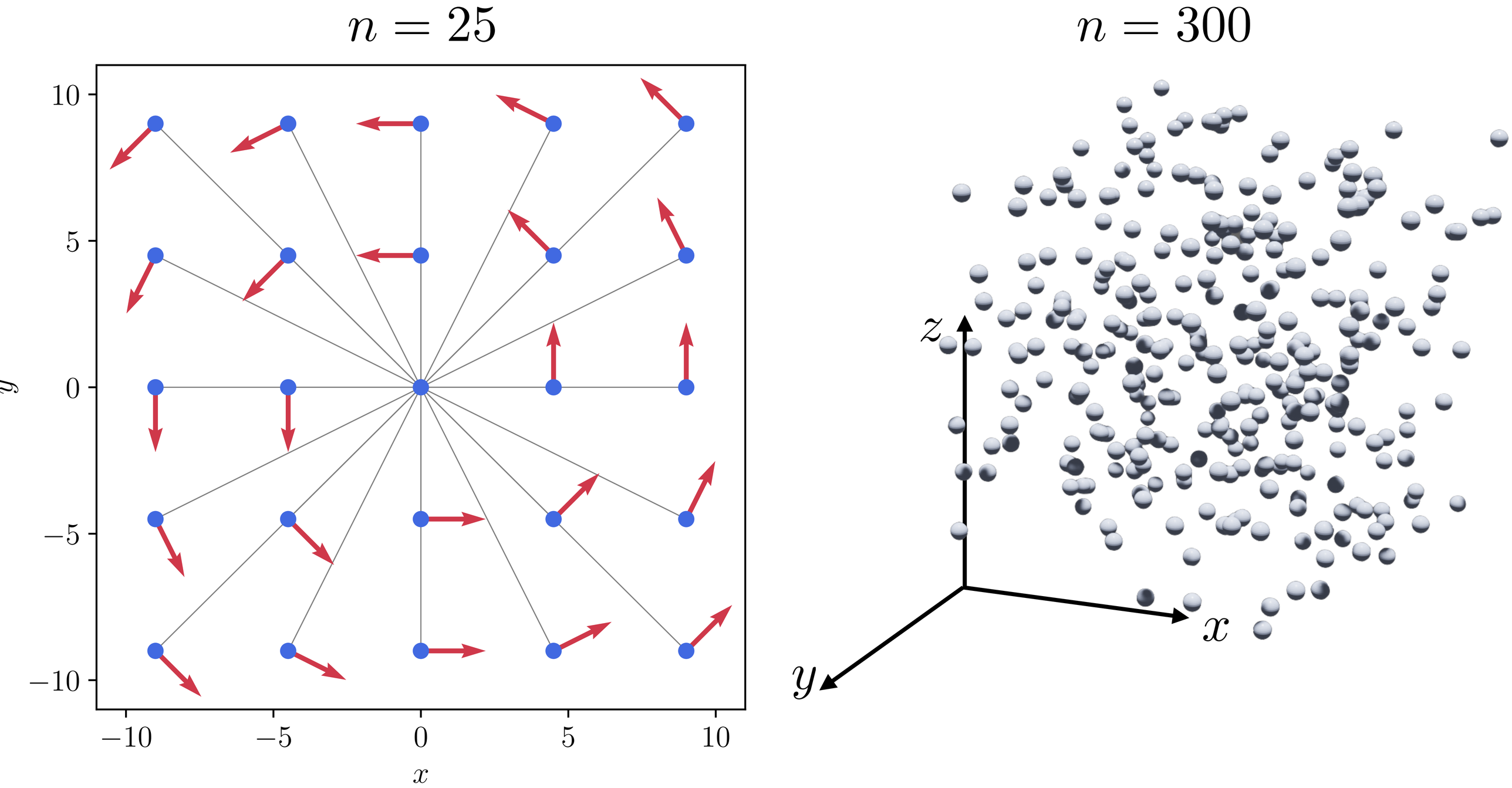
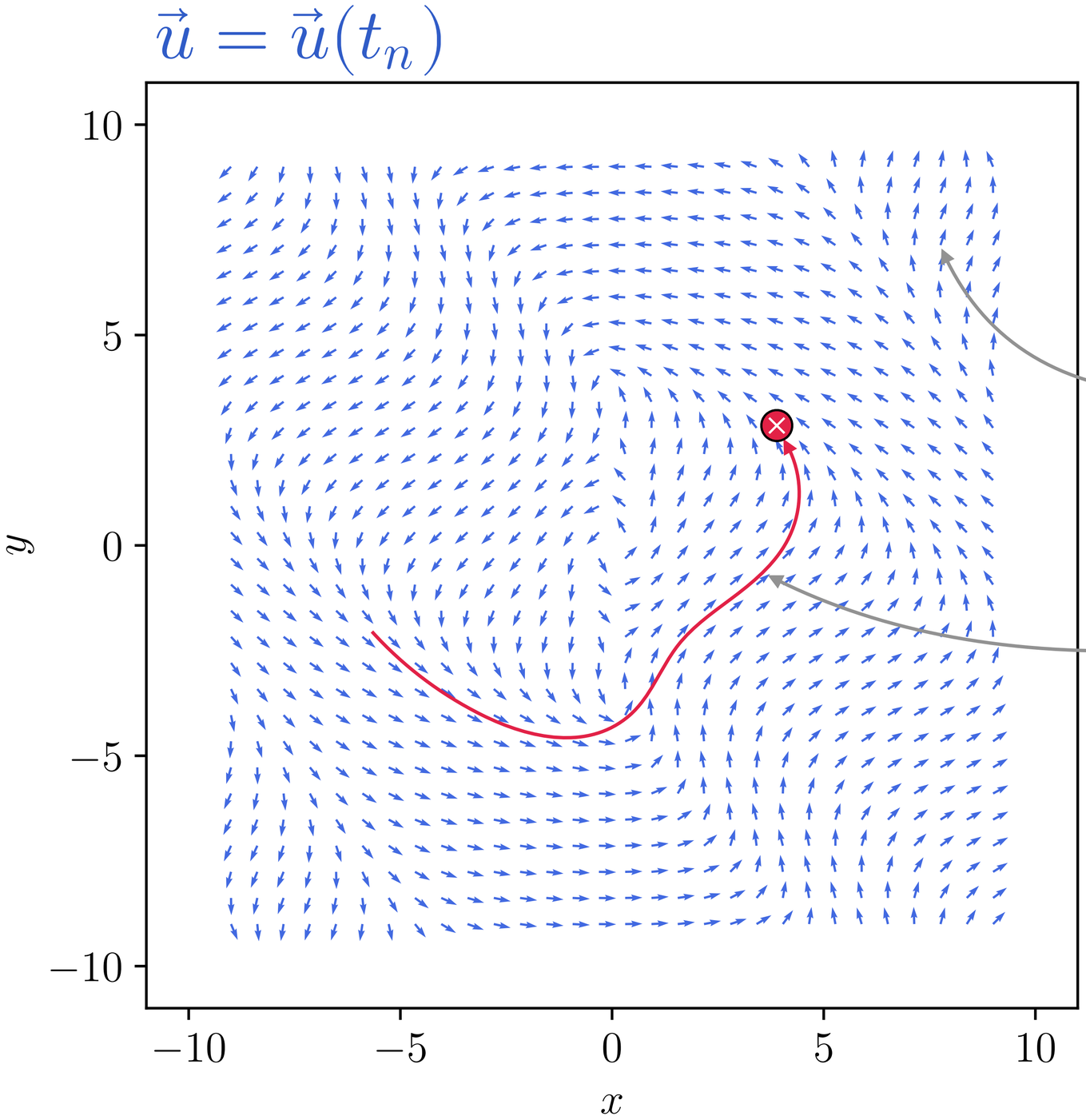

$t = t_3$
$t = t_2$
$t = t_{n-1}$
$t = t_1$
$t = t_n$


$t = t_3$
$t = t_2$
$t = t_{n-1}$
$t = t_1$
$t = t_n$
y
x
$\theta(t_2) - \theta(t_1)$
$\theta(t_2)$
$\theta(t_1)$


z
$\vec{r} \times \vec{v}$
y
θ
$\vec{v} \cdot \hat{e}_\theta$
x
$\vec{v}$
$\vec{v} \cdot \hat{e}_r$
$\vec{r} \times \vec{v}$
z
$\vec{v}$
$\vec{r}$
y
x


$n = 25$
$n = 300$
10
5
0
−5
−10
−10
−5
0
5
10
x
y
z
x
y

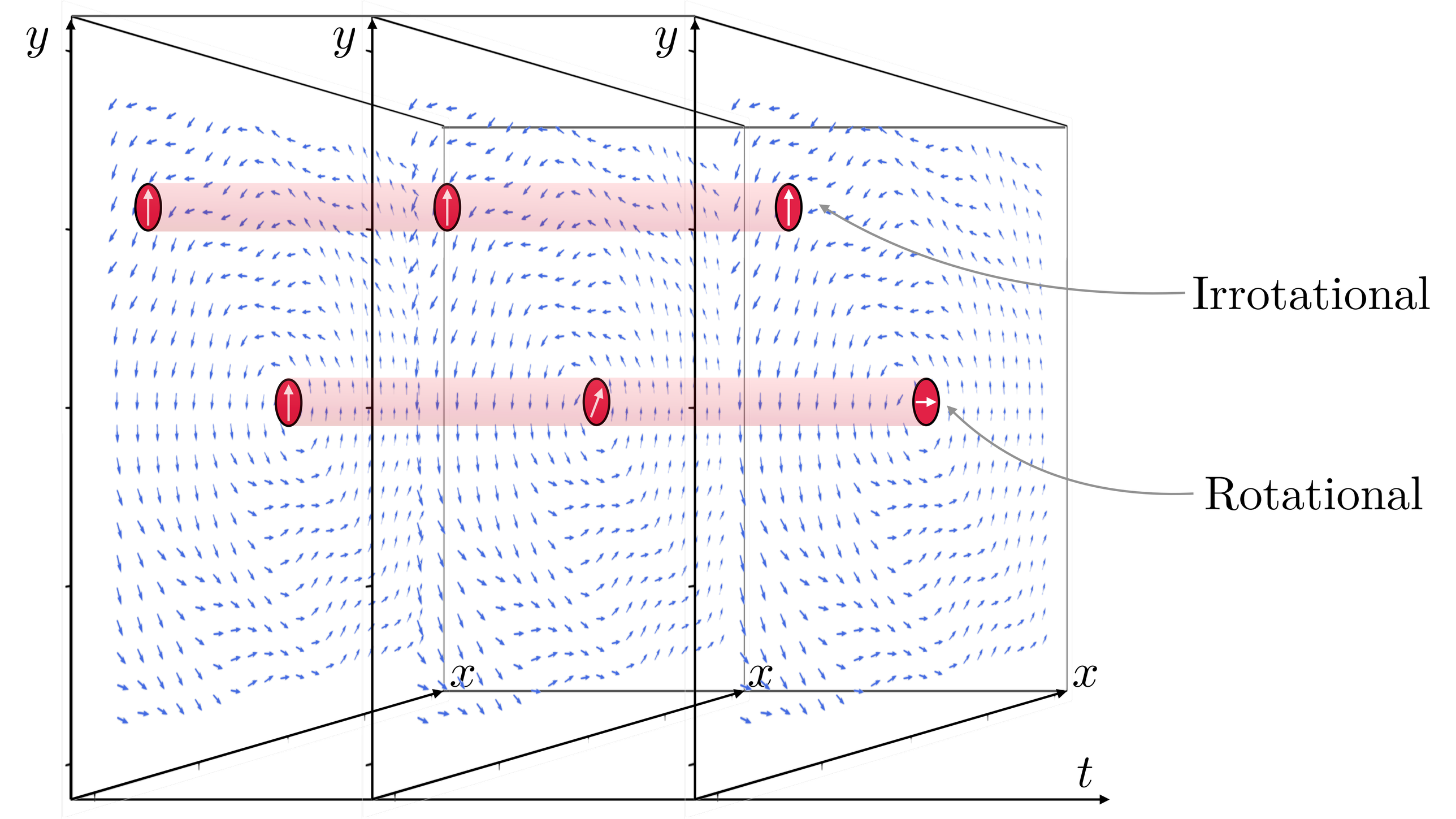


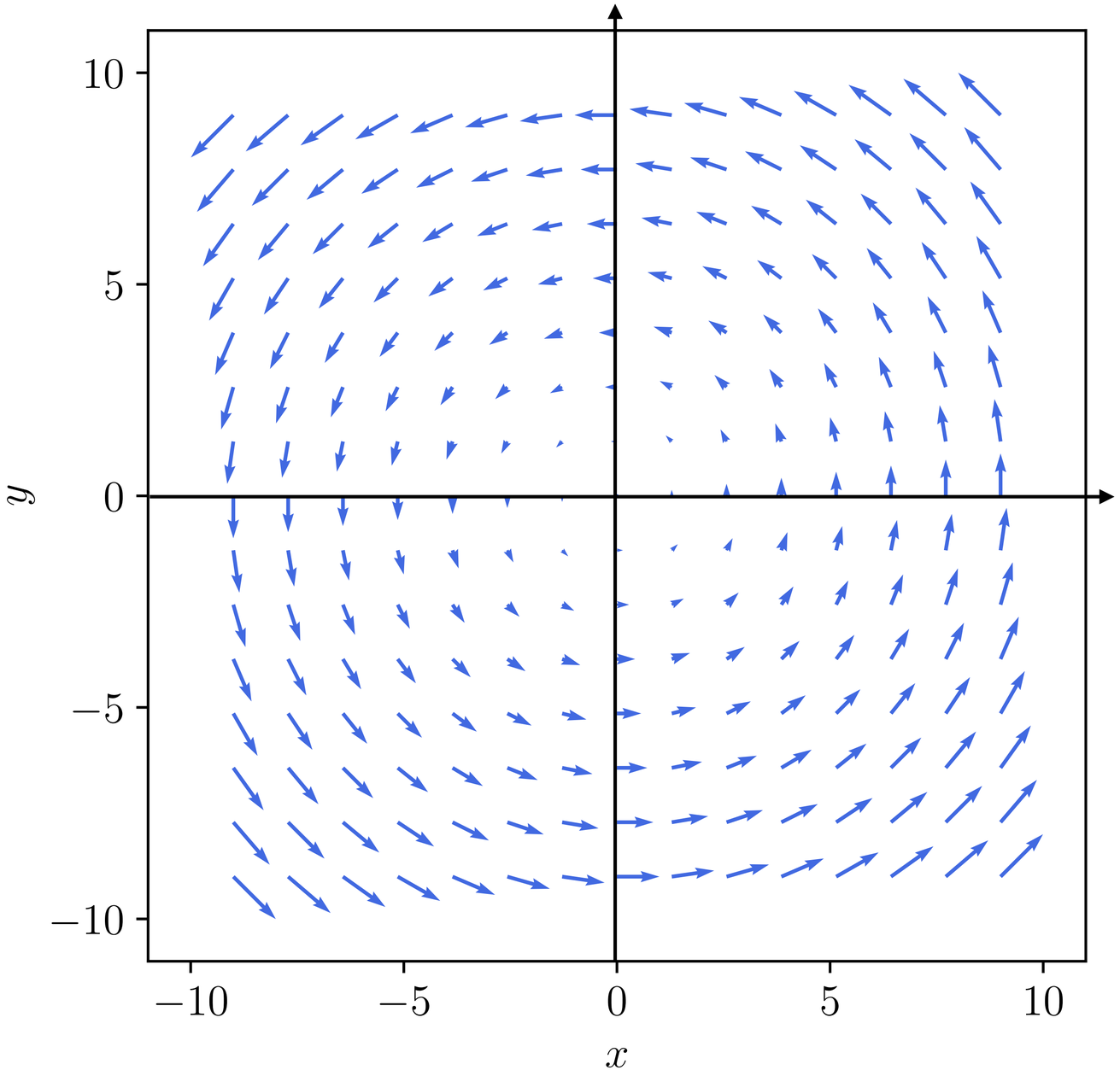
Stream line $\dfrac{dy}{dx} = \dfrac{u_y(x,y)}{u_x(x,y)}$

at specific time $t = t_n$

Path line

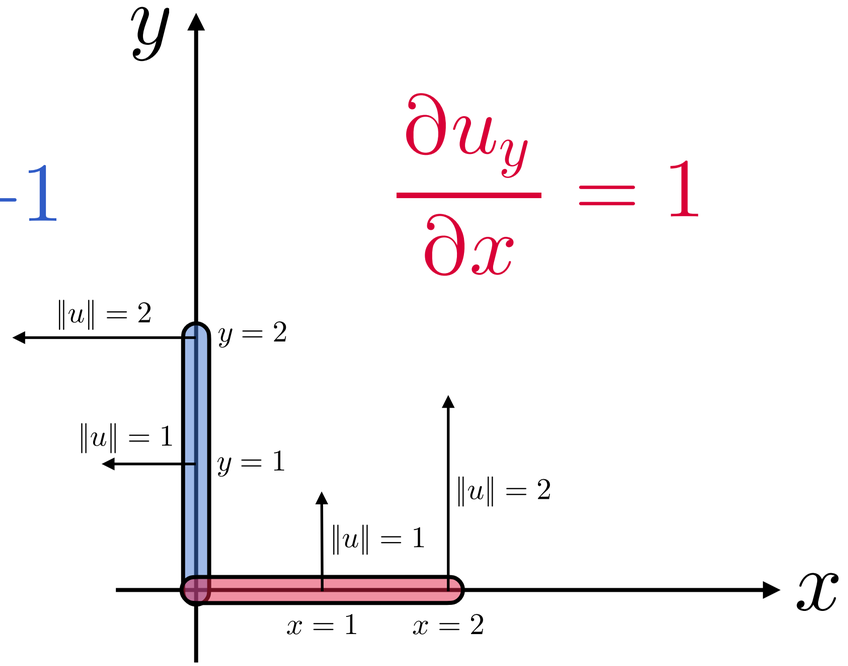
$$\frac{d\vec{r}(x(t), y(y))}{dt} = \vec{u}(x(t), y(t))$$


y
y
y
x
x
x
t
Irrotational
Rotational

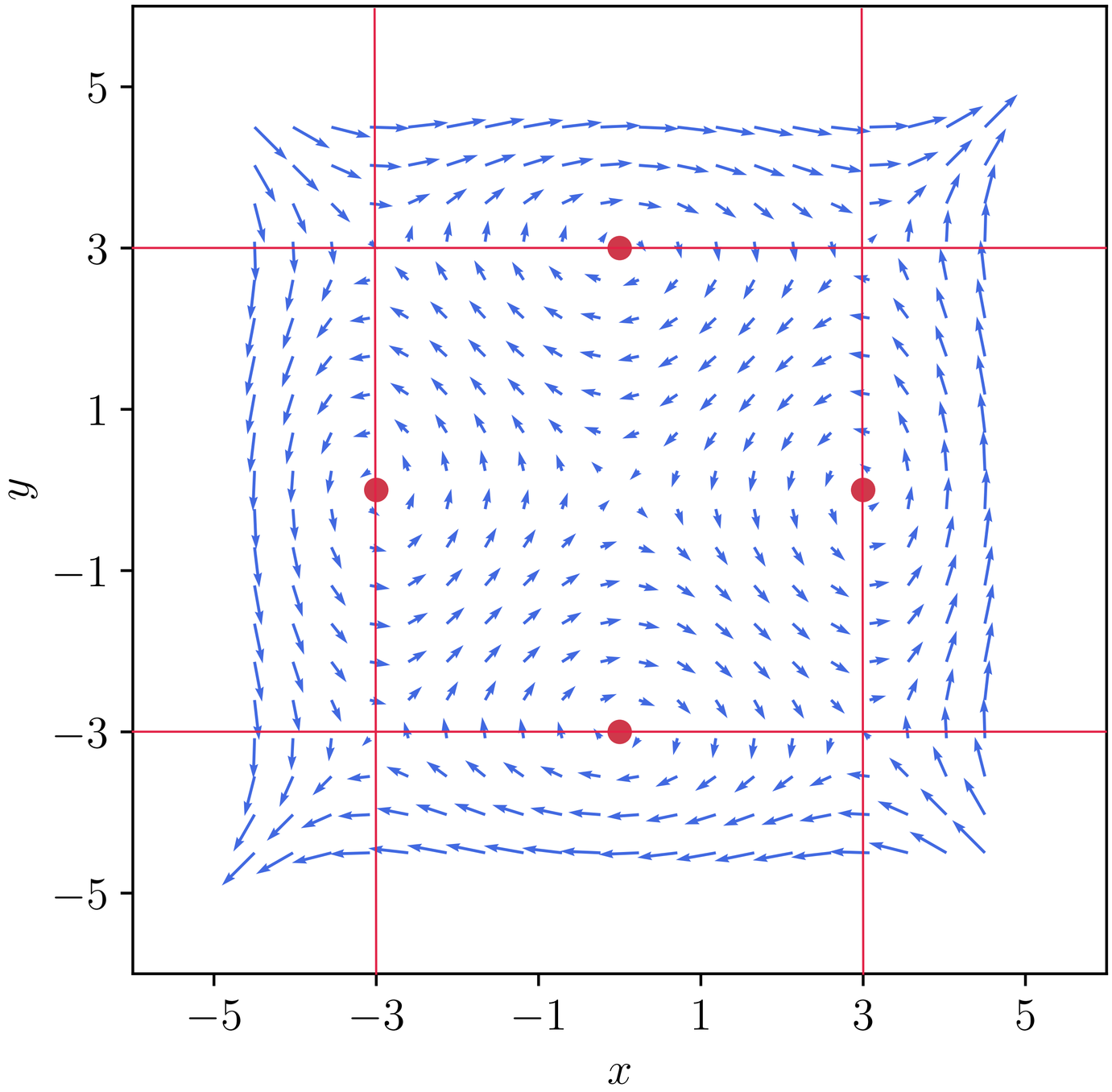


$$\frac{\partial u_x}{\partial y} = -1$$

$$\frac{\partial u_y}{\partial x} = 1$$



$$\operatorname{curl} \vec{u} = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 2$$



$$\vec{u}(x(t), y(y)) = \frac{d\vec{r}}{dt} \qquad \begin{cases} \dfrac{dx}{dt} = y^3 - 9y \\ \dfrac{dy}{dt} = x^3 - 9x \end{cases}$$

$\vec{r}_3 = x(t_3)\hat{e}_x + y(t_3)\hat{e}_y$

$\vec{u} = \vec{u}(\vec{r}_3)$

$t = t_3$

Path line

$\vec{r}_2 = x(t_2)\hat{e}_x + y(t_2)\hat{e}_y$

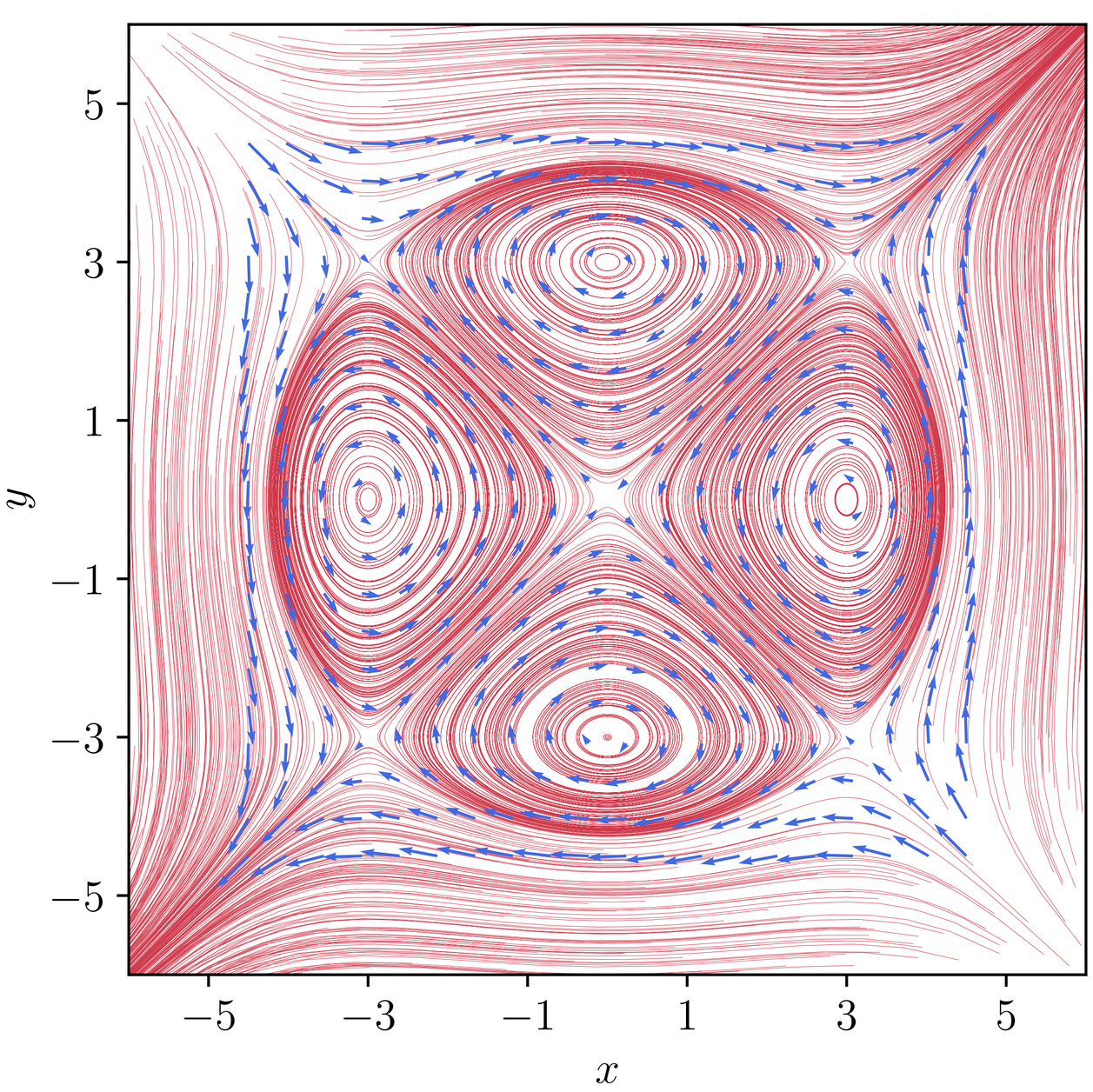
$\vec{u} = \vec{u}(\vec{r}_2)$

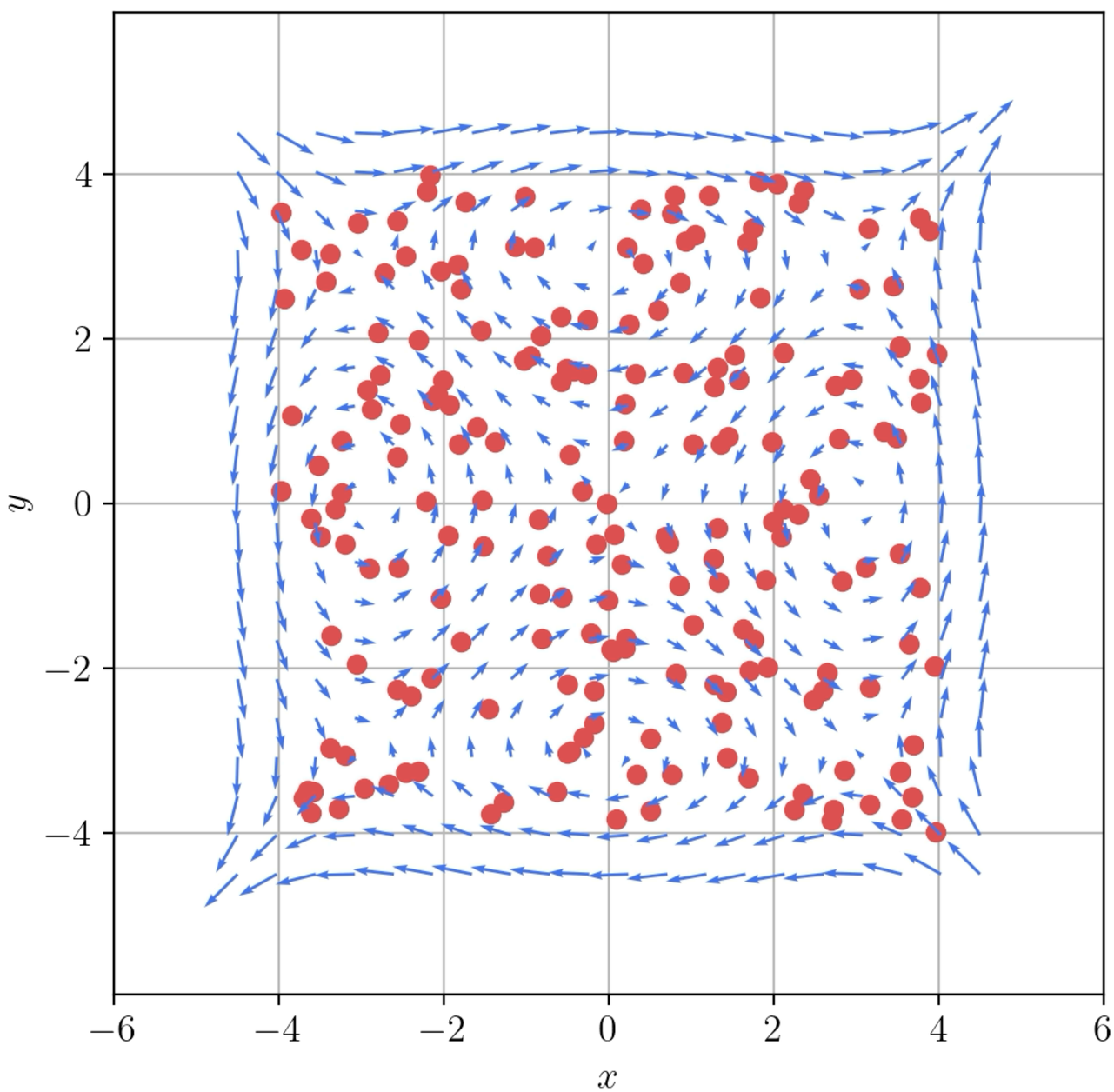
$t = t_2$

$\vec{u} = \vec{u}(\vec{r}_1)$

$t = t_1$

$\vec{r}_1 = x(t_1)\hat{e}_x + y(t_1)\hat{e}_y$

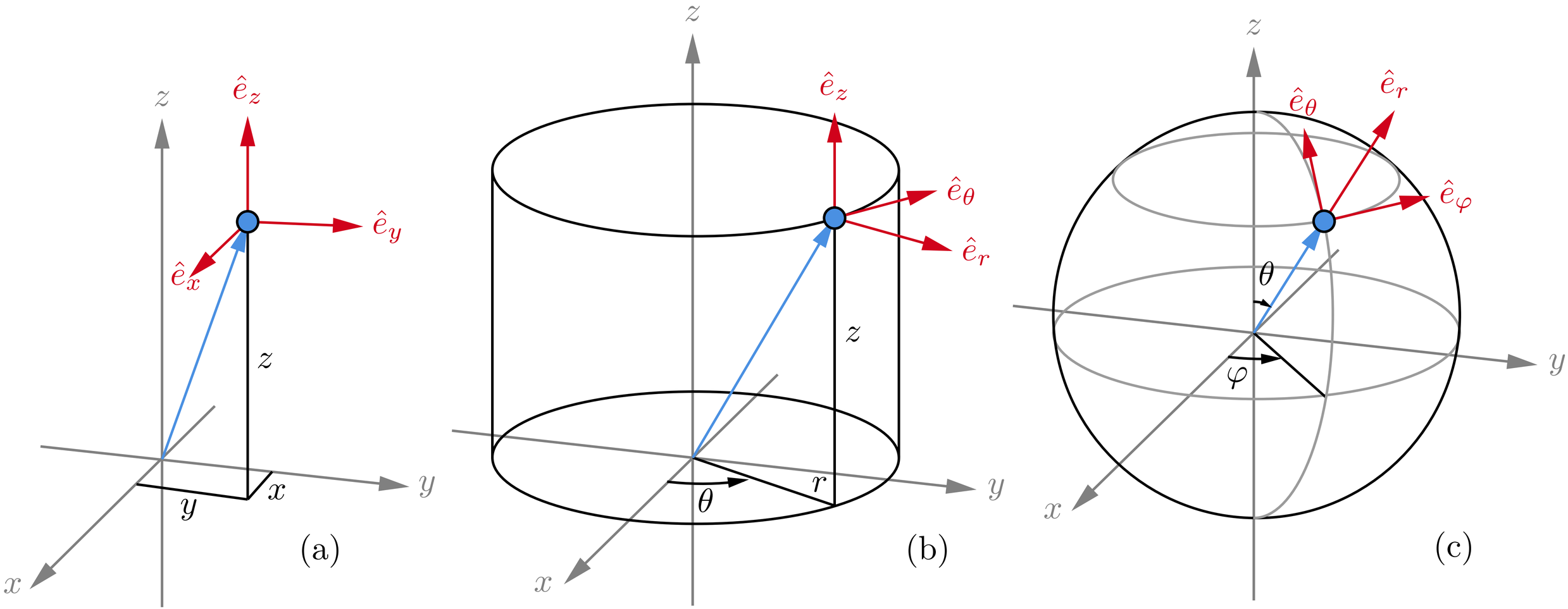

5
3
1
−1
−3
−5
y
−5
−3
−1
1
3
5
x

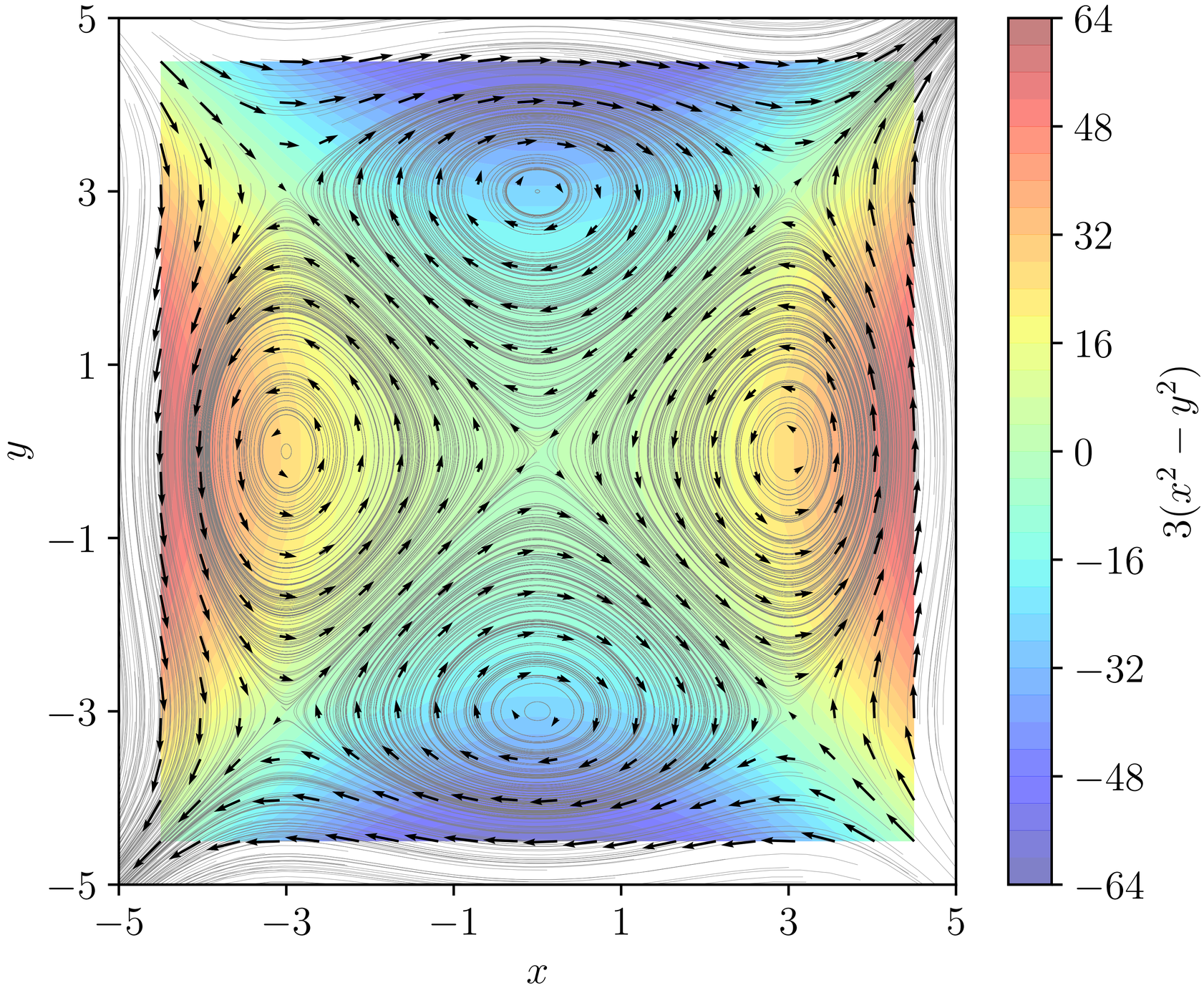

4
2
0
−2
−4
y
−6
−4
−2
0
2
4
6
x

Cartesian coordiante

Cylindrical coordiante

Spherical coordiante




5
3
1
−1
−3
−5
y
−5
−3
−1
1
3
5
x
64
48
32
16
0
−16
−32
−48
−64
$3(x^2 - y^2)$