EBU4375: SIGNALS AND SYSTEMS

TOPIC 3-5: DISCRETE TIME SIGNALS IN THE FREQUENCY DOMAIN

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AGENDA

1. Frequency or Time?
2. Manipulating LTI systems – examples.
3. What's next?

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1: TIME OR FREQUENCY?

- The **frequency-domain** characterization of an LTI system in terms of its frequency response represents an **alternative** to the **time-domain** characterization through convolution.
- In **analysing LTI systems**, it is often particularly **convenient** to utilise the **frequency domain** because convolution operations in the time domain becomes algebraic operation in the frequency domain.
- Moreover, concepts such as **frequency-selective filtering** are readily and simply visualized in the frequency domain.
- However, in **system design**, there are typically **both time-domain and frequency-domain** considerations.

1: LTI SYSTEMS - PROPERTIES

- **Only** LTI systems can characterised **completely** by the impulse response:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n]$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

$$Y(\Omega) = X(\Omega) \times H(\Omega)$$

$$Y(\omega) = X(\omega) \times H(\omega)$$

- **Commutative** property:

$$x[n] * h[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k],$$

$$X(\Omega) \times H(\Omega) = H(\Omega) \times X(\Omega)$$

$$x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau.$$

$$X(\omega) \times H(\omega) = H(\omega) \times X(\omega)$$

1: LTI SYSTEMS - PROPERTIES

- **Distributive** property:

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n],$$
$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t).$$

$$X(\Omega) \times \big(H_1(\Omega) + H_2(\Omega)\big) = \big(X(\Omega) \times H_1(\Omega)\big) + (X(\Omega) \times H_2(\Omega))$$
$$X(\omega) \times \big(H_1(\omega) + H_2(\omega)\big) = \big(X(\omega) \times H_1(\omega)\big) + (X(\omega) \times H_2(\omega))$$

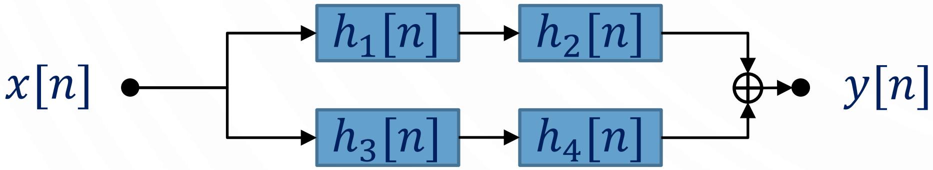
- **Associative** property:

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n],$$
$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t).$$

$$X(\Omega) \times (H_1(\Omega) \times H_2(\Omega)) = (X(\Omega) \times H_1(\Omega)) \times H_2(\Omega)$$
$$X(\omega) \times (H_1(\omega) \times H_2(\omega)) = (X(\omega) \times H_1(\omega)) \times H_2(\omega)$$

1: WHAT DO YOU THINK?

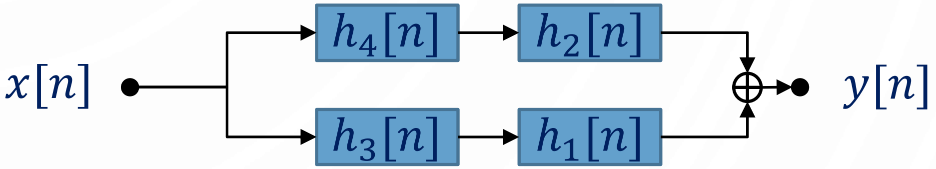
- Given the following LTI system:



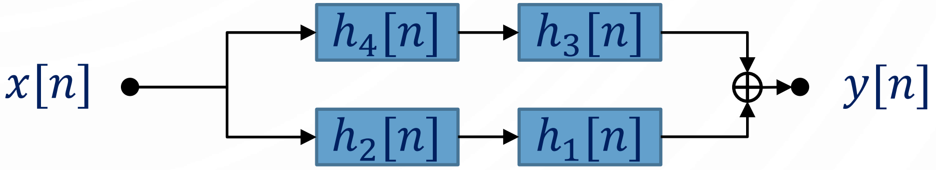
Let's go to Mentimeter!!!

- Which of these LTI systems are equivalent:

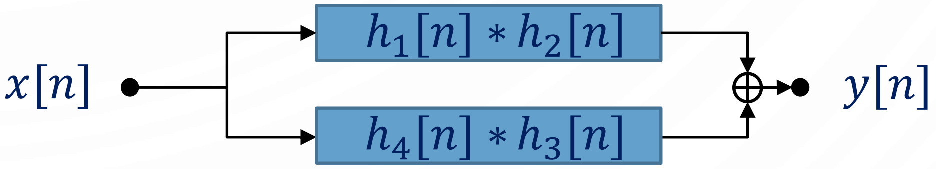
(A)



(B)



(C)



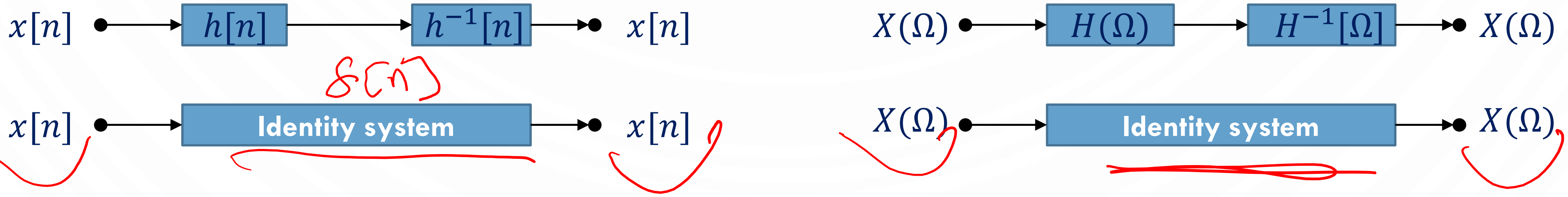
(D)

1: INVERTIBILITY OF LTI SYSTEMS

Consider a continuous-time LTI system with impulse response h(t). This system is invertible only if an inverse system exists that, when connected in series with the original system, produces an output equal to the input to the first system.

- if an LTI system is invertible, then it has an LTI inverse.

$h(n) * h^{-1}(n) = \delta(n)$ $\qquad$ $H(\Omega) \times H^{-1}(\Omega) = 1$



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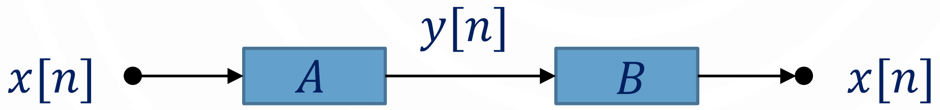
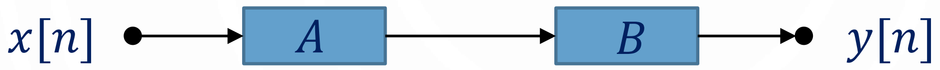
EXAMPLE 1



- A is an LTI system
- B is the inverse of system A
- Find the output of system B for input:
  - $y_1[n] + y_2[n]$
  - $y[n - n_0]$

EXAMPLE 2

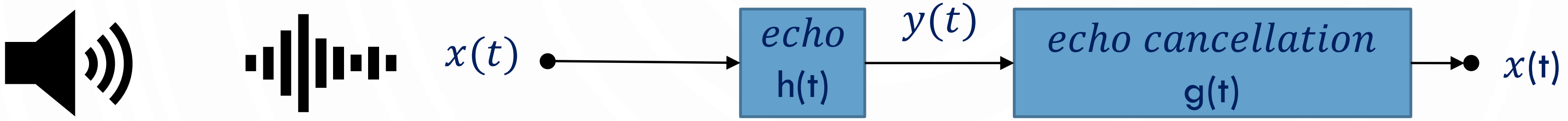


- A is an LTI system such as $h[n] = \left(\frac{1}{2}\right)^n u[n]$
- B is linear but time variant such that for an input w[n], the output is: $y[n] = n \times w[n]$

a) Show that the commutativity property does not hold

b) Replace system B with $y[n] = w[n] + 2$ and repeat a)

EXAMPLE 3



- The effect of echo can be modelled using the LTI system with impulse response:

$$h(t) = \sum_{k=0}^{\infty} h_k \delta(t - kT)$$

- Assume that $g(t) = \sum_{k=0}^{\infty} g_k \delta(t - kT)$

a) Determine the algebraic equations that the successive g_k must satisfy, and solve these equations for g_0, g_1, and g_2 in terms of h_k .

b) Suppose that $h_0 = 1$, $h_1 = 1/2$, and $h_i = 0$ for all $i \geq 2$. What is $g(t)$ in this case?

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DT & CT ARE TWO DISTINCT WORLDS OR …?

- We have mastered CT signals and systems, time domain and frequency domain, filters, etc…
- We have mastered DT signals and systems, time domain and frequency domain, filters, etc…
- Do CT and DT ever meet?
- Is it possible to convert from CT to DT?
- Is it possible to convert back from DT to CT?
- Why would we want to do that?