

EBU4375: SIGNALS AND SYSTEMS

LAB3: DISCRETE-TIME SYSTEMS IN THE FREQUENCY DOMAIN



ACKNOWLEDGMENT

These slides are partially from Labs prepared by
Dr Jesus Raquena Carrion.

YOUR TASKS

- BEFORE THE LAB:

- Read the slides carefully.
- Create a ID_FS.txt file where ID is your QMUL ID number, F is the first letter of your forename and S is the first letter of your surname.
- **Type all the code** in a red frame in the **ID_FS.txt file** and submit to the QMplus link.

- DURING THE LAB:

- Copy/paste the code from ID_FS.txt into Matlab command window as required- indicated by 
- Take note of the results and your answers to questions indicated by 

- **Make sure you do the work yourself as there will be questions in the class tests and exam related to Matlab.**

BACKGROUND

Given a periodic discrete-time signal $x_N[n]$ of period N :

1. Its fundamental frequency is $\Omega_0 = \frac{2\pi}{N}$.
2. According to the synthesis equation, $x_N[n]$ can be expressed as the sum of N harmonically related complex exponentials of frequencies $\Omega_k = k \frac{2\pi}{N}$.

$$x_N[n] = \sum_{k=\langle N \rangle} a_k e^{jk\Omega_0 n}$$

3. The Fourier coefficients a_k can be determined by using the analysis equation as:

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_N[n] e^{-jk\Omega_0 n}$$

BACKGROUND

In order to obtain the Fourier series decomposition of a periodic signal $x_N[n]$ of period N , we need to:

1. Identify the fundamental frequency Ω_0 .
2. Determine the N harmonic frequencies $\Omega_k = k\Omega_0$.
3. Obtain the Fourier coefficients a_k .

BACKGROUND

- Determining the fundamental frequency Ω_0 and its harmonics k is very easy.
- The Fourier coefficients a_k can be obtained analytically. For instance, for the periodic square wave of period N defined within one period centred around $n = 0$ as:

$$x_N[n] = \begin{cases} 1 & |n| \leq N_1 \\ 0 & \text{otherwise} \end{cases}$$

we can obtain its Fourier coefficients as:

$$a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} e^{-jk\Omega_0 n} = \begin{cases} \frac{2N_1 + 1}{N} & k = 0 \\ \frac{1}{N} \frac{\sin[2\pi k(N_1 + 1/2)/N]}{\sin(\pi k/N)} & k \neq 0 \end{cases}$$

BACKGROUND

You will have noticed that the analysis equation in DT:

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_N[n] e^{-jk\Omega_0 n}$$

is essentially a mathematical operation in which we:

1. Calculate N products of complex numbers.
2. Add N complex numbers.

Computers are very good at doing additions and multiplications so, why don't we let computers calculate the Fourier coefficients for us?

STEP 1 - Definition of periodic DT signal

In this lab, we will work with the periodic square wave $x[n]$ with period N defined as:

$$x_N[n] = \begin{cases} 1 & |n| \leq N_1 \\ 0 & \text{otherwise} \end{cases}$$

Your job now is to:

Based on you're the last digit of you QMUL ID number x , define

$$N = 2 \times \max(10, 10 \times x) + 1$$

$$N_1 = \max(10, 10 \times x) / 2$$

Identify its fundamental frequency Ω_0 and its harmonics k .

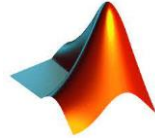
Draw on a piece of paper three periods of $x[n]$.

STEP 2- Plotting individual harmonic components

The following lines of code calculate the Fourier coefficients a_k of $x[n]$.

Based on you're the last digit of you QMUL ID number x , find the values of $n1$ and $n2$ such that $n2 = \max(10, 10 \times x)$ and $n1 = -n2$.

```
n1 = ?;%Fill in the value based on your ID number
n2 = ?;%Fill in the value based on your ID number
n = n1:1:n2; % Definition of the time vector
x = zeros(size(n));
nmin= n2-n2/2+1;
nmax= n2+n2/2+1;
x(nmin:nmax)=1; % Definition of x[n] as square wave
%Plot the signal x[n] in the time interval n using stem function.
N = 2*n2+1; % Period of x[n]
omega0=2*pi/N; % Fundamental frequency of x[n]
ak=zeros(1,N);
for k=0:2*n2; % This loop calculates the Fourier coefficients ak
ak(k+1)=(1/N)*sum(x.*exp(-k*1i*omega0*n));
end
```



Question 1:

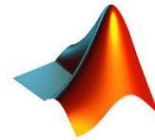
Save the figure in
which you plot $x[n]$



STEP 3- Plotting the Fourier Coefficients

The following lines of code plot the magnitude of the coefficients a_k .

```
Figure
nAxix=[0:n2,-n2:-1];
stem(nAxix,abs(ak)) % plots ak against n
xlabel('k') % adds text below the X-axis
ylabel('ak') % adds text beside the Y-axis
```



Question 2:

Save the figure in which you plot a_k



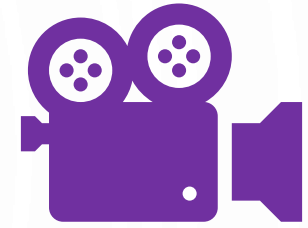
Question 3:

Make sure you understand how $nAxix$ is defined. Hint, try $nAxix=[0:2*n2];$ instead.

Question 4:

Take note of the value of N , the greatest value a_k , and the corresponding value of k . Can you justify?

STEP 3- Plotting the Fourier Coefficients

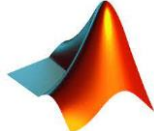


- Explain the function of $nAxix=[0:n2,-n2:-1]$ in plotting a_k (Hint: you can compare to $nAxix=[0:2*n2]$):
- Based on the plot of a_k please answer the following:
- What is the value of N ?
- What is the maximum value of a_k ?
- What is the corresponding values of k ?
- What is your justification for the maximum value of a_k ?

STEP 4- Synthesising $x[n]$

The following lines of code synthesise 3 periods of $x[n]$ by using the Synthesis equation:

```
m1 = -3*n2-1;  
m2 = 3*n2+1;  
n = m1:1:m2; % New time vector  
xsyn = zeros(size(n));  
k=0:2*n2;  
for m=m1:m2 % Synthesis of x  
    xsyn(m+m2+1)=sum(ak.*exp(k*1i*omega0*m));  
end  
figure  
stem(n,abs(xsyn))
```



Question 5:



Save the figure in which you plot $x_{syn}[n]$

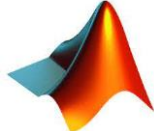
Explain briefly what you see



STEP 5- Lowpass Filtering $x[n]$

In the following lines of code, we filter out the frequencies $|\Omega| > 2\pi/N$ of $x[n]$, producing the signal $y[n]$ with Fourier coefficients b_k :

```
bk=zeros(size(ak));  
bk(1)=ak(1);  
bk(2)=ak(2);  
bk(2*n2+1)=ak(2*n2+1);  
for m=m1:m2 % Synthesis of y  
y(m+m2+1)=sum(bk.*exp(k*1i*omega0*m));  
end  
figure  
stem(n,abs(y)) % plots a k against n  
xlabel('n') % adds text below the X-axis  
ylabel('y') % adds text beside the Y-axis  
axis tight
```



Question 6:

Save the figure in which you plot $y[n]$



Explain briefly what you see

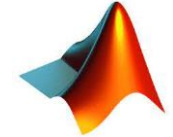


STEP 6- Changing the duty cycle

The duty cycle of the proposed discrete-time periodic signal is $\rho = \frac{n_2}{N} \approx 50\%$

Your job now is to:

- Plot the $x[n]$ for $\rho \approx 1/5$;
- Plot the Fourier coefficients a_k for $\rho \approx 1/5$;



Change the values of $x[n]$ to implement the duty cycle.

Explain your method
changing the duty cycle



Question 7:



Save the corresponding plot for $x[n]$
and a_k .

LAB3 DELIVERABLES

1. Pre LAB submission .txt file (Qmplus) **NOT MARKED**
2. Post LAB Answer sheet submission on QMplus **MARKED (10 marks)**
3. Post LAB **5-minute** video recording in which you explain the code and the results that you obtain **(20 marks)**

A brief self introduction - your name and QMUL student ID. **Your face must be seen in this part.** Give a detailed explanation of your code and obtained results. Do not exceed the time nor increase the speed of the recording.

