EBU4375: SIGNALS AND SYSTEMS

LECTURE 5

Queen Mary
University of London

More on Basic Time Signals

- Rectangular Function
- Signum Function
- Ramp Function
- Sinc Function
- CT Unit Impulse Function (recap)
- DT Unit Impulse Sequence (recap)
- Representation of Signals using Impulse Sequence (DT Signals)
- Representation of Signals using Impulse Function (CT Signals)

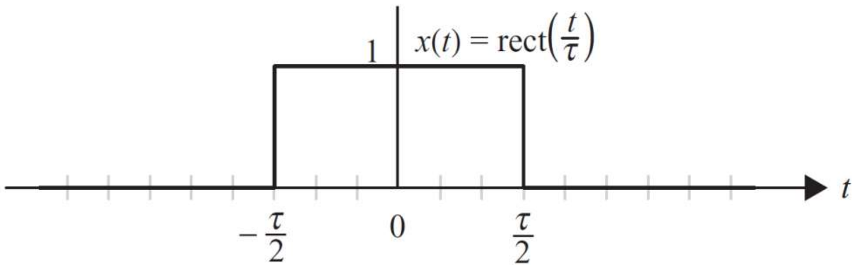
Basic Time Signals – Rectangular Function

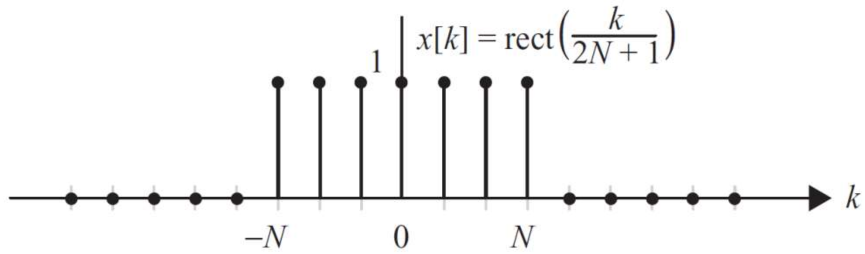
The CT rectangular pulse $\text{rect}(t/\tau)$ is defined as follows:

$$\text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| \le \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

The DT rectangular pulse $\text{rect}(k/(2N+1))$ is defined as follows:

$$\text{rect}\left(\frac{k}{2N+1}\right) = \begin{cases} 1 & |k| \le N \\ 0 & |k| > N \end{cases}$$





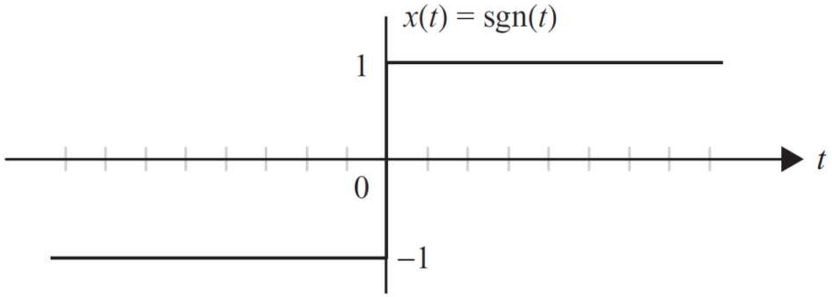
Basic Time Signals – Signum Function

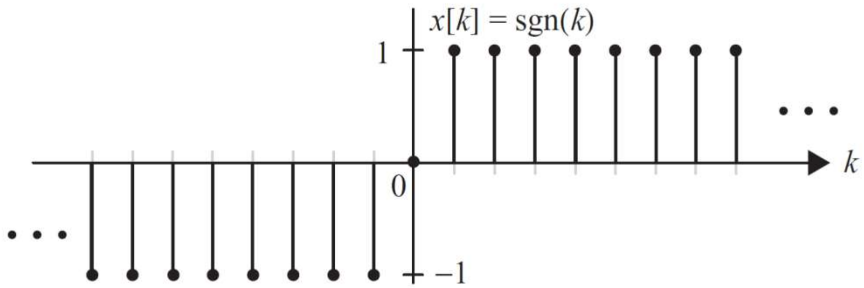
The *signum* (or *sign*) function, denoted by sgn(t), is defined as follows:

$$\mathrm{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0. \end{cases}$$

The DT signum function, denoted by sgn(k), is defined as follows:

$$\mathrm{sgn}[k] = \begin{cases} 1 & k > 0 \\ 0 & k = 0 \\ -1 & k < 0 \end{cases}$$

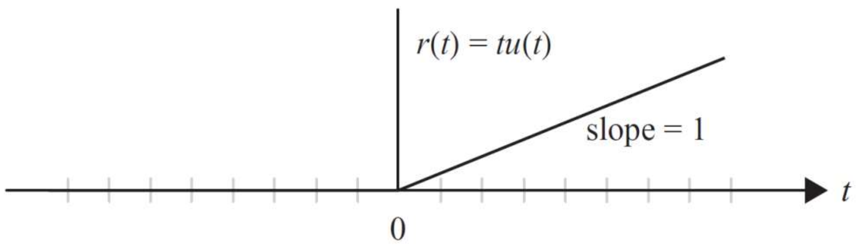


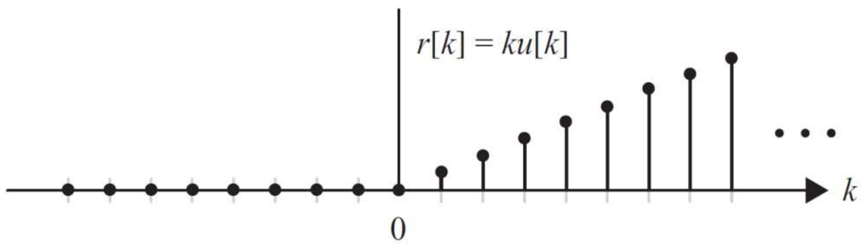


Basic Time Signals – Ramp Function

$$r(t) = tu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0, \end{cases}$$

$$r[k] = ku[k] = \begin{cases} k & k \geq 0 \\ 0 & k < 0, \end{cases}$$

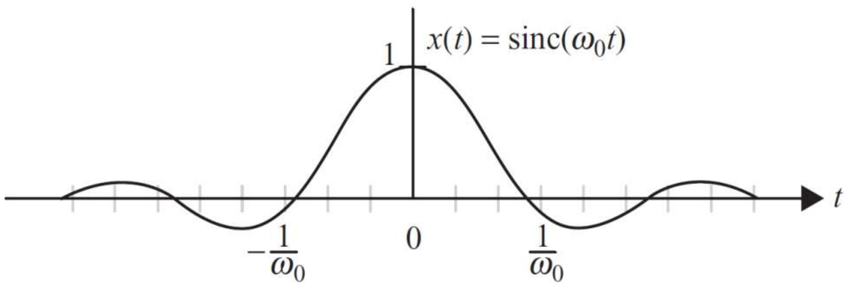


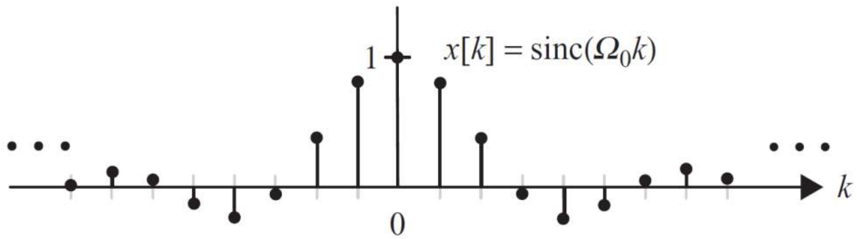


Basic Time Signals – Sinc Function

$$\mathrm{sinc}(\omega_0 t) = \frac{\sin(\pi \omega_0 t)}{\pi \omega_0 t}$$

$$\mathrm{sinc}(\Omega_0 k) = \frac{\sin(\pi \Omega_0 k)}{\pi \Omega_0 k}$$



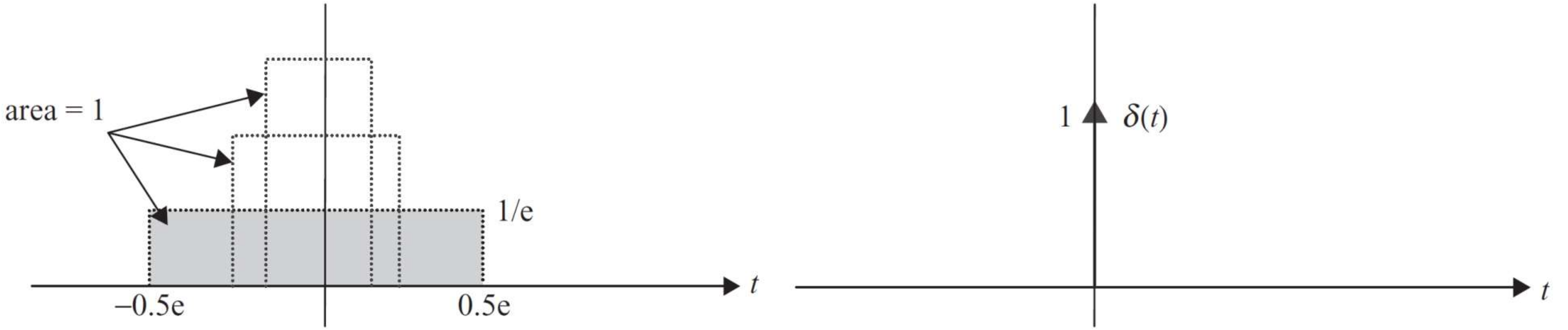


Basic Time Signals – CT Unit Impulse Function (Recap)

The *unit impulse* function $\delta(t)$, also known as the *Dirac delta* function or simply the *delta* function, is defined in terms of two properties as follows:

(1) amplitude $\quad \delta(t) = 0, \quad t \neq 0;$

(2) area enclosed $\quad \int_{-\infty}^{\infty} \delta(t)\mathrm{d}t = 1.$



(a) (b)

Impulse function $\delta(t)$.
(a) Generating the impulse function $\delta(t)$ from a rectangular pulse. (b) Notation used to represent an impulse function.

Basic Time Signals – CT Unit Impulse Function (Recap)

Properties of impulse function

(i) The impulse function is an even function, i.e. $\delta(t) = \delta(-t)$.

(ii) Integrating a unit impulse function results in one, provided that the limits of integration enclose the origin of the impulse. Mathematically,

$$\int_{-T}^{T} A\delta(t - t_0)\mathrm{d}t = \begin{cases} A & \text{for } -T < t_0 < T \\ 0 & \text{elsewhere.} \end{cases}$$

(iii) The scaled and time-shifted version $\delta(at + b)$ of the unit impulse function is given by

$$\delta(at + b) = \frac{1}{a}\delta\left(t + \frac{b}{a}\right).$$

(iv) When an arbitrary function $\phi(t)$ is multiplied by a shifted impulse function, the product is given by

$$\phi(t)\delta(t - t_0) = \phi(t_0)\delta(t - t_0).$$

Basic Time Signals – CT Unit Impulse Function (Recap)

In other words, multiplication of a CT function and an impulse function produces an impulse function, which has an area equal to the value of the CT function at the location of the impulse. Combining properties (ii) and (iv), it is straightforward to show that

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)\mathrm{d}t = \phi(t_0).$$

(v) The unit impulse function can be obtained by taking the derivative of the unit step function as follows:

$$\delta(t) = \frac{\mathrm{d}u}{\mathrm{d}t}.$$

(vi) Conversely, the unit step function is obtained by integrating the unit impulse function as follows:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau.$$

Basic Time Signals – DT Unit Impulse Sequence (Recap)

- recall the *unit impulse* signal: a sequence that is zero everywhere except at sample 0:

$$\delta(n) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \neq 0 \end{cases}$$

- any discrete-time signal can be represented as a sum of scaled and shifted unit impulses, i.e.

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

- this may seem complicated, but will become handy in a minute!

Basic Time Signals – DT Unit Impulse Sequence (Recap)

The DT impulse function, also referred to as the Kronecker delta function or the DT unit sample function, is defined as follows:

$$\delta[k] = u[k] - u[k-1] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0. \end{cases}$$