HW 3

Example 1.6.4

Solution:

The sample space $\Omega = \{HH, HT, TH, TT\}$

and the events listed are $A=\{HH, HT\}$ $B=\{HH, TH\}$,

$C=\{HH, TT\}$

Since the coin is balanced, that means all outcomes are assigned the same probability, each ~~are~~ equals to $\frac{1}{4}$

$\therefore P(A)=P(B)=P(C)=\frac{1}{2}$

$P(AB)=P(AC)=P(BC)=\frac{1}{4}$ $\quad P(ABC)=P(\{HH\})=\frac{1}{4}$

$\because P(AB)=P(A)\cdot P(B)$ $\quad P(AC)=P(A)\cdot P(C)$ $\quad P(BC)=P(B)\cdot P(C)$

$P(ABC)\neq P(A)P(B)P(C)$

$\therefore$ A, B and C are pairwise independent.

Ex.

1.38

(a) Let event A ~~dento~~ denotes for question a

$P(A)=\frac{6}{6+4}\cdot\frac{8}{8+3}=\frac{3}{5}\cdot\frac{8}{11}=\frac{24}{55}$

(b) Let event B denotes for question b

so the number of r & g are identical to the numbers at the beginning iff the two balls ~~are~~ have same colors

$\therefore P(B)=\frac{24}{55}+\frac{4}{6+4}\cdot\frac{4}{7+4}=\frac{24}{55}+\frac{2}{5}\cdot\frac{4}{11}=\frac{32}{55}$

1.39

We first define event A denotes a people has a disease.

event B denotes the people gain positive outcome from test.

$\therefore P(A)=\frac{1}{2}$ $\quad P(B|\bar{A})=\frac{3}{100}$ $\quad P(\bar{B}|A)=\frac{1}{50}$

~~(a) $P(B)=P(A)\cdot P(B|A)=P(A)\cdot(1-P(\bar{B}|A))$~~

~~$=\frac{1}{2}\cdot\frac{49}{50}=49\%$~~

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$\cancel{(b)\ P(A|B) = \frac{P(AB)}{P(B)} = \frac{}{P(A)\cdot P(B|A) + P(\bar{A})\cdot P(B|\bar{A})}}$

(a) $P(B) = P(A)\cdot P(B|A) + P(\bar{A})\cdot P(B|\bar{A})$

$= P(A)\cdot(1 - P(\bar{B}|A)) + P(\bar{A})\cdot P(B|\bar{A})$

$= \frac{1}{2}\cdot\left(1 - \frac{1}{50} + \frac{3}{100}\right) = \frac{101}{200} = 55.5\%$

(b) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)\cdot P(B|A)}{P(B)} = \frac{49}{100} \Big/ \frac{101}{200} = \frac{98}{101}$

1.45

$\because$ A and B are independent, $P(A) = 1 - P(\bar{A}) = P(\bar{B})$

$\therefore\ P(\bar{A}\bar{B}) = P(\bar{A})\cdot P(\bar{B}) = P(\bar{A})\cdot(P(A)) = P(A)(1-P(A)) = \frac{1}{9}$

$\cancel{\therefore\ P^2(A) - P(A) + \frac{1}{4} = \frac{1}{36} \Rightarrow (P(A) - \frac{1}{2})^2 = (\frac{1}{6})^2}$

$\cancel{\therefore\ P(A) - \frac{1}{2} = \pm\frac{1}{6}}$

$\cancel{\therefore\ P(A) = \frac{2}{3} \text{ or } P(A) = \frac{1}{3}}$

$\therefore\ P^2(A) - P(A) + \frac{1}{9} = 0 \quad \therefore\ P(A) = \frac{1}{2}\cdot\left(1 \pm \sqrt{1 - \frac{4}{9}}\right) = \frac{1}{2}\left(1 \pm \frac{\sqrt{5}}{3}\right)$

$\therefore\ P(A) = \frac{3+\sqrt{5}}{6}$ or $P(A) = \frac{3-\sqrt{5}}{6}$

1.48

Set event A denotes a success on the $i$th trial

event B denotes there are $r$ success in $n$ independent trials.

$P(B) = C_n^r \cdot p^r (1-p)^{n-r}$

$P(B|A)\ \cancel{P(AB)} = C_{n-1}^{r-1}\cdot p^{r-1}(1-p)^{n-r}$

$\cancel{\therefore\ P(A|B) = \frac{P(AB)}{P(B)} = \frac{C_{n-1}^{r-1}}{C_n^r\cdot p} = \frac{C_{n-1}^{r-1}}{\frac{n}{r}C_{n-1}^{r-1} p} = \frac{r}{np}}$

$\therefore\ P(A|B) = \frac{P(A)\cdot P(B|A)}{P(B)} = \frac{p\cdot C_{n-1}^{r-1} p^{r-1}(1-p)^{n-r}}{C_n^r p^r (1-p)^{n-r}} = \frac{C_{n-1}^{r-1}}{C_n^r} = \frac{r}{n}$

~~1.49~~

~~Define event A denotes the event which a player repeated 50 times (toss)~~

~~(a) gain exceeds \$1: P(>\$1) = P(A =~~

1.49

Define event A denotes the winning times of the repeated 50 times toss.

(a) gain exceeds \$1: $P(A \geq 2) = 1 - P(A=0) - P(A=1) = 1 - (\frac{1}{2})^{50} - C_{50}^{1}(\frac{1}{2})^{50}$

$= 1 - 51 \times (\frac{1}{2})^{50}$

~~gain exc~~ lose exceeds \$1: $P(A=50) = (\frac{1}{2})^{50}$

(b) because the player can lose at least $50 \times \frac{1}{50} = 1$ dollar, so he can never lose exceeds \$5.

gain exceeds \$5: $P(A \geq 6) = 1 - \sum_{k=0}^{5} P(A=k)$

$= 1 - (\frac{1}{2})^{50} - C_{50}^{1}(\frac{1}{2})^{50} - C_{50}^{2}(\frac{1}{2})^{50} - \cdots - C_{50}^{5}(\frac{1}{2})^{50}$

1.50

(a) ~~$P(A) = (\frac{1}{2})^3 + (\frac{1}{2})^3 = \frac{1}{4}$~~ ~~$P(B) = (\frac{1}{2})^3 + (\frac{1}{2})^3$~~

$P(A) = \frac{2}{2^3} = \frac{1}{4}$ $P(B) = \frac{4}{2^3} = \frac{1}{2}$ $P(C) = \frac{3+3}{2^3} = \frac{3}{4}$

$P(AB) = \frac{1}{2^3} = \frac{1}{8}$ $P(BC) = \frac{1+1+1}{2^3} = \frac{3}{8}$

$\because P(AB) = P(A)P(B)$ $P(BC) = P(B) \cdot P(C)$

$\therefore$ A and B, B and C are independent.

(b) $P(AC) = 0 \neq P(A) \cdot P(C)$ $\therefore$ A and C are dependent.

(c) No. The ~~trials~~ event and conclusion are based on the binomial distribution. If boy or girl doesn't have the same probability, the conclusion will not be correct.

(d) No. $P(AB) = \frac{1}{16}$ $P(A) = \frac{1}{8}$ $P(B) = \frac{5}{16}$

$P(AB) \neq P(A)P(B)$