

Defect detection in pipes using guided waves

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Abstract

The detection of corrosion in insulated pipes is of major importance to the oil and chemical industries. Current methods involving point-by-point inspection are expensive because of the need to remove the insulation. An alternative method which is being developed at Imperial College is to propagate guided waves in the walls of the pipes, and to look for reflections from defects. The test configuration is essentially pulse-echo; the insulation is removed at just one location on a pipe and the signals are then transmitted and received using a single transducer unit. The technique is currently undergoing field trials. This paper presents a review of the studies of the propagation of the waves and their sensitivity to defects which have been conducted in order to provide a sound scientific basis for the method. Issues of importance were the selection of the optimum guided wave modes and the establishment of relationships between the defect size and the strength of wave reflection. Analytical and numerical studies were conducted in parallel with an extensive experimental programme. © 1998 Elsevier Science B.V.

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1. Introduction

The oil, gas, chemical and petro-chemical industries operate hundreds of kilometres of pipelines, often transporting corrosive substances. A high proportion of these industrial pipelines are insulated, so that even external corrosion cannot readily be detected without the removal of the insulation, which in most cases is prohibitively expensive. There is therefore an urgent need for the development of a quick, reliable method for the detection of corrosion under insulation (CUI).

The use of cylindrical Lamb waves propagating along the pipe wall is potentially a very attractive solution to this problem since they can propagate a long distance under insulation and may be excited and received using transducers positioned at a location where only a small section of insulation has been removed.

The authors are working on the development of such a guided wave testing technique for the inspection of pipework in chemical plant, the original target being to detect any areas of corrosion larger than $3T \times 3T$ in area and $T/2$ deep where T is the pipe wall thickness. Pipes in the range of 2–12 in. diameter are being considered at present, but the technique could also be applied

to other sizes. The testing scheme employs a pulse-echo arrangement from a single location on a pipe, using waves which are guided along the pipe wall, illustrated in Fig. 1. The waves are excited and received using a ring transducer made up of mechanically independent dry-coupled piezo-electric elements distributed around the circumference. By exciting all of the elements equally and concurrently, an axially symmetric mode is launched. The presence and axial location of defects in the pipe wall are determined by any reflections and their arrival times. The scheme offers rapid inspection of long sections of pipe, up to 50 m. Site trials of the system have been undertaken successfully and it is due to be operated in the field under licence in the near future.

A number of authors have reported work on the use

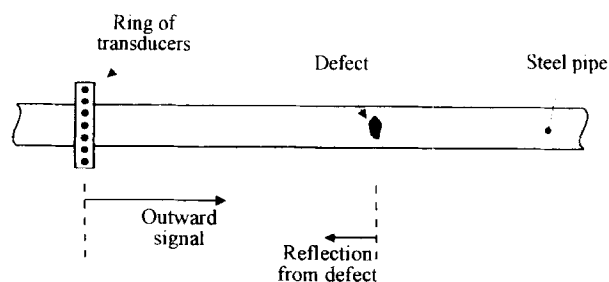


Fig. 1. Pulse echo inspection of pipes using Lamb waves.

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of Lamb waves for the inspection of pipes and tubes [1–7]. Much of this has addressed the requirements for inspection of heat exchanger tubing which is typically 1 in. in diameter and is therefore somewhat smaller than the chemical plant pipework. However, studies have included the inspection of larger diameter gas pipelines using circumferentially travelling waves [6,7]. Relevant studies have also been reported previously and elsewhere on the interaction of Lamb waves with defects in plate and pipe structures [8–11]. Regarding the present project, publications have reported the development of the dry-coupled piezoelectric transducers for the excitation and detection of the guided waves [12], studies of the reflection and mode conversion behaviour of the guided waves [13–16], studies of the influence of features such as flanges and pipe supports [17], and the results of field trials [18].

As an important part of the project, a considerable amount of work has been undertaken to study the physics of the propagation of the modes, and their reflection and conversion at defects, and much of this has been reported in stages throughout Refs. [12–18] cited above. The work included mode property calculations, precise laboratory experiments, and finite element simulations. The purpose of this paper is to present a summary of the findings of these fundamental studies which underpin the pipe inspection system.

2. Guided mode properties and mode selection

Wave propagation properties in pipes are extremely complicated, much more so than in plates. Figs. 2 and 3 show the phase and group velocity dispersion curves for a nominal 3 in. (inside diameter 76 mm, wall thickness 5.5 mm) chemical plant pipe. The curves were

calculated using a general purpose computer program which was developed by the authors [19,20]. The modes include a family of axially symmetric longitudinal, flexural and torsional motion of the pipe wall, as would be expected from an understanding of Lamb and Love modes if the pipe was considered simply as a curved plate. However, in addition to the axially symmetric (zero-order) modes, there are modes which have harmonic variation of displacements and stresses around the circumference. The order 1 modes have one cycle of variation around the circumference, order 2 have two, and so on. For each of this infinite series of orders there is a family of modes. The modes in the figures are labelled after the convention of Silk and Bainton [1]: the first integer of the integer pair in each mode label gives the harmonic order of circumferential variation; thus the axially symmetric modes have zero as their first integer. The letters L, F and T denote longitudinal, flexural and torsional modes, respectively. For illustration, the mode shapes of the displacements of three of the modes which are important to this study are shown in Fig. 4. All three are shown at 70 kHz, indicated by the dashed line in Fig. 3. Fig. 4(a) shows the L(0,1) mode which consists of axially symmetric pipe wall bending and is therefore similar to the a_0 plate mode at low frequency. Fig. 4(b) shows the L(0,2) mode which consists predominantly of axially symmetric uniform axial motion through the wall thickness and is therefore very similar to the S_0 plate mode at low frequency. The F(1,3) mode, in Fig. 4(c), has practically the same through-thickness shape as the L(0,2) mode, but has one cycle of harmonic variation around the circumference. This is the distribution of axial displacement which would be found if the whole pipe was subjected to bending.

The phase velocity dispersion curves show the velocity

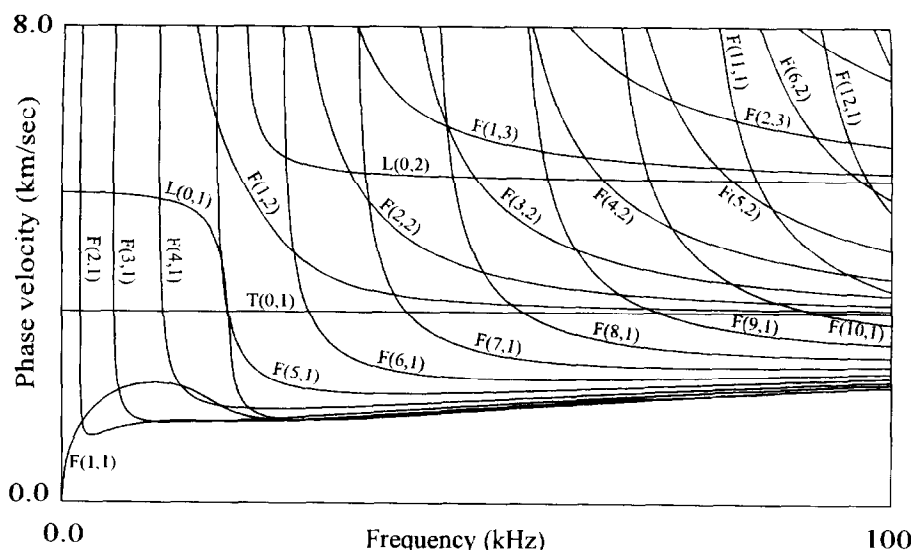


Fig. 2. Phase velocity dispersion curves for 76 mm (nominal 3 in.) diameter pipe.

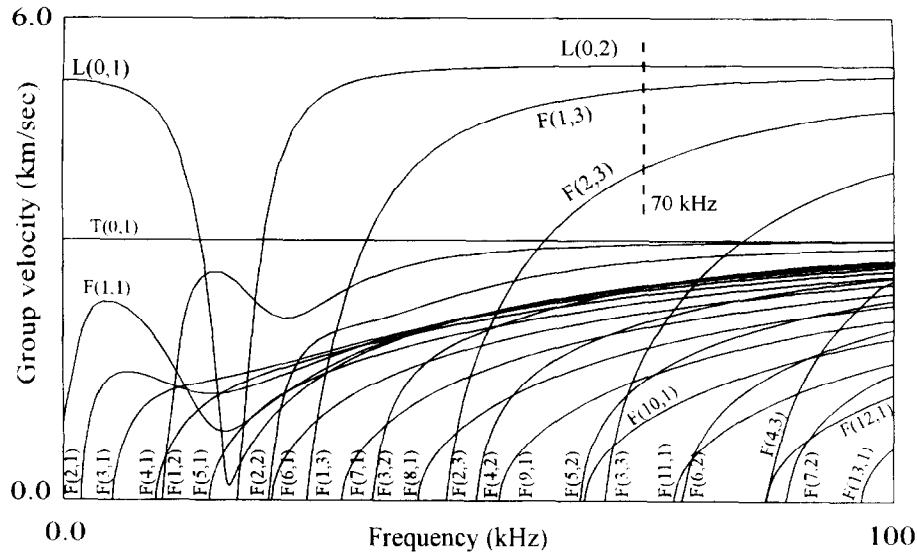


Fig. 3. Group velocity dispersion curves for 76 mm (nominal 3 in.) diameter pipe.

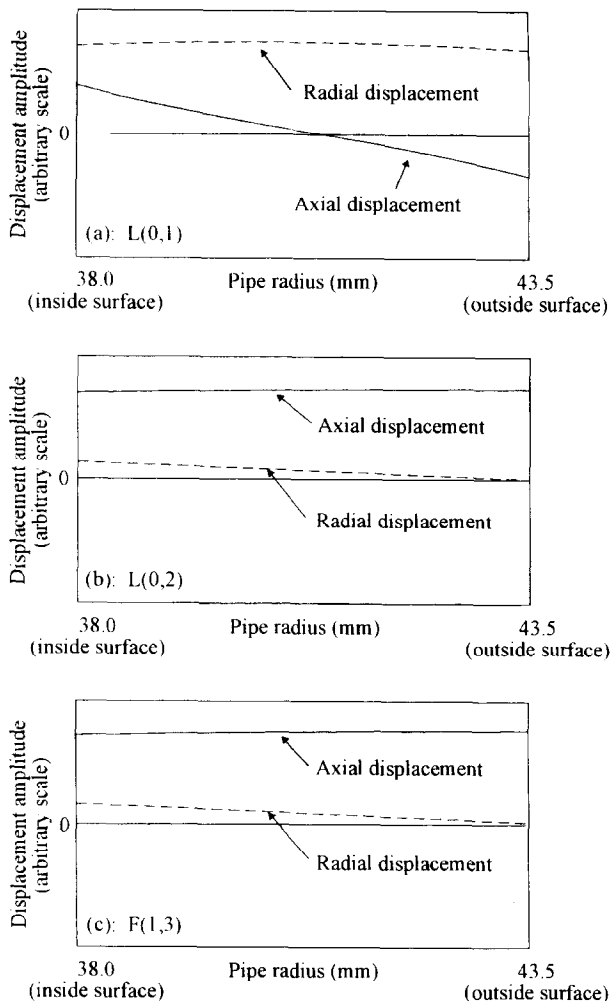


Fig. 4. Mode shapes for 76 mm (nominal 3 in.) diameter pipe at 70 kHz: (a) L(0,1); (b) L(0,2); (c) F(1,3).

of the harmonic wave cycles in the direction of propagation along the pipe; they provide useful information about wave speeds of single tones and of wavelengths of the modes. The group velocity dispersion curves show the velocity at which finite-time wave packets travel; they are therefore useful for the calculation of the travel times of the wave signals which are used in the long-range testing.

A key element of the inspection system is the selection and exploitation of a single mode. In general, an excitation source can excite all of the modes which exist within its frequency bandwidth, resulting in a signal which is much too complicated to interpret. Indeed, even with a single mode, great care is needed for the correct identification of the reflections from defects and from normal pipe features such as welds. Therefore, although troublesome to achieve, it is essential to design the transducers and the signal to excite only the chosen mode. Then, since defects and normal pipe features can convert energy to other modes, it is important also to be able to receive selectively.

The mode which was chosen for excitation in the inspection system is the axially symmetric L(0,2) mode, at about 70 kHz. This mode is very attractive for testing for several reasons: it is practically non-dispersive over a wide bandwidth around this frequency – that is to say its velocity does not vary significantly with frequency – so that the signal shape and amplitude are retained as it travels; it is the fastest mode so that any unwanted mode converted signals arrive after it has been received; and its mode shape [Fig. 4(b)] makes it equally sensitive to internal or external defects at any circumferential location.

The first step in selective excitation is to use a narrow band signal. This also gives good signal strength and

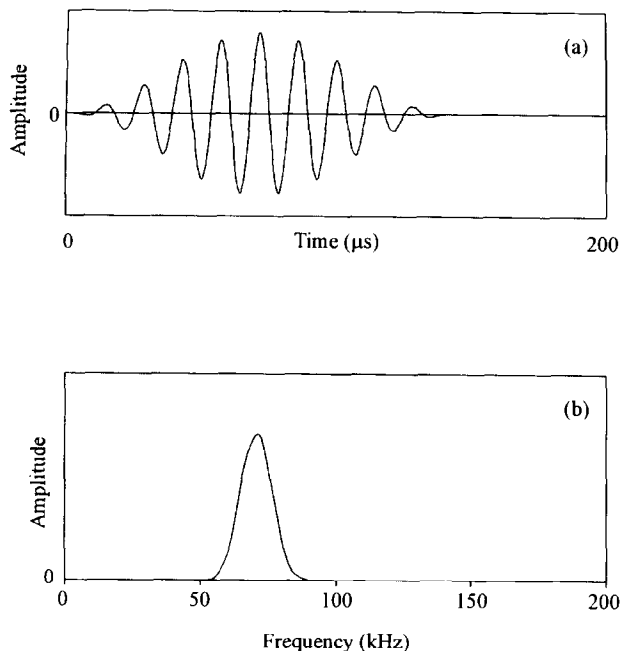


Fig. 5. Narrow band signal consisting of 10 cycles modulated by a Hanning window: (a) time record; (b) frequency spectrum.

avoids dispersion over long propagation distances. A tone burst of 5 or 10 cycles in a Hanning window achieves this ideally. A typical time record for a 10 cycle signal and its frequency spectrum are shown in Fig. 5.

Spatial selectivity depends primarily on the use of a ring of transducers around the pipe. If a sufficient number of equally spaced and equally driven transducers is excited around the circumference, then only the axially symmetric (order 0) modes are excited. The number of transducers must be greater than the highest order of the modes which can be present in the frequency range of the signal. For the 3 in. pipe, the highest order for frequencies up to 100 kHz is 13; the testing system uses 16 transducer elements for this size of pipe. Having avoided all non-zero order modes, there remain the $T(0,1)$, $L(0,1)$ and $L(0,2)$ modes within the chosen frequency range. The $T(0,1)$ mode consists of torsion of the pipe and can only be excited by circumferential motion. Its excitation is therefore avoided by using

transducers which excite axial motion [12]. Strong excitation of the $L(0,2)$ mode whilst only weakly exciting the $L(0,1)$ mode can be achieved by using two transmitter rings with wavelength axial separation. Fig. 2 shows that the phase velocity (and therefore wavelength) of $L(0,1)$ is markedly different to that of $L(0,2)$. On the same basis, the reception of signals is equally selective, the received signals of the elements of the transducers being summed. A further refinement of the system which is of great practical value is to excite a forward travelling wave without much energy being propagated in the backward direction. This can be achieved by using transducer rings which are spaced a quarter of a wavelength apart and separated by $\pi/2$ in phase [18].

Finally, mode conversion to the $(F1,3)$ mode has also been found to be useful when detecting defects [15,16] near welded pipe joints. The reflections of the $L(0,2)$ mode from welds tend to be quite strong and they therefore mask any reflections from defects at these locations. The welds are axially symmetric so the energy which they reflect is predominantly in the $L(0,2)$ mode. However, corrosion defects are invariably located at one side of the pipe and they mode convert some of the incident energy to reflect the non-axially symmetric $F(1,3)$ mode. Therefore detection of a reflected $F(1,3)$ mode can be used to indicate a defect at or near a weld. The reception of the $F(1,3)$ mode in isolation from $L(0,2)$ is achieved by introducing an angularly dependent phase lag to each transducer element of the receiver ring before summing their signals.

3. Experimental studies of reflections from defects

A number of different laboratory experiments have been conducted, using both 3 and 6 in. pipes, to investigate the reflections of $L(0,2)$ and of $F(1,3)$ at defects due to an incident $L(0,2)$ mode. Here we present the setup and the key results, making use solely of the experiments on 3 in. pipes. The results of the experiments on 6 in. pipes are in complete agreement with the results presented here.

The experiments were performed on 2.6 m lengths of

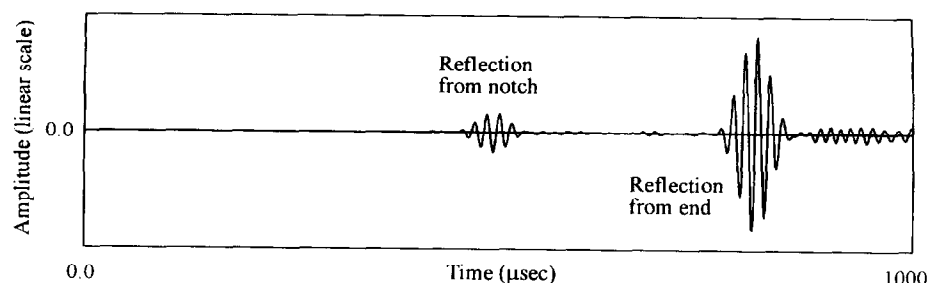


Fig. 6. Typical experimental time record: $L(0,2)$ reflection from a through-thickness notch extending around one-eighth of the circumference of a 76 mm (nominal 3 in.) diameter pipe.

nominal 3 in. Schedule 40 steel pipe (internal diameter 76 mm, wall thickness 5.5 mm). In each case the $L(0,2)$ mode was excited using transducer rings clamped near one end, and employing a windowed five- or 10-cycle, 70 kHz toneburst. The pipe was supported on steel Vee-blocks; these did not cause significant reflections. Before introducing any defects, the reflection from the remote end of the pipe was recorded as a reference signal. The received signal was amplified and then transferred to a digital oscilloscope. Typically 200 successive signals were captured by the oscilloscope for each measurement, and these were averaged to improve the signal-to-noise ratio. The measurement of the amplitude of the reflected signal was usually made simply by recording the peak-to-peak value in the time domain, though in some cases, the amplitude at 70 kHz in the frequency domain was recorded. Since the signals were narrow band, there is no significant difference between the results for these two approaches.

A notch was then machined approximately $2/3$ of the distance along each pipe, using a 3.2 mm slot drill. The cutter axis was aligned to lie on a radial axis of the pipe, and the notch was extended by rotating the pipe about its own axis. The transducer rings remained in position throughout each experiment. A machined notch is a rather different geometry than a representative corrosion trough. However, it was shown [17] that since both have dimensions which are significantly smaller than the 80 mm wavelength, the reflection from a trough is almost identical to that from a notch of the same depth and circumferential extent. The notch was much easier to create accurately than a trough would have been.

We report here experimental results for two separate pipes, using the following notches: (a) a notch of short circumferential extent (11%) over a range of depths; and (b) a through-thickness notch over a range of circumferential extent. In the former case, the reflection of the $L(0,2)$ mode was measured and in the latter case both $L(0,2)$ and $F(1,3)$ were measured. All measurements were converted to a reflection coefficient for presentation; the reflection coefficient was defined as the amplitude of the reflected signal divided by the amplitude of the incident signal.

Fig. 6 shows a typical time record from an experiment. The pipe had a through-thickness notch which extended around one-eighth of the circumference and the receiver ring was configured to receive the $L(0,2)$ mode. The time record clearly shows the reflected signal from the notch and later from the end of the pipe.

Fig. 7 shows a summary of the reflection coefficient results for the part-through notches. Here it can be seen that the reflection coefficient is modest at small depths and rises sharply as the depth is increased.

Fig. 8 shows a summary of the reflection coefficient results for the through-thickness notches. The $L(0,2)$

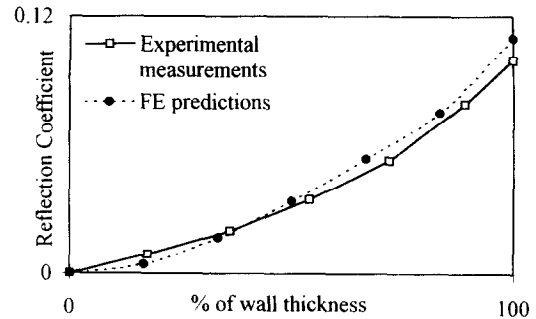


Fig. 7. Reflection of the $L(0,2)$ mode as a function of notch depth for a notch extending around 11% of the circumference of a 76 mm (nominal 3 in.) diameter pipe.

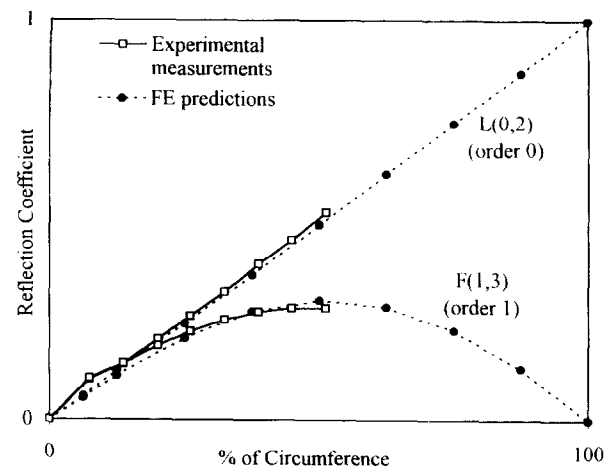


Fig. 8. Reflection of the $L(0,2)$ and $F(1,3)$ modes as functions of the circumferential extent of a through-thickness notch in a 76 mm (nominal 3 in.) diameter pipe.

reflection coefficient is evidently a roughly linear function of the circumferential extent of the notch. The reflection coefficient for the $F(1,3)$ mode approximates to the shape of a rectified half-sine wave. The limiting values for the $F(1,3)$ mode are intuitive: zero reflection should be expected for no notch or for a full-circumference notch; maximum reflection should be expected for a 50% notch.

4. Finite element studies of reflections from defects

Two finite element idealizations were used, one representing a notch which extended around the full circumference for a range of depths, and the other representing a through-thickness notch for a range of circumferential extent. Time-domain simulations were performed using the program Finel, developed at Imperial College [21]. Further details of the analyses are given in Refs. [14, 16].

The first simulations used an axisymmetric model to predict the reflection coefficient from the part-through notch which extended over the full circumference of the pipe. A mesh of identically sized linear quadrilateral

axisymmetric solid elements was used, with six elements through the wall thickness of the pipe, and an axial element length of 1 mm. This model was therefore capable of predicting the propagation and mode conversion of any axially symmetric modes whose displacements are in the radial-axial plane, including both $L(0,1)$ and $L(0,2)$. The excitation of the $L(0,2)$ mode was achieved by prescribing axial displacements equally at all of the nodes through the thickness at one end of the pipe. As in the experimental work, a five-cycle windowed tone burst was used. On reception, the $L(0,2)$ mode was detected without any other mode-converted modes by summing the axial displacements at all of the nodes through the thickness. A range of depths of a circumferential notch on the outside of the pipe was studied. The notch was modelled accurately [14] as a zero-width crack simply by disconnecting adjacent elements.

The simulations of the through-thickness notch extending around part of the circumference were achieved using a three-dimensional membrane finite element model. The basis for such a simplification is the simple nature of the mode shapes of the incident $L(0,2)$ mode and the reflected $L(0,2)$ and $F(1,3)$ modes at 70 kHz. All three modes are described accurately by membrane stresses and strains of the pipe wall. The justification is reasoned in detail in Ref. [14]. Half of the circumferential extent of a length of pipe was modelled, assuming one plane of symmetry. A mesh of identically sized linear quadrilateral membrane elements was used, with 16 elements around the 180° circumference of the model. The five-cycle toneburst was applied as a sequence of prescribed displacements in the axial direction at all nodes at the end of the pipe. Axial displacements around the circumference were monitored and were processed to measure the $L(0,2)$ and $F(1,3)$ reflections in the same way as in the experimental work. A series of analyses incorporating through-thickness notches of various circumferential lengths was conducted. Again the notches were represented simply by disconnecting adjacent elements in the model.

The results of the membrane model are presented in Fig. 8 for comparison with the experimental measurements. They demonstrate excellent agreement between the predictions and the experiments.

Although the axisymmetric models could not include a notch which extends for only part of the circumference at the same time as only part of the thickness, there is a very reasonable argument to infer the reflection coefficients for any such cases from the other results. The reflection coefficient is linear with respect to the circumferential extent for a through-thickness notch. Similarly, although not reported here, a linear relationship with circumference has also been demonstrated experimentally for a notch of 50% depth [14]. Therefore it is reasonable to assume that the reflection coefficient is linear with circumference for any depth of notch. The

reflection coefficient for a notch of any depth and any circumferential extent is then found simply by scaling the results of the axisymmetric models according to the circumferential extent of the notch. Thus, taking the case of the experimental measurements for the part-through notch covering 11% of the circumference of the nominal 3 in. pipe, the reflection coefficients are predicted simply by scaling the axisymmetric values by a factor of 0.11. These scaled results are presented in Fig. 7, together with the experimental measurements, showing the non-linear variation with notch depth and close agreement with the measurements.

5. Analysis of reflection functions

It was surprising initially to discover that the reflection function of the $L(0,2)$ mode is a linear function of the circumferential extent of the notch, rather than a curve of increasing sensitivity such as was shown for the relationship with depth (Fig. 7), and this prompted further investigation of the nature of the reflection functions. We present a summary description of the findings here; full details are given in Refs. [14,16].

Intuitively, there are two reasons why a non-linear relationship may be expected. First, an analysis of the crack opening displacement of a notched bar which is subjected to remote tension shows that the opening displacement rises sharply as the notch depth is increased. This can be understood physically by observing that, although only tension is applied remotely, the bar bends in the vicinity of the notch, as illustrated in Fig. 9. This provides an additional opening displacement to that which would be present simply because of axial displacements, and this effect increases with the notch depth. Clearly this analysis is also appropriate to a pipe with a through-thickness notch. Indeed, a fracture mechanics analysis of a notched pipe which is subjected to remote tension shows that the opening displacement at the centre of the notch increases with the square of the circumferential extent [22]. Second, even in the case of a flat plate in tension with a centre crack oriented normal to the tensile stress, the opening displacement at the centre of the crack increases in a non-linear manner as the crack length is increased [23]. Therefore, even without the bar bending effect we may expect a non-linear reflection function. In fact, neither of these analyses is valid, and the nature of the reflection of the $L(0,2)$ mode from the notch is found to be very much simpler.

Taking the bar bending effect, the difficulty with the

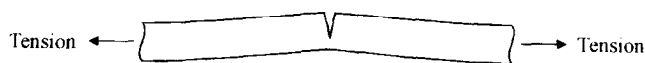


Fig. 9. Opening of notch due to pipe bending.

static analysis is that it assumes that the pipe moves to a static equilibrium position under the remotely applied tensile load. The bending effect which enhances the crack opening displacement can only occur if there is lateral movement (movement of the whole pipe in the direction normal to its axis) of the pipe near the location of the notch. Significant lateral movement could not occur when the $L(0,2)$ mode is incident because of the short duration and wavelength of the passing wave cycle. This was demonstrated by performing a finite element study of a flat plate, modelled using membrane elements, with a centrally positioned through-thickness notch. The S_0 Lamb mode was excited such that it was normally incident at the notch. The reflection of S_0 from such notches is closely representative of the behaviour which is studied in the pipe, except that there is no possibility of bending, yet the reflection coefficient showed the same linear relationship as that which was found from the pipe models. It is therefore concluded that the extent of any notch opening associated with bending of the pipe is insignificant.

The difficulty with the second argument for a non-linear function is that the wavelength is not long enough with respect to the crack length for the static crack opening displacements to be achieved. The static crack opening profile is valid only for a remote stress which remains constant as the crack opens. In the case of the 76 mm diameter pipe, the 70 kHz wave has a wavelength of about 80 mm, equal to one diameter of the pipe. The axial stress varies over the wavelength and only has the same sign in a 40 mm length, equal to the pipe radius. Clearly therefore the wavelength is rather short with respect to all but the shortest circumferential notches, and the stress state cannot be approximated as a remote stress. Thus, for a notch in either a flat plate or a pipe, the opening profile due to the incident wave does not match the remote stress static profile. This effect is illustrated in Fig. 10 where two crack opening profiles are shown for a through-thickness notch extending over 50% of the circumference of a pipe. One is from a remote-stress static analysis [11,22], and the other shows

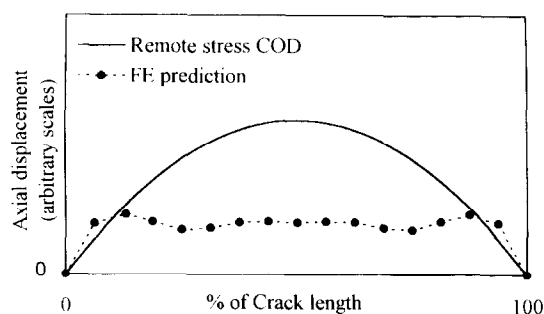


Fig. 10. Crack opening displacement shape for a through-thickness notch extending over 50% of the circumference of a 76 mm (nominal 3 in.) diameter pipe. Comparison of shape in finite element prediction with shape given by remote-stress fracture mechanics analysis.

the recorded displacements at the notch from the finite element analysis as the wave is reflected.

The notch opening displacements from the finite element analysis appear as a first approximation to be constant over the whole length of the notch. This suggests that the wavelength may be sufficiently short compared to the notch length that the wave simply reflects from the notch surface as it would from a free surface. In this case the problem is amenable to a very simple analysis to approximate the reflection functions. Following the analysis of Ditre [11], based on the S-parameter technique developed by Auld [24], the strength of conversion to each mode by a circumferential crack may be determined from the degree to which the crack opening profile (COD) matches the stress mode shape of the mode. Since the through-thickness axial stress mode shapes of the $L(0,2)$ and $F(1,3)$ modes are approximately constant, a spatial Fourier decomposition of the axial displacements around the circumference at the location of the notch gives the excitation strengths of the reflected and mode-converted waves [16]. Assuming that the axial displacements are constant over the length of the notch and zero elsewhere, this simple calculation was performed for a range of notch lengths. The results are shown in Fig. 11 together with the finite element reflection functions, showing good agreement. It is therefore concluded that a reasonable description of the nature of the reflection is that total reflection of the signal occurs at circumferential locations where the notch is present and perfect transmission elsewhere.

Finally, this analysis of the simple nature of the reflection implies a relationship for the reflection of the $F(1,3)$ mode from a part-through notch, which so far has not been studied. The reflection function must be zero when the depth is zero and equal to the appropriate value for the $F(1,3)$ mode in Fig. 8 when the notch is through-thickness. For intermediate depths, the signal

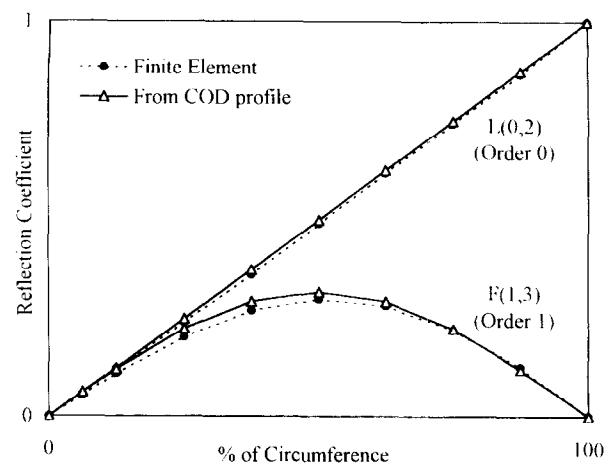


Fig. 11. Prediction of reflection of the $L(0,2)$ and $F(1,3)$ modes from a through-thickness notch, using simple assumption of constant COD profile, with finite element curves for comparison.

reflection from the notch which determines the amplitude of the reflected $L(0,2)$ mode should similarly govern the $F(1,3)$ mode. Therefore the $F(1,3)$ reflection function should follow the shape which was established for $L(0,2)$ in Fig. 7, the values being scaled to give the correct reflection when the notch is through-thickness.

6. Conclusions

The establishment of the reflection functions is clearly of great value for determining the sensitivity of the measurements to defects. The results provide a basis for predicting the strengths of $L(0,2)$ and $F(1,3)$ reflections from defects of any combination of circumferential and radial extent, and may be applied to pipes of other dimensions provided that the same mode is employed and the defects are small compared to the wavelength. Furthermore, the reflections of the $F(1,3)$ mode from through-thickness notches are comparable to those of $L(0,2)$ for short notches. It is for short notches that sensitivity has greatest practical value, and it appears therefore that the $F(1,3)$ mode is a useful alternative to $L(0,2)$ when attempting to detect defects in the vicinity of axially symmetric features such as butt-welds.

Analysis of the nature of the reflection functions for through-thickness notches shows that the behaviour is described very simply and accurately by perfect reflection of the signal at circumferential locations where the notch is present and perfect transmission elsewhere. Thus, assuming that the notch opening displacement is constant for all locations along the length of the notch, it is simple to make accurate predictions of the reflection functions of the modes.

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