HW 13

EX 3.50

(a) $\mu = 12\,cm$ $\quad \sigma_x = 0.04\,cm$

$E(\bar{X}) = \mu = 12$ $\quad \sigma_{\bar{X}} = \frac{\sigma_x}{\sqrt{n}} = 0.01\,cm$

(b) $E(\bar{X}) = 12$ $\quad \sigma_{\bar{X}} = \frac{0.04}{8} = 0.005\,cm$

(c) $\bar{X}$ is more likely be within 0.01 cm of 12 cm with the second one. Because $\bar{X}$ has a decreased variability with a larger sample size.

4.11

Let $S_n = X_1 + X_2 + \dots + X_n$, $Y_n = \frac{1}{n}(X_1 + \dots + X_n)$

$P(|\bar{Y}_n - \mu| \geq \varepsilon) \leq \frac{Var(\bar{Y}_n)}{\varepsilon^2}$

$\Rightarrow P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| \geq \varepsilon\right) \leq \frac{1}{n^2\varepsilon^2} \cdot n \cdot \frac{1}{12} = \frac{1}{12n\varepsilon^2}$

4.12

$X_1, X_2, \dots, X_{900}$ IID, $\sim B(1, \frac{1}{2})$

$X \sim B(900, \frac{1}{2})$ $\quad \mu = np = 450$ $\quad \sigma^2 = np(1-p) = 225$

$X \sim N(450, 225)$

$P(425 \leq X \leq 465) = P\left(\frac{425-450}{15} \leq \frac{X-450}{15} \leq \frac{465-450}{15}\right)$

$= \Phi(1) - \Phi(-2) = \Phi(1) - (1 - \Phi(2)) = 0.81859$

4.23 $\quad E(X) = \frac{1}{2} + (-\frac{1}{2}) = 0$ $\quad Var(X) = E(X^2) - E^2(X) = 1$

$S_n = X_1 + X_2 + \dots + X_{100}$ $\quad S_n^* = \frac{S_n - 0}{10}$

$P(-10 \leq S_n \leq 10) = P(-1 \leq S_n^* \leq 1) = 2\Phi(1) - 1 = 0.6826$

$P(|S_n| > 10) = 1 - P(-10 \leq S_n \leq 10) = 0.3174$

Eg 5.3.3 $\quad X_t = a\sin(\omega t + \Theta)$, $t \in (-\infty, +\infty)$

pdf of Θ is $f(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 < \theta < 2\pi \\ 0, & \text{other} \end{cases}$

$\mu_X(t) = E[a\sin(\omega t + \Theta)] = \frac{a}{2\pi}\int_0^{2\pi} \sin(\omega t + \theta)\, d\theta = 0$

$R_X(t_1, t_2) = E[a\sin(\omega t_1 + \Theta) \cdot a\sin(\omega t_2 + \Theta)]$

$= \frac{a^2}{2\pi}\int_0^{2\pi} \sin(\omega t_1 + \theta)\sin(\omega t_2 + \theta)\, d\theta = \frac{a^2}{2}\cos(\omega(t_2 - t_1))$

Let $t_2 = t_1 = t$ $\quad \sigma_X^2(t) = R_X(t,t) - \mu_X^2(t) = \frac{a^2}{2}$

Eg 5.3.4

$E(Z)=\mu_X(5)=3 \quad E(W)=3$

$E(Z^2)=R_X(5,5)=9+4=13 \quad E(W^2)=13$

$R_X(5,8)=E(ZW)=R(5,8)=9+4e^{-0.2\times3}=11.195$

$\sigma_X^2(t)=R_X(t,t)-\mu_X^2(t)=13-9=4$

$C_X(5,8)=R_X(5,8)-\mu_X(5)\mu_X(8)=11.195-9=2.195$

5.3.5

(a) $X_{2.5}=Y+Z\cdot2.5\sim N(0,7.25)$

(b) $X_t\sim N(0,1+t^2)$

(c) for $s\neq t$

$$\begin{pmatrix}X_s\\X_t\end{pmatrix}=\begin{pmatrix}1&s\\1&t\end{pmatrix}\begin{pmatrix}Y\\Z\end{pmatrix}$$

$E(X_s)=E(X_t)=0$

$Var(X_s)=1+s^2 \quad Var(X_t)=1+t^2$

$Cov(X_s,X_t)=E((Y+sZ)(Y+Zt))=1+st$

$\therefore (X_s,X_t)\sim N\left(0,0,1+s^2,1+t^2,\frac{1+st}{\sqrt{1+s^2}\cdot\sqrt{1+t^2}}\right)$

Ex 5.3 $\quad X_t=X\cos w_0t \quad X\sim N(2,4)$

$X_t\sim N(2\cos w_0t,\ 4\cos^2 w_0t)$

$f_{X_t}(x)=\frac{1}{\sqrt{2\pi}\cdot2\cos w_0t}e^{-\frac{(x-2\cos w_0t)^2}{8\cos^2w_0t}}$

5.6

(a) $X_t\sim(0,4t^2) \quad f_{X_t}(x)=\frac{1}{2\sqrt{2\pi}t}e^{-\frac{x^2}{8t^2}}$

(b) $\mu_X(t)=0 \quad R_X(t_1,t_2)=E(At_1\cdot At_2)=t_1t_2E(A^2)=t_1t_2\cdot4$

$C_X(s,t)=R_X(s,t)-\mu_X(s)\cdot\mu_X(t)=4st$

$\sigma_X^2(t)=C_X(t,t)=4t^2$

5.8 $\quad T\sim U(0,1) \quad X_t=T+(1-t)$

(a) $F(x;t)=P(X_t\le x)=P(T+1-t\le x)=P(T\le x+t-1)=F_T(x+t-1)$

$$\therefore F(x;t)=\begin{cases}0 & x<1-t\\ x+t-1 & 1-t\le x\le 2-t\\ 1 & x>2-t\end{cases}$$

(b) $\mu_X(t)=E(X_t)=\frac{1-t+2-t}{2}=\frac{3-2t}{2} \quad \mu_T=E(T)=\frac{1}{2}$

$\sigma_X(t)=Var(X_t)=\frac{(2-t-1+t)^2}{12}=\frac{1}{12}$

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\begin{aligned}
C_X(t_1,t_2) &= Cov(X_{t_1},X_{t_2}) = E(X_{t_1}\cdot X_{t_2}) - E(X_{t_1})E(X_{t_2}) \\
&= E\left((T+1-t_1)(T+1-t_2)\right) - E(X_{t_1})\,E(X_{t_2}) \\
&= E\left(T^2 + (2-t_1-t_2)T + (1-t_1)(1-t_2)\right) - \frac{(3-2t_1)}{2}\cdot\frac{3-2t_2}{2} \\
&= E(T^2) + (2-t_1-t_2)E(T) + (1-t_1)(1-t_2) - \frac{(3-2t_1)(3-2t_2)}{4} \\
&= \frac{1}{12}+\frac{1}{4}+\frac{2-t_1-t_2}{2} + 1 - t_1 - t_2 + t_1t_2 - \frac{9}{4} + \frac{3}{2}t_1 + \frac{3}{2}t_2 - t_1t_2 \\
&= \frac{1}{12}
\end{aligned}
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