3.1 a) $X=\{1,2,3,4,5,6\}$, $Y=\{2,3,4,5,\cdots,12\}$

$Y>X$

let $X=\alpha$, $Y=\beta$. $\alpha<\beta\le 2\alpha$

if $\beta=2\alpha$. $P(X,Y)=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

if $\beta<2\alpha$ $P(X,Y)=\frac{1}{6}\times\frac{1}{6}\times 2=\frac{1}{18}$

so $P(X=\alpha, Y=\beta)=\begin{cases}\frac{1}{18} & \alpha<\beta<2\alpha \\ \frac{1}{36} & \cancel{\alpha<\beta\le 2}\ \beta=2\alpha\end{cases}$

b) It's easy to find that $\beta\geq\alpha$

if $\beta=\alpha$ $P(X,Y)=\frac{1}{6}\times\frac{\alpha}{6}=\frac{\alpha}{36}$

$\beta>\alpha$ $P(X,Y)=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

$\therefore P(X=\alpha, Y=\beta)=\begin{cases}\frac{\alpha}{36} & \beta=\alpha \\ \frac{1}{36} & \beta>\alpha\end{cases}$

c) $\beta\geq\alpha$

$P(X=\alpha, Y=\beta)=\begin{cases}\frac{1}{18} & \beta>\alpha \\ \frac{1}{36} & \beta=\alpha\end{cases}$

3.2 $P(X_1=m)=(1-p)^m\cdot p$

$P(X_2=n)=(1-p)^n\cdot p$

$P(X_1=m, X_2=n)=(1-p)^{m+n}\cdot p^2$

3.4 a) $f(x,y) = ce^{-(x+y)}$ $(x>0, y>0)$

$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,dxdy = 1 \Rightarrow \int_0^{+\infty}\int_0^{+\infty} ce^{-(x+y)}dxdy$

$= c\int_0^{+\infty} -e^{-(x+y)}\Big|_0^{+\infty} dy = c\int_0^{+\infty} e^{-y}dy$

$= c\cdot(-e^{-y})\Big|_0^{+\infty} = c = 1$

b) $P(X+Y>1) = 1 - P(X+Y\le 1)$

$= 1 - \int_0^1\int_0^{1-x} e^{-(x+y)}dydx$

$= 1 - \int_0^1 -e^{-(x+y)}\Big|_0^{1-x} dx$

$= 1 - \int_0^1 (e^{-x} - e^{-1})dx = 1 - (-e^{-x} - e^{-1}x)\Big|_0^1$

$= \frac{2}{e}$

c) $\int_0^{y}\int_0^{+\infty} f(x,y)\,dydx = \int_0^{+\infty}\int_0^{y} e^{-(x+y)}dxdy$

$= \int_0^{+\infty} -e^{-(x+y)}\Big|_0^{y} dy$

$= \int_0^{+\infty} (e^{-y} - e^{-2y})dy$

$= (-e^{-y} + \frac{1}{2}e^{-2y})\Big|_0^{+\infty}$

$= 1 - \frac{1}{2} = \frac{1}{2}$

3.6 a) $X_i \sim N(1.5, 6)$ $\quad i=1,2,3,4$ $\quad \sum_{i=1}^{4} X_i \sim N(6,24)$

$$P(X_1+X_2+X_3+X_4>0) = P(Z\cdot 2\sqrt{6}+6>0)$$

$$P(Z>-\frac{\sqrt{6}}{2}) = 1-\Phi(-\frac{\sqrt{6}}{2}) \approx 0.8897$$

b) $P(\sum_{i=1}^{4} X_i>0 \mid \sum_{i=1}^{2} X_i=-5) = P(X_3+X_4>5)$

$$=P(Z\cdot 2\sqrt{3}+3>5) = P(Z>\frac{\sqrt{3}}{3}) = 1-\Phi(\frac{\sqrt{3}}{3}) \approx 0.2818$$

c) $P(\sum_{i=1}^{4} X_i>0 \mid X_1>5) = P(X_2+X_3+X_4>-5)$

$$=P(Z\cdot 3\sqrt{2}+4.5>-5) = P(Z>\frac{-9.5}{3\sqrt{2}}) = 1-\Phi(\frac{-9.5}{3\sqrt{2}}) \approx 0.9874$$

3.7 a) $\int_0^{+\infty}\int_0^{+\infty} c e^{-ax-by}\,dx\,dy = -\frac{c}{a}\int_0^{+\infty} e^{-ax-by}\Big|_0^{+\infty} dy$

$$=\frac{c}{a}\int_0^{+\infty} e^{-by}\,dy = \frac{c}{ab} = 1 \implies c=ab$$

b) $P(Y\le X) = \int_0^{+\infty}\int_0^{x} f(x,y)\,dy\,dx = -a\int_0^{+\infty} e^{-ax-by}\Big|_0^{x} dx$

$$= -a\int_0^{+\infty}(e^{-x(a+b)} - e^{-ax})\,dx = -a\left[-\frac{1}{a+b}e^{-x(a+b)}+\frac{1}{a}e^{-ax}\right]\Big|_0^{+\infty}$$

$$= -a\left(\frac{1}{a+b}-\frac{1}{a}\right) = -\left(\frac{a}{a+b}-1\right) = \frac{b}{a+b}$$

c) if $x\le 0$ or $y\le 0$ $\quad F(x,y)=0$

if $x>0$ and $y>0$ $\quad F(x,y) = \int_0^{x}\int_0^{y} ab\,e^{-au-bv}\,dv\,du$

$$= -a\int_0^{x} e^{-au-by}\Big|_0^{y} du = e^{-ax-by} - e^{-by} - e^{-ax} + 1$$

$$F(x,y) = \begin{cases} 0 & x\le 0 \text{ or } y\le 0 \\ e^{-ax-by} - e^{-by} - e^{-ax} + 1 & x>0 \text{ and } y>0 \end{cases}$$

3.9 a) $U^2-4V \geq 0$ for $U, V = \pm 1$

when $V=-1$, $U^2-4V>0$

$\therefore P=\frac{1}{2}$

b)

$V \backslash U$	1	-1
1	$\frac{1}{6}$	$\frac{1}{3}$
-1	$\frac{1}{3}$	$\frac{1}{6}$

$P(U=1, V=1)=\frac{1}{6}$

$X=\frac{-U\pm\sqrt{U^2-4V}}{2}$

$\begin{cases} U=1 & V=-1 \\ U=-1 & V=-1 \end{cases}$

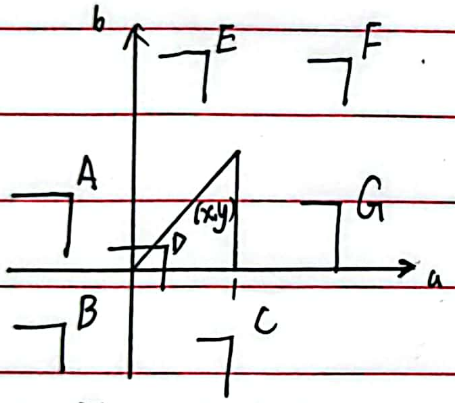
$E(X)=\frac{\sqrt{5}-1}{2}\times\frac{2}{3}+\frac{\sqrt{5}+1}{2}\times\frac{1}{3}=\frac{3\sqrt{5}-1}{6}$

c) $(U+V)^2-4(U+V)\geq 0$

$\Rightarrow (U+V-2)^2 \geq 4$

$\therefore P=1-P(U=1, V=1)=\frac{5}{6}$

3.1.8(c)



i) For A, B, C $X\leq 0$ or $Y\leq 0$

$\Rightarrow F(x,y)=0$

ii) For F, $X\geq 1$ and $Y\geq 1$

$\Rightarrow F(x,y)=1$

iii) For D, $0<x<1$, $0<y<1$ $0<y<x<1$

$F(x,y)=\int_0^y\int_b^x 8ab\,da\,db=2x^2y^2-y^4$

iv) For E, $0<x<1$, $y>1$ $y\geq x$

$$F(x,y)=\int_0^x\int_0^a 8ab\,db\,da=\int_0^x 4a^3\,da=x^4$$

v) For F, $x>1$, $0<y<1$

$$F(x,y)=\int_0^y\int_b^1 8ab\,da\,db=\int_0^y(4b-4b^3)\,db=2y^2-y^4$$

$$\therefore F(x,y)=\begin{cases}0 & x\leq 0 \text{ or } Y\leq 0\\ 2x^2y^2-y^4 & 0<y<x<1\\ x^4 & 0<x<1,\ y\geq x\\ 2y^2-y^4 & x>1,\ 0<y<1\\ 1 & x\geq 1 \text{ or } y\geq 1\end{cases}$$