Section 1.2 Experiment, Sample Space and Random Event

Ω

$$\phi \subseteq A \subseteq \Omega$$

ω: outcome.

$\Omega = \{\omega\}$: sample space.

$A \subset \Omega$: subset of S.S.

1.2.1 Basic Definitions

A random phenomenon is a situation in which we know what outcomes could happen, but we do not know which particular outcome did or will happen. ‘Random’ in statistics is not a synonym for ‘haphazard’ but a description of a kind of order that emerges only in the long run.

Experiment, Sample Space and Random Event

- 1.2.1 Basic Definitions
- 1.2.2 Events as Sets

$$A \cap B = AB = \emptyset \Longrightarrow P(A \cup B) = P(A) + P(B)$$

Independent : $P(AB) = P(A)P(B)$

1.2.1 Basic Definitions

Random experiment usually has the following three characteristics.

(i) Repeatability: it can be repeated under the same conditions.

(ii) Predictability: it has more than one outcome and we know all possible outcomes before the experiment.

(iii) Uncertainty: the outcome of the experiment will not be known in advance.

$$\omega \in \Omega \xrightarrow[\text{r.v.}]{X(\omega)} x \in \mathbb{R} \xrightarrow[\text{d.f.}]{F(x)} [0,1]$$

In this book, we shall abbreviate “random experiment” to experiment and denote it by E.

σ-algebra

$P(\cdot)$: set-function

(Probability) measure-theory

1.2.1 Basic Definitions

Some examples of random experiment:

E_1 : Determination of the sex of a newborn child. $\Omega = \{g, b\}$.

E_2 : Roll a die and observe which number appears.

E_3 : Flip two coins and observe the outcomes.

E_4 : Roll two dice and observe the outcomes.

E_5 : Observe call times for a call center.

E_6 : Measure the lifetime of cars. $\Omega = \{T \geq 0\}$.

tail prob. $P(T > t)$

How to record the experiment data?

Each possible outcome: ***sample point*** (ω)

The set of all possible outcomes: ***sample space*** (Ω)

1.2.1 Basic Definitions

Example

E_1 : *Determination of the sex of a newborn child.*

For E_1, $\Omega_1 = \{g, b\}$, where the outcome g means that the child is a girl and b that it is a boy.

Example

E_2 : *Roll a die and observe which number appears.*

For E_2, $\Omega_2 = \{1, 2, 3, 4, 5, 6\}$, where the outcome i means that i appeared on the die, $i = 1, 2, 3, 4, 5, 6$.

Example

E_3 : *Flip two coins and observe the outcomes.*

For E_3, $\Omega_3 = \{(H, H), (H, T), (T, H), (T, T)\}$, where H means head and T means tail.

Example

E_4 : Roll two dice and observe the outcomes.

For E_4,

$$\Omega_4 = \begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{pmatrix}$$

where the outcome (i, j) is said to occur if i appears on the first die and j appears on the second die, $i, j = 1, 2, \cdots, 6$.

1.2.1 Basic Definitions

Example

E_6 : *Measure the lifetime of cars.*

For E_6, $\Omega_6 = [0, \infty)$, where the outcome t is the lifetime of a car, $0 \leqslant t < \infty$.

- Any subset of the sample space Ω is known as an ***random event*** or ***event***. Events are usually denoted by capital letters $A, B, C, \cdots$.
- Those events must occur in the experiment are called the ***inevitable events***.

  Sample space Ω is an inevitable event.

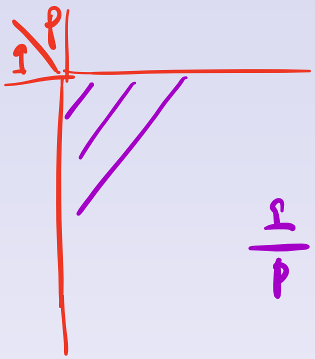
  $P(\{x = \frac{1}{2}\}) = 0$
- Those could not happen anytime are said to be ***impossible events***. We usually denote impossible events by $\emptyset$.

  $= \frac{1}{\infty}$

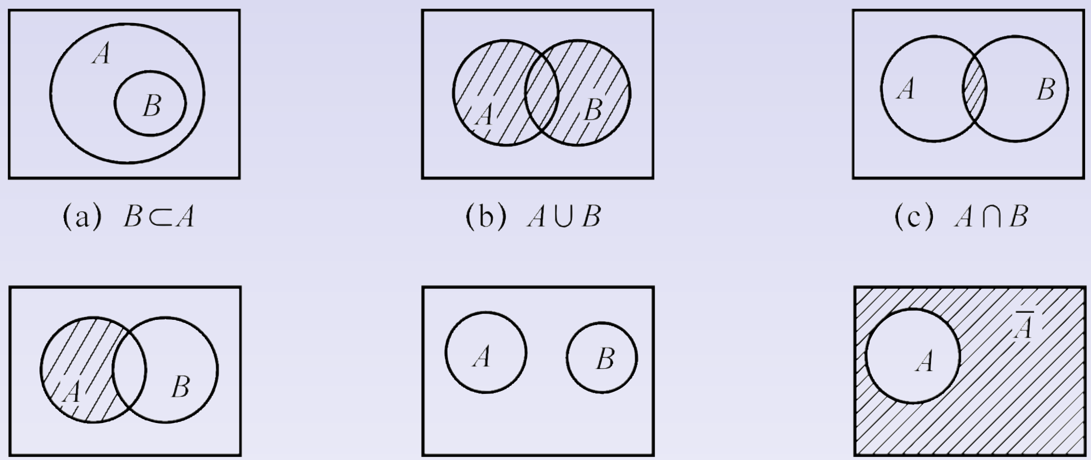
1.2.2 Events as Sets

$P(\{x \in [0,1] \cap \mathbb{Q}\}) = 0$

Figure: The relationships between events A and B



$\frac{1}{p} \to k$



(a) $B \subset A$ (b) $A \cup B$ (c) $A \cap B$

(d) $A - B$ (e) $AB = \varnothing$ (f) $A \cup \bar{A} = \Omega$, $A \cap \bar{A} = \varnothing$

σ-algebra

$$\bigcup_{i=1}^{n} A_i \triangleq A_1 \cup A_2 \cup \cdots \cup A_n, \quad \bigcup_{i=1}^{\infty} A_i \triangleq A_1 \cup A_2 \cup \cdots$$

$P(\ )$

$$\bigcap_{i=1}^{n} A_i \triangleq A_1 \cap A_2 \cap \cdots \cap A_n, \quad \bigcap_{i=1}^{\infty} A_i \triangleq A_1 \cap A_2 \cap \cdots$$

1.2.2 Events as Sets

Table: The jargons in set theory and probability theory

notation	Set jargon	Probability jargon
Ω	Collection of objects	Sample space
ω	Member of Ω	Elementary event, outcome
A	Subset of Ω	Events that some outcome in A oc
A or $\overline{A}$	Complement of A	Event that no outcome in A occurs
$A \cap B$	Intersection	Both A and B
$A \cup B$	Union	Either A or B or both
$A - B$	Difference	A, but no B
$A \subseteq B$	Inclusion	If A, then B
$\emptyset$	Empty set	Impossible event
Ω	Whole space	Certain set

1.2.2 Events as Sets

$\cup$: $+$ $\qquad$ $\cap$: $\times$

Let A, B and C be the random events of experiment E. The operations of the events will satisfy the following rules:

(i) Commutatively $\quad A \cup B = B \cup A,\ AB = BA.$

(ii) Associatively $\quad A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C.$

$A(BC) = (AB)C = ABC.$

(iii) Distributively $\quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$

对偶律 $\quad A(B \cup C) = (AB) \cup (AC).$

(iv) De Morgan's law $\quad \overline{A \cup B} = \overline{A} \cap \overline{B},\ \overline{A \cap B} = \overline{A} \cup \overline{B}.$

More generally, $\overline{(\bigcup_i A_i)} = \bigcap \overline{A_i},\ \overline{(\bigcap_i A_i)} = \bigcup \overline{A_i}.$

1.2.2 Events as Sets

In addition, there are some common properties, such as

(1) $\overline{\overline{A}} = A$.

(2) $A \cup B \supset A$ and $A \cup B \supset B$. In particular, if $A \subset B$, then $A \cup B = B$.

(3) $A \cap B \subset A$ and $A \cap B \subset B$. In particular, if $A \subset B$, then $A \cap B = A$.

(4) $A - B = A - AB = A\overline{B}$.

(5) $A \cup B = A \cup \overline{A}B$.

1.2.2 Events as Sets

Example

Suppose that A, B and C are three events and $D = \{\text{at least one of the three events will occur}\}$. Try to describe the event D by events A, B and C.

Solution. We describe the event D by three different ways:

(i) directed method: $D = A \cup B \cup C$,

(ii) decomposition method:

$$D = A\overline{B}\,\overline{C} \cup \overline{A}B\overline{C} \cup \overline{A}\,\overline{B}C \cup \overline{A}BC \cup A\overline{B}C \cup AB\overline{C} \cup ABC,$$

(iii) inverse method:

$$D = \overline{\overline{D}} = \overline{\{A,\ B \text{ and } C \text{ will not occur}\}} = \overline{\overline{A}\,\overline{B}\,\overline{C}}.$$

Example

Select an integer from 1 to 1000 at random. How to describe the event that the integer is not divisible by 6 and 8.

$A = \{x : 6 \mid x\}$

$B = \{x : 8 \mid x\}$

$\overline{A} \cap \overline{B} = \overline{A \cup B}$

$P(\overline{A} \cap \overline{B})$

$= 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(AB))$

The end

Thank you for your patience !