Analyze the Method to Eradicate Ebola by Mathematical Model

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Abstract

This paper mainly solved 3 problems:1. describe the spread of Ebola Virus and forecast the effect of Vaccines 2.Set the delivery location of vaccine 3.Provide a feasible plan for Vaccine distribution. We will carry out the 3 goals in the following 3 steps. STEP 1: Develop two models for spread and cure Inspired by the SEIRD epidemic model , we use the method of differential equa- tion. Unlike other epidemic such as influenza and varicella, Ebola has high rate of death and spread relatively, once diagnosed, patients are extremely dangerous to oth- ers and they are soon quarantined to other susceptible people which means the main source of infection is the infected but undetected people. Aware of this characteristic we set a new group of people called undetected on the base of SEID and SEIRD and specifically designed Model 1(SUID) and Model 2(SUIRD) to cater for Ebola Virus's characteristic. STEP 2: Locate the delivery location Firstly we used the Dijkstra method to get the shortest distance from each city to other cities. Then we consider the severity of epidemic in each city and the distance to determine the delivery location. STEP 3: Find the optimal solution by dynamic programming We firstly qualify the severity of the city epidemic into a cost. Our goal is to dis- tribute the vaccines to every city, and minimize the total cost. This problem can be solved by dynamic programming. In STEP 1 and STEP 2, we obtain the delivery posi- tion, initial conditions and the state transition equation. After setting the progress, we found that every determination is a linear programming problem. With the help of MATLAB, we solved this problem and obtained the vaccine distribution of every city. By the 3 steps above, we can roughly forecast the spread trend of Ebola with and without vaccines and we can provide a general method to solve the distribution of vaccines in different conditions.

Contents

1 Introduction

The outbreak of the Ebola Virus Disease in western Africa in 2013 is the first and the most terrible one ever since.

According to early analysis of transmission dynamic of Ebola[1], we get following conclusions:

- The only mode of transmission is spread via liquid.
- The virus is highly lethal, people infected die very fast.
- The number of infected people is in exponential growth.
- We cannot produce the vaccines with a satisfactory velocity at the first stage.

In order to eradicate Ebola, it is necessary to analyze the spread and cure of it first. Thus, we construct our model to describe the spread trend of the Ebola based on SEIRD Model.

However, the producing velocity of vaccines is limited. Thus we have to balance the amount of vaccine to distribute to different cities. It is sure that the virus will be controlled and eradicated eventually, but we should find the best way to minimize the cost. And this paper will give the Optimize Solution.

2 Previous work — SEIRD Model

It assumes the study area is close and stable. The stable means there is no population's migration. It just consider three objects, **Susceptible** people **(S)**, **Exposed** people **(E)**, **Infective** people **(I)**, **Recovered** (R) and **Deaths** (D) with immunity people. It uses 5 differential equations to describe the epidemic model.

$$\begin{cases} dS = -\beta SI - kS \\ dE = -\beta SI - \delta E \\ dI = \delta E - \gamma I - \lambda I \\ \frac{dR}{dt} = \gamma I + kS \\ \frac{dD}{dt} = \lambda I \end{cases}$$

Where k is the rate of vaccination in the susceptible people; β the rate of infection; δ the rate of diagnosed; γ the cure rate; λ mortality.

3 Symbols and Definitions

- Symbols for main variables

Symbols	Definition
S	The number of the susceptible people
U	The number of undetected patients
I	The number of patients (quarantined)
R	The number of recovered people
D	The number of deaths
V_{ai}	Infect velocity after injection
N	The total number of people
N_{max}	The maximum of the number of the total people
N_n	The number of people alive
T_i	Injection time
V_0	Infect velocity before injection
K_d	The ratio of demand for vaccines
M_a	The demand for vaccines
V_{low}	Low risk λ infect velocity
V_{mid}	Medium risk λ infect velocity
V_{high}	High risk λ infect velocity
D_{low}	The minimum time of entering low risk interval
D_{mid}	The minimum time of entering medium risk interval
D_{high}	The minimum time of entering high risk interval
W	The severity of urban epidemics
L	The distance between two cities
V_M	The relative velocity of the ith city at M th determination
M	An integer from 1 to 5
i	An integer from 1 to 10
P_{Demo}	The current patients of the ith city before determination
D_{is}	The distance from delivery location Bo (a city) to other cities
C_m	The cost of money we have to pay
C_{sum}	The total cost
V_{pro}	Vaccine produced in each interval

- Symbols for normal coefficients

Symbols	Definitions
β	The probability of the undetected infecting other healthy people
k	The ratio of healthy crowd successful vaccination
δ	The carriers confirmed probability
λ	The mortality of patients
γ	The cure rate of diagnosed patients
t	Time
a	some real positive number
b	some real positive number, but bigger than a
c	some real positive number, but bigger than b
ζ	The cost of vaccine delivery per unit distance

4 Constructing Models

4.1 Simplifying Assumptions

- **Assume hospitals can isolate diagnosed patients in time**: Since Ebola is highly lethal, we assume that once diagnosed, patients will be quarantined by the hospital. It makes these people **(Factor: I)** cannot spread virus.
- **Assume the birth rate is equal to the mortality of the local people**: While analyzing the spread velocity of Ebola and other factors such as harmfulness and the difficulty to cure, the influence of birth rate and mortality is negligible.
- **Assume the possibility for infected people of Ebola to be cured is really low**: The vaccine is only able to preserve susceptible people from undetected virus carriers.
- **Once vaccinated, susceptible people will obtain permanent immunity**: In order to simplify our problems, we think vaccines can protect vaccinees once for all.
- **Assume the vaccine have the strong specificity (Ignore the variation of the virus).**
- **Assume there is no mobility of population**: We think local people are afraid of Ebola. As thus, they are not willing to travel to other cities.
- **All the unit transportation costs between two cities are same.**
- **There is only one proper delivery location.**

4.2 SUIRD Model

4.2.1 SUIRD-Ebola Spread Model (Ideal Model:With Abundant Vaccines)

The Ebola's incubation period is 2 to 21 days. Compared with other severer virus, Ebola's incubation period is really short. Meanwhile, governments from all over the world are very alert with Ebola. Thus, all the patients detected are well quarantined (It means that the quarantined people will lose infect ability). **Thus the main source of infection is the undetected infected people(U)**. We specifically build a improved SEIRD model —- SUIRD Model to describe the process of the Ebola's spread. The variable E is replaced by U, where U is the undetected patients (Perhaps in *incubation period* or they cannot be quarantined in time). Thus virus can spread through the undetected group. The Figure 1 is our mind map

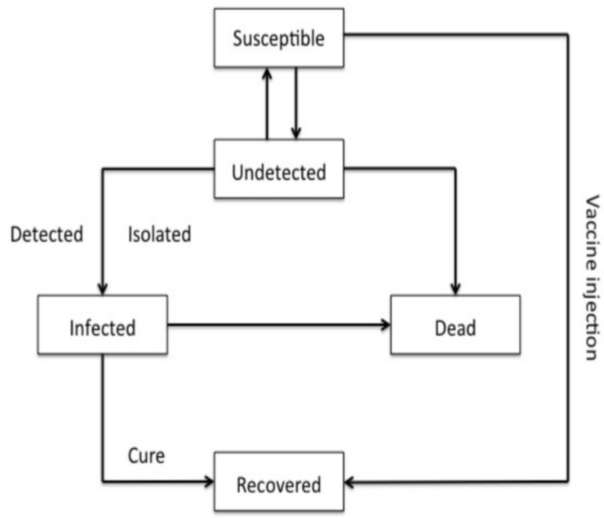

Figure 1: Mind map

Since we have ignored the influence of birth rate and mortality, we divide the total amount of the population to 5 categories: the susceptible people (S), the undetected (U), the infected (I), the recovered(R), the dead (D). Thus the amount of the local people can be expressed as equation 4.1:

$$N(t) = S(t) + U(t) + I(t) + R(t) + D(t) \tag{3.1}$$

Through differential equations, we can express each variable's changing ratio. The system of differential equations is made by 5 ordinary differential equations (ODE).

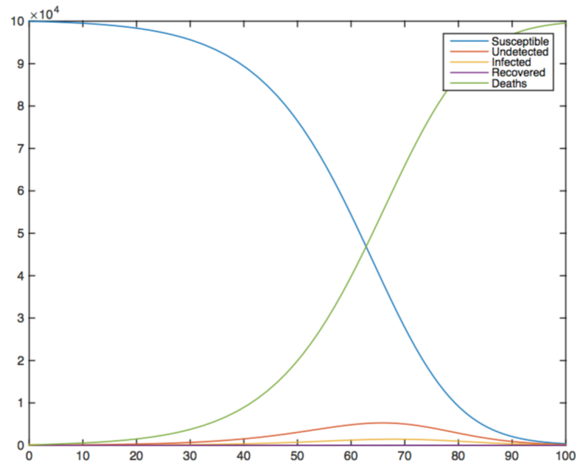

Figure 2: The curves of changing 5 kinds of people before using vaccines

$$\begin{cases} \frac{dS}{dt} = & -\beta SU - kS \\ \frac{dU}{dt} = & -\beta SU - \delta U - \lambda I - \gamma I \\ \frac{dI}{dt} = & \delta U - \lambda I - \gamma I \\ \frac{dR}{dt} = & \gamma I + kS \\ \frac{dD}{dt} = & \lambda U + \lambda I \end{cases} \tag{3.2}$$

This system of differential equations express **the spread process of Ebola** (Before use vaccines). In the system of differential equations.

4.2.2 The Figure of Ebola Spread Model

According to Table 7, there are 170 people infected on average for a city with a population of 100,000. Thus, by solving the differential equations with Runge-Kutta method. We obtain the Figure 2 describes the spread trend of Ebola.

Observing Figure 2,we can easily draw the following conclusions:

1.The infectious ratio can be seen as the gradient of the D.Thus by the Figure2 **The spread velocity first increase and reach a maximum and then decrease**. The initial spread velocity is relatively low. But because of the high infectious ratio and the death rate.Once the U gets to over 2%, the infectious ratio grow much faster,and the infectious ratio reaches the maximum while D reached its maximum.

2.All the people will be killed in the Ebola Spread Model in 100days, the spread velocity reaches to a maximum at day 67.Thus we can derive that the golden time to **control the disease is before day 30.**

3.The amount of **I and U change at a little margin**, during all time of the Ebola Spread Model (This conclusion will use as a crucial condition in the followed models).

While debugging the MATLAB files, we found an interesting fact that in the epidmic spread proccess, higher death rate does not always lead to fast spread velocity.

That is because if the death rate is too high ,the I and U will soon “extinct” because they died too soon to have the chance to infect others.

5 Conclusion

In the end, conclude the whole article:

• **Ebola has a highly lethal**: According to the SUIRD spread Model and its last figure (See Figure 2), if the virus spreads without any prevent, a city with 100,000 population, will be destroyed. Perhaps, there are some lucky dogs.

• **After using vaccine, Ebola will be controlled quickly, and will be eradicated no longer**: From the Figure 3, we have that the Ebola nearly loses its infect ability. The immunes take the place of the susceptible people. Thus, the virus must be eliminated, except a terrible distribution plan.

• **City Bo is the best choice, to be the delivery station**: From table 8, 9, 10, we obtain paths drew from the Bo can be cheap and short.

• Basing on Dynamic Programming Model, the article gives the following plan: use Pulse delivery method, give vaccine every 40 days, more than 15%, but less than 40%. And the total times of distributions is 10, and 1million vaccines are given per times. (See Table 11).

References

[1] Eric Beauregard, Jean Proulx, and D. Kim Rossmo. Spatial patterns of sex offenders: Theoretical, empirical, and practical issues. *Aggression and Violent Behavior*, 10(5):579–603, 2005.

[2] Scotia J. Hicks and Bruce D. Sales. *Crime Analysis: From First Report to Final Arrest*. American Psychological Association, Washington, DC, 2006.

[3] Spotcrime.com. `http://www.spotcrime.com`. Accessed on january 14, 2015.

[4] Doboszczak, Stefan and Virginia Forstall. Mathematical modeling by differential equations, 2013. `http://www.norbertwiener.umd.edu/Education/m3cdocs/Presentation2.pdf`. Accessed on january 14, 2015.

Appendices

A Appendix(data)

The City Name	Patients	Demograph
Kailahun	565	358190
Kenema	502	497948
Bo	314	463668
Port Loko	1348	453746
Western Area	3173	947122
Bombali	992	408390
Kono	247	335401
Kambia	161	270462
Tonkolili	449	347197
Moyamba	206	260910