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# LABORATORY REPORT

# REPRESENTATION OF CONTINUOUS-TIME PERIODIC SIGNALS USING FOURIER SERIES AND FOURIER ANALYSIS

## INTRODUCTORY SUMMARY

Last week we have a laboratory about Representation of Continuous-time Periodic Signals by using MATLAB to analysis the signals. By this laboratory, we demonstrated the properties of the continuous-time Fourier series, explored analysing and synthesising periodic signals with complex exponentials, computed the discrete-time Fourier series (DTFS) of a periodic discrete-time signal, synthesised a signal with its DTFS, and studied the numerical approximation to the CTFT.

## LAB MATERIALS

Our lab analyse the given signals and given functions, by using MATLAB we draw diagrams to show the result of signals and functions, to test and verify the rules of signals.

## LAB PROCEDURE

Once we get a signal or a function, we first theoretically analyse it, and assume a result which could be generated by the test. Then we write the simulative code of MATLAB for test and verify. Finally we generate the result diagram and analyse it.

Step 1

$$x_1(t) = \cos(\omega_0 t) + \sin(2\omega_0 t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{2j}e^{j2\omega_0 t} - \frac{1}{2j}e^{-j2\omega_0 t}$$

Analysis this function, we got this result:

$$c_k = \begin{cases} \frac{1}{2}, k = \pm 1 \\ \frac{1}{2j}, k = 2 \\ -\frac{1}{2j}, k = -2 \\ 0, otherwise \end{cases}$$

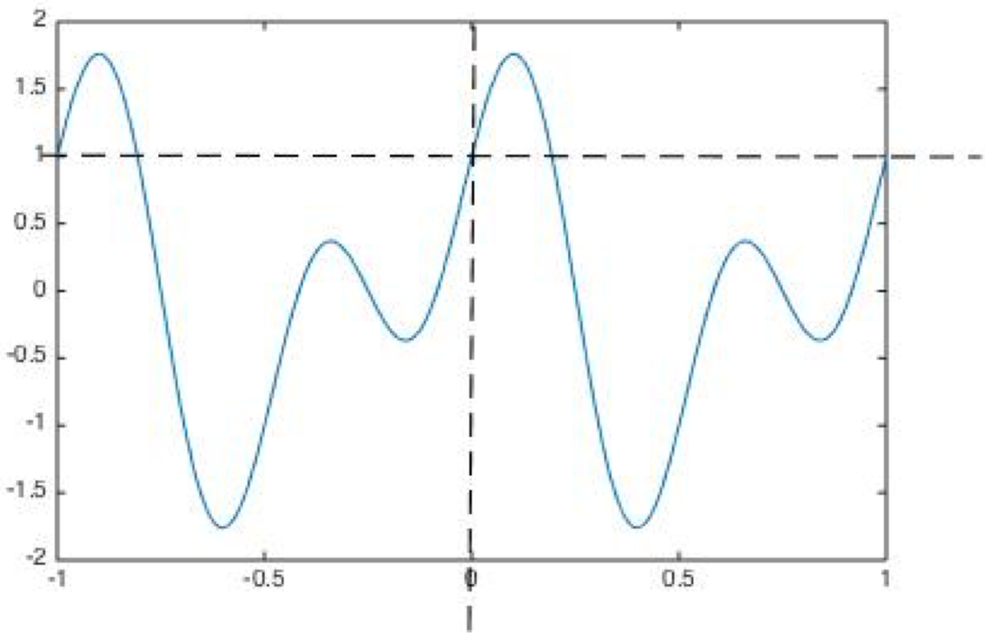
Step 2

The result of theoretically analysis is not very complex, so we can easily get the simulative code:

```
t=linspace(-1, 1, 1000);
w0=2*pi;
x1=cos(w0*t)+sin(2*w0*t);
plot(t,x1)
```

Step 3

Running this Code on MATLAB, we got a diagram like this:




It's easy to see that the fundamental period of this signal is 1. And we verified that:

For Continuous-Time Fourier Series:

$$x(t+T) = x(t), \quad \textit{for all } t$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \quad \text{where} \quad \omega_0 = \frac{2\pi}{T_o}.$$

## PROBLEMS ENCOUNTERED

The entire lab procedure went as planned. The only problem was that some original signal does not have imaginary part at all, but when we use MATLAB to transform it by using the Fourier Transform and transform it back. The imaginary part appears. This tells us that MATLAB may make errors. So we should deal with the result data properly.

## CONCLUSION

For Continuous-Time Fourier Series:

$$x(t+T)=x(t), \quad \textit{for all } t$$

$$x(t)=\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}\,, \quad \text{where} \quad \omega_0=\frac{2\pi}{T_o}.$$

The complex Fourier coefficient $a_k$ is given by

$$c_k=\frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}dt=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j\omega_0 t}dt$$

$$x(-t)=\sum_{k=-\infty}^{\infty} c_k e^{-jk\omega_0 t}=\sum_{k=-\infty}^{\infty} c_{-k} e^{jk\omega_0 t}$$

For Synthesizing Continuous-Time Signals with the Fourier Series:

$$x_N(t)=\sum_{k=-N}^{N} c_k e^{jk\left(\frac{2\pi}{T_0}\right)t}$$

For Discrete-time Fourier Series:

$$x[n]=\sum_{k=0}^{N-1} c_k e^{jk(\frac{2\pi}{N})n}$$

$$c_k=\frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(\frac{2\pi}{N})n}$$

For Continuous-Time Fourier Transform:

$$X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

This conclusion is based on our textbook and our test and verify, so we suppose that this conclusion is reliable. We would be glad to do more labs by using MATLAB.

I will call you this week to discuss our study and possible follow-up you may wish us to do.

Sincerely,

Weidu Wang

Weidu Wang

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