Question 1

a) Consider the system shown in Figure 1, which consists of two sinusoidal waves of unit amplitude and frequencies 100 Hz and 200 Hz. The spectrum of the input signal $x(t)$ is $X(f)$.

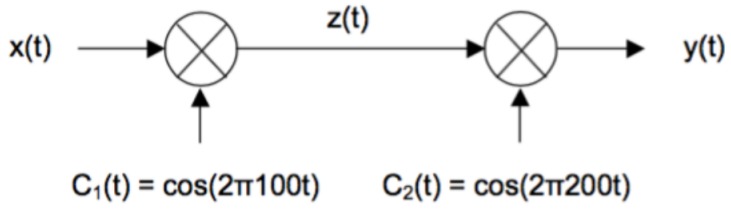


Figure 1: A system consisting of two sinusoidal waves

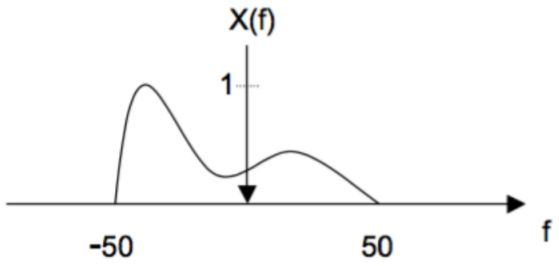


Figure 2: Fourier transform of the input signal $x(t)$

[16 marks]

i) Find the expression for the Fourier transform of the intermediate signal z(t).
ii) Plot the Fourier transform of the intermediate signal z(t).
iii) Find the expression for the Fourier transform of the output signal y(t).
iv) Plot the Fourier transform of the output signal y(t).

b) For the following signal in frequency domain

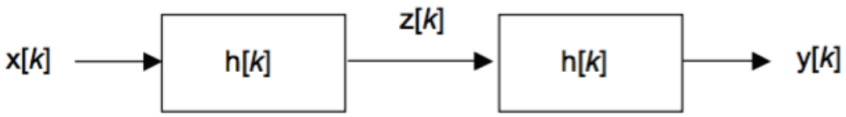
$$G(f) = 4\text{rect}[(f - 6000)/8000]\text{rect}[(f - 8000)/8000]$$

[9 marks]

i) Plot G(f).
ii) Find the expression for the inverse Fourier transform of the signal G(f).

Question 2

a) This question is about convolution. Use the **Table Method** to solve this question. Given the following interconnection of systems



where

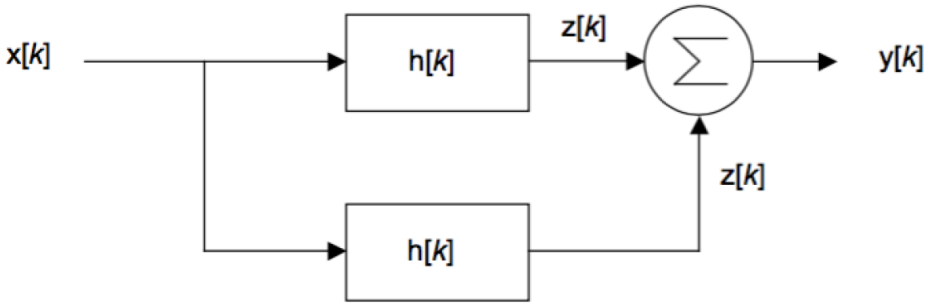
$$x[k] = \begin{cases} 2, 0 \le k \le 2 \\ 0, \text{elsewhere} \end{cases}$$

$$h[k] = \begin{cases} 1, 0 \le k \le 2 \\ 0, \text{elsewhere} \end{cases}$$

[16 marks]

i) Determine the intermediate signal z[k].
ii) Plot the intermediate signal z[k].
iii) Determine the output signal y[k].
iv) Plot the output signal y[k].

b) Determine and plot the output response $y[k]$ for the following system, with input signal $x[k]$ and impulse responses $h[k]$ given in part a) of Question 2:



[9 marks]

Question 3

a) Consider two discrete-time LTI systems with impulse responses

$$h_1[n] = (n+4)u[n+3]u[-n-1]$$
$$h_2[n] = (-n+4)u[-n+4]u[n]$$

[9 marks]

Draw the impulse responses h1[n] and h2[n].

b) Consider now a new discrete-time LTI system as the parallel connection of $h_1[n]$ and $h_2[n]$.

[8 marks]

i) Obtain the impulse response h[n] of this new system and draw it.

ii) Obtain the frequency response H(Ω) of the LTI system defined by the impulse response h[n].

c) Suppose that the input of the LTI system defined by $h[n]$ is

$$x[n] = 2\delta[n+8] - 2\delta[n+4] + 2\delta[n-4] - 2\delta[n-8]$$

[8 marks]

i) Draw the input signal x[n].

ii) Obtain the modulus of the Fourier transform of x[n].

i) Obtain the Fourier transform Y (Ω) of the output of the LTI system defined by h[n] when the input is x[n].

Question 4

a) This question concerns Sampling Theory.

[11 marks]

i) Explain the Sampling Theorem for band-limited signals.

ii) Determine the Nyquist rate in Hz of $x_1(t) = 17 + 4\cos(2\pi t + \frac{7\pi}{8}) + 8\cos(\pi t + \frac{5\pi}{8}) + 2\cos(6\pi t + \frac{\pi}{8})$.

iii) Determine the Nyquist rate in Hz of $x_2(t) = \frac{\sin(2\pi t)}{\pi t} * \frac{\sin(4\pi t)}{\pi t}$.

b) This question concerns AD/DA conversion.

[14 marks]

i) Describe the main functional blocks in an AD converter and express the binary rate as a function of the parameters of such blocks.

ii) Assume that the signal $x(t) = \left(\frac{\sin(2\pi t)}{\pi t}\right)^2$ is sampled at the Nyquist rate, resulting in signal $x_p(t)$. Obtain the Fourier transform $X_p(\omega)$ of $x_p(t)$. **(5 marks)**

iii) Describe an interpolator that recovers $x(t)$ from $x_p(t)$.

iv) Assume you sample the signal $z(t) = x(2t)$ at the same rate as $x(t)$ and interpolate the sampled its sampled version $z_p(t)$ using the interpolator that you designed for $x_p(t)$. Would you recover $z(t)$? Why? **(2 marks)**