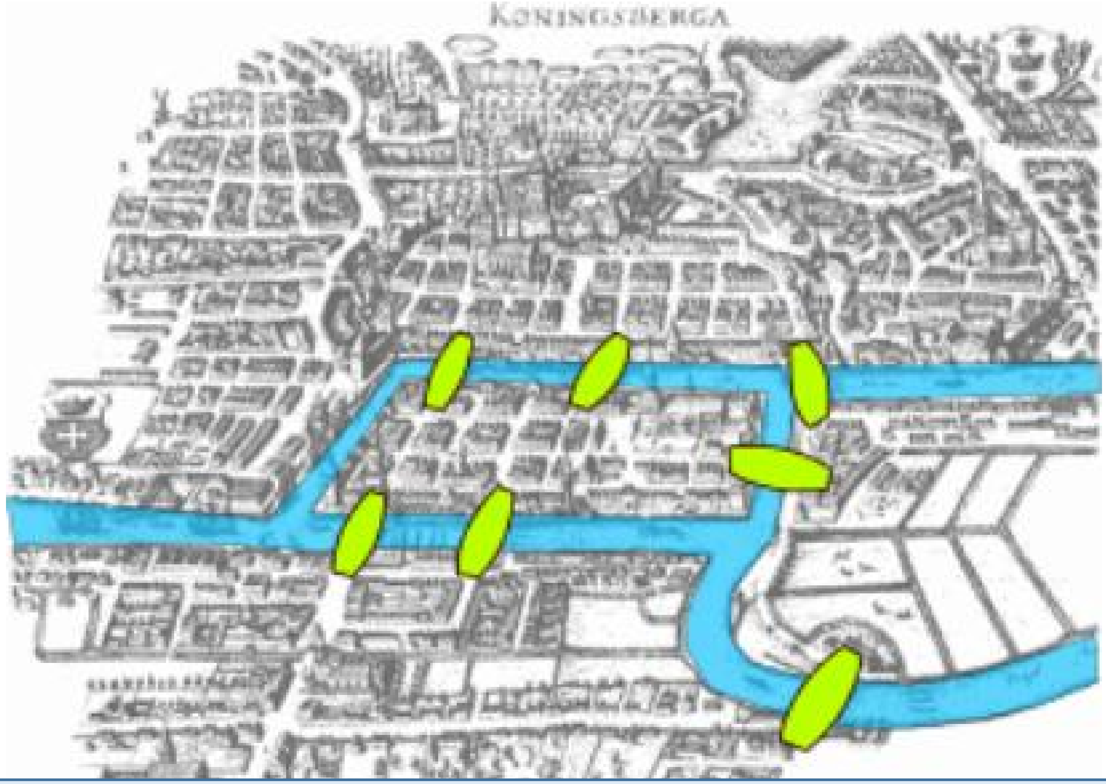
Chapter 8 Graph Theory 图论

Lu Han

hl@bupt.edu.cn

8.1 Introduction 简介

Seven Bridges of Königsberg 柯尼斯堡七桥问题



Is it possible to walk with a route that crosses each bridge exactly once?

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Seven Bridges of Königsberg 柯尼斯堡七桥问题

So Euler showed that the **"Seven Bridges of Königsberg"** is unsolvable.

When is it possible to have a walk that visits every edge exactly once?

Eulerian Path

Euler's theorem: A graph has an Eulerian path if and only if it is "connected" and has at most two vertices with an odd number of edges.

This theorem was proved in 1736,

and was regarded as the starting point of graph theory.

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graph (or undirected graph) (无向图)

digraph (or directed graph) (有向图)

incident on (相关联的)

adjacent vertices (相邻顶点)

parallel edges (平行边/并行边)

loop (圈)

isolated vertex (孤立顶点)

simple graph (简单图)

weighted graph (权重图)

complete graph (完全图)

bipartite (二部图 / 二分图)

complete bipartite graph (完全二部图)

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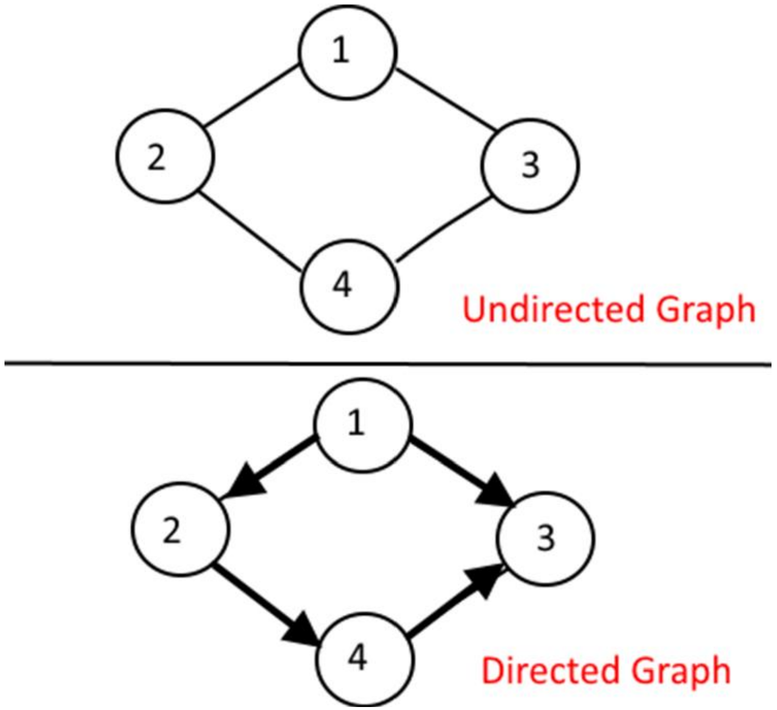

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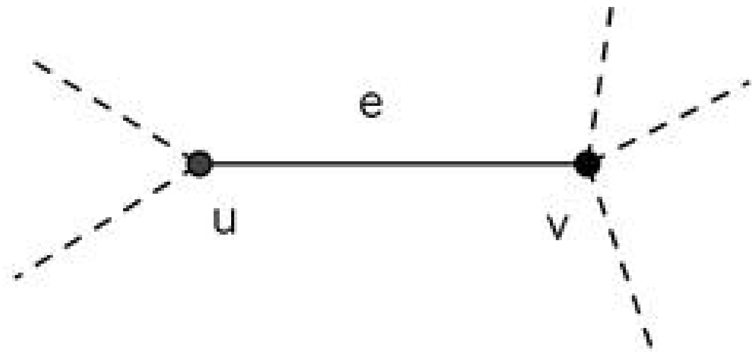

- Two vertices are called **adjacent** if they are connected by an edge.
- Two edges are called **incident**, if they share a vertex.
- A vertex and an edge are called **incident**, if the vertex is one of the two vertices the edge connects.

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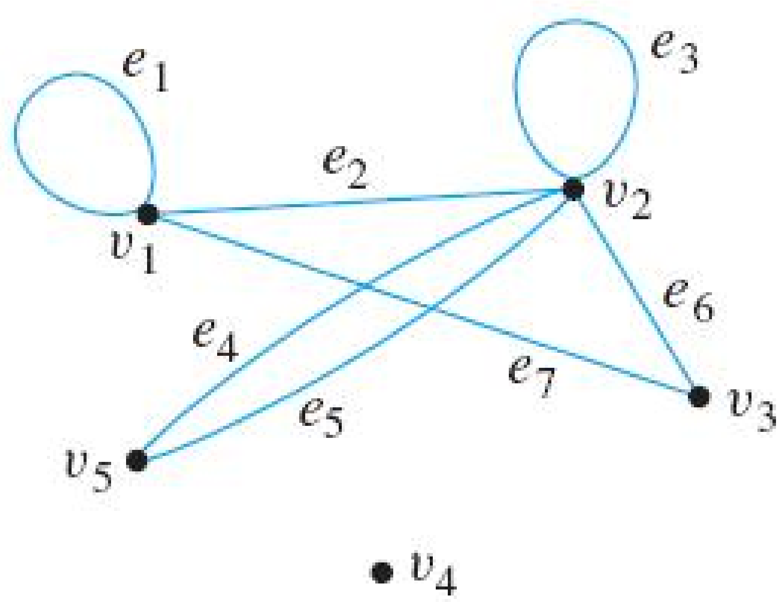

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8.2 Paths and Cycles 路径和回路

Definition 8.2.1 Let v_0 and v_n be vertices in a graph. A **path (路径)** from v_0 to v_n of length n is an alternating sequence of $n + 1$ vertices and n edges beginning with vertex v_0 and ending with vertex v_n,

$$(v_0, e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n),$$

in which edge e_i is incident on vertices v_{i-1} and v_i for $i = 1, \ldots, n$.

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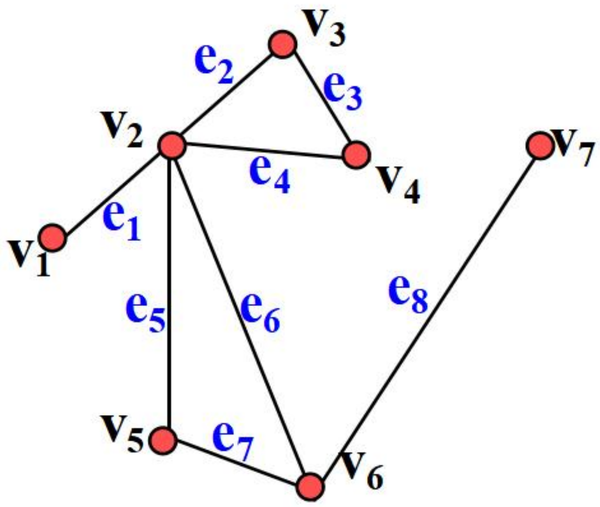

$(v_1, e_1, v_2, e_2, v_3, e_3, v_4)$

$(v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_2)$

$(v_2, e_2, v_3, e_3, v_4, e_4, v_2)$

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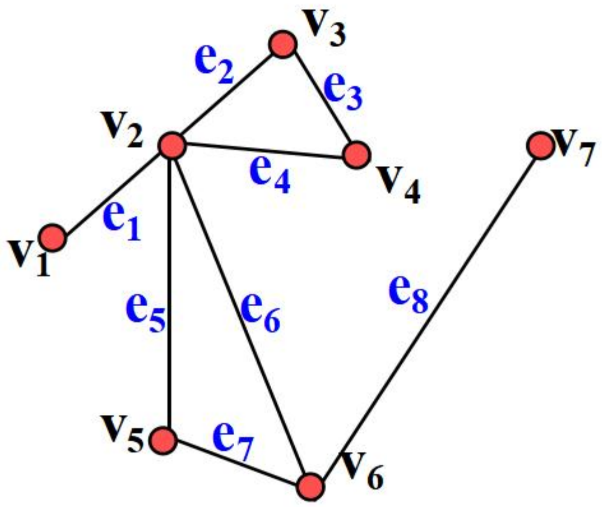

$(v_1, e_1, v_2, e_2, v_3, e_3, v_4)$

$(v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_2)$

$(v_2, e_2, v_3, e_3, v_4, e_4, v_2)$ closed path

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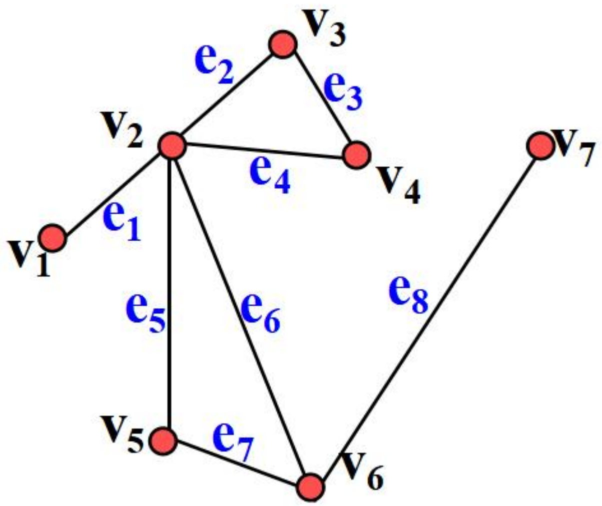

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(v_6)

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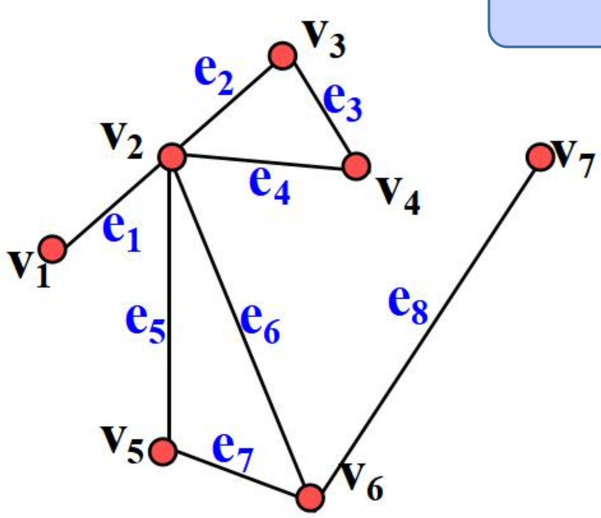
in which edge e_i is incident on vertices v_{i-1} and v_i for $i = 1, \ldots, n$.

In absence of parallel edges, in denoting a path we may suppress the edges.



$(v_1, e_1, v_2, e_2, v_3, e_3, v_4) \rightarrow (v_1, v_2, v_3, v_4)$

$(v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_2)$

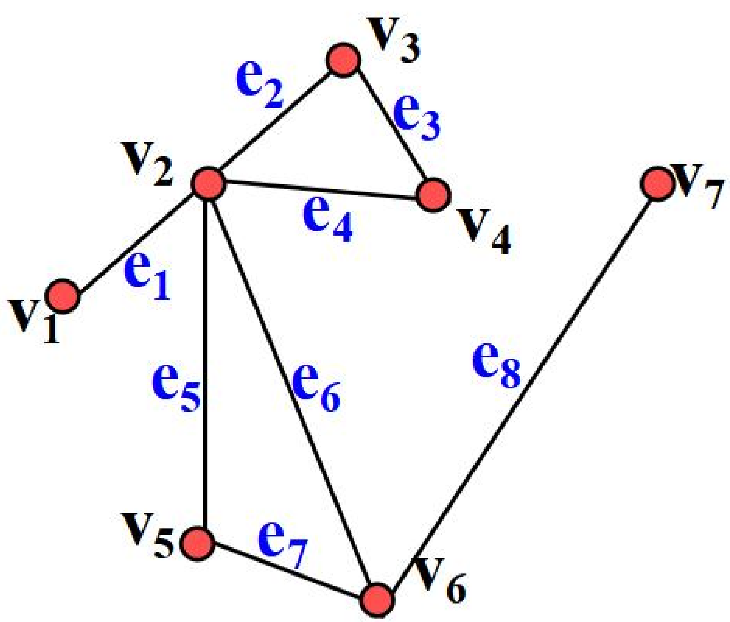
$(v_2, e_2, v_3, e_3, v_4, e_4, v_2)$ closed path

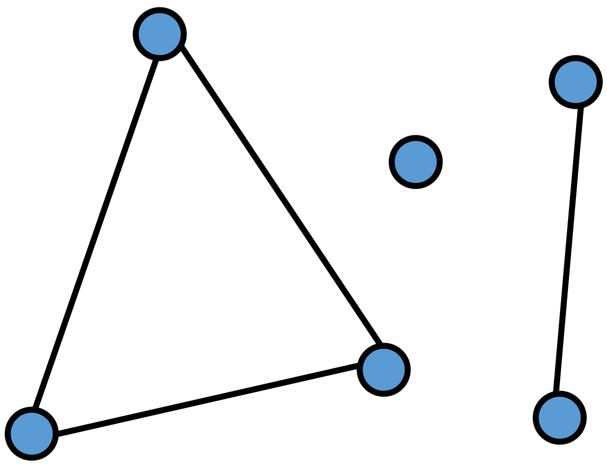
(v_6)

8.2 Paths and Cycles 路径和回路

connected graph (连通图)

Definition 8.2.4 A graph G is **connected (连通的)** if given any vertices v and w in G, there is a path from v to w.





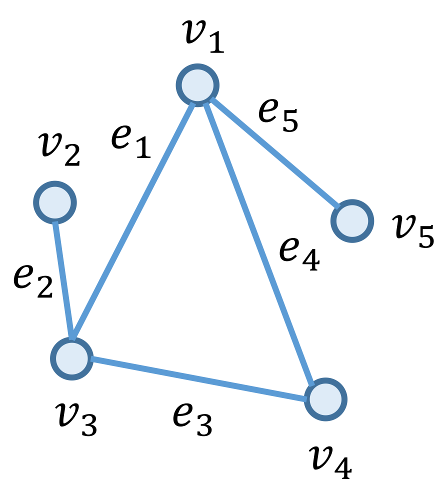
8.2 Paths and Cycles 路径和回路

Definition 8.2.8 Let $G = (V, E)$ be a graph, we call (V', E') a **subgraph (子图)** G if

(a) $V' \subseteq V$ and $E' \subseteq E$.

(b) For every edge $e' \in E'$, if e' is incident on v' and w', then $v', w' \in V'$.

Example



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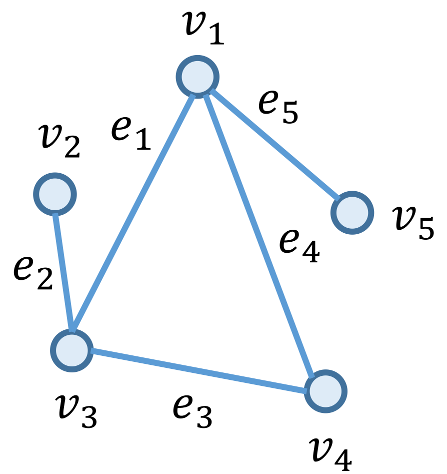

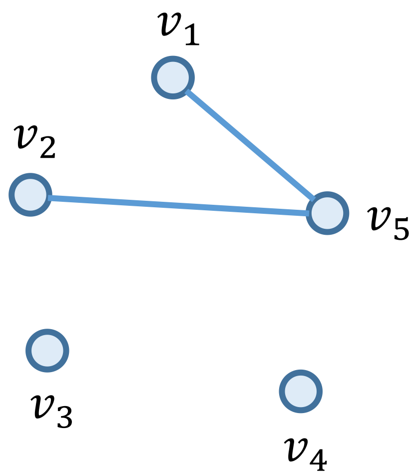

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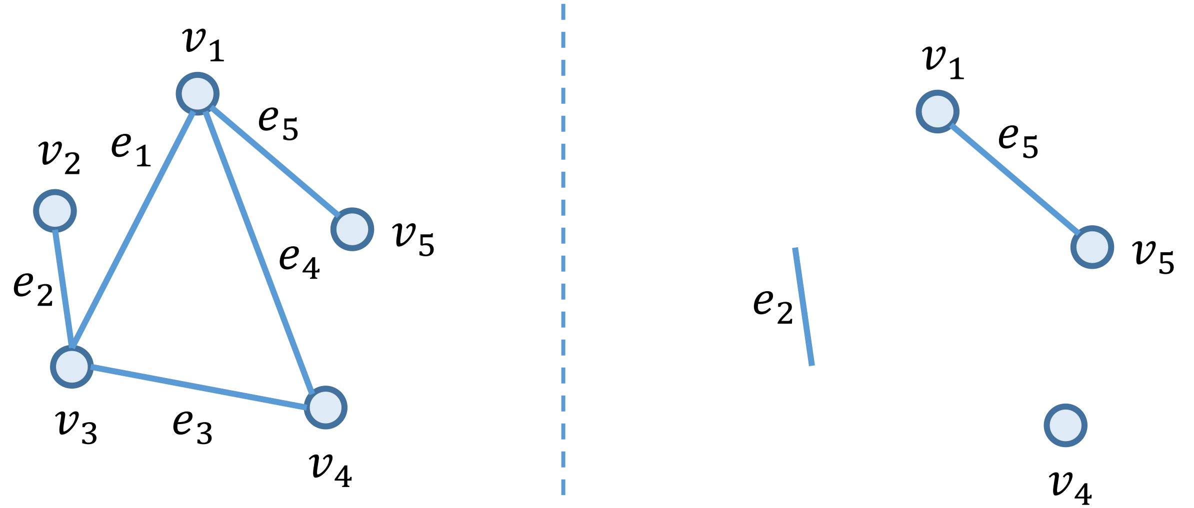

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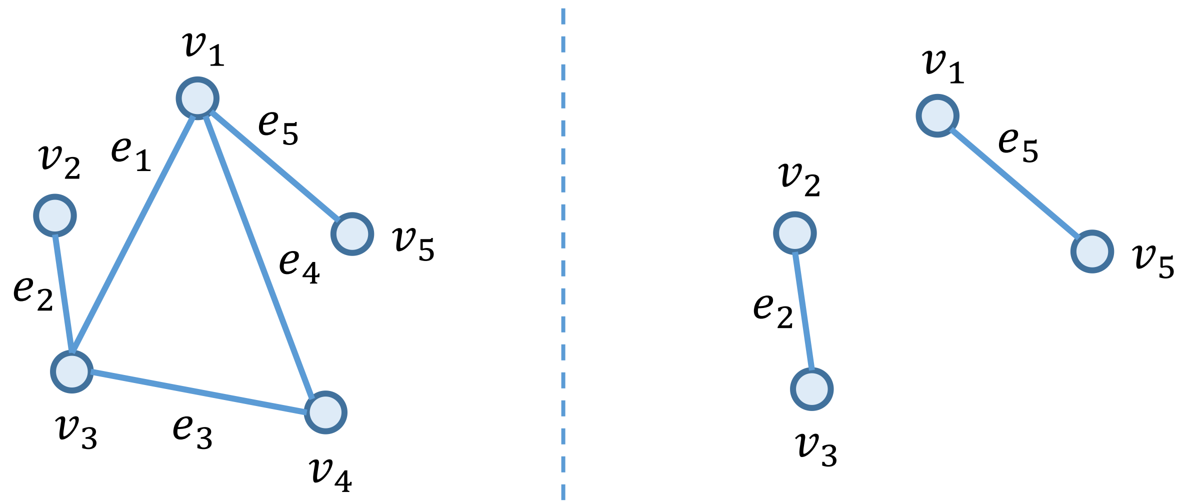

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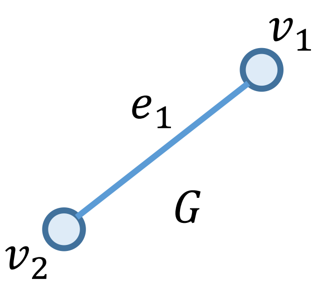
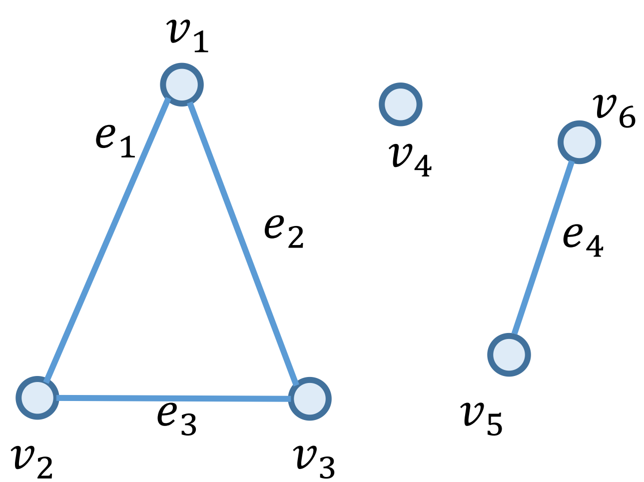
Example 8.2.10 Find all subgraphs of the graph G having at least one vertex.



8.2 Paths and Cycles 路径和回路

Definition 8.2.11 Let G be a graph and let v be a vertex in G. The subgraph G' of G consisting of all edges and vertices in G that are contained in some path beginning at v is called the **component (分支)** of G containing v.

Example 8.2.13 The components of G



The component of G containing v_2?

The component of G containing v_4?

The component of G containing v_5?

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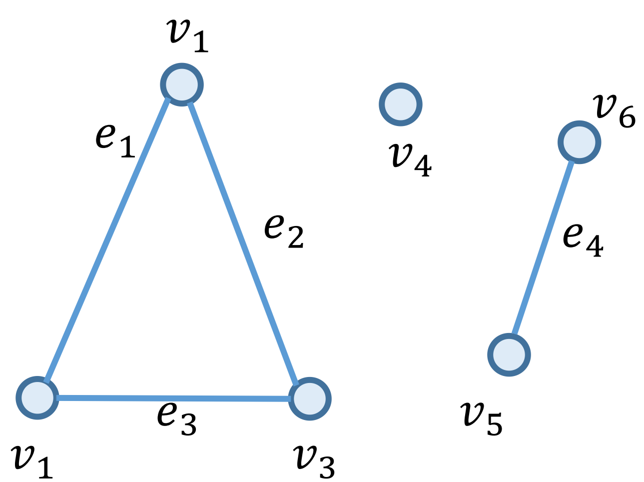

A graph is **connected** if and only if it has only **1 connected component**.

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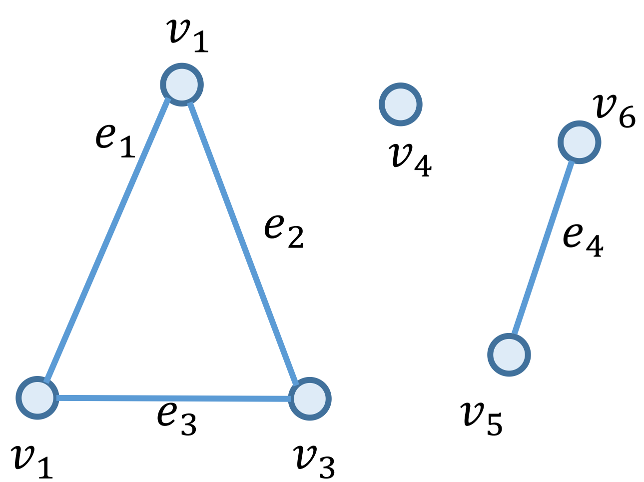

If we define a relation R on the set of vertices V by the rule
vRw* if there is a path from *v* to *w
then R is equivalence relation on V .

If $v \in V$, the set of vertices in the componeent containing v is the equivalence class

$$[v] = \{w \in V \mid wRv\}.$$

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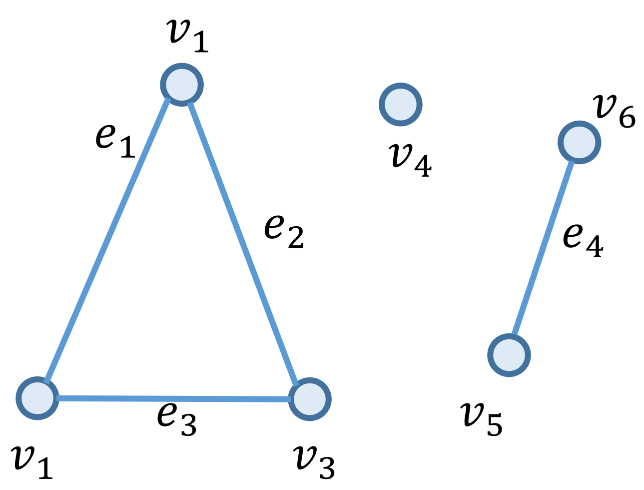

If we define a relation R on the set of vertices V by the rule

vRw if there is a path from v to w

then R is equivalence relation on V .

$[v_1] =$

$[v_4] =$

$[v_5] =$

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Definition 8.2.14 Let v and w be vertices in a graph G.

- A **simple path(简单路径)** from v to w is a path from v to w with no repeated vertices.

No repeated edges?

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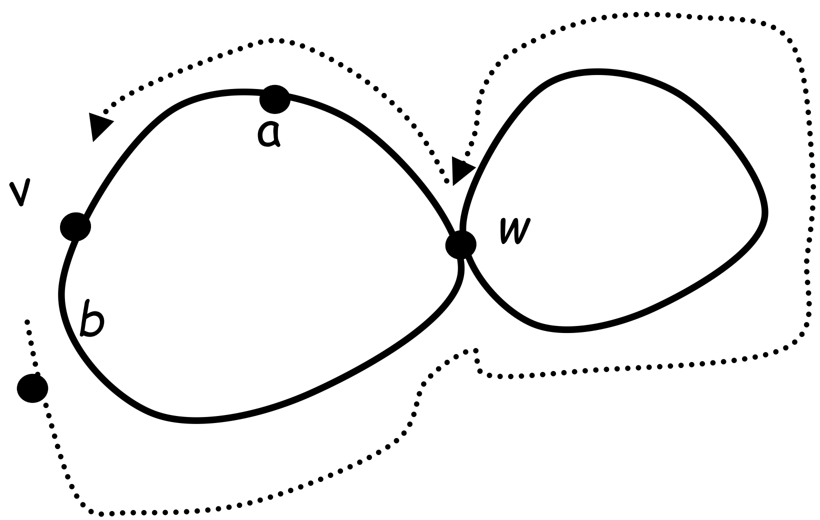
Definition 8.2.14 Let v and w be vertices in a graph G.

- A **simple path（简单路径）** from v to w is a path from v to w with no repeated vertices.
- A **cycle (or circuit) (回路或者环路)** is a path of nonzero length from v to v with no repeated edges.



cycle: $v \cdots b \cdots w \cdots w \cdots a \cdots v$

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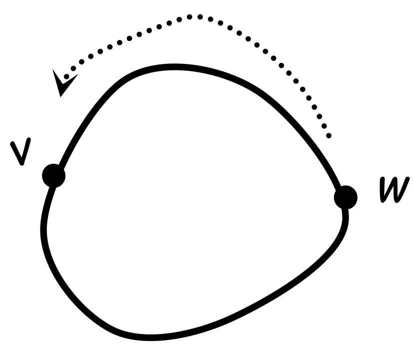

cycle: $v \cdots w \cdots v$

In a simple cycle, every vertex is of degree exactly 2.

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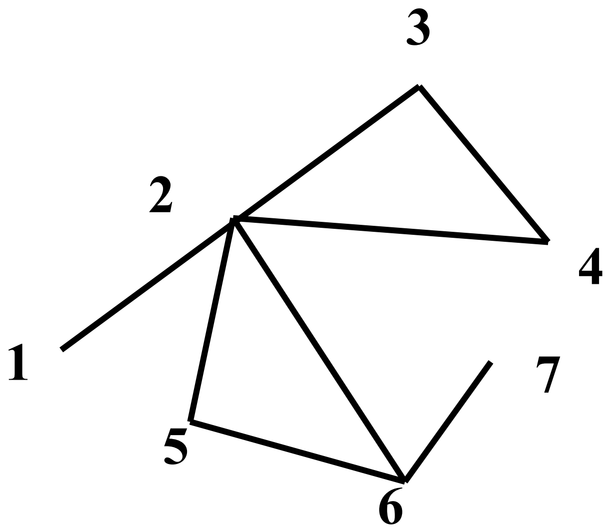

Path	Simple path	Cycle	Simple cycle
6 5 2 4 3 2 1			
6 5 2 4			
2 6 5 2 4 3 2			
5 6 2 5			
7			

8.2 Paths and Cycles 路径和回路

Eulerian Path (欧拉路径)

Euler Cycle (欧拉回路): a cycle in a graph G that includes all of the edges and all of the vertices of G.

The **degree of a vertex (顶点度)** v, denoted by $\delta(v)$, is the number of edges incident on v.

Each loop on v contributes 2 to the degree of v.

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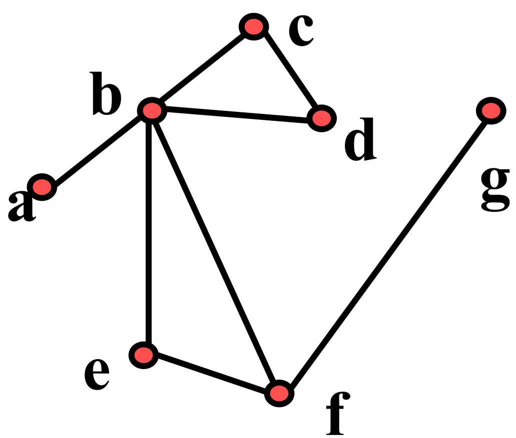

$\delta(a) =$
$\delta(b) =$
$\delta(c) =$
$\delta(d) =$
$\delta(e) =$
$\delta(f) =$
$\delta(g) =$

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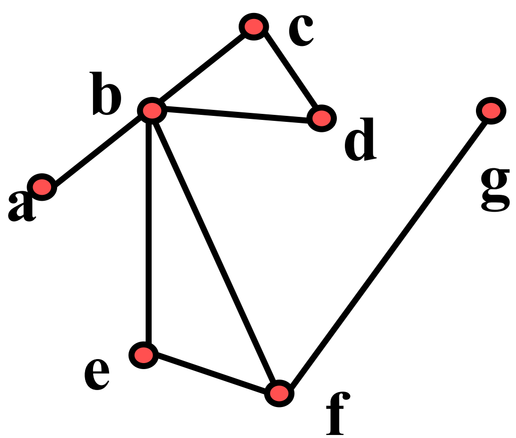

δ(a) = 1,
δ(b) = 5,
δ(c) = 2,
δ(d) = 2,
δ(e) = 2,
δ(f) = 3,
δ(g) =1.

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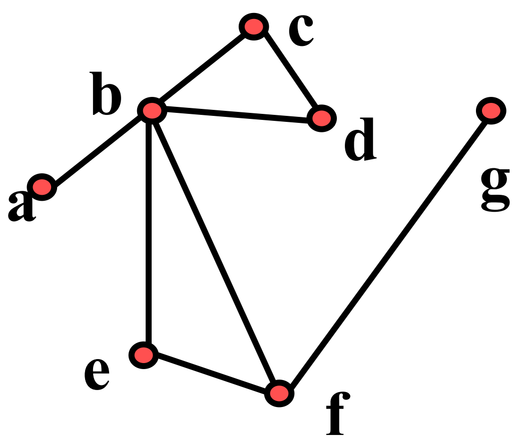

$\delta(a) = 1$,
$\delta(b) = 5$,
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$\delta(d) = 2$,
$\delta(e) = 2$,
$\delta(f) = 3$,
$\delta(g) = 1$.

The ***neighbour set*** $N(v)$ of a vertex v is the set of vertices adjacent to it.

The degree of a vertex v = the number of neighbours of v?

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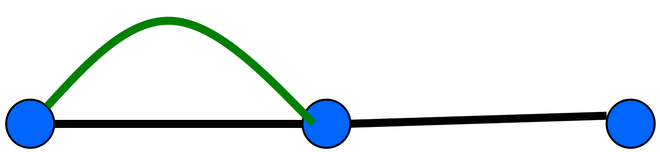

NO!

The ***neighbour set*** $N(v)$ of a vertex v is the set of vertices adjacent to it.

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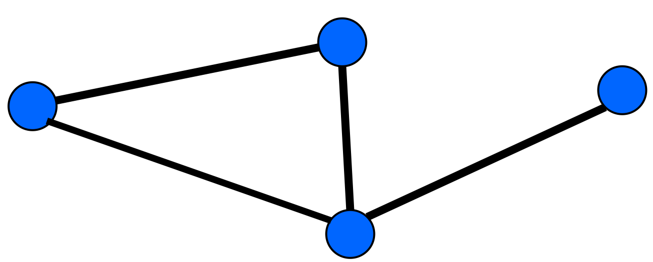

Yes for any simple graph.

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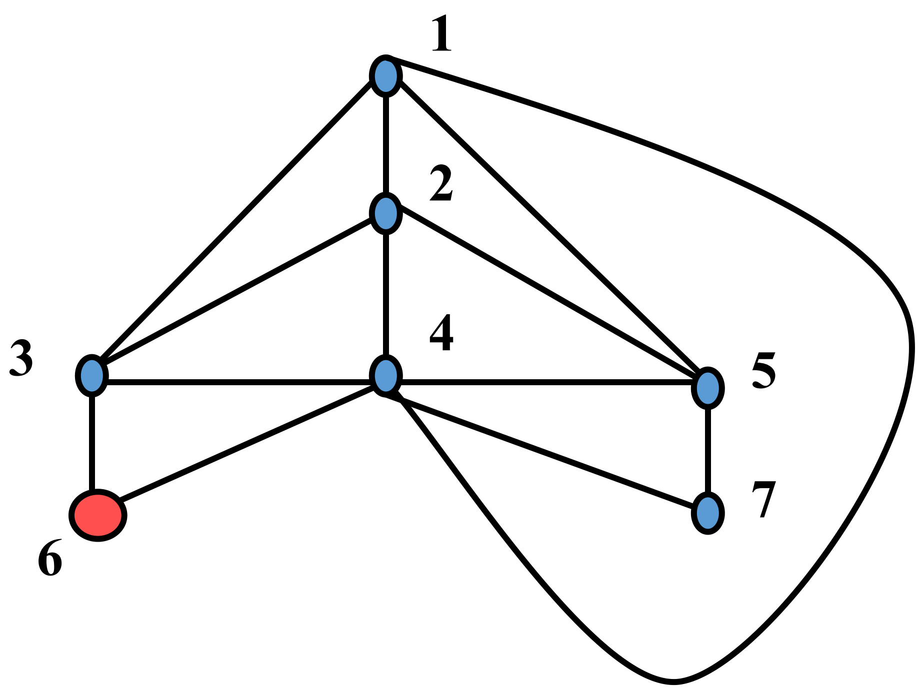
The **degree of a vertex (顶点度)** v, denoted by $\delta(v)$, is the number of edges incident on v.

Each loop on v contributes 2 to the degree of v.

Theorem 8.2.17 If a graph G has an Euler cycle, then G is connected and every vertex has even degree.

8.2 Paths and Cycles 路径和回路

Theorem 8.2.18 If G is a connected graph and every vertex has even degree, then G has an Euler cycle.



8.2 Paths and Cycles 路径和回路

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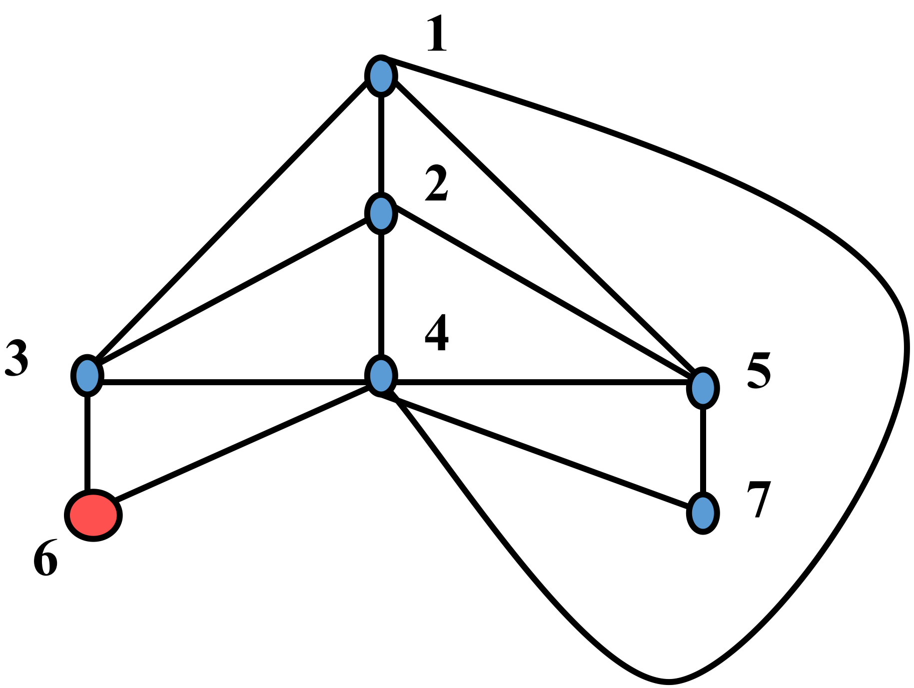

6 4 7 5 1 3 4 1 2 5 4 2 3 6

8.2 Paths and Cycles 路径和回路

A graph G is an **Euler graph (欧拉图)** if it has an Euler cycle.

Theorems 8.2.17 and 8.2.18: G is an Euler graph if and only if G is connected and all its vertices have even degree.

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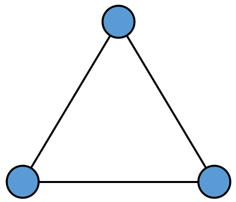
8.2 Paths and Cycles 路径和回路

Is there a graph with degree sequence (2, 2, 2)? YES.



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8.2 Paths and Cycles 路径和回路

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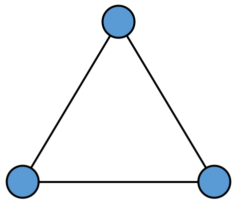

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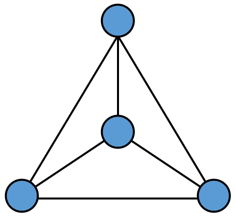

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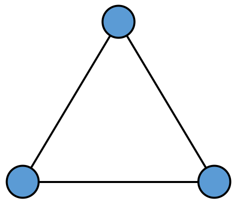

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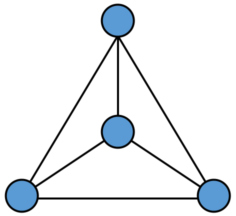

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8.2 Paths and Cycles 路径和回路

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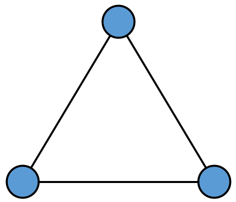
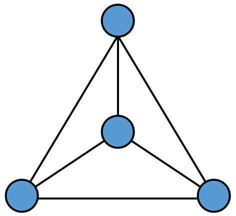


Is there a graph with degree sequence (3, 3, 3, 3)? YES.



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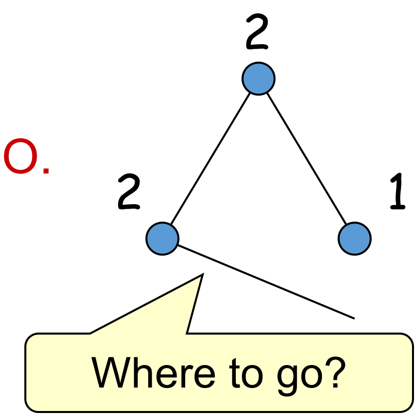

8.2 Paths and Cycles 路径和回路

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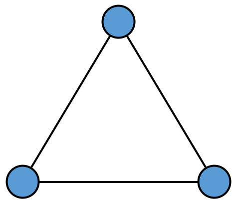

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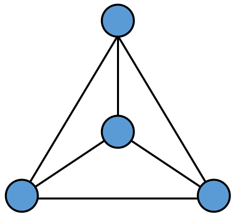

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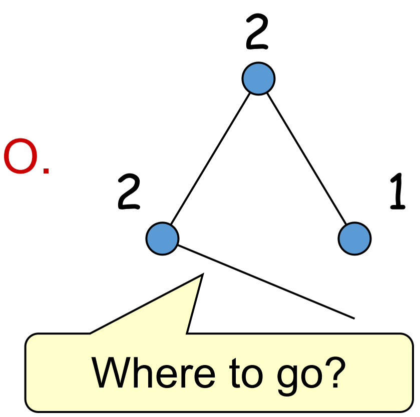

Is there a graph with degree sequence (2, 2, 2, 2, 1)?

8.2 Paths and Cycles 路径和回路

Theorem 8.2.21 The handshaking theorem

If several people shake hands, the total number of hands shake must be even.

8.2 Paths and Cycles 路径和回路

Theorem 8.2.21 The handshaking theorem

If G is a graph with m edges and n vertices $v_1, v_2, \ldots, v_n$, then

$$\sum_{i=1}^{n} \delta(v_i) = 2m$$

In particular, the sum of the degrees of all the vertices of a graph is even.

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In particular, the sum of the degrees of all the vertices of a graph is even.

Proof When we sum over the degrees of all the vertices, we count each edge (vi, vj) twice—once when we count it as (vi, vj) in the degree of vi and again when we count it as (vj, vi) in the degree of vj. The conclusion follows.

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8.2 Paths and Cycles 路径和回路

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Examples

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2+2+1 = odd, so impossible.

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8.2 Paths and Cycles 路径和回路

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Is there a graph with degree sequence (1, 2, 3)?

Question. Given a degree sequence, if the sum of degree is even, is it true that there is a simple graph with such a degree sequence?

8.2 Paths and Cycles 路径和回路

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NO! (3, 3, 3, 1), (3, 5)...

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Examples

Is there a graph with degree sequence (1, 2, 3)?

YES!

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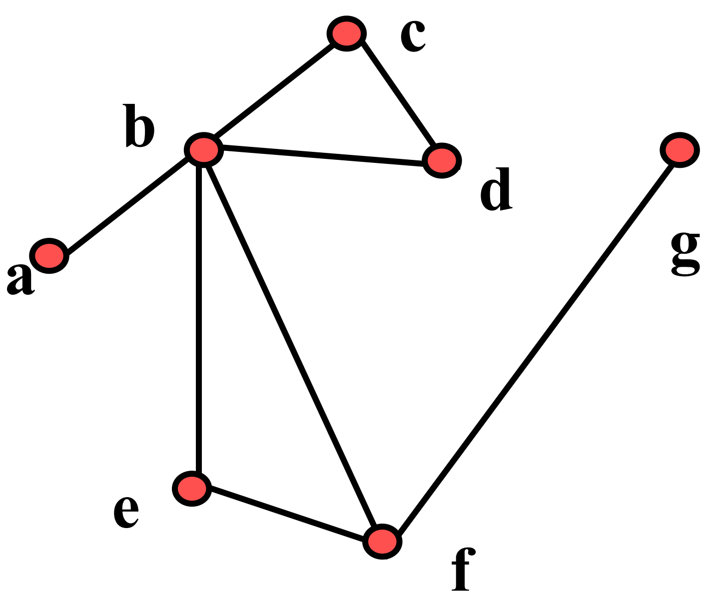
8.2 Paths and Cycles 路径和回路

Corollary 8.2.22 In any graph, the number of vertices of odd degree is even.



$\delta(a) = 1$,
$\delta(b) = 5$,
$\delta(c) = 2$,
$\delta(d) = 2$,
$\delta(e) = 2$,
$\delta(f) = 3$,
$\delta(g) = 1$.

8.2 Paths and Cycles 路径和回路

Corollary 8.2.22 In any graph, the number of vertices of odd degree is even.

Proof Let us divide the vertices into two groups: those with even degree $x_1, \ldots, x_m$ and those with odd degree $y_1, \ldots, y_n$.
Let $S = \delta(x_1) + \delta(x_2) + \ldots + \delta(x_m)$, $T = \delta(y_1) + \delta(y_2) + \ldots + \delta(y_n)$.
By **Theorem 8.2.21 (The handshaking theorem)**, $S + T$ is even. Since S is the sum of even numbers, S is even. Thus T is even. But T is the sum of n odd numbers, and therefore n is even.

8.2 Paths and Cycles 路径和回路

Theorem 8.2.23 A graph has a path with no repeated edges from v to w ($v \neq w$) containing all the edges and vertices if and only if it is connected and v and w are the only vertices having odd degree.

8.2 Paths and Cycles 路径和回路

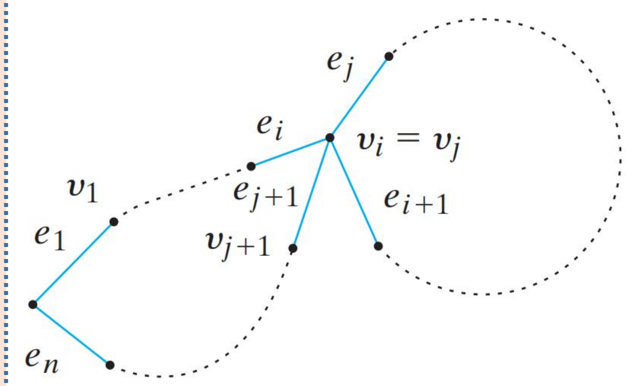
Theorem 8.2.24 If a graph G contains a cycle from v to v, G contains a simple cycle from v to v.

A **cycle (or circuit) (回路或者环路)** is a path of nonzero length from v to v with no repeated edges.

8.2 Paths and Cycles 路径和回路

Theorem 8.2.24 If a graph G contains a cycle from v to v, G contains a simple cycle from v to v.

Proof Let
$C = (v_0, e_1, v_1, \ldots, e_i, v_i, e_{i+1}, \ldots, e_j, v_j, e_{j+1}, v_{j+1}, \ldots, e_n, v_n)$
be a cycle from v to v where $v = v_0 = v_n$. If C is not a simple cycle, then $v_i = v_j$, for some $i < j < n$. We can replace C by the cycle
$C' = (v_0, e_1, v_1, \ldots, e_i, v_i, e_{j+1}, v_{j+1}, \ldots, e_n, v_n)$
If C' is not a simple cycle from v to v, we repeat the previous procedure. Eventually we obtain a simple cycle from v to v.



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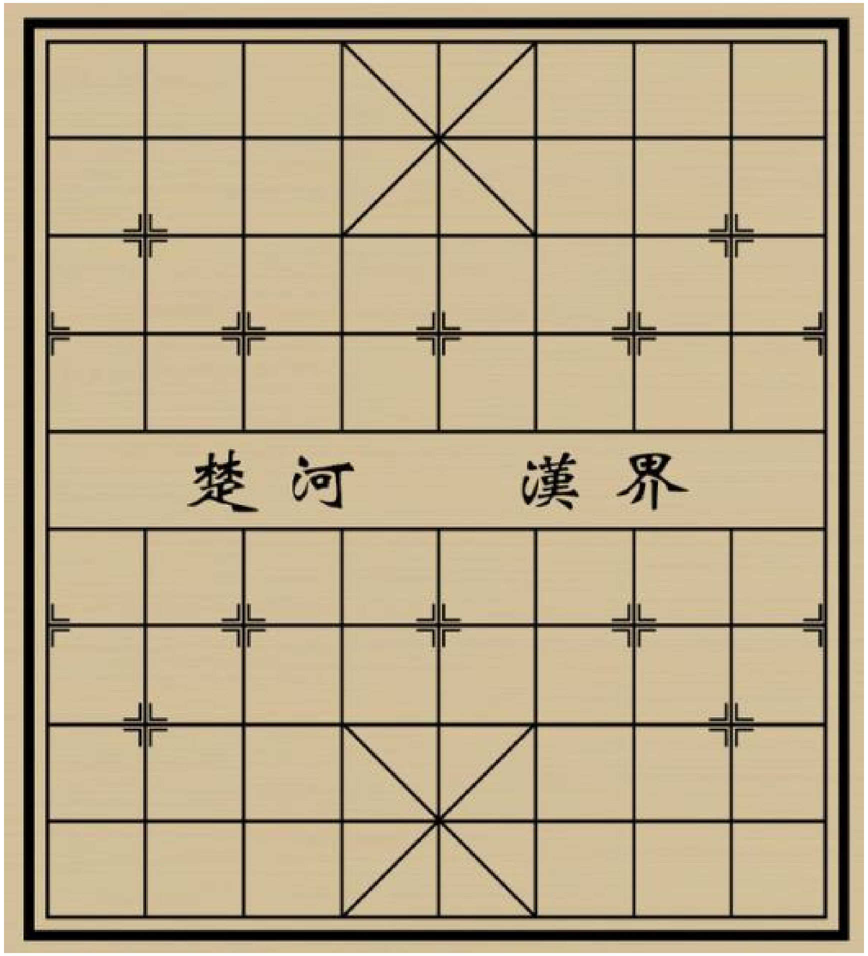
8.2 Paths and Cycles 路径和回路



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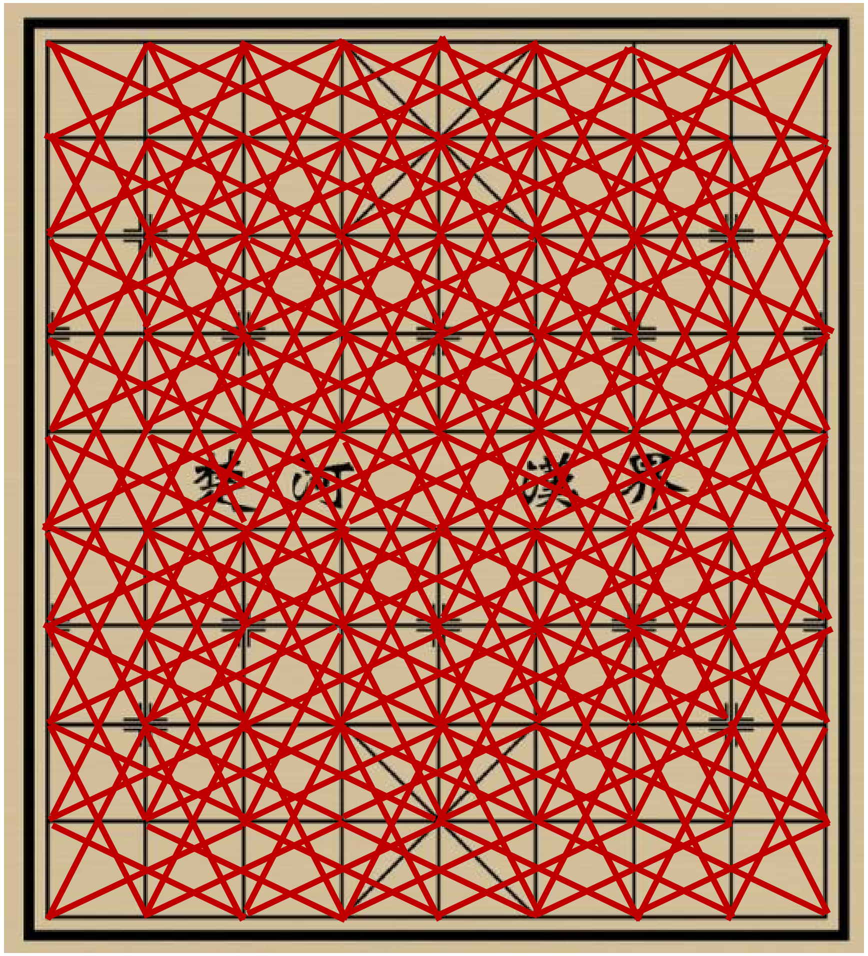
8.2 Paths and Cycles 路径和回路



Problem-Solving Corner

Is it possible in a department of 25 person，racked by dissension, for each person to get along with exactly four others?

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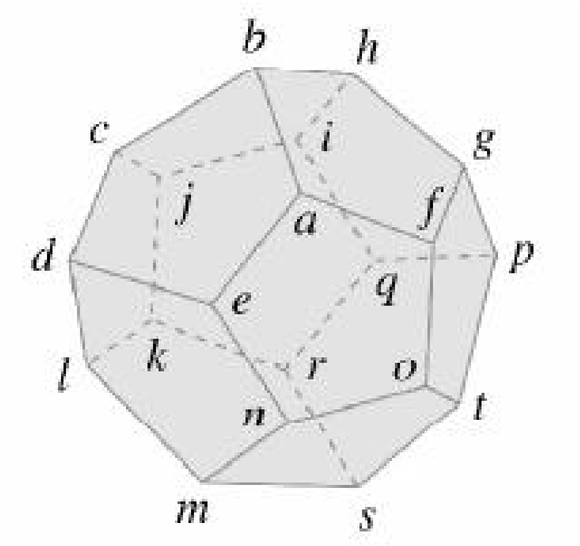
8.3 Hamiltonian Cycles and the Traveling Salesperson Problem 哈密顿回路和旅行商问题

Sir William Rowan Hamilton marketed a puzzle in the mid-1800s in the form of a dodecahedron.

Hamilton's puzzle (哈密顿难题): Can we find a cycle in the graph of the dedecahedron that contains each vertex exactly once？



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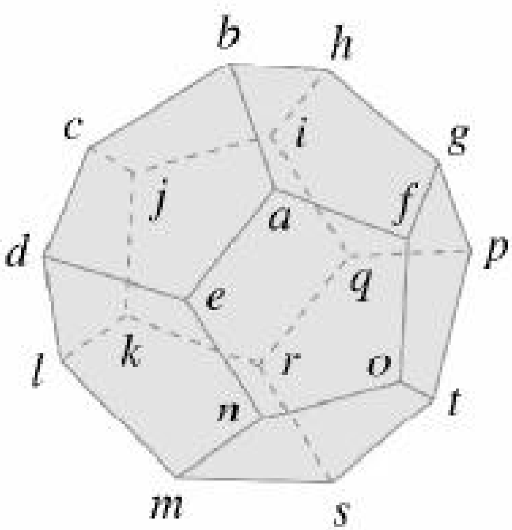

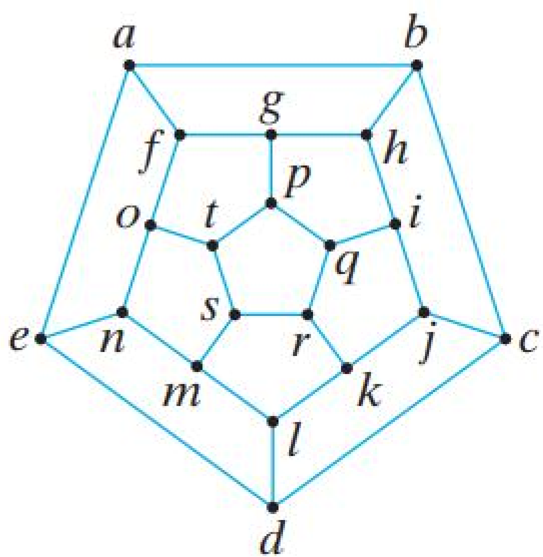

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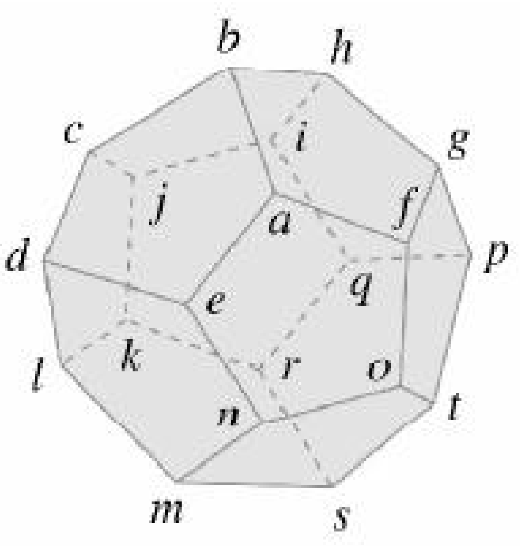

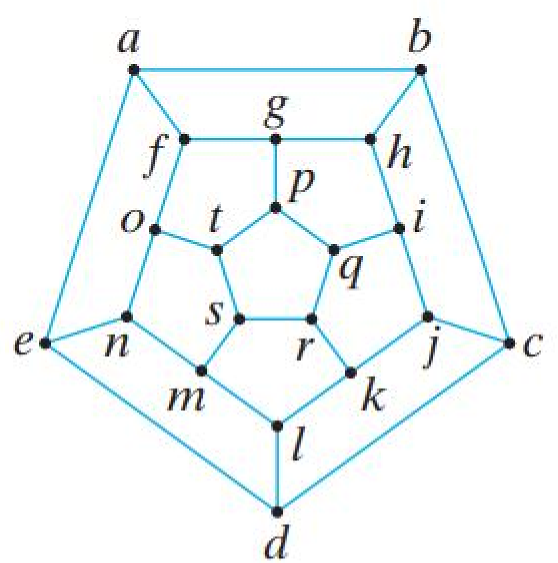

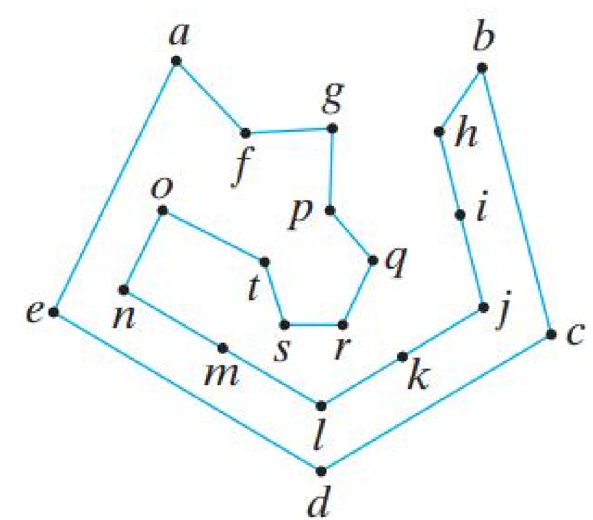

8.3 Hamiltonian Cycles and the Traveling Salesperson Problem 哈密顿回路和旅行商问题

Hamiltonian Cycle (哈密顿回路): a cycle in a graph G that visits each vertex once.

- A **cycle (or circuit) (回路或者环路)** is a path of nonzero length from v to v with no repeated edges.
- **Euler Cycle (欧拉回路)**: a cycle in a graph G that includes all of the edges and all of the vertices of G.

8.3 Hamiltonian Cycles and the Traveling Salesperson Problem 哈密顿回路和旅行商问题

Hamiltonian Cycle (哈密顿回路): a cycle in a graph G that visits each vertex once.

(Unlike the situation for Euler cycles, no easily verified necessary and sufficient conditions are known for the existence of a Hamiltonian cycle in a graph.)

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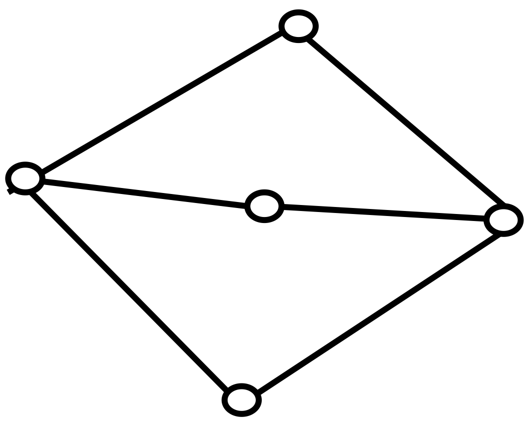
Example 8.3.2 Does the following graph contain a Hamiltonian cycle?



8.3 Hamiltonian Cycles and the Traveling Salesperson Problem 哈密顿回路和旅行商问题

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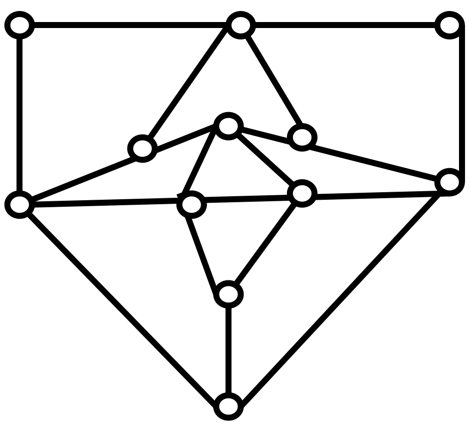

8.3 Hamiltonian Cycles and the Traveling Salesperson Problem 哈密顿回路和旅行商问题

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Exercise Does the following graphs contain a Hamiltonian cycle?