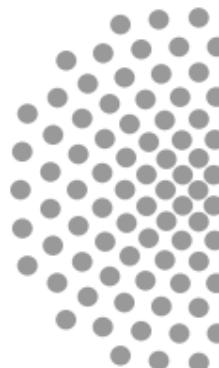


Uncertainty-Aware Dimensionality Reduction for Channel Charting with Geodesic Loss



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University of Stuttgart
Institute of Telecommunications
Prof. Dr. Ing. Stephan ten Brink



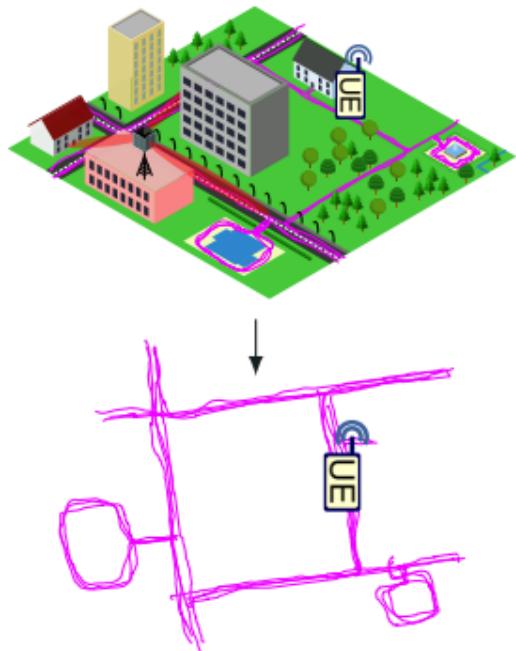
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- ① Introduction, Motivation, State of the Art
- ② New Training Architecture
- ③ Geodesic Loss
- ④ Uncertainty (Concept Overview)
- ⑤ Evaluation and Results
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Objective of Channel Charting [Studer et al., 2018]



Channel Charting is a **dimensionality reduction** technique that learns a mapping from a high-dimensional **channel state information (CSI)** manifold to a low-dimensional space, called **Channel Chart (CC)**, purely from data available at the base station.

- The mapping is called the **forward charting function**, usually $D = 2$ or $D = 3$:

$$\mathbb{C}^{B \times M \times N_{\text{sub}}} \rightarrow \text{FCF } C_{\Theta} \rightarrow \mathbb{R}^D$$

- Channel Charting is self-supervised

What is a Channel Chart?

A channel chart is a *map* and C_{Θ} localizes the UE on the map:

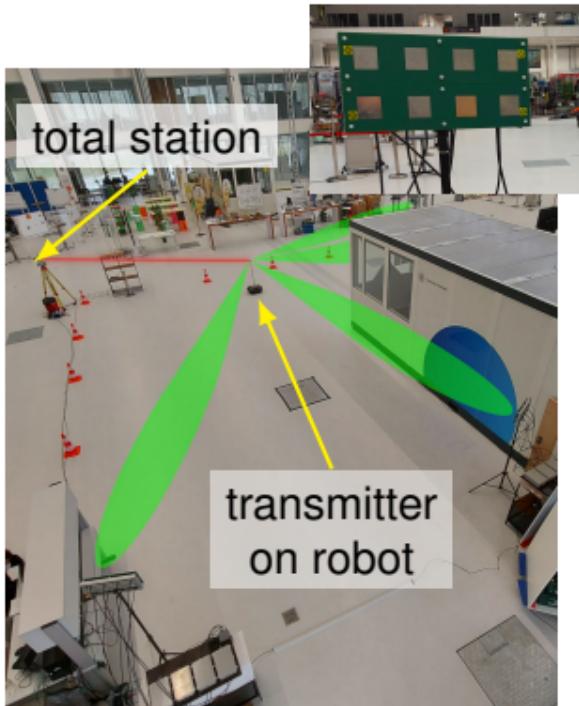
- Maps depict (spatial) relationships
 - Different maps preserve different properties, serve different purposes
 - Applications:
 - UE clustering, pilot allocation
 - Channel prediction
 - CSI compression
 - **Localization (our focus!)**
 - ...



TfL Tube Map, OpenTopoMap



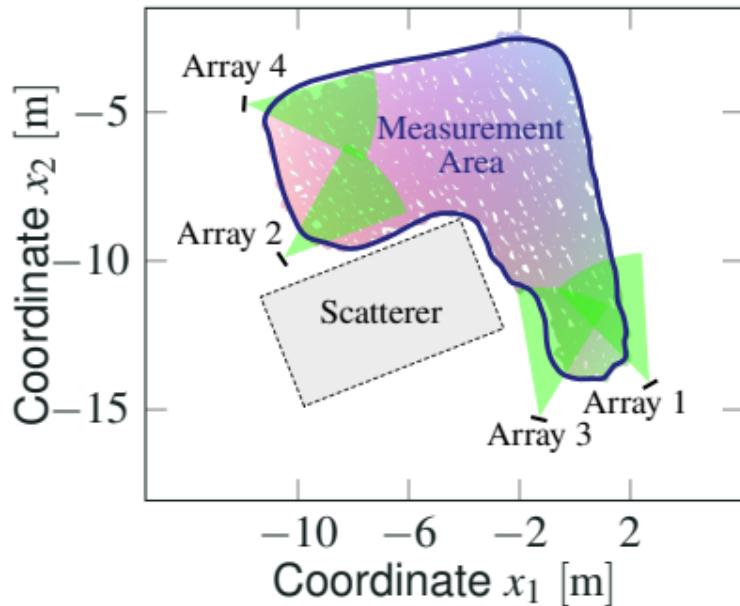
CSI Dataset: Distributed mMIMO in Factory



- Common benchmark:
[Stahlke et al., 2023, Taner et al., 2023, Euchner et al., 2023, Taner et al., 2024, Vindas and Guillaud, 2024, Stephan et al., 2023, Chaaya et al., 2024, Euchner et al., 2024]
- DICHASUS (Distributed Channel Sounder by University of Stuttgart) dataset
dichasus-cf0x, publicly available^a
- Single position-tracked TX on robot
- $B = 4$ arrays with $M = 2 \times 4$ antennas
- 50MHz bandwidth, $f_c = 1.27$ GHz, OFDM

^a<https://dichasus.inue.uni-stuttgart.de/datasets/data/dichasus-cf0x/>

CSI Dataset: Distributed mMIMO in Factory Top View



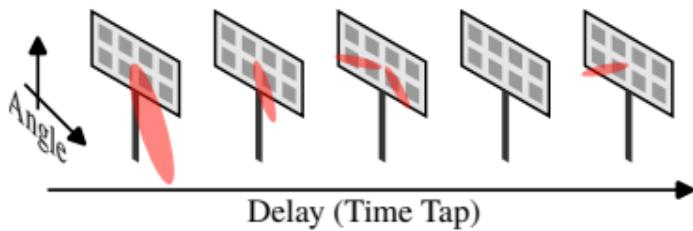


State of the Art *of dissimilarity-metric based Channel Charting*

Dissimilarity Metrics

Angle-Delay-Profile Δ_{ADP}

- Same power from same angle at same delay → similar location



Timestamps Δ_{time}

- Use time difference as dissimilarity

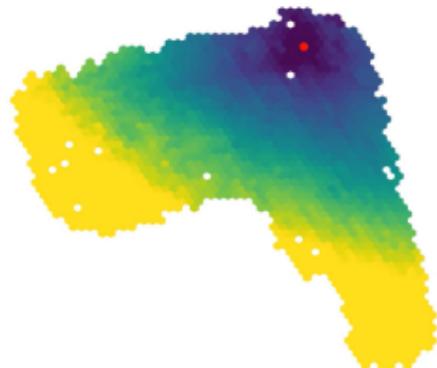
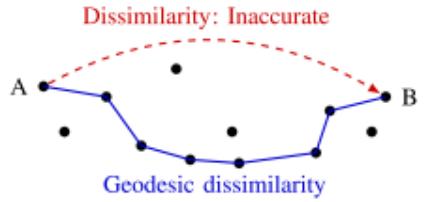
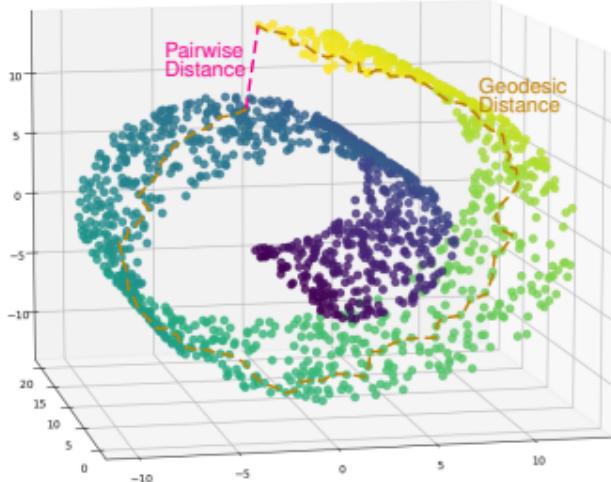
$$\Delta_{\text{time},i,j} = |t^{(i)} - t^{(j)}|$$

- Uses *time-domain* CSI $\mathbf{H} \in \mathbb{C}^{B \times M \times T}$:

$$\Delta_{\text{ADP},i,j} = \sum_{b=1}^B \sum_{\tau=1}^T \left(1 - \frac{\left| \sum_{m=1}^M \left(\mathbf{H}_{b,m,\tau}^{(i)} \right)^* \mathbf{H}_{b,m,\tau}^{(j)} \right|^2}{\left(\sum_{m=1}^M \left| \mathbf{H}_{b,m,\tau}^{(i)} \right|^2 \right) \left(\sum_{m=1}^M \left| \mathbf{H}_{b,m,\tau}^{(j)} \right|^2 \right)} \right)$$



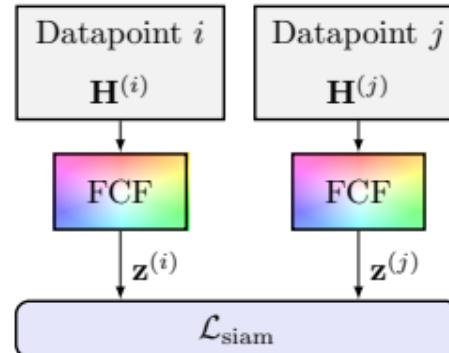
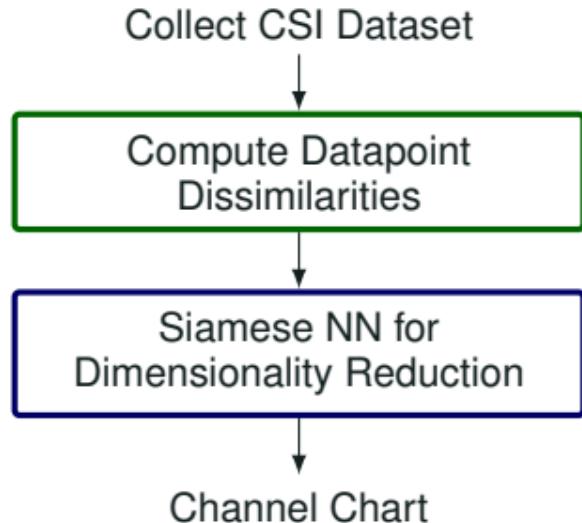
Geodesic Dissimilarities Δ_{geo}



Dissimilarity metrics only locally accurate? Apply **shortest path algorithm** to a neighborhood graph!



Siamese Neural Network Training



$$\mathcal{L}_{\text{siam}} = \frac{1}{L^2} \sum_{i,j} \frac{\left(\left\| \mathbf{z}^{(i)} - \mathbf{z}^{(j)} \right\| - \Delta_{\text{geo},i,j} \right)^2}{\Delta_{\text{geo},i,j} + \beta}$$

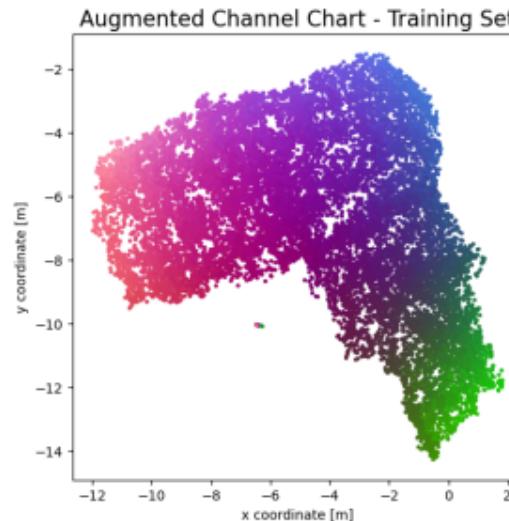
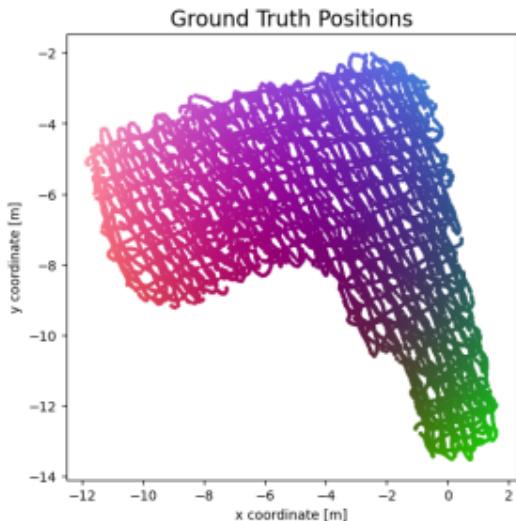
with CC locations $\mathbf{z}^{(l)} \in \mathbb{R}^2$, $l = 1, \dots, |\mathcal{S}|$.



Exemplary Channel Charts

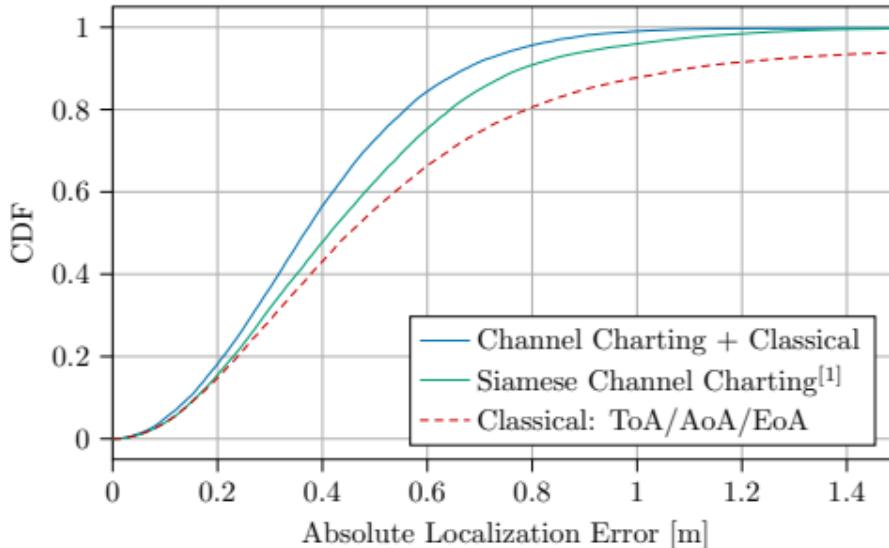


The Good (Asilomar '23): CC + Classical Techniques



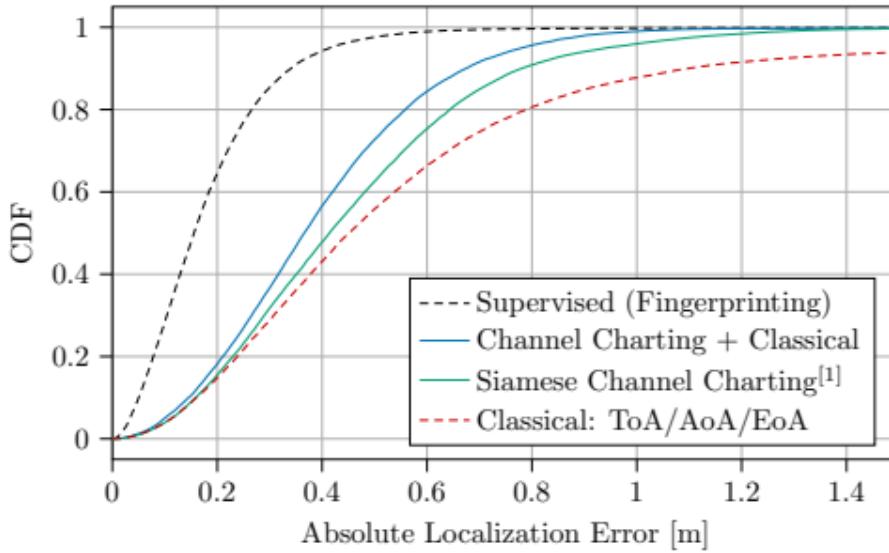
CC + AoA + ToA produces channel chart with $\text{MAE} \approx 0.40 \text{ m}$ [\[Euchner et al., 2023\]](#)

The Good (Asilomar '23): CC + Classical Techniques



[1]Evaluated after optimal affine transform

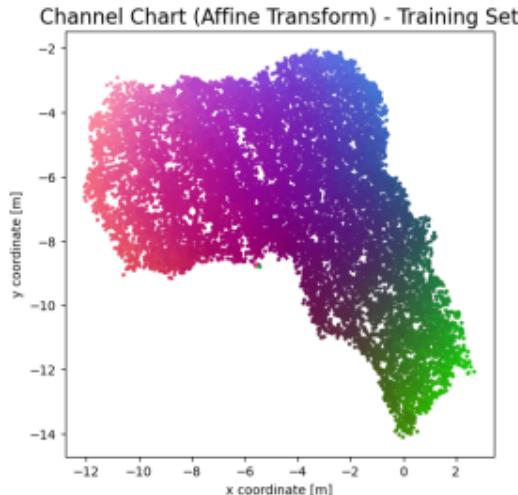
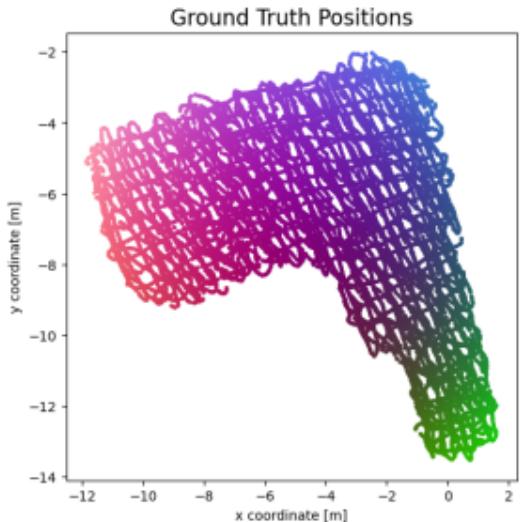
The Good (Asilomar '23): CC + Classical Techniques



→ Still room for improvement!

[1]Evaluated after optimal affine transform

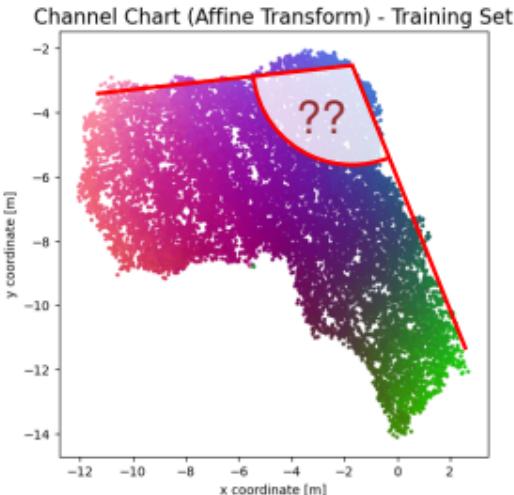
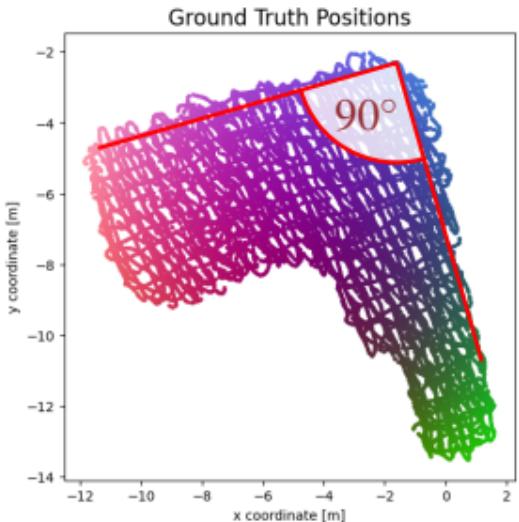
The Bad (TCOM '23): CC on nonconvex shape



CC produces channel chart with $MAE \approx 0.44\text{ m}$ [Stephan et al., 2023]

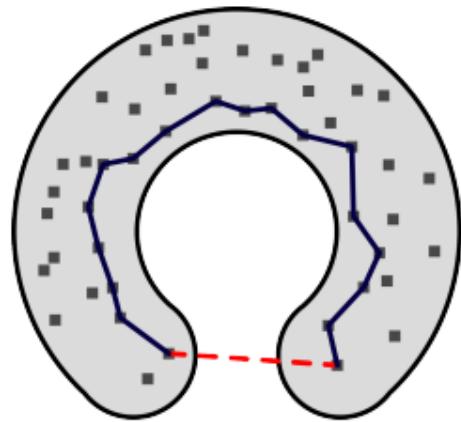


The Bad (TCOM '23): CC on nonconvex shape



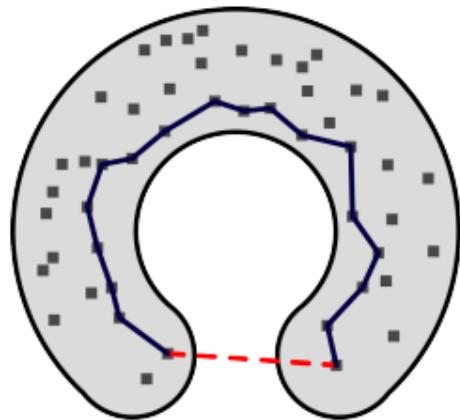
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Euclidean Distance \neq Geodesic Distance

The Bad (TCOM '23): CC on nonconvex shape



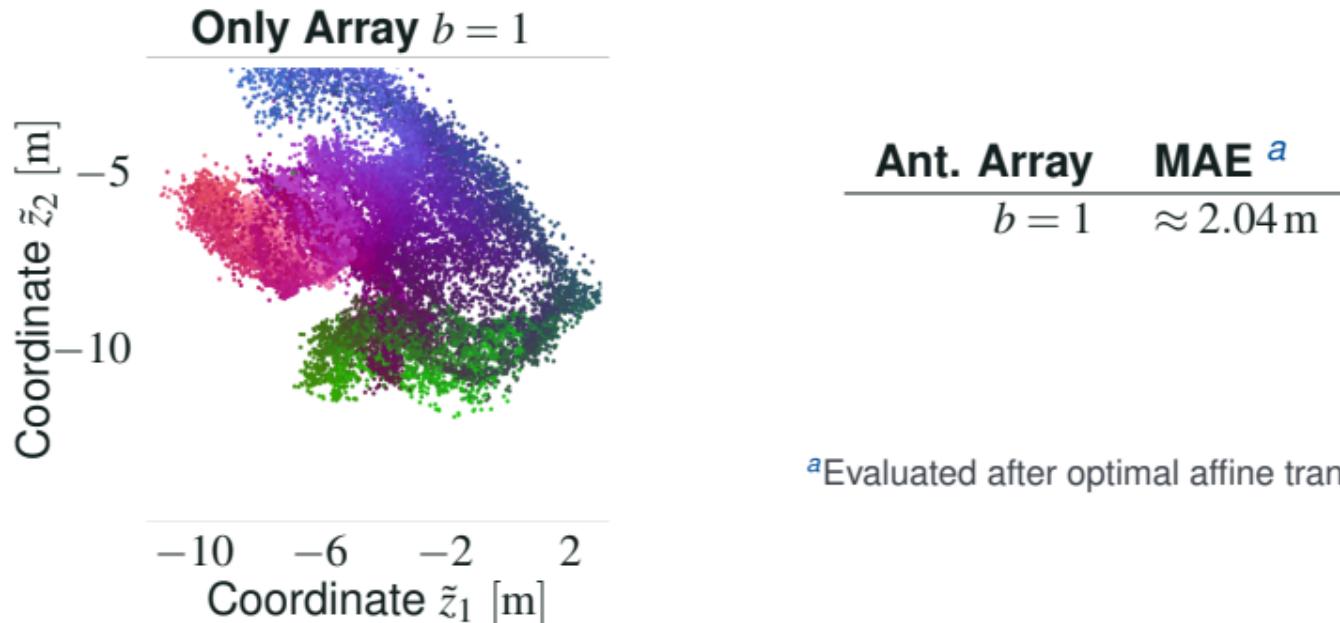
Euclidean Distance \neq Geodesic Distance

→ Well-known fundamental dimensionality reduction issue!

[Budninskiy et al., 2019, Rosman et al., 2010, Zha and Zhang, 2003, Schwartz and Talmon, 2019]



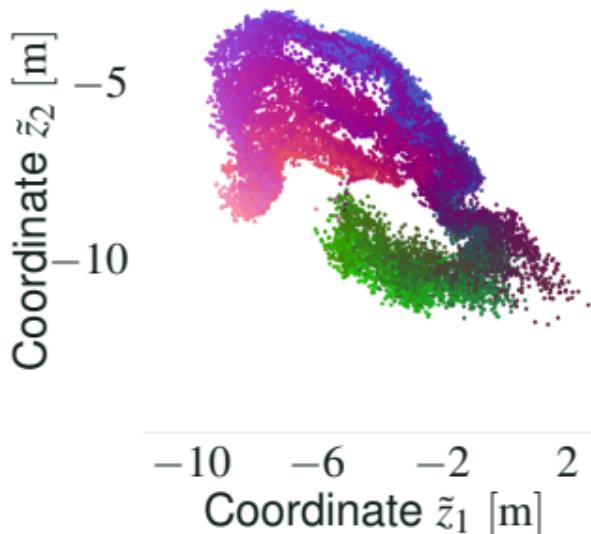
The Ugly: CC with a Single Antenna Array





The Ugly: CC with a Single Antenna Array

Only Array $b = 2$



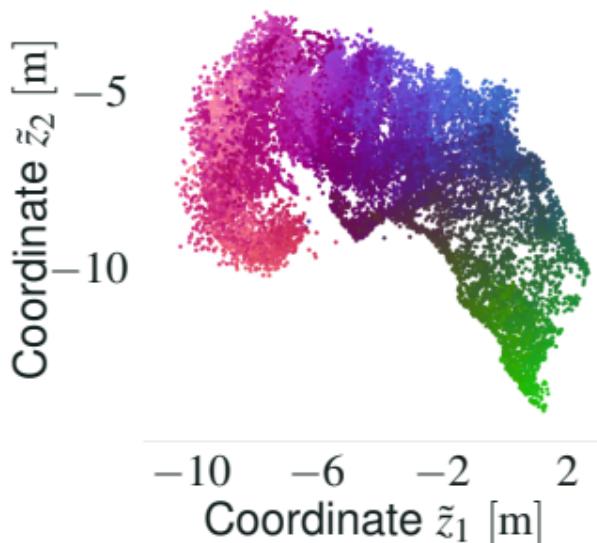
Ant. Array	MAE ^a
$b = 1$	≈ 2.04 m
$b = 2$	≈ 2.23 m

^aEvaluated after optimal affine transform



The Ugly: CC with a Single Antenna Array

Only Array $b = 3$

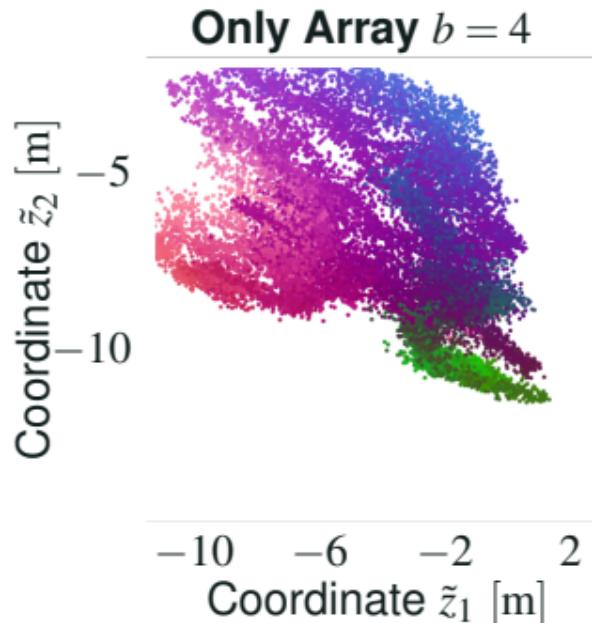


Ant. Array	MAE ^a
$b = 1$	≈ 2.04 m
$b = 2$	≈ 2.23 m
$b = 3$	≈ 1.37 m

^aEvaluated after optimal affine transform



The Ugly: CC with a Single Antenna Array



Ant. Array	MAE ^a
$b = 1$	≈ 2.04 m
$b = 2$	≈ 2.23 m
$b = 3$	≈ 1.37 m
$b = 4$	≈ 1.62 m

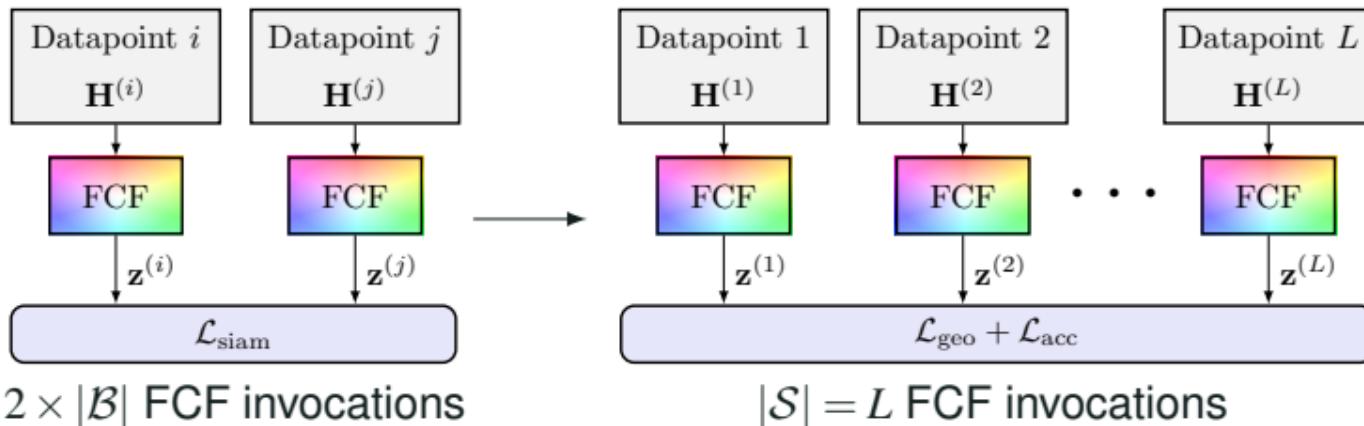
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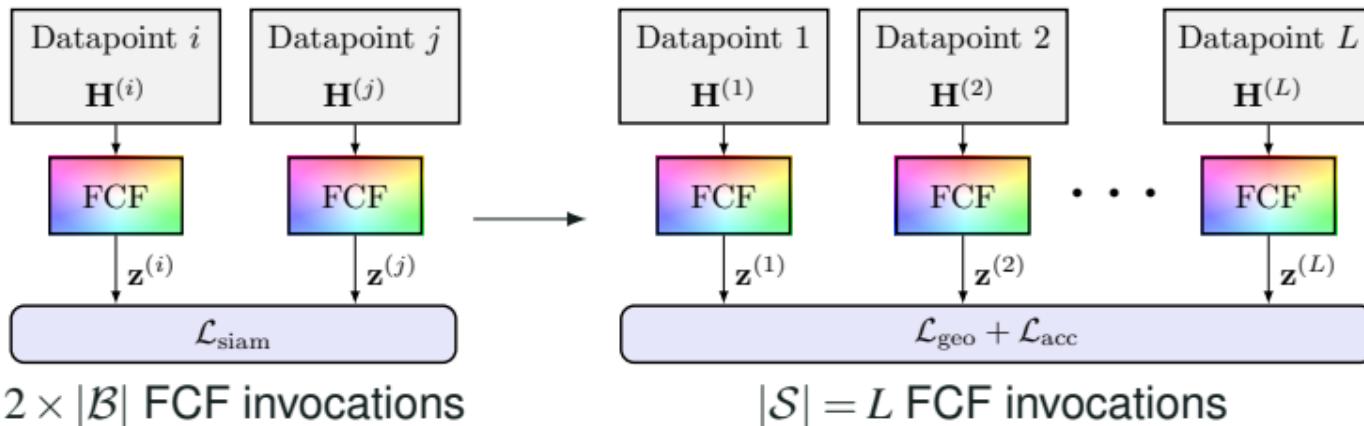
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Siamese to Batch-Wise Loss



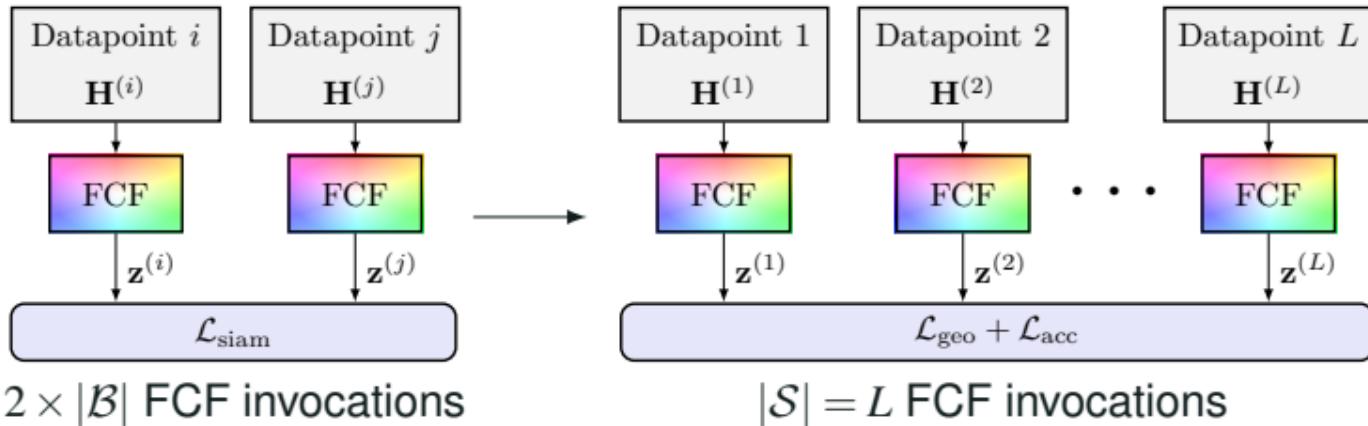
Siamese to Batch-Wise Loss



- Concept not entirely new [Hermans et al., 2017, Sohn, 2016]
 - Not more complex if batches \mathcal{B} were already large: $2 \times |\mathcal{B}| \approx |\mathcal{S}| = L$
 - *Global view* (whole channel chart) of the problem in loss function



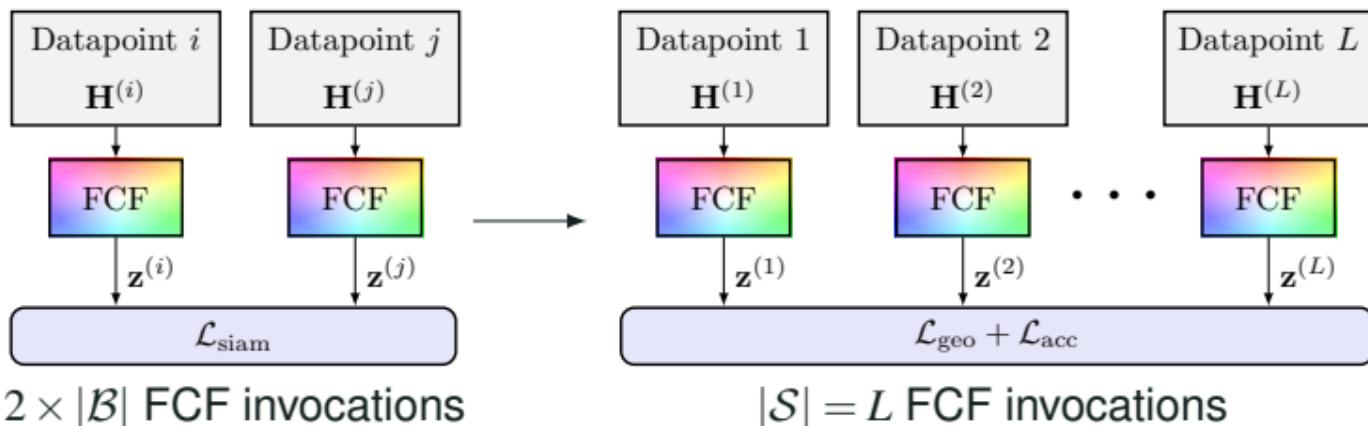
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- *Global view* (whole channel chart) of the problem in loss function



Improvement 1: Acceleration Constraint

- Exploit physical *inertia* of transmitter [Rappaport et al., 2021]
- Instantaneous velocity and acceleration in Channel Chart:

$$\mathbf{v}^{(l)} = \frac{\mathbf{z}^{(l)} - \mathbf{z}^{(l-1)}}{t^{(l)} - t^{(l-1)}} \quad \mathbf{a}^{(l)} = \frac{\mathbf{v}^{(l)} - \mathbf{v}^{(l-1)}}{t^{(l)} - t^{(l-1)}}$$

- Enforce log-likelihood acceleration constraint based on assumed distribution $a = \|\mathbf{a}\| \sim \mathcal{N}(\mu_{\text{acc}}, \sigma_{\text{acc}}^2)$:

$$\mathcal{L}_{\text{acc}} = \frac{1}{L} \sum_{l=1}^L \frac{\left(\|\mathbf{a}^{(l)}\| - \mu_{\text{acc}} \right)^2}{2\sigma_{\text{acc}}^2}$$

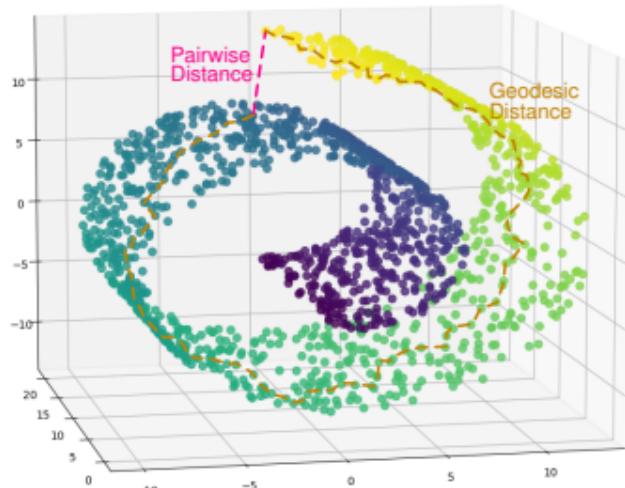


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Improvement 2: Path-based Loss Function



Remember path $q_{i,j}[k]$ through manifold (*predecessor matrix*)!

- Paths $q_{i,j}[k]$: Index l of k -th datapoint in path from datapoint i to j
- Length of path through channel chart:

$$d_{\text{path},i,j} = \sum_{k=1}^{K_{i,j}-1} \left\| \mathbf{z}^{(q_{i,j}[k])} - \mathbf{z}^{(q_{i,j}[k+1])} \right\|$$

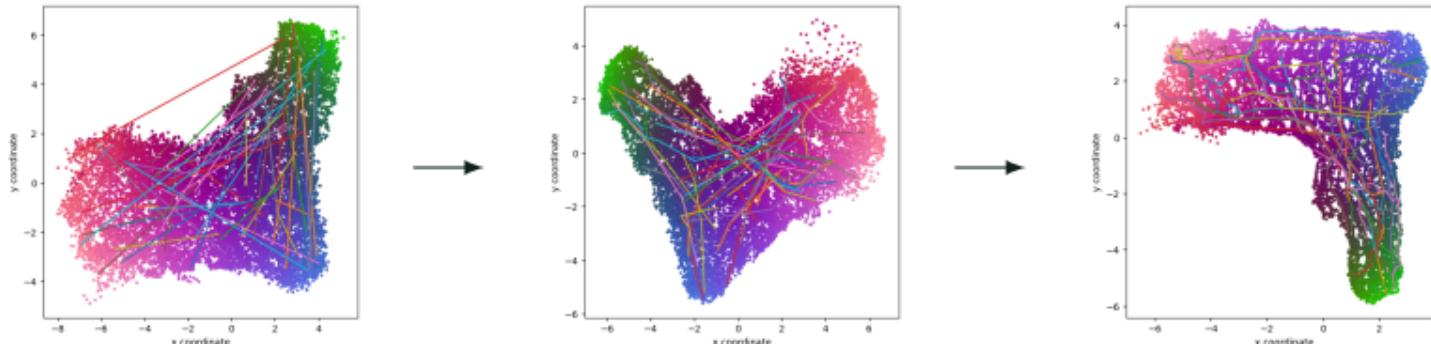
- Compare geodesic distance on manifold to path length:

$$\mathcal{L}_{\text{geo}} = \frac{1}{L^2} \sum_{i,j} \frac{(d_{\text{path},i,j} - \Delta_{\text{geo},i,j})^2}{\Delta_{\text{geo},i,j} + \beta}$$



Implementation Issues

- $d_{\text{path},i,j}$ depends on $\mathbf{z}^{(q_{i,j}[1])}$ to $\mathbf{z}^{(q_{i,j}[K_{i,j}])}$ → many degrees of freedom
- Training may become unstable (optimizer runs into local minima)
- Solution: Train with sub-sampled paths $q_{i,j}^{(s)}[k] = q_{i,j}[k \cdot s]$, sub-sampling factor s depends on $d_{\text{path},i,j}$ and decreases during training



- Additional constraint to prevent loops / local minima, omitted here



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Sammon's Loss vs Gaussian Log-Likelihood

Sammon's / Siamese Loss:

$$\mathcal{L}_{\text{Siam}} = \frac{1}{L^2} \sum_{i,j} \frac{(\|\mathbf{z}_i - \mathbf{z}_j\| - \Delta_{i,j})^2}{\Delta_{i,j} + \beta}$$

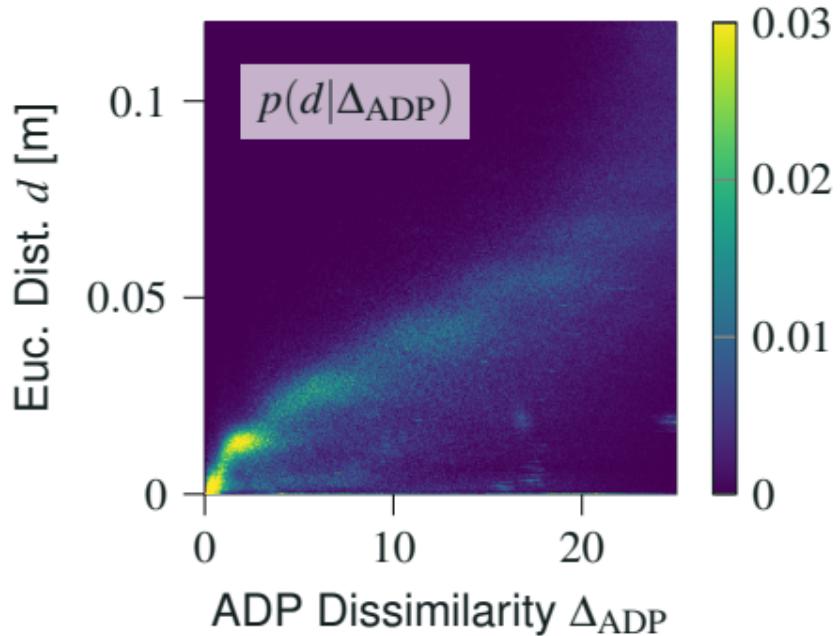
Gaussian Log-Likelihood:

$$\mathcal{L}_{\text{ML}} = \text{const.} + \frac{1}{L^2} \sum_{i,j} \frac{(\|\mathbf{z}_i - \mathbf{z}_j\| - \Delta_{i,j})^2}{2\sigma_{i,j}^2}$$

- Comparison reveals implicit assumption $\sigma_{i,j}^2 = \frac{\Delta_{i,j} + \beta}{2}$
- May have more information about uncertainty in $\Delta_{i,j}$
- Idea: **Probabilistic** model for dissimilarities Δ and distances d



Uncertainty: Formalization



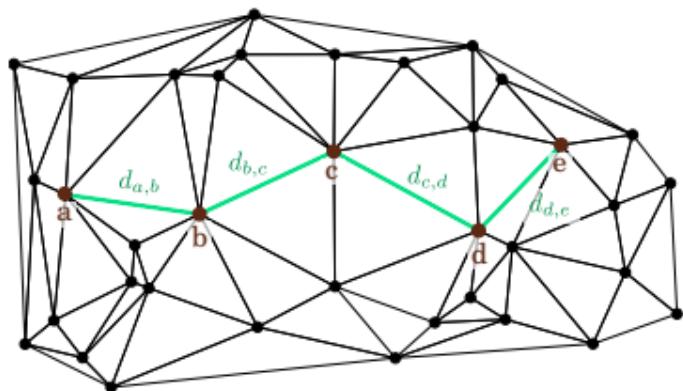
- Model dissimilarities Δ , distances d as random variables
- Assume known model

$$p(d_{i,j} | \Delta_{i,j})$$

for every type of dissimilarity



Many Questions



Q1. How to run shortest path algorithm if $d_{i,j}$ is non-deterministic?

Q2. What is the distribution of

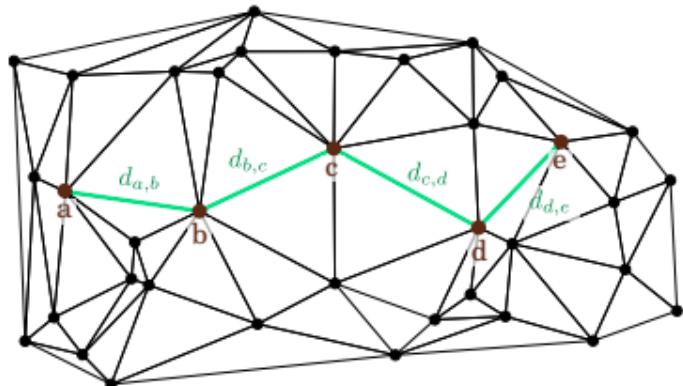
$$d_{\text{geo},i,j} = \sum_{k=1}^{K_{i,j}-1} d_{q_{i,j}[k],q_{i,j}[k+1]},$$

which is needed for \mathcal{L}_{geo} ?

Q3. Correlations between $d_{a,b}$, $d_{b,c}$, ...?



Many Questions



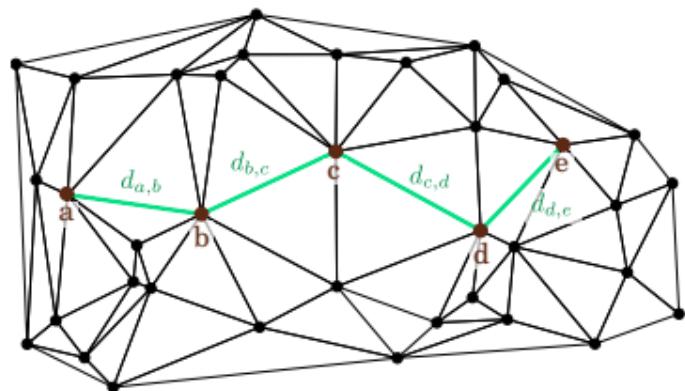
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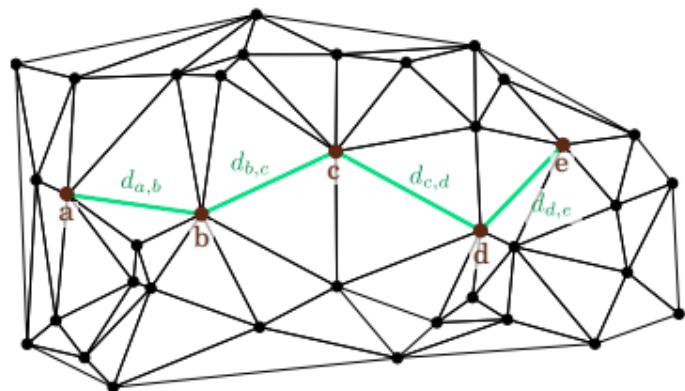
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Many Questions



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$$d_{\text{geo},i,j} = \sum_{k=1}^{K_{i,j}-1} d_{q_{i,j}[k], q_{i,j}[k+1]},$$

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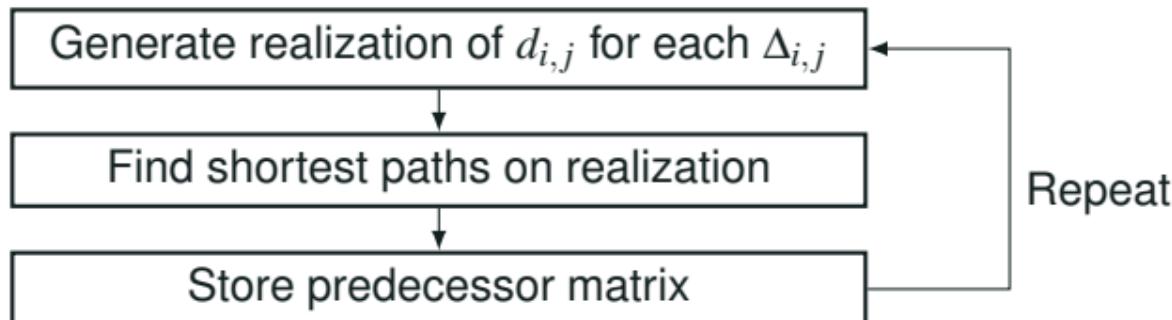
- Q3. Correlations between $d_{a,b}$, $d_{b,c}$, ...?

Need pragmatic answers!



(Q1) Shortest Path or Short Path

- Isomap, Siamese CC rely on *shortest* path and convex shape
- Geodesic Loss: Some *short* (no *shortest*) path is sufficient
- Algorithm to find *some short paths*:

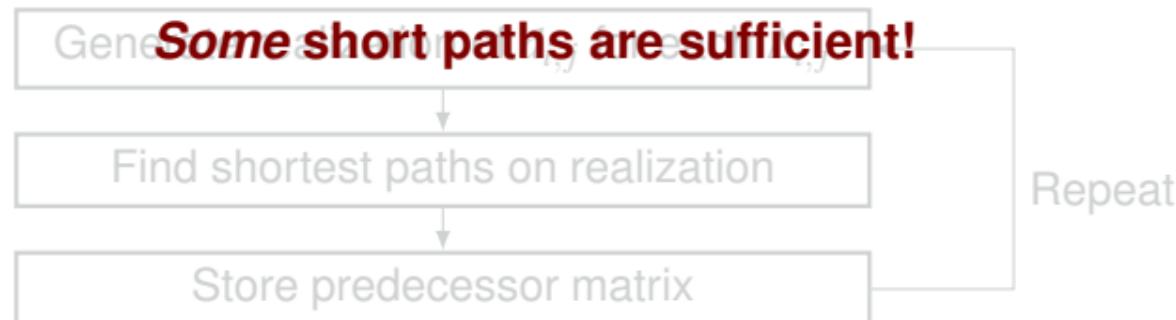


- Now: Figure out assumed distribution of path lengths $d_{geo,i,j}$



(Q1) Shortest Path or Short Path

- Isomap, Siamese CC rely on *shortest* path and convex shape
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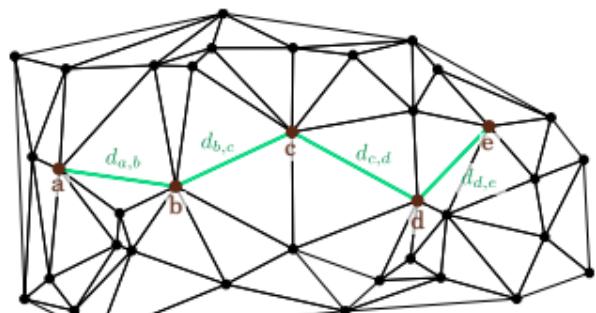


(Q2) Distribution of $d_{\text{geo},i,j}$

- Arbitrary distributions $p(d|\Delta)$ are too hard to handle
- Assumption: Pairwise distances $d_{i,j}|\Delta_{i,j} \sim \mathcal{N}(\mu_{i,j}, \sigma_{i,j}^2)$
- Hence: $d_{\text{geo},i,j} \sim \mathcal{N}(\mu_{\text{geo},i,j}, \sigma_{\text{geo},i,j}^2)$ with

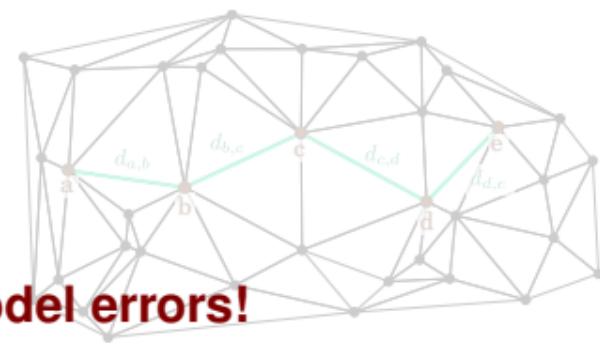
$$\mu_{\text{geo},i,j} = \sum_{k=1}^{K_{i,j}-1} \mu_{q_{i,j}[k], q_{i,j}[k+1]}$$

$$\sigma_{\text{geo},i,j}^2 = \sum_{k=1}^{K_{i,j}-1} \sigma_{q_{i,j}[k], q_{i,j}[k+1]}^2 + 2 \sum_{1 \leq a < b \leq k} \text{Cov}(d_{q_{i,j}[a], q_{i,j}[a+1]}, d_{q_{i,j}[b], q_{i,j}[b+1]})$$



(Q2) Distribution of $d_{\text{geo},i,j}$

- Arbitrary distributions $p(d|\Delta)$ are too hard to handle
 - Assumption: Pairwise distances $d_{i,j}|\Delta_{i,j} \sim \mathcal{N}(\mu_{i,j}, \sigma_{i,j}^2)$
 - Hence: **Use normal distribution to m**



$$\mu_{\text{geo},i,j} = \sum_{k=1}^{K_{i,j}-1} \mu_{q_{i,j}[k], q_{i,j}[k+1]}$$

(Q3) Modeling Correlations

- **CSI-based** dissimilarities: Low correlation
→ Assume: $d|\Delta_{\text{ADP}}$ **perfectly uncorrelated** to one another
 - **Timestamp** dissimilarities: Displacement $d|\Delta_{\text{time}}$ highly correlated to each other (unknown velocity)
→ Worst-case assumption: $d|\Delta_t$ **perfectly correlated**
 - Timestamp dissimilarities with velocity measurement: Displacement $d|\Delta_{\text{time}}, v$ correlated only if velocity measurement error is correlated

(details omitted)

(Q3) Modeling Correlations

- **CSI-based** dissimilarities: Low correlation
→ Assume: $d|\Delta_{ADP}$ **perfectly uncorrelated** to one another
 - **Timestamp** dissimilarities: Displacement $d|\Delta_{time}$ highly correlated to each other (unknown velocity)
Make reasonable worst-case assumptions!
→ Worst-case assumption: $d|\Delta_t$ **perfectly correlated**
 - Timestamp dissimilarities with velocity measurement: Displacement $d|\Delta_{time}, v$ correlated only if velocity measurement error is correlated

(details omitted)



Improvement 3: Maximum Likelihood Geodesic Loss Function

- Take variance $\sigma_{\text{geo},i,j}^2$ of geodesic path length into account:

$$\mathcal{L}_{\text{geo,ML}} = \frac{1}{L^2} \sum_{i,j} \frac{\left(\sum_{k=1}^{K_{i,j}^{(s)}-1} \left\| \mathbf{z}^{(q_{i,j}^{(s)}[k])} - \mathbf{z}^{(q_{i,j}^{(s)}[k+1])} \right\| - \mu_{\text{geo},i,j} \right)^2}{2\sigma_{\text{geo},i,j}^2}$$

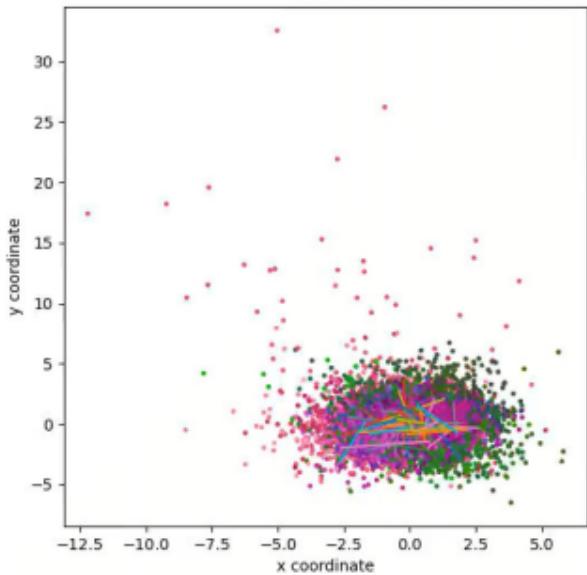
compares length of sub-sampled path through channel chart to likelihood function describing path length through manifold.



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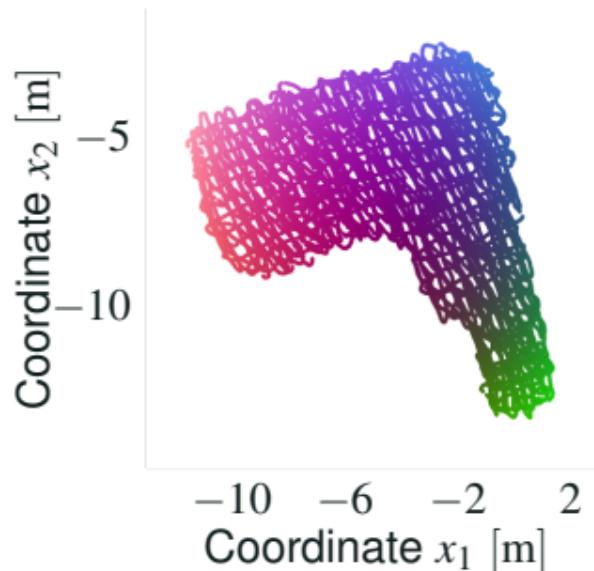
Training Animation - All Arrays



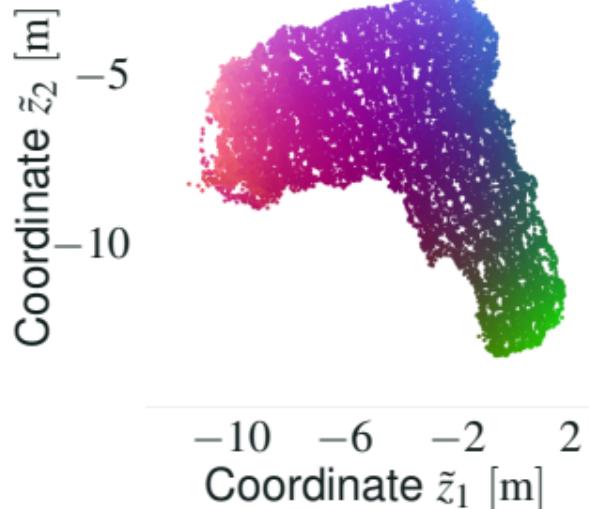


Evaluation - All Antenna Arrays

Ground Truth



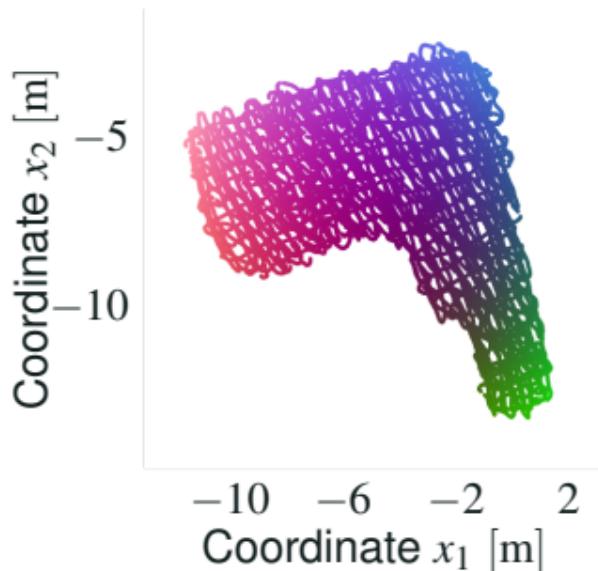
Channel Chart



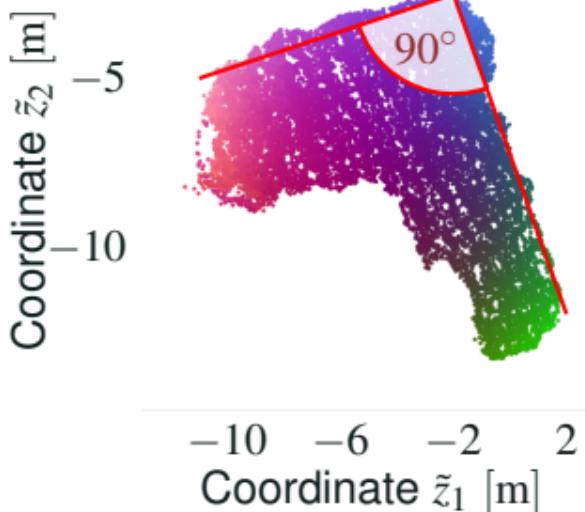


Evaluation - All Antenna Arrays

Ground Truth

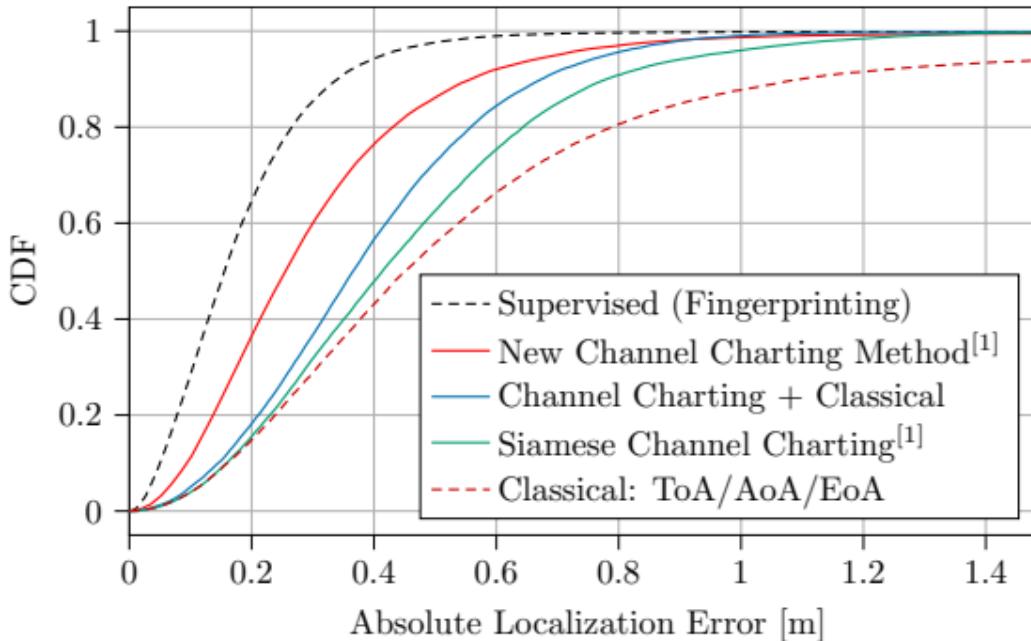


Channel Chart





Evaluation - All Antenna Arrays



^[1]Evaluated after optimal affine transform

Evaluation - All Antenna Arrays

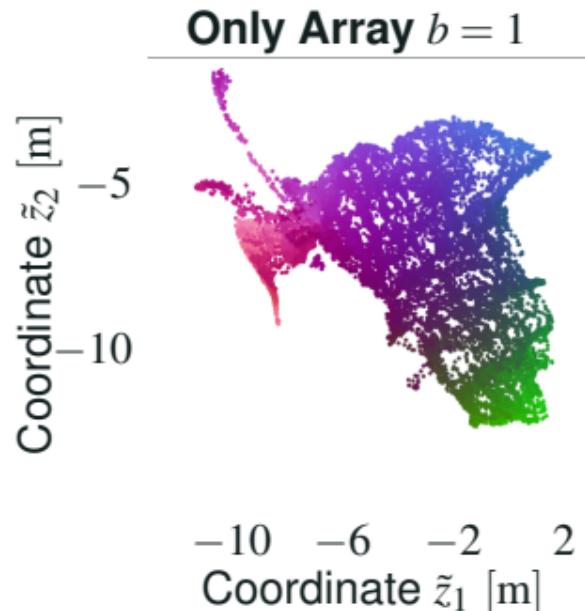
	MAE ↓ [2]	DRMS ↓ [2]	CEP ↓ [2]	R95 ↓ [2]	KS ↓ [2]	CT/TW ↑ [2]
Classical ToA + AoA	0.676m	1.228m	0.462m	1.763m	0.214	0.965/0.970
Siamese CC [1]	0.490m	0.584m	0.441m	1.026m	0.071	0.996/0.996
Classical + CC	0.401m	0.483m	0.369m	0.789m	0.070	0.995/0.995
New Method [1]	0.295m	0.375m	0.234m	0.719m	0.069	0.998/0.998

[1]MAE / DRMS / CEP / R95 evaluated after optimal affine transform

[2]MAE = mean absolute error, DRMS = distance root mean squared, CEP = circular error probable, R95 = 95th error percentile, KS = Kruskal Stress, CT/TW = Continuity / Trustworthiness



Evaluation - Single Antenna Array



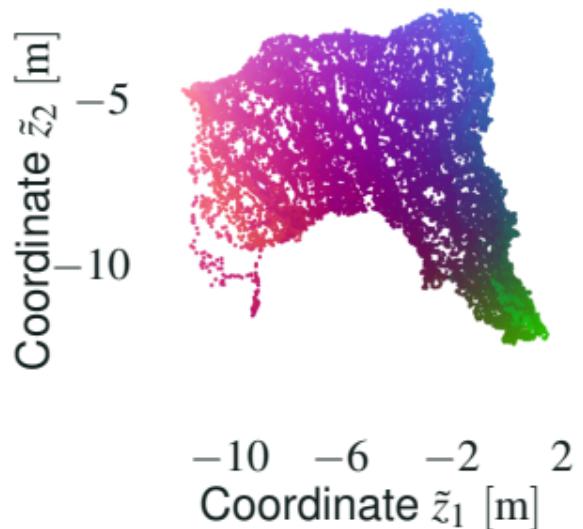
Ant. Array	MAE [1]	MAE Siam. [1]
$b = 1$	≈ 1.05 m	≈ 2.04 m
$b = 2$		≈ 2.23 m
$b = 3$		≈ 1.37 m
$b = 4$		≈ 1.62 m

[1] Evaluated after optimal affine transform



Evaluation - Single Antenna Array

Only Array $b = 2$



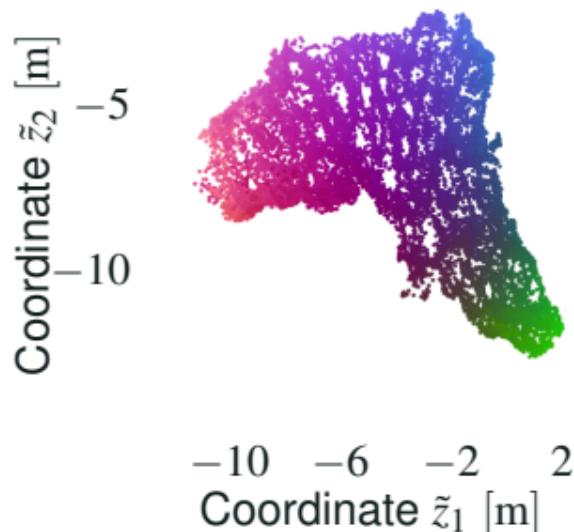
Ant. Array	MAE [1]	MAE Siam. [1]
$b = 1$	≈ 1.05 m	≈ 2.04 m
$b = 2$	≈ 0.44 m	≈ 2.23 m
$b = 3$		≈ 1.37 m
$b = 4$		≈ 1.62 m

[1] Evaluated after optimal affine transform



Evaluation - Single Antenna Array

Only Array $b = 3$



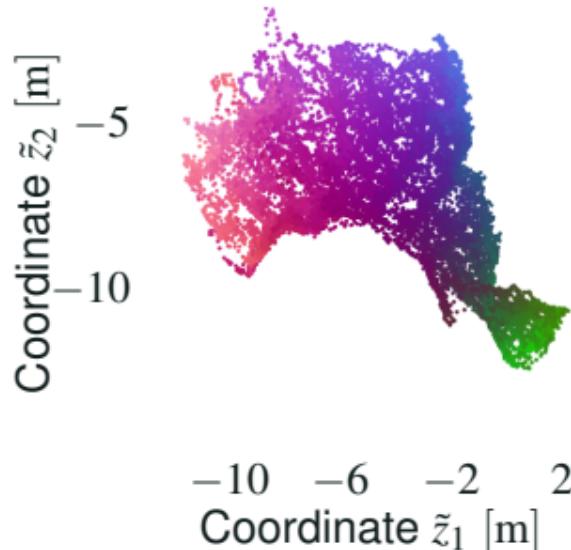
Ant. Array	MAE [1]	MAE Siam. [1]
$b = 1$	≈ 1.05 m	≈ 2.04 m
$b = 2$	≈ 0.44 m	≈ 2.23 m
$b = 3$	≈ 0.44 m	≈ 1.37 m
$b = 4$		≈ 1.62 m

[1] Evaluated after optimal affine transform



Evaluation - Single Antenna Array

Only Array $b = 4$



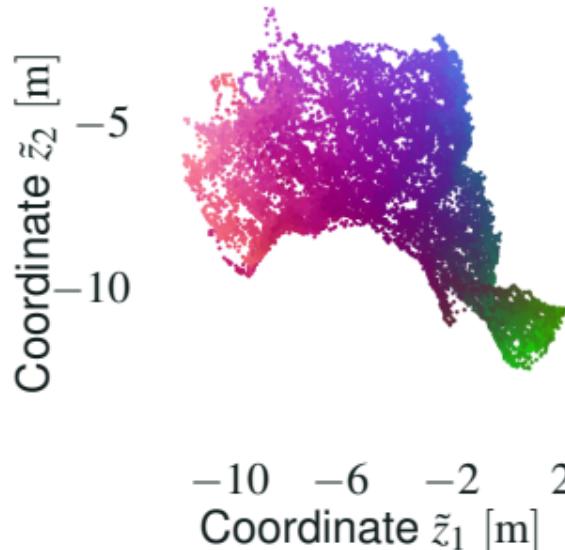
Ant. Array	MAE [1]	MAE Siam. [1]
$b = 1$	≈ 1.05 m	≈ 2.04 m
$b = 2$	≈ 0.44 m	≈ 2.23 m
$b = 3$	≈ 0.44 m	≈ 1.37 m
$b = 4$	≈ 0.73 m	≈ 1.62 m

[1] Evaluated after optimal affine transform



Evaluation - Single Antenna Array

Only Array $b = 4$



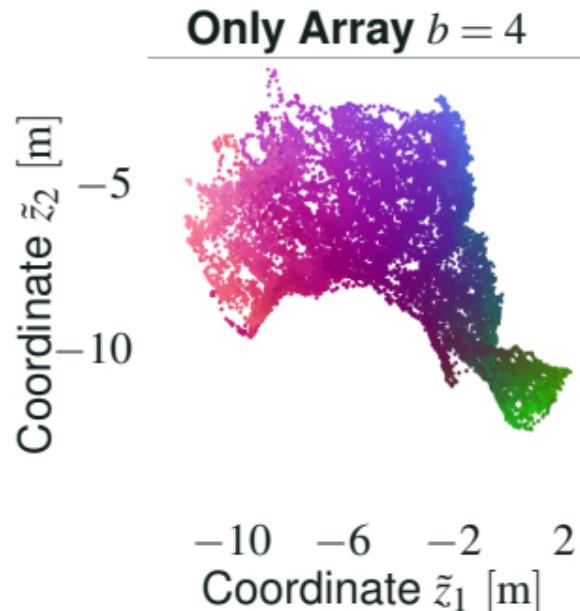
Ant. Array	MAE [1]	MAE Siam. [1]
$b = 1$	$\approx 1.05 \text{ m}$	$\approx 2.04 \text{ m}$
$b = 2$	$\approx 0.44 \text{ m}$	$\approx 2.23 \text{ m}$
$b = 3$	$\approx 0.44 \text{ m}$	$\approx 1.37 \text{ m}$
$b = 4$	$\approx 0.73 \text{ m}$	$\approx 1.62 \text{ m}$

→ Good single array LoS localization!

[1] Evaluated after optimal affine transform



Evaluation - Single Antenna Array



Ant. Array	MAE [1]	MAE Siam. [1]
$b = 1$	≈ 1.05 m	≈ 2.04 m
$b = 2$	≈ 0.44 m	≈ 2.23 m
$b = 3$	≈ 0.44 m	≈ 1.37 m
$b = 4$	≈ 0.73 m	≈ 1.62 m

→ Good single array LoS localization!
 → Some NLoS localization capability!

[1] Evaluated after optimal affine transform



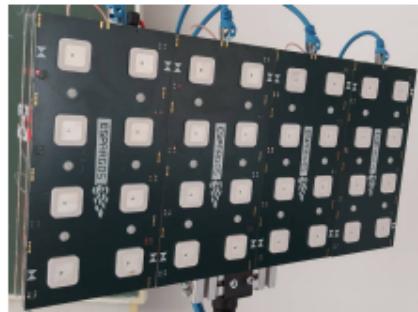
Agenda

- ① Introduction, Motivation, State of the Art
- ② New Training Architecture
- ③ Geodesic Loss
- ④ Uncertainty (Concept Overview)
- ⑤ Evaluation and Results
- ⑥ Conclusion and Outlook



Conclusion and Outlook

- New batch-wise training architecture
- New geodesic loss taking into account uncertainty
- Significant localization performance improvements!
- Uncertainty model: Integrate additional information sources (e.g., velocity measurements)
- Reproduce results on other CSI datasets (WiFi/ESPARGOS, DICHASUS, third-party)





Thank you for your attention! Questions?



Source Code (GitHub)



Dataset (DICHASUS)



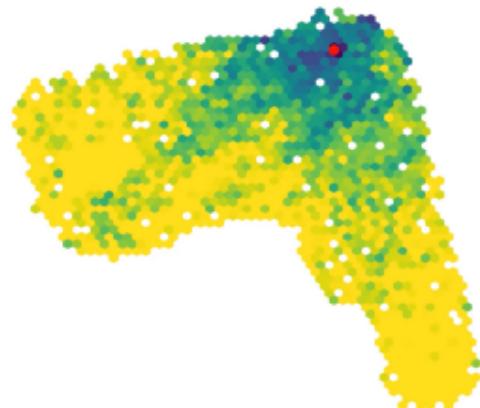
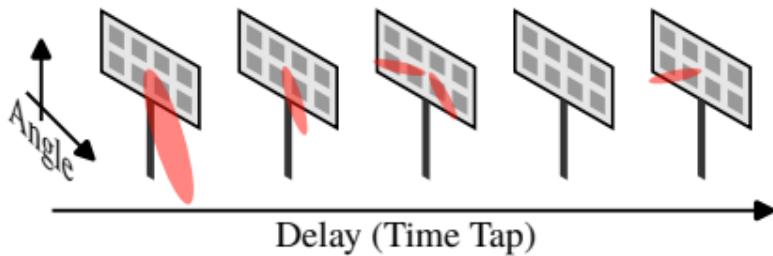
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Dissimilarity Metric: Angle-Delay-Profile Δ_{ADP}

- Same power from same angle at same delay \rightarrow similar location



- Uses *time-domain CSI* $\mathbf{H} \in \mathbb{C}^{B \times M \times T}$:

$$\Delta_{ADP,i,j} = \sum_{b=1}^B \sum_{\tau=1}^T \left(1 - \frac{\left| \sum_{m=1}^M \left(\mathbf{H}_{b,m,\tau}^{(i)} \right)^* \mathbf{H}_{b,m,\tau}^{(j)} \right|^2}{\left(\sum_{m=1}^M \left| \mathbf{H}_{b,m,\tau}^{(i)} \right|^2 \right) \left(\sum_{m=1}^M \left| \mathbf{H}_{b,m,\tau}^{(j)} \right|^2 \right)} \right)$$



Dissimilarity Metric: Timestamp Difference Δ_{time}

- Use time difference as dissimilarity

$$\Delta_{\text{time},i,j} = |t^{(i)} - t^{(j)}|$$





Fusing Dissimilarity Metrics

- Metrics may be combined into a unified dissimilarity matrix, e.g.:

$$\Delta_{\text{fuse},i,j} = \min \{\Delta_{\text{ADP},i,j}, \Delta_{\text{time},i,j}\}$$

- Requires scaling (e.g., express all dissimilarities in meters)

