DT Convolution Examples

x[k]

$h_1[k]$

$h_2[k]$

$h_3[k]$
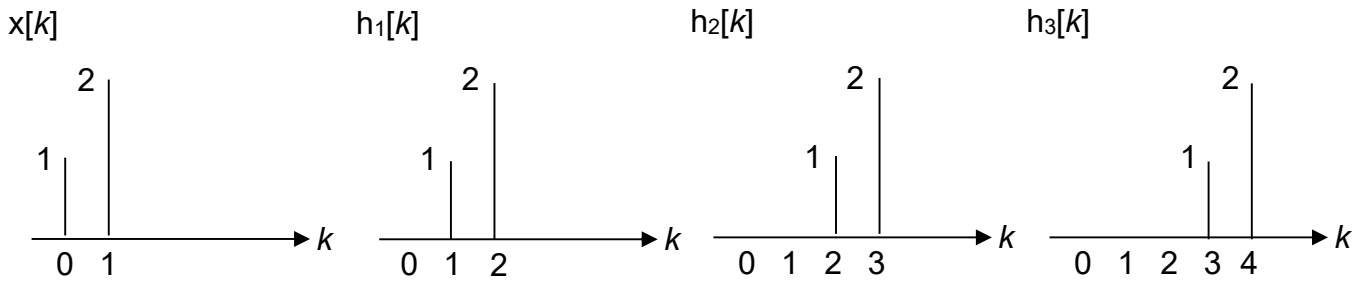
Question 1

(i) Determine and plot the overall impulse response $h_{total}[k]$.
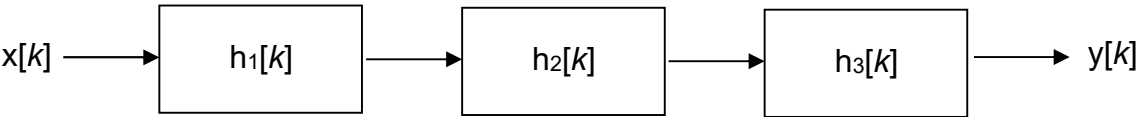
k	-3	-2	-1	0	1	2	3	4	5	
h1[k]				0	1	2				
h2[-k]	2	1	0	0						h12[0] = 0
h2[-k+1]		2	1	0	0					h12[1] = 0
h2[-k+2]			2	1	0	0				h12[2] = 0
h2[-k+3]				2	1	0	0			h12[3] = 1
h2[-k+4]					2	1	0	0		h12[4] = 4
h2[-k+5]						2	1	0	0	h12[5] = 4

k	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	
h12[k]					0	0	0	1	4	4					
h3[-k]	2	1	0	0	0	0									hT[0] = 0
h3[-k+1]		2	1	0	0	0									hT[1] = 0
h3[-k+2]			2	1	0	0	0								hT[2] = 0
h3[-k+3]				2	1	0	0	0							hT[3] = 0
h3[-k+4]					2	1	0	0	0						hT[4] = 0
h3[-k+5]						2	1	0	0	0					hT[5] = 0
h3[-k+6]							2	1	0	0	0				hT[6] = 1
h3[-k+7]								2	1	0	0	0			hT[7] = 6
h3[-k+8]									2	1	0	0	0		hT[8] = 12
h3[-k+9]										2	1	0	0	0	hT[9] = 8

$h_T[k]$
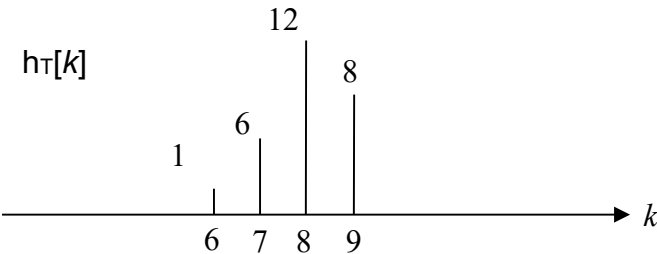
(ii) Determine and plot the output signal y[k].

k	-1	0	1	2	3	4	5	6	7	8	9	10			
hT[k]								1	6	12	8				
x[-k]	2	1													y[0] = 0
x[-k+1]		2	1												y[1] = 0
x[-k+2]			2	1											y[2] = 0
x[-k+3]				2	1										y[3] = 0
x[-k+4]					2	1									y[4] = 0
x[-k+5]						2	1								y[5] = 0
x[-k+6]							2	1							y[6] = 1
x[-k+7]								2	1						y[7] = 8
x[-k+8]									2	1					y[8] = 24
x[-k+9]										2	1				y[9] = 32
x[-k+10]											2	1			y[10] = 16
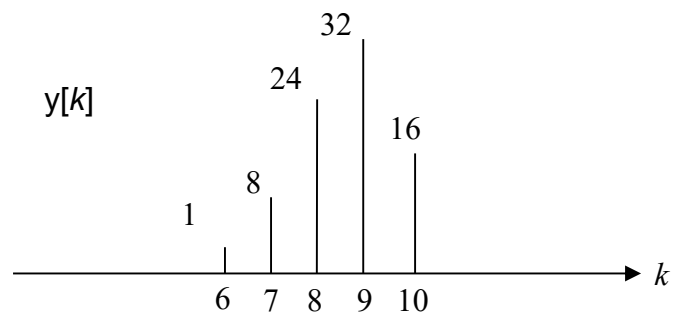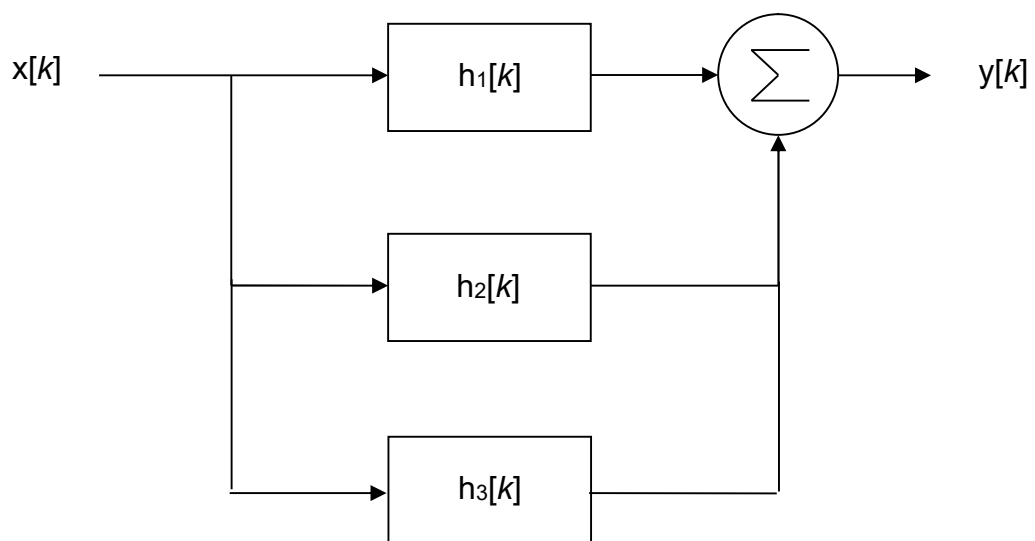
y[k]

32

24

16

8

1

6 7 8 9 10   k

Question 2

x[k] → h₁[k], h₂[k], h₃[k] → Σ → y[k]

(i) Determine and plot the total impulse response h$_{total}$[k].

h$_T$[k]

```
          3   3
              
              2
  1
                    
          │ │ │ 
  │   │ │ │ │ │           → k
  1   2 3   4
```
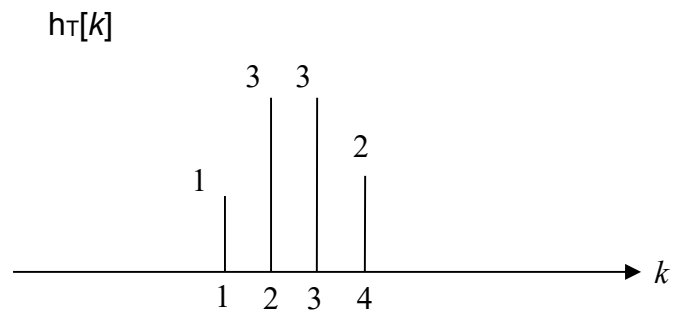
(ii) Determine and plot the output signal y[k].

k	-1	0	1	2	3	4	5	
hT[k]		0	1	3	3	2		
x[-k]	2	1						y[0] = 0
x[-k+1]		2	1					y[1] = 1
x[-k+2]			2	1				y[2] = 5
x[-k+3]				2	1			y[3] = 9
x[-k+4]					2	1		y[4] = 8
x[-k+5]						2	1	y[5] = 4

y[k]

```
              9
              
              8
                    
       5      
              4
  1           
                    
  │   │ │ │ │ │       → k
  1   2 3 4 5
```
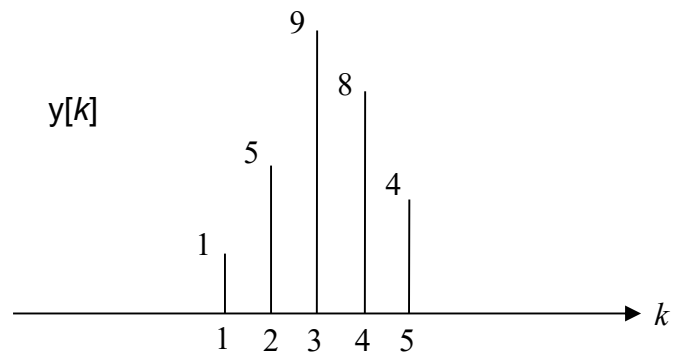
Question 3
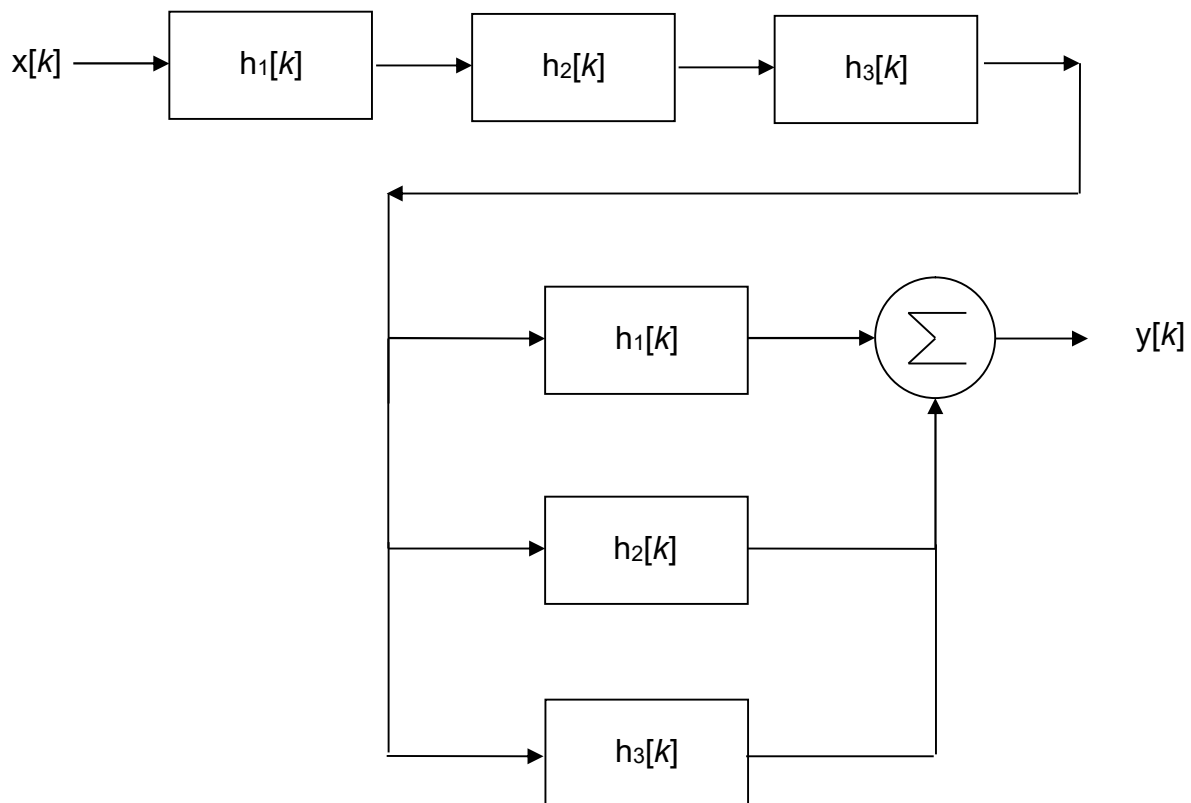
(i) Determine and plot the total impulse response $h_{total}[k]$.

k	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
hT1[k]											1	6	12	8						
hT2[-k]	2	3	3	1	0															hT[0]=0
hT2[-k+1]		2	3	3	1	0														hT[1]=0
hT2[-k+2]			2	3	3	1	0													hT[2]=0
hT2[-k+3]				2	3	3	1	0												hT[3]=0
hT2[-k+4]					2	3	3	1	0											hT[4]=0
hT2[-k+5]						2	3	3	1	0										hT[5]=0
hT2[-k+6]							2	3	3	1	0									hT[6]=0
hT2[-k+7]								2	3	3	1	0								hT[7]=1
hT2[-k+8]									2	3	3	1	0							hT[8]=9
hT2[-k+9]										2	3	3	1	0						hT[9]=33
hT2[-k+10]											2	3	3	1	0					hT[10]=64
hT2[-k+11]												2	3	3	1	0				hT[11]=72
hT2[-k+12]													2	3	3	1	0			hT[12]=48
hT2[-k+13]														2	3	3	1	0		hT[13]=16

$h_T[k]$

72
64
48
33
16
9
1

7 8 9 10 11 12 13 k

(ii) Determine and plot the output signal y[k].

$x[k]$

2
1

0 1 k

$h_T[k]$

72
64
48
33
16
9
1
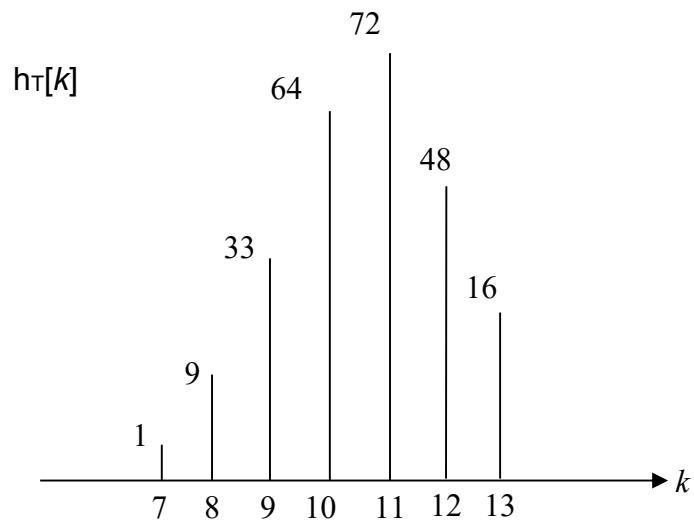
7 8 9 10 11 12 13 k

$x[k]$ → | $h_T[k]$ | → $y[k]$
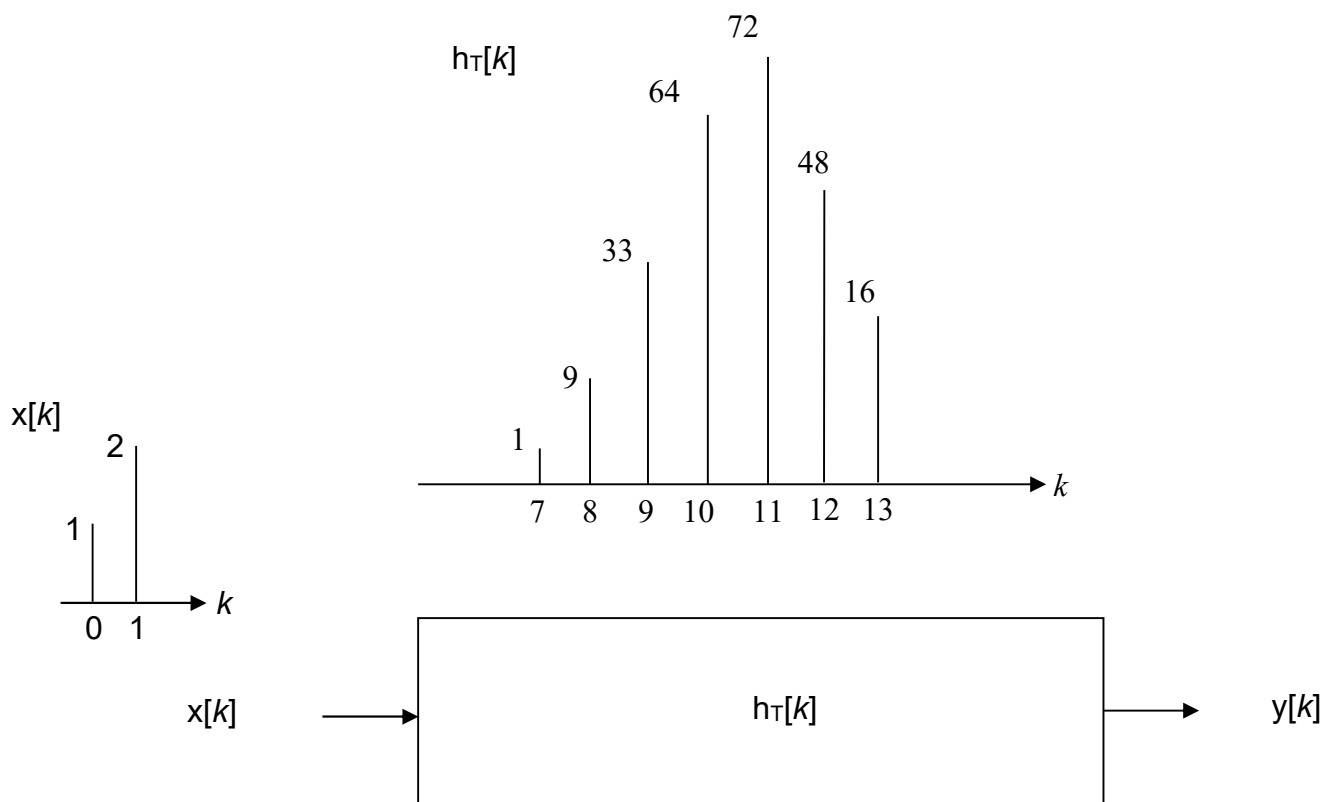
k	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
hT[k]									1	9	33	64	72	48	16		
x[-k]	2	1															y[0]=0
x[-k+1]		2	1														y[1]=0
x[-k+2]			2	1													y[2]=0
x[-k+3]				2	1												y[3]=0
x[-k+4]					2	1											y[4]=0
x[-k+5]						2	1										y[5]=0
x[-k+6]							2	1									y[6]=0
x[-k+7]								2	1								y[7]=1
x[-k+8]									2	1							y[8]=11
x[-k+9]										2	1						y[9]=51
x[-k+10]											2	1					y[10]=130
x[-k+11]												2	1				y[11]=200
x[-k+12]													2	1			y[12]=192
x[-k+13]														2	1		y[13]=112
x[-k+14]															2	1	y[14]=32

y[k]

200
192
130
112
51
32
11
1

7 8 9 10 11 12 13 14   k