EBU4375: SIGNALS AND SYSTEMS

LECTURE 4: PART 1

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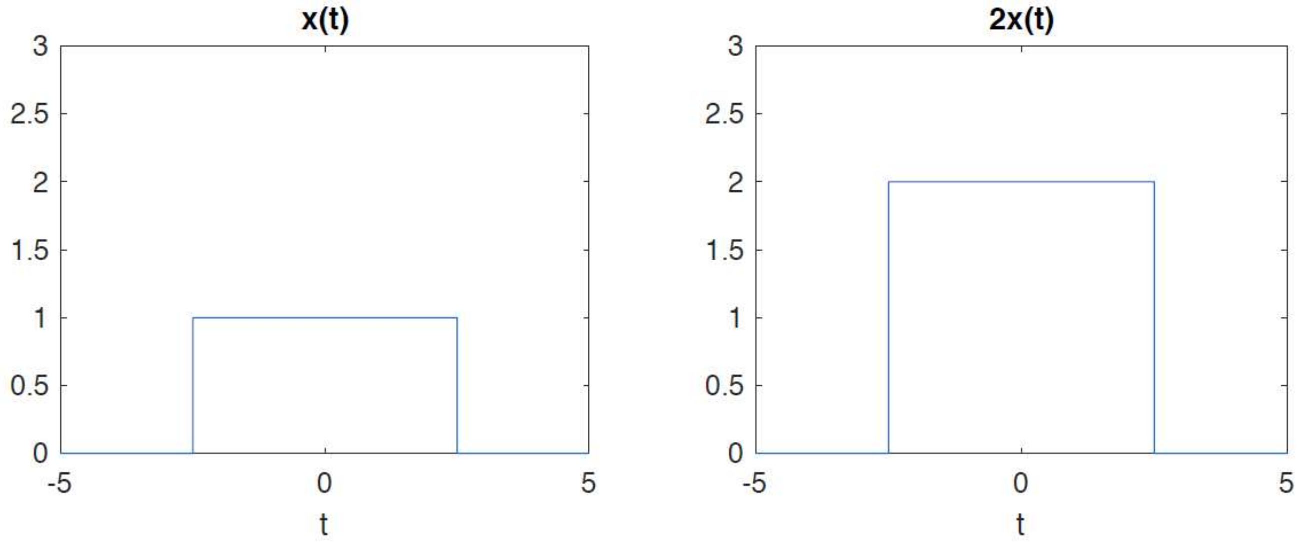
Operations – Amplitude Scaling (CT Signals)

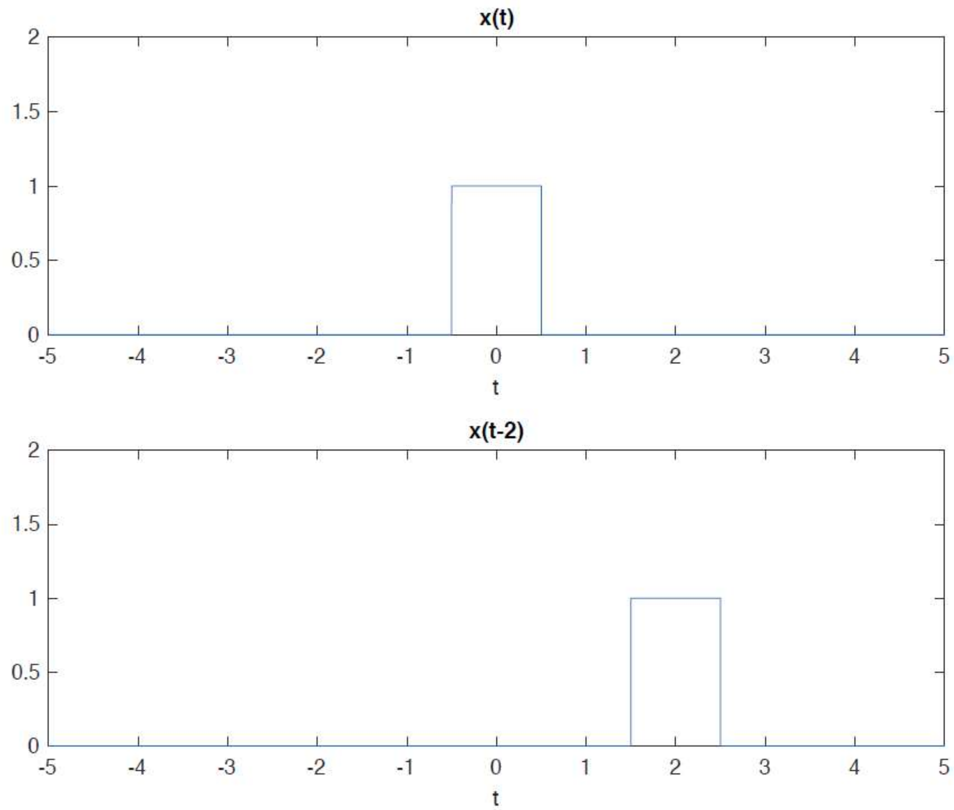
Given a CT signal $x(t)$, scaling consist of multiplying it by a scalar value a, producing the new signal $y(t) = ax(t)$.



Scaling is defined in an analogous way for DT signals.

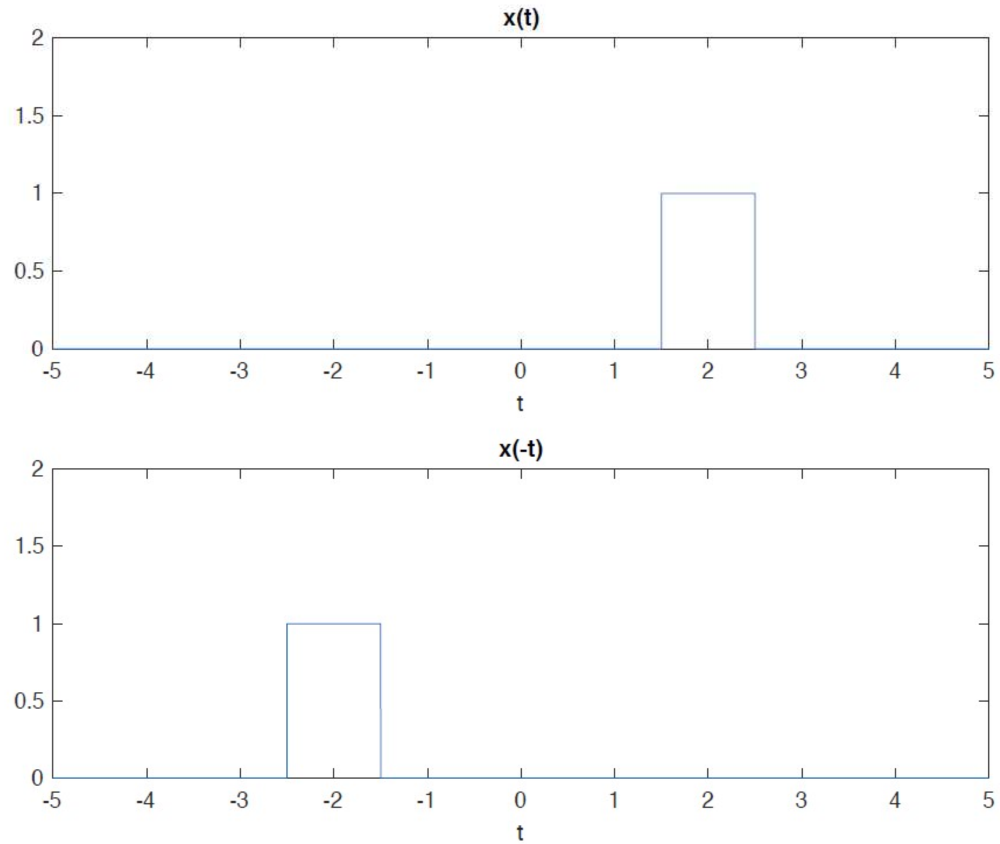
Operations – Time Shift (CT Signals)

Given a CT signal $x(t)$, time shifting by t_0 units of time produces the new signal $y(t) = x(t - t_0)$ (DT shifting is defined in a similar way).



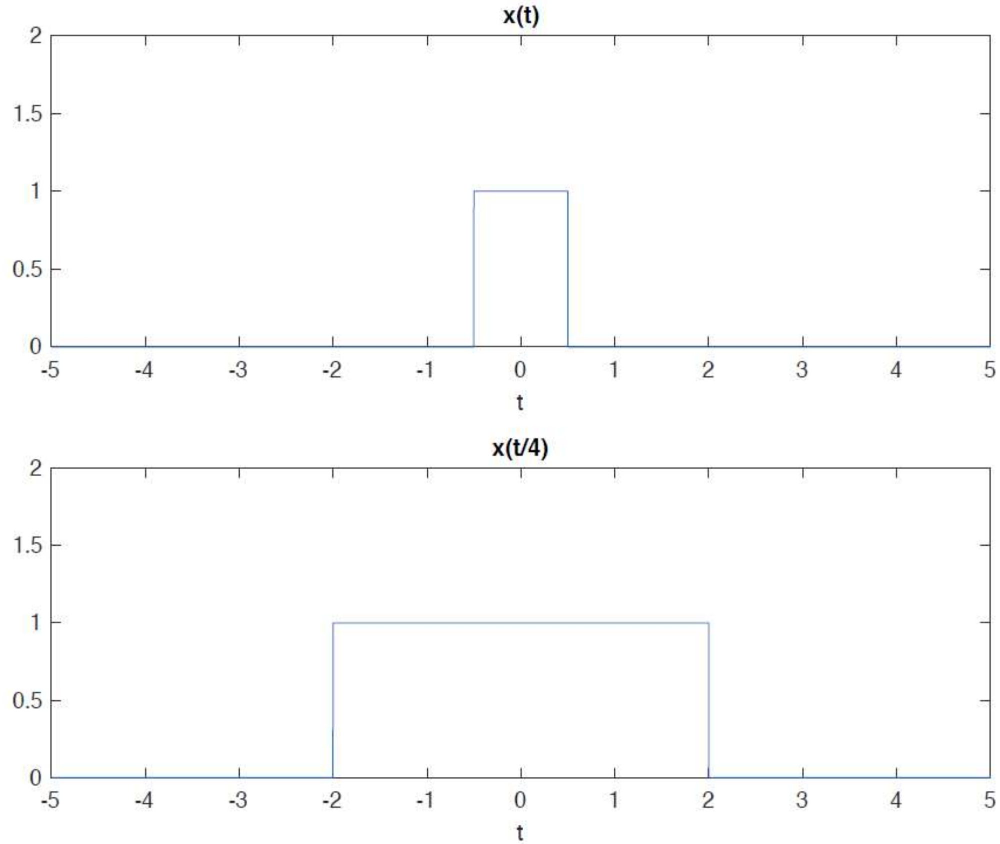
Operations – Time Inversion/Reversal (CT Signals)

Time reversal *flips* the time axis producing the signal $y(t) = x(-t)$.



Operations – Time Scaling (CT Signals)

Time scaling **expands** or **compresses** the time axis. Signal $y(t) = x(at)$ is a compressed version of $x(t)$ if $|a| > 1$, and an expanded version if $|a| < 1$.



Operations – Combined Time Operations (CT Signals)

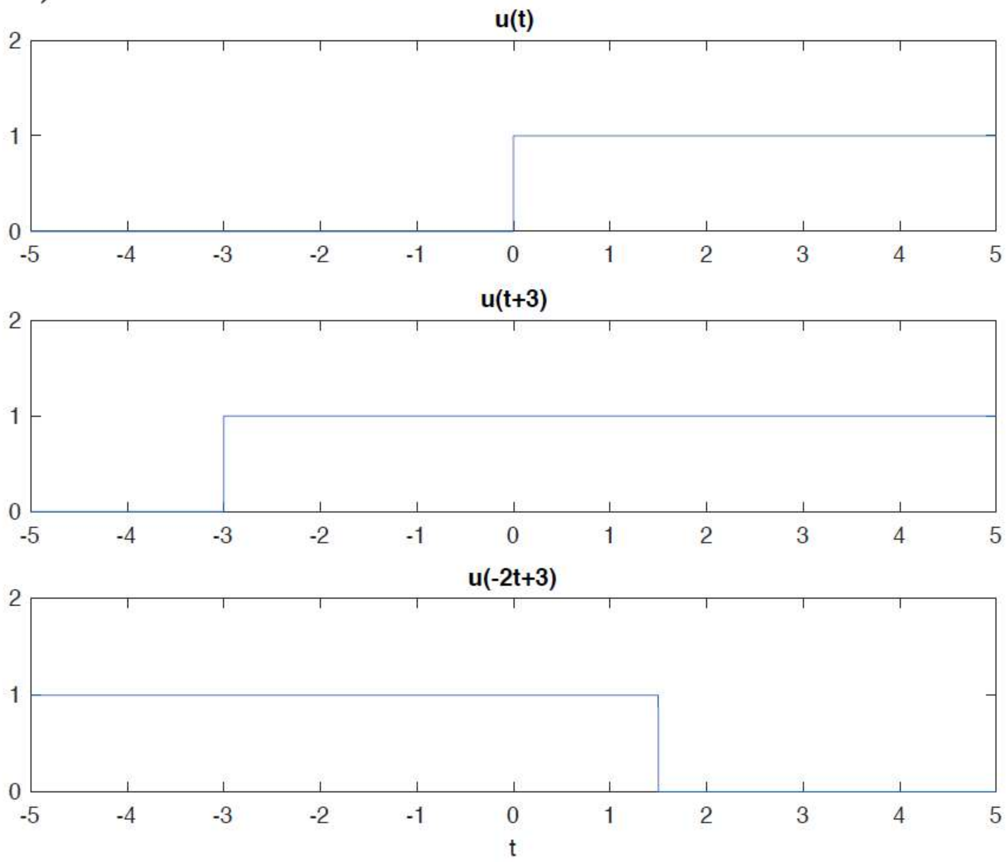
Consider the signal $x(t) = u(-2t + 3)$. In order to obtain $x(t)$ we will take the following steps:

- Define the time-shift $y(t) = u(t + 3)$.
- Define the time scaling and reverse $z(t) = y(-2t)$.

As we can see, $z(t) = y(-2t) = u(-2t + 3)$ and therefore $x(t) = z(t)$.

Operations – Combined Time Operations (CT Signals)

In general, we can obtain the signal $y(t) = x(-at + t_0)$ by shifting $x(t)$ first and then by scaling and time reversing the result. Graphically, the signal $u(-2t + 3)$ can be obtained as follows:



You can see that $u(-2(1.5) + 3) = u(0)$.

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LECTURE 4: PART 2

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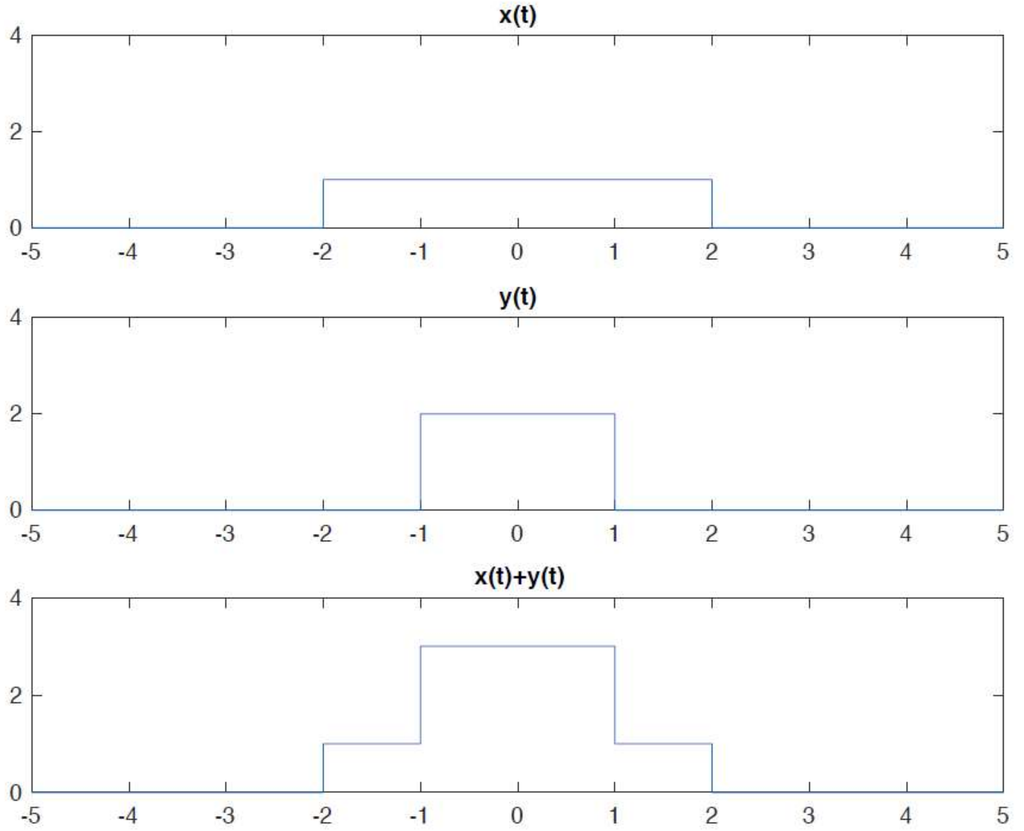
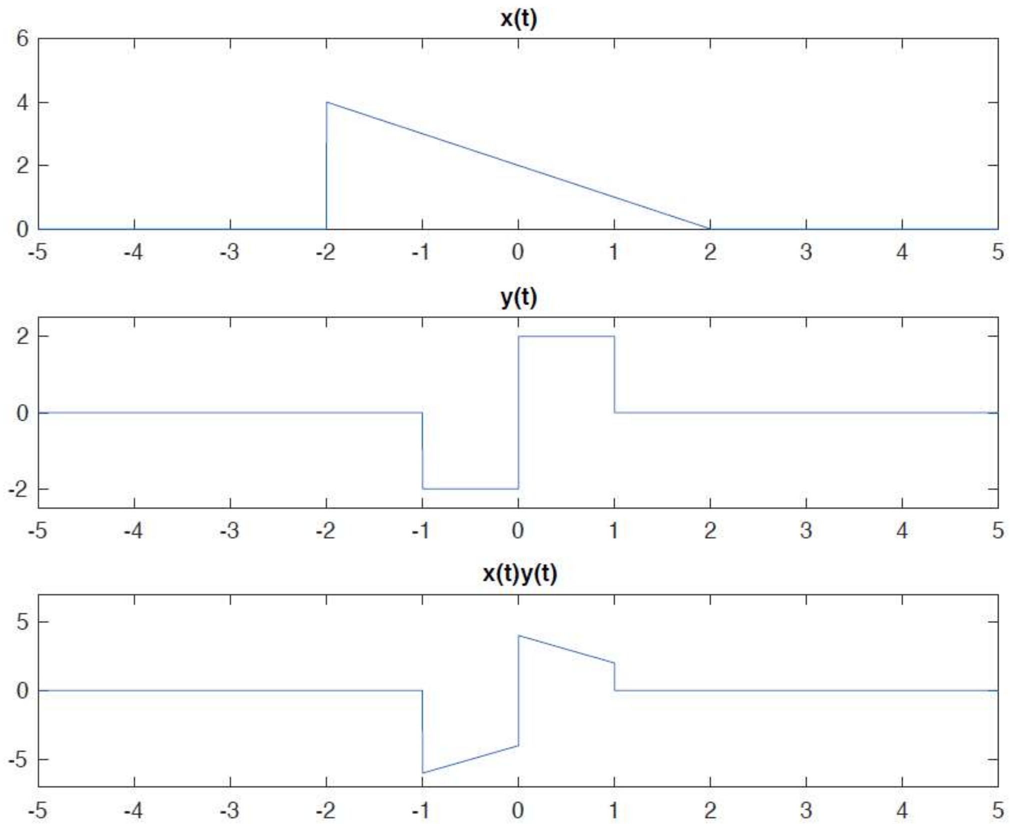
Operations – Sum (CT Signals)

Adding two signals $x(t)$ and $y(t)$ means adding their values each time instant.



Operations – Product (CT Signals)

Similarly, we multiply signals by multiplying their values each time instant.

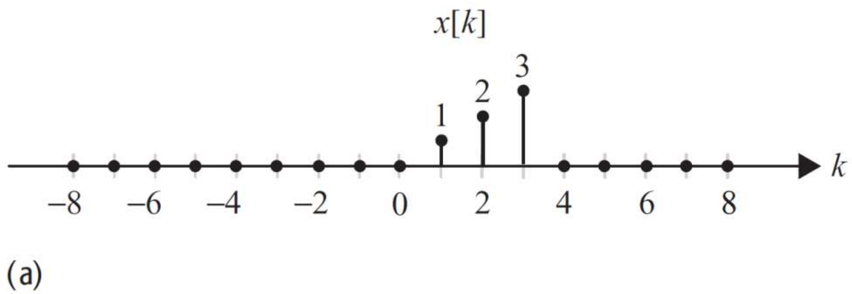


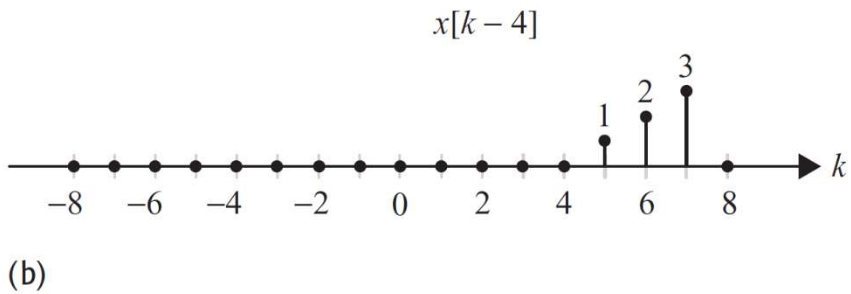
Operations – Time Shift (DT Signals)

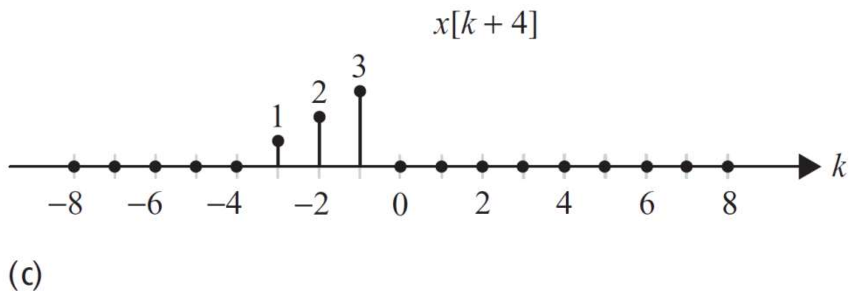
When a DT signal $x[k]$ is shifted by m time units, the delayed signal $\phi[k]$ is expressed as

$$\phi[k] = x[k+m]$$

If $m < 0$, the signal is said to be delayed in time.





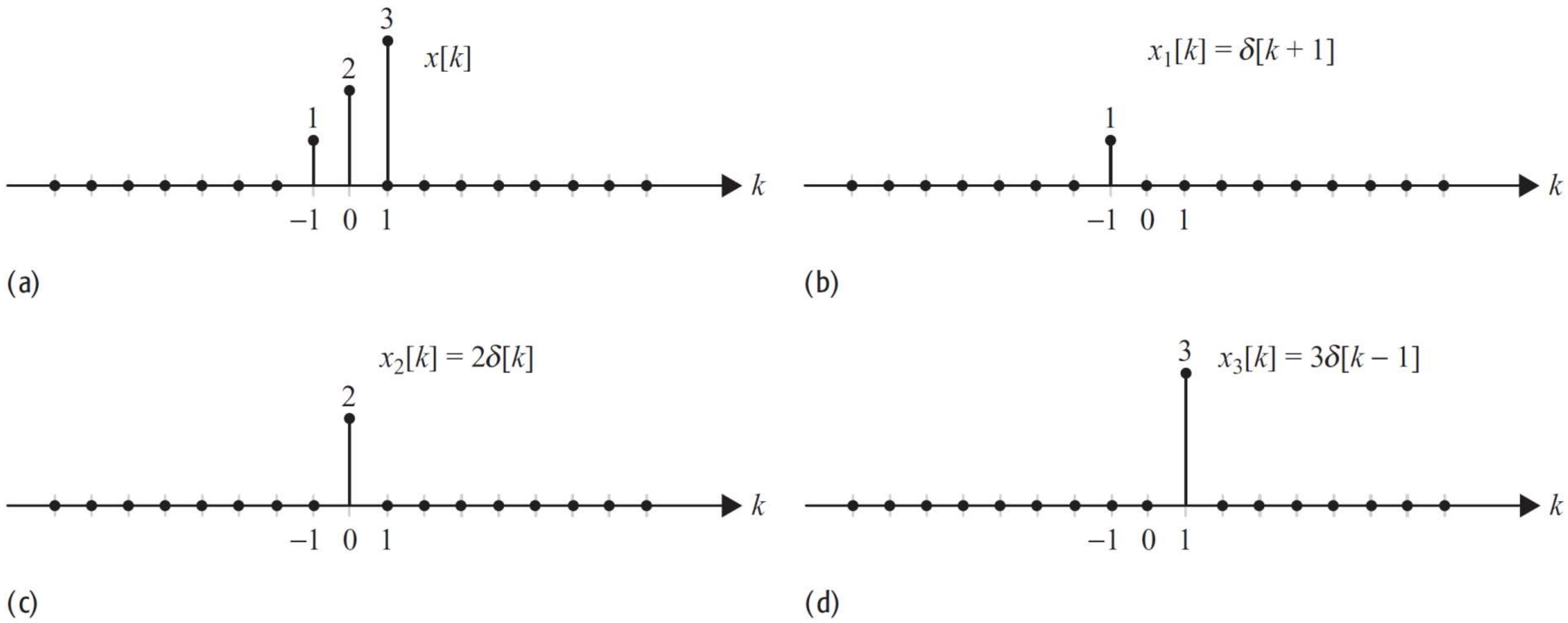


Time shifting of a DT signal. (a) Original DT signal $x[k]$. (b) Time-delayed version $x[k-4]$ of the DT signal $x[k]$. (c) Time-advanced version $x[k+4]$ of the DT signal $x[k]$.

Operations – Time Shift (DT Signals)

Example

Represent the DT sequence shown in (a) as a function of time-shifted DT unit impulse functions.



$$x[k] = \delta[k+1] + 2\delta[k] + 3\delta[k-1]$$

Operations – Time Inversion/Reversal (DT Signals)

$$y(n) = x(-n)$$

positive time switches to negative time and vice versa

Example

Sketch the time-inverted version of the following DT sequence:

$$x[k] = \begin{cases} 1 & -4 \leq k \leq -1 \\ 0.25k & 0 \leq k \leq 4 \\ 0 & \text{elsewhere,} \end{cases}$$

Operations – Time Inversion/Reversal (DT Signals)

Solution

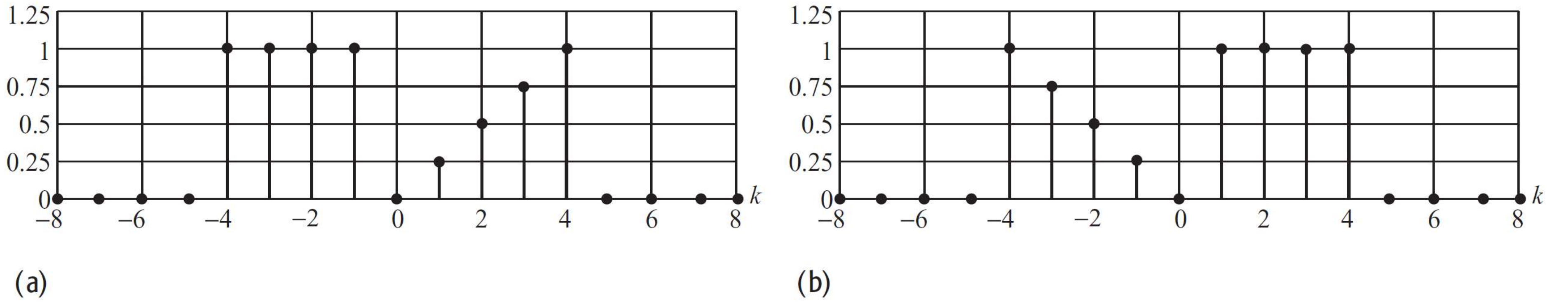
To derive the expression for the time-inverted signal $x[-k]$, substitute $k = -m$

$$x[-m] = \begin{cases} 1 & -4 \leq -m \leq -1 \\ -0.25m & 0 \leq -m \leq 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Simplifying the above expression and expressing it in terms of the independent variable k yields

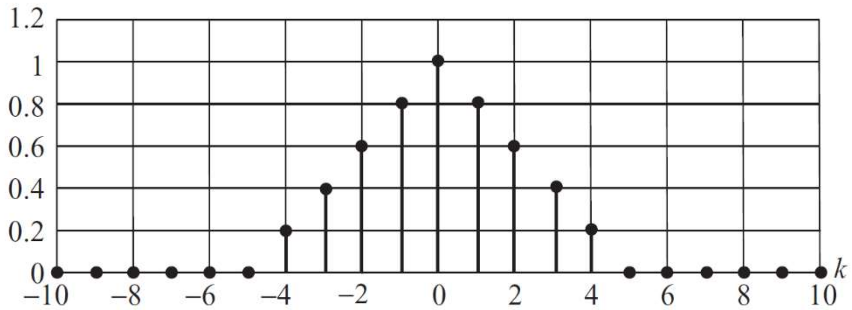
$$x[-m] = \begin{cases} 1 & 1 \leq m \leq 4 \\ -0.25m & -4 \leq \quad m \leq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Operations – Time Inversion/Reversal (DT Signals)

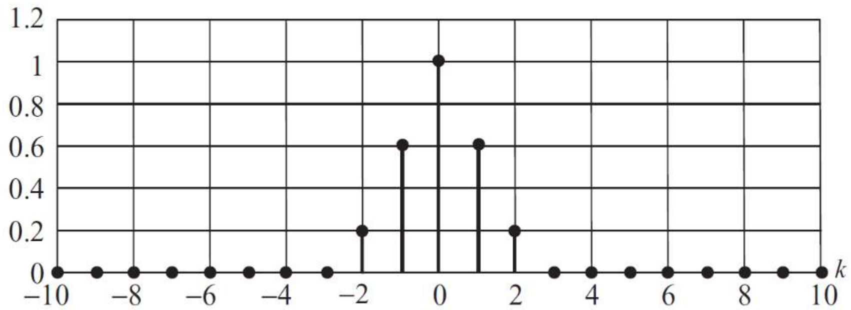


(a) Original CT sequence $x[k]$
(b) Time-inverted version $x[-k]$

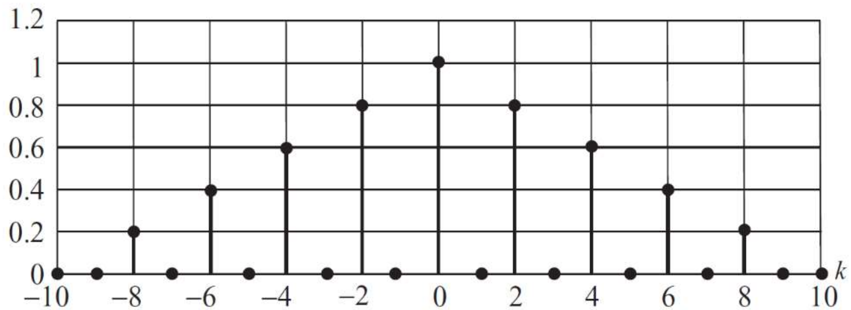
Operations – Time Scaling (DT Signals) also known as Decimation and Interpolation



(a)



(b)



(c)

(a) Original DT sequence $x[k]$. (b) Decimated version $x[2k]$, of $x[k]$. (c) Interpolated version $x[0.5k]$ of signal $x[k]$.

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LECTURE 4: PART 3

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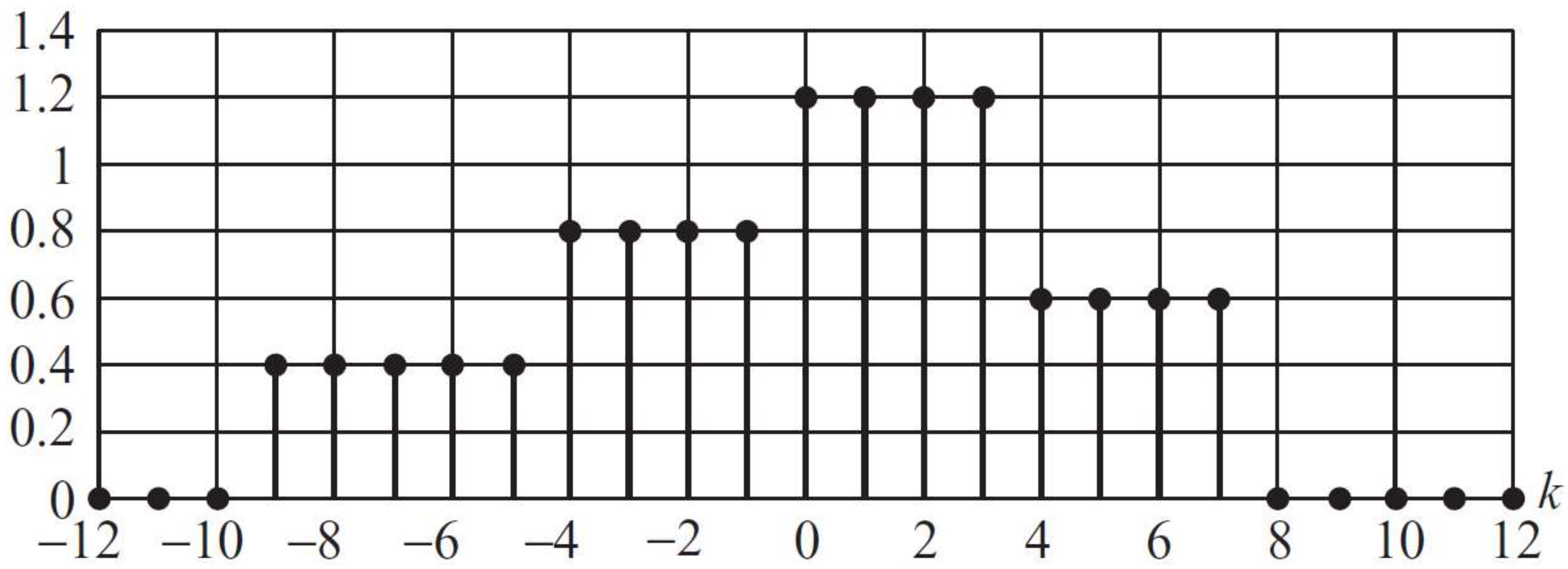
Operations – Combined Time Operations (DT Signals)

Example

Sketch the waveform for $x[-15 - 3k]$ for the DT sequence $x[k]$



Operations – Combined Time Operations (DT Signals)

Solution

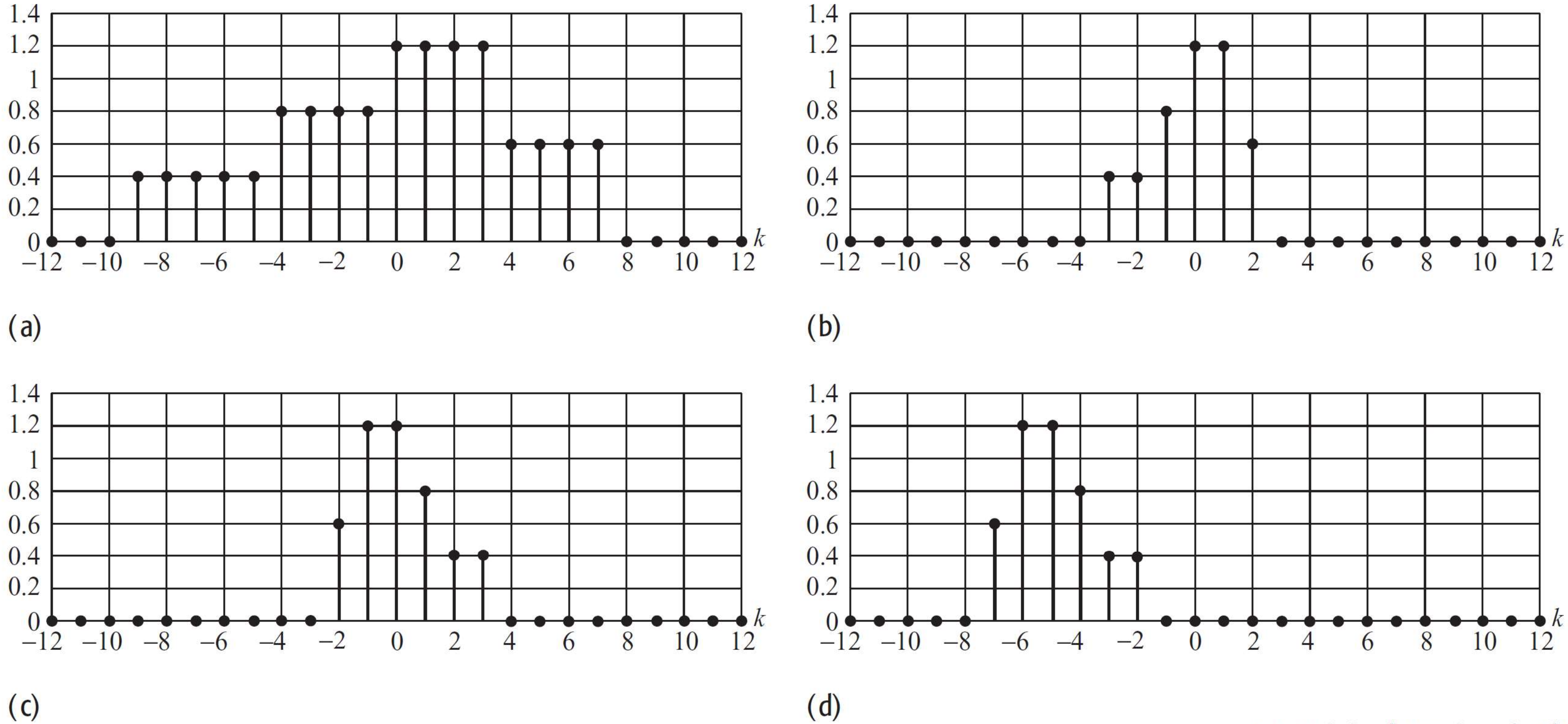
Express $x[-15 - 3k] = x[-3(k + 5)]$ and follow steps (i)–(iii) as outlined below.

(i) Compress $x[k]$ by a factor of 3 to obtain $x[3k]$.

(ii) Time-reverse $x[3k]$ to obtain $x[-3k]$.

(iii) Shift $x[-3k]$ towards the left-hand side by five time units to obtain $x[-3(k + 5)] = x[-15 - 3k]$.

Operations – Combined Time Operations (DT Signals)



(a) Original DT signal $x[k]$.
(b) Time-scaled version $x[3k]$.
(c) Time-inverted version $x[-3k]$ of (b). (d) Time-shifted version $x[-15 - 3k]$ of (c).