BBC4113 A (BBC4114 A) 2022/23

Question 1

[15 marks]

(a) Let U be a universal set and let A, B be subsets of U. Prove that $\overline{(A \cap B)} = \overline{A} \cup \bar{B}$. **(10 marks)**

(b) Determine whether the argument

$$\begin{array}{c} p \rightarrow q \\ p \wedge \neg q \\ \hline \therefore q \rightarrow p \end{array}$$

is valid. **(5 marks)**

Question 2

Prove that for any positive integer n, $\left(1+\frac{1}{n}\right)^n \leq \left(1+\frac{1}{n+1}\right)^{n+1}$. **[10 marks]**

Question 3

List relations satisfying the following properties. If you can, please list a relation with the domain set; otherwise, give some explanations. **[15 marks]**

(a) Not reflexive, not symmetric, not antisymmetric, and transitive. **(5 marks)**

(b) Not reflexive, symmetric, antisymmetric, and not transitive. **(5 marks)**

(c) Reflexive, symmetric, not antisymmetric, and not transitive. **(5 marks)**

Question 4

Select a theta notation for $f(n) = \sum_{k=1}^{n} \left(1+\frac{1}{k}\right)^{3k} k \ln k$ and prove it. **[10 marks]**

Question 5

Solve the recurrence relation

$$a_n = 2a_{n-1} + n \cdot 2^n + 3^n \quad \text{for all } n \geq 2$$

with initial condition $a_1 = 11$. **[15 marks]**

Question 6

Find the number of integer solutions of

$$x_1 + x_2 + x_3 = 15$$

subject to $x_1 \geq 1, 0 < x_2 \leq 5, 1 \leq x_3 < 4$. **[10 marks]**

Question 7

Consider an n by n grid of points at positions $\{(i, j): 1 \leq i \leq n, 1 \leq j \leq n)\}$ in the plane. Each point is colored black or white. How large must n be such that for every way to color the points, there is a rectangle whose four corners all have the same color? The answer requires a proof for the upper bound (**7 marks**) and an example for the lower bound (**3 marks**).

[10 marks]

Question 8

G is a simple graph with 60 vertices with each of vertex degree > 40. Prove that G must contain the 4-order complete graph K_4 as a subgraph. **[15 marks]**