EBU4375: SIGNALS AND SYSTEMS

LECTURE 7: PART 1

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Response and the Convolution

- Response to an Impulse **(CT Systems)**
- Response to a General Input **(CT Systems)**
- Convolution Integral **(CT Systems)**
- Convolution Algebra **(CT Systems)**
- Response to an Impulse **(DT Systems)**
- Response to a General Input **(DT Systems)**
- Convolution Sum **(DT Systems)**
- Sequence Convolution Algebra **(DT Systems)**

Response to an Impulse (CT Systems)

The impulse-response (IR) $h(t)$ of a continuous-time LTI system is defined to be the response following excitation by the signal $\delta(t)$ i.e.

$$h(t) = T\{\delta(t)\}$$

system response to the impulse

"impulse" excitation

Transfer Function

Response to a General Input (CT Systems)

Recall the earlier result that a general signal could be expressed by

$$x(t) = \int_{-\infty}^{\infty} d\tau \, x(\tau)\delta(t-\tau)$$

Since the system is **linear**, the response $y(t)$ to an excitation $x(t)$ can be written as

$$y(t) = T\{x(t)\} = T\left\{ \int_{-\infty}^{\infty} d\tau \, x(\tau)\delta(t-\tau) \right\}$$

$$= \int_{-\infty}^{\infty} d\tau \, x(\tau) T\{\delta(t-\tau)\} \qquad (1)$$

Time-invariance implies

$$h(t-\tau) = T\{\delta(t-\tau)\} \qquad (2)$$

$$(2) \to (1) \Rightarrow y(t) = \int_{-\infty}^{\infty} d\tau \, x(\tau) h(t-\tau) \qquad (3)$$

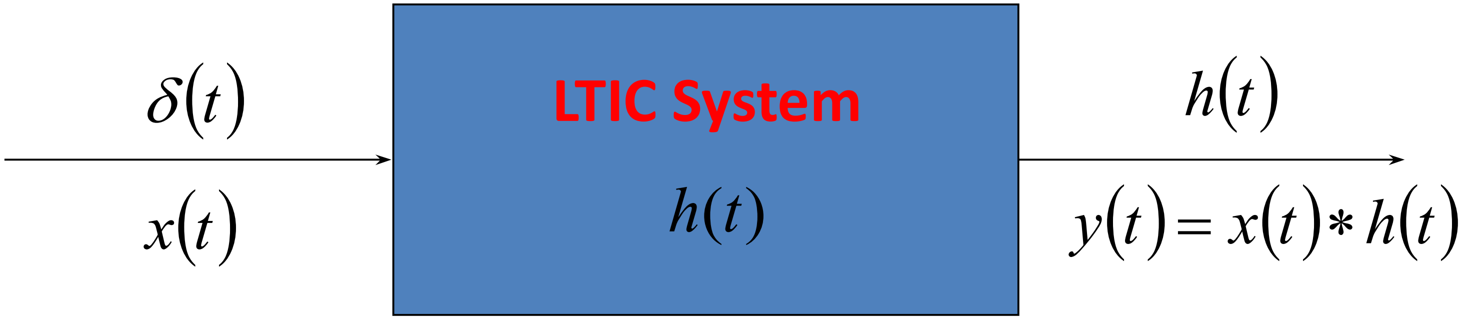
(3) says that the continuous-time response of an LTI system is entirely characterised by it's impulse response $h(t)$.

Convolution Integral (CT Systems)

Equation (3) ***defines*** the convolution operation, i.e.

$$y(t) = x(t) * h(t) \equiv \int_{-\infty}^{\infty} d\tau \, x(\tau) h(t-\tau) \qquad (4)$$

so that (4) is the *convolution integral*.



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LECTURE 7: PART 2

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Convolution Algebra (CT Systems)

Commutation: $$x(t)*h(t)=h(t)*x(t)$$

Association: $$\{x(t)*h_1(t)\}*h_2(t)=x(t)*\{h_1(t)*h_2(t)\}$$

Distribution: $$x(t)*\{h_1(t)+h_2(t)\}=x(t)*h_1(t)+x(t)*h_2(t)$$

Goto http://mathworld.wolfram.com/Convolution.html to get a dynamic appreciation of convolution.