HW 5

1

P	Q	P ⇒ NOT Q	Q ⇒ NOT P
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

P	Q	P < = > NOT Q	((P AND NOT Q) OR (NOT P AND Q))
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

2

S	F	(S ⇒ F) ⇒ (NOT S ⇒ NOT F)
T	T	T
T	F	T
F	T	F
F	F	T

The sentence holds w1, w2, and w4. It is satisfiable but not valid since it doesn't satisfy w3.

S	F	H	(S ⇒ F) ⇒ ((S OR H) ⇒ F)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

The sentence holds w1, w2, w3, w4, w5, w6, and w8. It is satisfiable but not valid since it doesn't satisfy w7.

S	F	H	((S AND H) ⇒ F) < = > ((S ⇒ H) OR (H ⇒ F))
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

The sentence is satisfiable in all worlds and thus valid.

3

a.

Propositional symbols:

A: Mythical

B: Immortal

C: Mammal

D: Horned

E: Magical

$$1. A \implies B$$
$$2. \neg A \implies (\neg B \cup C)$$
$$3. (B \cup C) \implies D$$
$$4. D \implies E$$

b.

$$CNF = (\neg A \cup B) \cap (A \cup \neg B) \cap (A \cup C) \cap (\neg B \cup D) \cap (\neg C \cup D) \cap (\neg D \cap E)$$

4

The first NNF is

- decomposable, since variables in all AND gates don't overlap
- non-deterministic, since the root OR gate has more than 1 true input, e.g. A=T, B=F, C=T, D=F
- not smooth, since not all variables in OR gate are shared, e.g. 2 OR gates in 3rd level

The second NNF is

- decomposable, since variables in all AND gates don't overlap
- non-deterministic, since not all OR gates only receive 1 true input, e.g. first OR gate on 3rd level when A=F, B=T
- smooth, since all variables in OR gate are shared

5

a.

$$WMC = (.9)(.3) + (.7)(.1) = .34$$

b.

Since the formula for the root node is $(\neg A \cap B) \cup (\neg B \cap A)$, the count for the root node is equal to WMC for the formula.

c.

$$NNF = (((\neg A \cap B) \cup (\neg B \cap A)) \cap ((C \cap D) \cup (\neg C \cup \neg D))) \cup (((\neg A \cap \neg B) \cup (A \cap B)) \cap ((C \cap \neg D) \cup (\neg C \cap D)))$$
$$WMC = ((.9)(.3) + (.7)(.1))((.5)(.7) + (.5)(.3)) + ((.9)(.7) + (.1)(.3))((.5)(.3) + (.5)(.7)) = .5$$