

ROBOTIC CONTROL

Assignment 1

Question 1

$$\xi = \frac{\ln(0.09)}{\sqrt{T^2 + \ln^2(0.09)}} = 0.6 \quad \omega_n = \frac{4}{(2\cdot 3)(0.6)} = 3$$

Dominant pole

$$s = -5 + \omega_n \sqrt{1 - \xi^2} j \\ = -1.8 \pm 2.4j$$

Steady State Error for plant.

$$K_v = \lim_{s \rightarrow 0} s G(s) = s \times \frac{1}{s(s+1)(s+4)} = \frac{1}{1 \times 4} = 0.25$$

so ESS = 4

meaning we must increase low frequency gain by a larger factor of 10+

* There's going to be a cascade of two lead stages and a lag
 1 - lead stage is to improve damping
 2 - add extra phase at higher frequency to reduce settling time

$$\text{Lead 1} = \frac{s+1}{s+5} \quad \text{Lead 2} = \frac{s+2}{s+40} \quad \text{Lag} = \frac{s+0.02}{s+0.0004}$$

* NOTE: The poles and zero I used Matlab 'sisotool' to find it

Open loop transfer function

$$1 + G_c G(s) = K \frac{(s+1)(s+2)(s+0.02)}{(s+5)(s+40)(s+0.0004)} \times \frac{1}{s(s+1)(s+4)} + 1 = 0$$

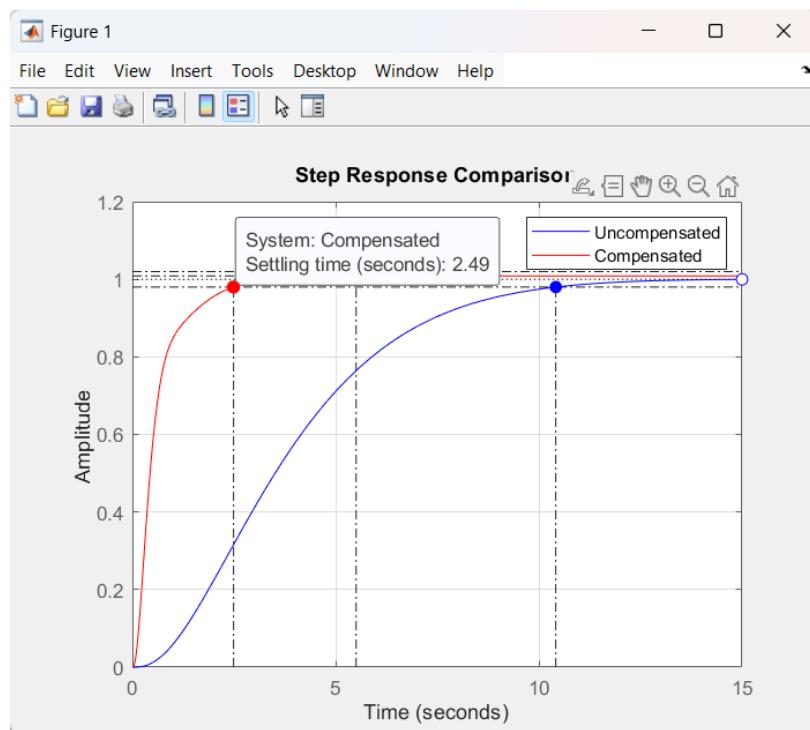
$$K = \left| \frac{(s+5)(s+40)(s+0.0004)s(s+1)(s+4)}{(s+1)(s+2)(s+0.02)} \right|$$

$$= 623$$

$$\therefore G_c = 623 \frac{(s+1)(s+2)(s+0.02)}{(s+5)(s+40)(s+0.0004)}$$

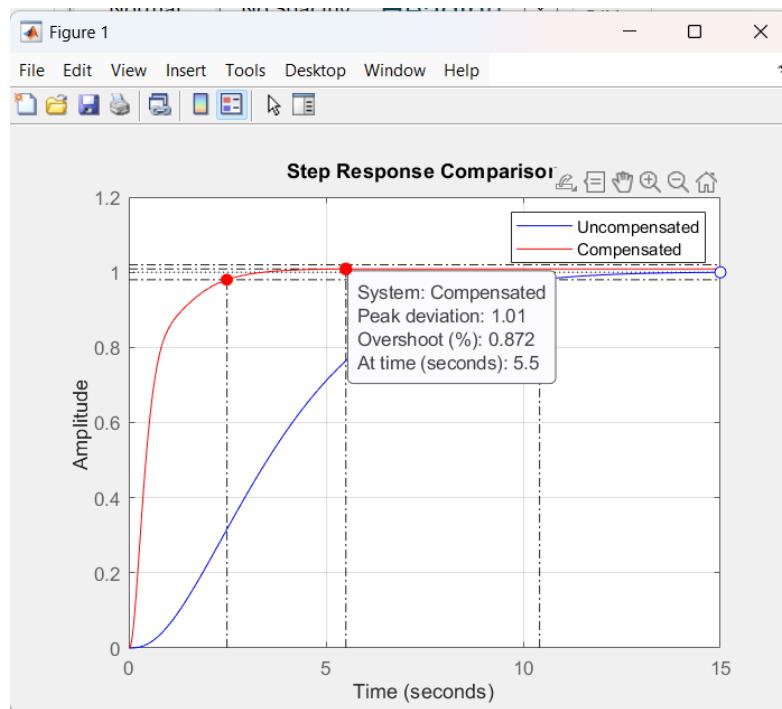
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Note: The settling time is slightly above 2.3 because of the approximation I made in my calculations where I rounded to the whole number

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Question 2

From Maron's Gain Formula

$$OLTF = \frac{1}{R_a + L_a S} \frac{1}{J_L S + B_L} \frac{1}{S} N$$

$$1 + \frac{K_i K_2}{R_a + L_a S} + \frac{K_b K_i}{(R_a + L_a S)(J_L S + B_L)} + \frac{K K_i K_i K_t}{(R_a + L_a S)(J_L S + B_L)}$$

Sub Values

$$K_v = 150 \Rightarrow L_a S G(s) H(s) = 150$$

$$\lim_{S \rightarrow 0} \frac{K_s K K_i K_i N}{B (R_a + L_a S)(J_L S + B_L)} = 150$$

$$1 + \frac{K_i K_2}{R_a + L_a S} + \frac{K_b K_i}{(R_a + L_a S)(J_L S + B_L)} + \frac{K K_i K_i K_t}{(R_a + L_a S)(J_L S + B_L)}$$

$$\Rightarrow \frac{K_s K K_i K_i N}{R_a B_L} = 150$$

$$1 + \frac{K_i K_2}{R_a} + \frac{K_b K_i}{R_a B_L} + \frac{K K_i K_i K_t}{R_a B_L} = 150$$

$$\Rightarrow \frac{K_s K K_i K_i N}{R_a B_L + K_i K_2 B_L + K_b K_i + K K_i K_i K_t} = 150$$

$$\Rightarrow \frac{(1) K (10) (9) (0.1)}{(5)(1) + (10)(0.5)(1) + (0.0636)(9) + K(10)(9)K_t} = 150$$

$$\frac{9K}{10.5724 + 90K_t} = 150$$

$$9K = 1057.24 + 9000K_t \quad \text{--- ①}$$

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$$CLTF = \frac{OLTF}{1+OLTF}$$

Characteristic equation will

$$= (R_a + L_a s)(J_L s + B_L) + K_1 K_2 (J_L s + B_L) + K_b K_i + K K_1 K_i K_t$$

$$= L_a J_L s^2 + s(R_a J_L + B_L L_a) + R_a B_L + K_1 K_2 J_L s + K_1 K_2 B_L + K_b K_i + K K_1 K_i K_t$$

$$= L_a J_L s^2 + s[R_a J_L + B_L L_a + K_1 K_2 J_L] + R_a B_L + K_1 K_2 B_L + K_b K_i + K K_1 K_i K_t$$

$$= L_a J_L \left[s^2 + \frac{s[R_a J_L + B_L L_a + K_1 K_2 J_L]}{L_a J_L} + \frac{R_a B_L + K_1 K_2 B_L + K_b K_i + K K_1 K_i K_t}{L_a J_L} \right]$$

Sub in values

$$= (0.003)(0.01) \left[s^2 + s(3433.3) + \frac{10.5724 + 90K K_t}{(0.003)(0.01)} \right]$$

$$= 3 \times 10^5 \left[s^2 + s(3433.3) + \frac{10.5724 + 90K K_t}{3 \times 10^5} \right]$$

Compare with $s^2 + 2\zeta w_n s + w_n^2$

$$2\zeta w_n = 3433.3$$

$$w_n = \frac{3433.3}{2(0.6)} = 2861$$

$$w_n^2 = \frac{10.5724 + 90K K_t}{3 \times 10^5}$$

$$285.6 = 10.5724 + 90K K_t$$

$$K K_t = 2.6$$

From eq ①

$$K = \frac{10.5724 + 9000 K K_t}{9} = 2717$$

and

$$K_t = 2.6/K = 9.6 \times 10^{-4}$$