Tutorial

1)

i) $g(t) = e^{-t} \sin(2\pi f_0 t)\, u(t)$

$= e^{-t} u(t) \sin(2\pi f_0 t)$

We know that

$$e^{-t} u(t) \rightleftharpoons \frac{1}{1+j2\pi f}$$

We also know that

$$\sin(2\pi f_0 t) = \frac{1}{2j}\left[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right]$$

Therefore

$$g(t) = \frac{e^{-t}u(t)}{2j}\left[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right]$$

frequency shift property

$\therefore$

$$G(f) = \frac{1}{2j}\left[\frac{1}{1+j2\pi(f-f_0)} - \frac{1}{1+j2\pi(f+f_0)}\right]$$

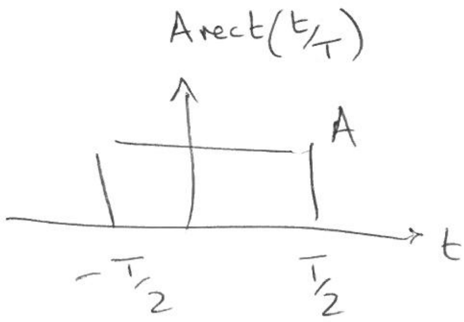
1) ii) $$g(t) = 8\,\mathrm{rect}(t/4)\cos(2\pi 10^6 t)$$

$$= 8\,\mathrm{rect}\left(\frac{t}{4}\right)\left[\frac{e^{j2\pi 10^6 t} + e^{-j2\pi 10^6 t}}{2}\right]$$

$$= 4\,\mathrm{rect}\left(\frac{t}{4}\right)\left[e^{j2\pi 10^6 t} + e^{-j2\pi 10^6 t}\right]$$

frequency shift property

We know that



$\rightleftharpoons$ AT sinc (fT)

Therefore

$$G(f) = 16\,\mathrm{sinc}((f-10^6)4) + 16\,\mathrm{sinc}((f+10^6)4)$$

2) Find the inverse Fourier transform of

$$G(f) = 12 \operatorname{sinc}(4f)\sin(4\pi f)$$

We can rewrite $G(f)$ as

$$G(f) = \frac{6}{j}\operatorname{sinc}(4f)\left[e^{j2\pi(2)f} - e^{-j2\pi(2)f}\right]$$

time shift property

$$= \frac{6}{j}\left[\operatorname{sinc}(4f)\,e^{j2\pi(2)f} - \operatorname{sinc}(4f)\,e^{-j2\pi(2)f}\right]$$

Note that

$$A \operatorname{rect}(t/T) \rightleftharpoons A\,T \operatorname{sinc}(fT)$$

therefore

$$\operatorname{rect}(t/4) \rightleftharpoons 4 \operatorname{sinc}(4f)$$

$$\frac{1}{4}\operatorname{rect}(t/4) \rightleftharpoons \operatorname{sinc}(4f)$$

$$\therefore\ g(t) = \frac{6}{j}\left[\frac{1}{4}\operatorname{rect}\left(\frac{t+2}{4}\right) - \frac{1}{4}\operatorname{rect}\left(\frac{t-2}{4}\right)\right]$$

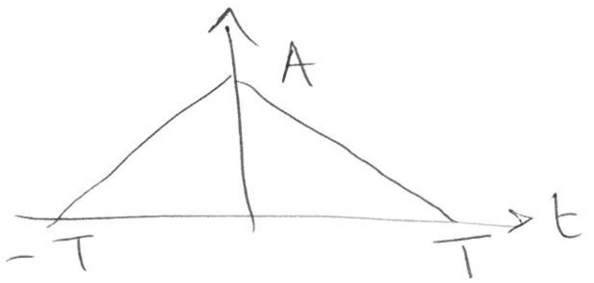
3) Find the inverse Fourier transform of

$$G(f) = 16\,\text{sinc}^2(4(f-10^6)) + 16\,\text{sinc}^2(4(f+10^6))$$

frequency shift property frequency shift property

We know that

$$A\,\text{tri}(t/T) \rightleftharpoons AT\,\text{sinc}^2(fT)$$



therefore

$AT = 16$

$T = 4$

& $A = \frac{16}{4} = 4$

Therefore

$$g(t) = 4\,\text{tri}\left(\tfrac{t}{4}\right)e^{j2\pi 10^6 t} + 4\,\text{tri}\left(\tfrac{t}{4}\right)e^{-j2\pi 10^6 t}$$

$$= 8\,\text{tri}(t/4)\cos(2\pi 10^6 t)$$

4) Find Fourier transform of

$$g(t) = 10\,\text{tri}(2t - 1/2)$$

$$= 10\,\text{tri}(2(t - 1/4))$$

↳ time shift property

$\therefore$

$$G(f) = 5\,\text{sinc}^2(f/2)\, e^{-j2\pi f/4}$$

4) ii) $g(t) = 8 \, \text{tri}(t/2) \cos(2\pi 10^6 t)$

We can rewrite $g(t)$ as

$$g(t) = \frac{8}{2} \text{tri}\left(\frac{t}{2}\right) \left[e^{j2\pi 10^6 t} + e^{-j2\pi 10^6 t} \right]$$

frequency shift property

$\therefore$

$$G(f) = 8 \, \text{sinc}^2((f - 10^6)2) + 8 \, \text{sinc}^2((f + 10^6)2)$$

Note that:

$$A \, \text{tri}(t/T) \rightleftharpoons AT \, \text{sinc}^2(fT)$$