



Queen Mary

University of London

Science and Engineering

EBU4202: Digital Circuit Design

Sequential Circuits Analysis

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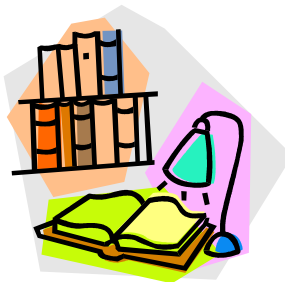
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Overview: Sequential Logic Design Principles

- * Introduction
- * Bistable Elements
- * Latches & Flip-Flops
- * Analysis Procedure
- * Design Procedure



Chapter 7 – “Digital Design: Principles and Practices” book

Analysis and Synthesis

In the context of electronic circuits:

- **Analysis** means that we have a circuit, and we want to know what it does
- **Synthesis** means that we know what we want a circuit to do, and we then need to *design and realise* (put a design into practice) a circuit to do it.
- In this set of lectures, we will look at the *analysis* process of Synchronous Sequential Circuits and the *design* of Sequential Circuits (limited to Synchronous Sequential Circuits)

Analysis Procedure: What this is about

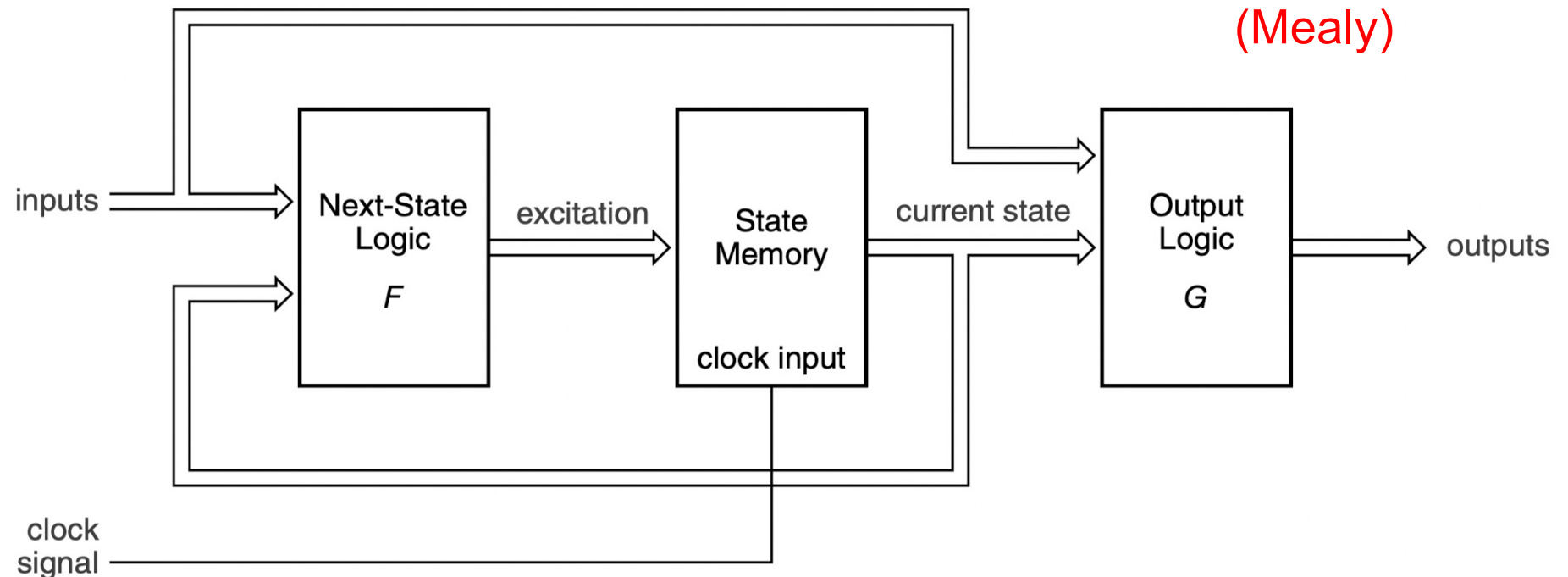
- **Behavior of sequential circuits:**
 - Determined from *inputs*, *outputs*, and the *state of flip-flops*.
- **Outputs** and **next state** are both a function of the *inputs* and the *present state*.
- **Analysis consists of:**
 - Obtaining a *suitable* description that demonstrates the time sequence of *inputs*, *outputs*, and *flip-flop states*.

Clocked Synchronous State Machines: Operation

- **Latches & Flip-Flops**: basic building blocks of sequential circuits and can be formally analysed.
- Usually, sequential circuits may not just include latches & flip-flops; they may also include combinational circuits.
- **Clocked Synchronous State Machines**:
 - *State Machine*: general sequential circuit.
 - *Clocked*: their storage elements (i.e., flip-flops) use a clock input.
 - *Synchronous*: all the flip-flops use the same clock signal.
 - Its state changes only when a change occurs on the clock signal.

Mealy Machines

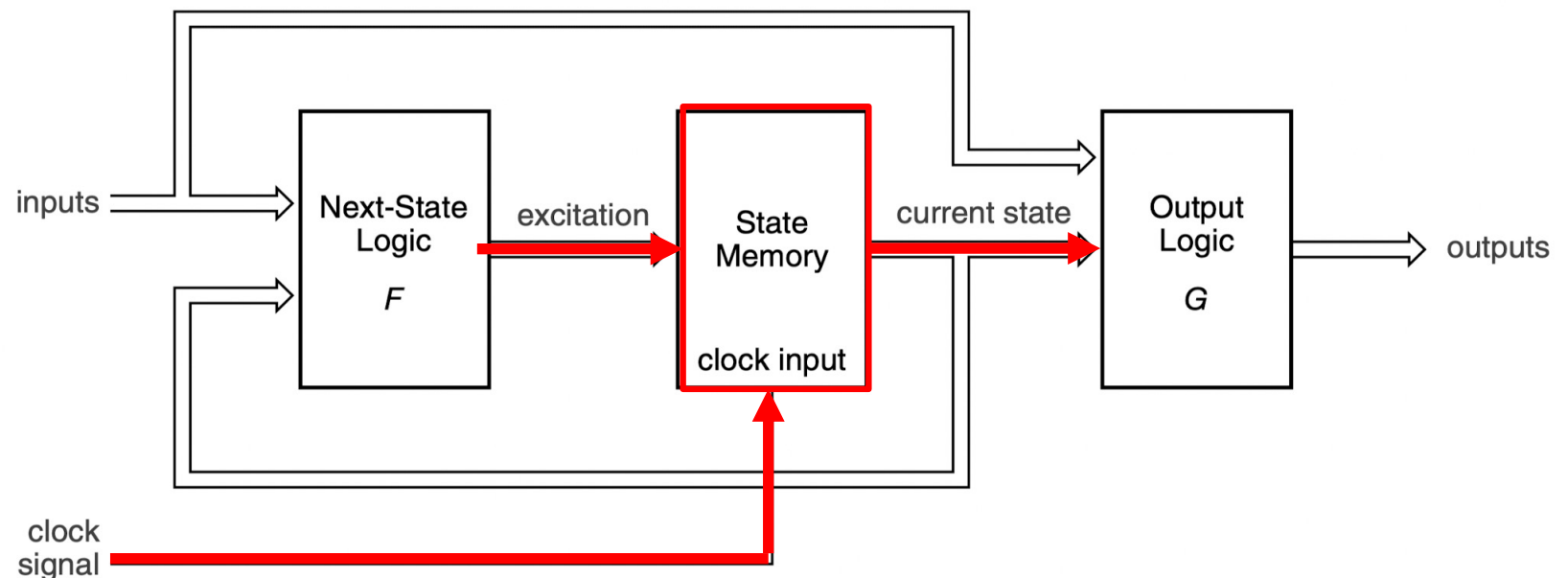
- Mealy model for a finite state machine (FSM)
- Outputs depend on **inputs and current state**



State Memory/Register

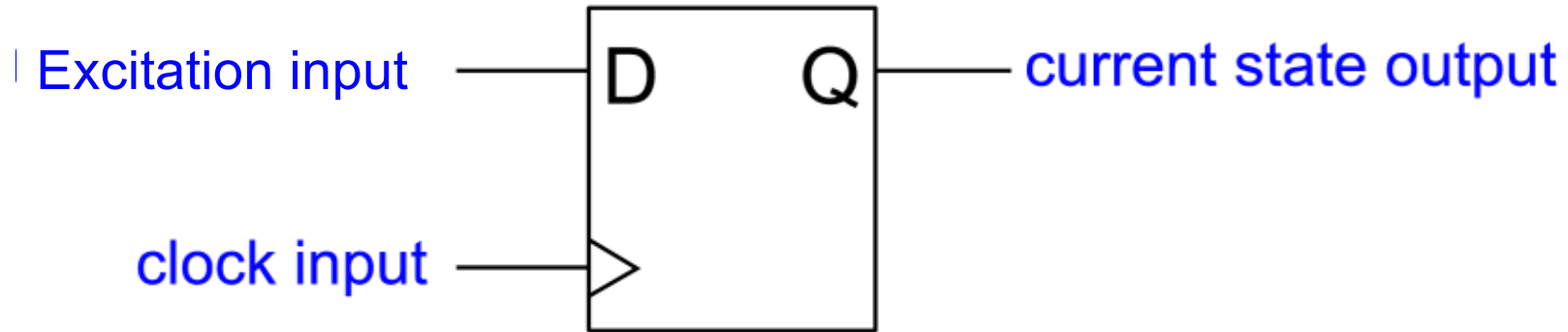
State Memory:

- A set of flip-flops that store the current state.
- n flip-flops can store 2^n distinct states.
- Output of the flip-flop is determined by the characteristic equation for the flip-flop type; also called the *Characteristic Equation*.



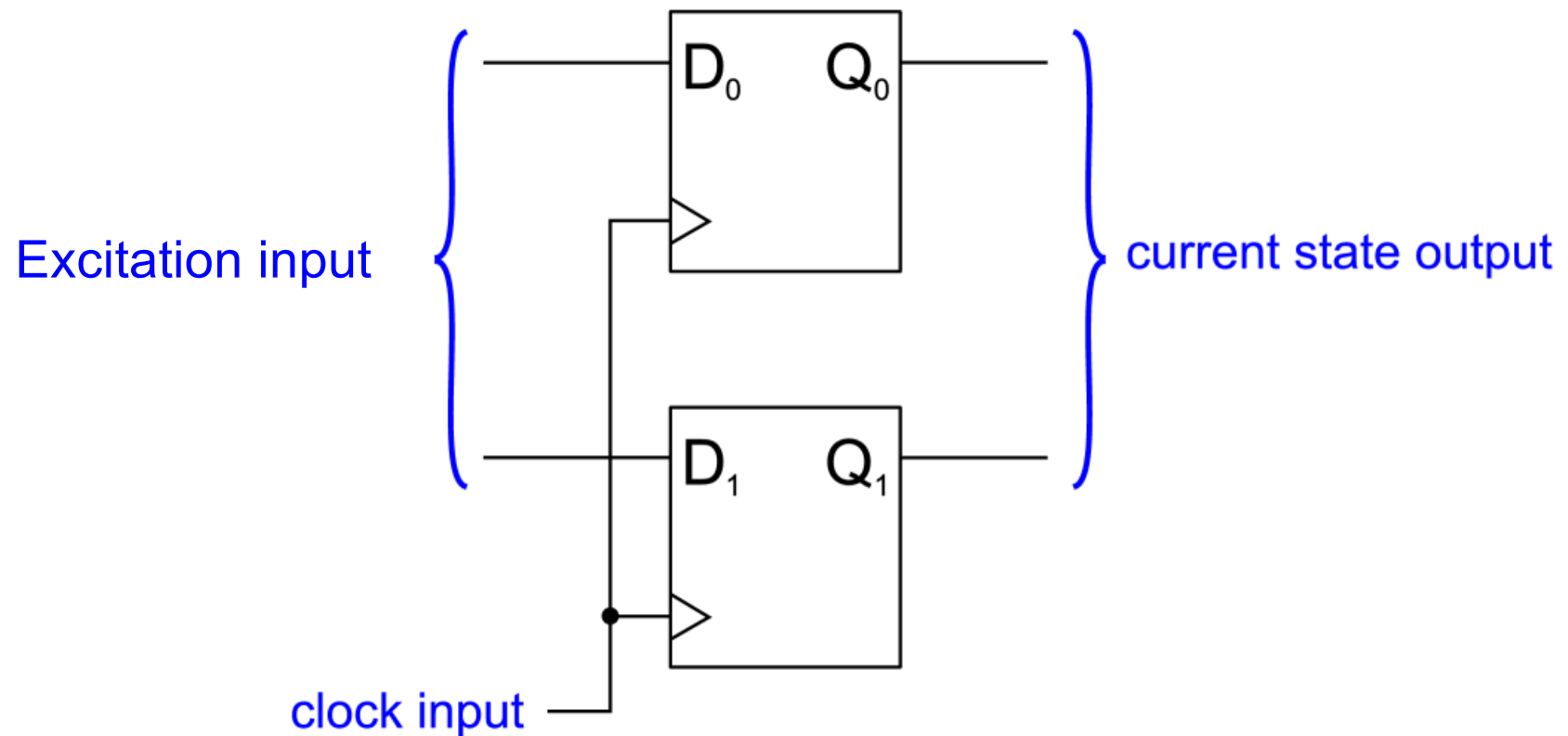
State Memory/Register

- One D-type flip-flop has how many states?



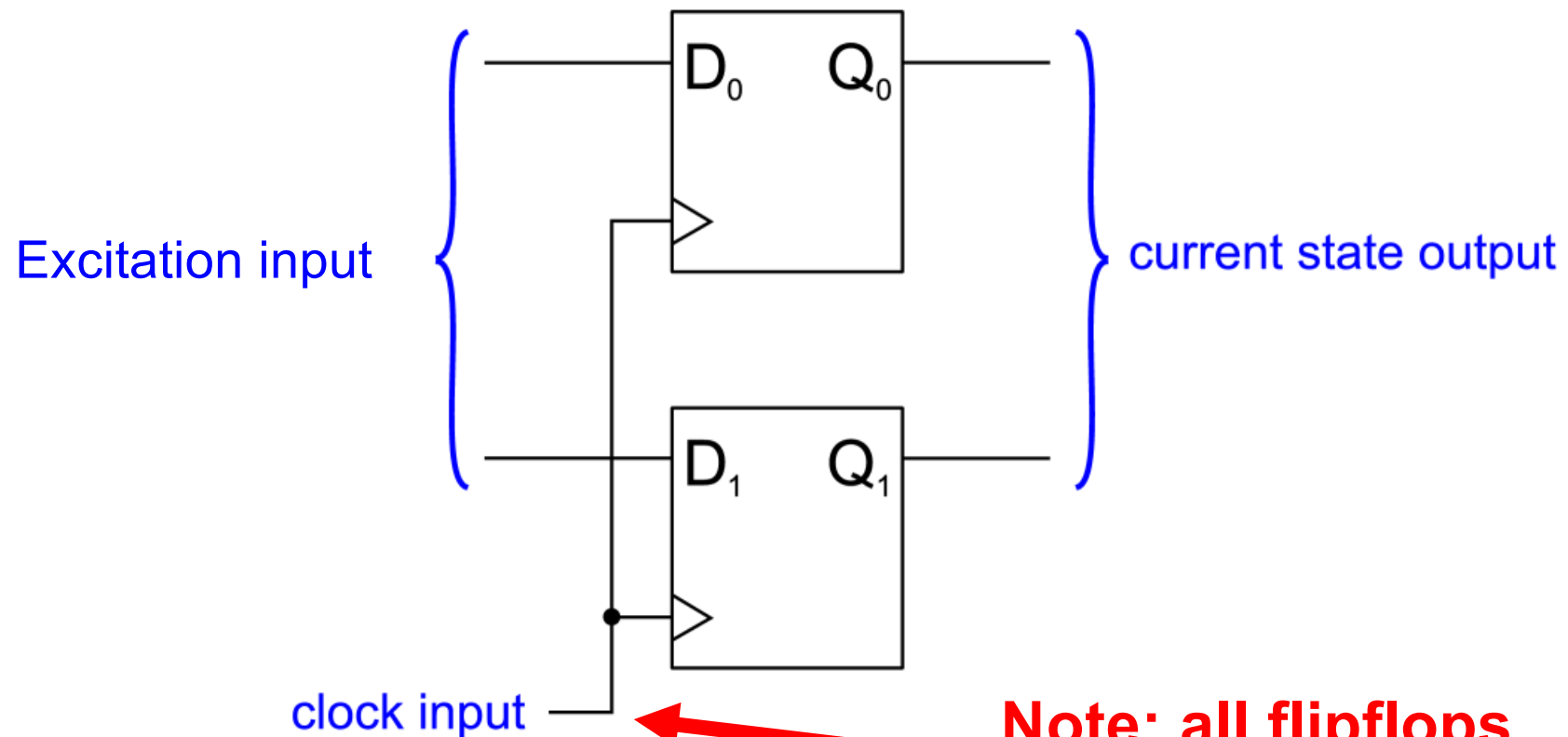
State Memory/Register

- Two D-type flip-flops have how many states?



State Memory/Register

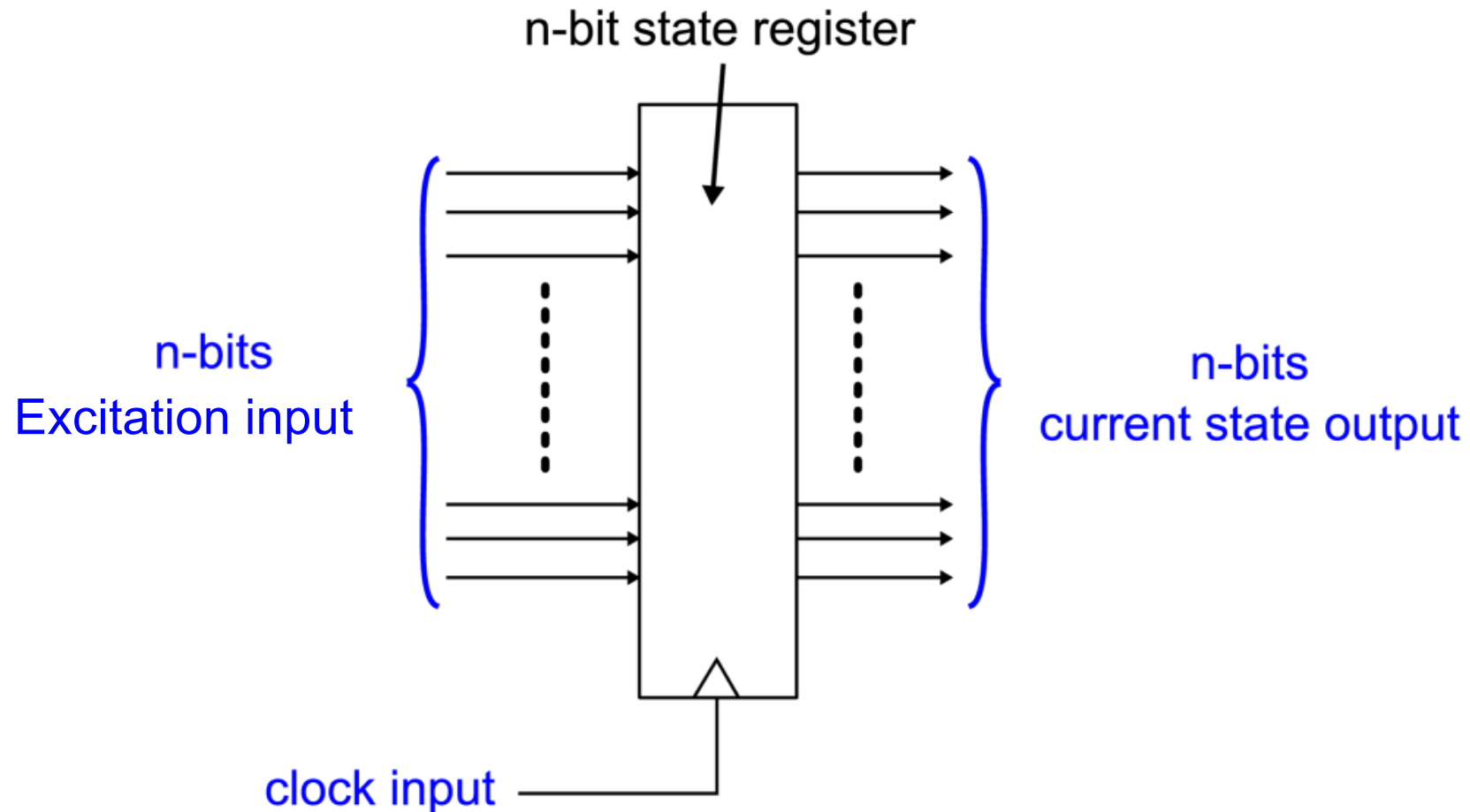
- Two D-type flip-flops have how many states?



**Note: all flipflops
share common clock**

State Memory/Register

- A state register with n flipflops has 2^n possible states



Characteristic Equations: Definition

- Describe the **functional behaviour** of a latch or flip-flop.
- Specify the **flip-flop's next state** as a function of its current state and inputs.
- Q*** means "*the next value of Q*".

MS = Master/Slave
ET = Edge-Triggered

<i>Device Type</i>	<i>Characteristic Equation</i>
S-R Latch MS S-R Flip-Flop	$Q^* = S + R'Q$
D Latch	$Q^* = D$
JK Latch MS J-K Flip-Flop ET J-K Flip-Flop	$Q^* = JQ' + K'Q$
ET D Flip-Flop	$Q^* = D$
D Flip-Flop w/ Enable	$Q^* = E_N D + E_N' Q$
T Flip-Flop	$Q^* = Q'$
T Flip-Flop w/ Enable	$Q^* = E_N Q' + E_N' Q$

Characteristic Equations: How they are Derived

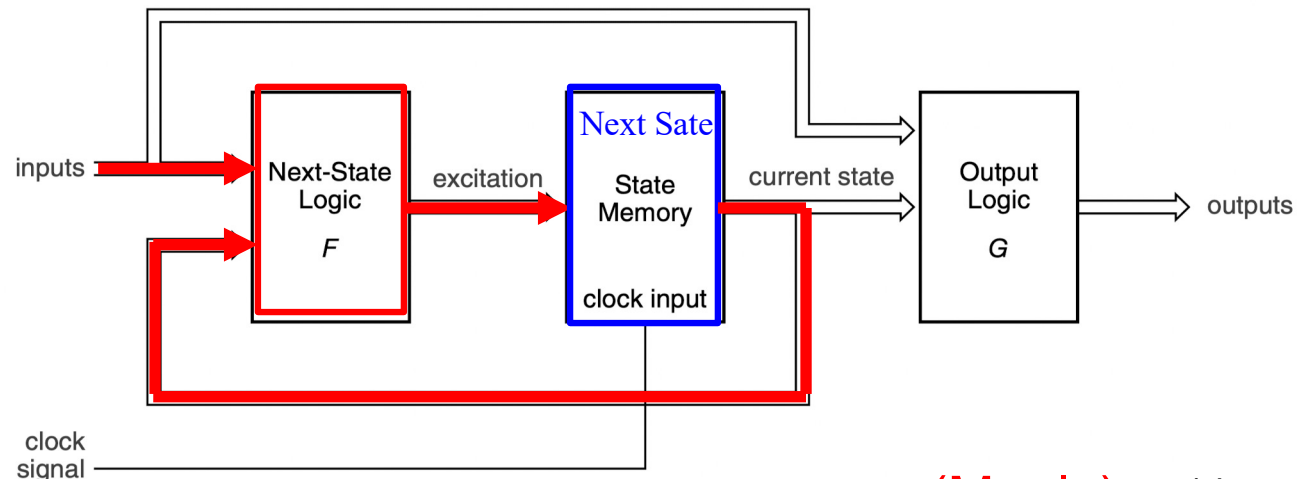
- Can derive the characteristic equations by considering the behaviour of the flip-flop/latch, **but these equations**:
 - do not describe in detail the flip-flop/latch timing behaviour.
 - only describe the functional behaviour of the flip-flop/latch upon changes to the control inputs.
- **Examples**:
 - *D (Data) flip-flop* – what goes in comes out. Thus, $Q^* = D$.
 - *T (Trigger) flip-flop* – the next Q is always the complement of the current Q . Thus, $Q^* = Q'$.

Next-State Logic

Next-State Logic:

- Combinational logic applied to the inputs before being put into the “state memory” (i.e., flip-flops).
- The next state of the state machine is **determined** by the next-state logic F as a function of the current state and input.
- Expresses the input (excitation) to the state memory as a boolean equation called either *Input Equation* or *Excitation Equation*.

It is a function of the input and the current state i.e.,
 $\text{Next State} = F(\text{Current State}, \text{Input})$.

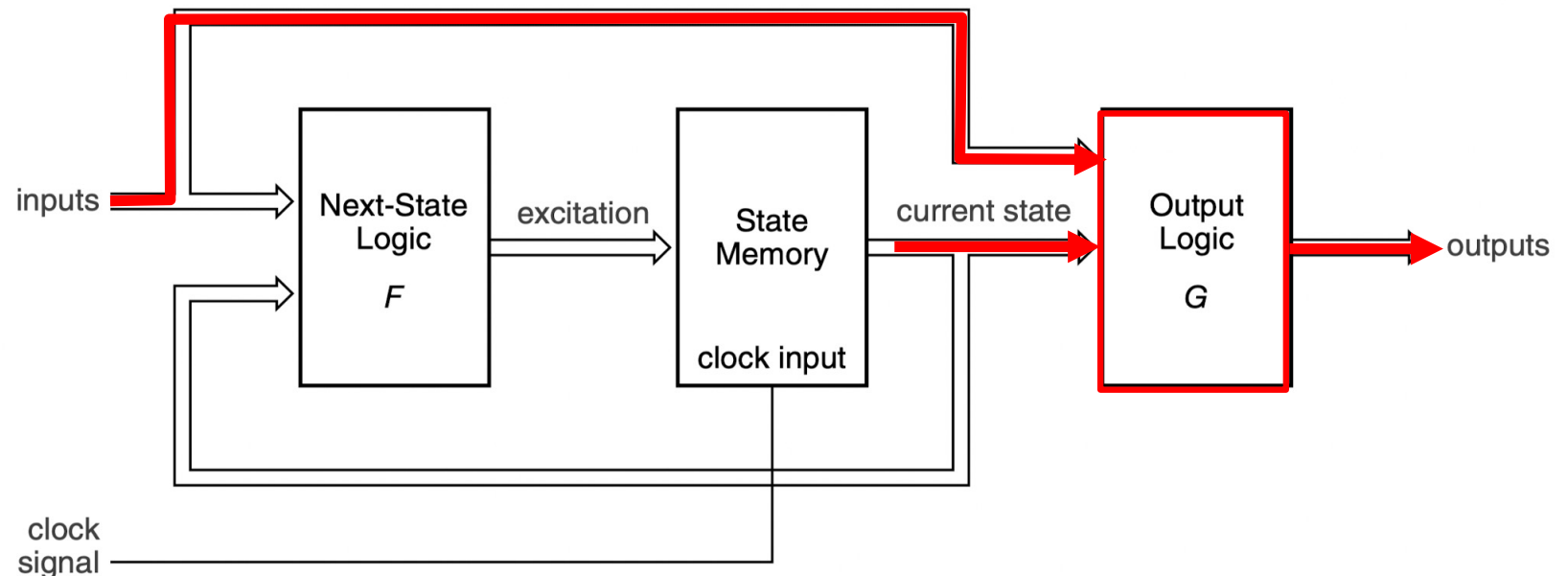


Output Logic

Output Logic:

- Combination logic that determines the output of the state machine.
- ***Output Equation*** is a function of the input & current state.

$\text{Output} = G(\text{Current State}, \text{Input})$.



State Machines (1/2)

- A **state machine** (or a *clocked synchronous state machine*) consists of three components:

1. Next-State Logic:

- Combinational logic applied to the inputs before being put into the “state memory” (i.e., flip-flops).
- Expresses the input to the state memory as a boolean equation called either *Input Equation* or **Excitation Equation**.
 - It is a function of the input and the current state i.e.,
 $\text{Next State} = F(\text{Current State}, \text{Input})$.

State Machines (2/2)

2. State Memory:

- A set of flip-flops that store the current state.
- n flip-flops can store 2^n distinct states.
- Output of the flip-flop is determined by the characteristic equation for the flip-flop type; also called the ***Characteristic Equation***.

3. Output Logic:

- Determines the output of the state machine.
- ***Output Equation*** is a function of the input & current state.
 - $\text{Output} = G(\text{Current State}, \text{Input})$.

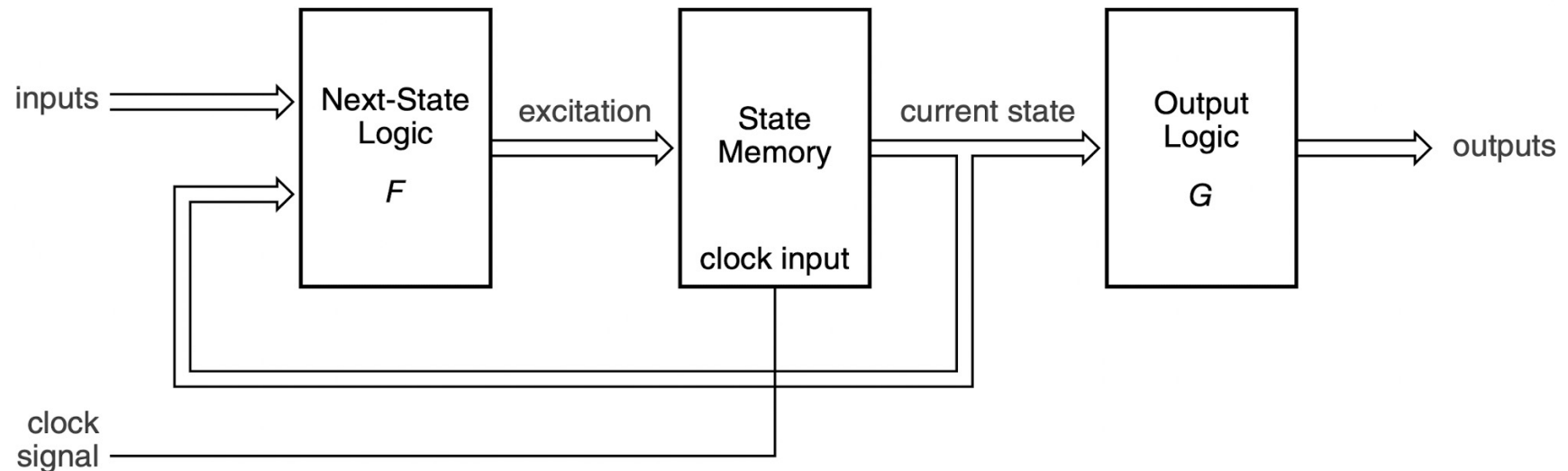
Moore Machines

- Moore model for a finite state machine (FSM)
- Outputs only depend on current state

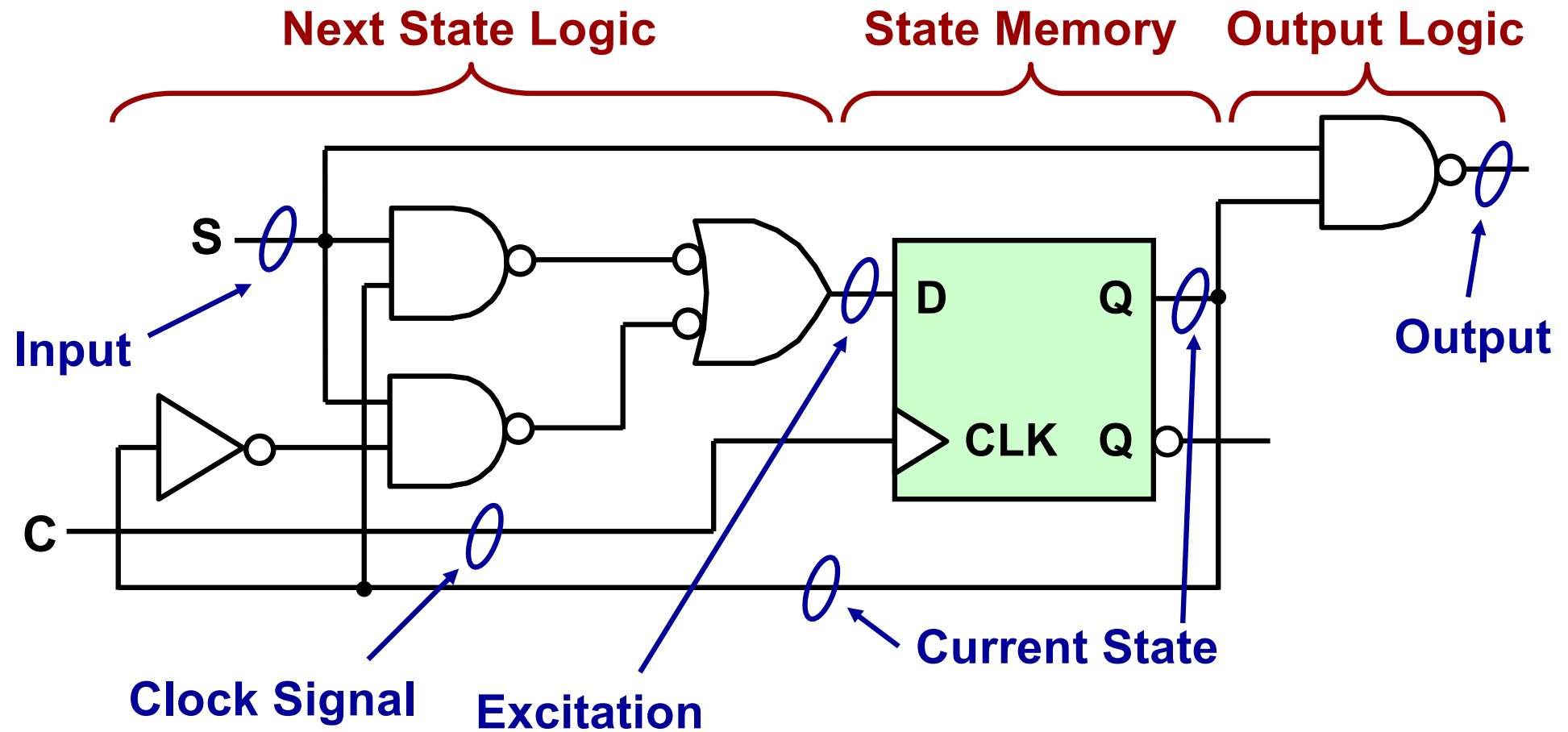
Output Logic:

- Determines the output of the state machine.
- **Output Equation** is a function of the input & current state.

Output = $G(\text{Current State})$.



Example: State Machine Diagram



(Mealy)

Analysis Procedure: Steps

1. Obtain the *input* (or **excitation**) **equations**.
2. Obtain the **output equations**.
3. Obtain the *next state* (or **characteristic**) **equations**.
4. Substitute the *excitation equations* into the *characteristic equations* to obtain **transition equations**.
5. Develop a **transition table** from the *transition equations*.
6. Develop a **state table** that relates the possible states in terms of the *present* and *next state*.
7. Develop a **state/output table** that relates the possible states in terms of the *present* and *next state*, *together with the outputs*.
8. Draw the **state diagram**.



Table Development Process in CSSMs

- **Tables of Clocked Synchronous State Machines** (CSSMs) have up to four sections:
 - **Current state**: shows state of the flip-flops at any given time t .
 - **Input**: gives the value of the inputs for each possible state.
 - **Next state**: shows the state of the flip-flops one clock period later, i.e., at $(t+1)$.
 - **Output**: gives the system output values for each present state.

Table Development Process: Transition Table

- There are **three types of tables** to develop:

- **Transition Table**

- Expresses the *next state* as a function of the *current state* and the *input*.
- Uses **Transition Equations** to determine entries.

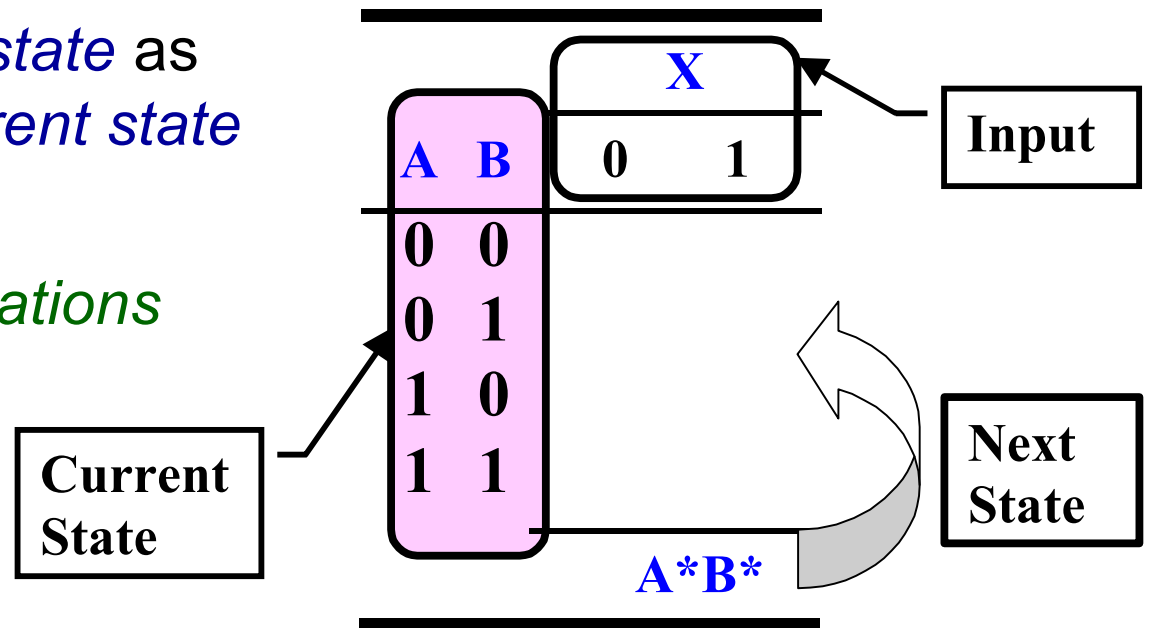


Table Development Process: State Table

- There are **three types of tables** to develop (**cont.**):

- **State Table**

- Expresses the *next state* as a function of the *current state* and the *input* using alphanumeric state names.

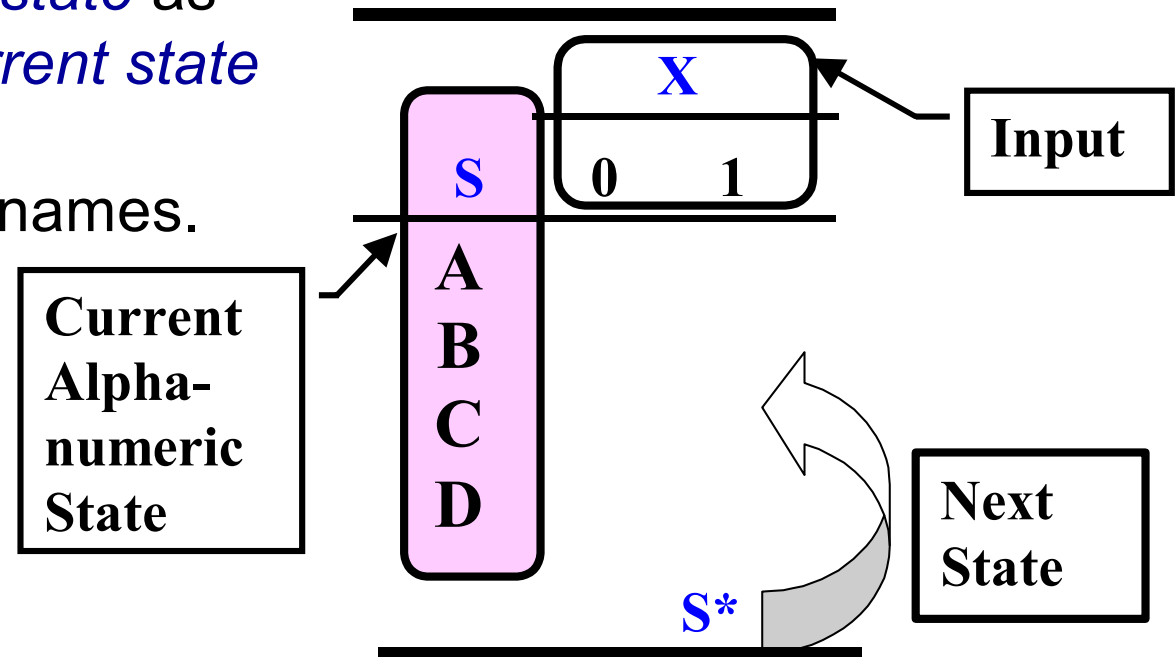
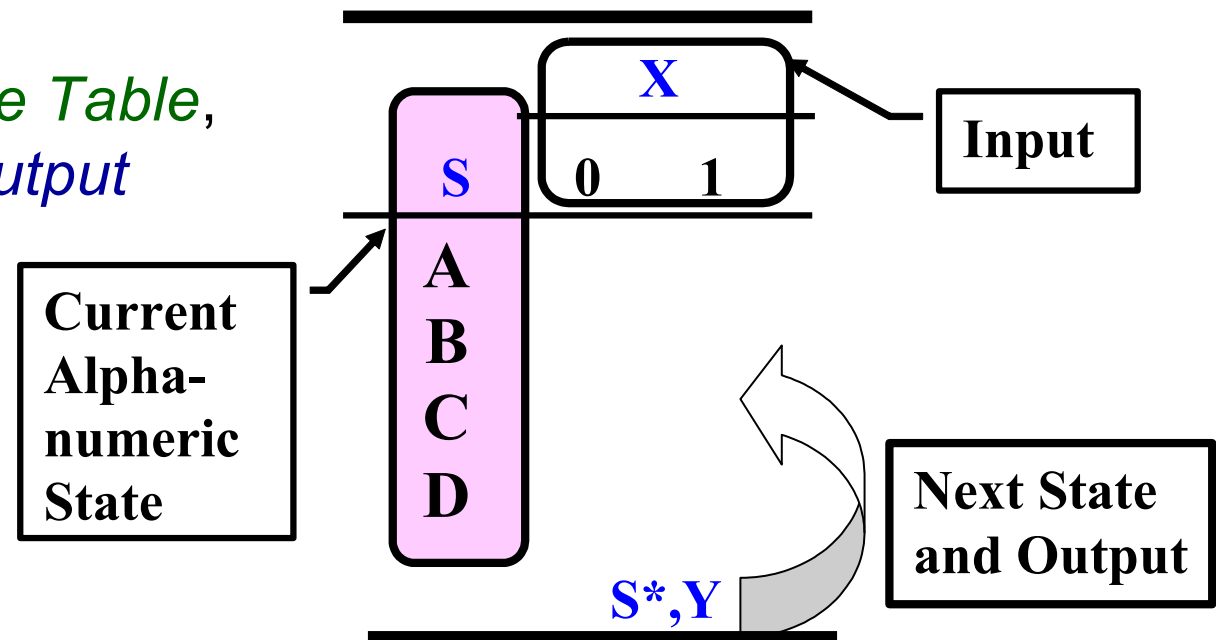


Table Development Process: State/Output Table

- There are **three types of tables** to develop (**cont.**):

- **State/Output Table**

- Similar to the *State Table*, but includes the *output* as well.



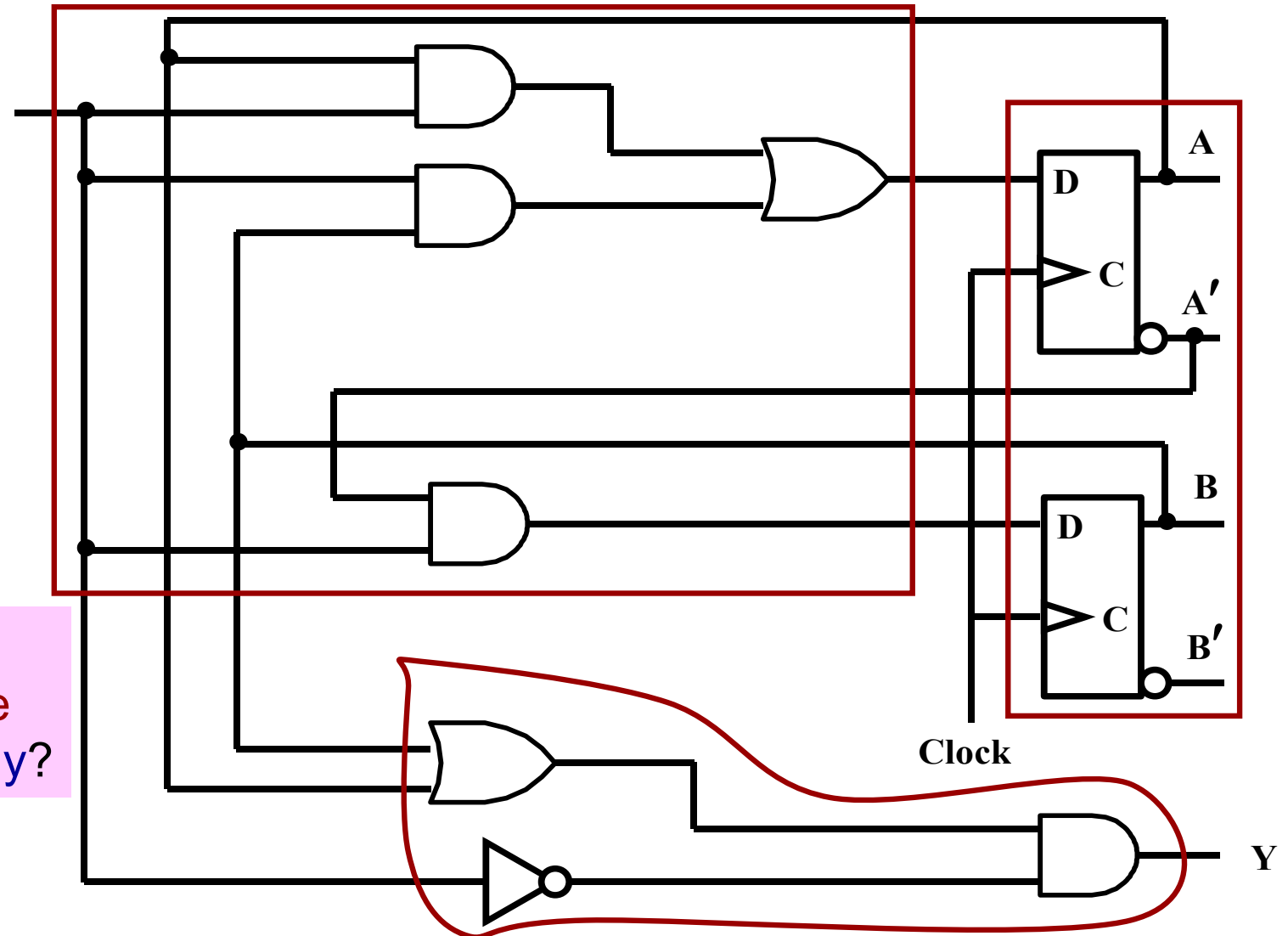
Things to remember about tables ...

- Always **correctly label your tables** – you will lose marks on your coursework/exam if this is not done!
- Always provide a key to your **alphanumeric states** e.g.:
 - If you have *four states*, each described by *2 bits*, then your states could be labelled as, *A = 00; B = 01; C = 10; D = 11*. (Try to avoid 'I' and 'O' as labels as they could look like 1 and 0.)
- **State/Output tables** for *Moore Machines* (i.e., where the output depends only on the current state) are simpler than those of *Mealy Machines* (i.e., where the output depends both on the current state and the inputs):
 - Simpler *output equation* as it is not a function of the input(s).

Analysis Example: State Machine with D Flip-Flops

What do the areas in red represent?

What kind of state machine is this and why?



Analysis Example: Table Development Process

- *First*, determine the **Excitation (input)** and **output equations**:

Excitation (Input)

Equations:

$$D_A = AX + BX$$

$$D_B = A'X$$

Output Equation:

$$Y = (A + B)X'$$

What is another name for the “Input Equations”?

- How do we determine what the *output* of the D flip-flops is or the *Next State Equations*?

- Use the **Characteristic Equation** for a D flip-flop: $Q^* = D$

- So what goes into a D flip-flop comes out again! Thus,

Transition $A^* = D_A = AX + BX$ (substituting in for D_A)

Equation: $B^* = D_B = A'X$ (substituting in for D_B)

Analysis Example: Transition Table

Transition Equations:

$$A^* = AX + BX$$

$$B^* = A'X$$

Output Equation: $Y = (A + B)X'$

1. List all possible states

		X	
B	A	0	1
0	0	00	10
0	1	00	01
1	0	00	11
1	1	00	01
		B*A*	

Determine B*A* using the Transition Equations for the appropriate flip-flops.

For A=0, B=0, X=0:

$$A^* = AX + BX$$

$$= 0 \bullet 0 + 0 \bullet 0$$

$$= 0$$

$$B^* = A'X$$

$$= 1 \bullet 0$$

$$= 0$$

Thus entry is 00

2. Determine Next State of BA for X=0 and X=1

Analysis Example: State Table

Give each numerical state an alphanumeric name:

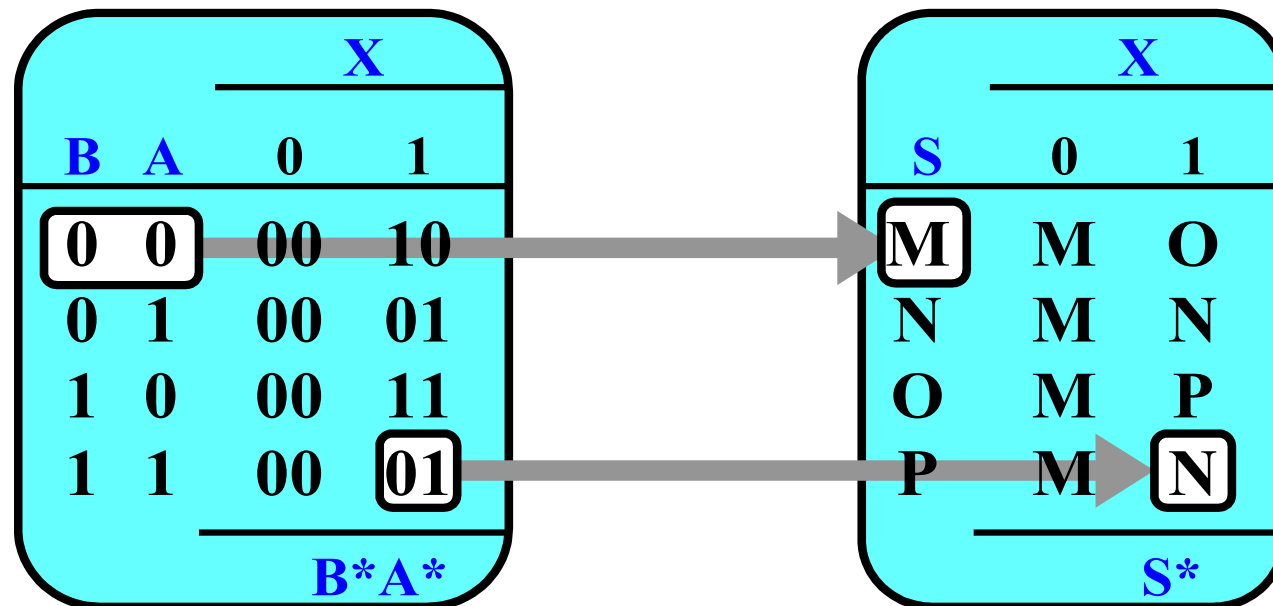
00 = M

01 = N

10 = O

11 = P

Substitute into the **Transition Table** to form the **State Table**.



Analysis Example: State/Output Table

- Expand your *State Table* to include the output **Y**, to form the *State/Output Table*:

X		
S	0	1
M	M,0	O,0
N	M,1	N,0
O	M,1	P,0
P	M,1	N,0
S*,Y		

Determine Y using the Output Equation.

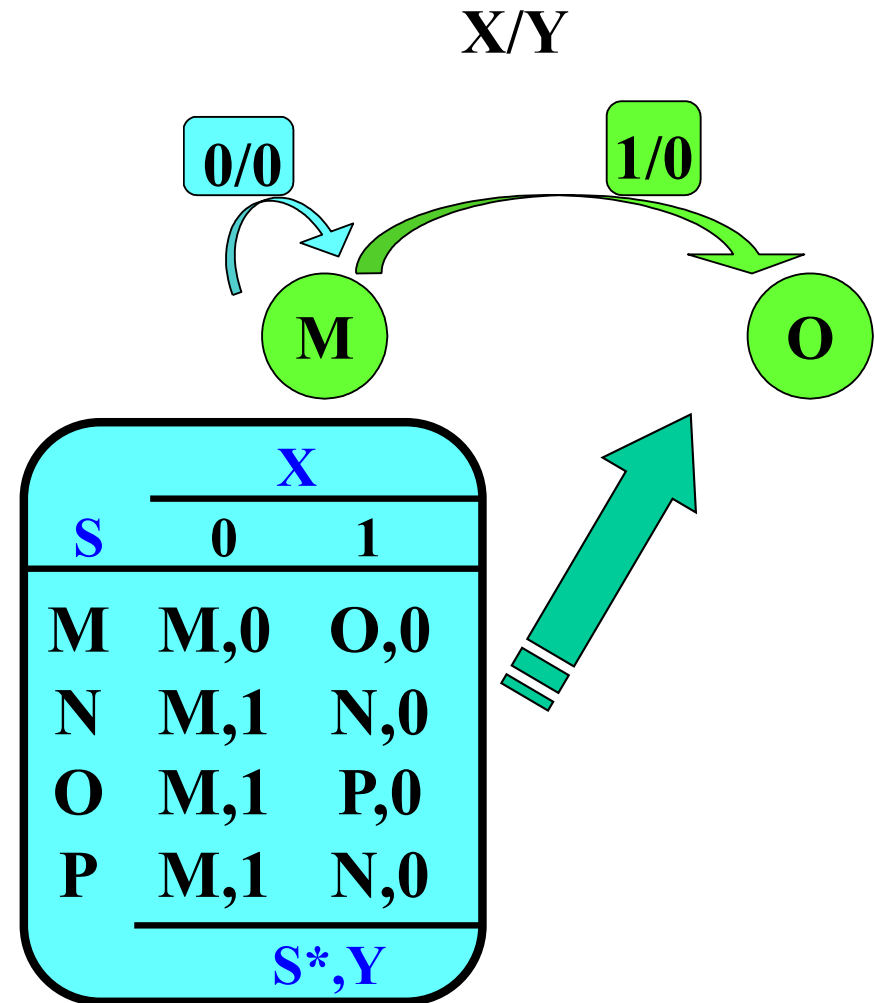
For A=0, B=0, X=0:

$$\begin{aligned} Y &= (A + B)X' \\ &= (0 + 0) 1 \\ &= 0 \end{aligned}$$

Thus entry is 0.

Analysis Example: State Diagram Development Process (1/2)

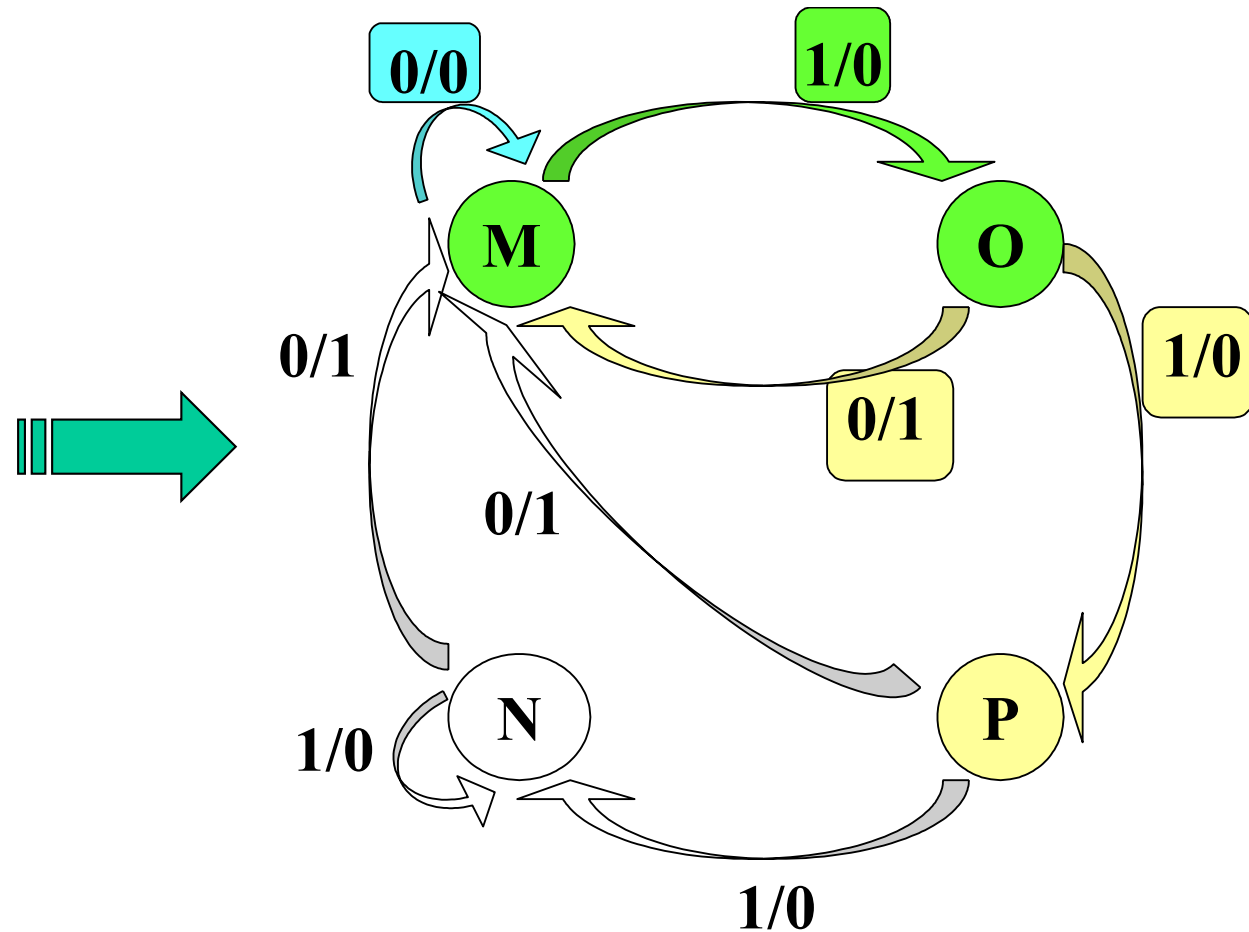
- **Development Process:**
 - Develop the *State Diagram* from the *State/Output Table*.
 - Represent the *Present States* by circles.
 - Represent the transition between states by *directed lines*.
 - Label the *directed lines* with *input/output values*.



Analysis Example: State Diagram Development Process (2/2)

S	X	
	0	1
M	M,0	O,0
N	M,1	N,0
O	M,1	P,0
P	M,1	N,0

S^*, Y

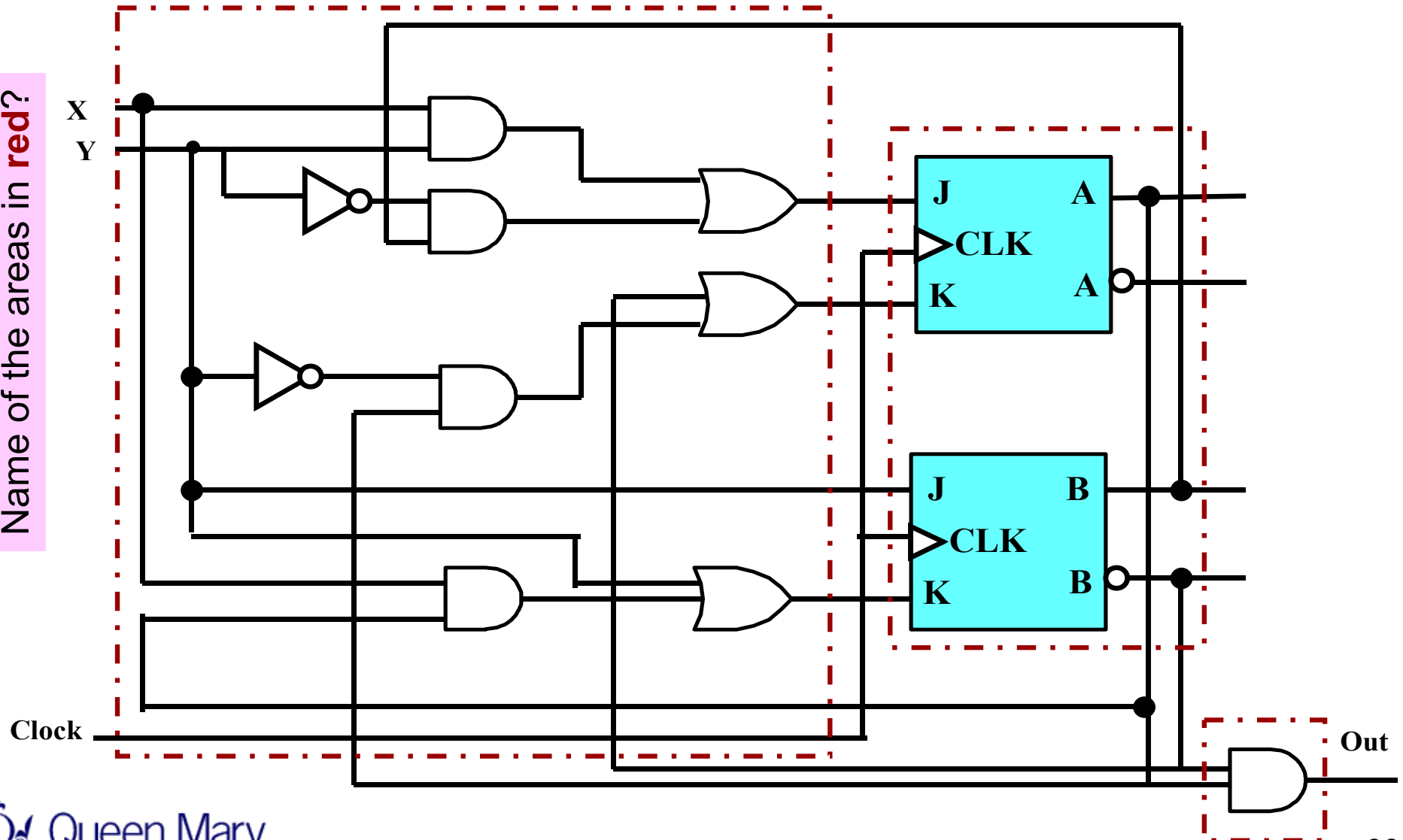


Analysis with JK Flip-Flops

- The analysis example we just went through was only for **D Flip-Flops** with only one data input.
 - The *Next State* values could be obtained directly from the excitation (input) equations!
- What about if there are 2 inputs, i.e., with **JK Flip-Flops**?
 - We need to use the appropriate *Characteristic Equations*!

Analysis Example: JK Flip-Flop

Name of the areas in red?



JK Flip-Flops Analysis Example: Obtain Input & Output Equations

Reading directly from the diagram:

$$\text{Excitation (Input) Equations: } \left\{ \begin{array}{ll} J_A = XY + Y'B & K_A = Y'A + B' \\ J_B = Y & K_B = XA + Y \end{array} \right.$$

$$\text{Output Equation: } \text{OUT} = AB'$$

For *Next State*, use the **Characteristic Equation** for a JK flip-flop!

$$- Q^* = JQ' + K'Q$$

$$\begin{aligned} \text{Transition Equations: } A^* &= J_A A' + K_A' A \\ &= (XY + Y'B)A' + (Y'A + B')'A = XYA' + Y'BA' + ABY \end{aligned}$$

$$\begin{aligned} B^* &= J_B B' + K_B' B \\ &= YB' + (XA + Y)'B = YB' + X'Y'B + Y'A'B \end{aligned}$$

JK Flip-Flops Analysis Example: Transition Table

- Transition* equations:

$$A^* = XYA' + Y'A'B + ABY$$

$$B^* = YB' + X'Y'B + Y'A'B$$

1. List all possible states

		XY			
B	A	00	01	10	11
0	0	00	10	00	11
0	1	00	10	00	10
1	0	11	00	11	01
1	1	10	01	00	01
		B*A*			

For B*A*, plug in appropriate equations for each entry.

For A=0, B=1, X=0, Y=0:

$$\begin{aligned} A^* &= XYA' + Y'A'B + ABY \\ &= 0 \cdot 0 \cdot 1 + 1 \cdot 1 \cdot 1 + 0 \cdot 1 \cdot 0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} B^* &= YB' + X'Y'B + Y'A'B \\ &= 0 \cdot 0 + 1 \cdot 1 \cdot 1 + 1 \cdot 1 \cdot 1 \\ &= 1 \end{aligned}$$

Thus entry is 11.

2. Determine Next State of BA for all permutations of the inputs

JK Flip-Flops Analysis Example: State Table

Give each numerical state an alphanumeric name:

00 = H

01 = I

10 = J

11 = K

Substitute into **Transition Table**
to form the **State Table**:

S	XY			
	00	01	10	11
H	H	J	H	K
I	H	J	H	J
J	K	H	K	I
K	J	I	H	I
S*				

JK Flip-Flops Analysis Example: State/Output Table

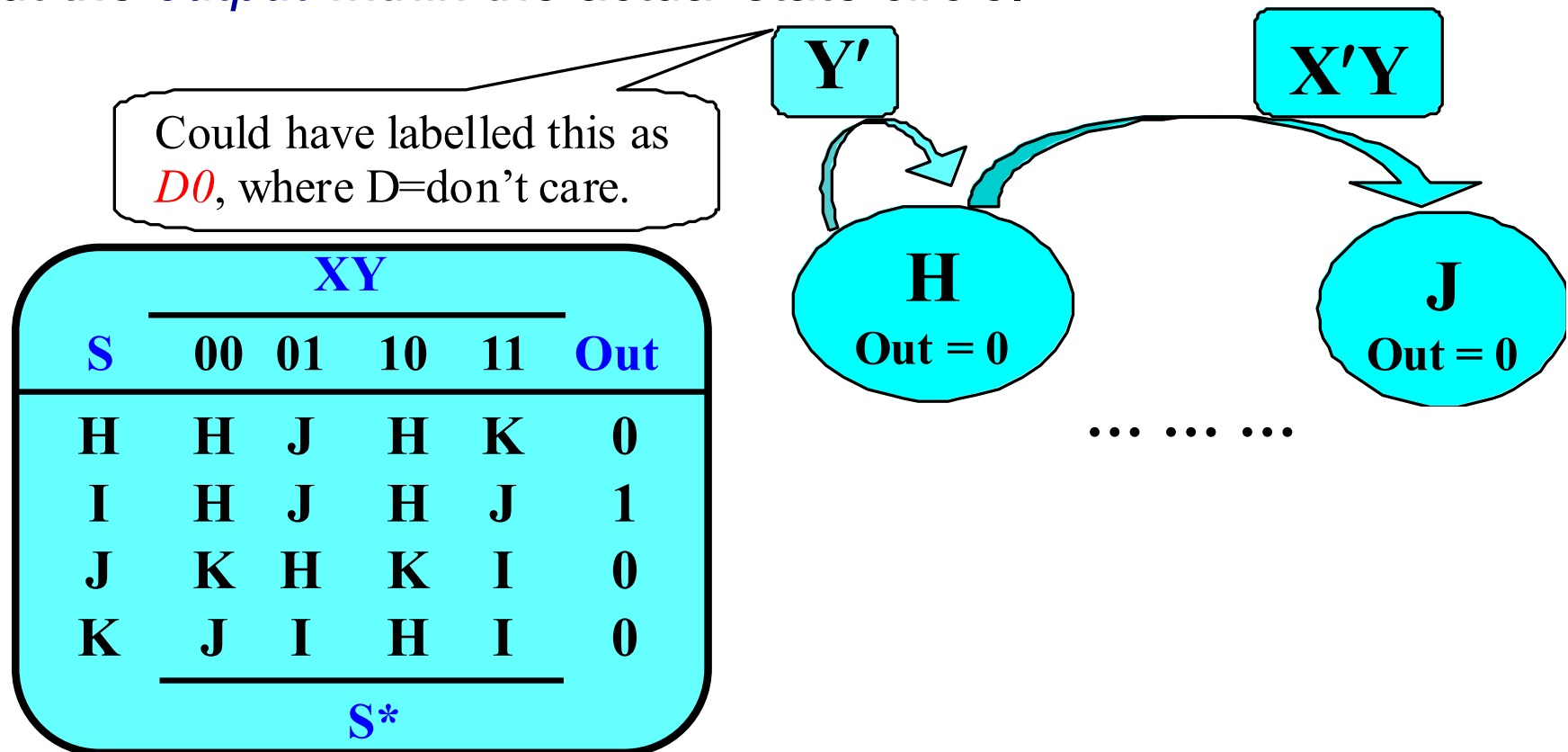
Expand your **State Table** to include the output **Out** to form the **State/Output Table**.

This is a Moore Machine – the outputs are *only* a function of the state. Thus, they *do not change* per input and can be listed in their own column.

S	XY				Out
	00	01	10	11	
H	H	J	H	K	0
I	H	J	H	J	1
J	K	H	K	I	0
K	J	I	H	I	0
S*					

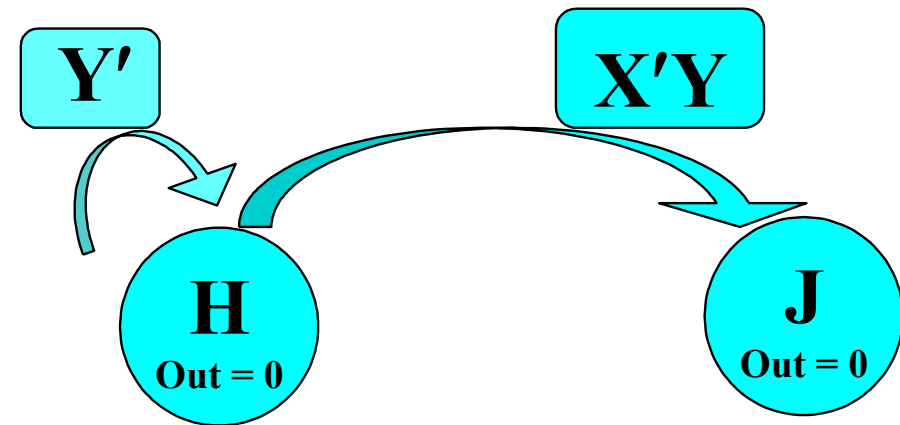
JK Flip-Flops Analysis Example: State Diagram Development Process (1/2)

- Development Process:** Same as before with a Mealy Machine, but can put the *output* within the actual state circle.



JK Flip-Flops Analysis Example: State Diagram Development Process (2/2)

- Complete the State Diagram, based on the information in the State/Output Table.



To be completed in class ...

Time Delay & Maximum Clock Speed (1/3)

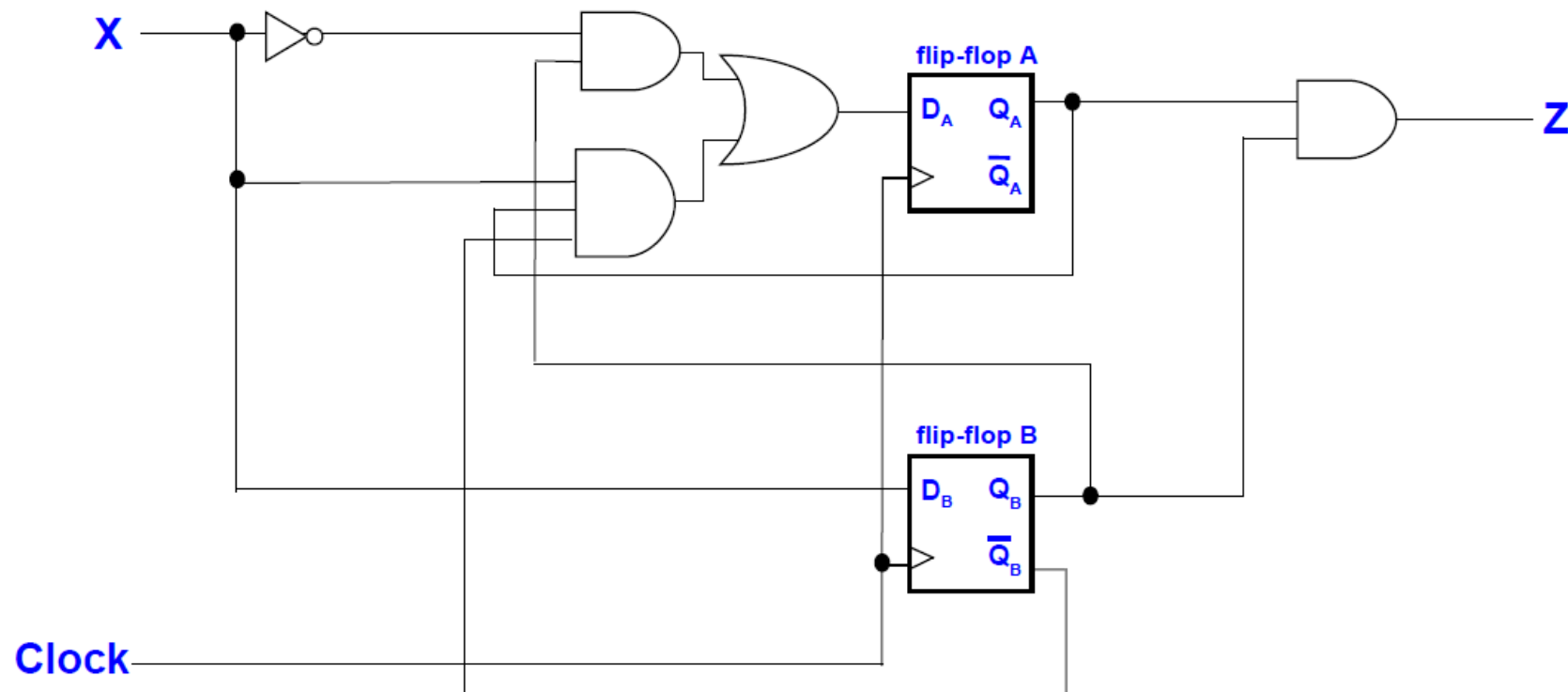
- Gate delay limits the speed of state machines...
- Feedback logic delay dictates **highest clock frequency**
- For circuit with **max delay** T_D , **max clock frequency** is:

$$F_{max} < \frac{1}{T_D}$$

- Attempt to **clock** $> F_{max}$ and circuit will be **unstable**

Time Delay & Maximum Clock Speed (2/3)

- Assume all gates have 7ns delay ...



Time Delay & Maximum Clock Speed (3/3)

- Assume all gates have 7ns delay ...

