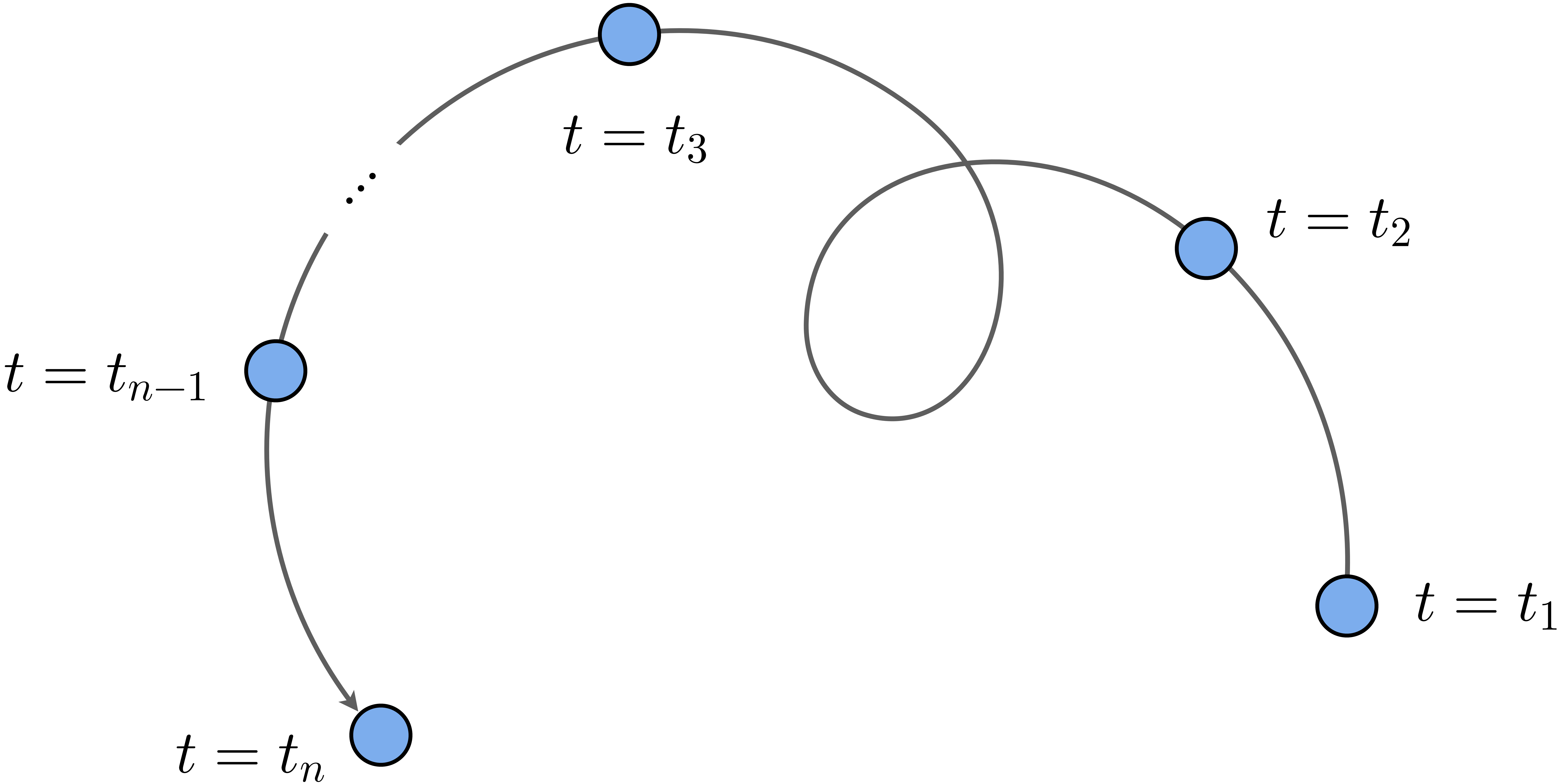
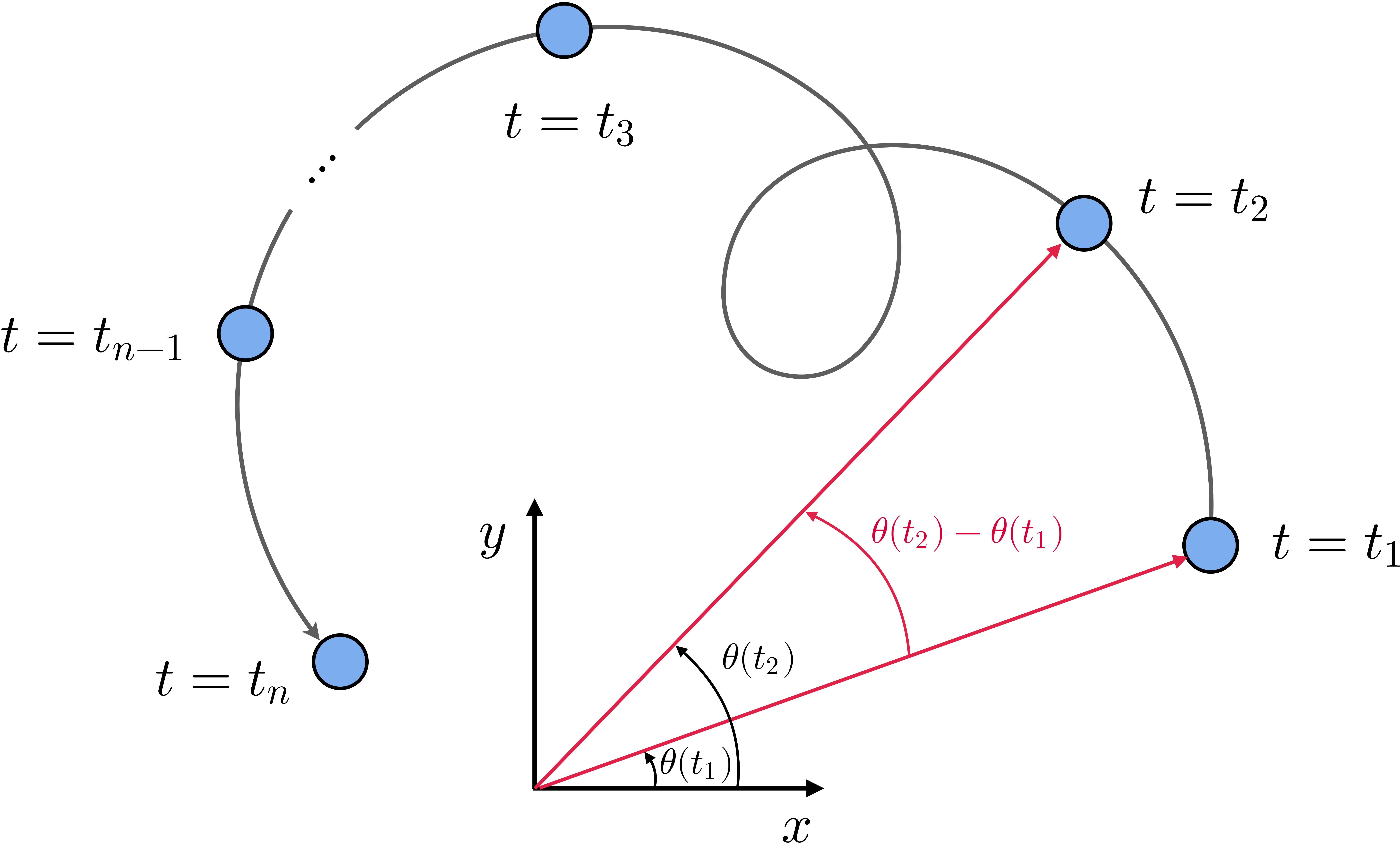
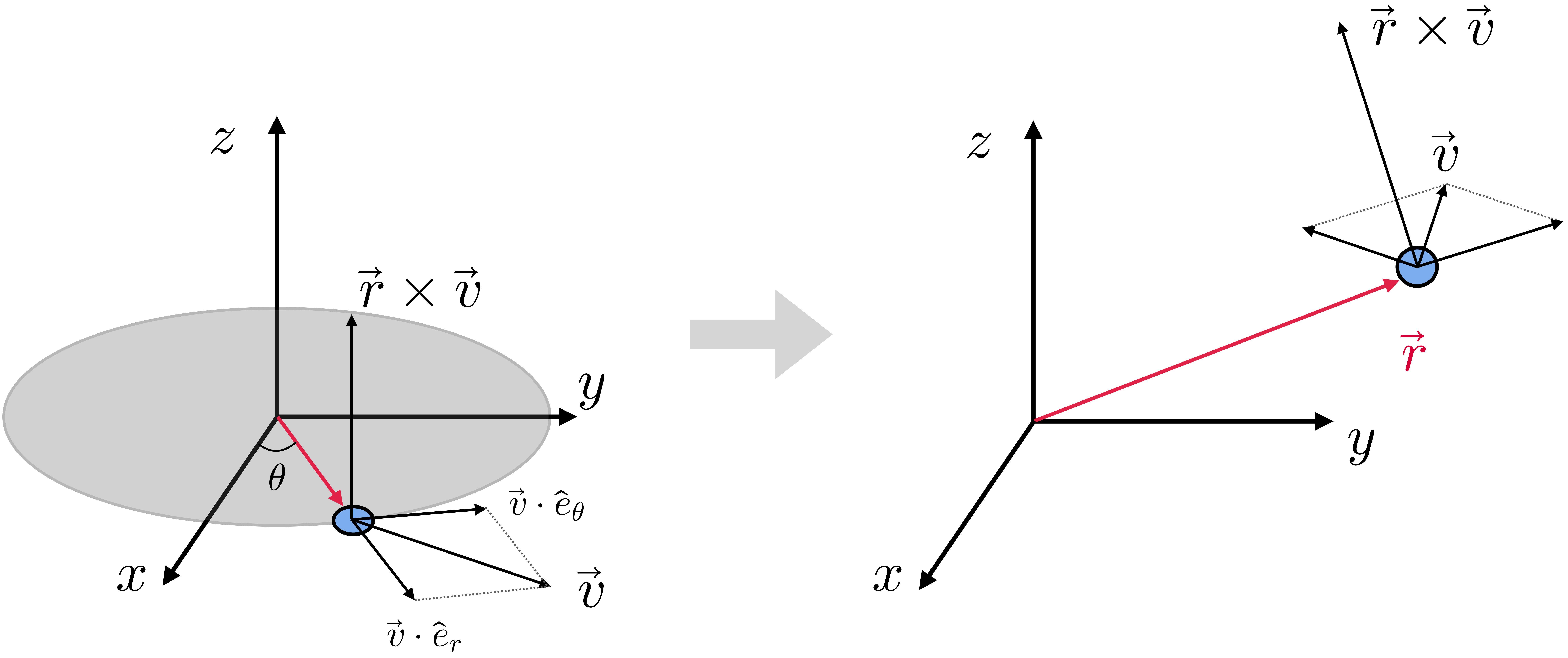
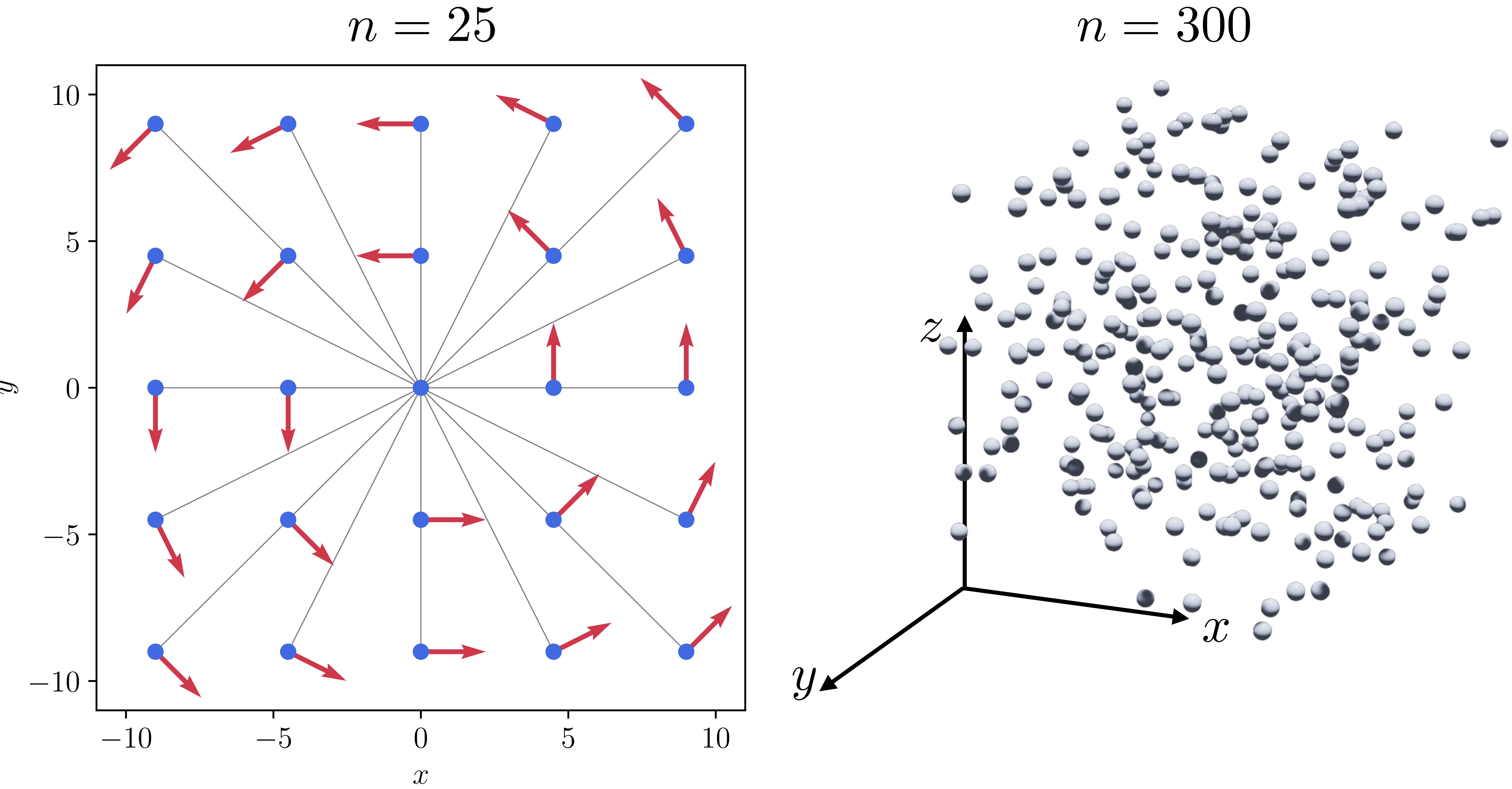

$t = t_3$
$t = t_2$
$t = t_{n-1}$
$t = t_1$
$t = t_n$

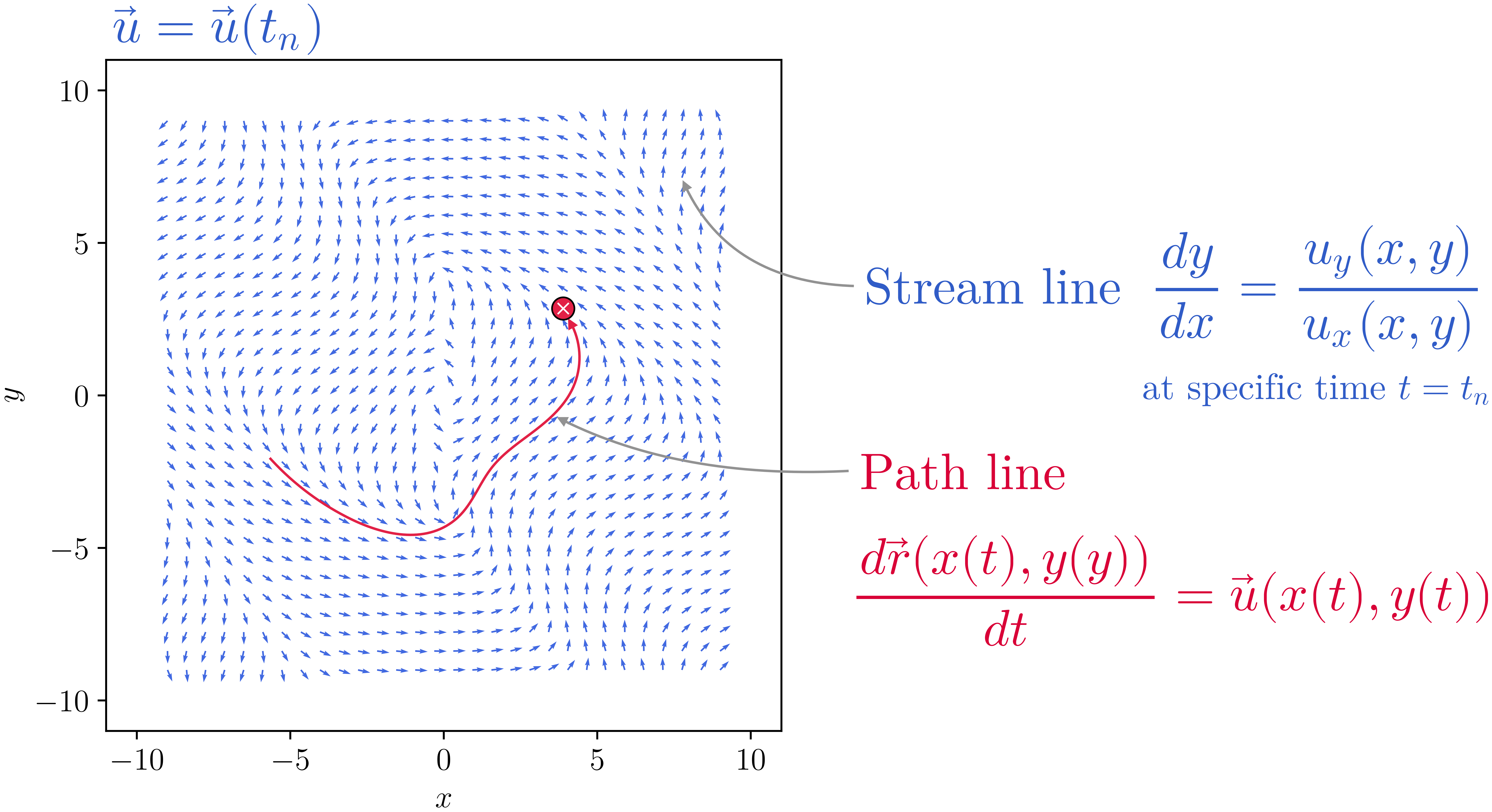


$t = t_3$
$t = t_2$
$t = t_{n-1}$
$t = t_1$
$\theta(t_2) - \theta(t_1)$
y
$\theta(t_2)$
$t = t_n$
$\theta(t_1)$
x



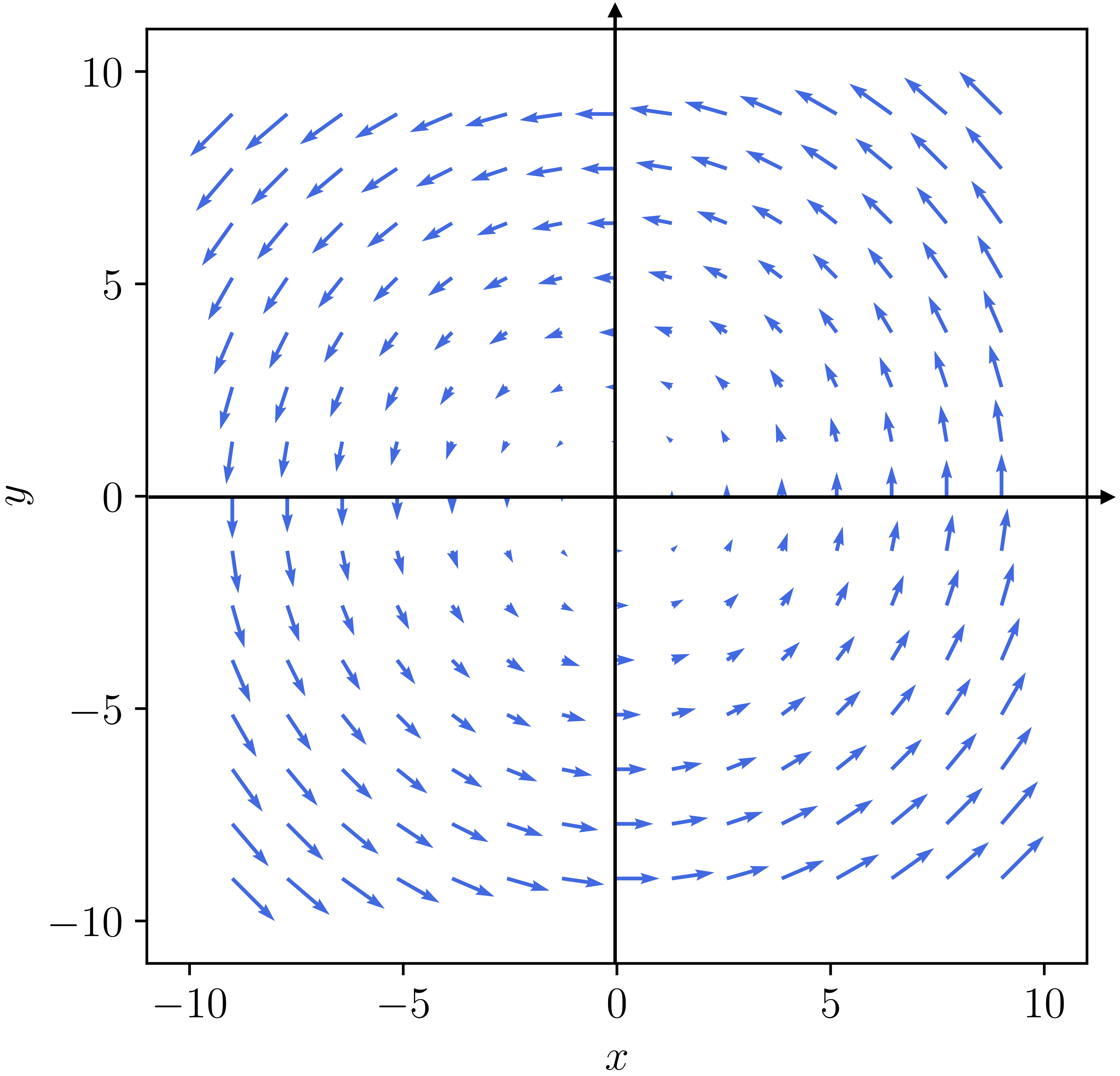
z
$\vec{r} \times \vec{v}$
y
θ
$\vec{v} \cdot \hat{e}_\theta$
x
$\vec{v}$
$\vec{v} \cdot \hat{e}_r$
$\vec{r} \times \vec{v}$
z
$\vec{v}$
$\vec{r}$
y
x


$n = 25$
$n = 300$
10
5
0
−5
−10
y
−10
−5
0
5
10
x
z
x
y

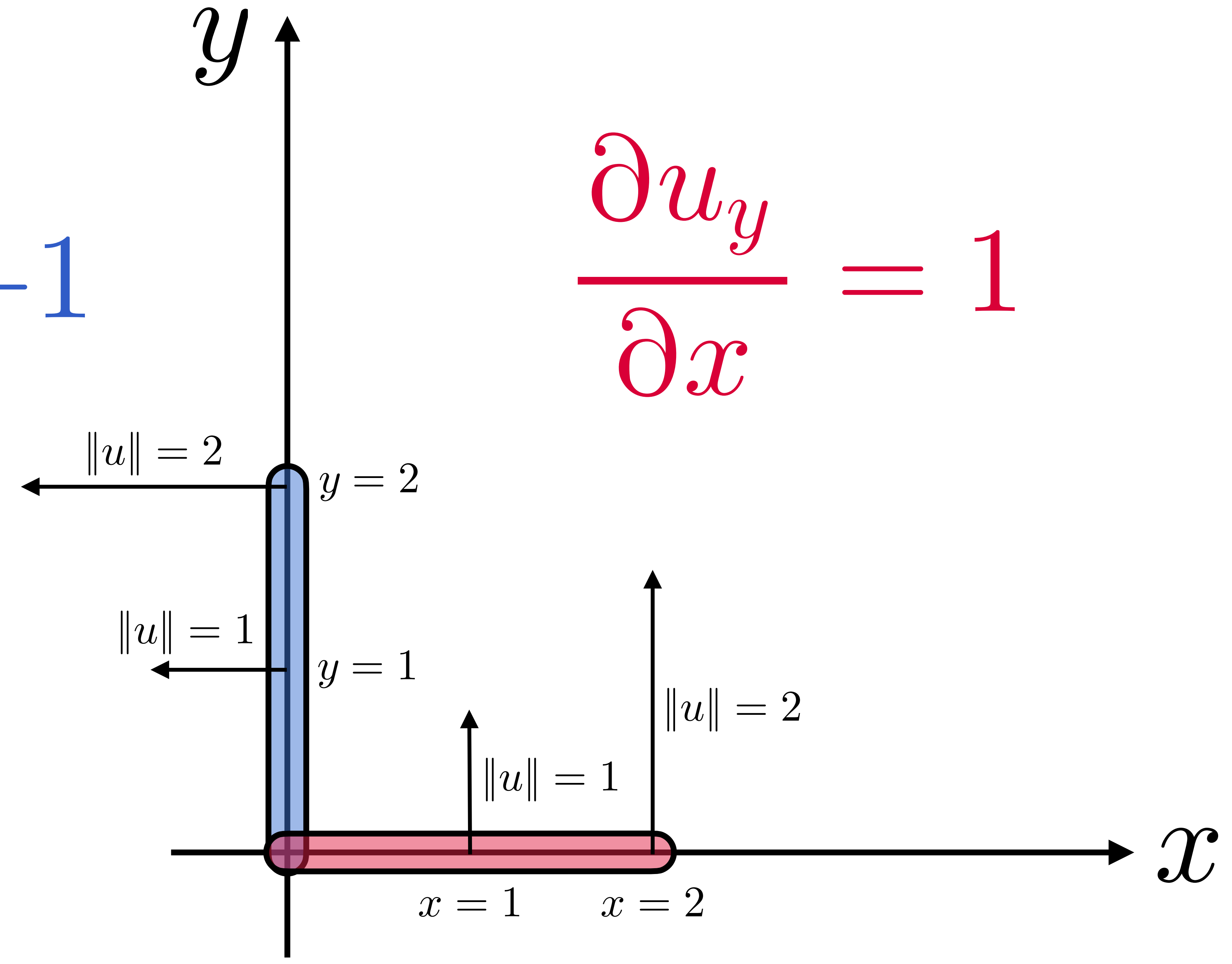


$\vec{u} = \vec{u}(t_n)$
10
5
0
−5
−10
y
−10
−5
0
5
10
x
Stream line $\frac{dy}{dx} = \frac{u_y(x,y)}{u_x(x,y)}$
at specific time $t = t_n$
Path line
$\frac{d\vec{r}(x(t),y(y))}{dt} = \vec{u}(x(t),y(t))$

y
y
y
Irrotational
Rotational
x
x
x
t

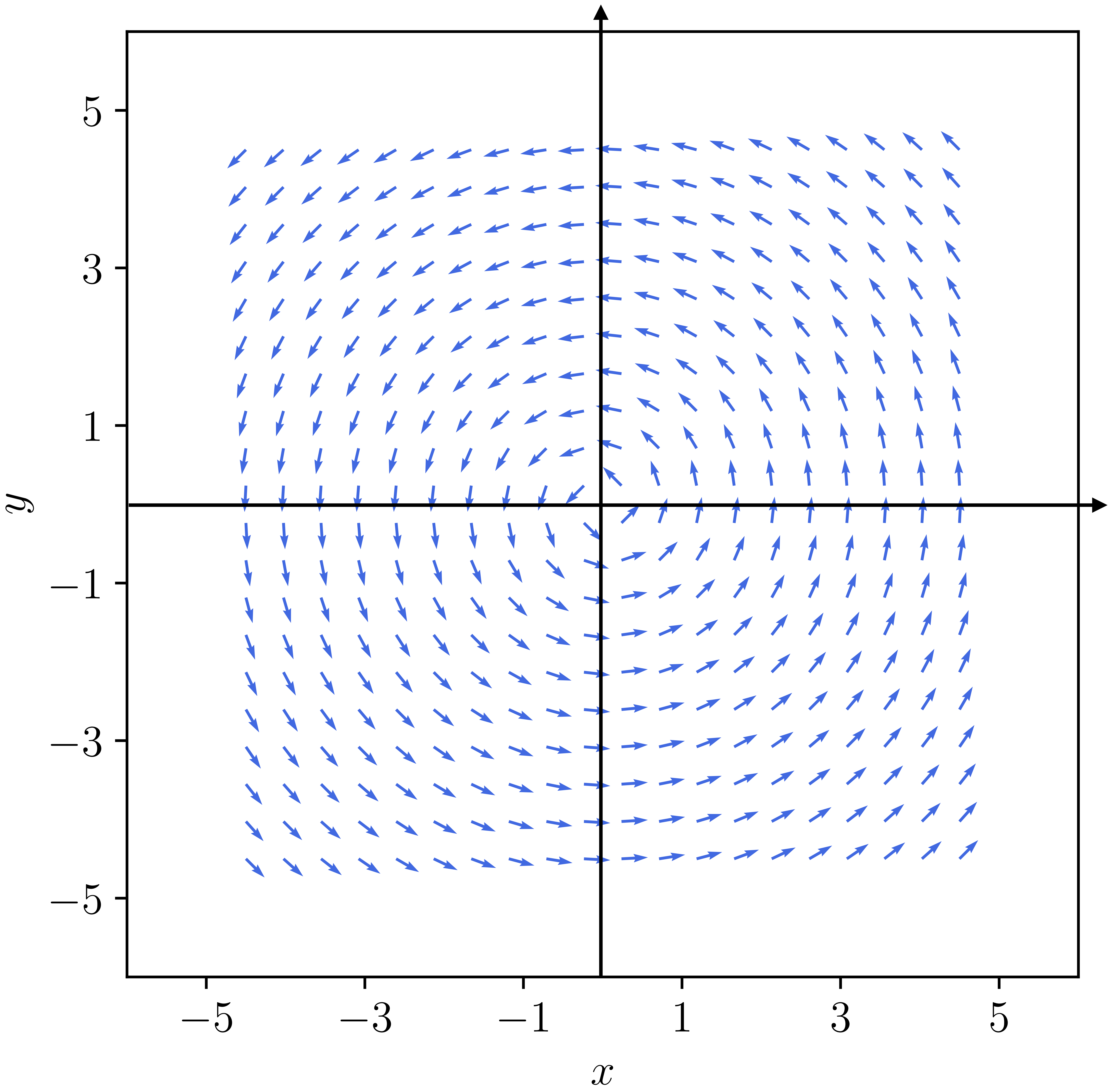


$$\frac{\partial u_x}{\partial y} = -1$$

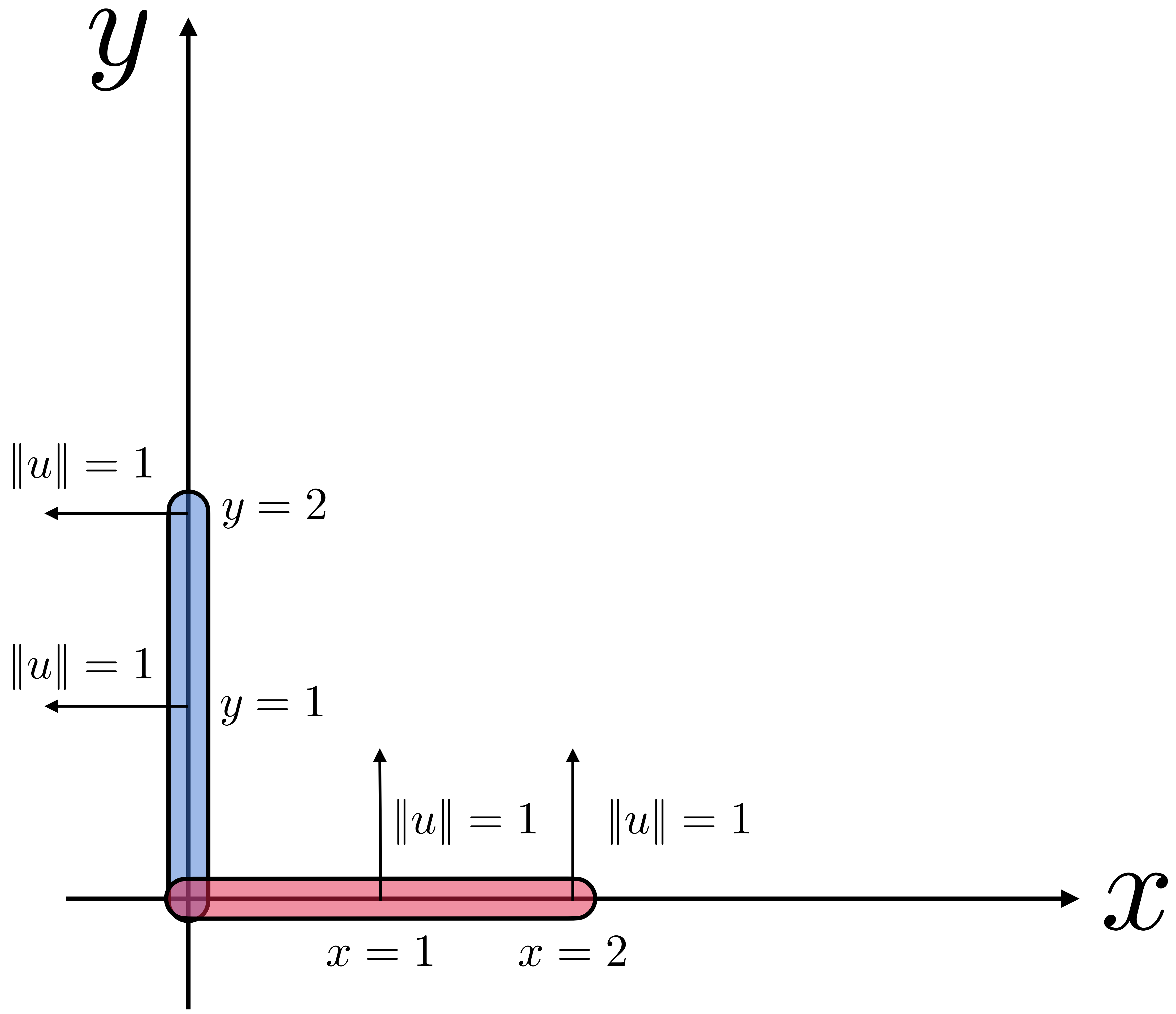


$$\frac{\partial u_y}{\partial x} = 1$$

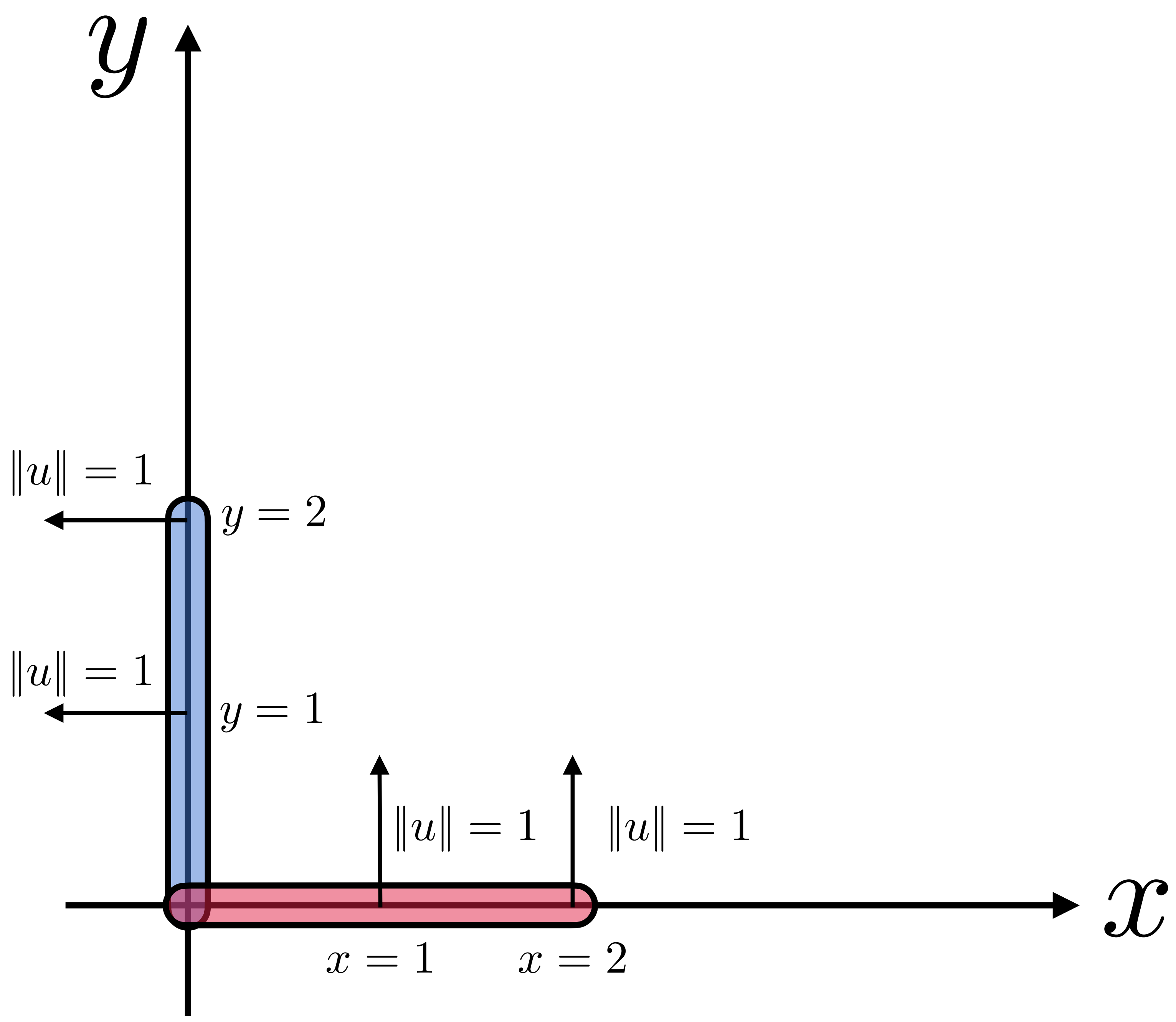
$$\operatorname{curl} \vec{u} = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 2$$



5
3
1
−1
−3
−5
−5
−3
−1
1
3
5
x
y



y
x
‖u‖ = 1
y = 2
‖u‖ = 1
y = 1
‖u‖ = 1
‖u‖ = 1
x = 1
x = 2

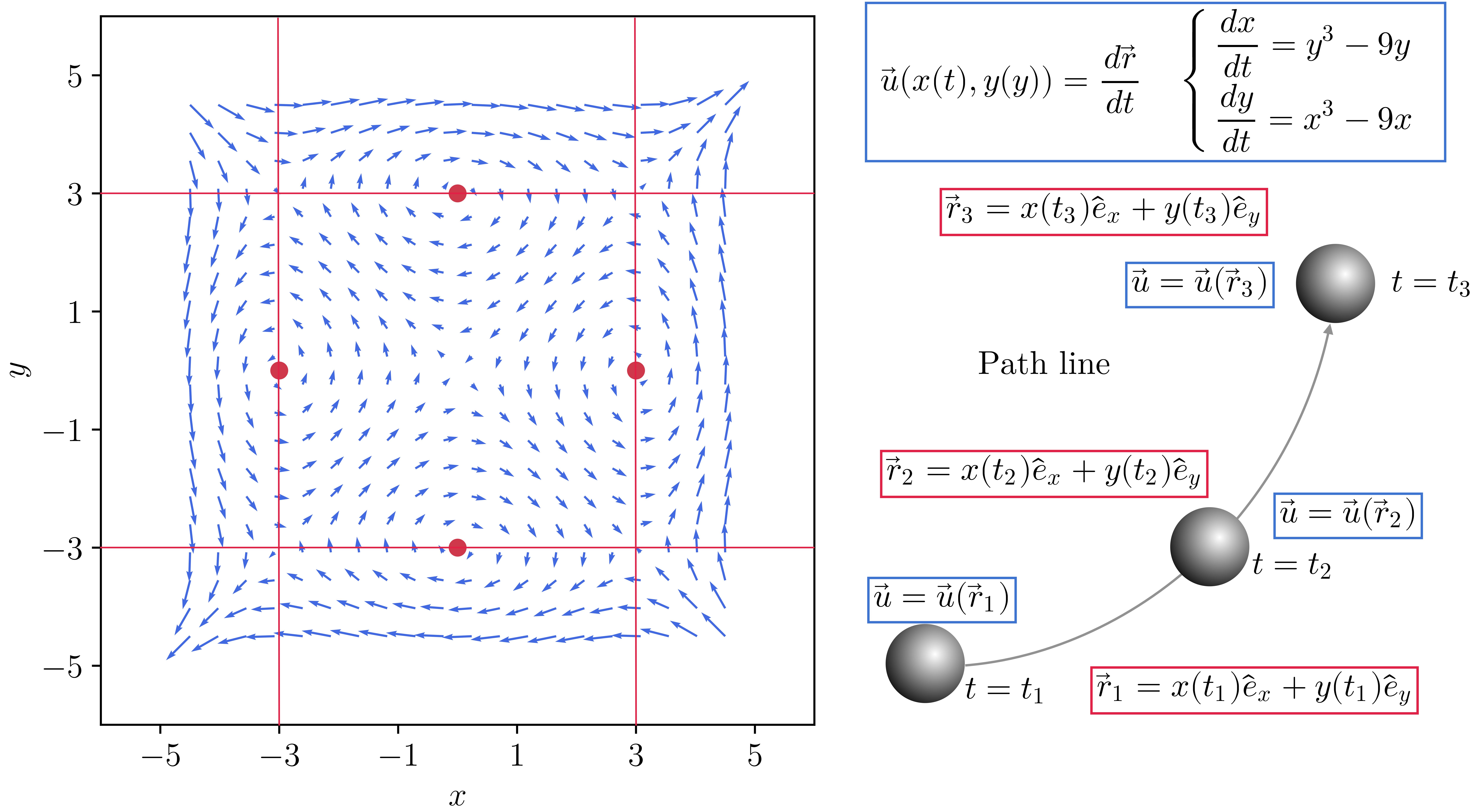


$$\frac{\partial u_y}{\partial x} = -\frac{y^2}{\left(x^2+y^2\right)^{3/2}} \xrightarrow{x=0} -\frac{1}{y}$$

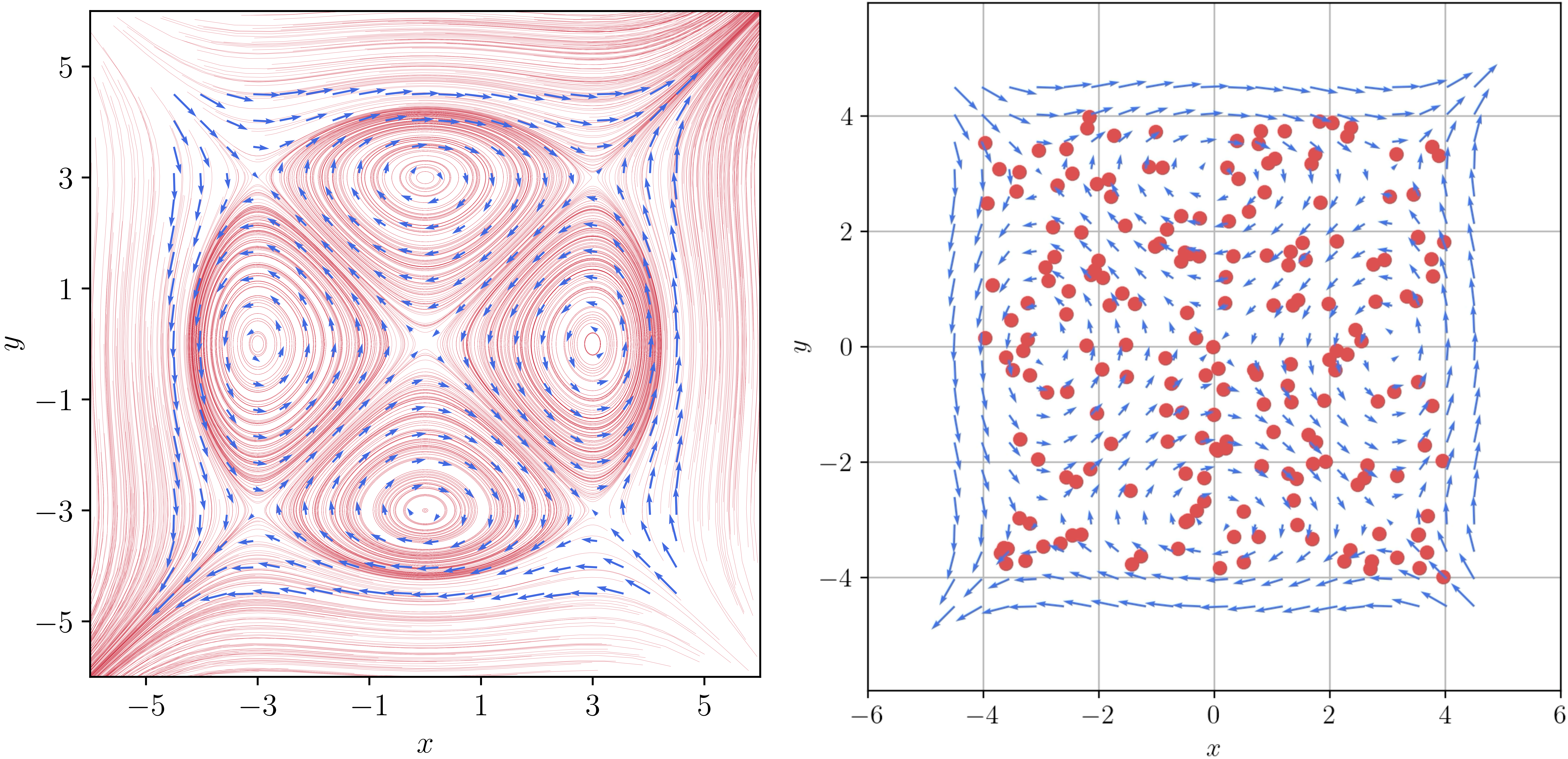
$$\frac{\partial u_x}{\partial y} = \frac{x^2}{\left(x^2+y^2\right)^{3/2}} \xrightarrow{y=0} \frac{1}{x}$$

$$\operatorname{curl}\vec{u} = \left(\frac{y^2}{\left(x^2+y^2\right)^{3/2}}\right) - \left(-\frac{x^2}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$= \frac{1}{\sqrt{x^2+y^2}}$$

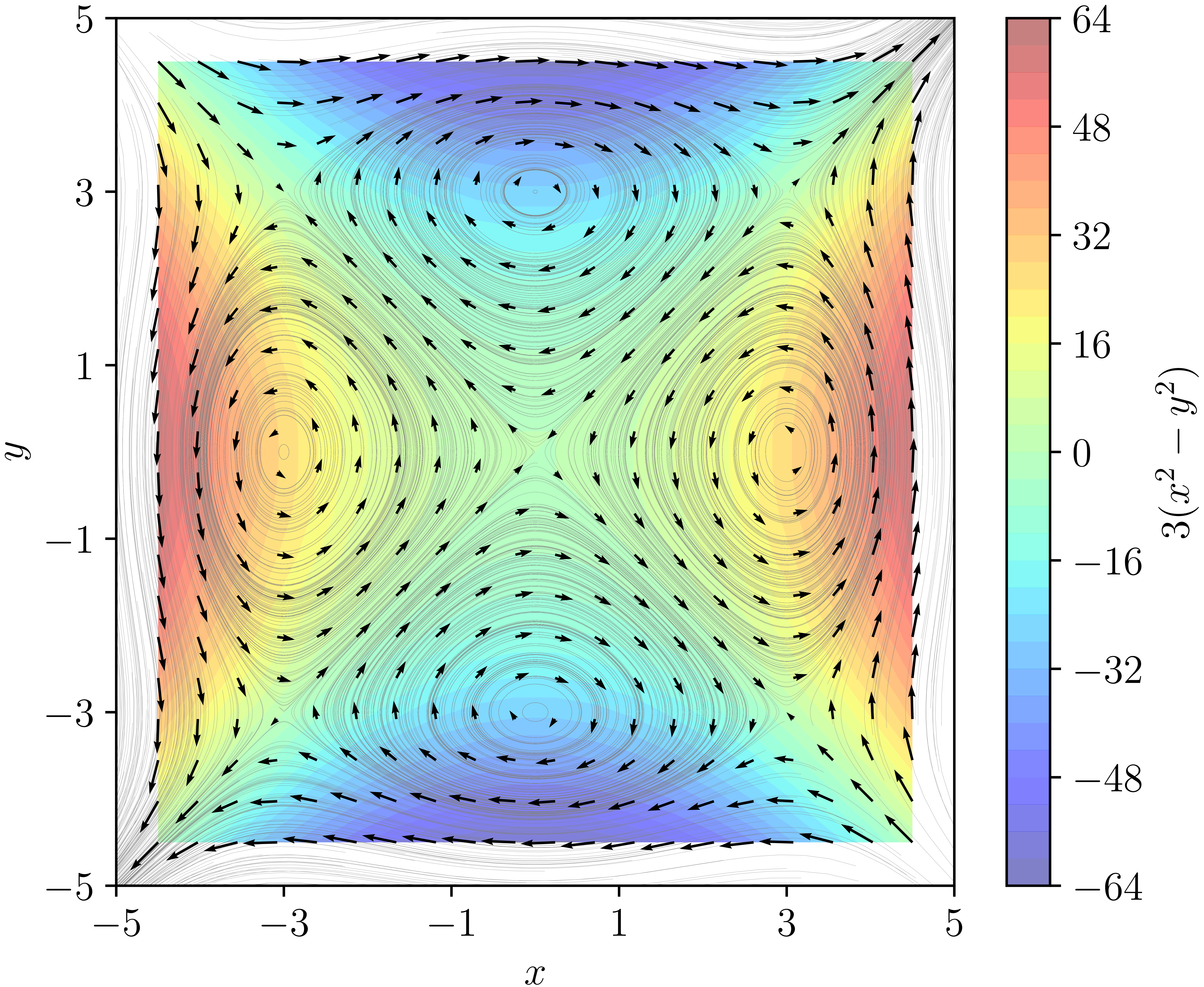


$\vec{u}(x(t), y(y)) = \frac{d\vec{r}}{dt} \quad \begin{cases} \frac{dx}{dt} = y^3 - 9y \\ \frac{dy}{dt} = x^3 - 9x \end{cases}$
$\vec{r}_3 = x(t_3)\hat{e}_x + y(t_3)\hat{e}_y$
$\vec{u} = \vec{u}(\vec{r}_3)$
$t = t_3$
Path line
$\vec{r}_2 = x(t_2)\hat{e}_x + y(t_2)\hat{e}_y$
$\vec{u} = \vec{u}(\vec{r}_2)$
$t = t_2$
$\vec{u} = \vec{u}(\vec{r}_1)$
$t = t_1$
$\vec{r}_1 = x(t_1)\hat{e}_x + y(t_1)\hat{e}_y$
y
5
3
1
-1
-3
-5
x



5
3
1
−1
−3
−5
y
−5
−3
−1
1
3
5
x
4
2
0
−2
−4
y
−6
−4
−2
0
2
4
6
x

Cartesian coordiante
Cylindrical coordiante
Spherical coordiante
z
$\hat{e}_z$
$\hat{e}_y$
$\hat{e}_x$
z
x
y
y
x
(a)
z
$\hat{e}_z$
$\hat{e}_\theta$
$\hat{e}_r$
z
r
θ
y
x
(b)
z
$\hat{e}_\theta$
$\hat{e}_r$
$\hat{e}_\varphi$
θ
φ
y
x
(c)



5
3
1
−1
−3
−5
y
−5
−3
−1
1
3
5
x
64
48
32
16
0
−16
−32
−48
−64
$3(x^2 - y^2)$