$P(X \geq \varepsilon) \leq \frac{E(X)}{\varepsilon}$

HW7 $\quad P(|X-\mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$

2.33

(a) Let ~~event~~ A = # automobiles sold weekly

$P(X>18) = P(X \geq 19) \leq \frac{E(X)}{19} = \frac{16}{19}$

(b) $P(X>25) = P(X \geq 26) \leq \frac{16}{26} = \frac{8}{13}$

2.34

(a) $P(|X-10| \geq 2) \leq \frac{\sigma^2}{\varepsilon^2} = \frac{100}{3 \times 2^2} = \frac{25}{3}$

$\Rightarrow P(|X-10| \geq 2) \leq 1$

(b) $P(|X-10| \geq 5) \leq \frac{100}{3 \times 25} = \frac{4}{3} \leq 1$

(c) $P(|X-10| \geq 9) \leq \frac{100}{3 \times 81} = \frac{100}{243}$

(d) $P(|X-10| \geq 20) \leq \frac{100}{3 \times 400} = \frac{1}{12}$

3.1

(a) $X = \{1,2,3,4,5,6\}$ $\quad Y = \{2,3,4,\cdots,12\}$

Let $X = p$ $\quad Y = q$ $\quad p < q \leq 2p$

1° $q = 2p$ $\quad P(X=p, Y=q) = \frac{1}{6} \times \frac{1}{11} = \frac{1}{66}$

2° $q < 2p$ $\quad P(X, Y) = \frac{1}{6} \times \frac{1}{11} \times 2 = \frac{1}{33}$

$\therefore P(X=p, Y=q) = \begin{cases} \frac{1}{66} & q = 2p \\ \frac{1}{33} & q < 2p \end{cases}$

(b) $\underset{\alpha}{X} = \{1,2,3,4,5,6\}$ $\quad \underset{\beta}{Y} = \{1,2,\cdots,6\}$ $\quad \beta \geq \alpha$

if $\alpha = \beta$ $\quad P(X=\alpha, Y=\beta) = \frac{1}{6} \times \frac{\alpha}{6} = \frac{\alpha}{36}$

if $\alpha < \beta$ $\quad P(X=\alpha, Y=\beta) = \frac{1}{6} \times \frac{6-\alpha}{6} = \frac{6-\alpha}{36}$

$\therefore P(X=\alpha, Y=\beta) = \begin{cases} \frac{\alpha}{36} & \beta = \alpha \\ \frac{6-\alpha}{36} & \beta \geq \alpha \end{cases}$

(c) $X, Y = \{1,2,\cdots,6\}$ $\quad \alpha \leq \beta$

$P(X=\alpha, Y=\beta) = \begin{cases} \frac{1}{36} & \alpha = \beta \\ \frac{1}{5} \times \frac{6-\alpha}{5} & \alpha < \beta \end{cases}$

$\frac{1}{5} \times \frac{6-\alpha}{5} = \frac{6-\alpha}{25}$

$16+32+24+8+1$

3.2

$P(X_1=m) = (1-p)^m \cdot p \qquad P(X_2=n) = (1-p)^n \cdot p$

$\therefore P(X_1=m, X_2=n) = (1-p)^{m+n} \cdot p^2$

~~3.3~~

3.3

(a) $3c+6C=1 \qquad C=\frac{1}{9}$

(b) ~~$P(X=x, Y=y)=$~~

Y \ X	0	1
0	$\frac{2}{9}$	$\frac{2}{9}$
1	$\frac{1}{9}$	$\frac{4}{9}$

(c) $P(X) = C_{12}^2 \times C_{10}^2 \times C_8^2 \times C_6^2 \times C_4^2 \times C_2^2 \times (\frac{1}{9})^6 \times (\frac{2}{9})^6$

$= \frac{12!}{2^5} \cdot \frac{2^6}{9^{12}} = \frac{12! \times 2}{9^{12}}$

(d) Let event A be number of one in 4 trials

B — — — two — — —.

$\frac{2}{9}$

B \ A	0	1	2	3	4
0					
1					
2					
3					
4					

$\frac{1}{9}$

$P(A=0, B=4) = (\frac{2}{9})^4 = \frac{2^4}{9^4}$

$P(A=1, B=3) = 4 \times \frac{1}{9} \times (\frac{2}{9})^3$

$P(A=2, B=2) = C_4^2 \times (\frac{1}{9})^2 \times (\frac{2}{9})^2$

$P(A=3, B=1) = C_4^3 \cdot (\frac{1}{9})^3 \times \frac{2}{9}$

$P(A=4, B=0) = (\frac{1}{9})^4$

$P(A,B) = \frac{1}{9^4} \times (2^4 + 4 \times 2^3 + 6 \times 4 + 8 + 1) = \frac{1}{9^4} \times 81 = \frac{1}{9^2} = \frac{1}{81}$

$\therefore P(X) = C_{12}^4 \times \frac{1}{81} \times C_8^4 \times \frac{1}{81} \times \frac{1}{81} = \frac{12!}{4!} \times \frac{1}{9^6}$

(e) Let A = # tosses land on even numbers.

$P(A \geq 8) = P(A=8) + P(A=9) + \cdots + P(A=12)$

$= C_{12}^8 \times (\frac{6}{9})^8 \times (\frac{3}{9})^4 + C_{12}^9 \times (\frac{6}{9})^9 \times (\frac{3}{9})^3 + C_{12}^{10} \times (\frac{6}{9})^{10} \times (\frac{3}{9})^2 + C_{12}^{11} \times (\frac{6}{9})^{11} \times \frac{3}{9}$

$+ (\frac{6}{9})^{12}$

$$Z=\sum_{i=1}^{4} X_i \sim N(6, \cancel{36}\ 24)$$

$$\frac{Z-6}{2\sqrt{6}} \sim \phi(0,1)$$

3.6

(a) $X_i \sim N(1.5, \cancel{36}\ 6)$ $\mu=1.5$ $\sigma^2=6$, $i=1,2,3,4$

$P(X_1+X_2+X_3+X_4>0)$

$= P\left(\frac{X-6}{2\sqrt{6}} > \frac{0-6}{2\sqrt{6}}\right) = P\left(Z > -\frac{\sqrt{6}}{2}\right)$

$= 1-\Phi\left(-\frac{\sqrt{6}}{2}\right) = \Phi\left(\frac{\sqrt{6}}{2}\right) = 0.8897$

(b) $P\left(\sum_{i=1}^{4} X_i > 0 \mid \sum_{i=1}^{2} X_i = -5\right) = P(X_3+X_4>5)$

$= P\left(\frac{X_3+X_4-3}{2\sqrt{3}} > \frac{5-3}{2\sqrt{3}}\right)$

$= P\left(Z > \frac{\sqrt{3}}{3}\right) = 1-\Phi\left(\frac{\sqrt{3}}{3}\right) \approx 0.2818$

(c) $P\left(\sum_{i=1}^{4} X_i > 0 \mid X_1 > 5\right) = P(X_2+X_3+X_4 > -5)$

$= P\left(Z > \frac{-9.5}{3\sqrt{2}}\right) = \Phi\left(\frac{9.5}{3\sqrt{2}}\right) \approx 0.9874$

3.8 Let X be the life of car. $X \sim Exp(\lambda)$

$E(X) = \frac{1}{\lambda} = 300000$ $f(x) = \lambda e^{-\lambda x}$ $x>0$

$P(X>125000 \mid X>100000)$

$= P(X>25000) = 1-P(X\le 25000)$

$= 1-\int_0^{25000} \frac{1}{300000} e^{-\frac{1}{300000}x}\, dx$

$= 1+\int_0^{25000} e^{-\frac{1}{300000}x}\, d\left(-\frac{1}{300000}x\right)$

$= 1+ e^{-\frac{1}{300000}x}\Big|_0^{25000}$

$= 1+e^{-\frac{1}{12}}-1 = e^{-\frac{1}{12}}$