STAT 230
PROBABILITY

PROFESSOR N. MOHAMMAD • WINTER 2014 • UNIVERSITY OF WATERLOO

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Future Modifications

1 Introduction

Note. Assignment are handed in on learn. Electronic copy only

1.1 Introduction

Definition 1.1. Probability is the study of randomnesses and uncertainty.

- Variability in population, processes or phenomena

Definition 1.2. Classic Interpretatation makes assumptions about the physical world to deduce probability.

$$\frac{\text{\# ways an event can occur}}{\text{Total \# of outcomes}}$$

Definition 1.3. Relative-frequency interpretation is when the probability of specific outcome is defined as the proportion of times it occurs over the long run. May only be used if the experiment may be repeated.

Definition 1.4. Frequency is just the amount of times an event occurs while **relative frequency** is a fraction of the amount of times an event occurs over the total amount of possible outcomes.

Definition 1.5. Personal-probability interpretation is the degree to which a given individual believes the event wil happen.

Quote. Coherent means that personal probability of one event does not contradict personal probability of another.

Example 1.1. If the probability of finding a parking space is 0.2, the probability of not finding one should be 0.8.

1.2 Probability Models

Definition 1.6. Experiment is any action, phenomenon, or process whose outcome is subject to uncertainty.

Definition 1.7. Trial is a single repetition of an experiment.

Definition 1.8. Sample space, denoted by S, is the set of possible distinct outcomes. In a single trial, only one outcome may occur. The sample space may be either **discrete** or **continuous**.

Definition 1.9. An **event** is any subset of outcomes contained in the sample space S.

- **Simple** - one outcome
- **Compound** - more than one outcome

Probability notation

$P(\text{event})$ is used to denote the probability of an event occurring. The **compliment** is the probability of an event not happening and is denoted as $P(A^c)$ or $P(\overline{A})$

Definition 1.10. The odds in **favour** of an event is the odds of an event occurring compared to its compliment.

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1 - P(A)}$$

The odds against is the reciprocal of odds in favour.

Theorem 1.1.

$$\sum_{i=0}^{k} P(A_i) = 1$$

The set $P(A_i), i = 1, 2, \ldots$ is the probability distribution on S.

Theorem 1.2. If A and B are two events wirh $A \subseteq B$, then $P(A) \leq P(B)$

Definition 1.11. Two events are **mutually exclusive** or **disjoint** if they cannot happen simultaneously.

$$A \cap B = \emptyset$$

1.3 Counting Techniques

Counting Principle • Permutations • Combinations

Definition 1.12. Addition rule: When there are m ways to perform A, and n ways to perform B, there are $m + n$ ways to perform A **OR** B.

Definition 1.13. Product rule: Where there are p ways to perform A, and q ways to perform B, there are $m \times n$ ways to perform A **AND** B.

Definition 1.14. Uniform Probability Model:

$$P(A) = \frac{\text{Outcomes in } A}{\text{Outcomes in } S}$$

Definition 1.15. Permutation is an arrangement of elements in an ordered list.

Note. The amount of ways to arrange n items (all of them have to be used) is $n!$

$$P_{k,n} = \frac{n!}{(n-k)!}$$

Definition 1.16. Any unordered sequence of k objects taken from a set of n distinct objects is called a **combination**.

$$C_{k,n} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Example 1.2. Binomial Theorem:

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$$