Kriging and Spline Method

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Introduction to Kriging

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  - These covariances decrease with the distances between the observations.

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- The assumption of a second-order satiationary covariance process implies that the variogram is a function of the distance h between two locations.

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  - Simple

Ordinary Kriging

Suppose that measurements of the variable of interest Z at the points x_1 of the study region are made, that is to say they have realizations of the variables $Z_1, ..., Z_n$, and if it is desired to predict Z_0 at the point X_0, where there was no measurement. In this circumstance, the ordinary Kriging method proposes that the value of the variable can be predicted as a linear combination of the n variables as:

$$Z^*(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i)$$

Ordinary Kriging

- λ_i are the weights of the original values.

The idea now is to determine the spatial dependence between the measured data of a variable, we can know this by means of the variogram in its effect an estimate of the variogram (semivariogram).

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- λ contains spatial autocorrelation.

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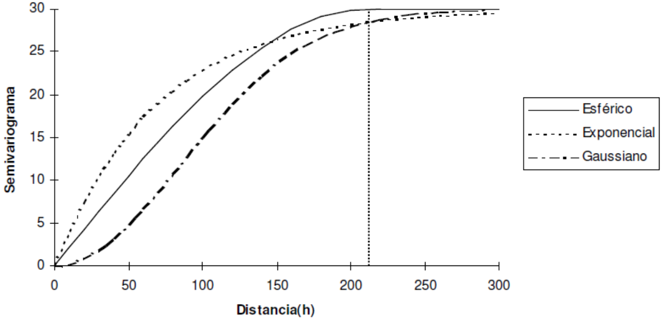
Types of Semivariogram

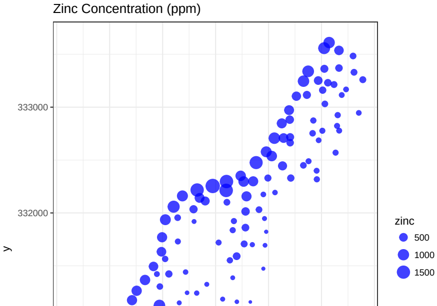


Let's go into R

Loading the information

```
library(sp)
library(gstat)
library(tidyverse)


meuse <- read_csv(file = 'data/meuse_kriging.csv')
meuse %>%
  ggplot(aes(x, y)) +
  geom_point(aes(size=zinc), color="blue", alpha = 3/4) +
  ggtitle("Zinc Concentration (ppm)") +
  coord_equal() +
  theme_bw()
```



Converting to an SPDF

To convert it to a spatial dataframe, we must first specify which of the columns contain the coordinates of the data. This is done by using R's formula notation as follows:

```
coordinates(meuse) <- ~ x + y
```

Here we see that a couple of things happen when we specify the coordinates. First, the dataframe becomes an SPDF

Fitting a variogram

To perform kriging, you must first have a variogram model, from which the data can be interpolated. There are a couple steps involved:

- Calculate the sample variogram. This is done with the variogram function.

For example, a variogram could be fit as simply as the following code:

```
lzn.vgm <- variogram(zinc~1, meuse) # calculates sample variogram values
Sph.fit <- fit.variogram(lzn.vgm, model= vgm("Sph")) # fit model
Exp.fit <- fit.variogram(lzn.vgm, model= vgm("Exp"))
Gau.fit <- fit.variogram(lzn.vgm, model= vgm("Gau"))

# plot(lzn.vgm, Sph.fit)
# plot(lzn.vgm, Exp.fit)
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- Fit a model to the sample variogram.

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```

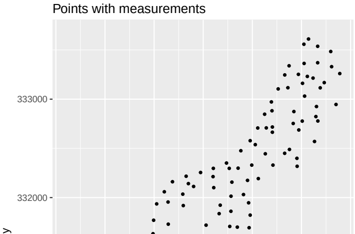
Performing Kriging

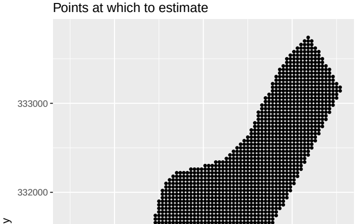
```
# load spatial domain to interpolate over
meuse.grid <- read_csv("data/meuse_grid.csv")

# to compare, recall the bubble plot above; those points were what there were values for. this is much more
plot1 <- meuse %>%  as.data.frame %>%
  ggplot(aes(x, y)) + geom_point(size=1) + coord_equal() +
  ggtitle("Points with measurements")

# this is clearly gridded over the region of interest
plot2 <- meuse.grid %>%
  ggplot(aes(x, y)) + geom_point(size=1) + coord_equal() +
  ggtitle("Points at which to estimate")

library(gridExtra)
grid.arrange(plot1, plot2, ncol = 2)
```





Computation

Once we have the prepared all of the above, we are now ready to krige. This can be done with the gstat::krige function, which usually takes four arguments:
The model formula.

- An SPDF of the spatial domain that has measurements.

Note that the second and third arguments have to be SPDF's and cannot just be dataframes.

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The model formula.

- An SPDF of the spatial domain that has measurements.
- An SPDF of the spatial domain to krige over.
- A variogram model fitted to the data.

Note that the second and third arguments have to be SPDF's and cannot just be dataframes.

Fitting

```
coordinates(meuse.grid) <- ~ x + y # step 3 above
lzn.kriged <- krige(zinc ~ 1, meuse, meuse.grid, model=Sph.fit)
```

```
## [using ordinary kriging]
```

```
library(scales)
lzn.kriged %>% as.data.frame %>%
  ggplot(aes(x=x, y=y)) + geom_tile(aes(fill=var1.pred)) + coord_equal() +
  scale_fill_gradient(low = "yellow", high="red") +
  scale_x_continuous(labels=comma) + scale_y_continuous(labels=comma) +
  theme_bw()
```