② Найти производные 1-го порядка фун-и:

$$z = \left(1 + \frac{\ln x}{\ln y}\right)^3 = 1 + 3\frac{\ln x}{\ln y} + 3\left(\frac{\ln x}{\ln y}\right)^2 + \left(\frac{\ln x}{\ln y}\right)^3$$

$$dx = 0 + \frac{(3\ln x)'}{\ln y} + \frac{(3\ln^2 x)'}{\ln y} + \frac{(\ln^3 x)'}{\ln y} =$$

$$= \frac{3}{x\ln y} + \frac{6\ln x}{x\ln y} + \frac{3\ln^2 x}{x\ln y} = \frac{3(\ln^2 x + 2\ln x + 1)}{x\ln y} = \boxed{\frac{3(\ln x + 1)^2}{x\ln y}}$$

$$3\ln x = 0\cdot\ln x + 3\cdot\frac{1}{x} = \frac{3}{x}$$

$$3\ln^2 x = 0\cdot\ln^2 x + 3\cdot\frac{2\ln x}{x} = \frac{6\ln x}{x}$$

$$(\ln x\cdot\ln x)' = \frac{1}{x}\cdot\ln x + \ln x\cdot\frac{1}{x} = \frac{\ln x}{x} + \frac{\ln x}{x} = \frac{2\ln x}{x}$$

$$\ln x\cdot\ln^2 x = \frac{1}{x}\cdot\ln^2 x + \ln x\cdot\frac{2\ln x}{x} = \frac{\ln^2 x}{x} + \frac{2\ln^2 x}{x} = \frac{3\ln^2 x}{x}$$

$$dy = 0 + \frac{3\ln x}{(\ln y)'} + \frac{3\ln^2 x}{(\ln^2 y)'} + \frac{\ln^3 x}{(\ln^3 y)'} =$$

$$= -\frac{3\ln x \cdot y}{\ln^2 y} - \frac{3\ln^2 x \cdot y}{2\ln^5 y} - \frac{\ln^3 x \cdot y}{3\ln^8 y}$$

$$\left(\frac{1}{\ln y}\right)' = \frac{0 \cdot \ln y - 1 \cdot \frac{1}{y}}{\ln^2 y} = \frac{-\frac{1}{y}}{\ln^2 y} = -\frac{\ln^2 y}{y}$$

$$\left(\frac{1}{\ln^2 y}\right)' = \frac{-1 \cdot \frac{2\ln y}{y}}{\ln^4 y} = -\frac{2\ln^5 y}{y}$$

$$\left(\frac{1}{\ln^3 y}\right)' = \frac{-1 \cdot \frac{3\ln^2 y}{y}}{\ln^6 y} = -\frac{3\ln^8 y}{y}$$

1 Найти ООФ:

$$z = \sqrt{1-x^3} + \ln(y^2-1)$$

$(1-x^3) \geq 0 \quad -x^3 \geq -1 \quad x^3 \leq 1 \quad x \leq 1$

$(y^2-1) > 0 \quad y^2 > 1 \quad y > \pm 1$

$x \in (-\infty; 1]$

$y \in (-1; 1)$

③ Найти полный дифф-л ф-и в т...

$$z = \sqrt{2xy + \cos\frac{x}{y}} =$$

$$\boxed{(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'} \qquad \boxed{(\cos u)' = -\sin u}$$

$(\cos\frac{x}{y})'_x = -\sin\frac{x}{y} \cdot x' \cdot \frac{1}{y}$

$(\cos\frac{x}{y})'_y = -\sin\frac{x}{y} \cdot (\frac{x}{y})' = -\sin\frac{x}{y} \cdot \frac{x}{y^2}$

$$\frac{dz}{dx} = \frac{1}{2\sqrt{2xy + \cos\frac{x}{y}}} \cdot \left(xy + \left(-\sin\frac{x}{y}\right.\right.$$

$$\frac{dz}{dy} = \frac{1}{2\sqrt{2xy + \cos\frac{x}{y}}} \cdot \left(2x + \sin\frac{x}{y} \cdot \frac{1}{y^2}\right.$$

$$d\cancel{y}\Big|_{1,1} = \frac{1}{2\sqrt{2 + \cos 1}} \cdot (2 + \sin 1)$$

$(y^{-1})' = -1y^{-2} = -\frac{1}{y^2}$

$$d\cancel{x}\Big|_{1,1} = \frac{(1 - \sin 1)}{2\sqrt{2 + \cos 1}}$$

$U''_{xx} = -2$ $\qquad \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix}$ $\qquad \Delta_1 = -2 <$

$U''_{yy} = -2$ $\qquad \Delta_2 = 4 - 4$

4. Иссл-ть на экстремум ф-ю

$z = x^2 + xy + y^2 - 6x - 9y$ $\quad f(x)$

$dx = 2x + y - 6 = 0$

$dy = x + 2y - 9 = 0$

$2x + y = 6 \qquad x + 2y = 9$

$y = 6 - 2x \qquad x + 2(6 - 2x) = 9$

$x + 12 - 4x = 9$

$y = 4 \qquad 3 = 3x \quad x = 1$

$dx'' = 2$

$dy' = 2$

$$\begin{pmatrix} U''_{xx} & U''_{xy} \\ U''_{yx} & U'_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$$

$\Delta_1 = 2 > 0, \quad \Delta_2 = 4 - 4 = 0 \quad = 0$

$-f(x) \quad z = -x^2 - xy - y^2 + 6x + 9y$

$U'_x = -2x - y + 6 \qquad U'_y = -x - 2y + 9$

$-2x = -6 + y \quad x = \frac{6+y}{2} \qquad -\left(\frac{6+y}{2}\right) - 2y = -9$

$-6 - 9y$ ...