EBU4375: SIGNALS AND SYSTEMS

LAB2: FOURIER SERIES IN MATLAB

Queen Mary
University of London

ACKNOWLEDGMENT

These slides are partially from Labs prepared by

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YOUR TASKS

- BEFORE THE LAB:
  - Read the slides carefully.
  - Create a ID_FS.txt file where ID is your QMUL ID number, F is the first letter of your forename and S is the first letter of your surname.
  - Type all the code in a red frame in the ID_FS.txt file and submit to the QMplus link.
- DURING THE LAB:
  - Copy/paste the code from ID_FS.txt into Matlab command window as required- indicated by
  - Take note of the results and your answers to questions indicated by
- **Make sure you do the work yourself as there will be questions in the class tests and exam related to Matlab.**

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BACKGROUND

According to Fourier Theory, a periodic CT signal x(t) with period T can be expressed as a linear combination of harmonically related complex exponentials:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Where:

- ω_0 is the fundamental frequency.
- k = 0;±1;±2 : : :
- $k\omega_0$ is the k-th harmonic frequency.
- a_k is the Fourier coefficient of the k-th harmonic frequency.

This expression is called the synthesis equation.

BACKGROUND

Given a periodic CT signal x(t), its Fourier series representation is obtained as follows:

1. Identify its period T.
2. Identify its fundamental frequency ω_0 and its harmonics $k\omega_0$.
3. Calculate the Fourier coefficient a_k by using the **analysis equation:**

$$a_k = \int_T x(t)e^{-jk\omega_0 t}dt$$

4. Express x(t) as a linear combination of harmonic frequencies (synthesis equation).

BACKGROUND

For instance, take the periodic square wave defined over one period T as:

$$x(t) = \begin{cases} 1 & |t| < \tau/2 \\ 0 & \tau/2 < |t| < T/2 \end{cases}$$

1. Its period is by definition T, and $\frac{\tau}{T}$ is the duty cycle
2. Its fundamental frequency $\omega_0 = 2\pi/T$, its harmonics $\omega_\mathrm{k} = k2\pi/T$.
3. The Fourier coefficients are: $a_k = \begin{cases} \frac{\tau}{T} & k = 0 \\ \frac{\sin(k\pi\tau/T)}{k\pi} & k \neq 0 \end{cases}$
4. The periodic square wave can be expressed as:

$$x(t) = \frac{\tau}{T} + \sum_{k=-\infty,k\neq 0}^{\infty} \frac{\sin(k\pi\tau/T)}{k\pi} e^{jk\omega_0 t} \text{ or } x(t) = \frac{\tau}{T} + \sum_{k=-\infty,k\neq 0}^{\infty} \frac{\sin(k\tau\omega_0/2)}{k\pi} e^{jk\omega_0 t}$$

STEP 1- Definition of periodic CT signal

$$x(t) = \begin{cases} 1 & |t| < \tau/2 \\ 0 & \tau/2 < |t| < T/2 \end{cases}$$

- Your job now is to identify the values of τ and T based on your QMUL ID number and your BUPT ID number, such as:
  - T = last digit of QMUL ID number (if last digit=0, T=10)
  - If last digit of BUPT ID number is even, τ =T/2, else τ =T/3
- Identify the fundamental frequency ω_0
- Draw on a piece of paper three periods of x(t).
- Write down the Fourier series representation of x(t), identifying the value of its Fourier coefficients and its harmonic frequencies.

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OBJECTIVE OF THE LAB

In this lab, we will use Matlab to define and plot CT signals. Specifically, we will work with periodic CT signals defined as Fourier series. We will:

1. Obtain the Fourier series of a periodic square wave with period T.
2. Define the time vector corresponding to three periods of the periodic square wave.
3. Define and plot individual harmonic components for different harmonic frequencies $\omega_k = k\omega_0$.
4. Synthesise the periodic square wave as a Fourier series by adding harmonic components.

STEP 2- Plotting individual harmonic components

The following lines of code combine the Fourier series components corresponding to k=1 and k=−1 into the sinusoidal signal c1. Then, c1 is plotted. Note that $1i = \sqrt{-1}$. Based on your ID numbers, fill in the missing values in the code.

```
T=?;%TO BE COMPLETED BY YOU
tau=?%TO BE COMPLETED BY YOU
omega0=2*pi/T;
ts =0;
te = 3*T;
dt = 0.001;
t = ts:dt:te; % Variable t denotes time
ek1 = (sin(omega0*tau/2)/(pi))*exp(1i*omega0*t); % k=1 exp component
ekn1= (sin(-omega0*tau/2)/(-pi))*exp(-1i*omega0*t); % k=-1 exp component
c1=ek1+ekn1; % k=1 sinusoidal component
figure
plot(t,c1) % plots x agains n
xlabel('t') % adds text below the X-axis
ylabel('c1(t)') % adds text beside the Y-axis
```

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STEP 2- Plotting individual harmonic components

Before the LAB, type in your .txt file:

- %Code from LAB1Step2 – QMULID= 191234567 – BUPTID= 191234567
- Then the boxed code below with your values.

```
T=?;%TO BE COMPLETED BY YOU
tau=?%TO BE COMPLETED BY YOU
omega0=2*pi/T;
ts =0;
te = 3*T;
dt = 0.001;
t = ts:dt:te; % Variable t denotes time
ek1 = (sin(omega0*tau/2)/(pi))*exp(1i*omega0*t); % k=1 exp component
ekn1= (sin(-omega0*tau/2)/(-pi))*exp(-1i*omega0*t); % k=-1 exp component
c1=ek1+ekn1; % k=1 sinusoidal component
figure
plot(t,c1) % plots x agains n
xlabel('t') % adds text below the X-axis
ylabel('c1(t)') % adds text beside the Y-axis
```

Question 1:

What is the period of c1(t)?

- τ
- T
- $\tau/2$
- T/2

STEP 3- Plotting individual harmonic components

Before the LAB, type in your .txt file:

- %Code from LAB1Step3 – QMULID= 191234567 – BUPTID= 191234567
- Then the boxed code below with your values for c2(t) defined as the combination of the Fourier series components of x(t) corresponding to k = 2 and k = −2.

```
T=?;%TO BE COMPLETED BY YOU
tau=?%TO BE COMPLETED BY YOU
omega0=2*pi/T;
ts =0;
te = 3*T;
dt = 0.001;
t = ts:dt:te; % Variable t denotes time
ek2 = ?; % k=2 exp component
ekn2= ?; % k=-2 exp component
c2=ek2+ekn2; % k=2 sinusoidal component
figure
plot(t,c2) % plots x agains n
xlabel('t') % adds text below the X-axis
ylabel('c2(t)') % adds text beside the Y-axis
```

Question 2:
What is the period of c2(t)?

- τ
- T
- $\tau/2$
- T/2

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STEP 4- Plotting a truncated version of x(t)

Given a time vector t, the following lines of code synthesise truncated versions of x(t). The value K corresponds to the number of harmonic frequencies included in the synthesis equation.

```
T=?;%TO BE COMPLETED BY YOU
tau=?%TO BE COMPLETED BY YOU
omega0=2*pi/T;
ts =0;
te = 3*T;
dt = 0.001;
t = ts:dt:te; % Variable t denotes time
K =?;%TO BE COMPLETED BY YOU
x = zeros(size(t));%define x as zero
x = tau/T; %add the component k=0 of its fourier series
for k=1:K
   ekn = (sin(k*omega0*tau/2)/(k*pi))*exp(1i*k*omega0*t);
   eknn= (sin(-k*omega0*tau/2)/(-k*pi))*exp(-1i*k*omega0*t);
   x=x+ekn+eknn;
End
Gibbs=max(x)-1;%Max(x) gives the overshoot value, 1 is the
maximum value of the rectangular pulse.
figure
plot(t,x) % plots x agains n
title(['Truncated signal for K=',num2str(K)])
xlabel('t') % adds text below the X-axis
ylabel('x(t)') % adds text beside the Y-axis
```

STEP 4- Plotting a truncated version of x(t)

Before the LAB, type in your .txt file:

- %Code from LAB1Step4 – QMULID= 191234567 – BUPTID= 191234567
- Then the boxed code below with your values.
- Generate plots for each value of

  K=1,5,10,50,100

```
T=?;%TO BE COMPLETED BY YOU
T1=?%TO BE COMPLETED BY YOU
omega0=2*pi/T;
ts =0;
te = 3*T;
dt = 0.001;
t = ts:dt:te; % Variable t denotes time
K =?;%TO BE COMPLETED BY YOU
x = zeros(size(t));%define x as zero
x = tau/T; %add the component k=0 of its fourier series
for k=1:K
   ekn = (sin(k*omega0*tau/2)/(k*pi))*exp(1i*k*omega0*t);
   eknn= (sin(-k*omega0*tau/2)/(-k*pi))*exp(-1i*k*omega0*t);
   x=x+ekn+eknn;
End
Gibbs=max(x)-1;%
figure
plot(t,x) % plots x agains n
title(['Truncated signal for K=',num2str(K)])
xlabel('t') % adds text below the X-axis
ylabel('x(t)') % adds text beside the Y-axis
```

Question 3:

Which of the following statement is correct?

- As K increases the frequency of the output signal increases
- As K increases the shape of the output signal looks more like a square
- As K increases the frequency of the output signal decreases
- As K increases the shape of the output signal looks more like a sinusoid

Question 4:

What is the duty cycle for each value of K (in the form DD.dd%):

- K=1
- K=5
- K=10
- K=50
- K=100

TIPS FOR THE LAB

- Prepare well and upload your .txt file before coming to the LAB.
- In the LAB, open you .txt file and copy/paste into the *Command Window*. Make sure you take note of the questions and answers
- If you have any questions during the preparation, please post them on the QMplus forum.
- If you have questions during the LAB please ask the supervisors.

LAB2 DELIVERABLES

1. Pre LAB submission .txt file (Qmplus) NOT MARKED
2. Post LAB Answer sheet submission on QMplus MARKED (10 marks)
3. Post LAB 5-minute video recording in which you explain the code and the results that you obtain (10 marks)

A brief self introduction - your name and QMUL student ID. Your face must be seen in this part. Give a detailed explanation of your code and obtained results. Do not exceed the time nor increase the speed of the recording.