EBU4375 Signals and Systems:
The CT Fourier transform in the ω and f domains

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The CT Fourier transform

The Fourier transform allows us to describe signals in the frequency domain. In other words, it allows us to express signals in terms of their frequency components.

We usually distinguish between:

- The ordinary frequency f (measured in Hertz).
- The angular frequency ω (measured in radians per second).

The ordinary frequency and the angular frequency are related by $\omega = 2\pi f$ and we can express the Fourier transform as a function of both. The purpose of this slides is to show the relationship between the Fourier transform in ω and f.

Notation

We will denote a CT signal by $x(t)$, its Fourier transform in the f domain by $X_f(f)$ and its Fourier transform in the ω domain by $X_\omega(\omega)$. We will express the relationships between $x(t)$ and its Fourier transforms in f and ω like this:

$$x(t) \overset{FT_f}{\Longleftrightarrow} X_f(f)$$
$$x(t) \overset{FT_\omega}{\Longleftrightarrow} X_\omega(\omega)$$

Definition of the Fourier transform

The Fourier transforms in the f and ω domains are defined as follows:

$$
\begin{aligned}
X_f(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\
X_\omega(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt
\end{aligned}
$$

As you can see, since $\omega = 2\pi f$, $X_\omega(\omega) = X_f\left(\frac{\omega}{2\pi}\right)$.

Definition of the inverse Fourier transform

The inverse Fourier transform from the f domain is defined as

$$x(t) = \int_{-\infty}^{\infty} X_f(f) e^{j2\pi f t} df$$

Let us now change the integration variable, using the substitution $f = \omega/2\pi$:

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty} X_f(f) e^{j2\pi f t} df \\ &= \int_{-\infty}^{\infty} X_f\left(\frac{\omega}{2\pi}\right) e^{j\omega t} \frac{d\omega}{2\pi} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_\omega(\omega) e^{j\omega t} d\omega \end{aligned}$$

This derivation shows the connection between the inverse Fourier transforms from the f and ω domains.