HW2

Theorem 1.4.3 $P(\bar{A}) = 1 - P(A)$

$\because$ A and $\bar{A}$ are disjoint events and $A \cup \bar{A} = \Omega$

$\therefore P(A) + P(\bar{A}) = P(\Omega) = 1$ then $P(\bar{A}) = 1 - P(A)$

1.4.4 if $A \subset B$ then $P(B-A) = P(B) - P(A)$ and $P(A) \le P(B)$

prove: $\because A \subset B$

$\therefore B = A + \bar{A}B$ and $A \cap \bar{A}B = \phi$

$\therefore P(B) = P(A) + P(\bar{A}B)$ and $P(\bar{A}B) \ge 0$, $B - A = \bar{A}B$

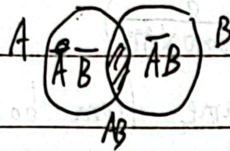
$\therefore P(B-A) = P(\bar{A}B) = P(B) - P(A)$, $P(B) - P(A) \ge 0$

1.4.5

it is obvious that $P(A) \ge 0$. Because $A \subset \Omega$

$\therefore P(A) \le P(\Omega) = 1$

1.4.6 for every events A, B $P(A \cup B) = P(A) + P(B) - P(AB)$



$A \cup B = A\bar{B} \cup AB \cup \bar{A}B$

$P(A \cup B) = P(A\bar{B}) + P(AB) + P(\bar{A}B)$

$P(A) = P(A\bar{B}) + P(AB)$ $P(B) = P(\bar{A}B) + P(AB)$ $\square$.

Ex.

1.25

$P(A-B) = P(A\bar{B}) = 0.32$

$P(AB) = P(A) - P(A\bar{B}) = 0.5 - 0.32 = 0.18$

$P(A \cup B) = P(A) + P(B) - P(AB) = 0.5 + 0.4 - 0.18 = 0.72$

1.31

(a) Suppose event A means that first ball drawn is blue.

event B means that second ball drawn is cyan.

So the possibility of the second ball is cyan

$P(B) = P(B|A)P(A) + P(B|\bar{A}) \cdot P(\bar{A}) = \frac{c}{b+c+d} \cdot \frac{b}{b+c} + \frac{c+d}{b+c+d} \cdot \frac{c}{b+c} = \frac{c}{b+c}$

(b) The probability is $P(\bar{A}|B) = \frac{P(\bar{A}B)}{P(B)} = \frac{P(\bar{A})\cdot P(B|\bar{A})}{P(B)} = \frac{\frac{c(c+d)}{(b+c)(b+c+d)}}{\frac{c}{b+c}} = \frac{c+d}{b+c+d}$

1.32

Suppose that event A means the coin chosen is fair.

event B means head shows both times

$P(A|B) = \frac{P(AB)}{P(B)}$

$P(B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A}) = \frac{1}{4}\times\frac{1}{2} + 1\times\frac{1}{2} = \frac{5}{8}$

$P(AB) = P(B|A)\cdot P(A) = \frac{1}{8}$

$\therefore P(A|B) = \frac{1}{5}$

1.33

(a) suppose event A means both bulbs are defective

$P(A) = \frac{1}{2}\times\frac{C_{100}^{2}}{C_{1000}^{2}} + \frac{1}{2}\times\frac{C_{200}^{2}}{C_{2000}^{2}} = \frac{100\times 99}{4}\cdot\left(\frac{2}{1000\times 999} + \frac{2}{2000\times 1999}\right)$

(b) suppose event B means the defective bulbs came from box 1

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{C_{100}^{2}}{C_{1000}^{2}}}{\frac{C_{100}^{2}}{C_{1000}^{2}} + \frac{C_{200}^{2}}{C_{2000}^{2}}} = \frac{C_{2000}^{2}}{C_{1000}^{2} + C_{2000}^{2}} \approx 0.8$

1.36

(a) suppose event A means he use compact car.

event B means he use minivan,

event C means he gets home before 5:30 pm

$P(C) = P(C|A)\,P(A) + P(C|B)\cdot P(B) = 0.75\times 0.75 + 0.25\times 0.6 = 0.7125$

~~(b) $P(A|\bar{C}) = \frac{P(A\bar{C})}{P(\bar{C})} = \frac{P(A)\cdot P(\bar{C}|A)}{P(\bar{C})} = \frac{0.75\times 0.25}{0.75\times 0.25 + 0.25\times 0.4} = \frac{0.5625}{0.2875}$~~

(b) $P(A|\bar{C}) = \frac{P(A)\cdot P(\bar{C}|A)}{P(\bar{C})}$

$= \frac{P(A)\cdot P(\bar{C}|A)}{P(\bar{C}|A)P(A) + P(\bar{C}|B)P(B)} = \frac{0.75\times 0.25}{0.25\times 0.75 + 0.4\times 0.25} \approx 0.652$

~~(c)~~ $\cancel{P(BC) = P(C|B)\cdot P(B) = \frac{0.35\times 0.4}{1} = 0.1}$

~~(d)~~ $\cancel{2\times[P(BC)+P(AC)] = 2\cdot(0.75\times 0.75 + 0.}$

(c)

$$P(B\bar{C}) = P(\bar{C}|B)\cdot P(B) = 0.4\times 0.25 = 0.1$$

(d) suppose event D means he gets home before 5:30 pm on two consecutive days.

event E means he doesn't use same car.

$$P(DE) = \cancel{P(D|E)\cdot P(E)}\ P(E)\cdot P(D|E)$$

$$= (1-0.75^2-0.25^2)\cdot(2\times 0.75\times 0.6) = 0.3375$$