

Теорема о пределах.

$$\textcircled{1} \lim_{x \rightarrow 0} \frac{\sin(2x)}{4x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{2 \cdot 2x} = \frac{1}{2}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{x}{\sin(x)} = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{-1} = 1^{-1} = 1$$

$$\textcircled{3} \lim_{x \rightarrow 0} \frac{x}{\arcsin(x)} = 1^{-1} = 1$$

$$\begin{aligned} \textcircled{4} \lim_{x \rightarrow \infty} \left( \frac{4x+3}{4x-3} \right)^{6x} &= \lim_{x \rightarrow \infty} \left( \frac{4x-3+6}{4x-3} \right)^{6x} \\ &= \lim_{x \rightarrow \infty} \left( 1 + \frac{6}{4x-3} \right)^{\frac{4x-3}{6} \cdot \frac{6}{4x-3} \cdot 6x} = e^{\lim_{x \rightarrow \infty} \frac{36}{4x-3}} \\ &= e^9 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \lim_{x \rightarrow \infty} \frac{\sin x + \ln x}{x} &= \lim_{x \rightarrow \infty} (\sin x + \ln x) \cdot \frac{1}{x} \\ &= \lim_{x \rightarrow \infty} \frac{1}{x} \rightarrow 0 \quad \lim_{x \rightarrow \infty} (\sin x + \ln x) = -\infty \end{aligned}$$

$$\begin{aligned} \textcircled{6} \lim_{x \rightarrow 0} \frac{\sin(x) + \ln(x)}{x} &= \lim_{x \rightarrow 0} (\sin x + \ln x) \cdot \frac{1}{x} \\ &= \lim_{x \rightarrow 0} \frac{1}{x} = +\infty \quad \lim_{x \rightarrow 0} (\sin x + \ln x) = -\infty \end{aligned}$$