Final Review <u>BBC4941</u>

	Prob	Stoc.	
Q1 填空	10~12	3~5	$3' \times 15 = 45'$
Q2-Q6 解答	Q2~4	Q5, Q6	$55'$

prob

- Ch1: A, $P(A)$
- Ch2. X, $F(x) = P(X \le x)$
- Ch3. (X, Y) $f_{Y|X}(y|x)$ $\iint dx\,dy$
- Ch4 $\dfrac{S_n - n\mu}{\sqrt{n}\,\sigma} \sim N(0,1)$

Ch1

Event = Set: AB, $A \cup B$, $\bar{A}$, $\overline{A \cap B} = \bar{A} \cup \bar{B}$

prob: $P(A \cup B) = P(A) + P(B) - P(AB)$

A, B independent

$P(A\bar{B}) = P(A)P(\bar{B})$, $P(A|B) = P(A)$

$P(\bar{A}) = 1 - P(A)$

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$$

Conditional prob.:

$$P(A|B) = \frac{P(AB)}{P(B)} \quad ①$$

formulars

$P(AB) = P(A)P(B|A)$ ②

$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$ ③

$P(B_i|A) =$ by ①~③

ch2

X.

$F(x)$: d.f.

(1) $F(-\infty)=0, \quad F(+\infty)=1$

(2) $F(x+) = \lim_{y\to x, y>x} F(y) = F(x)$

(3) $F \uparrow$

"天选"

$P(X=x) = F(x) - F(x-)$

$P(a < X \le b) = F(b) - F(a)$

discrete: $p(x_i)$ p.f. :

$\Sigma p(x_i) = 1$

$F(x) \Leftrightarrow p(x_i)$

continuous: $f(x)$ p.d.f.

$\begin{cases} f(x) = F'(x) \\ F(x) = \int_{-\infty}^{x} f(t)\,dt \end{cases}$

$P(X \in A) = \int_A f(x)dx$

$Y = g(X)$:

$F_Y(y) = P(Y \le y)$

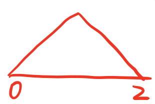
$= P(g(X) \le y)$

$\overset{g\uparrow\uparrow}{=} P(X \le \square)$

Table 2.9

$$= F_X(\square)$$

Typical distributions and their NC

distribution	p.f / p.d.f	$E(X)$	$Var(X)$
$b(1,p)$	$p^k(1-p)^{1-k}$, $k=0,1$.	p	$p(1-p)$
$b(n,p)$	$C_n^k p^k (1-p)^{n-k}$, $k=0,1,\dots,n$.	$n\cdot p$	$n\cdot p(1-p)$
$Pois(\lambda)$ $\pi(\lambda)$	$\frac{\lambda^k}{k!}e^{-\lambda}$, $k=0,1,\dots$	λ	λ
$Geom(p)$	$(1-p)^{k-1}\cdot p$, $k=1,2,\dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$U(a,b)$	$\frac{1}{b-a}$, $a<x<b$ $Z=X+Y$ 	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$Exp(\lambda)$	$\lambda e^{-\lambda x}$, $x>0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $-\infty < x < +\infty$	μ	σ^2

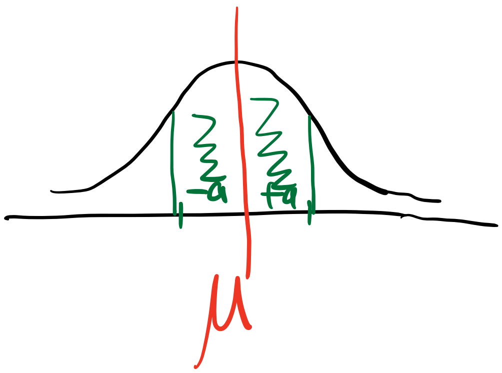
Memorylessness: $X \sim Exp(\lambda)$

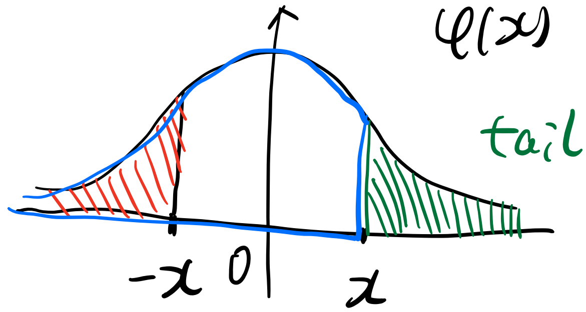
$$\begin{cases} 1-P(X > s+t \mid X > s) = 1-P(X > t) \\ P(X \le s+t \mid X > s) = P(X \le t) \end{cases}$$

$$F(x) = \begin{cases} 1-e^{-\lambda x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

$$S(x) = P(X > x) = e^{-\lambda x}, \quad x > 0$$

Symetry: $X \sim N(\mu, \sigma^2)$





$$\Phi(-x) = 1 - \Phi(x)$$

Ch3 (X,Y)

- discrete: $Y \backslash X$
- Continuous:
  - $f(x,y)$: $P((X,Y)\in A)=\iint_A f(x,y)\,dx\,dy$ // ...
  - ① $a<x<b$
    $f_X(x)$: $\int_{-\infty}^{+\infty} f(x,y)\,dy = \int_{c(x)}^{d(x)} \cdots \,dy$
  - ① $a<x<b$
    $f_{Y|X}(y|x)$: $\dfrac{f(x,y)}{f_X(x)}$, $c(x)<y<d(x)$
  - $P(Y\in A \mid X=x)=\int_A f_{Y|X}(y|x)\,dy$
- $Z=g(X,Y)$
  - ① $z \Rightarrow x$
    $Z=X+Y$: $f_Z(z)=\int_{-\infty}^{+\infty} \underbrace{f_X(x)}_{>0}\cdot \underbrace{f_Y(z-x)}_{>0}\,dx$ ($\cap$)
    $= \begin{cases} \int_{a_1(z)}^{b_1(z)} \cdots \,dx & z\in I_1 \\ \int_{a_2(z)}^{b_2(z)} \cdots \,dx & z\in I_2 \end{cases}$
  - $Z=X\vee Y$: $F_Z(z)=F^2(z)$
  - $Z=X\wedge Y$: $F_Z(z)=1-(1-F(z))^2$

Numerical Characteristics

Small letter: Const.
Capital letter: r.v

d=1.

$$E(X) = \begin{cases} \sum_i x_i \cdot p(x_i) & (d.) \\ \int_{-\infty}^{+\infty} x \cdot f(x)\, dx & (c.) \end{cases}$$

$$Var(X) = E\left((X - EX)^2\right)$$

$$= \underbrace{E(X^2)}_{E(g(X))} - E^2(X)$$

?

$$E(g(X)) = \begin{cases} \sum_i g(x_i) \cdot p(x_i) & (d) \\ \int_{-\infty}^{+\infty} g(x) \cdot f(x)\, dx & (c.) \end{cases}$$

d=2.

$$E(aX \pm bY) = aE(X) \pm bE(Y)$$

$$Var(aX \pm bY)$$

$$(a \pm b)^2$$

$$= Var(aX) + Var(bY) \pm 2Cov(aX, bY)$$

$$= a^2 Var(X) + b^2 Var(Y) \pm 2ab\, Cov(X, Y)$$

$$Cov(X, Y) = \underbrace{E(XY)}_{E(g(X,Y))} - EX \cdot EY$$

$$E(g(X,Y)) = \iint g(x,y) \cdot \underline{f(x,y)}\, dx\,dy$$

$$Cov(aX + bY, cU + dV)$$

$$= ac\, Cov(X, U) + ad\, Cov(X, V)$$

$$+ bc\, Cov(Y, U) + bd\, Cov(Y, V).$$

$$(ax + by)(cu + dv) =$$

un correlated: $\rho_{XY}=0 = \dfrac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y}$

$\Longleftrightarrow Cov(X,Y)=0$

$\Longleftrightarrow E(XY) = EX \cdot EY.$

$Cov(X,Y) = \rho_{XY} \cdot \sigma_X \cdot \sigma_Y$

$Cov(X,X) = Var(X).$

$X(t)$

d=1 $\mu_X(t) = E(X(t))$
- E(大写)
- 小写 E(⋯)
- $E(g(X))$

aX : $(\cos t)\,\theta$

$$\left\{ \; g(\Theta),\ \text{eg: } \cos(t\Theta) \right.$$

$d=2$

$$R_X(s,t) = E\big(X(s)\,X(t)\big)$$

$$R_X(t,\ t+\tau)$$

depends only on τ.

$$\begin{cases} E(X^2) = \sigma_X^2 + \mu_X^2 \\ \qquad\quad\ = Var(X) + E^2(X) \\ E(XY) = Cov(X,Y) + \mu_X \mu_Y \end{cases}$$

$\square$

Stoc

Ch5-8

- Ch5: $X(t)$: $F(\vec{x};\vec{t}) = P(\cap X_{t_i} \le x_i)$
  - $\mu_X(t)$. $R_X(s,t)$
  - $\sigma_X^2(t)$. $C_X(s,t)$
- Ch6. Stationary
  - def: $\begin{cases} \mu_X(t) = \text{constant.} \\ R_X(\tau) = R_X(t,t+\tau) : \text{depends only on } \tau. \end{cases}$
  - $Q = R_X(0) \, (\tau=0) = E(X_t^2)$
  - $R_X(\tau) \Leftrightarrow S_X(\omega)$, $Ae^{-\beta|\tau|} \Leftrightarrow \frac{2A\beta}{\omega^2+\beta^2}$
- Ch8. I-I, $X_0=0$
  - $C_X(s,t) = Var_X(s \wedge t)$
  - $N_t \sim \pi(\lambda t)$.
  - $W_t \sim N(0,\sigma^2 t)$.

Ch7. MC

$S = \{0, 1, 2\}$ or $S = \{1, 2, 3\}$.

$$\vec{q}(0) = \Big(P(X_0 = 0), P(X_0 = 1), P(X_0 = 2) \Big)$$

$$X_0 \sim \vec{q}(0)$$

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} \; & \; & \; \\ \; & \; & \; \\ \boxed{\;} & \; & \; \end{pmatrix}_{3\times3} \end{array}$$

$p_{20}(1)$

1. ergodic ? why.

$P^2 > 0 \quad \therefore$ ergodic.

i.e. $\quad p_{ij}(2) > 0, \quad \forall i, j \in S.$

2. $X_n \sim ?$

$$\boxed{\vec{q}(n) = \vec{q}(0) \cdot P^n}$$

3. $P(X_{n_1}=i_1,\ X_{n_2}=i_2,\ X_{n_3}=i_3)$

$$= P(早) \cdot \underbrace{P(再|早)}_{p_{i_1 i_2}^{(n_2-n_1)}} \cdot \underbrace{P(后|再)}_{p_{i_2 i_3}^{(n_3-n_2)}}$$

$$P^2 \qquad\qquad P$$

4. limit / invariant / stationary distr ?

$$\begin{cases} \vec{\pi} = \vec{\pi} P \\ \sum \pi_i = 1 \end{cases}$$

$$\begin{cases} (\pi_0 \quad \pi_1 \quad \pi_2) = (\pi_0 \quad \pi_1 \quad \pi_2)\begin{pmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{pmatrix} \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$\square$