SWINBURNE UNIVERSITY TECHNOLOGY

RME30002 AUTOMATION AND CONTROL

EXPERIMENT 4

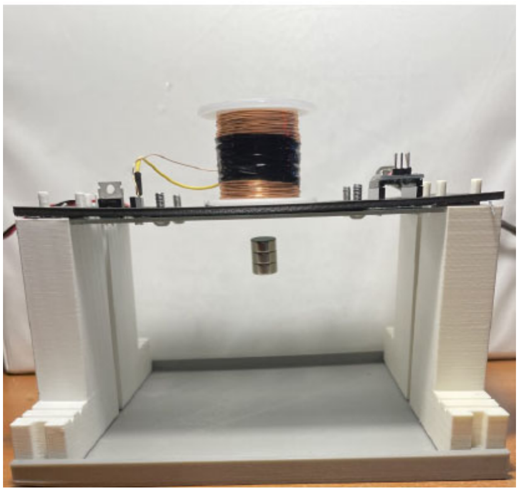
NONLINEAR SYSTEM ANALYSIS

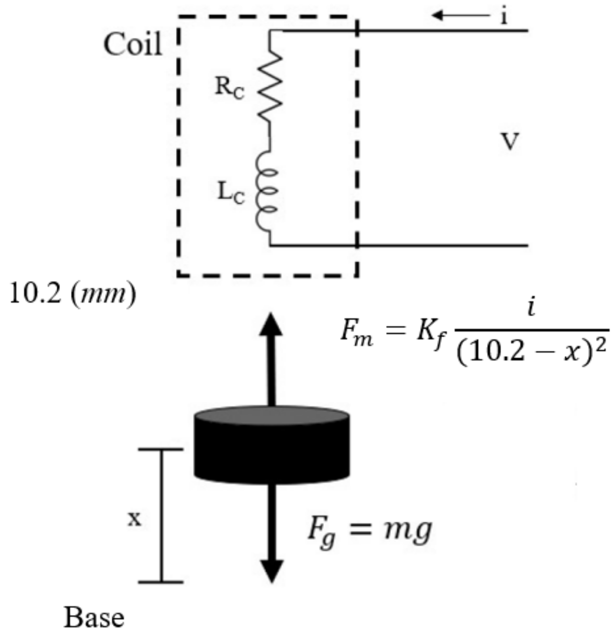
OBJECTIVE

- To be familiar with the system analysis using Matlab.
- To study the root locus techniques.
- To understand effects of the controllers (proportional, integral, and derivative) on the system performance.
- To linearize a nonlinear system

PRELIMINARY

The experimental rig of the Magnetic Levitation (Maglev) system shown in Figure 1a is used to levitate a magnet in air by an electromagnetic force generated by an electromagnet. The position of the magnet is measured by a sensor on the maglev rig and available for feedback. The simplified attractive Maglev system is shown in Figure 1b, where x is the distance of the magnet from the base (mm), i is the coil current (Ampere), g is the gravitational constant 9810 ($\frac{mm}{sec^2}$), F_m is an electromagnetic force (mN), K_f is the electromagnetic force coefficient ($\frac{mN-mm^2}{Amp}$), m is the mass of the magnet 0.0282 (Kg) and the distance from the base to the surface of the electromagnet is 10.2 (mm).





(a) (b)

Figure 1. Magnetic levitation system

The dynamic equation of the above system is as follows:

$$m\frac{d^2x}{dt^2} = K_f\frac{i}{(10.2-x)^2} - mg \tag{1}$$

Now perform the following tasks:

1. Eq. (1) can be used to computer K_f at the equilibrium point when $\frac{d^2x}{dt^2} = 0$ as follows:

$$K_f = \frac{mg(10.2-x_0)^2}{i_0} \tag{2}$$

   where the electromagnet coil current is i_0 and the magnet position is x_0. At the equilibrium point $x_0 = 6.94mm, i_0 = 0.3A$, calculate K_f.

2. The presence of the term $K_f\frac{i}{(10.2-x)^2}$ makes Eq. (1) nonlinear. Since we want to linearize the equation about the equilibrium point x_0 *and* i_0, we let $x = x_0 + \hat{x},\ i = i_0 + \hat{\imath}$, where $\hat{x}$ *and* $\hat{\imath}$ are small excursions from x_0 *and* i_0 respectively. Substituting x *and* i into Eq. (1) and dividing *m* on both sides yields:

$$\frac{d^2(x_0+\hat{x})}{dt^2} = K_f\frac{i_0+\hat{\imath}}{m[10.2-(x_0+\hat{x})]^2} - g \tag{3}$$

   But

$$\frac{d^2(x_0+\hat{x})}{dt^2} = \frac{d^2\hat{x}}{dt^2} \tag{4}$$

   Let $f(x,i) = K_f\frac{i}{m(10.2-x)^2} - g = K_f\frac{i_0+\hat{\imath}}{m[10.2-(x_0+\hat{x})]^2} - g$ which can be linearized with the truncated Taylor series as follows:

$$f(x,i) = f(x_0,i_o) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0,i=i_0}\hat{x} + \left.\frac{\partial f}{\partial i}\right|_{x=x_0,i=i_0}\hat{\imath} \tag{5}$$

   Note that $f(x_0,i_o) = 0$ at the equilibrium point due to a zero acceleration. Substituing Eq. (4) and Eq. (5) into Eq.(3) yields:

$$\frac{d^2\hat{x}}{dt^2} = \left.\frac{\partial f}{\partial x}\right|_{x=x_0,i=i_0}\hat{x} + \left.\frac{\partial f}{\partial i}\right|_{x=x_0,i=i_0}\hat{\imath} \tag{6}$$

   Let $a = \left.\frac{\partial f}{\partial x}\right|_{x=x_0,i=i_0}$ *and* $b = \left.\frac{\partial f}{\partial i}\right|_{x=x_0,i=i_0}$, calculate a and b at the equilibrium point $x_0 = 6.94mm, i_0 = 0.3A$.

3. Derive the transfer function of the linearized system Eq. (6) at the equilibrium point: $P(s) = \frac{\hat{X}(s)}{\hat{I}(s)}$.

PROCEDURE

Simulation analysis of the Maglev system performance using Matlab

Consider the feedback control system in Figure 2:

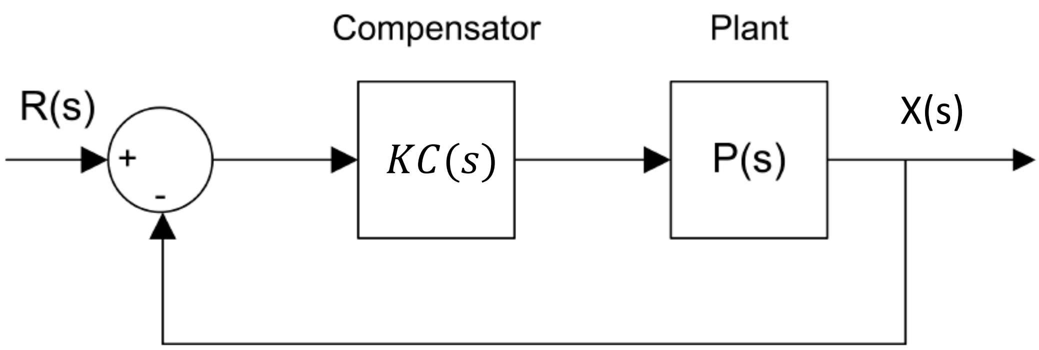


Figure 2. Control diagram of a Maglev system

where

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{7}$$

and $P(s)$ is the maglev plant transfer function.

1. Analyse the system performance with a P (proportional) controller where $K_p = 1, K_i = 0 \; and \; K_d = 0$ in Eq. (7).
   a) Plot the system root locus with the proportional controller above and $K \in (0, \infty)$. Analyse the system stability. (Hints: you may use 'rlocus' command to plot the root locus in Matlab.)
   b) Plot the system step response with the proportional controller above and *K=1*. Analyse the results.

2. Analyse the system performance with a PD controller where $K_p = 1, K_i = 0 \; and \; K_d = 0.01$ in Eq. (7).
   a) Plot the system root locus with the PD controller above and $K \in (0, \infty)$. Analyse the system stability and the accuracy of a unit step input. Comment on the effects of the PD controller.
   b) Plot the system step response with the PD controller above and *K=1*. Comment on the system performance.

3. Analyse the system performance with a PID controller where $K_p = 1, K_i = 1\ and\ K_d = 0.01$ in Eq. (7).
   a) Plot the system root locus with the PID controller above and $K \in (0, \infty)$. Analyse the system stability and the accuracy to a unit step input. Comment on the effects of the PID controller.
   b) Plot the system step response with the PID controller above and *K=1*. Comment on the system performance.

Experimental analysis of the Maglev system performance

Set up the experimental system and conduct the following steps. The detailed instructions are available in the lab.

1. Record the closed loop step response with the PD controller ($K_p = 1, K_i = 0\ and\ K_d = 0.01, and\ \ K = 1$).

2. Record the closed loop step response with the PID controller ($K_p = 1, K_i = 1\ and\ K_d = 0.01, and\ \ K = 1$).

3. Analyse the above results and comment on the system performances.

REPORT

Please refer to the guideline for laboratory works in the Canvas regarding how to write your report.