EBU4375: SIGNALS AND SYSTEMS

TOPIC 3-1: CONTINUOUS-TIME SYSTEMS IN THE FREQUENCY DOMAIN

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ACKNOWLEDGMENT

These slides are partially from lectures prepared by

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COURSE CONTENT AND SCHEDULE

- The main topics covered by this course are organised as follows:
  - Topic 1: Signals and systems in the time domain.
  - Topic 2: Continuous-time signals in the frequency domain.
  - **Topic 3: Discrete-time signals in the frequency domain.**
  - Topic 4: Sampling theory and communication systems.

BRIEF REVISION

- Before we move to DT signals in Frequency Domain, let's brush up on some ideas first!
    1. Introduction
    2. The Frequency domain….What for?
    3. The convolution theorem
    4. Introducing ***Filters***

BRIEF REVISION

- Before we move to DT signals in Frequency Domain, let's brush up on some ideas first!
  1. Introduction
  2. The Frequency domain….What for?
  3. The convolution theorem
  4. Introducing ***Filters***

1: WHAT HAVE WE LEARNED SO FAR?

1. CT and DT signals in the time domain: basic signals, representation, properties, classification, manipulations in the time domain (shift, reflection, amplification) . . .
2. CT and DT systems in the time domain: properties, LTI systems, impulse response, convolution . . .
3. CT signals in the frequency domain: Fourier series and Fourier transform.

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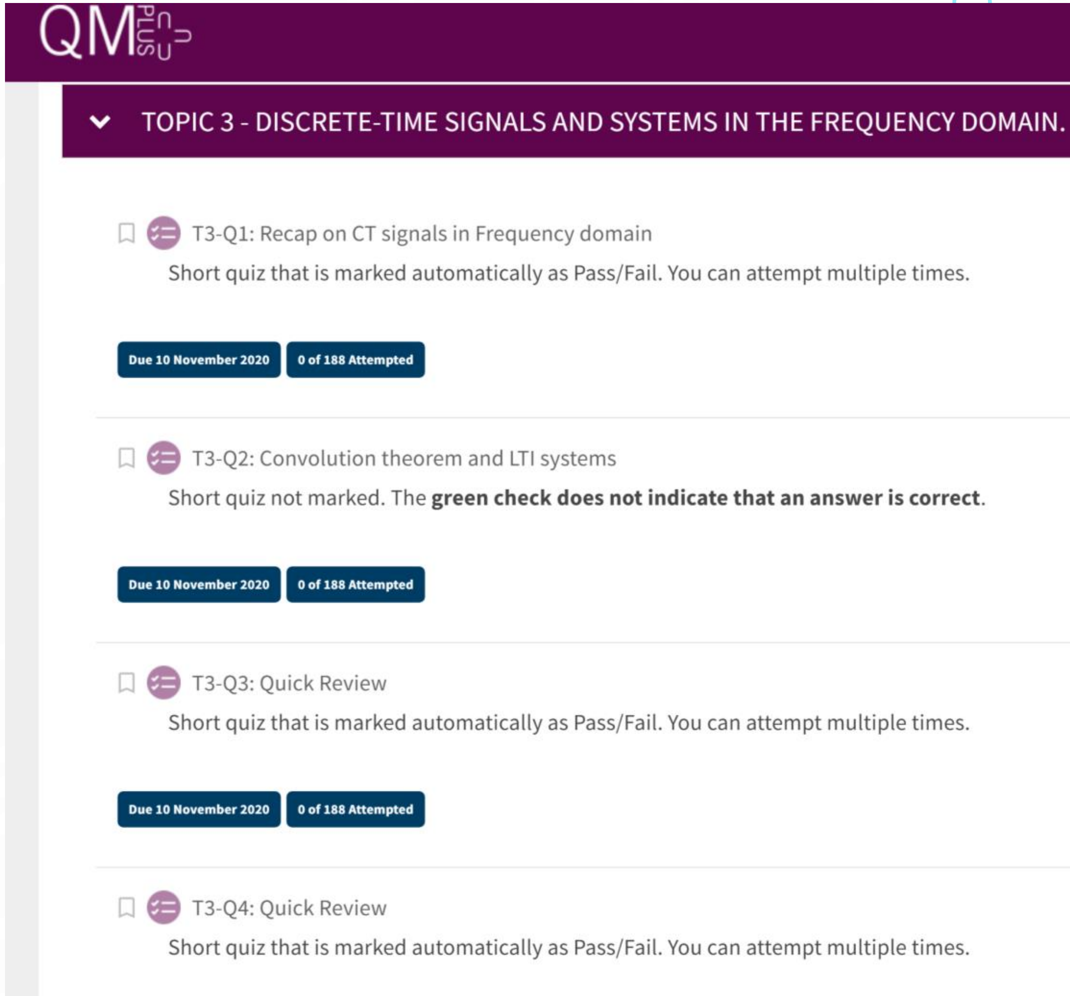
1: WHAT HAVE WE LEARNED SO FAR?

- Please put the recording on hold and login to QM+
- Go to Topic 3
- Take 10 minutes to answer the questions in T3-Q1
- You can retry as many times as you wish.
- Take 5 minutes to understand your mistakes and discuss with your friends
- You are still unsure? Post your question on the MS Teams channel or QM+ forum.



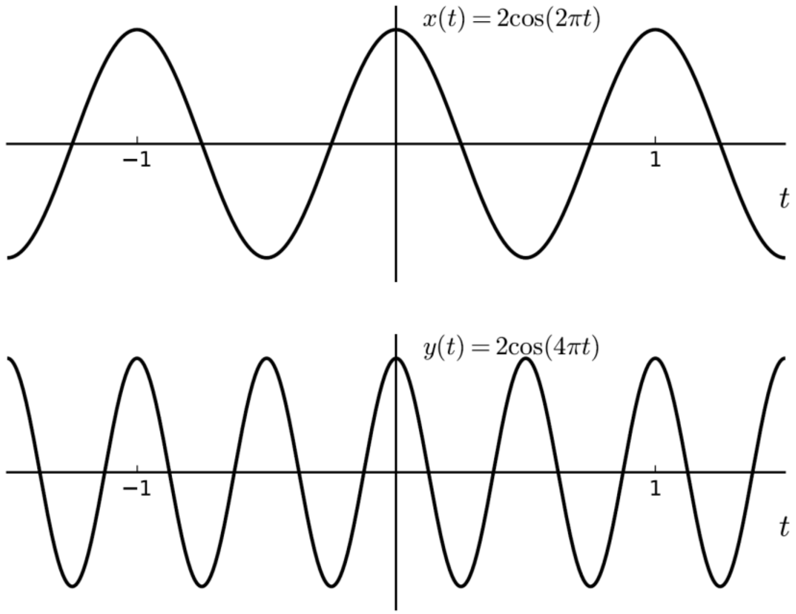
1: The frequency domain and the Fourier transform

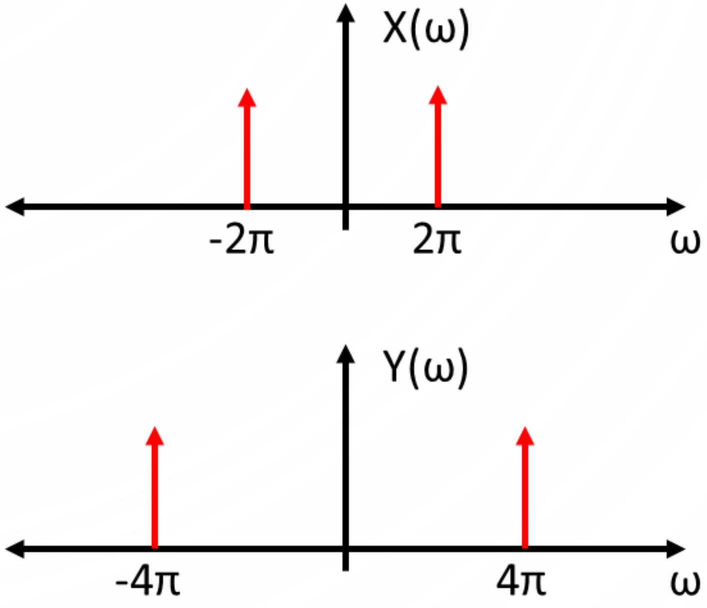
$$x(t) \overset{FT}{\Longleftrightarrow} X(f), X(\omega)$$

The f-domain		The ω-domain
$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$	**Analysis**	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$
$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$	**Synthesis**	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

1: The frequency domain and the Fourier transform

Sinusoidal Signals

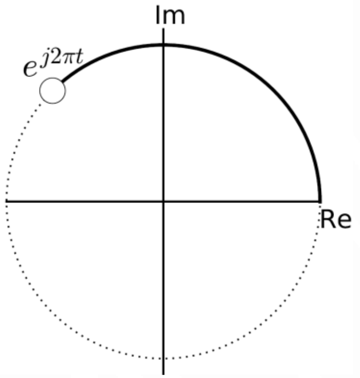


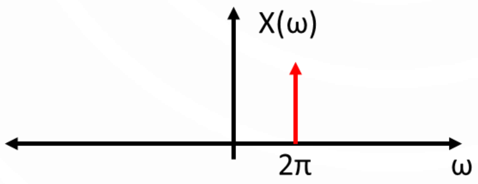


1: The frequency domain and the Fourier transform

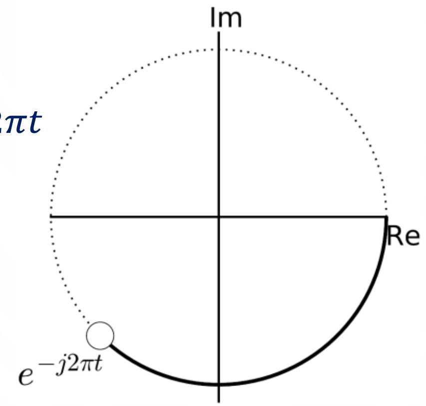
Complex Exponential Signals

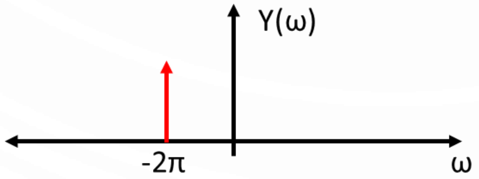
$$x(t) = e^{j2\pi t}$$



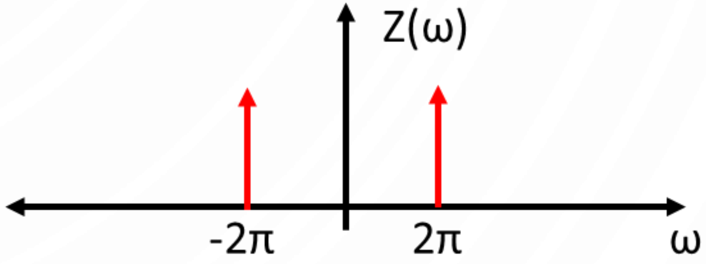


$$y(t) = e^{-j2\pi t}$$





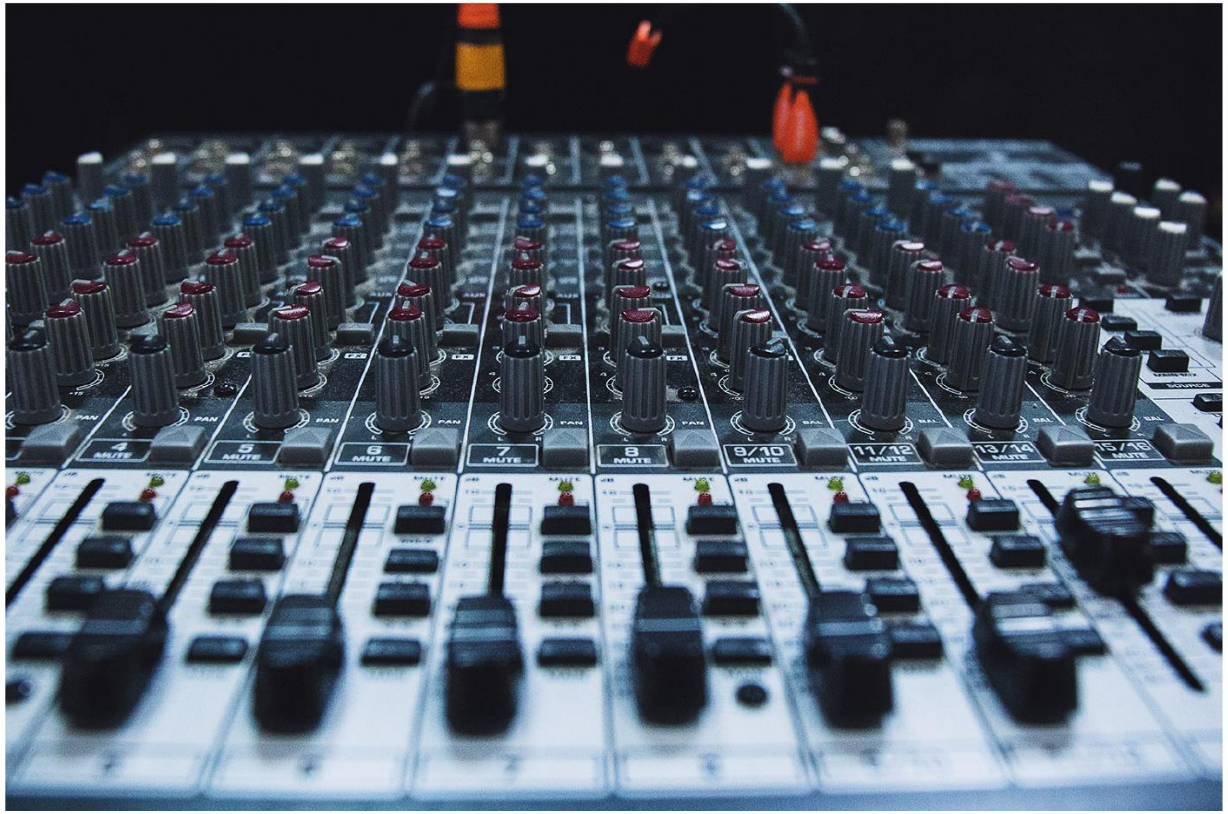
$$z(t) = x(t) + y(t) = e^{j2\pi t} + e^{-j2\pi t} = 2\cos 2\pi t$$



AGENDA

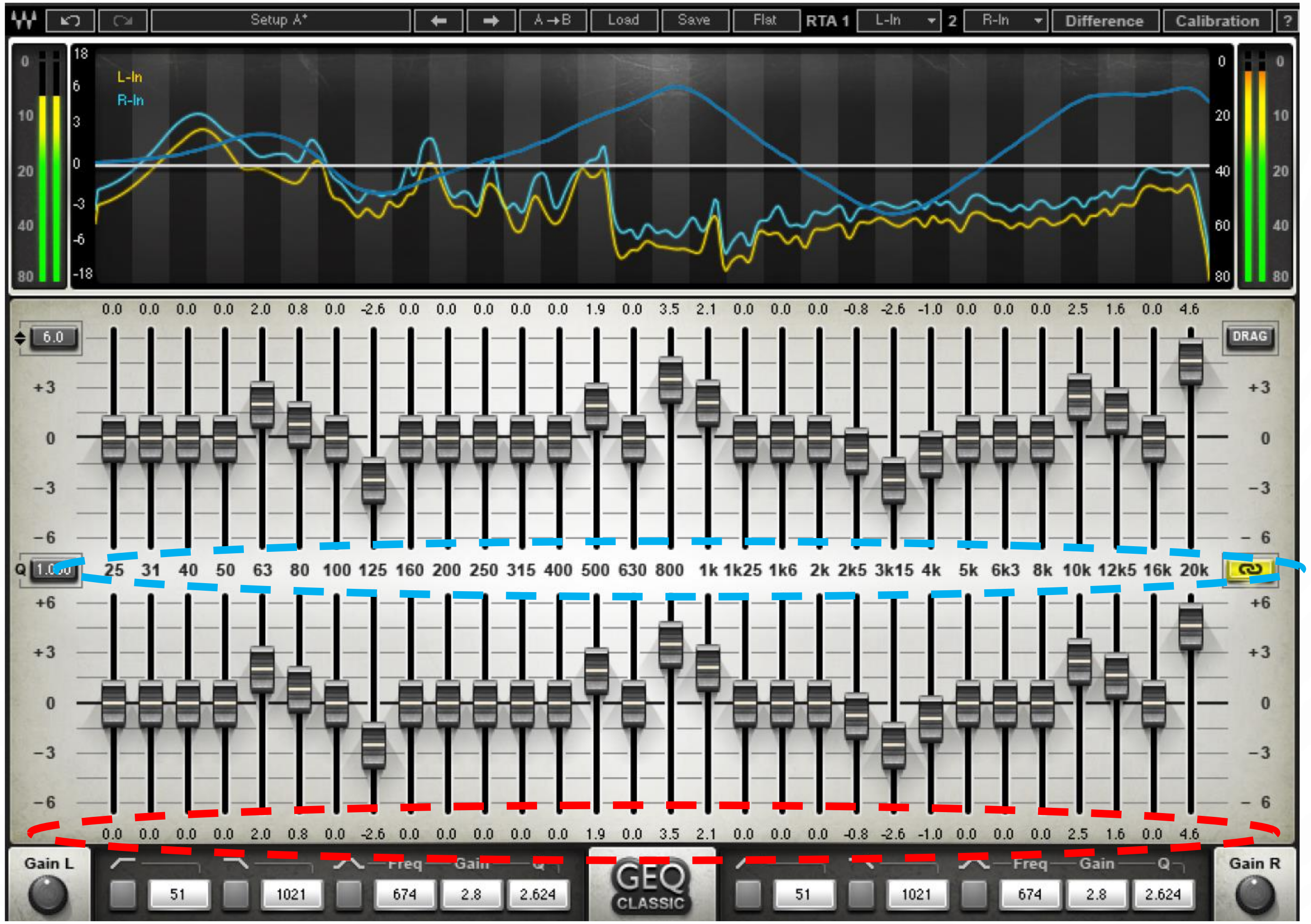
1. Introduction
2. The Frequency domain….What for?
3. The convolution theorem
4. Introducing ***Filters***

2: WHY FREQUENCY DOMAIN?



2: APPLICATION: DIGITAL EQUALISER

This is level of attenuation or amplification per central frequency



Humans can hear sound signals from 20Hz to 20kHz

Q is often used to refer to the bandwidth of frequencies around the central to fe affected

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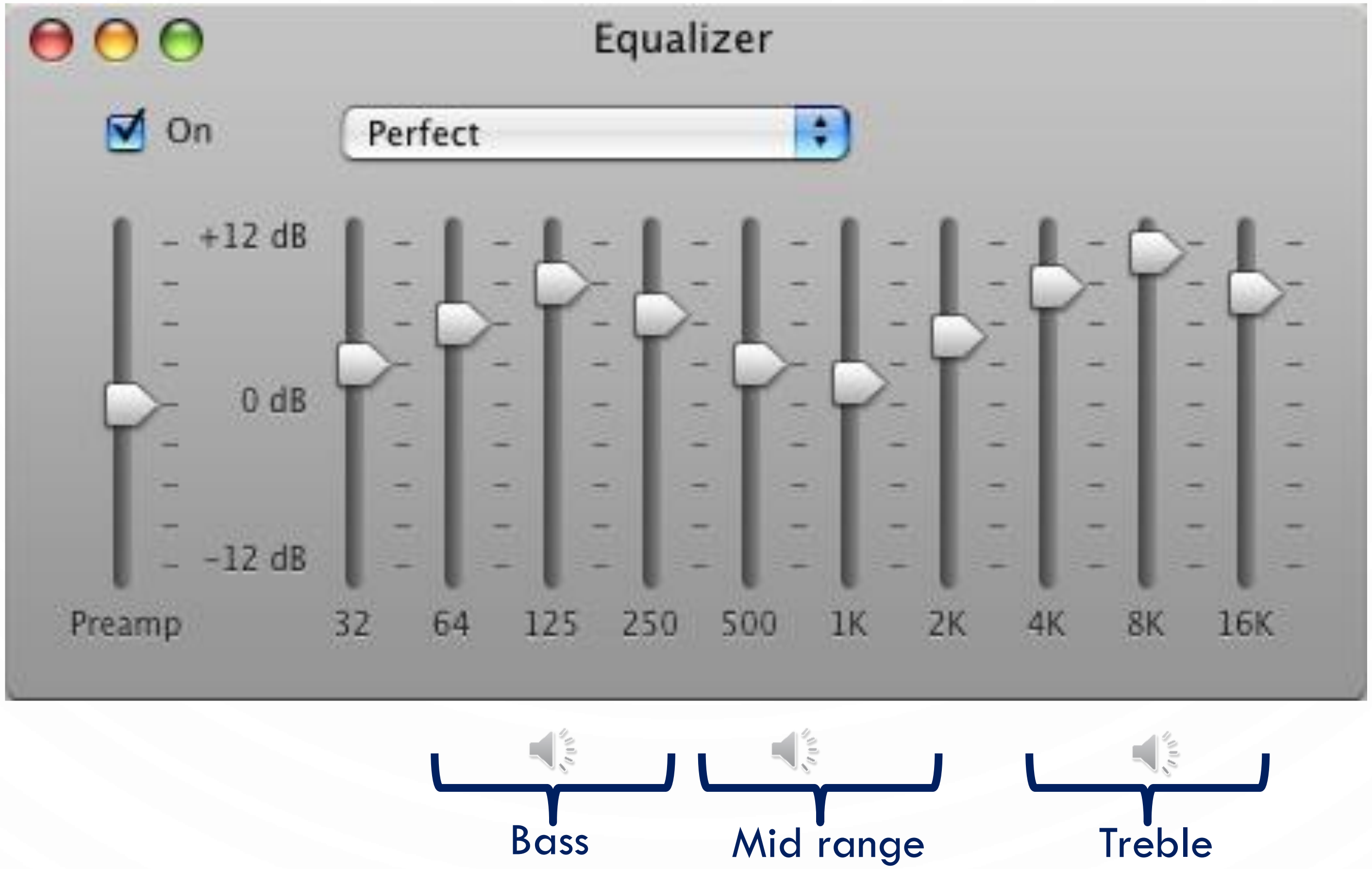
2: APPLICATION: DIGITAL EQUALISER (SIMPLIFIED)



2: OTHER APPLICATIONS

- By looking at **the frequency domain we can extract useful information**.
  - Because many natural phenomena are **cyclic**
  - (electromagnetic radiation, movement of planets, circadian rhythms. . .).
- It can be easier to understand **signal distortions caused by physical media** in the frequency domain.
  - Because media can often be described as linear and time-invariant => can be **modeled as LTI system**.
- **Modulation techniques** for transmitting data can be best understood in the frequency domain.
- **Signal processing techniques** can be best understood in the frequency domain.

2: APPLICATION- INTERNET OF THINGS

IoT devices:

- **Measure and process physical signals**, and information might be more apparent in the frequency domain.
- **Digitise physical signals**, and the process of digitisation can be best understood in the frequency domain.
- **Transmit information** by using modulation techniques and they can be understood in the frequency domain.

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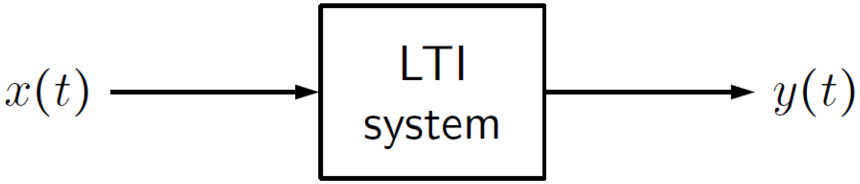
3: LTI SYSTEMS

Linear Time Invariant systems are defined by two basic properties:

1. **Linearity**: Combinations of inputs produce combinations of their outputs.

2. **Time invariance**: Delayed inputs produce delayed outputs.

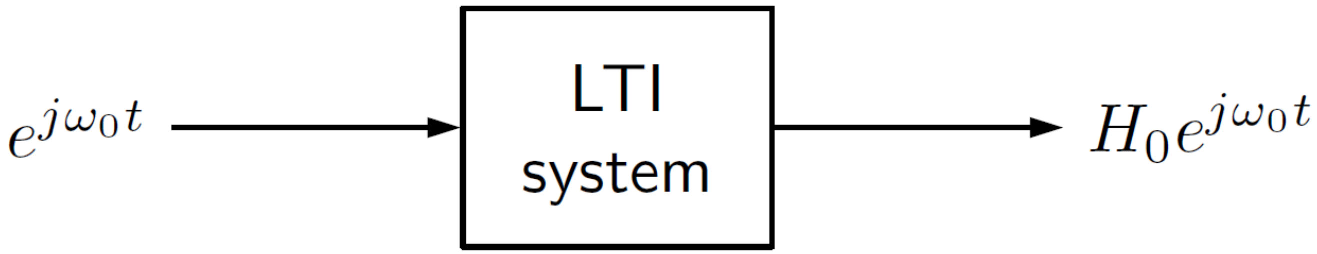


$$\begin{aligned} x_1(t) &\rightarrow y_1(t) \\ x_2(t) &\rightarrow y_2(t) \end{aligned}$$

$$\begin{aligned} A_1 x_1(t) + A_2 x_2(t) &\rightarrow A_1 y_1(t) + A_2 y_2(t) \\ x_1(t - t_0) &\rightarrow y_1(t - t_0) \end{aligned}$$

3: LTI SYSTEMS WITH COMPLEX EXPONENTIALS

A pure frequency ω at the input produces the same pure frequency ω at the output (with different amplitude and phase):



$$
\begin{aligned}
e^{j\omega_0 t} &\longrightarrow H_0 e^{j\omega_0 t} \\
e^{j\omega_1 t} &\longrightarrow H_1 e^{j\omega_1 t} \\
&\vdots \\
e^{j\omega t} &\longrightarrow H(\omega) e^{j\omega t}
\end{aligned}
$$

$$A_0 e^{j\omega_0 t} + A_1 e^{j\omega_1 t} \longrightarrow$$

3: LTI SYSTEMS WITH COMPLEX EXPONENTIALS

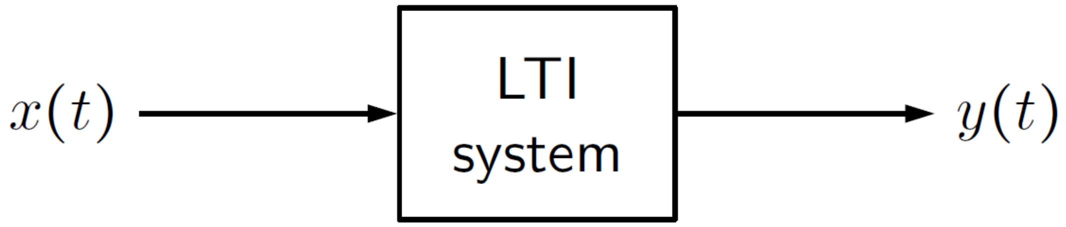
A pure frequency ω at the input produces the same pure frequency ω at the output (with different amplitude and phase):

$$e^{j\omega_0 t} \longrightarrow \boxed{\text{LTI system}} \longrightarrow H_0 e^{j\omega_0 t}$$

$$\begin{array}{ccc} e^{j\omega_0 t} & \longrightarrow & H_0 e^{j\omega_0 t} \\ e^{j\omega_1 t} & \longrightarrow & H_1 e^{j\omega_1 t} \\ & \vdots & \\ e^{j\omega t} & \longrightarrow & H(\omega) e^{j\omega t} \end{array}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \longrightarrow$$

3: LTI SYSTEMS WITH GENERAL SIGNALS

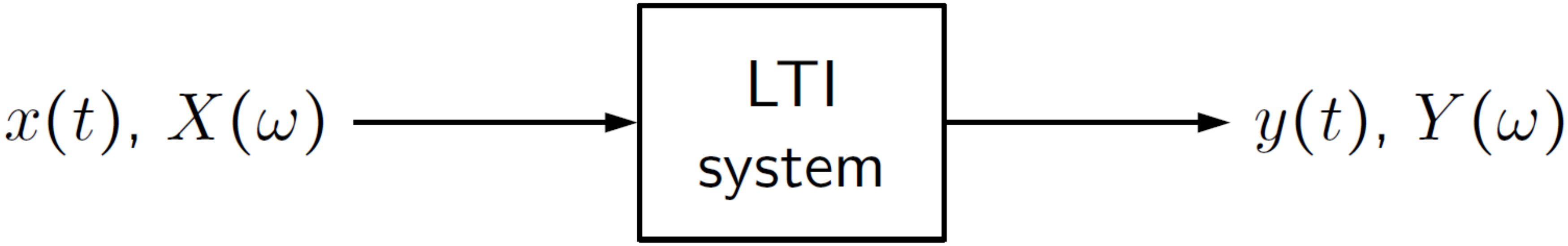


$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \longrightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)H(\omega)e^{j\omega t}d\omega$$

$$y(t) \overset{FT}{\Longleftrightarrow} Y(\omega) = X(\omega)H(\omega)$$

H(ω) is the frequency response or transfer function of the LTI system

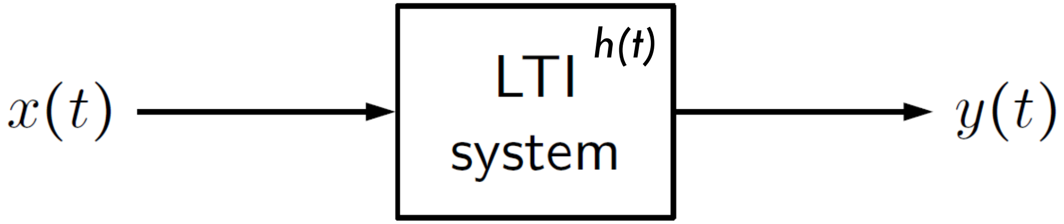
3: LTI SYSTEMS WITH GENERAL SIGNALS



$$y(t) = x(t) \star h(t) \overset{FT}{\Longleftrightarrow} Y(\omega) = X(\omega)H(\omega)$$

Is there a relation between H(ω) and h(t)? What could that be?

3: CONVOLUTION THEOREM



- Please put the recording on hold.
- Question: What is the Fourier transform of a convolution? Consider y(t) = x(t) * h(t).
- Calculate the Fourier transform of y(t). (take 5 minutes)

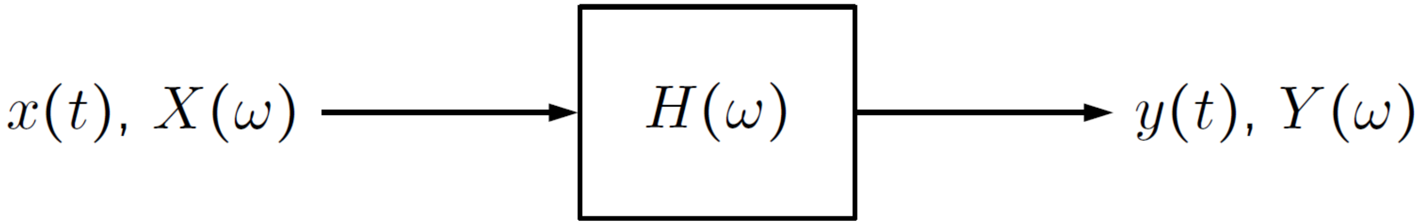
$$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt$$

- Now, login to QM+ and go to Topic 3
- Take 5 minutes to answer the questions in T3-Q2

3: CONVOLUTION THEOREM

$$
\begin{aligned}
Y(\omega) &= \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt \\
&= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \right] e^{-j\omega t}dt \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau)h(t-\tau)e^{-j\omega(t-\tau)}e^{-j\omega\tau}d\tau dt \\
&= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)e^{-j\omega(t-\tau)}dt \right] e^{-j\omega\tau}d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)H(\omega)e^{-j\omega\tau}d\tau \\
&= X(\omega)H(\omega)
\end{aligned}
$$

3: LTI SYSTEMS – A SUMMARY



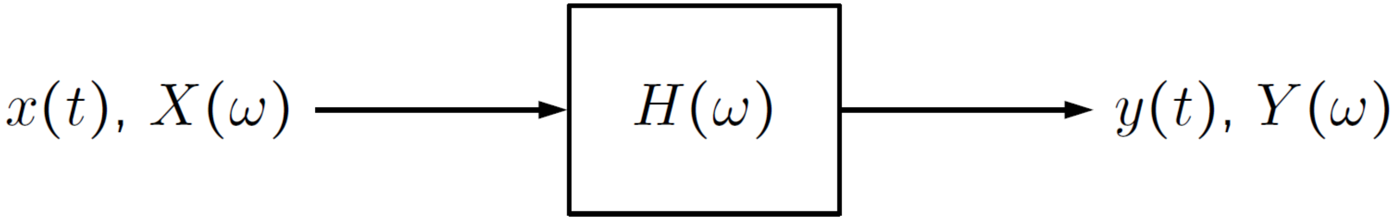
$$y(t) = x(t) \star h(t) \quad \overset{FT}{\Longleftrightarrow} \quad Y(\omega) = X(\omega)H(\omega)$$

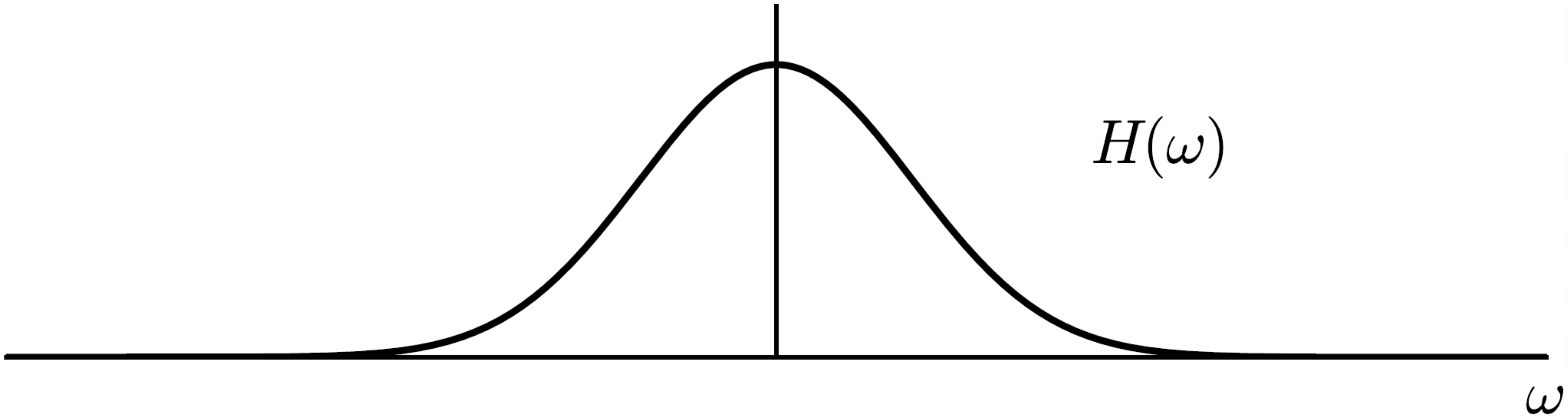
$$x(t) \quad \overset{FT}{\Longleftrightarrow} \quad X(\omega)$$

$$h(t) \quad \overset{FT}{\Longleftrightarrow} \quad H(\omega)$$

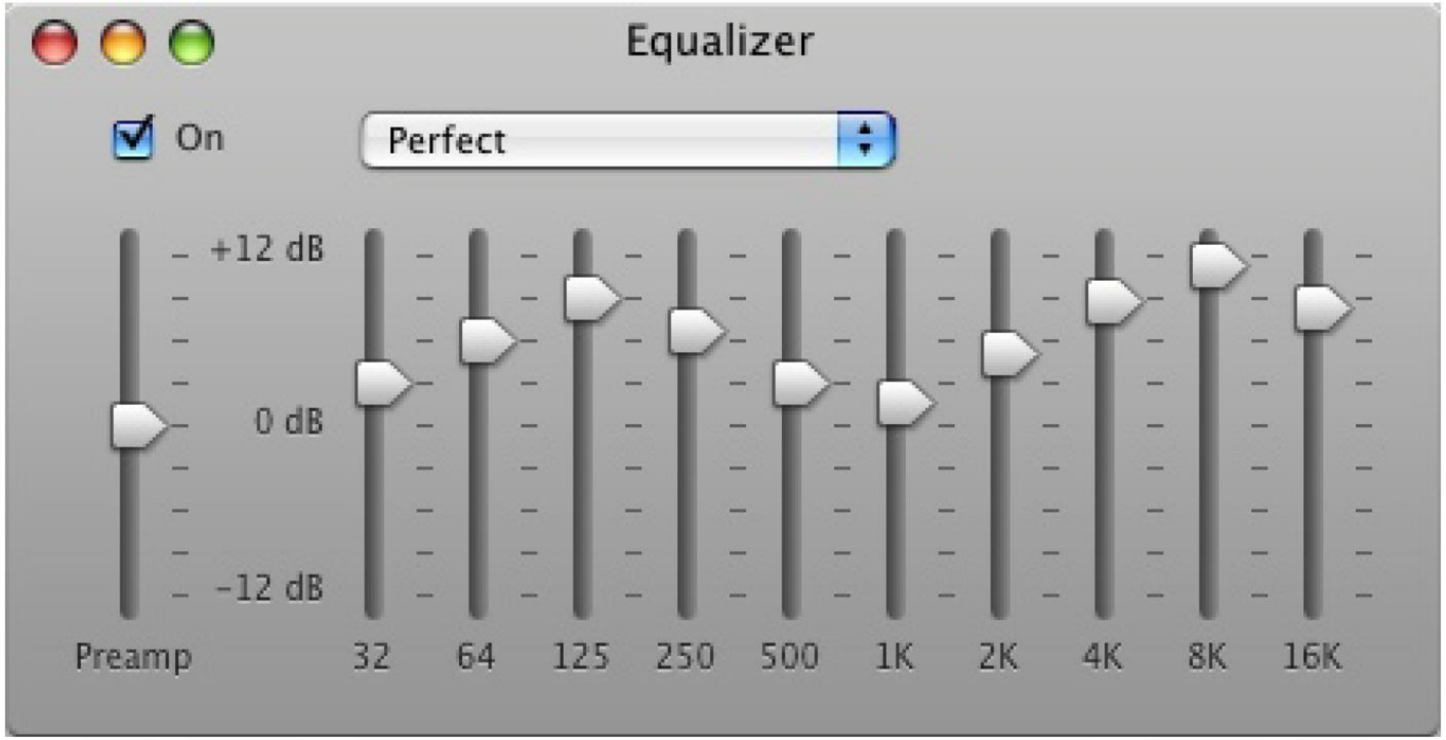
$$y(t) \quad \overset{FT}{\Longleftrightarrow} \quad Y(\omega)$$

3: FREQUENCY RESPONSE- WHAT DOES IT TELL US?





3: FREQUENCY RESPONSE- WHAT DOES IT TELL US?



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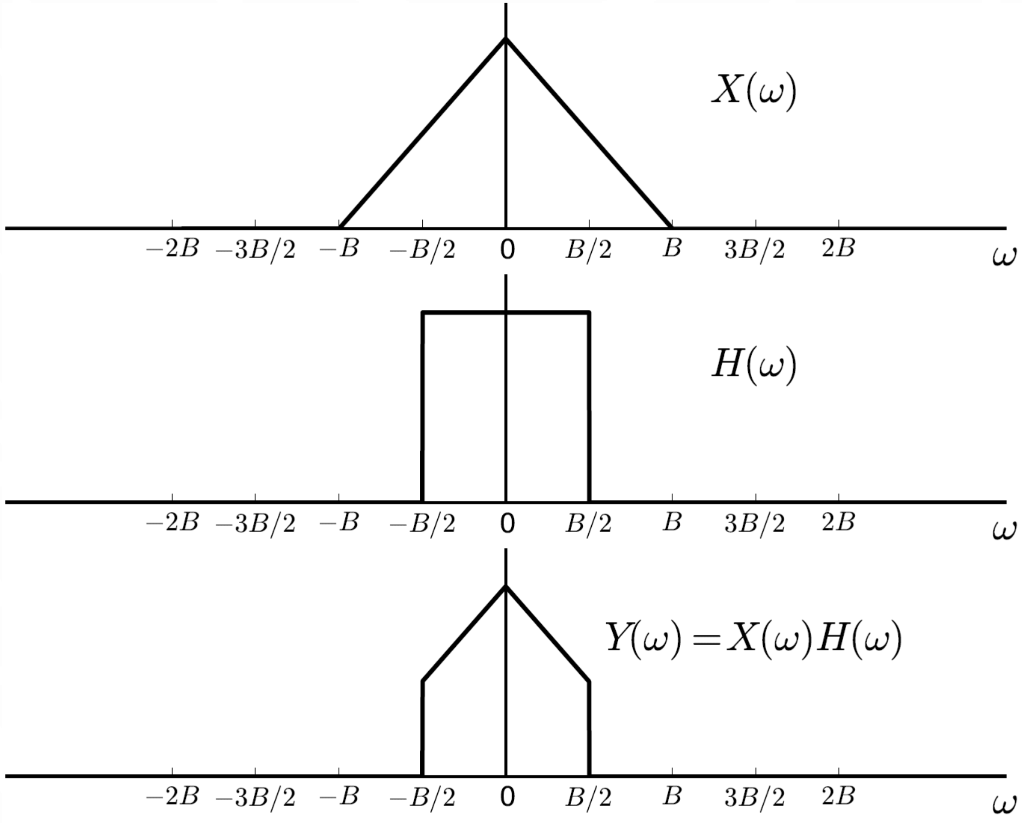
LTI SYSTEMS AS FILTERS

The frequency response H(ω) shows that LTI systems act as frequency filters since they allow certain frequencies at the input to pass whereas they stop other frequencies.

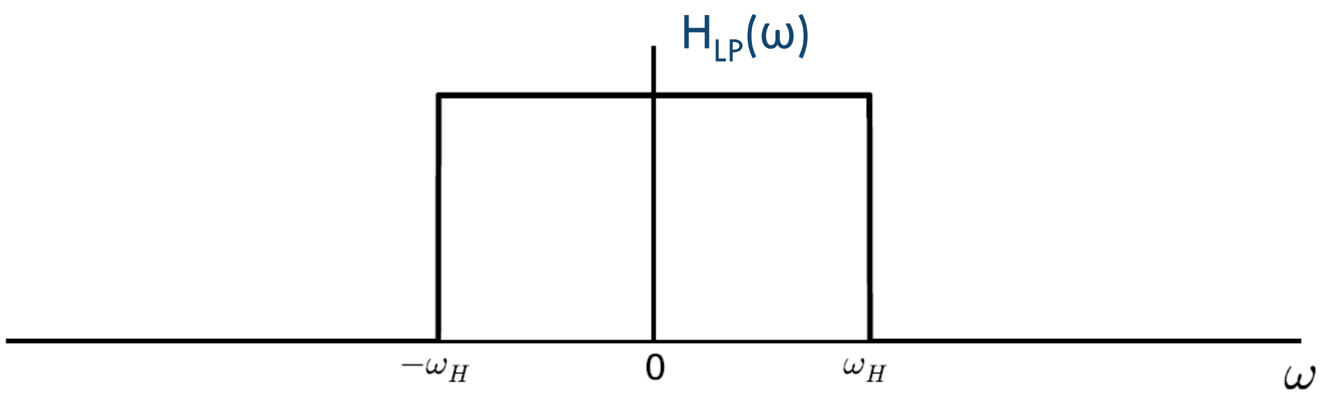


LTI SYSTEMS AS FILTERS

- The frequency response of LTI systems are characterised by
  - The **stopband**: interval of frequencies that are not allowed to pass.
  - The **passband**: interval of frequencies that are allowed to pass.
  - A **bandwidth**: width of the passband (ONLY POSITIVE FREQUENCIES ARE CONSIDERED).

- There are three basic types of filters:
  - **Lowpass** filters: Low frequencies pass.
  - **Highpass** filters: High frequencies pass.
  - **Bandpass** filters: Frequencies within an intermediate band pass.

4: IDEAL FILTERS – LOW PASS



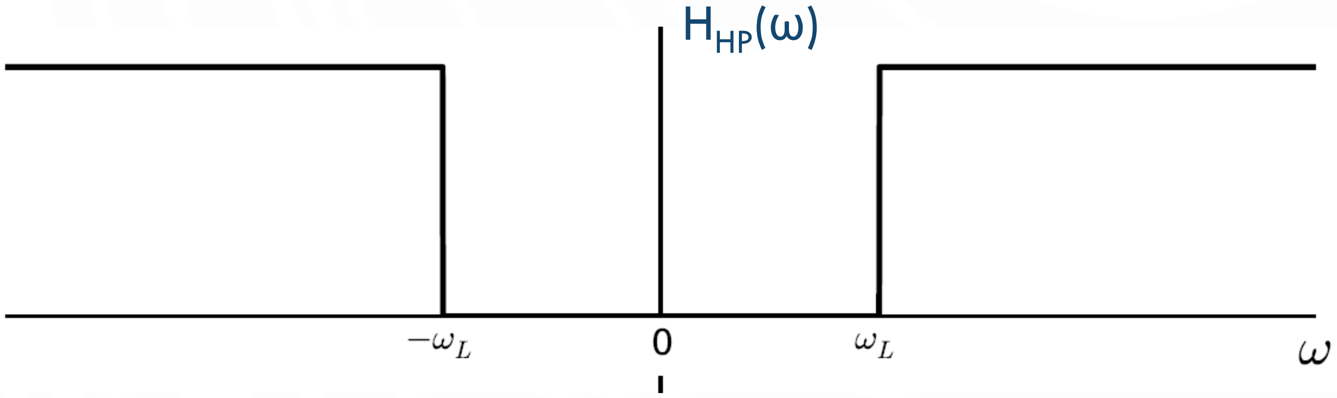
Lowpass filter:

$$H_{LP}(\omega)$$

$$B_{LP} = \omega_H - 0 = \omega_H$$

- $H_{LP}(\omega)$ is the frequency response
- ω_H is the upper cut off frequency
- B_{LP} is the bandwidth

4: IDEAL FILTERS – HIGH PASS



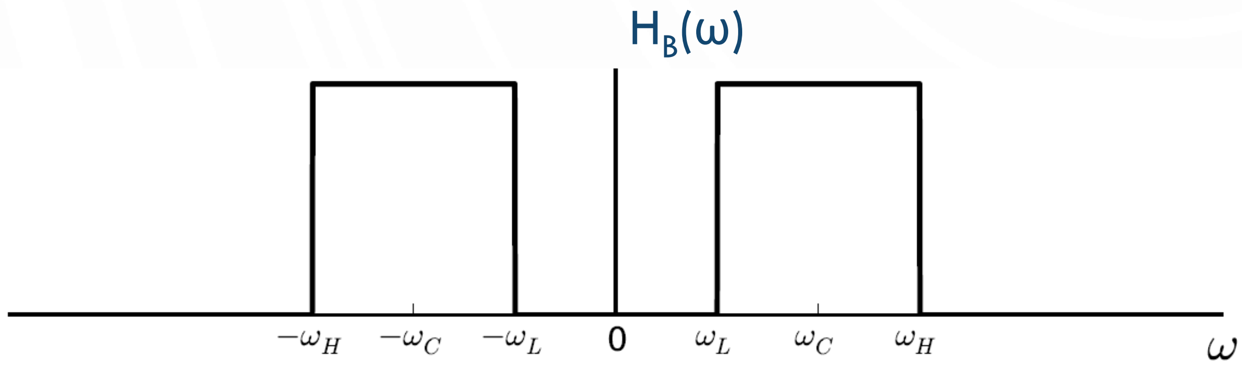
Highpass filter:

$$H_{HP}(\omega)$$

$$B_{HP} = \infty - \omega_L = \infty$$

- $HH_P(\omega)$ is the frequency response
- ω_L is the lower cut off frequency
- B_{HP} is the bandwidth

4: IDEAL FILTERS – BAND PASS



Bandpass filter:

$$H_{BP}(\omega)$$

$$B_{BP} = \omega_H - \omega_L$$

- $H_{BP}(\omega)$ is the frequency response
- ω_H is the cut off frequency
- ω_L is the lower cut off frequency
- B_{BP} is the bandwidth

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BUT! We have only covered CT signals and filters so far!

What happens if the signal and system are Discrete Time?