Введение в математический анализ

Интегралы. Дифференциальные уравнения.

Разбор ДЗ

$$\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2} \text{ по признаку д}'\text{Аламбера}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{\big((n+1)!\big)^2} : \frac{n^n}{(n!)^2} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)^2 \cdot n^n} =$$

$$= \lim_{n\to\infty} \frac{\left(\frac{n+1}{n}\right)^n}{n+1} = \lim_{n\to\infty} \frac{e}{n+1} = 0 < 1$$

Разбор ДЗ

$$\sum_{n=1}^{\infty} \frac{n}{2^n} \text{ по радикальному признаку Коши}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n+1}{2^{n+1}} : \frac{n}{2^n} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1$$

Разбор ДЗ

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \ln n} \text{ по признаку Лейбница}$$

$$\begin{cases} \lim\limits_{n\to\infty} |a_n| = 0 \\ |a_{n+1}| < |a_n| \\ \sum\limits_{n=1}^{\infty} \dfrac{1}{n + \ln n} \sim O\left(\dfrac{1}{n^1}\right) \end{cases} => \sum_{n=1}^{\infty} \frac{(-1)^n}{n + \ln n} - \text{сходится условно}$$

Разбор ДЗ

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n} \text{ по радикальному признаку Раабе}$$

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right) = \lim_{n\to\infty} n\left(\frac{3^n}{2^n} : \frac{3^{n+1}}{2^{n+1}} - 1\right) = \lim_{n\to\infty} n\left(-\frac{1}{3}\right) =$$

$$= -\infty < 1$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}}{2^{n+1}} : \frac{3^n}{2^n} = \frac{3}{2} > 1$$

Разбор ДЗ

Задание 1

$$\int(2x^2 - 2x - 1 + \sin x - \cos x + \ln x + e^x)dx =$$

$$= \frac{2x^3}{3} - x^2 - x - \cos x - \sin x + x\ln x - x + e^x + C$$

Разбор ДЗ

Задание 2

$$\int (2x + 6xz^2 - 5x^2y - 3\ln z)dx = x^2 + 3x^2z^2 - \frac{5x^3}{3}y - 3x\ln z + C$$

Разбор ДЗ

$$\int_0^{\pi} 3x^2 \sin 2x \, dx = \left(-\frac{3}{2}x^2 \cos 2x\right)\Bigg|_0^{\pi} + 3\int_0^{\pi} x \cos 2x \, dx =$$

$$U = 3x^2 \quad => \quad dU = 6xdx$$

$$dV = \sin 2x \, dx \quad => \quad V = -\frac{1}{2}\cos 2x$$

$$= -\frac{3}{2}\pi^2 + 3\int_0^{\pi} x \cos 2x \, dx = -\frac{3}{2}\pi^2 + \left(\frac{3}{2}x \sin 2x\right)\Bigg|_0^{\pi} - \frac{3}{2}\int_0^{\pi} \sin 2x \, dx = -\frac{3}{2}\pi^2$$

$$U = x \quad => \quad dU = dx$$

$$dV = \cos 2x \, dx \quad => \quad V = \frac{1}{2}\sin 2x$$

Разбор ДЗ

Задание 4

$$\int \frac{1}{\sqrt{x+1}}\,dx\Big|_{t=\sqrt{x+1}} = \int \frac{1}{t}\,d(t^2-1) = 2\int dt = 2t + C = 2\sqrt{x+1} + C$$

$$\int \frac{1}{\sqrt{x+1}}\,dx = \int \frac{1}{\sqrt{x+1}}\,d(x+1)\Big|_{t=x+1} = \int \frac{1}{\sqrt{t}}\,dt = \int t^{-\frac{1}{2}}\,dt = 2\sqrt{x+1} + C$$

Ряд Тейлора. Пример из ДЗ

Разложить $\ln(16x^2)$ в ряд Тейлора в окресности 1

$$\ln(16x^2) = \frac{\ln 16}{0!}(x-1)^0 + \frac{2}{1!}(x-1)^1 -$$

$$-\frac{2}{2!}(x-1)^2 + \frac{2\cdot 2}{3!}(x-1)^3 - \frac{2\cdot 3!}{4!}(x-1)^4 + \cdots =$$

$$= \ln 16 + 2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-1)^n}{n}$$

$$(\ln(16x^2))' = \frac{2}{x} \qquad (\ln(16x^2))'' = -\frac{2}{x^2}$$

$$(\ln(16x^2))''' = \frac{2\cdot 2}{x^3} \qquad (\ln(16x^2))^{IV} = -\frac{2\cdot 2\cdot 3}{x^4}$$

Разбор ДЗ

$$f(x) = x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\frac{\pi^3}{3} = \frac{2\pi^2}{3}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(xn)\,dx = \frac{4\cdot(-1)^n}{n^2}$$

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos(nx)$$

Разбор ДЗ

$$f(x) = x^2 \text{ на отрезке } [-\pi; \pi]$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{2\pi}\left(\frac{x^3}{3}\right)\Bigg|_{-\pi}^{\pi} =$$

$$= \frac{\pi^3}{6\pi} - \frac{(-\pi)^3}{6\pi} = \frac{2\pi^3}{6\pi} = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos nx\,dx = \left(\frac{1}{\pi n}x^2\sin nx\right)\Bigg|_{-\pi}^{\pi} - \frac{2}{\pi n}\int_{-\pi}^{\pi} x\sin nx\,dx =$$

$$U = x^2 \quad => \quad dU = 2xdx$$

$$dV = \cos nx\,dx \quad => \quad V = \frac{1}{n}\sin nx$$

$$= -\frac{2}{\pi n}\int_{-\pi}^{\pi} x\sin nx\,dx = \left(\frac{2}{\pi n^2}x\cos nx\right)\Bigg|_{-\pi}^{\pi} - \frac{2}{\pi n^2}\int_{-\pi}^{\pi}\cos nx\,dx =$$

$$U = x \quad => \quad dU = dx$$

$$dV = \sin nx\,dx \quad => \quad V = -\frac{1}{n}\cos nx$$

$$= \frac{2}{\pi n^2}\pi\cos n\pi - \frac{2}{\pi n^2}(-\pi)\cos n\pi = 4\frac{(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx\,dx = \left(-\frac{1}{\pi n}x^2\cos nx\right)\Big|_{-\pi}^{\pi} + \frac{2}{\pi n}\int_{-\pi}^{\pi} x\cos nx\,dx =$$

$$U = x^2 \quad => \quad dU = 2xdx$$

$$dV = \sin nx\,dx \quad => \quad V = -\frac{1}{n}\cos nx$$

$$= \frac{2}{\pi n}\int_{-\pi}^{\pi} x\cos nx\,dx = \left(\frac{2}{\pi n^2}x\sin nx\right)\Big|_{-\pi}^{\pi} - \frac{2}{\pi n^2}\int_{-\pi}^{\pi}\sin nx\,dx =$$

$$U = x \quad => \quad dU = dx$$

$$dV = \cos nx\,dx \quad => \quad V = \frac{1}{n}\sin nx$$

$$= \left(\frac{2}{\pi n^3}\cos nx\right)\Big|_{-\pi}^{\pi} = \frac{2}{\pi n^3}\cos n\pi - \frac{2}{\pi n^3}\cos n\pi = 0$$