EBU4375: SIGNALS AND SYSTEMS

LECTURE 3: PART 1

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Basic Time Signals

Basic Continuous-Time Signals

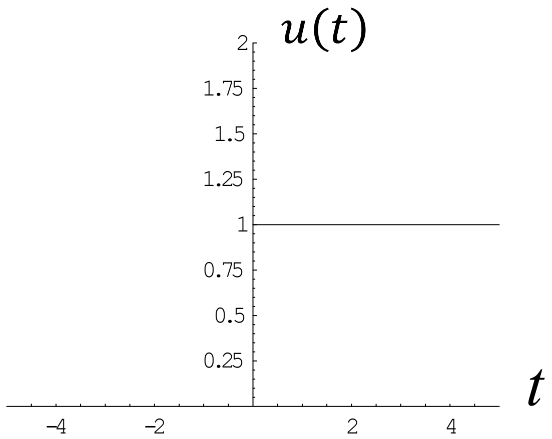
- The Unit-Step Function
- The Unit-Impulse Function
- Complex Exponential and Sinusoidal Signals

Basic Discrete-Time Signals

- The Unit-Step Sequence
- The Unit-Impulse Sequence
- Complex Exponential and sinusoidal Sequence

The Unit-Step Function (CT Signals)

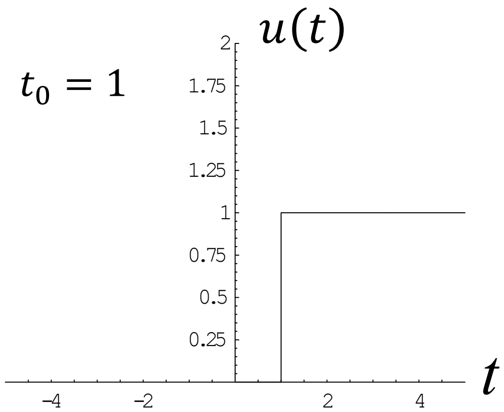
- the unit (or Heaviside) step function is <u>defined</u> as



$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- the shifted (retarded) step function is similarly defined as

e.g. $t_0 = 1$

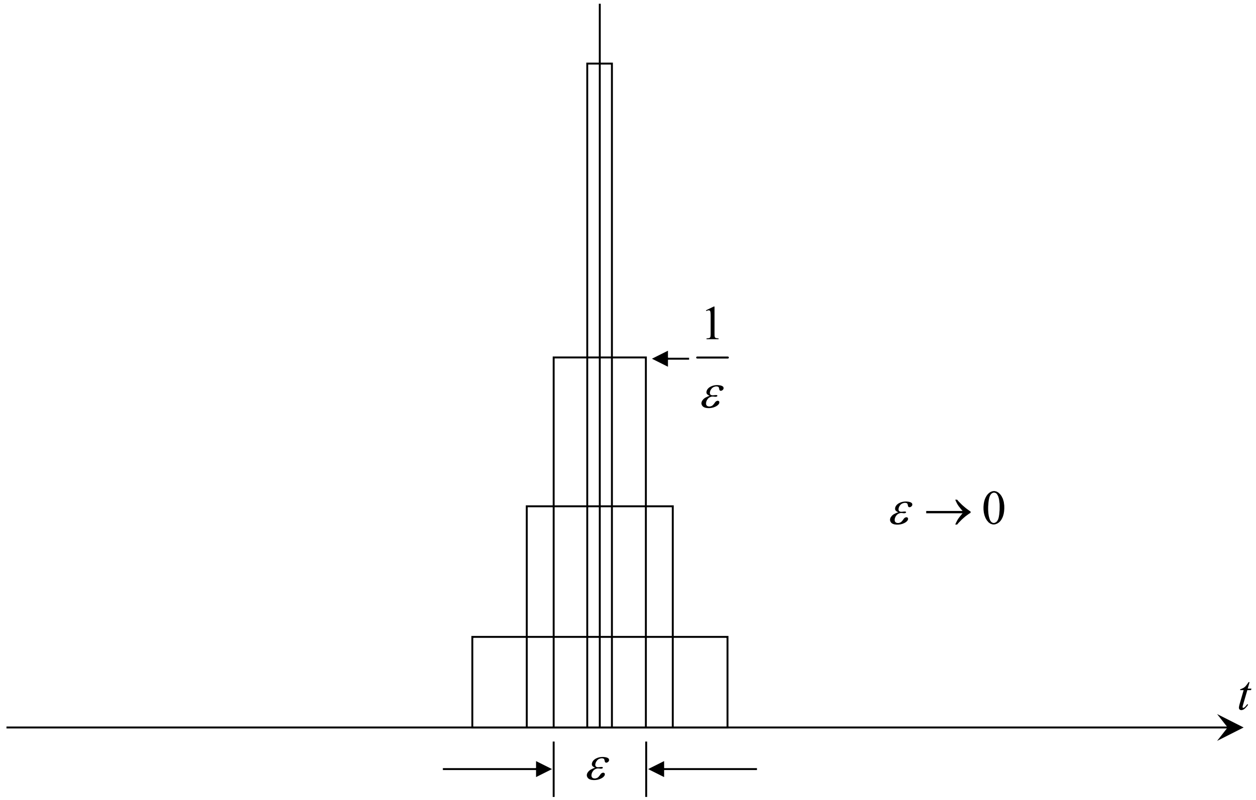


$$u(t - t_0) = \begin{cases} 1 & t \geq t_0 \\ 0 & t < t_0 \end{cases}$$

The Unit-Impulse Function (CT Signals)

The unit-impulse (Dirac-delta) function is defined as

$$\delta(t) \equiv \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \rightarrow \quad \int_{-\varepsilon}^{\varepsilon} dt\, \delta(t) \equiv 1$$

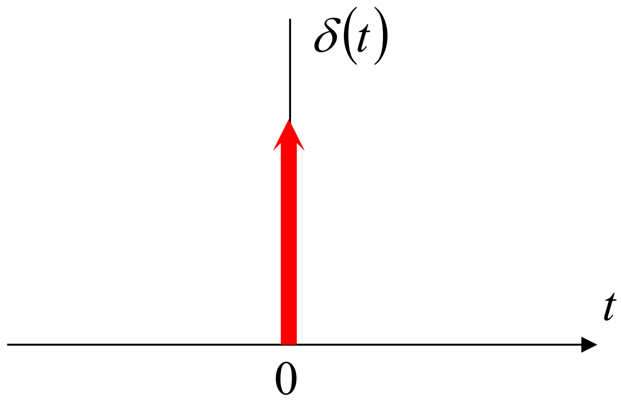


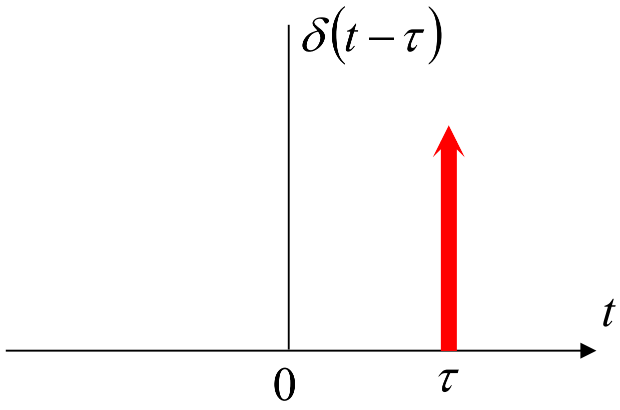
The Unit-Impulse Function (CT Signals)

- It is also defined by

$$\int_a^b dt\, \phi(t)\delta(t) \equiv \begin{cases} \phi(0) & a<0<b \\ 0 & a<b<0 \; or \; 0<a<b \\ undefined & a=0 \; or \; b=0 \end{cases}$$

- A delayed (retarded) delta function $\delta(t-\tau)$ is defined by $\int_{-\infty}^{\infty} dt\, \phi(t)\delta(t-\tau) = \phi(\tau)$ (1)





The Unit-Impulse Function (CT Signals)

Properties of $\delta(t)$:

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\delta(-t) = \delta(t) \qquad \text{(2)}$$

$$x(t)\delta(t) = x(0)\delta(t) \qquad \text{(if } x(t) \text{ is continuous at } t = 0\text{)}$$

$$x(t)\delta(t-\tau) = x(\tau)\delta(t-\tau) \qquad \text{(if } x(t) \text{ is continuous at } t = \tau\text{)}$$

A continuous-time signal $x(t)$ may be expressed as (we prove this in the following lecture)

$$x(t) = \int_{-\infty}^{\infty} d\tau \, x(\tau)\delta(t-\tau)$$

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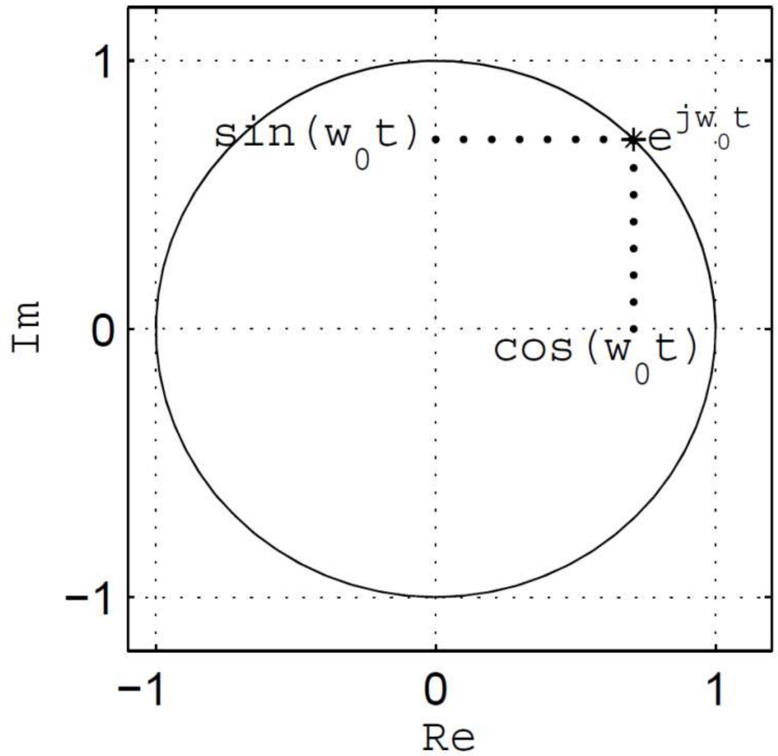
Complex Exponential and Sinusoidal (CT Signals)

Euler's formula: $e^{jw_0t} = \underbrace{\cos(w_0t)}_{\mathrm{Re}\{e^{jw_0t}\}} + j\underbrace{\sin(w_0t)}_{\mathrm{Im}\{e^{jw_0t}\}}$

where $j = \sqrt{-1}$, $w_0 \neq 0$ is real, and t is the time.



Complex Exponential and Sinusoidal (CT Signals)

Since

$$e^{jw_0\left(t+\frac{2\pi}{|w_0|}\right)} = e^{jw_0 t}e^{j2\pi\frac{w_0}{|w_0|}} \longrightarrow e^{jw_0 t}$$

we have

$e^{jw_0 t}$ is periodic with fundamental period $\dfrac{2\pi}{|w_0|}$

Note that

$$e^{j2\pi k} = 1, \text{ for } k = 0, \pm 1, \pm 2, \ldots.$$

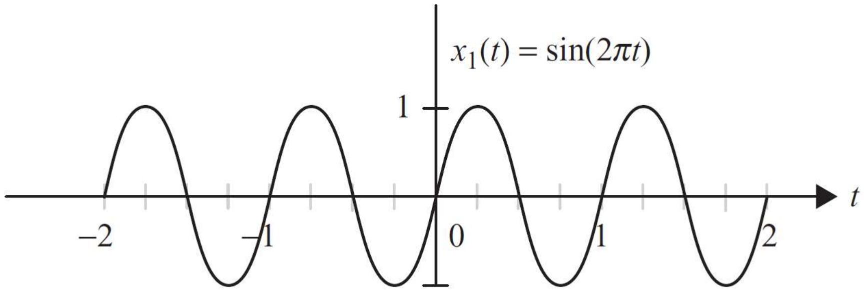
Complex Exponential and Sinusoidal (CT Signals)

- e^{jw_0t} and e^{-jw_0t} have the same fundamental period
- Energy in e^{jw_0t}: $\int_{-\infty}^{\infty} |e^{jw_0t}|^2 dt = \int_{-\infty}^{\infty} 1.dt = \infty$
- Average Power in e^{jw_0t}: $\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{jw_0t}|^2 dt$

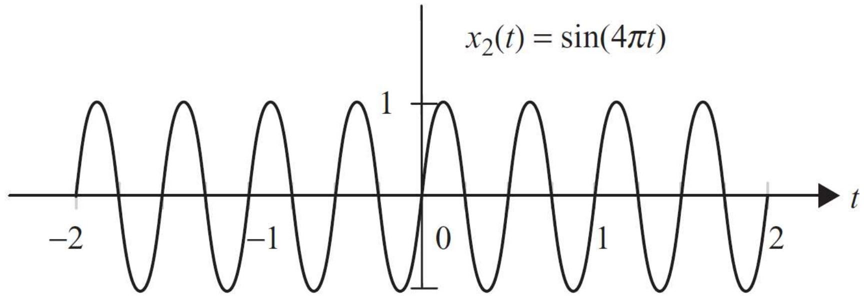
$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1.dt = 1$$

Periodicity and Fundamental Period (CT Signals)

Consider two sinusoidal functions $x(t) = \sin(\omega_0 t + \theta)$ and $x_m(t) = \sin(m\omega_0 t + \theta)$. The fundamental angular frequencies of these two CT signals are given by ω_0 and $m\omega_0$ radians/s, respectively. In other words, the angular frequency of the signal $x_m(t)$ is m times the angular frequency of the signal $x(t)$. In such cases, the CT signal $x_m(t)$ is referred to as the mth harmonic of $x(t)$.



(a)



(b)

Examples of harmonics.
(a) Waveform for the sinusoidal signal $x(t) = \sin(2\pi t)$; (b) waveform for its second harmonic given by $x_2(t) = \sin(4\pi t)$.

Periodicity and Fundamental Period (CT Signals)

Proposition *A signal $g(t)$ that is a linear combination of two periodic signals, $x_1(t)$ with fundamental period T_1 and $x_2(t)$ with fundamental period T_2 as follows:*

$$g(t) = ax_1(t) + bx_2(t)$$

is periodic iff

$$\frac{T_1}{T_2} = \frac{m}{n} = \text{rational number}.$$

The fundamental period of $g(t)$ is given by $nT_1 = mT_2$ provided that the values of m and n are chosen such that the greatest common divisor (gcd) between m and n is 1.

Periodicity and Fundamental Period (CT Signals)

Example

Determine if the following signals are periodic. If yes, determine the fundamental period.

$$g_1(t) = 3\sin(4\pi t) + 7\cos(3\pi t);$$

Solution

$$\frac{T_1}{T_2} = \frac{1/2}{2/3} = \frac{3}{4}$$

fundamental period of $g_1(t)$ is given by $nT_1 = 4T_1 = 2\,\text{s}$.

fundamental period of $g_1(t)$ can also be evaluated from $mT_2 = 3T_2 = 2\,\text{s}$.

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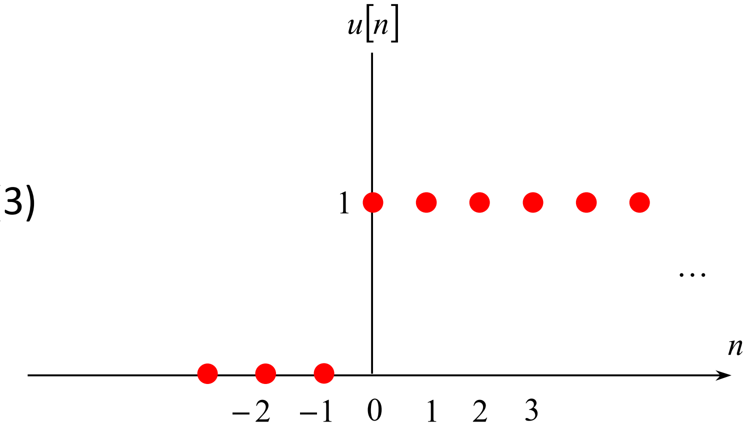
The Unit-Step Sequence (DT Signals)

- The unit-step sequence $u[n]$ is defined by

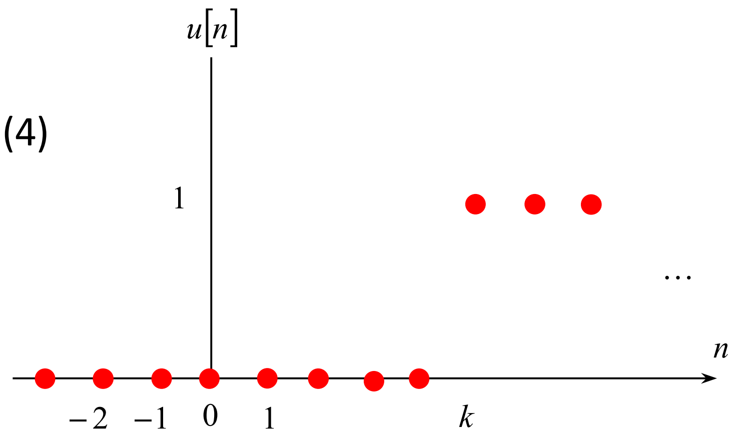
$$u[n] \equiv \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \qquad (3)$$

Unlike $u(t)$, $u[n]$ is defined at $n = 0$



- The shifted unit-step sequence $u[n-k]$ is similarly defined by

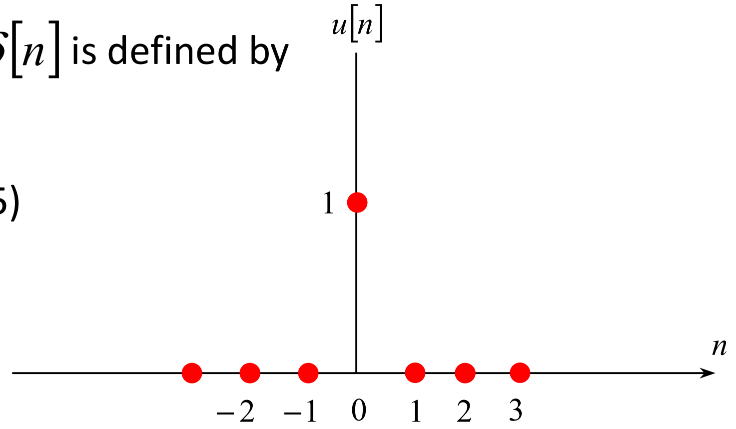
$$u[n-k] \equiv \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases} \qquad (4)$$



The Unit-Impulse Sequence (DT Signals)

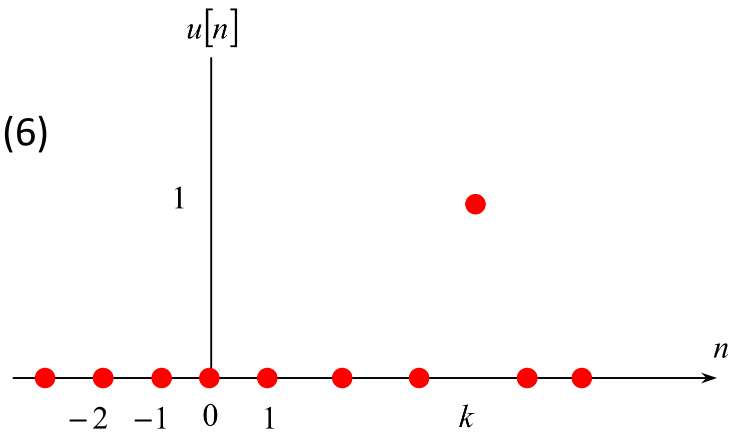
- The unit-impulse (or unit-sample) sequence $\delta[n]$ is defined by

$$\delta[n] \equiv \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \qquad (5)$$



- The shifted unit-impulse (sample) sequence $\delta[n-k]$ is similarly defined by

$$\delta[n-k] \equiv \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases} \qquad (6)$$



The Unit-Impulse Sequence (DT Signals)

- Unlike $\delta(t)$, $\delta[n]$ *is* readily defined. From (5) and (6) it is evident that

$$x[n]\delta[n] = x[0]\delta[n]$$
$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

are the discrete-time counterparts of

$$x(t)\delta(t) = x(0)\delta(t)$$

$$x(t)\delta(t-\tau) = x(\tau)\delta(t-\tau)$$

- from (3) and (4), $\delta[n]$ and $u[n]$ are related by

$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

A discrete-time signal $x[n]$ may be expressed as (we prove this in the following lecture)

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$