EBU4375: SIGNALS AND SYSTEMS

LAB3: DISCRETE-TIME SYSTEMS IN THE FREQUENCY DOMAIN

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ACKNOWLEDGMENT

These slides are partially from Labs prepared by

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YOUR TASKS

- BEFORE THE LAB:
  - Read the slides carefully.
  - Create a ID_FS.txt file where ID is your QMUL ID number, F is the first letter of your forename and S is the first letter of your surname.
  - Type all the code in a red frame in the ID_FS.txt file and submit to the QMplus link.
- DURING THE LAB:
  - Copy/paste the code from ID_FS.txt into Matlab command window as required- indicated by
  - Take note of the results and your answers to questions indicated by
- **Make sure you do the work yourself as there will be questions in the class tests and exam related to Matlab.**

BACKGROUND

Given a periodic discrete-time signal $x_N[n]$ of period N:

1. Its fundamental frequency is $\Omega_0 = \frac{2\pi}{N}$.
2. According to the synthesis equation, $x_N[n]$ can be expressed as the sum of N harmonically related complex exponentials of frequencies $\Omega_k = k\frac{2\pi}{N}$.

$$x_N[n] = \sum_{k=\langle N \rangle} a_k e^{jk\Omega_0 n}$$

3. The Fourier coefficients a_k can be determined by using the analysis equation as:

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_N[n] e^{-jk\Omega_0 n}$$

BACKGROUND

In order to obtain the Fourier series decomposition of a periodic signal $x_N[n]$ of period N, we need to:

1. Identify the fundamental frequency Ω_0.
2. Determine the N harmonic frequencies $\Omega_k = \mathrm{k}\Omega_0$.
3. Obtain the Fourier coefficients a_k.

BACKGROUND

- Determining the fundamental frequency Ω_0 and its harmonics k is very easy.
- The Fourier coefficients a_k can be obtained analytically. For instance, for the periodic square wave of period N defined within one period centred around n = 0 as:

$$x_N[n] = \begin{cases} 1 & |n| \leq N_1 \\ 0 & otherwise \end{cases}$$

we can obtain its Fourier coefficients as:

$$a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} e^{-jk\Omega_0 n} = \begin{cases} \frac{2N_1 + 1}{N} & k = 0 \\ \frac{1}{N} \frac{\sin[2\pi k (N_1 + 1/2)/N]}{\sin(\pi k/N)} & k \neq 0 \end{cases}$$

BACKGROUND

You will have noticed that the analysis equation in DT:

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_N[n] e^{-jk\Omega_0 n}$$

is essentially a mathematical operation in which we:

1. Calculate N products of complex numbers.

2. Add N complex numbers.

Computers are very good at doing additions and multiplications so, why don't we let computers calculate the Fourier coefficients for us?

STEP 1- Definition of periodic DT signal

In this lab, we will work with the periodic square wave x[n] with period N defined as:

$$x_N[n] = \begin{cases} 1 & |n| \leq N_1 \\ 0 & otherwise \end{cases}$$

Your job now is to:

Based on you're the last digit of you QMUL ID number x, define

$$\text{N}=2 \times \max(10,10 \times x) +1$$

$$\text{N1}= \max(10,10 \times x) /2$$

Identify its fundamental frequency Ω_0 and its harmonics k.

Draw on a piece of paper three periods of x[n].

STEP 2- Plotting individual harmonic components

The following lines of code calculate the Fourier coefficients a_k of x[n].

Based on you're the last digit of you QMUL ID number x, find the values of n1 and n2 such that n2=$\max(10,10 \times x)$ and n1=-n2.

```
n1 = ?;%Fill in the value based on your ID number
n2 = ?;%Fill in the value based on your ID number
n = n1:1:n2; % Definition of the time vector
x = zeros(size(n));
nmin= n2-n2/2+1;
nmax= n2+n2/2+1;
x(nmin:nmax)=1; % Definition of x[n] as square wave
%Plot the signal x[n] in the time interval n  using stem function.
N = 2*n2+1; % Period of x[n]
omega0=2*pi/N; % Fundamental frequency of x[n]
ak=zeros(1,N);
for k=0:2*n2; % This loop calculates the Fourier coefficients ak
ak(k+1)=(1/N)*sum(x.*exp(-k*1i*omega0*n));
end
```

Question 1:

Save the figure in which you plot x[n]

STEP 3- Plotting the Fourier Coefficients

The following lines of code plot the magnitude of the coefficients a_k.

```
Figure
nAxix=[0:n2,-n2:-1];
stem(nAxix,abs(ak)) % plots ak against n
xlabel('k') % adds text below the X-axis
ylabel('ak') % adds text beside the Y-axis
```

Question 2:

Save the figure in which you plot ak

Question 3:

Make sure you understand how nAxix is defined. Hint, try *nAxix=[0:2*n2];* instead.

Question 4:

Take note of the value of N, the greatest value ak, and the corresponding value of k. Can you justify?

STEP 3- Plotting the Fourier Coefficients

- Explain the function of nAxix=[0:n2,-n2:-1] in plotting ak (Hint: you can compare to nAxix=[0:2*n2]):
- Based on the plot of ak please answer the following:
- What is the value of N?
- What is the maximum value of ak?
- What is the corresponding values of k?
- What is your justification for the maximum value of ak?

STEP 4- Synthesising x[n]

The following lines of code synthesise 3 periods of x[n] by using the Synthesis equation:

```
m1 = -3*n2-1;
m2 = 3*n2+1;
n = m1:1:m2; % New time vector
xsyn = zeros(size(n));
k=0:2*n2;
for m=m1:m2 % Synthesis of x
xsyn(m+m2+1)=sum(ak.*exp(k*1i*omega0*m));
end
figure
stem(n,abs(xsyn))
```

Question 5:

Save the figure in which you plot xsyn[n]

Explain briefly what you see

STEP 5- Lowpass Filtering x[n]

In the following lines of code, we filter out the frequencies $|\Omega| > 2\pi/N$ of x[n], producing the signal y[n] with Fourier coefficients b_k:

```
bk=zeros(size(ak));
bk(1)=ak(1);
bk(2)=ak(2);
bk(2*n2+1)=ak(2*n2+1);
for m=m1:m2 % Synthesis of y
y(m+m2+1)=sum(bk.*exp(k*1i*omega0*m));
end
figure
stem(n,abs(y)) % plots a k against n
xlabel('n') % adds text below the X-axis
ylabel('y') % adds text beside the Y-axis
axis tight
```

Question 6:

Save the figure in which you plot y[n]

Explain briefly what you see

STEP 6- Changing the duty cycle

The duty cycle of the proposed discrete-time periodic signal is $\rho = \frac{n2}{N} \approx 50\%$

Your job now is to:

- Plot the x[n] for $\rho \approx 1/5$;
- Plot the Fourier coefficients a_k for $\rho \approx 1/5$;

Change the values of x[n] to implement the duty cycle.

Explain your method changing the duty cycle

Question 7:

Save the corresponding plot for x[n] and ak.

LAB3 DELIVERABLES

1. Pre LAB submission .txt file (Qmplus) NOT MARKED
2. Post LAB Answer sheet submission on QMplus MARKED (10 marks)
3. Post LAB 5-minute video recording in which you explain the code and the results that you obtain (20 marks)

A brief self introduction - your name and QMUL student ID. Your face must be seen in this part. Give a detailed explanation of your code and obtained results. Do not exceed the time nor increase the speed of the recording.