

③ Найти проп.-ю гр.-и $U = x^2 + y^2 + z^2$ по напр. вектора $\vec{C}(-9, 8, -12)$ в точке $M(8, -12, 9)$

$$|\vec{C}| = \sqrt{x_0^2 + y_0^2 + z_0^2} = \sqrt{(-9)^2 + 8^2 + (-12)^2} = \sqrt{81 + 64 + 144} = \sqrt{289} = 17$$

$$\vec{C}_0 = \frac{\vec{C}}{|\vec{C}|} = \left(-\frac{9}{17}, \frac{8}{17}, -\frac{12}{17} \right)$$

$$U'_x = 2x$$

$$U'_y = 2y$$

$$U'_z = 2z$$

$$\text{grad } U \Big|_{(8, -12, 9)} = (16, -24, 18)$$

$$U'_\vec{C} \Big|_{(8, -12, 9)} = -\frac{9}{17} \cdot 16 + \frac{8}{17} \cdot (-24) - \frac{12}{17} \cdot 18 = -\frac{144}{17} - \frac{192}{17} - \frac{216}{17}$$

$$= \left(-\frac{552}{17} \right)$$

④ $U = e^{x^2 + y^2 + z^2}$ по напр. вектора $\vec{d}(4, -13, -16)$ в точке $M(8, -12, 9)$

$$|\vec{d}| = \sqrt{4^2 + (-13)^2 + (-16)^2} = \sqrt{441} = 21$$

$$\vec{d}_0 = \left(\frac{4}{21}, -\frac{13}{21}, -\frac{16}{21} \right)$$

$$U'_x = e^{x^2 + y^2 + z^2} \cdot 2x$$

$$U'_y = e^{x^2 + y^2 + z^2} \cdot 2y$$

$$U'_z = e^{x^2 + y^2 + z^2} \cdot 2z$$

$$\text{grad } U \Big|_{(-16, 4, -13)} = (e^{x^2 + y^2 + z^2} \cdot 2x, e^{x^2 + y^2 + z^2} \cdot 2y, e^{x^2 + y^2 + z^2} \cdot 2z) \Big|_{(-16, 4, -13)}$$

$$U'_\vec{d} \Big|_{(-16, 4, -13)} = \frac{4}{21} + \frac{-13}{21} + \frac{-16}{21} = -\frac{15}{21} = -\frac{5}{7}$$

$$U = x^2 + y^2 + z^2$$

12) в точке

$$(-12, 2) =$$

4) $U = e^{x^2 + y^2 + z^2}$ по комп. вект. $\vec{d} = (4, -13, -16)$
в точке $L(-16, 4, -13)$

$$|\vec{d}| = \sqrt{4^2 + (-13)^2 + (-16)^2} = \sqrt{16 + 169 + 256} = \sqrt{441} = 21$$

$$\vec{d}_0 = \left(\frac{4}{21}, -\frac{13}{21}, -\frac{16}{21} \right)$$

$$U'_x = e^{x^2 + y^2 + z^2} \cdot 2x = e^{(-16)^2 + 4^2 + (-13)^2} \cdot 2 \cdot (-16)$$

$$U'_y = e^{x^2 + y^2 + z^2} \cdot 2y = e^{(-16)^2 + 4^2 + (-13)^2} \cdot 2 \cdot 4$$

$$U'_z = e^{x^2 + y^2 + z^2} \cdot 2z = e^{(-16)^2 + 4^2 + (-13)^2} \cdot 2 \cdot (-13)$$

$$\text{grad } U \Big|_{(-16, 4, -13)} = \left(e^{441} \cdot (-32), e^{441} \cdot 8, e^{441} \cdot (-26) \right)$$

$$U'_{\vec{d}} \Big|_{(-16, 4, -13)} = \frac{4}{21} \cdot (-32e^{441}) + \left(-\frac{13}{21} \right) \cdot 8e^{441} + \left(-\frac{16}{21} \right) \cdot (-26e^{441})$$

$$(-24) -$$

$$\frac{192}{17} - \frac{216}{17} =$$