Direct Proof

Proof by Contradiction

Proof by Contrapositive

Proof by Cases

Proofs of Equivalence

Existence Proofs

Mathematical Induction

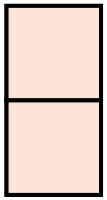
Strong Form of Induction

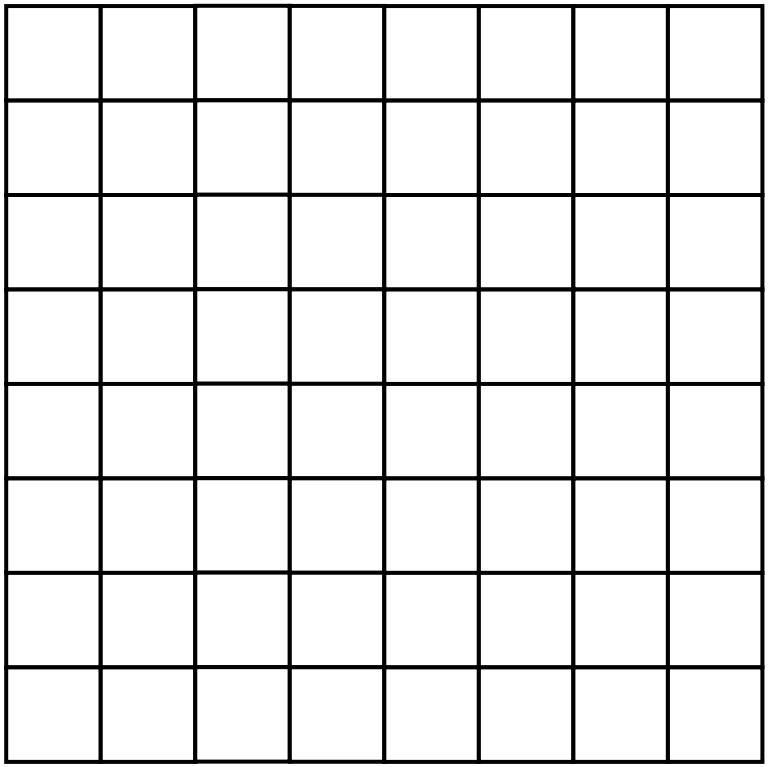
⋮

Domino Puzzle & Invariant Method

An 8x8 chessboard, 32 pieces of dominos.

Can we fill the chessboard?





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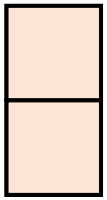

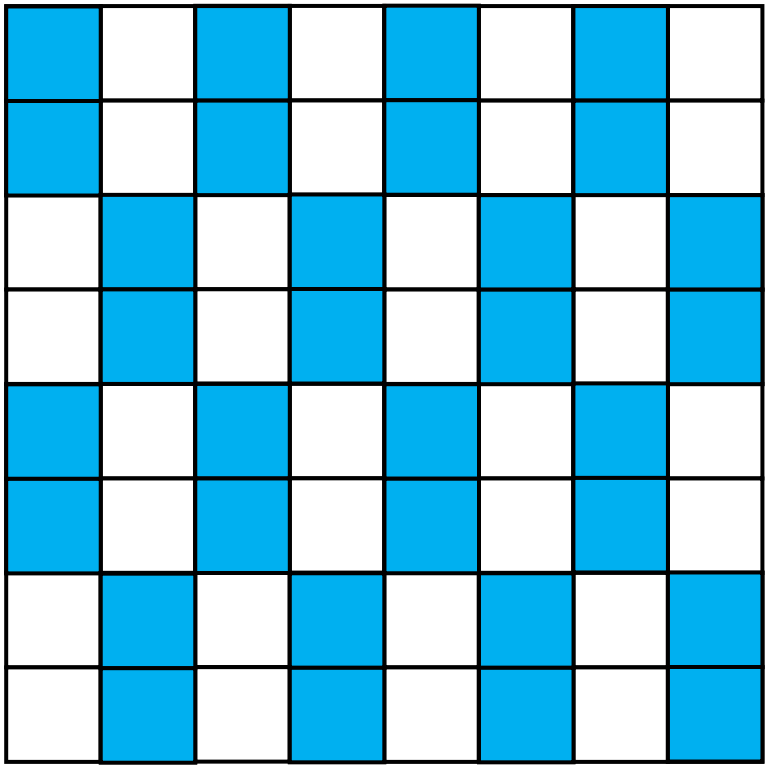

Domino Puzzle & Invariant Method

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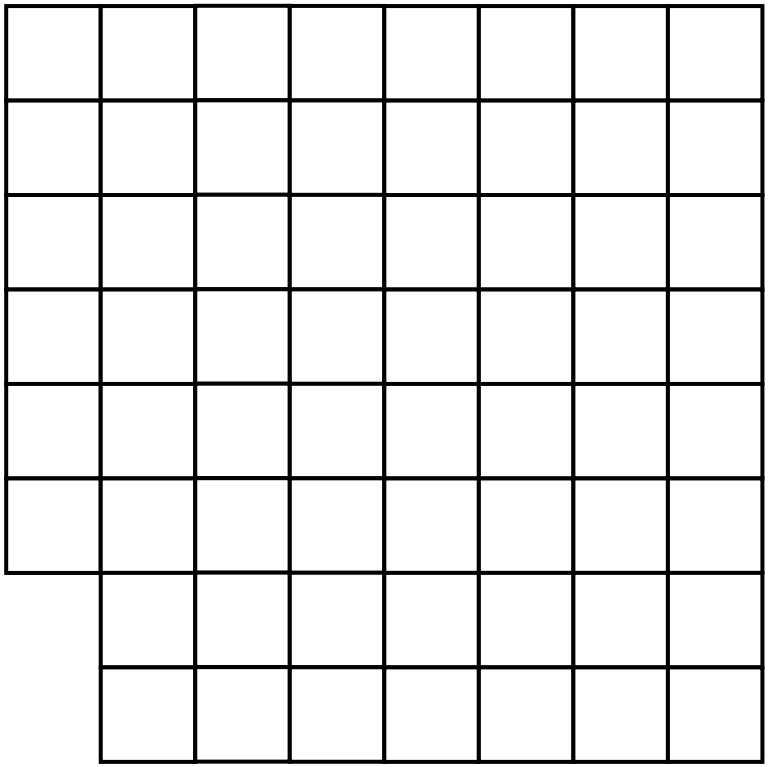

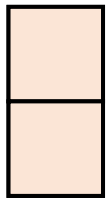

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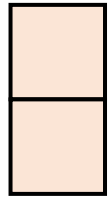

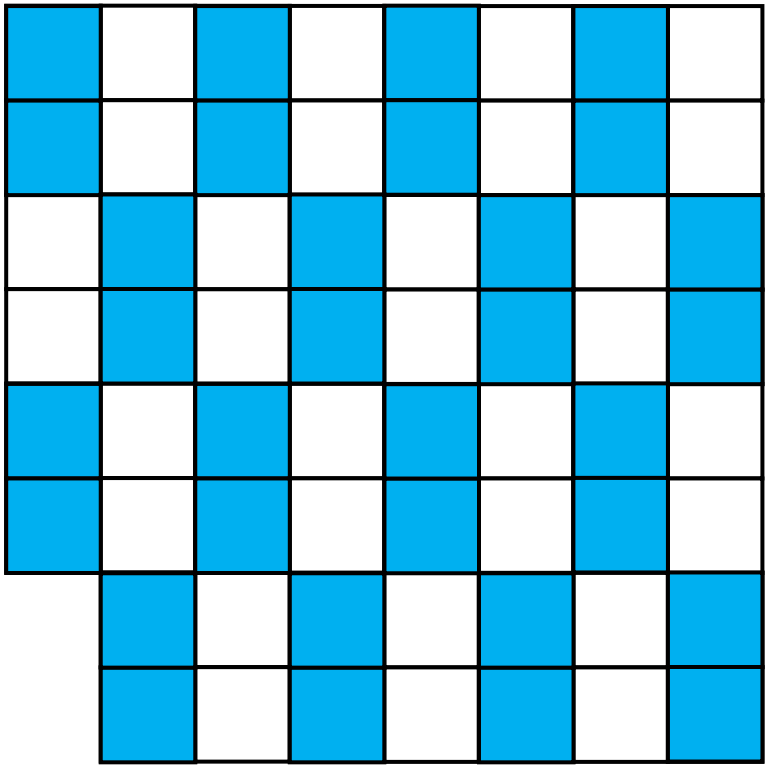

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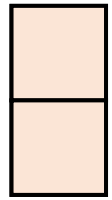
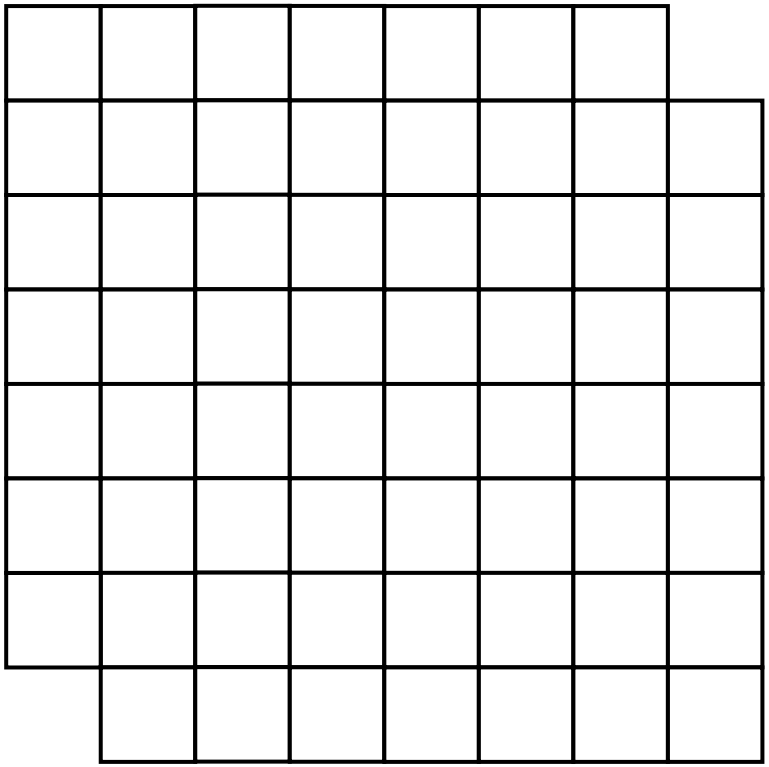
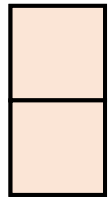


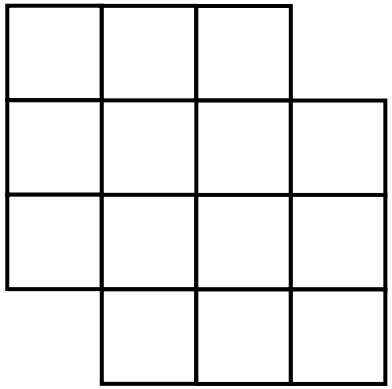


Domino Puzzle & Invariant Method

An 4x4 chessboard, 7 pieces of dominos.

Can we fill the chessboard?





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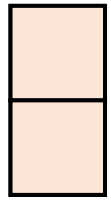

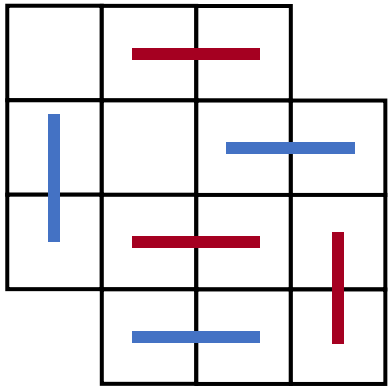

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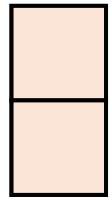
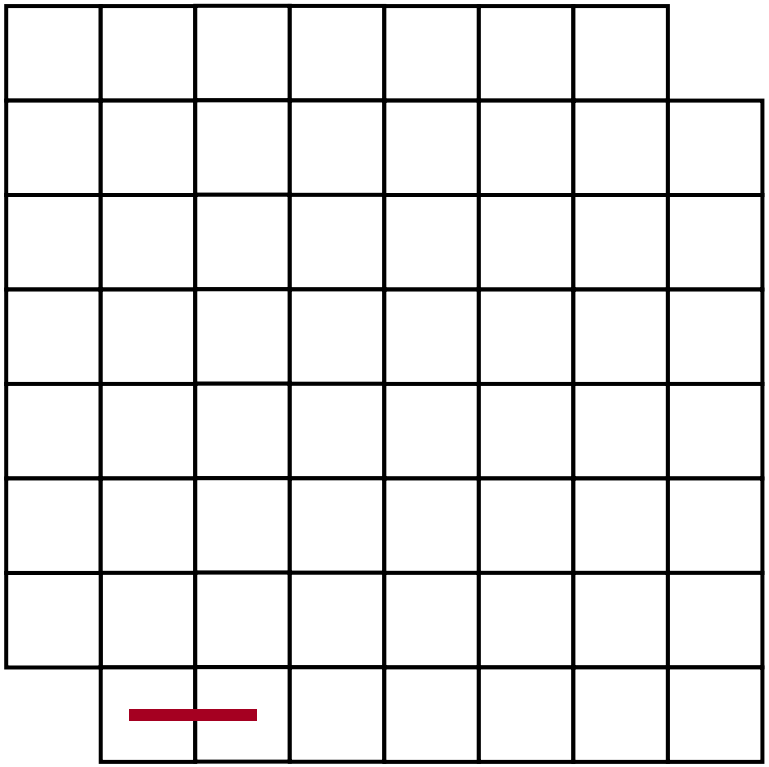
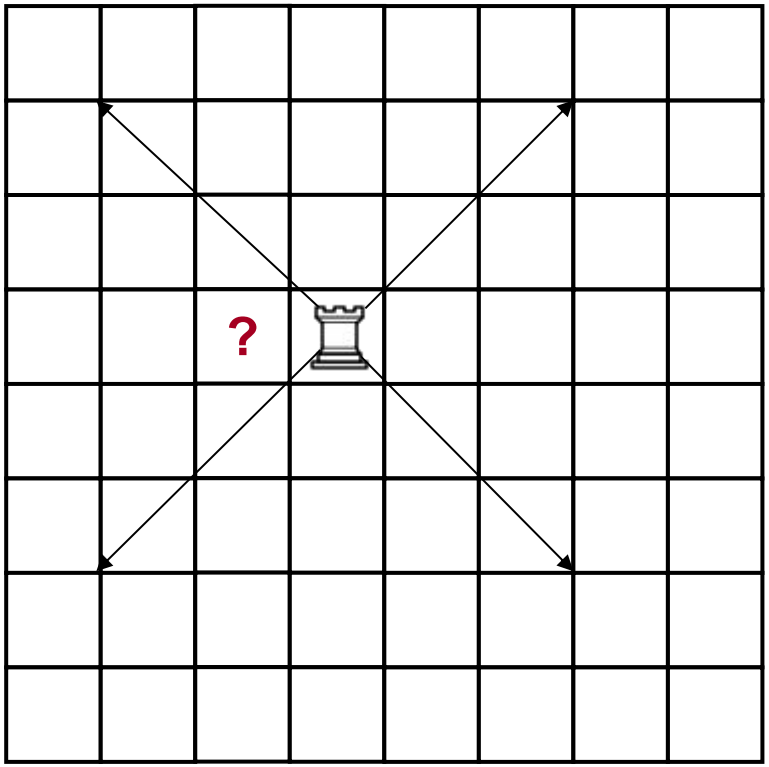




Domino Puzzle & Invariant Method

A Hint: a rook can only move along a diagonal.

Can a rook move from its current position to the question mark?



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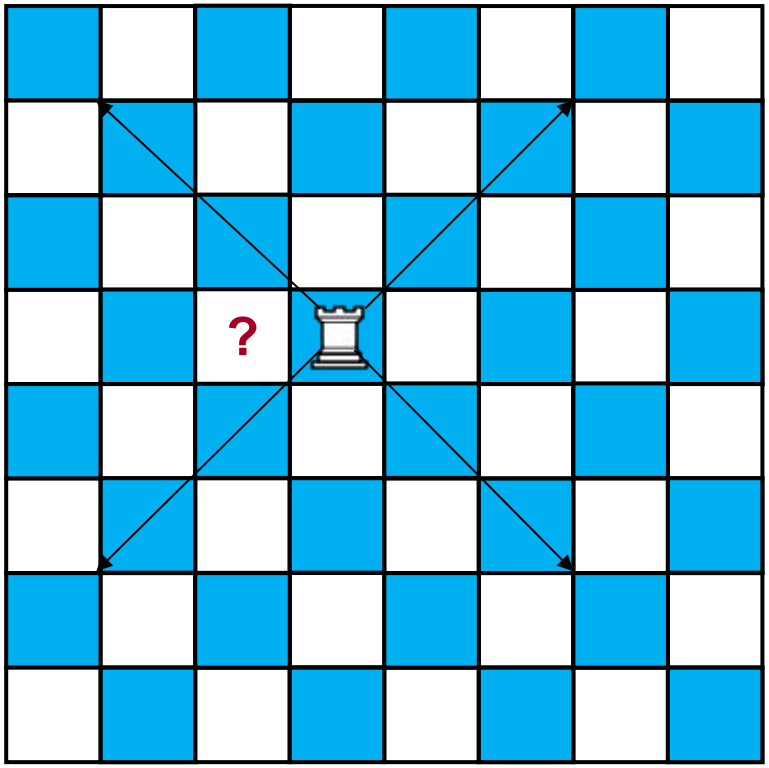

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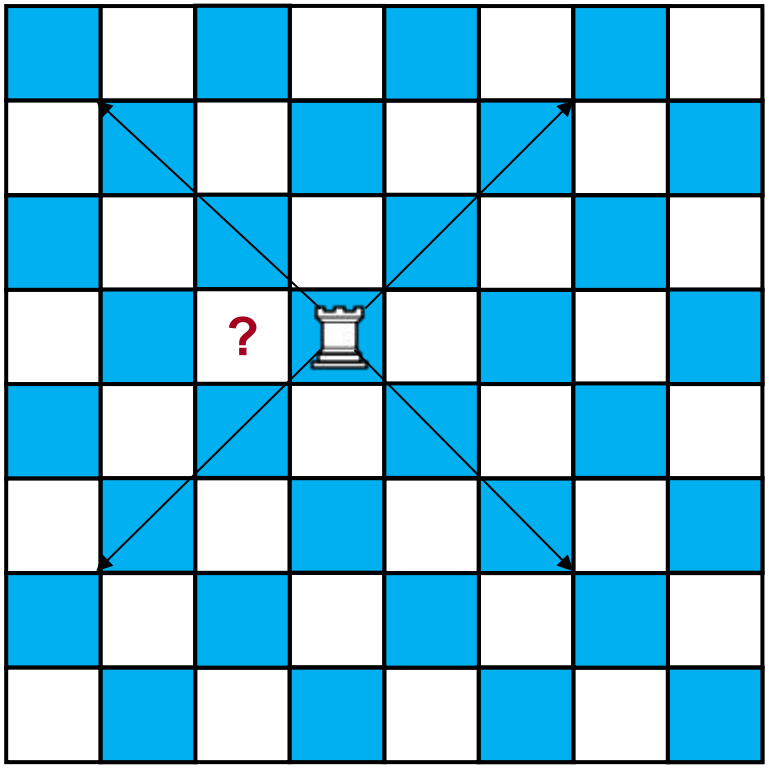
Can a rook move from its current position to the question mark?

Impossible!

Why?



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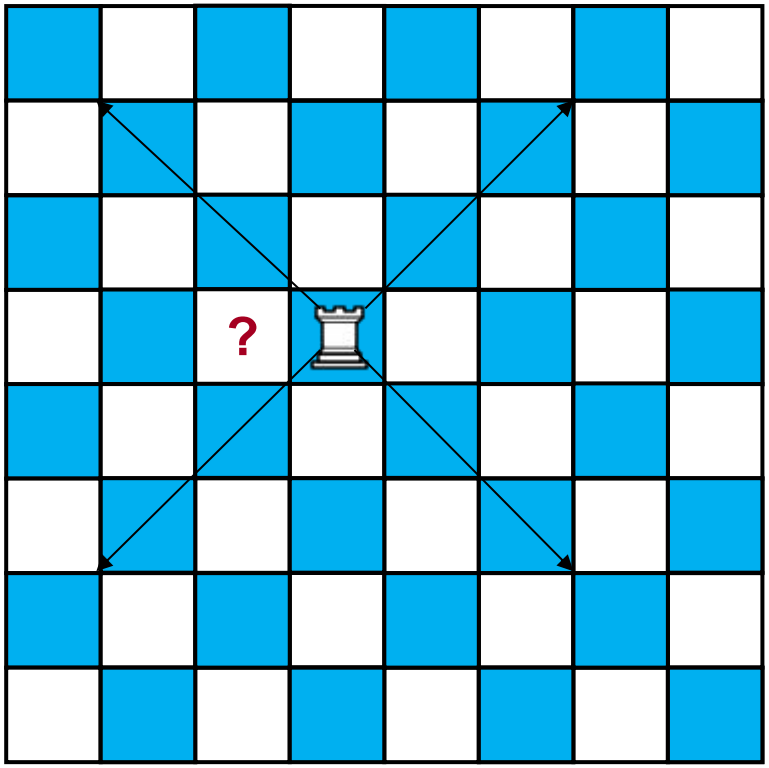


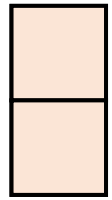
An example of the **invariant method**

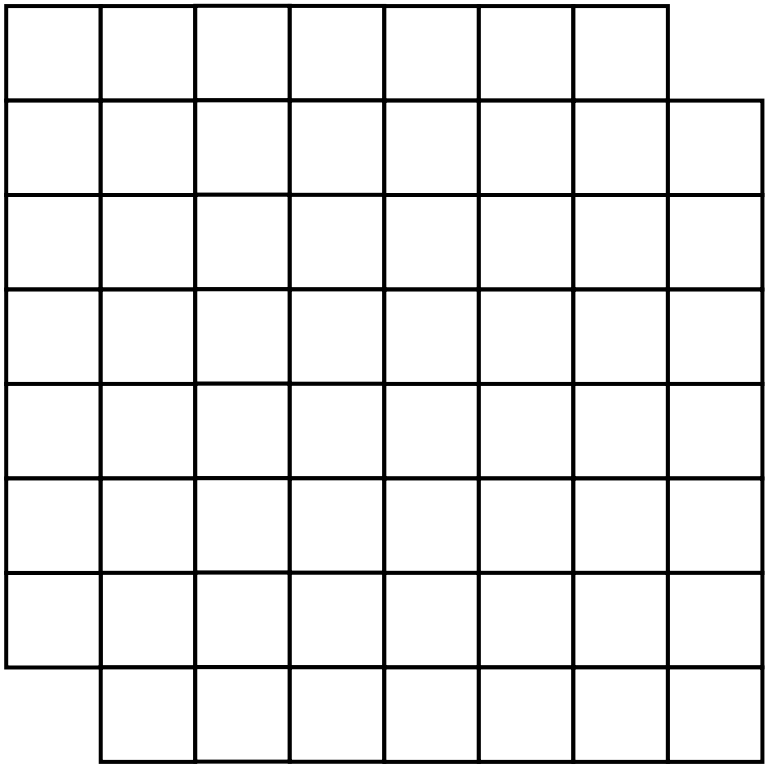
1. The rook is in a blue position.
2. A blue position can only move to a blue position by diagonal moves.
3. The question mark is in a white position.
4. So it is impossible for the rook to go there.

Domino Puzzle & Invariant Method

An 8x8 chessboard, 31 pieces of dominos.

Can we fill the chessboard?





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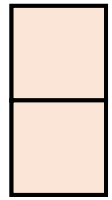

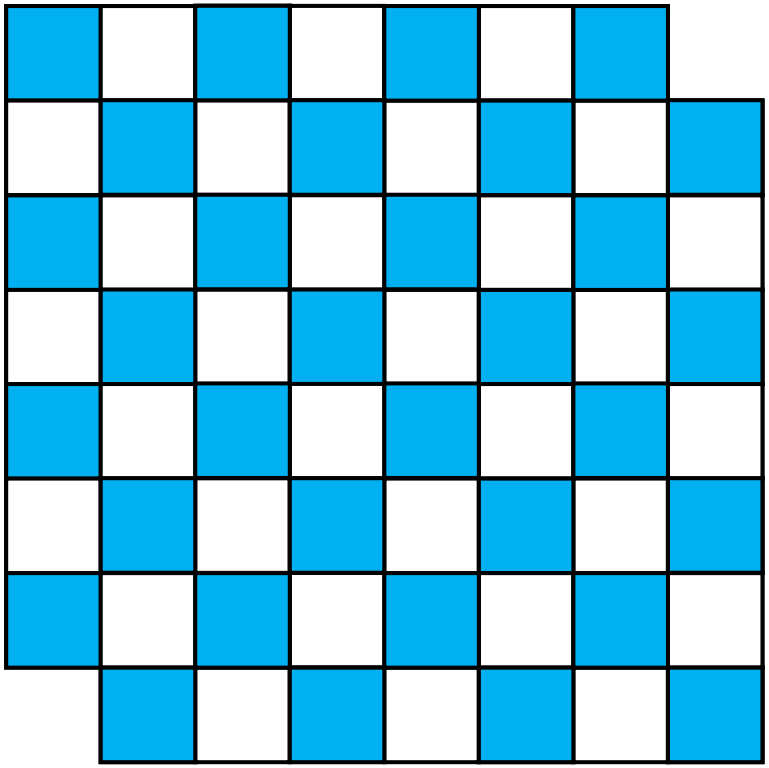

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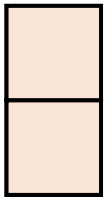

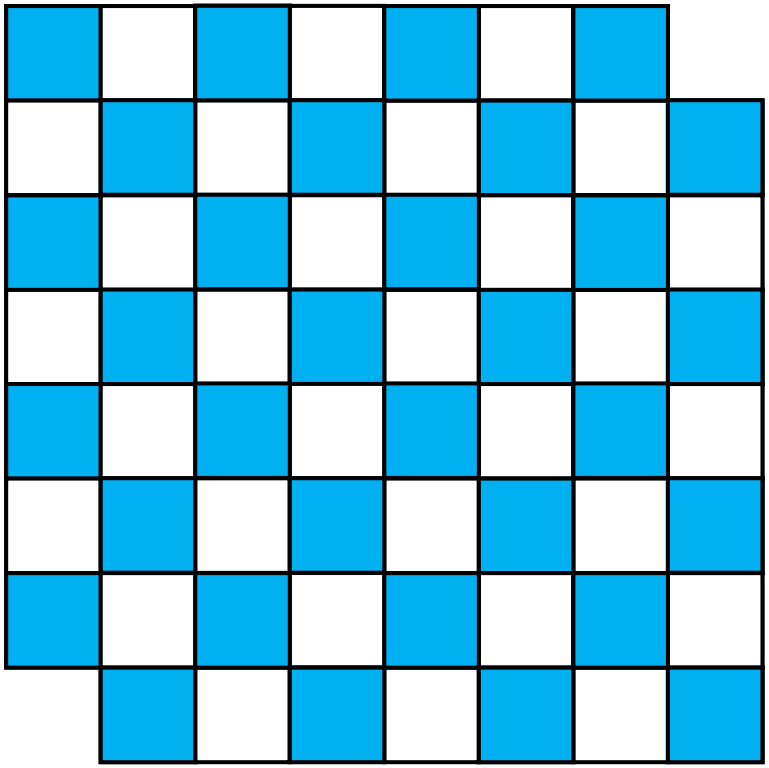

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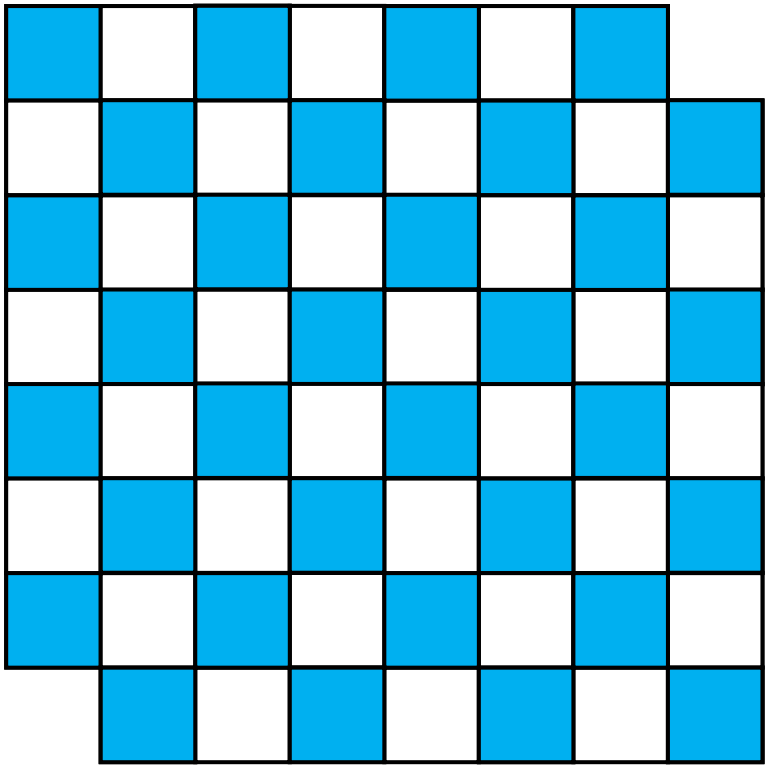
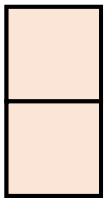




An example of the **invariant method**

Impossible!

Why?

1. Each domino will occupy one white square and one blue square.
2. There are 32 blue squares but only 30 white squares.
3. So it is impossible to fill the chessboard using only 31 dominos.

Invariant Method

1. Find properties (the **invariants**) that are satisfied throughout the whole process.

2. Show that the target do not satisfy the properties.

3. Conclude that the target is not achievable.

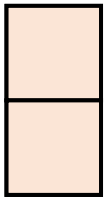
In the rook example, the invariant is the colour of the position of the rook.

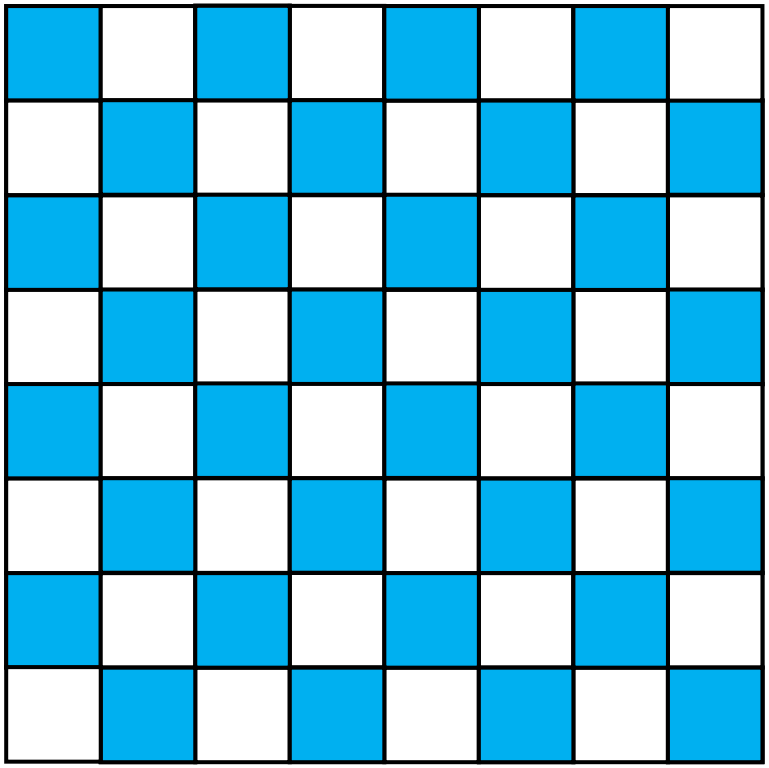
In the domino example, the invariant is that any placement of dominos will occupy the same number of blue positions and white positions.

Discuss the Possible

What if we take out two squares **of different colours**?

Would it be always possible to finish then?





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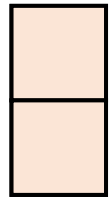

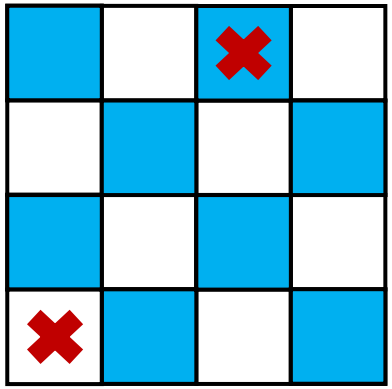

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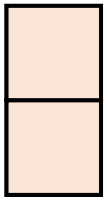

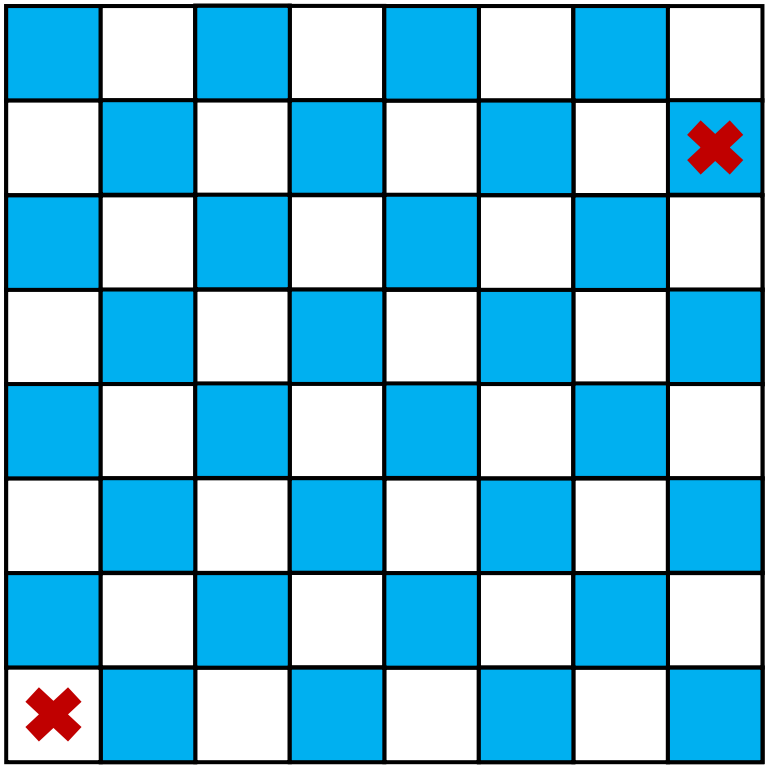

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