Homework-1 Solution Key
COT 5405 - Fall 2017

1.

Stable matching

In this version of the Gale-Shapley algorithm, the individual selected to propose will be the first individual that is not already engaged.

(a) **Gale-Shapley from the male's perspective**

The following table will be updated while stepping through the algorithm by,

- selecting the line(s) to be affected by the current iteration
- applying strikethrough to prospects who reject the proposal
- applying bold text to the prospects who accept the proposal

male	fiancée				
Von	Allie	Bobbie	Eleanor	Cathy	Deanna
Will	Allie	Cathy	Deanna	Bobbie	Eleanor
Xander	Deanna	Bobbie	Eleanor	Allie	Cathy
Yousef	Eleanor	Deanna	Cathy	Bobbie	Allie
Zack	Cathy	Deanna	Allie	Bobbie	Eleanor

From the final table, a matching (if one exists) may be easily read.

i. Select Von to propose to Allie who accepts.

Von	**Allie**	Bobbie	Eleanor	Cathy	Deanna

ii. Select Will to propose to Allie who declines.

Will	~~Allie~~	Cathy	Deanna	Bobbie	Eleanor

iii. Select Will to propose to Cathy who accepts.

Will	~~Allie~~	**Cathy**	Deanna	Bobbie	Eleanor

iv. Select Xander to propose to Deanna who accepts.

Xander	**Deanna**	Bobbie	Eleanor	Allie	Cathy

v. Select Yousef to propose to Eleanor who accepts.

Yousef	**Eleanor**	Deanna	Cathy	Bobbie	Allie

vi. Select Zack to propose to Cathy who accepts. Cathy breaks her engagement with Will.

Will	~~Allie~~	~~Cathy~~	Deanna	Bobbie	Eleanor
Zack	**Cathy**	Deanna	Allie	Bobbie	Eleanor

vii. Select Will to propose to Deanna who declines.

Will	~~Allie~~	~~Cathy~~	~~Deanna~~	Bobbie	Eleanor

viii. Select Will to propose to Bobbie who accepts.

Will	~~Allie~~	~~Cathy~~	~~Deanna~~	**Bobbie**	Eleanor

The final table for the male-centric algorithm is,

male	fiancée				
Von	**Allie**	Bobbie	Eleanor	Cathy	Deanna
Will	~~Allie~~	~~Cathy~~	~~Deanna~~	**Bobbie**	Eleanor
Xander	**Deanna**	Bobbie	Eleanor	Allie	Cathy
Yousef	**Eleanor**	Deanna	Cathy	Bobbie	Allie
Zack	**Cathy**	Deanna	Allie	Bobbie	Eleanor

(b) **Gale-Shapley from the female's perspective**

The following table will be updated while stepping through the algorithm by,

- selecting the line(s) to be affected by the current iteration
- applying strikethrough to prospects who reject the proposal
- applying bold text to the prospects who accept the proposal

female	fiancé				
Allie	Von	Will	Xander	Zack	Yousef
Bobbie	Xander	Will	Zack	Yousef	Von
Cathy	Yousef	Zack	Von	Xander	Will
Deanna	Zack	Xander	Yousef	Von	Will
Eleanor	Von	Yousef	Zack	Xander	Will

From the final table, a matching (if one exists) may be easily read.

i. Select Allie to propose to Von who accepts.

Allie	**Von**	Will	Xander	Zack	Yousef

ii. Select Bobbie to propose to Xander who accepts.

Bobbie	**Xander**	Will	Zack	Yousef	Von

iii. Select Cathy to propose to Yousef who accepts.

Cathy	**Yousef**	Zack	Von	Xander	Will

iv. Select Deanna to propose to Zack who accepts.

Deanna	**Zack**	Xander	Yousef	Von	Will

v. Select Eleanor to propose to Von who declines.

Eleanor	~~Von~~	Yousef	Zack	Xander	Will

vi. Select Eleanor to propose to Yousef who accepts. Yousef breaks his engagement with Cathy.

Cathy	~~Yousef~~	Zack	Von	Xander	Will
Eleanor	~~Von~~	**Yousef**	Zack	Xander	Will

vii. Select Cathy to propose to Zack who accepts. Zack breaks his engagement with Deanna.

Cathy	~~Yousef~~	**Zack**	Von	Xander	Will
Deanna	~~Zack~~	Xander	Yousef	Von	Will

viii. Select Deanna to propose to Xander who accepts. Xander breaks his engagement with Bobbie.

Bobbie	~~Xander~~	Will	Zack	Yousef	Von
Deanna	~~Zack~~	**Xander**	Yousef	Von	Will

ix. Select Bobbie to propose to Will who accepts.

Bobbie	~~Xander~~	**Will**	Zack	Yousef	Von

The final table for the female-centric algorithm is,

female	fiancé				
Allie	**Von**	Will	Xander	Zack	Yousef
Bobbie	~~Xander~~	**Will**	Zack	Yousef	Von
Cathy	~~Yousef~~	**Zack**	Von	Xander	Will
Deanna	~~Zack~~	**Xander**	Yousef	Von	Will
Eleanor	~~Von~~	**Yousef**	Zack	Xander	Will

2.

Graph node adjacency format

Here are the adjacency matrix and adjacency list,

node	A	B	C	D	E	F
A	0	**1**	1	**1**	0	0
B	**1**	0	1	1	0	0
C	1	1	0	1	1	1
D	**1**	1	1	0	**1**	0
E	0	0	1	**1**	0	1
F	0	0	1	0	1	0

A-**B**-C-**D**
B-**A**-C-D
C-A-B-D-E-F
D-**A**-B-C-**E**
E-C-**D**-F
F-C-E

3.

Topological orderings

Every ordering of the graph must begin with node a and end with node f. There exist $4!$ orderings of the remaining 4 nodes b through e. Not all of these orderings are valid topological orderings of the graph. For each valid ordering with b followed by c (not necessarily adjacent) there exists an invalid ordering with b and c reversed. Likewise for orderings with d followed by e.

The number of topological orderings for this graph can then be computed as,

$$n = \frac{4!}{2! \cdot 2!} = 6$$

with $4!$ total (valid plus not valid) orderings and $2!$ (valid plus not valid) pairings each of nodes b, c and d, e. The topological orderings are enumerated as,

```
abcdef   abdcef   abdecf   adbcef   adbecf   adebcf
```

4.

Shortest path

Comment	array	A	B	C	D	E	F
	$S = \{A\}$						
Initialize	D	-	-	-	-	-	-
	D'	∞	4	10	3	∞	∞
$D'(D)$ is minimum	$S = \{A, D\}$						
$D(D) \leftarrow D'(D)$	D	-	-	-	3	-	-
Set $D'(w)$ for neighbors $w \notin S$ of D. $D'(B) = \min\{4, 3+6\}$, $D'(C) = \min\{10, 3+7\}$, $D'(E) = \min\{\infty, 3+2\}$	D'	∞	4	10	~~3~~	5	∞
$D'(B)$ is minimum	$S = \{A, D, B\}$						
$D(B) \leftarrow D'(B)$	D	-	4	-	3	-	-
Set $D'(w)$ for neighbors $w \notin S$ of B. $D'(C) = \min\{10, 4+8\}$	D'	∞	~~4~~	10	~~3~~	5	∞
$D'(E)$ is minimum	$S = \{A, D, B, E\}$						
$D(E) \leftarrow D'(E)$	D	-	4	-	3	5	-
Set $D'(w)$ for neighbors $w \notin S$ of E. $D'(C) = \min\{10, 5+1\}$, $D'(F) = \min\{\infty, 5+5\}$	D'	∞	~~4~~	6	~~3~~	~~5~~	10
$D'(C)$ is minimum	$S = \{A, D, B, E, C\}$						
$D(C) \leftarrow D'(C)$	D	-	4	6	3	5	-
Set $D'(w)$ for neighbors $w \notin S$ of C. $D'(F) = \min\{10, 6+9\}$, $D'(F) = \min\{\infty, 5+5\}$	D'	∞	~~4~~	~~6~~	~~3~~	~~5~~	10
$D'(F)$ is minimum	$S = \{A, D, B, E, C, F\}$						
$D(F) \leftarrow D'(F)$	D	-	4	6	3	5	10
Set $D'(w)$ for neighbors $w \notin S$ of F.	D'	∞	~~4~~	~~6~~	~~3~~	~~5~~	~~10~~