Hw. 14 06.03.

6.11 Suppose that the process $\{X_t\}_{t\in R}$ is a stationary process with the autocorrelation function $R_X(t) = 4\cos\omega t$, where ω is constant. Find the average power of X_t.

Since the average power of a stationary process equals to $R_X(0)$

$\therefore \quad \bar{Q} = R_X(0) = 4\cos 0 = 4$

6.12 A stochastic stationary process has an autocorrelation function of

$$R_X(n) = 5\sin(n/80) + 4. \quad \text{even?}$$

(a) Find the variance of this stochastic process.

$$E(X_t^2) = R_X(0) = 5\sin 0/80 + 4 = 4$$

$$\therefore \quad D(X) = E(X_t^2) - E^2(X_t) = 4 - E^2(X_t)$$

6.13 A stationary stochastic process has a power spectral density of $S_X(\omega) = \frac{500}{\omega^2+9}$. Find the autocorrelation function of X_t.

$$R_X(\tau) = \frac{1}{\pi}\int_0^{+\infty} S_X(w)\cos w\tau\, dw$$

We know that

$$R_X(\tau) = Ae^{-\beta|\tau|} \longleftrightarrow S_X(w) = \frac{2A\beta}{w^2+\beta^2}$$

$$\therefore \quad S_X(w) = \frac{500}{w^2+9} \Rightarrow \begin{cases} 2A\beta = 500 \\ \beta^2 = 9 \end{cases} \Rightarrow A>0,\ \beta>0 \quad \begin{cases} A = \frac{250}{3} \\ \beta = 3 \end{cases}$$

$$\therefore \quad R_X(\tau) = \frac{250}{3} e^{-3|\tau|}$$