Chapter 9 Trees 树

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9.1 Introduction 简介

Semifinals and finals of classic tennis competition Wimbledon.

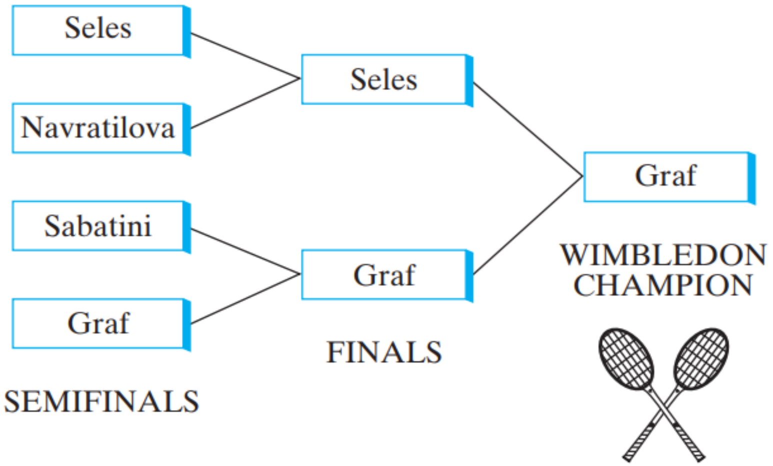

Figure 9.1.1 Semifinals and finals at Wimbledon.

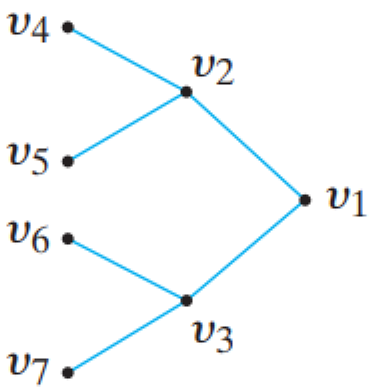

Figure 9.1.2 The tournament of Figure 9.1.1 as a tree.

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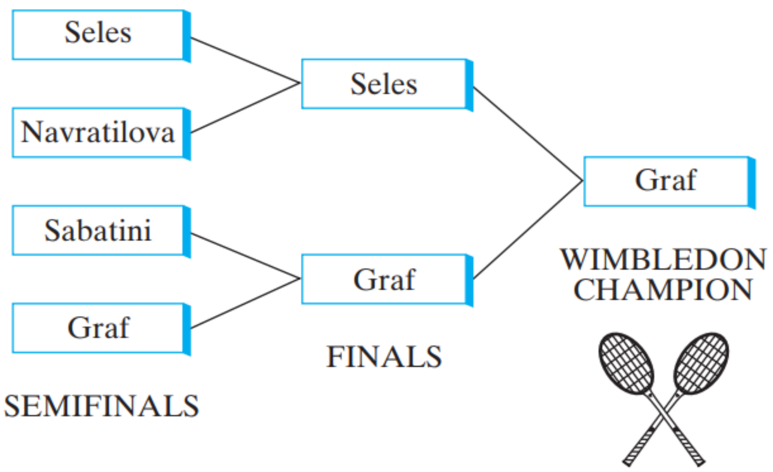

Figure 9.1.1 Semifinals and finals at Wimbledon.

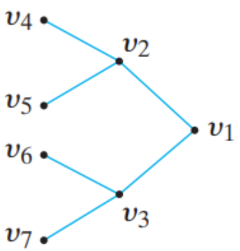

Figure 9.1.2 The tournament of Figure 9.1.1 as a tree.

Figure 9.1.3 The tree of Figure 9.1.2 rotated (a) compared with a natural tree (b).

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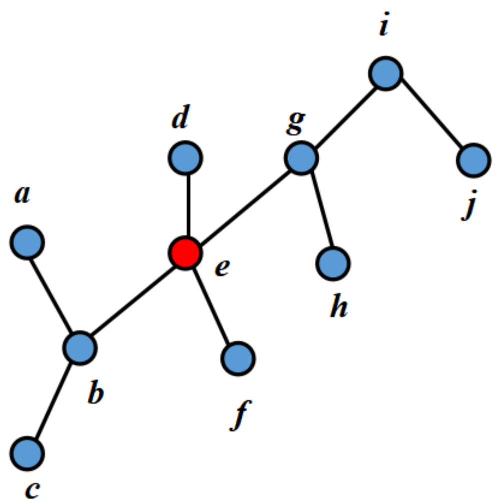
Definition 9.1.1 A **(free) tree (自由树)** T is a simple graph satisfying the following: If v and w are vertices in T, there is a unique simple path from v to w.

A **rooted tree (有根树)** is a tree in which a particular vertex is designated the root.



Designate e as the root in the tree.

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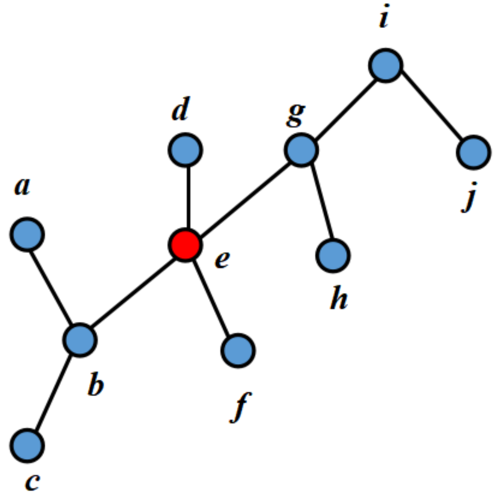
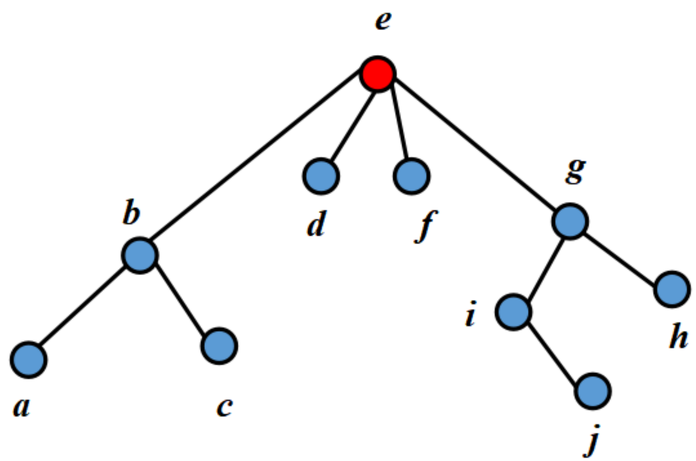
- In graph theory rooted trees are typically drawn with their roots at the top.





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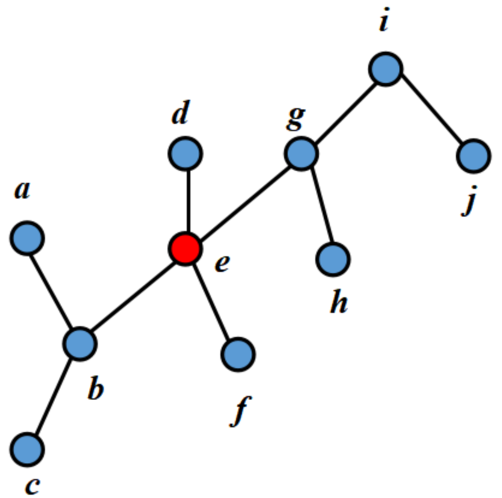

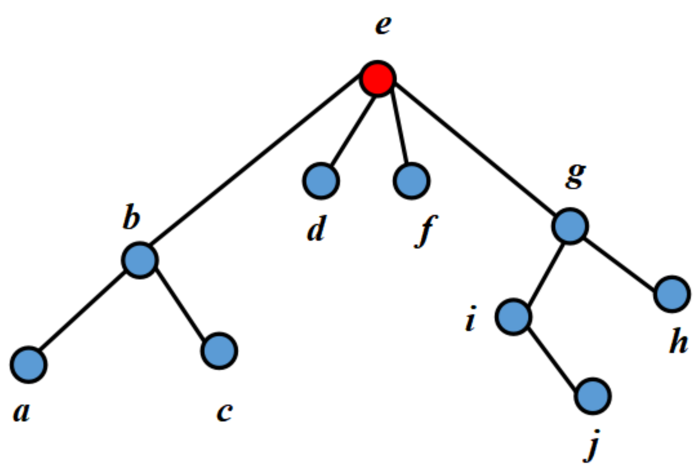

- We call the level of the root level 0.
- The vertices under the root are said to be on level 1, and so on.

9.1 Introduction 简介

The **level of a vertex v (顶点 v 所在的层次)**: the length of the simple path from the root to v.

The **height (高度)** of a rooted tree: the maximum level number that occurs.

A **rooted tree (有根树)** is a tree in which a particular vertex is designated the root.

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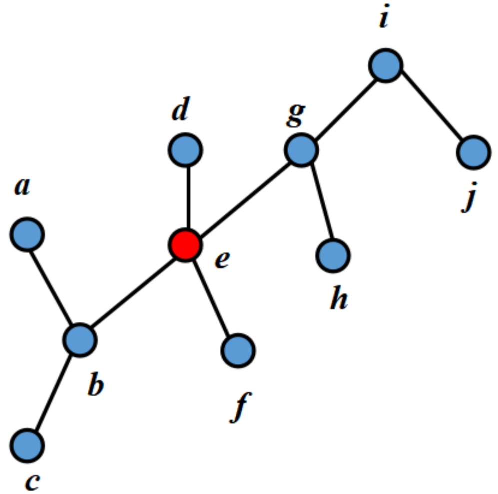

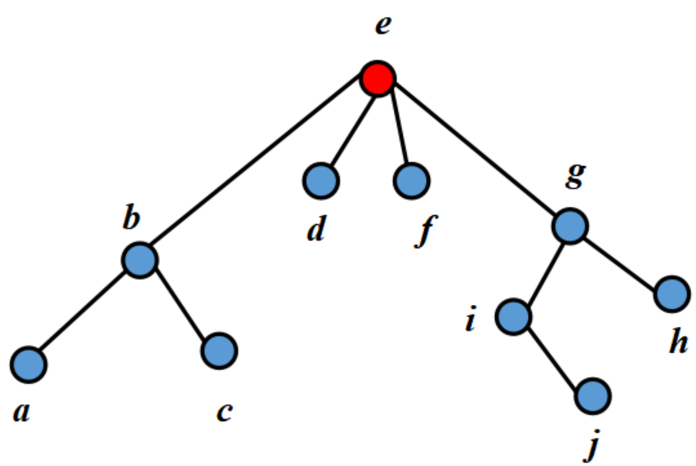

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9.2 Terminology and Characterizations of Trees 树的术语和性质

Definition 9.2.1 Let T be a tree with root v_0. Suppose that x, y, and z are vertices in T and that $(v_0, v_1, \ldots, v_n)$ is a simple path in T. Then

(a) v_{n-1} is the **parent (父节点)** of v_n.

(b) $v_0, \ldots, v_{n-1}$ are **ancestors (祖先节点)** of v_n.

(c) v_n is a **child (子节点)** of v_{n-1}.

(d) If x is an ancestor of y, y is a **descendant (后代节点)** of x.

(e) If x and y are children of z, x and y are **siblings (兄弟节点)**.

(f) If x has no children, x is a **terminal vertex (or a leaf) (终节点/叶节点)**.

(g) If x is not a terminal vertex, x is an **internal (or branch) vertex (中间节点/枝节点)**.

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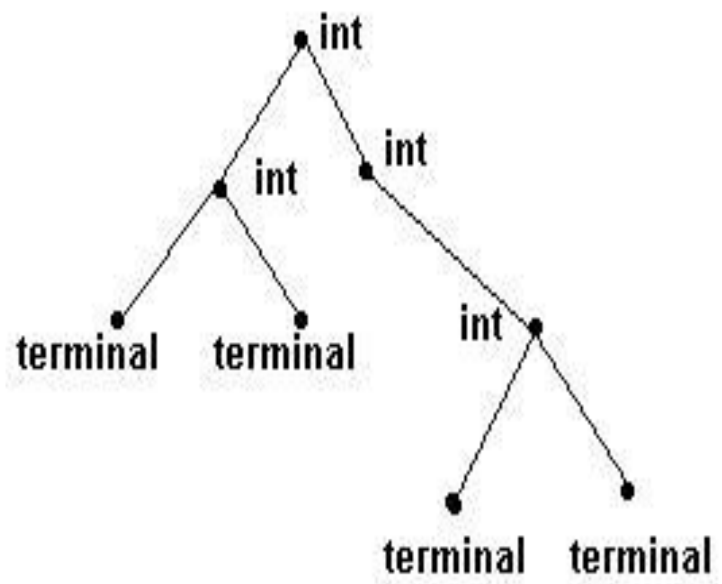

If x is not a terminal vertex, x is an **internal (or branch) vertex (中间节点/枝节点)**.

- An **internal vertex (中间节点)** is a vertex that has at least one child.
- A **terminal vertex (终节点)** is a vertex that has no children

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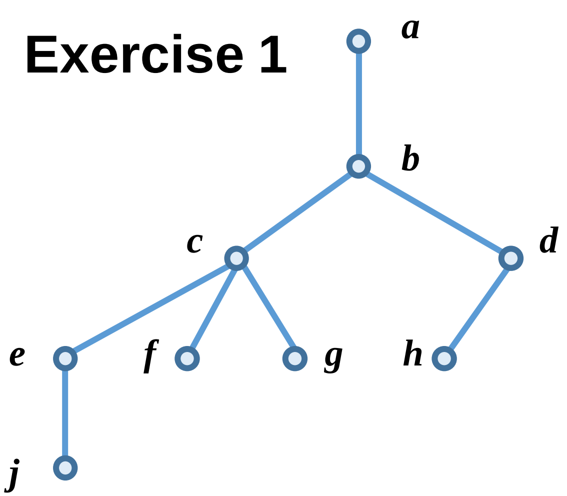
Definition 9.2.1 Let T be a tree with root v_0. Suppose that x, y, and z are vertices in T and that $(v_0, v_1, \ldots, v_n)$ is a simple path in T. Then

(h) The **subtree (子树)** of T rooted at x is the graph with vertex set V and edge set E, where V is x together with the descendants of x and

$E = \{e \mid e$ is an edge on a simple path from x to some vertex in $V\}$.

Exercise 1



Draw the subtree rooted at c.

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Definition 9.1.1 A **(free) tree (自由树)** T is a simple graph satisfying the following: If v and w are vertices in T, there is a unique simple path from v to w.

A tree is connected.

A tree cannot contain a cycle.

A graph with no cycles is called an **acyclic graph (非循环图)**.

Definition 9.2.3 Let T be a graph with n vertices. The following are equivalent.

(a) T is a tree.

(b) T is connected and acyclic.

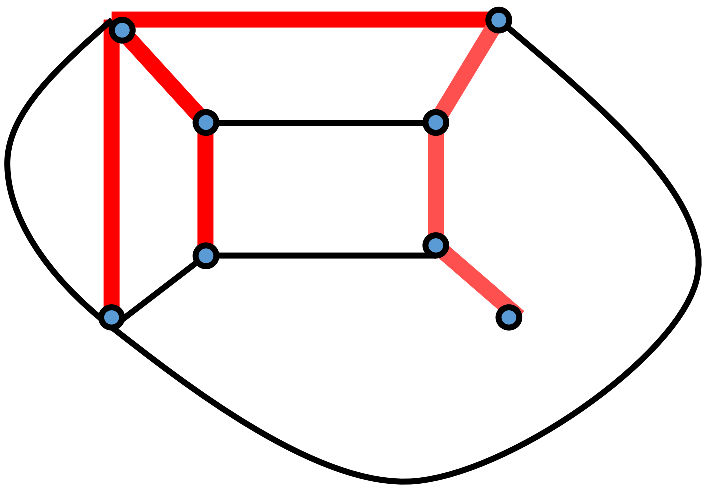
(c) T is connected and has $n-1$ edges.

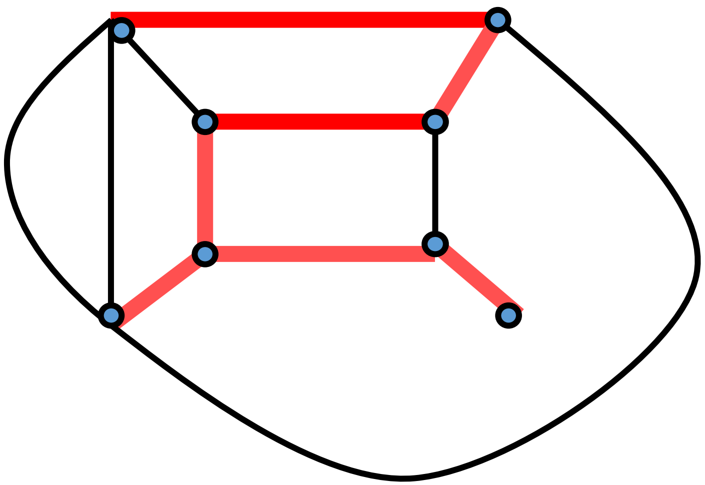
(d) T is acyclic and has $n-1$ edges.

9.3 Spanning Trees 生成树

Definition 9.3.1 A tree T is a **spanning tree (生成树)** of a graph G if T is a subgraph of G that contains all of the vertices of G.

In general, a graph will have several spanning trees.





9.3 Spanning Trees 生成树

Breadth-First Search 广度优先搜索

The idea of breadth-first search is to process all the vertices on a given level before moving to next-higher level.

Depth-First Search 深度优先搜索 → Backtracking 回溯

The idea of depth-first search is to proceeds to successive levels in a tree at the earliest possible opportunity.

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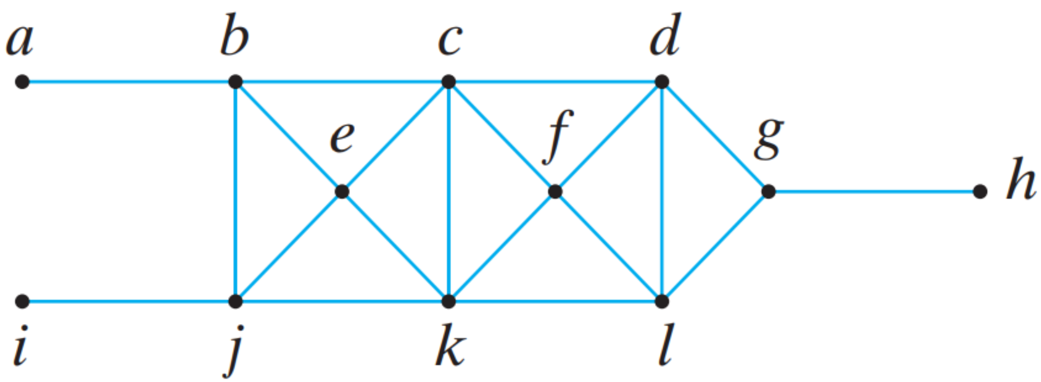
Exercise Find a spanning tree for the graph using Breadth-First Search.



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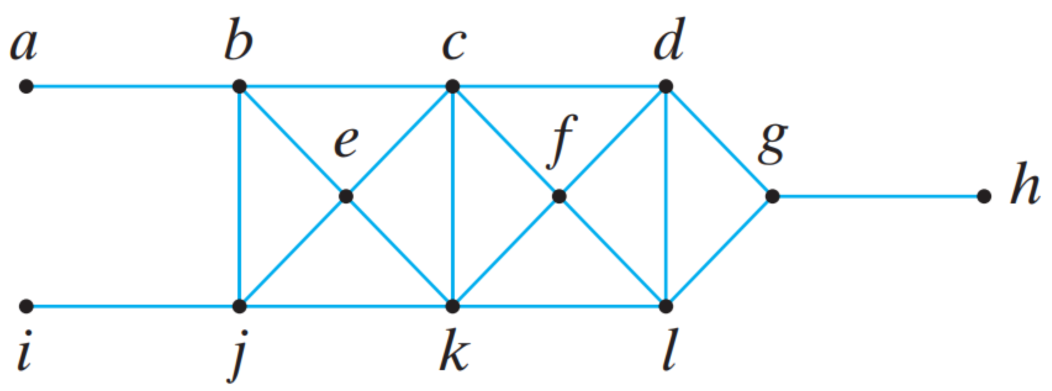
Exercise Find a spanning tree for the graph using Depth-First Search.



9.4 Minimal Spanning Trees 最小生成树

Definition 9.4.1 Given a weighted graph G, a **minimal spanning tree (最小生成树)** of G is a spanning tree of G that has minimum weight.

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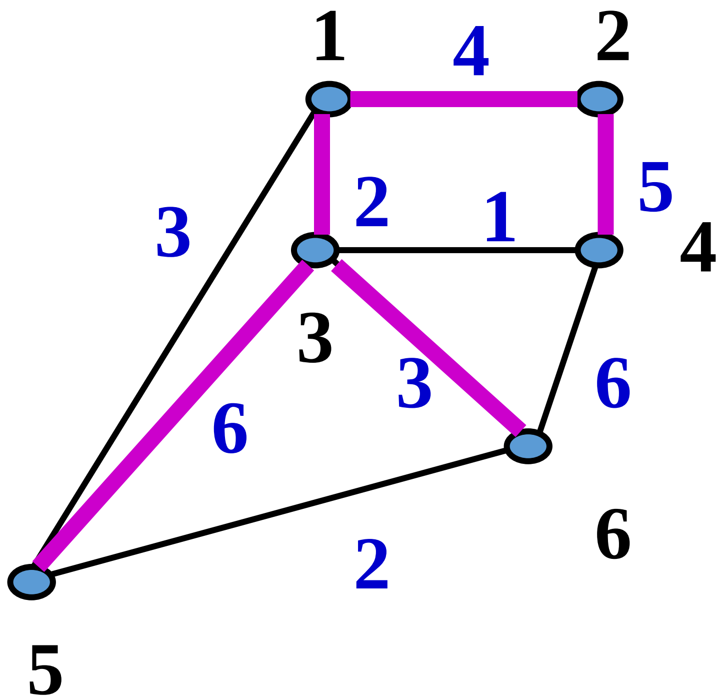
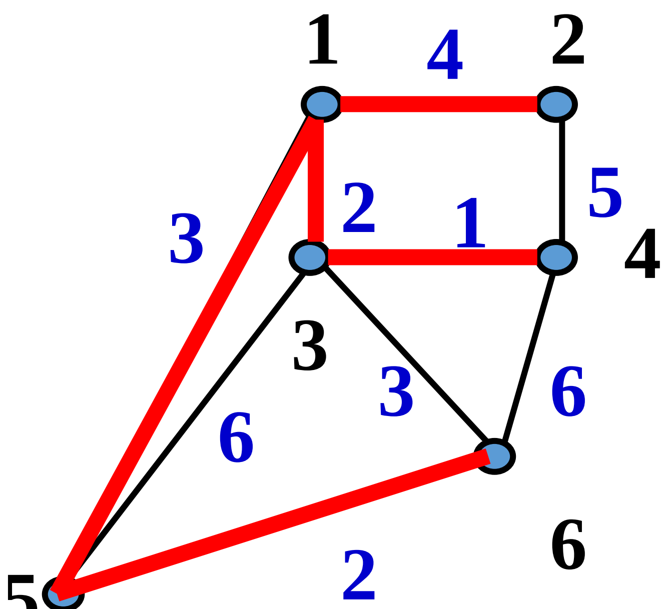


Weight=20



Weight=12

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Prim's Algorithm 普里姆算法
The algorithm begins with a single vertex. Then at each iteration, it adds to the current tree a minimum-weight edge that does not complete a cycle.

Kruskal's Algorithm 克鲁斯卡尔算法
It is a greedy algorithm in graph theory as in each step it adds the next lowest-weight edge that will not form a cycle to the minimum spanning forest.

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The algorithm begins with a single vertex. Then at each iteration, it adds to the current tree a minimum-weight edge that does not complete a cycle.

Step 0: Pick any vertex as a starting vertex (call it a). $T = \{a\}$.

Step 1: Find the edge with smallest weight incident to a. Add it to T. Also include in T the next vertex and call it b.

Step 2: Find the edge of smallest weight incident to either a or b. Include in T that edge and the next incident vertex. Call that vertex c.

Step 3: Repeat Step 2, choosing the edge of smallest weight that does not form a cycle until all vertices are in T. The resulting subgraph T is a minimum spanning tree.

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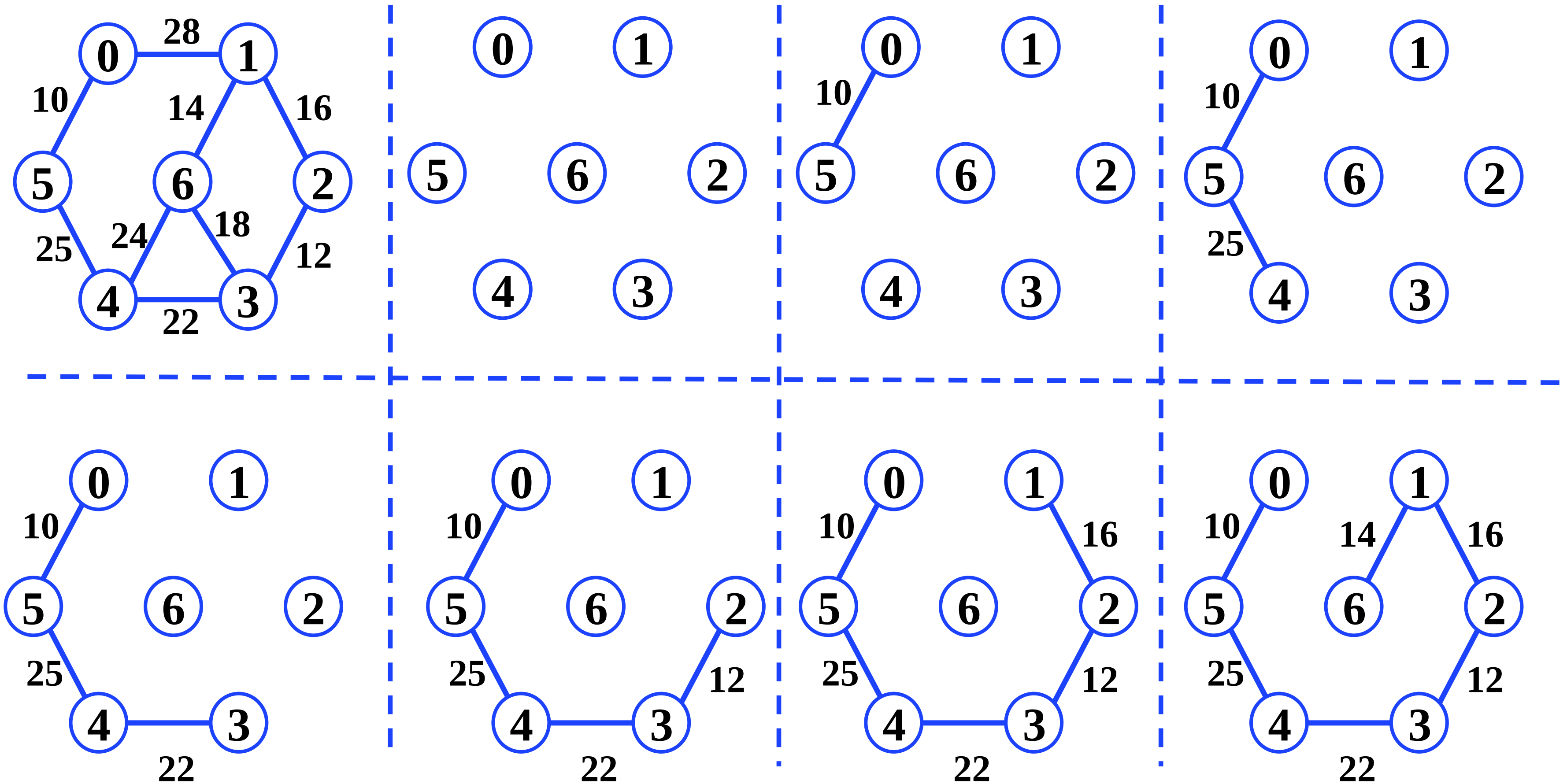

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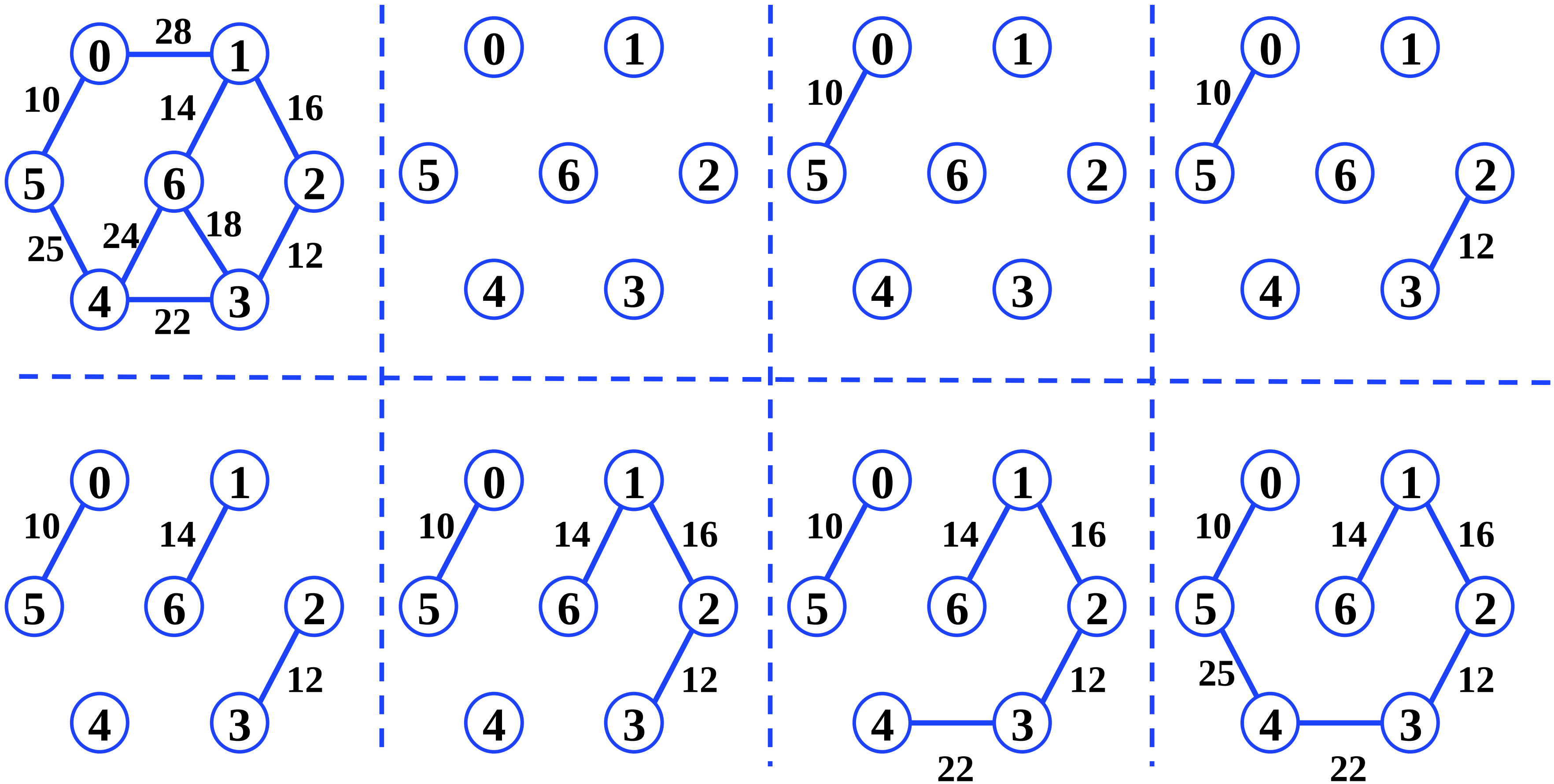
It is a greedy algorithm in graph theory as in each step it adds the next lowest-weight edge that will not form a cycle to the minimum spanning forest.

Step 1: Find the edge in the graph with smallest weight (if there is more than one, pick one at random).

Step 2: Find the next edge in the graph with smallest weight that doesn't close a cycle.

Step 3: Repeat Step 2 until you reach out to every vertex of the graph. The chosen edges form the desired minimum spanning tree.

Kruskal's Algorithm 克鲁斯卡尔算法



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Exercise Use Prim's or Kruskal's Algorithm to find a minimal spanning tree for the following graph.

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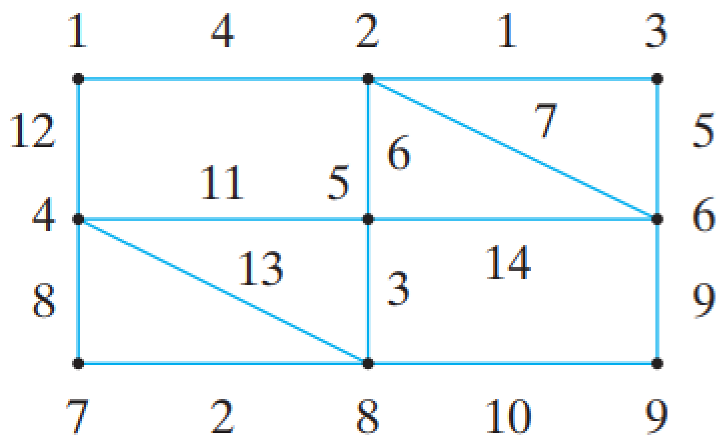

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