COT5405: Homework 3 (Fall 2017)

Instructions: study Solved Exercises 1 and 2 on pages 307 – 311. Complete the following exercises by defining the recurrence formulae, designing DP algorithms, and analyzing their running time and space requirements. The first two's analyses have been done for you.

1. Exercise 8(b) on pages 319-320.

    Hint: Let OPT(j) be the maximum number of robots that can be destroyed for the instance of the problem just on $x_1,\ldots, x_j$. Clearly, if the input ends x_j, there is no reason not to activate the EMP then (you are not saving it for anything), so the choice is just when to last activate it before step j. Thus OPT(j) is the best of these choices over all i.

2. Exercise 27 on pages 333-334.

    Hint: A solution is specified by the days on which orders of gas arrive, and the amount of gas that arrives on those days. In computing the total cost, we will take into account delivery and storage costs, but we can ignore the cost for buying the actual gas, since this is the same in all solutions. (At least all those where all the gas is exactly used up.)

    Consider an optimal solution. It must have an order arrive on day 1, and it the next order is due to arrive on day i, then the amount ordered should be $\sum_{j=1}^{i-1} g_j$. Moreover, the capacity requirements on the storage tank say that i must be chosen so that $\sum_{j=1}^{i-1} g_j \leq L$.

    What is the cost of the storing this first order of gas? We pay g_2 to store the g_2 gallons for one day until day 2, and $2g_3$ to store the g_3 gallons for two days until day 3, and so forth, for a total of $\sum_{j=1}^{i-1}(j-1)g_j$. More generally, the cost to store an order of gas that arrives on day a and lasts through day b is $\sum_{j=a}^{b}(j-a)g_j$. Let us denote this quantity by S(a,b).

    Let OPT(a) denote the optimal solution for days a through n, assuming that the tank is empty at the start of day a, and an order is arriving. We choose the next day b on which an order arrives: we pay P for the delivery, S(a,b-1) for the storage, and then we can behave optimally from day b onward. Thus we know how to compute OPT(a).

3. Exercise 2(a) & (b) on pages 313-314.