HW 6

2.42 $X \sim Geom(p)$ $p(k) = (1-p)^{k-1} \cdot p$

(a) $y = x^2 = g(x)$ $g^{-1}(y) = \sqrt{y}$

$\therefore f_Y(y) = f_X(\sqrt{y})\left(\frac{d}{dy}\sqrt{y}\right)$

$\cancel{P(Y=k)} \overset{||}{p(y)} = \frac{1}{2\sqrt{y}} \cdot (1-p)^{\sqrt{y}-1} \cdot p$

(b) $y = x+3$ $g^{-1}(y) = y-3$

$p(y)\ \cancel{P(Y=k)} = f_X(y-3) = (1-p)^{y-3} \cdot p$

2.49 $Y = g(X)$ $y = g(x) = 2F(x) + 4$ $g^{-1}(y) = F^{-1}\left(\frac{y-4}{2}\right)$

(a) $\cancel{f_Y(y) = f_X(g^{-1}(y)) \cdot \left|\frac{d}{dy} g^{-1}(y)\right|}$ $F'(x) = f_X(x)$

$\therefore f_Y(y) = f_X\left(F^{-1}\left(\frac{y-4}{2}\right)\right) \cdot \left|\frac{d}{dy} \cdot F^{-1}\left(\frac{y-4}{2}\right)\right|$

(b) $Y \sim N(8, 10)$ $\cancel{\mu_Y = 8 \quad \sigma_Y = \sqrt{10}}$

$\cancel{f_Y(y) = \frac{1}{\sqrt{20\pi}} e^{-\frac{(y-8)^2}{20}}}$ $f(y) = \frac{1}{2}$ $8 \le y \le 10$

$\cancel{\frac{1}{2} = g(x)}$ $y = g(x)$

$\frac{1}{2} = f(g^{-1}(y)) \cdot \frac{d}{dy}(g^{-1}(y))$

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2.50 $X \sim Exp(\lambda)$ $f_X(x) = \lambda e^{-\lambda x}$, $x \geq 0$

Prove: $cX \sim Exp(\frac{\lambda}{c})$

Let $Y = cX$ $y = cx$

$f_Y(y) = f_X(\frac{y}{c}) \cdot \frac{1}{c} = \lambda e^{-\lambda \frac{y}{c}} \cdot \frac{1}{c} = \frac{\lambda}{c} e^{-\frac{\lambda}{c} \cdot y} = Exp(\frac{\lambda}{c})$

2.52 $T \sim Exp(\lambda)$ $E(T) = \frac{1}{\lambda} = 5$ $\lambda = \frac{1}{5}$ $F_T(x) = 1 - e^{-\lambda x}$

$P(T > t+4 | T > t) = P(T > 4) = 1 - P(T \leq 4) = 1 - F_T(4)$

$= 1 - (1 - e^{-\frac{1}{5} \times 4}) = e^{-0.8}$

2.53

(a) $X \sim N(\mu, 0.01\mu^2)$ $\sigma = 0.1\mu$

$X(1.15\mu) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(1.15\mu - \mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi} \cdot 0.1\mu} e^{-\frac{0.15^2}{0.02}}$

(b) $P(X \leq b) = 90\% \Rightarrow \Phi(\frac{b-\mu}{\sigma}) - \cancel{\Phi(\frac{a-\mu}{\sigma})} = 90\%$

$\frac{b-\mu}{\sigma} = 1.29 \Rightarrow \frac{X-4}{0.4} = 1.29$

$X = 4.516$

2.54 $X \sim N(1, 4)$ $\mu = 1$ $\sigma = 2$ $Z = \frac{X-1}{2}$

(a) $P(X \leq 3) = \Phi(1) = 0.84134$ (b) $P(X > 1.5) = 1 - \Phi(0.25) \approx 0.41$

(c) $P(X = 1) = \Phi(0) - \Phi(0^-) = 0$

(d) $P(2 < X < 5) = \Phi(2) - \Phi(0.5) = 0.97 - 0.69 = 0.28$

(e) $P(X \geq 0) = 1 - \Phi(-0.25) = 1 - (1 - \Phi(0.25)) = 0.59$

(f) $P(-1 < X < 0.5) = \Phi(-0.25) - \Phi(-1) = \Phi(1) - \Phi(0.25) = 0.25$

(g) $P(-2 \leq X \leq 2) = \Phi(0.5) - \Phi(-1.5) = \Phi(0.5) + 1 - \Phi(1.5) = 1.69 - 0.93 = 0.76$

(h) $P(-2.5 \leq X \leq 1) = \Phi(0) - \Phi(-1.75) = -\frac{1}{2} + \Phi(1.75) = 0.95 - 0.5 = 0.45$

2.56 $\int_0^{+\infty} e^{-4x^2} dx$ $\because \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1$

$\Rightarrow \int_{-\infty}^{+\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$ $e^{-\frac{x^2}{2}}$ is an even function

$\int_0^{\infty} e^{-\frac{x^2}{2}} dx = \frac{\sqrt{2\pi}}{2}$

$\int_0^{\infty} e^{-4x^2} dx = \frac{1}{2\sqrt{2}} \int_0^{\infty} e^{-\frac{(2\sqrt{2}x)^2}{2}} d(2\sqrt{2}x) = \frac{\sqrt{\pi}}{4}$

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2.57 $X \sim N(0,2)$ $\mu=0$ $\sigma=\sqrt{2}$ $Z=\frac{X}{\sqrt{2}}$

(a) $P(1 \le X \le 2) = \Phi(\sqrt{2}) - \Phi\left(\frac{\sqrt{2}}{2}\right)$

(b) $P(1 \le X \le 2 \mid X \ge 1) = \frac{P(1 \le X \le 2)}{P(X \ge 1)} = \frac{\Phi(\sqrt{2}) - \Phi\left(\frac{\sqrt{2}}{2}\right)}{1-\Phi\left(\frac{\sqrt{2}}{2}\right)}$

2.58

(a) $f_Y(y) = f_X\left(\frac{y-4}{2}\right) \cdot \frac{1}{2} = \frac{1}{4\sqrt{\pi}} e^{-\frac{\left(\frac{y-4}{2}-5\right)^2}{4}}$

(b) $E(X)=5$ $E(Y)=2E(X)+4=14$

$Var(Y) = 4Var(X) = 8$

2.59 $X \sim N(\mu, \sigma^2)$ $Z=\frac{X-\mu}{\sigma}$

$P(X<116) = \Phi\left(\frac{X-\mu}{\sigma} < \frac{116-\mu}{\sigma}\right) = 0.2 \Rightarrow 1-\Phi\left(\frac{\mu-116}{\sigma}\right)=0.2$

$\Phi\left(\frac{328-\mu}{\sigma}\right)=0.9$ $\Phi\left(\frac{\mu-116}{\sigma}\right)=0.8$

$\begin{cases} \frac{\mu-116}{\sigma} = 0.85 \\ \frac{328-\mu}{\sigma} = 1.3 \end{cases}$ $\begin{cases} \mu-116 = 0.85\sigma \\ 328-\mu = 1.3\sigma \end{cases}$ $2.15\sigma = 212 \Rightarrow \sigma = 98.6$

$\mu = 199.81$

2.60

(a) $X \sim U(a,b)$ $m=\frac{b-a}{2}$

(b) $X \sim N(\mu, \sigma^2)$ $F(m)=\frac{1}{2} \Rightarrow \frac{X-\mu}{\sigma}=0$ $m=\mu$

(c) $X \sim Exp(\lambda)$ $F(m)=\frac{1}{2} \Rightarrow 1-e^{-\lambda m}=\frac{1}{2}$

$e^{-\lambda m}=\frac{1}{2}$ $\lambda m = \ln 2$ $m=\frac{\ln 2}{\lambda}$