1.4 Arguments and Rules of Inference 论证和推理规则

Consider the following sequence of propositions.

- The bug is either in module 17 or in module 81.
- The bug is a numerical error.
- Module 81 has no numerical error.

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“ The bug is in module 17”

This process of drawing a conclusion from a sequence of propositions is called **deductive reasoning (演绎推理)**.

Argument 论证

Definition1.4.1 An argument is a sequence of propositions written

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ \hline \therefore\ q \end{array}$$

or

$$p_1, p_2, \ldots, p_n / \therefore q$$

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hypotheses (假设)
or premises (前提)

conclusion (结论)

or

$$p_1, p_2, \ldots, p_n / \therefore\ q$$

$\therefore$ means therefore

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Argument is said to be **valid (有效的)** if the conclusion follows from the hypotheses; that is, if p_1 and p_2 and … and p_n are true, then q must also be true. otherwise, the argument is invalid (or a fallacy).

Rules of inference 推理规则

1. **Modus ponens rule of inference** or **law of detachment** (假言推理或分离定律)

$$\begin{array}{l} p \longrightarrow q \\ p \\ \hline \therefore q \end{array}$$

Rules of inference 推理规则

2. **Modus tollens**
(拒取)

$$\begin{array}{l} p \longrightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Rules of inference 推理规则

3. **Addtion**
(附加)

$$\frac{p}{\therefore p \vee q}$$

Rules of inference 推理规则

4. **Simplification**
   (化简)

$$\frac{p \land q}{\therefore p}$$

Rules of inference 推理规则

5. **Conjunction**
(合取)

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Rules of inference 推理规则

6. **Hypothetical syllogism**
   (假言三段论)

$$\begin{array}{c} p \longrightarrow q \\ q \longrightarrow r \\ \hline \therefore p \longrightarrow r \end{array}$$

Rules of inference 推理规则

7. **Disjunctive syllogism**
(析取三段论)

$$\begin{array}{l} \quad p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Rules of inference 推理规则

Example 1.4.5. Represent the argument

The bug is either in module 17 or in module 81.
The bug is a numerical error.
Module 81 has no numerical error.

$\therefore$ The bug is in module 17.

given at the beginning of this section symbolically and show that it is valid.

p: The bug is in module 17.
q: The bug is in module 81.
r: The bug is a numerical error.

Rules of inference 推理规则

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p: The bug is in module 17.
q: The bug is in module 81.
r: The bug is a numerical error.

$$\begin{array}{c} p \vee q \\ r \\ r \to \neg q \\ \hline \therefore p \end{array}$$

Rules of inference 推理规则

Exercise. It is known that

1. It is not sunny this afternoon, and it is colder than yesterday.
2. We will go swimming only if it is sunny this afternoon.
3. If we do not go swimming, we will play basketball.
4. If we play basketball, we will go home early.

Can you conclude “we will go home early”?

Rules of inference 推理规则

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1. It is not sunny this afternoon, and it is colder than yesterday.
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Can you conclude “we will go home early”?

p := It is sunny this afternoon
q := It is colder than yesterday
r := We will go swimming
s := We will play basketball
t := We will go home early

Rules of inference 推理规则

Exercise. A student is trying to prove that propositions p, q, and r are all true.

She proceeds as follows.

First, she proves three facts:

- p implies q
- q implies r
- r implies p.

Then she concludes,

``Thus p, q, and r are all true."

What does its form of argument is like?

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First, she proves three facts:

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Then she concludes,

``Thus p, q, and r are all true."

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ r \rightarrow p \\ \hline \therefore\ p \wedge q \wedge r \end{array}$$

What does its form of argument is like?

Rules of inference 推理规则

$$\begin{array}{l} p \to q \\ q \to r \\ r \to p \\ \hline \therefore p \wedge q \wedge r \end{array}$$

p	q	r	$p \to q$	$q \to r$	$r \to p$	$p \wedge q \wedge r$

To prove an argument is not valid, we just need to find a counterexample.

Valid Arguments?

$$\begin{array}{c} p \to q \\ q \\ \hline \therefore\ p \end{array}$$

Valid Arguments?

$$\begin{array}{c} p \rightarrow q \\ q \\ \hline \therefore\ p \end{array}$$

If you are a fish, then you drink water.
You drink water.

You are a fish.

Valid Arguments?

$$\begin{array}{c} p \to q \\ q \\ \hline \therefore\ p \end{array}$$

If you are a fish, then you drink water.
You drink water.

You are a fish.

$$\begin{array}{c} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

Valid Arguments?

$$
\begin{array}{c} p \rightarrow q \\ q \\ \hline \therefore\ p \end{array}
$$

If you are a fish, then you drink water.
You drink water.

You are a fish.

$$
\begin{array}{c} p \rightarrow q \\ \neg p \\ \hline \therefore \neg q \end{array}
$$

If you are a fish, then you drink water.
You are not a fish.

You do not drink water.

Exercises

$$\frac{p}{\therefore\ p \vee q}$$

Addition

$$\frac{p}{\therefore\ p \wedge q}$$

$$\frac{p \wedge q}{\therefore\ p}$$

Simplification

$$\frac{p \vee q}{\therefore\ p}$$

Exercises

(A)
$$\begin{array}{l} \neg p \rightarrow q \\ \neg q \\ \hline \therefore\ p \end{array}$$

(B)
$$\begin{array}{l} \neg p \rightarrow \neg q \\ \hline \therefore\ p \rightarrow q \end{array}$$

(C)
$$\begin{array}{l} \neg p \rightarrow \neg q \\ \hline \therefore\ q \rightarrow p \end{array}$$

Honest man and Liar

Honest man always tell the truth.

Liar always lie.

A says: “B is an honest man.”

B says: “A and I are of opposite type.”

What are the identities of A and B?

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Liar always lie.

A says: “B is a liar.”

B says: “A is an honest man.”

What are the identities of A and B?

A game

Imagine there are 3 coins on the table. Gold, silver and copper.

If you say a truthful sentence, you will get one coin. If you say a false sentence, you get nothing.

Which sentence can guarantee gaining the gold coin?

A game

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If you say a truthful sentence, you will get one coin. If you say a false sentence, you get nothing.

Which sentence can guarantee gaining the gold coin?

(A) You will give me the gold coin.
(B) You will give me all the coins.
(C) You will not give me any of the coins.
(D) You will give me either silver or copper coin.
(E) You will give me neither silver nor copper coin.

Problem-Solving Tips

The validity of a very short argument or proof might be verified using a truth table. In practice, arguments and proofs use rules of inference.

Problem-Solving Tips

TABLE 1.4.1 Rules of Inference for Propositions

Rule of Inference	*Name*	*Rule of Inference*	*Name*
$p \to q$ p $\therefore q$	Modus ponens	p q $\therefore p \wedge q$	Conjunction
$p \to q$ $\neg q$ $\therefore \neg p$	Modus tollens	$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
p $\therefore p \vee q$	Addition	$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \wedge q$ $\therefore p$	Simplification		