(1) $A(x) = (3x_1 + 2x_3, 0, -x_2)$, kur $x = (x_1, x_2, x_3)$

BRUNNEN

Lai pārveidojums būtu lineārais operātors jāizpildās:

$\forall x,y \in L (A(x+y) = A(x) + A(y))$
$\forall x \in L, \forall \alpha \in P\ (A(x\alpha) = \alpha A(x))$

$A(x+y) = A(x_1+y_1, x_2+y_2, x_3+y_3) = (3(x_1+y_1) + 2(x_3+y_3), 0, -(x_2+y_2)) =$

$= (3x_1 + 3y_1 + 2x_3 + 2y_3, 0, -x_2 - y_2) = (3x_1 + 2x_3, 0, -x_2) + (3y_1 + 2y_3, 0, -y_2) =$

$= A(x) + A(y)$

$A(\alpha x) = A(\alpha x_1, \alpha x_2, \alpha x_3) = (\alpha(3x_1 + 2x_3), 0, \alpha \cdot (-x_2)) = (3x_1\alpha + 2x_3\alpha, 0, -x_2\alpha) =$

$= \alpha(3x_1 + 2x_3, 0, -x_2) = \alpha A(x)$

Tātad ir lineārs operātors.

Bāze $s_1 = (1,0,0)$ $\quad As_1 = (3,0,0)$
$s_2 = (0,1,0)$ $\quad As_2 = (0,0,-1)$
$s_3 = (0,0,1)$ $\quad As_3 = (2,0,0)$

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\text{Bāze} = \left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \right\}$$, tātad tāds pats $\mathrm{Im}(A)$

$$\begin{bmatrix} 3 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} /3 \rightarrow \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \xrightarrow{R2-R3} \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \xrightarrow{R3+R2} \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$$

x_3 pieņemsim X
$x_2 = 0$
$x_1 = -\frac{2X}{3}$

Ievietojam 1.

$$\mathrm{Ker}(A) = \left\{ \begin{bmatrix} -\frac{2}{3} \\ 0 \\ 1 \end{bmatrix} \right\}$$

(2) Bāze: $e_1 = 1, e_2 = x, e_3 = x^2$ $\qquad A(P(x)) \overset{def}{=} P(x+3)$

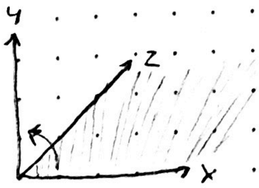
$A(e_1) = A(1) = 1+3 = 4 \Rightarrow A \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$ $\qquad A_e = \begin{bmatrix} 4 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A(e_2) = A(x) = x + 3 \Rightarrow A \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

$A(e_3) = A(x^2) = x^2 + 3 \Rightarrow A \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

A ir lineārs operātors, jo
$A: e \to P(e)$ un izpildās
$\forall x,y \in L, \forall \alpha, \beta \in F$
$(A(\alpha x + \beta y) = \alpha \cdot Ax + \beta \cdot Bx)$

3\. 1) Pagrieziens $\frac{(9-5)\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$



$$P = \begin{bmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} & 0 \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ pagrieziens

$$P_1(Oxz) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Projekcija uz Oxz

Operatora matrica $P \cdot P_1 = \begin{bmatrix} \cos(\frac{2\pi}{3}) & 0 & 0 \\ \sin(\frac{2\pi}{3}) & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Vektors $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \\ 2 \end{bmatrix}$ //

4\.

$c, d \in L_1$ $\quad A: L_1 \to L_2$ $\quad x_d \in L_1$

$e, f \in L_2$ $\quad B: L_2 \to L_3$

$g, h \in L_3$ $\quad y = B(A(x))$

Izteikt y_h ar A_{ec} un B_{gf}

P_{cd} P_{ef} P_{gh}

$x_c = P_{cd} \cdot x_d$

$x_e = A_{ec} \cdot x_c$

$x_f = P_{ef}^{-1} \cdot x_e = P_{fe} \cdot x_e$

$x_g = B_{gf} \cdot x_f$

$x_h = P_{gh}^{-1} \cdot x_g = P_{hg} \cdot x_g$

$y_h = P_{gh}^{-1}(B_{gf}(P_{ef}^{-1}(A_{ec}(P_{cd}(x_d)))))$

5\.

$A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & -3 \\ -1 & 2 & 1 \\ 2 & -3 & 2 \end{bmatrix}$ $\quad f = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 3 & 3 \end{bmatrix}$

$P_{fs} = \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R1-R2]{R3-2R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 2 & -1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \xrightarrow[R1-5R3]{R2+R3}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 9 & -5 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$

$$P_{fs}^{-1} = P_{sf} = \left(\begin{array}{ccc|ccc} -3 & 9 & -5 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -3 & 9 & -5 & 1 & 0 & 0 \end{array}\right) \xrightarrow{R3+3R1}$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 3 & -2 & 1 & 0 & 3 \end{array}\right) \xrightarrow[R3+3R2]{R1-2R2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -2 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 3 & 3 \end{array}\right) \xrightarrow[R2-R3]{R1+R3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 4 \\ 0 & -1 & 0 & -1 & -2 & -3 \\ 0 & 0 & 1 & 1 & 3 & 3 \end{array}\right) \xrightarrow{\cdot -1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 3 & 3 \end{array}\right)$$

$$P_{sf} = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 3 & 3 \end{bmatrix}$$

$$A' = P_{fs} \cdot A \cdot P_{sf} = \begin{bmatrix} -3 & 9 & -5 \\ 0 & -1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 & -3 \\ -1 & 2 & 1 \\ 2 & -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 35 & -22 \\ -1 & -4 & 0 \\ 1 & -7 & 9 \end{bmatrix}$$