EBU4375: SIGNALS AND SYSTEMS

LECTURE 2: PART 1

Queen Mary
University of London

Deterministic and Stochastic (Random) Signals

• *deterministic* signals have values that are *completely specified* at any given time (e.g. a sinusoid produced by a function-generator).

• *stochastic* signals take on random values at any given time and necessitate a statistical description (e.g. thermal noise in any device above absolute zero).

Even and Odd Signals (CT and DT Signals)

A CT signal $x_e(t)$ is said to be an even signal if

$$x_e(t) = x_e(-t).$$

Conversely, a CT signal $x_o(t)$ is said to be an odd signal if

$$x_o(t) = -x_o(-t).$$

A DT signal $x_e[k]$ is said to be an even signal if

$$x_e[k] = x_e[-k].$$

Conversely, a DT signal $x_o[k]$ is said to be an odd signal if

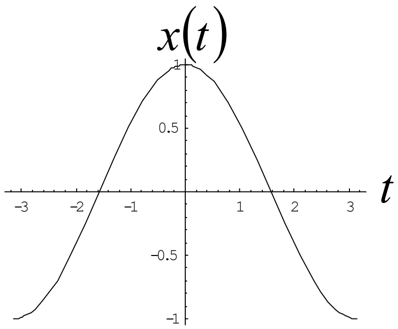
$$x_o[k] = -x_o[-k].$$

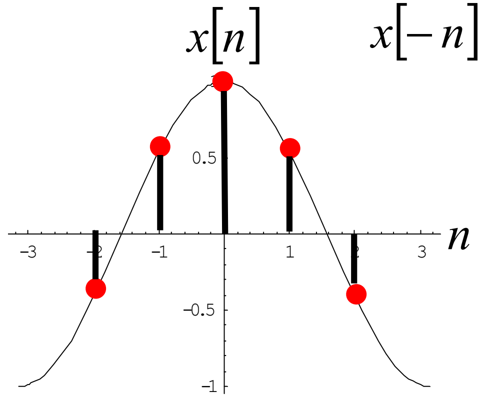
Even and Odd Signals (CT and DT Signals)

A signal $x(t)$ or $x[n]$ is termed an *even* signal if:

$$x(-t) = x(t)$$

$$x[-n] = x[n]$$



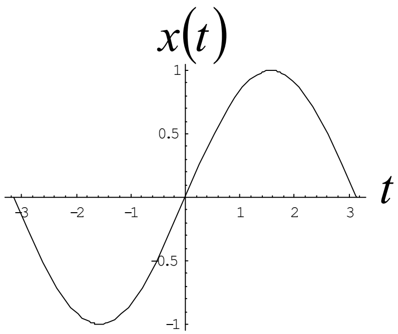


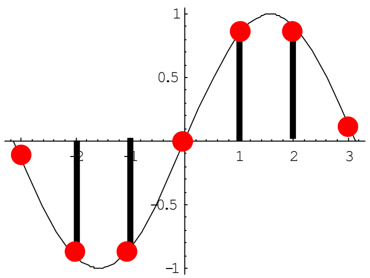
i.e. reflection symmetry in the ordinate axis

A signal $x(t)$ or $x[n]$ is termed an *odd* signal if:

$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$





i.e. inversion symmetry

Even and Odd Signals (CT and DT Signals)

Any signal $x(t)$ or $x[n]$ can be decomposed as the sum of an even $(x_e(t), x_e[n])$ and odd $(x_o(t), x_o[n])$ signal:

$$x(t) = x_e(t) + x_o(t)$$
$$x[n] = x_e[n] + x_o[n]$$

where

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} \qquad x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\} \qquad x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$$

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Energy and Power Signals (CT and DT Signals)

A signal $x(t)$, or $x[k]$, is called an *energy signal* if the total energy E_x has a non-zero finite value, i.e. $0 < E_x < \infty$. On the other hand, a signal is called a *power signal* if it has non-zero finite power, i.e. $0 < P_x < \infty$. Note that a signal cannot be both an energy and a power signal simultaneously. The energy signals have zero average power whereas the power signals have infinite total energy.

1. $x(t), x[n]$ are said to be an energy signal or sequence if $0 < E < \infty$, <u>and</u> $P = 0$

2. $x(t), x[n]$ are said to be a power signal or sequence if $0 < P < \infty \Rightarrow E = \infty$

Energy and Power Signals (CT and DT Signals)

The *total energy* of a CT signal is its energy calculated over the interval $t = [-\infty, \infty]$. Likewise, the total energy of a DT signal is its energy calculated over the range $k = [-\infty, \infty]$. The expressions for the total energy are therefore given by the following:

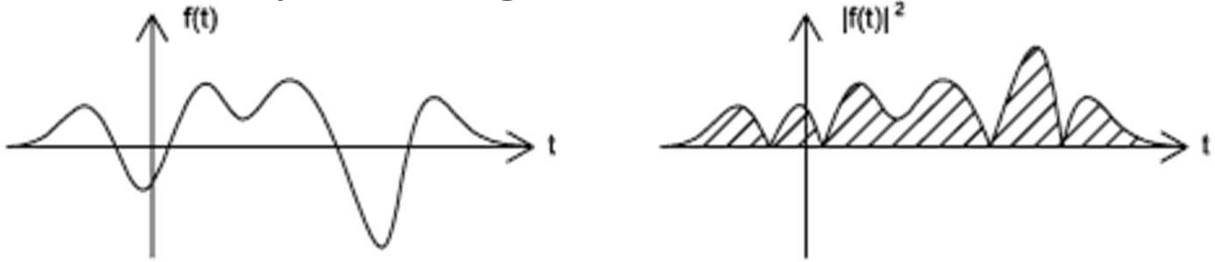
CT signals $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \mathrm{d}t;$$

DT sequences $$E_x = \sum_{k=-\infty}^{\infty} |x[k]|^2$$

Energy and Power Signals (CT and DT Signals)

• Since we often think of signal as a function of varying amplitude through time, it seems to reason that a good measurement of the strength of a signal would be the area under the curve.

• This suggests either squaring the signal or taking its absolute value, then finding the area under that curve. It turns out that what we call the ***energy*** of a signal is the area under the squared signal.



The energy of this signal is the shaded region.

$$E = \int_{-\infty}^{\infty} dt \left| f(t) \right|^2 \quad joules$$

Energy and Power Signals (CT and DT Signals)

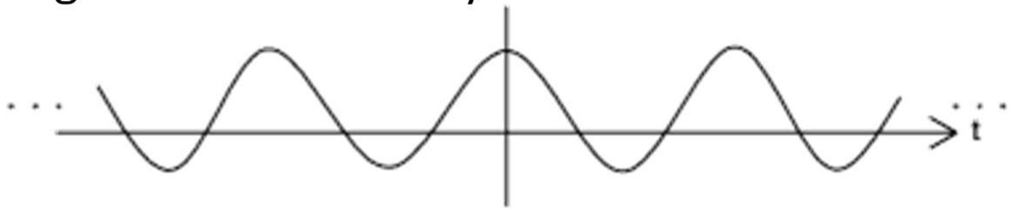
Since power is defined as energy per unit time, the *average power* of a CT signal $x(t)$ over the interval $t = (-\infty, \infty)$ and of a DT signal $x[k]$ over the range $k = [-\infty, \infty]$ are expressed as follows:

CT signals $$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \mathrm{d}t$$

DT sequences $$P_x = \lim_{k \to \infty} \frac{1}{2K + 1} \sum_{k=-K}^{K} |x[k]|^2$$

Energy and Power Signals (CT and DT Signals)

• Our definition of energy seems reasonable, and it is. However, what if the *signal does not decay*?



A simple, common signal with infinite energy.

• In this case we have infinite energy for any such signal. Does this mean that a fifty hertz sine wave feeding into your headphones is as strong as the fifty hertz sine wave coming out of your power outlet? Obviously not. This is what leads us to the idea of ***signal power***.

• Power is a time average of energy (energy per unit time). This is useful when the energy of the signal goes to infinity.

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \left| f(t) \right|^2 \quad watts$$

Energy and Power Signals (CT and DT Signals)

NB:

A periodic signal is a power signal if its energy content per period is finite, and then the average power of this signal need only be calculated over a period.

Energy and Power Signals (CT and DT Signals)

- **"Energy signals" have finite energy.**
- **"Power signals" have finite power.**

Are all energy signals also power signals?

No, any signal with finite energy will have zero power.

Are all power signals also energy signals?

No, any signal with finite power will have infinite energy.

Are *all* signals either energy or power signals?

No. Any infinite-duration, increasing-magnitude function will not be either. (eg $f(t) = t$ is neither)

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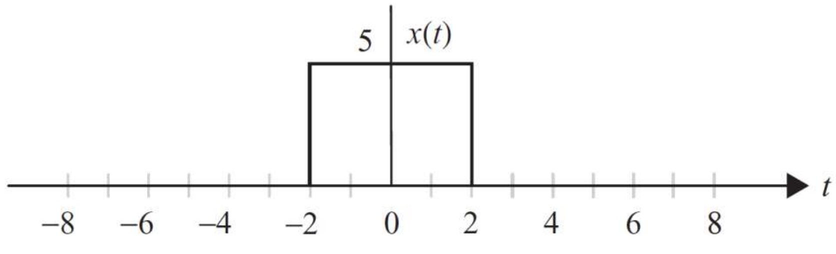
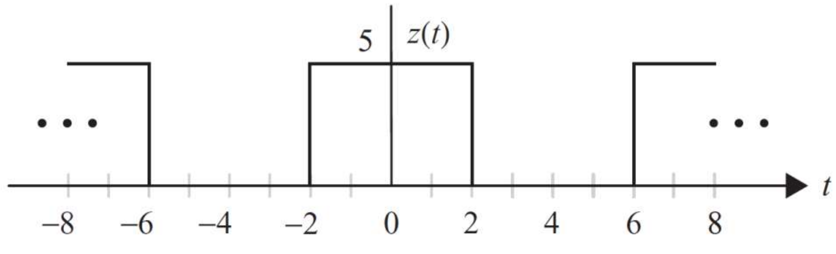
Energy and Power Signals (Example of CT Signals)

Example



(a)



(b)

Consider the CT signals shown in Figs. (a) and (b). Calculate the instantaneous power, average power, and energy present in the two signals. Classify these signals as power or energy signals.

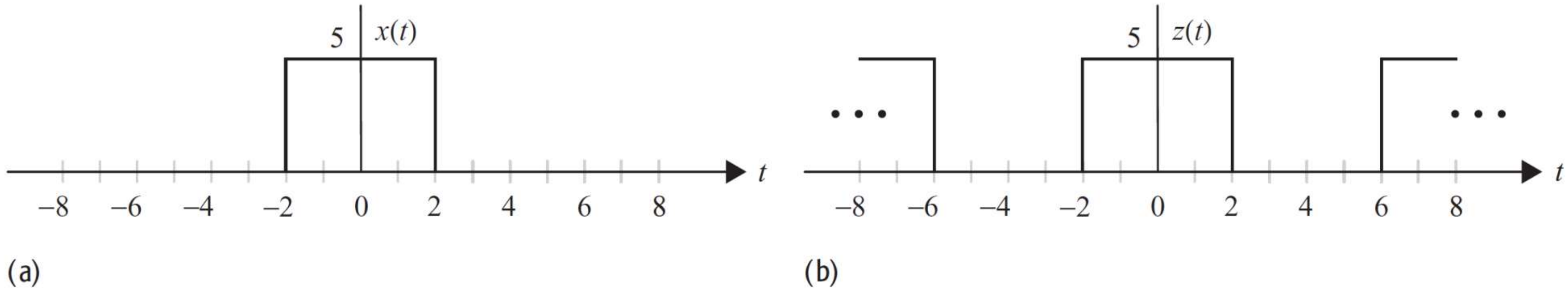
Solution

(a) The signal $x(t)$ can be expressed as follows:

$$x(t) = \begin{cases} 5 & -2 \leq t \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Energy and Power Signals (Example of CT Signals)

Example



instantaneous power $$P_x(t) = \begin{cases} 25 & -2 \le t \le 2 \\ 0 & \text{otherwise;} \end{cases}$$

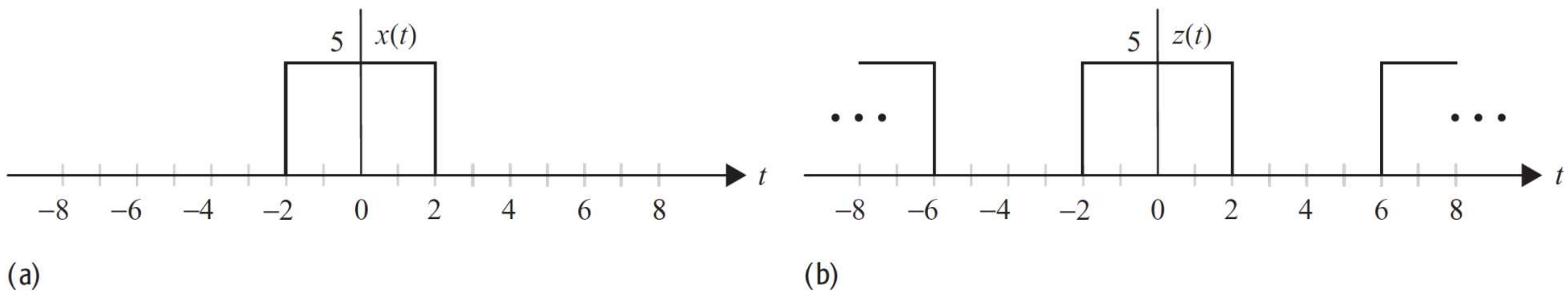
energy $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \mathrm{d}t = \int_{-2}^{2} 25 \, \mathrm{d}t = 100;$$

average power $$P_x = \lim_{T \to \infty} \frac{1}{T} E_x = 0.$$

Because $x(t)$ has finite energy ($0 < E_x = 100 < \infty$) it is an energy signal.

Energy and Power Signals (Example of CT Signals)

Example



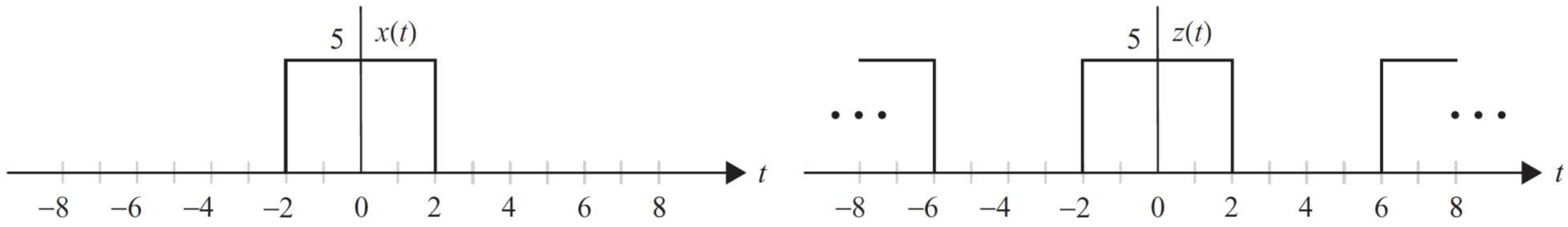
(b) The signal $z(t)$ is a periodic signal with fundamental period 8 and over one period is expressed as follows:

$$z(t) = \begin{cases} 5 & -2 \le t \le 2 \\ 0 & 2 < |t| \le 4, \end{cases}$$

with $z(t+8) = z(t)$.

Energy and Power Signals (Example of CT Signals)

Example



(a) (b)

instantaneous power $$P_z(t) = \begin{cases} 25 & -2 \le t \le 2 \\ 0 & 2 < |t| \le 4 \end{cases}$$

and $P_z(t+8) = P_z(t)$;

average power $$P_z = \frac{1}{8}\int_{-4}^{4} |z(t)|^2 \, dt = \frac{1}{8}\int_{-2}^{2} 25 \, dt = \frac{100}{8} = 12.5;$$

energy $$E_z = \int_{-\infty}^{\infty} |z(t)|^2 \, dt = \infty.$$

Because the signal has finite power ($0 < P_z = 12.5 < \infty$), $z(t)$ is a power signal.