HW 14

Ex. $X_n \sim N(0,1)$ $Y_n \sim U(-\sqrt{3},\sqrt{3})$

First prove $\{Z_n\}_{n\geq 1}$ is WSS.

$$E(Z_n) = \begin{cases} E(X_n) = 0 & 2\mid n \\ E(Y_n) = 0 & 2\nmid n \end{cases}$$

for all $m, n \geq 0$,

$$R_X(\cancel{Z_n, Z_m}\ n,m) = \begin{cases} E(X_n X_m) = E(X_n)E(X_m) = 0 & 2\mid n,\ 2\mid m \\ E(X_n Y_m) = E(X_n)E(Y_m) = 0 & 2\mid n \quad 2\nmid m \\ E(Y_n Y_m) = E(Y_n)E(Y_m) = 0 & 2\nmid m \quad 2\nmid n \end{cases}$$

$\therefore \{Z_n\}_{n\geq 1}$ is WSS

However, $Z_{2k+1} \neq Z_{2k+1-(2k+1)} = Z_0$ $(h = -(2k+1))$

$\therefore Z_h \neq Z_{h-h}$

$\therefore (Z_{2k+1}, Z_{2k+2}) \neq (Z_{2k+1+(2j+1)}, Z_{2k+2+(2j+1)})$

$\therefore \{Z_n\}_{n\geq 1}$ is not SSS.

Ex 6.3

$E(X_t) = E(A)\cdot \sin(2\pi t)$ is not a constant

$\therefore X_t$ is not WSS.

Ex 6.5 $X_t = t^2 + A\sin t + B\cos t$

$E(X_t) = t^2 + E(A)\sin t + E(B)\cos t$ is not a constant

$\therefore X_t$ is not a stationary process.

$\mu_X(t) = E(X_t) = t^2$

$\therefore Y_t = X_t - t^2$

$\therefore E(Y_t) = E(A)\sin t + E(B)\cos t = 0$

$R_Y(t, t+\tau) = E\left((X_t - t^2)(X_{t+\tau} - (t+\tau)^2)\right)$

$= E\left(X_t X_{t+\tau} - (t+\tau)^2 X_t - t^2 X_{t+\tau}\right) + t^2(t+\tau)^2$

$= E(X_t X_{t+\tau}) - E(X_t)E(X_{t+\tau}) = Cov(X_t, X_{t+\tau})$

$E(X_t X_{t+\tau}) = E\left((t^2 + A\sin t + B\cos t)((t+\tau)^2 + A\sin(t+\tau) + B\cos(t+\tau))\right)$

$= t^2(t+\tau)^2 + \sin t \sin(t+\tau) E(A^2) + \cos t \cos(t+\tau) E(B^2)$

$= t^2(t+\tau)^2 + 10\sin t\sin(t+\tau) + 10\cos t\cos(t+\tau)$

$= t^2(t+\tau)^2 + 10\cos\tau$

$R_Y(t, t+\tau) = E(X_t X_{t+\tau}) - E(X_t)E(X_{t+\tau}) = 10\cos\tau$

depends only on τ

$\therefore Y_t$ is a WSS.

Ex 6.11 $\quad \overline{Q} = R_X(0) = 4$

Ex 6.12 (a) $\quad E(X_t^2) = R_X(0) = 4 \qquad D(X) = \cancel{Var(X} E(X_t^2) - E^2(X_t)$

Ex 6.13

$R_X(t) = \cancel{\frac{1}{\pi}\int_0^\infty \frac{500}{\omega^2+9}d\omega} \ \frac{1}{\pi}\int_0^\infty S_X(\omega)\cos\omega t\, d\omega$

$Ae^{-\beta|\tau|} \leftrightarrow \frac{2A\beta}{\omega^2+\beta^2} \qquad \beta = 3 \quad A = \frac{250}{3}$

$\therefore R_X(t) = \frac{250}{3}e^{-3|t|}$