Optimization Methods **2024**

Student: Jeferson Morales Mariciano

Discussed with:

In-Class Exercise **Due date:** Wednesday, 8 May 2024

2.

Solve $\min_x f(x, y)$ by using both the implemented methods.

Matlab scripts are provided in */code* folder. The 3 main files to run are: main.m, cauchyPoint.m, dogleg.m. They handle both computations and visualization of the Rosenbrock's function with the corresponding trust region methods.

3.

Plot the obtained steps on the energy landscape and compare performance of the methods.

Hereby the example of the Rosenbrock function has the same parameters as assignment 2: $tol = 1e-6$, $maxIter = 50000$, $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. For Trust Region methods, the parameters chosen were: $\hat{\Delta} = 2$, $\Delta = 0.75$, $\eta = \frac{1}{4}$.

Clearly, the Dogleg method converges faster than the Cauchy Point method. The convergence plots in Figure 2 and 4 show that the Dogleg method converges in 16 iterations, while the Cauchy Point method converges in 13972 iterations. That is due to the fact that the Dogleg method is faster in terms of iterations because it uses second order information from the hessian. The Cauchy Point method is faster in terms of time per iteration because it is computationally inexpensive: no matrix factorization are required to compute the step p_k^C. An evaluation of whether an approximate solution for the trust region problem is acceptable is necessary before employing the Cauchy Point method.

4.

Compare performance of the trust region method based on Dogleg and on Cauchy point for three different x_0.

$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Cauchy point method for Figures 1, 2. Dogleg method for Figures 3, 4.

Figure 1: Cauchy Point method, $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

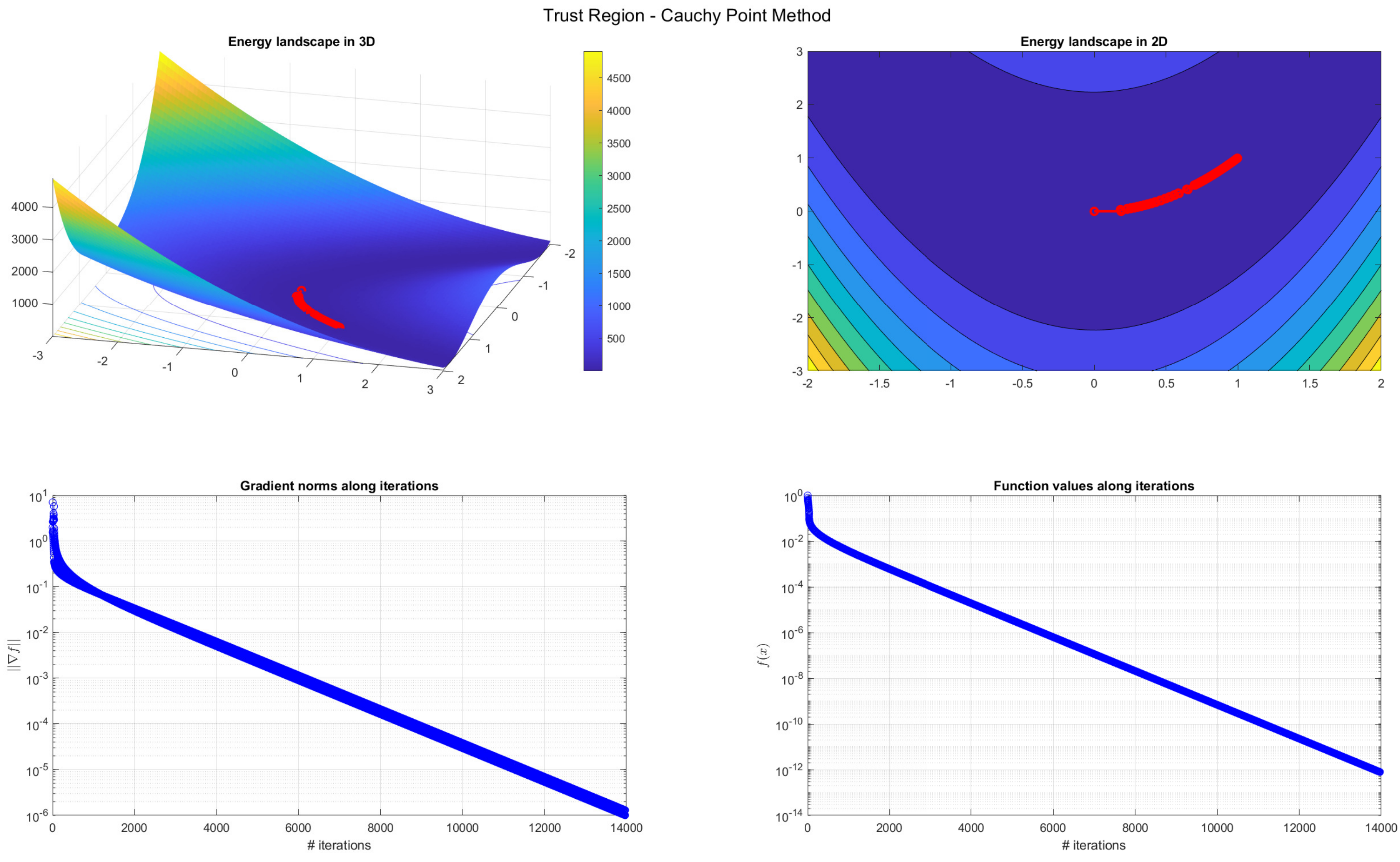


Figure 2: Cauchy Point method convergence, $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

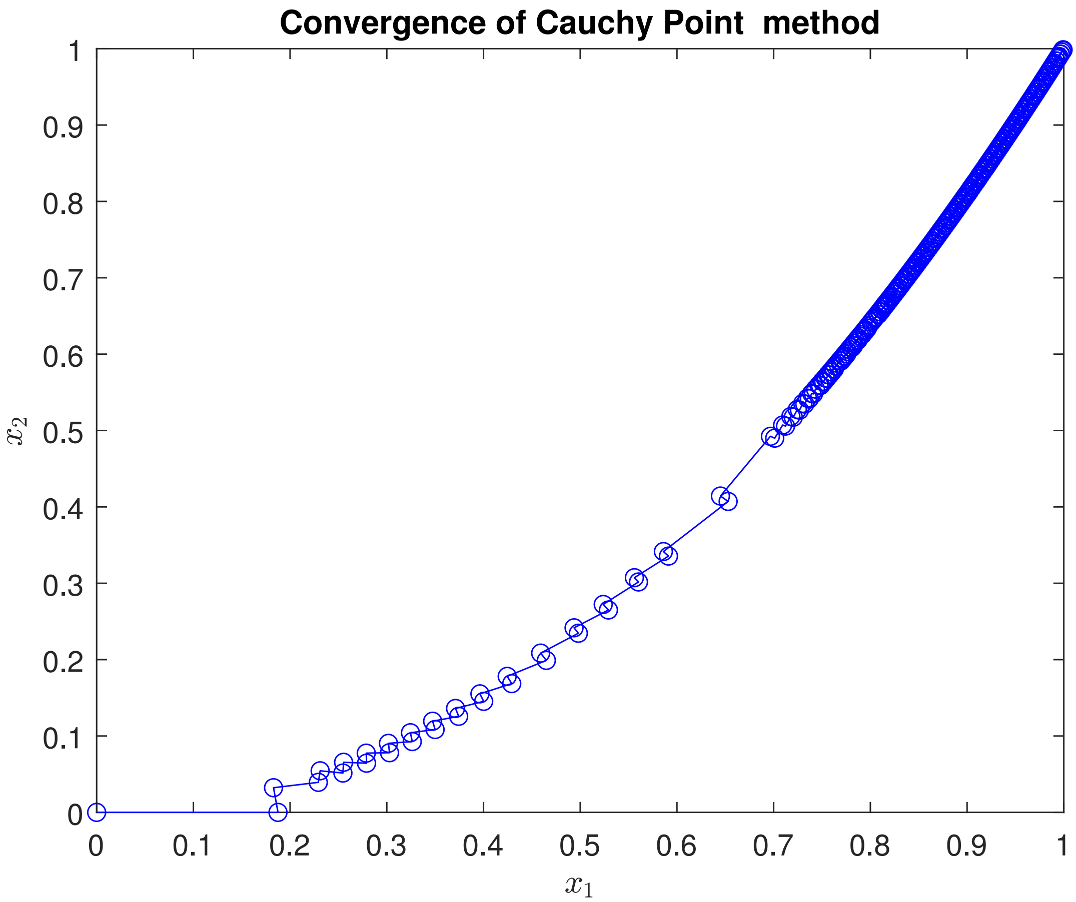


Figure 3: Dogleg method, $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

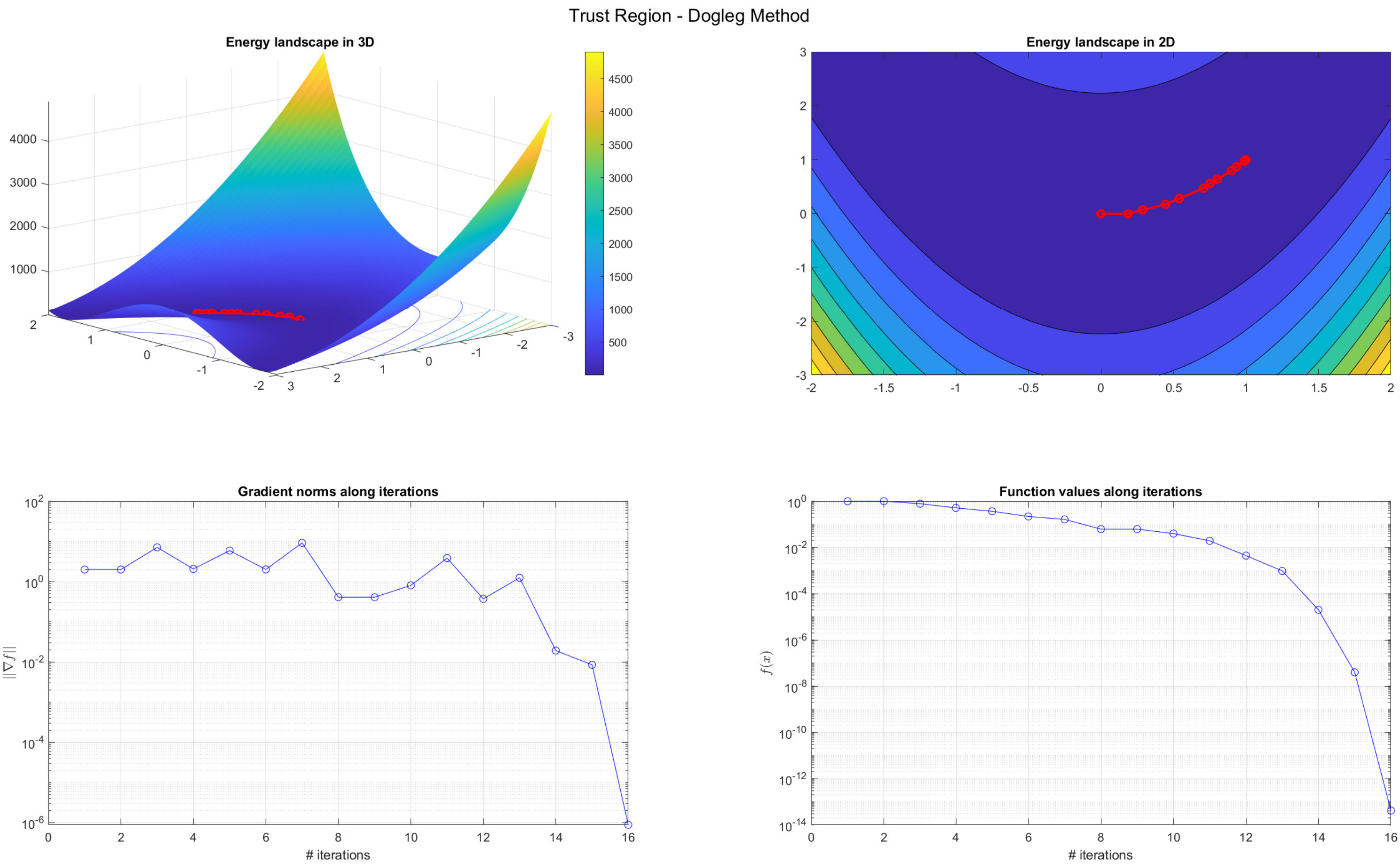
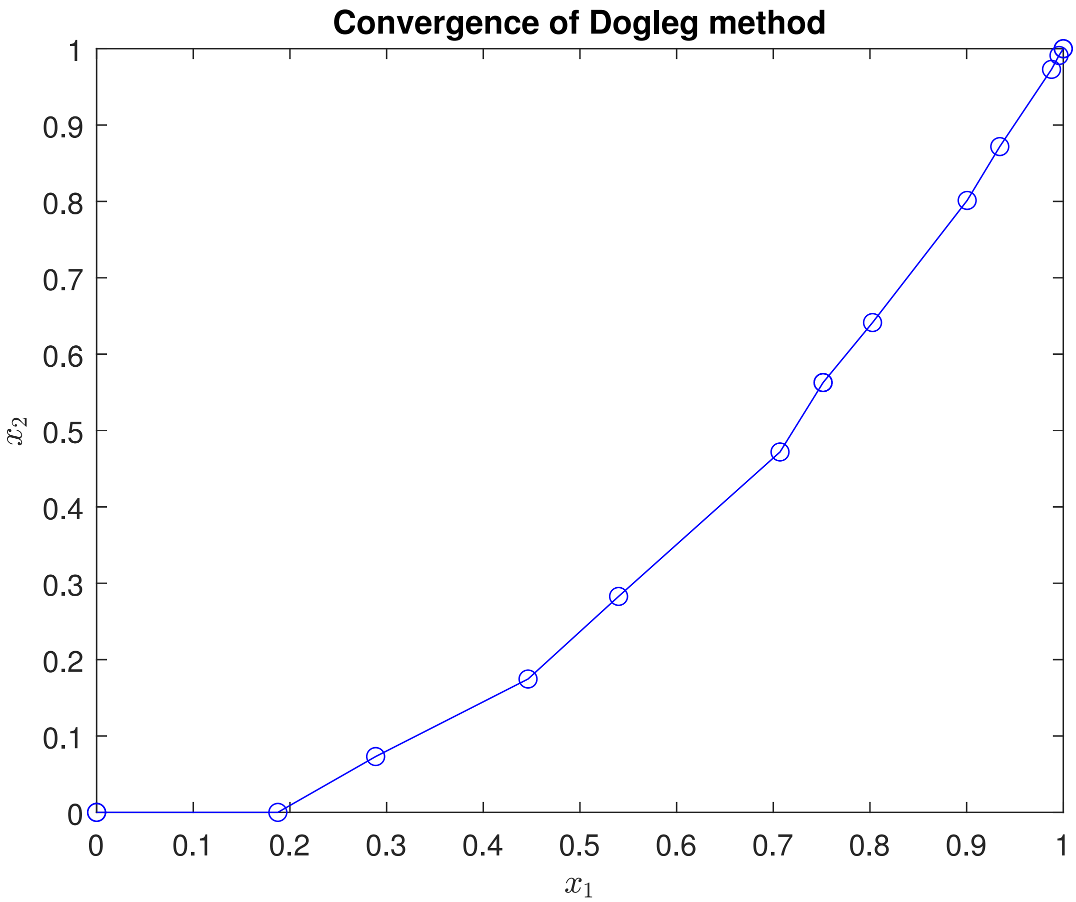


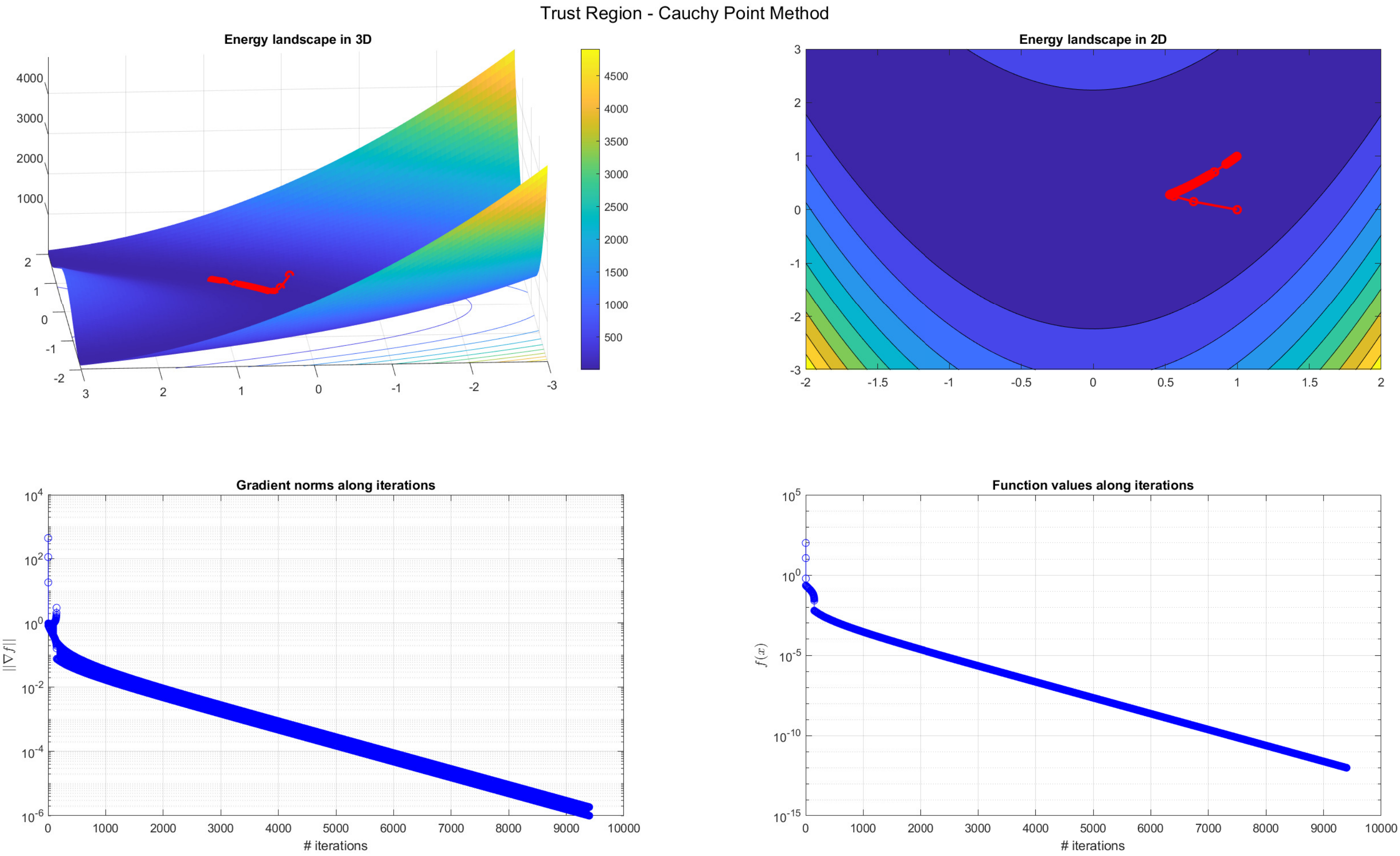
Figure 4: Dogleg method convergence, $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



$x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Cauchy point method for Figures 5, 6. Dogleg method for Figures 3, 4.

Figure 5: Cauchy Point method, $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



$x_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Cauchy point method for Figures 9, 10. Dogleg method for Figures 11, 12.

5.

Compare performance of the trust region based on Dogleg method for three different δ_0.

Argumented in previous answer 3.

6. (Bonus)

Report the convergence history i.e., for each iteration, report the values of objective function, trust-region-radius, and the ratio of the actual reduction to the predicted reduction. (Please make a table.)

Figure 6: Cauchy Point method convergence, $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

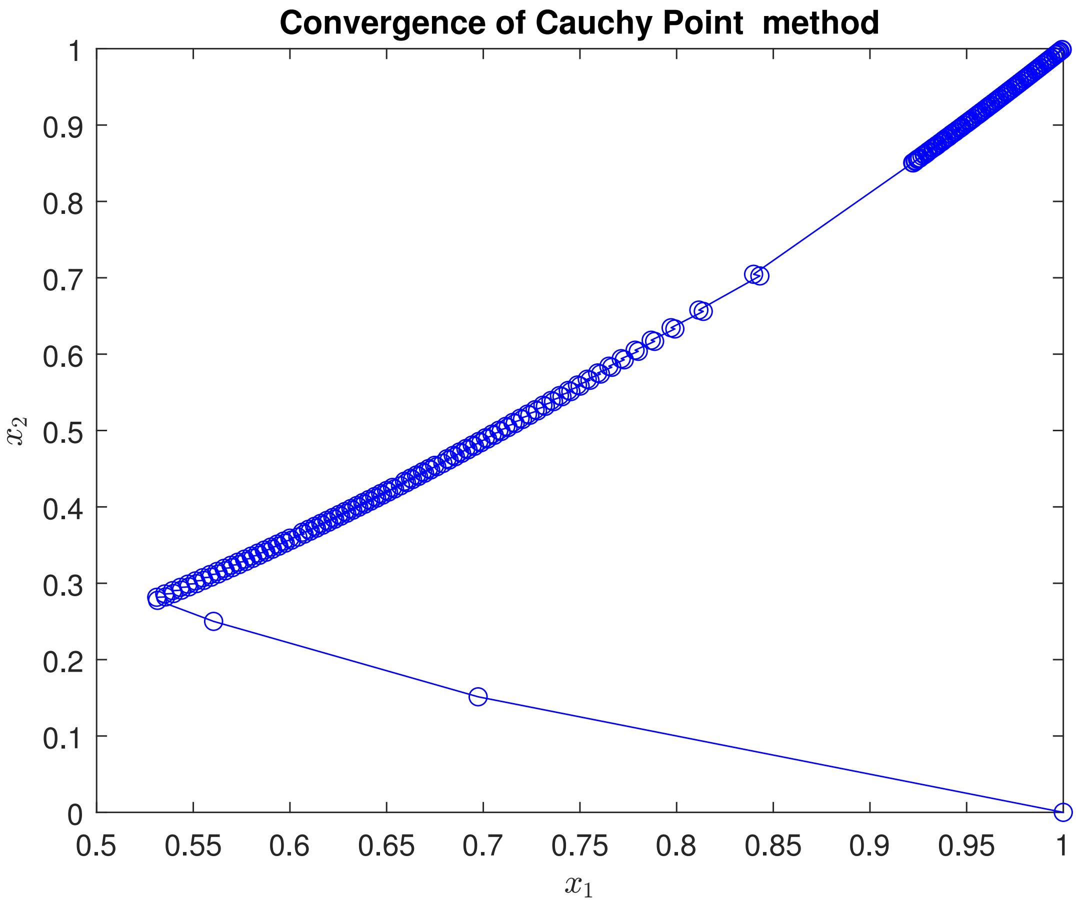


Figure 7: Dogleg method, $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

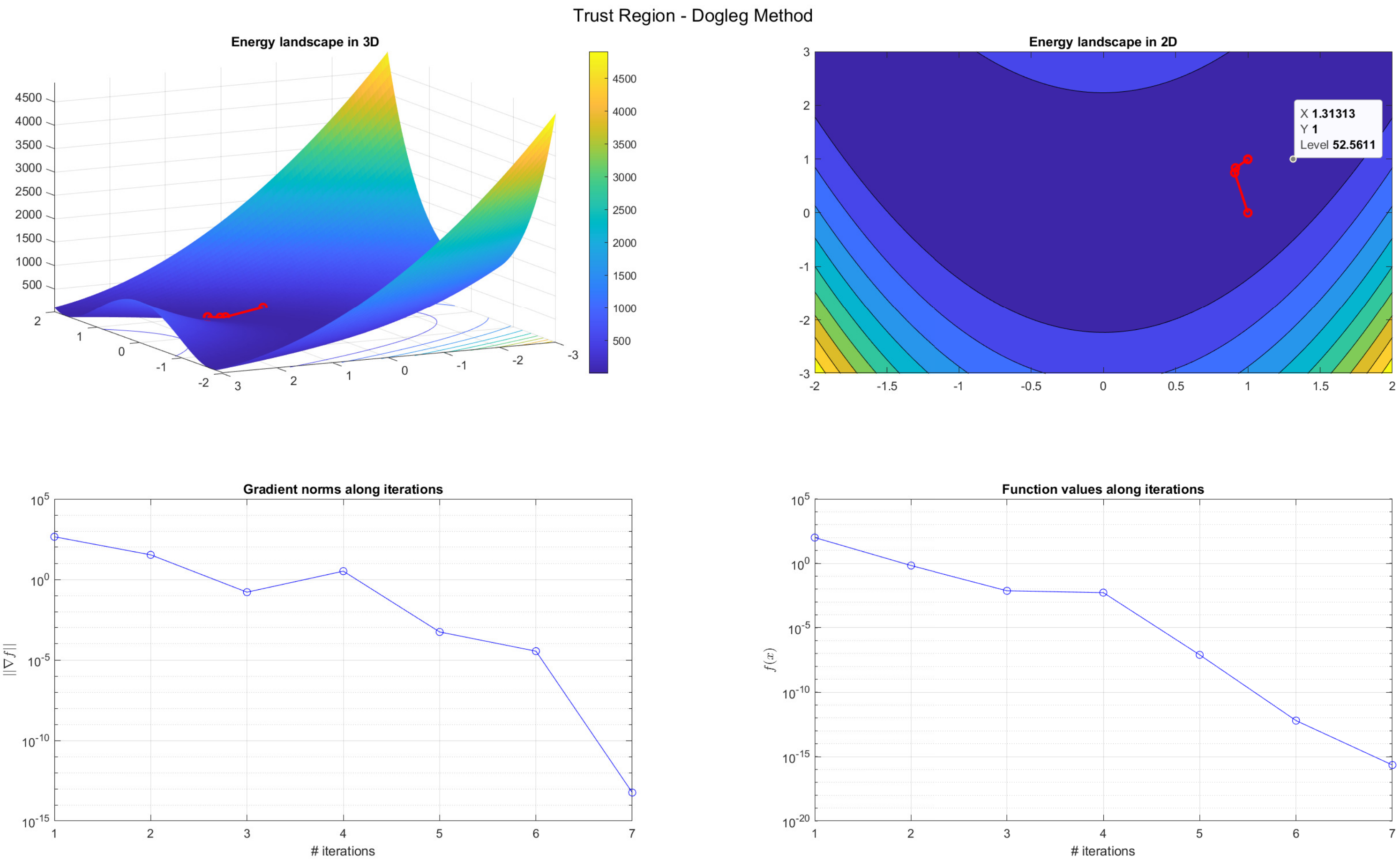


Figure 8: Dogleg method convergence, $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

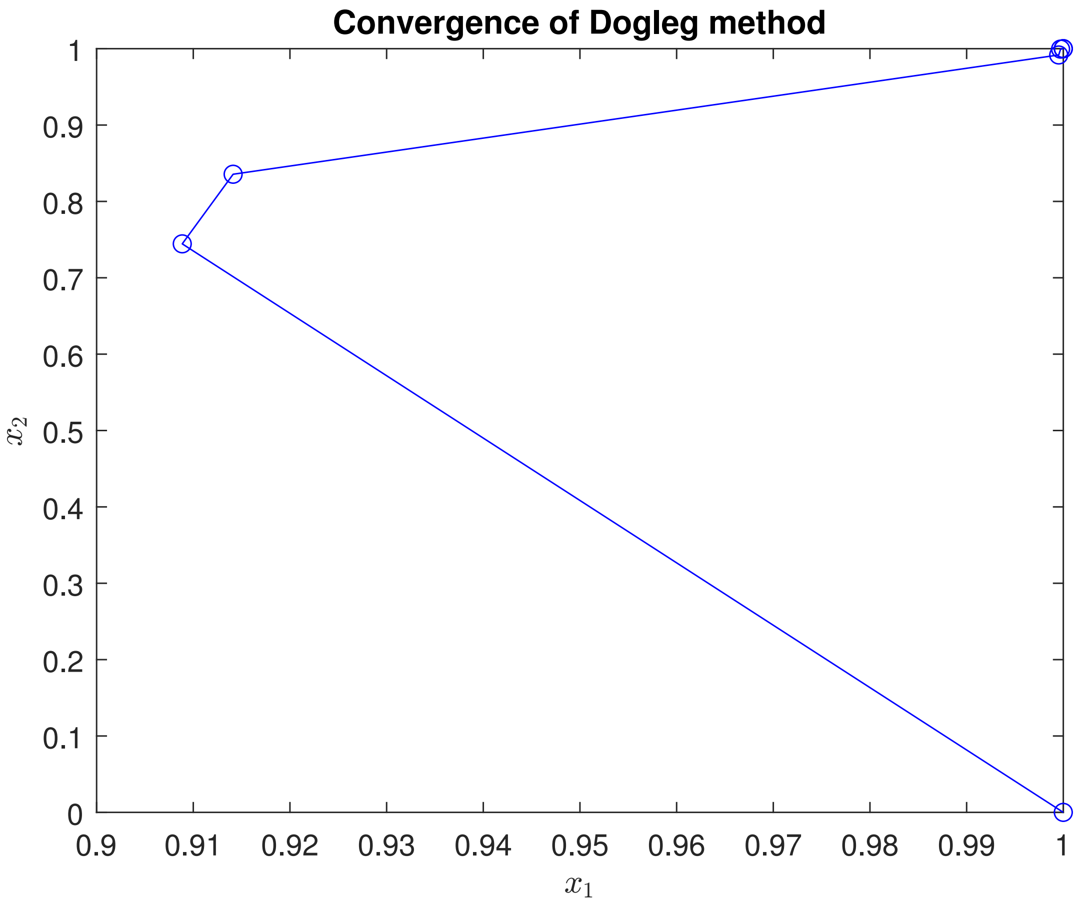


Figure 9: Cauchy Point method, $x_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

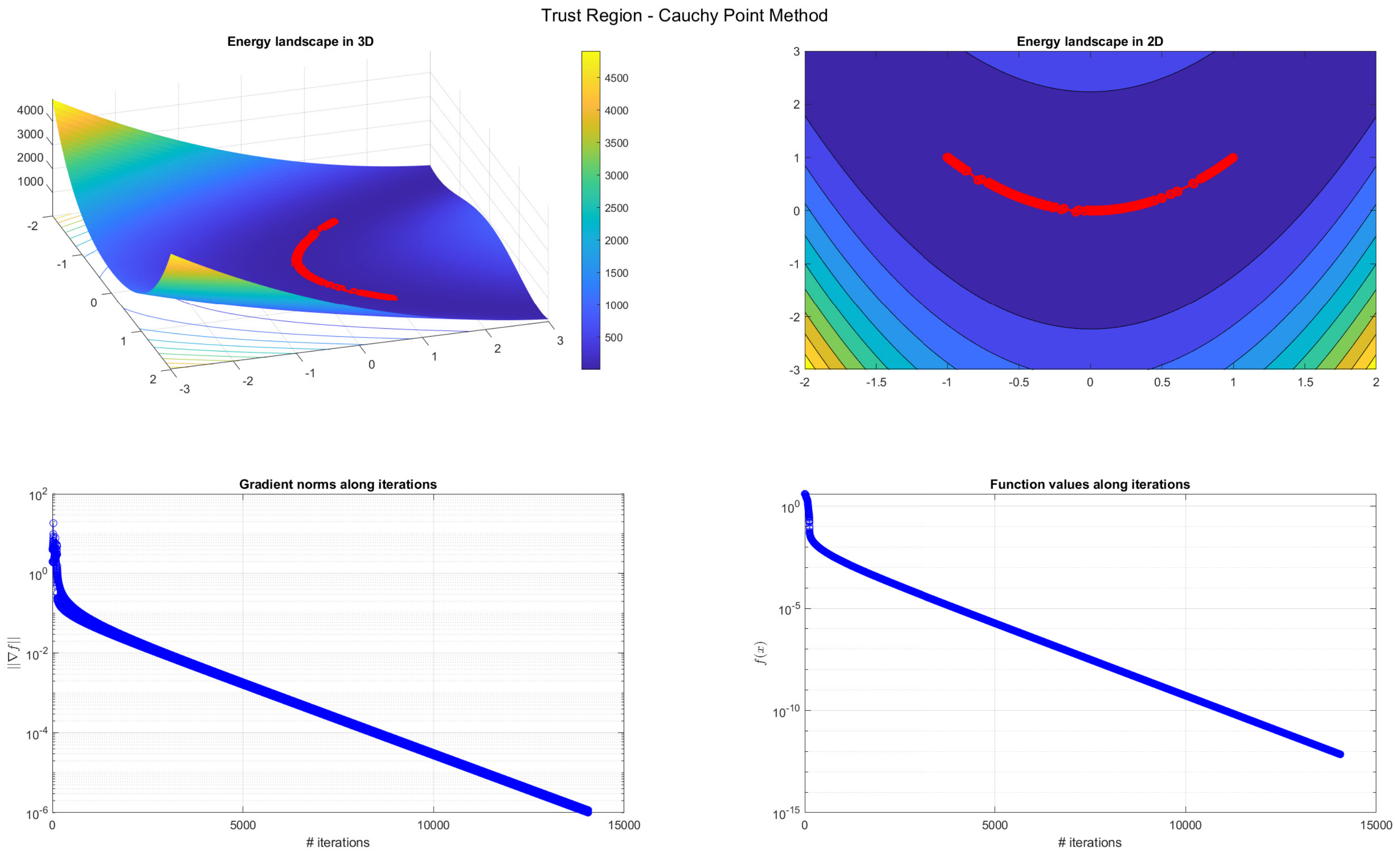


Figure 10: Cauchy Point method convergence, $x_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

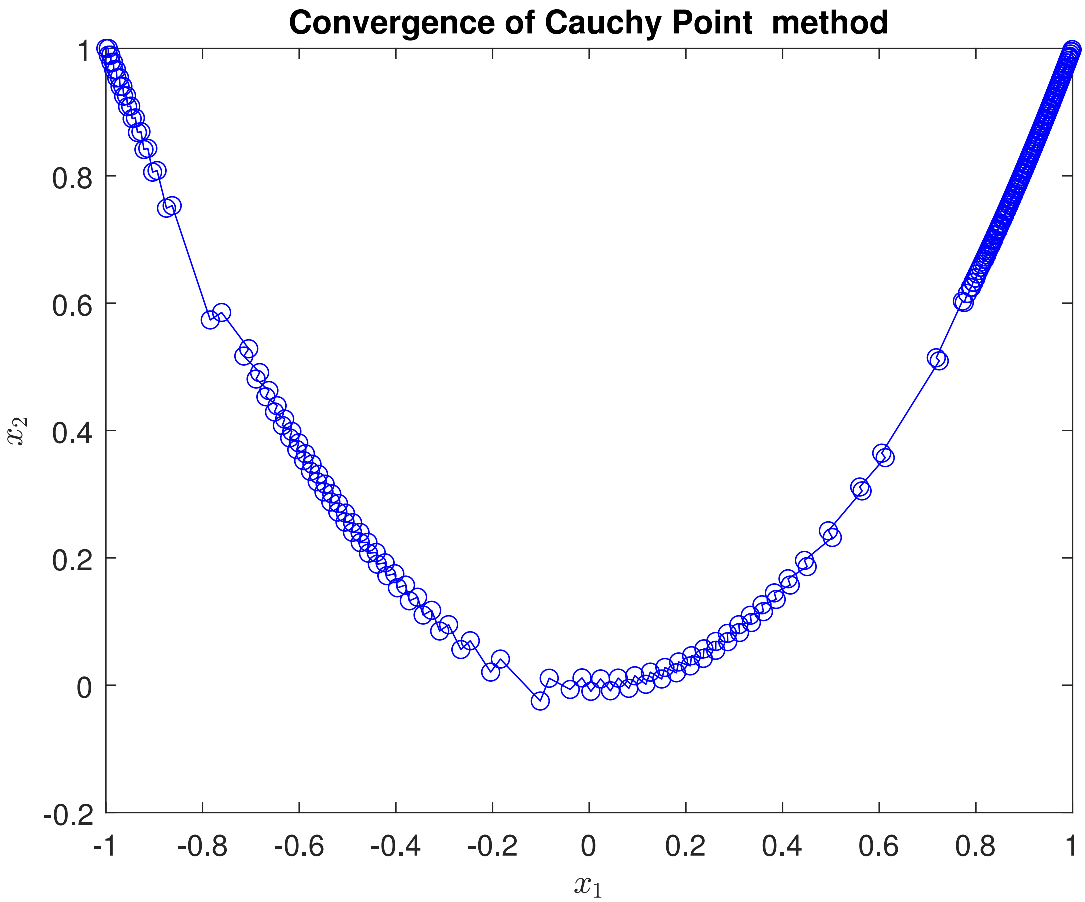


Figure 11: Dogleg method

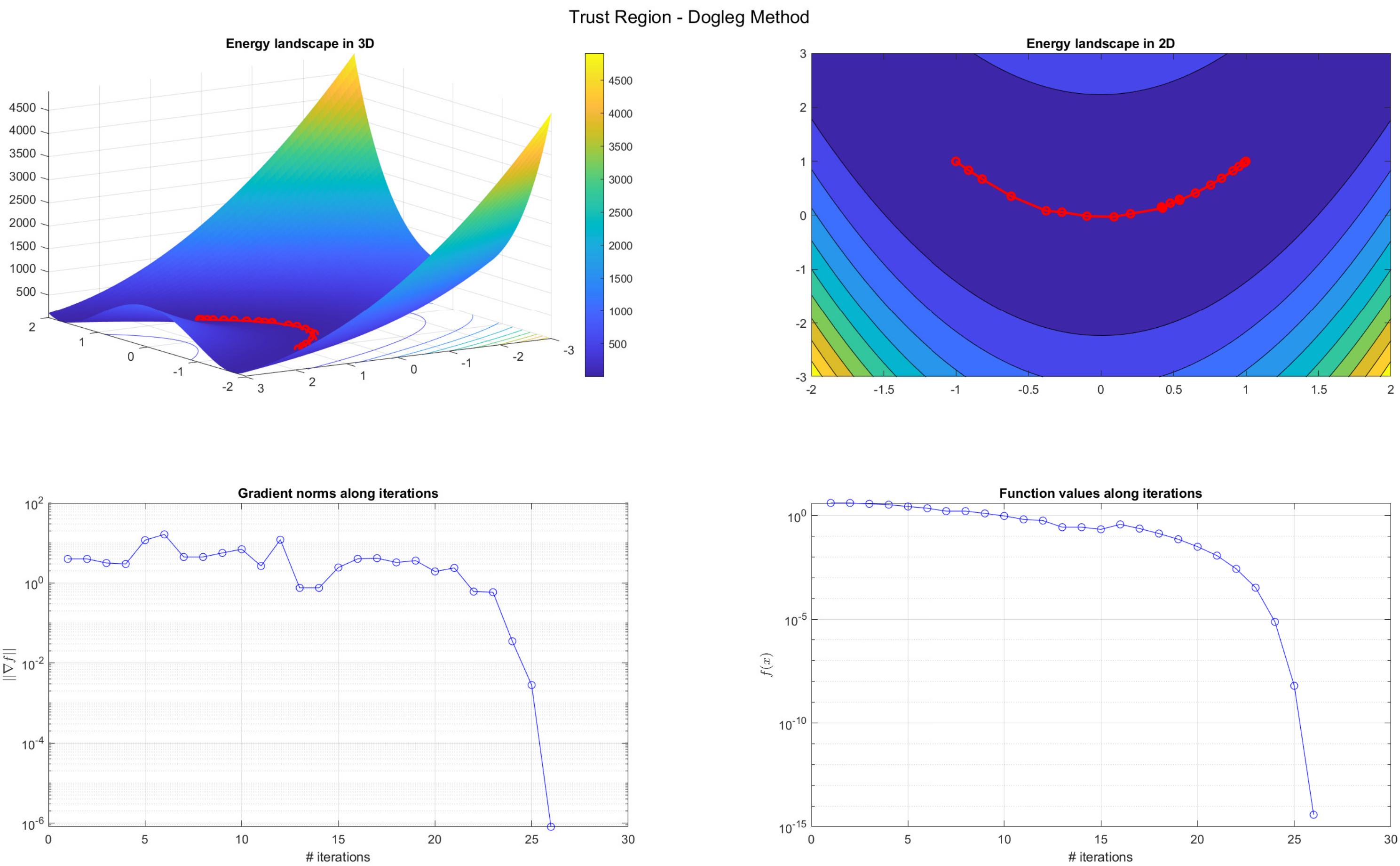
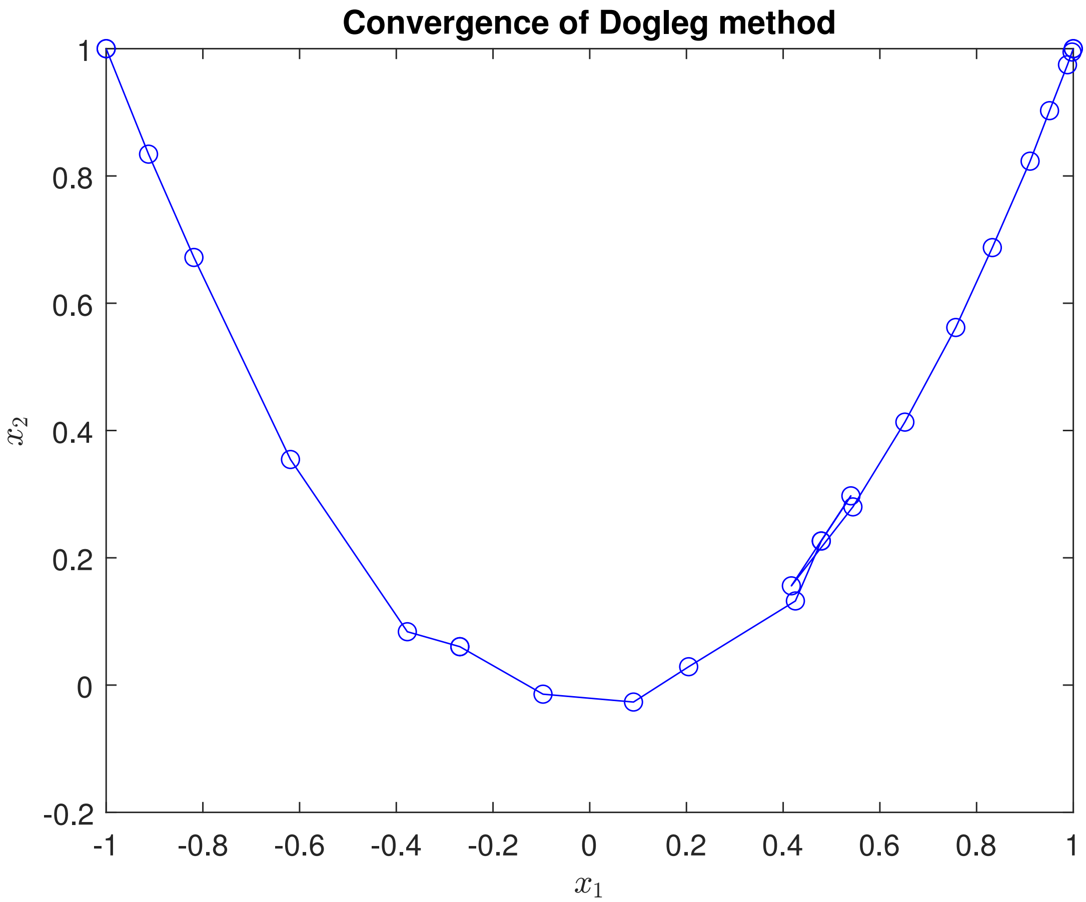


Figure 12: Dogleg method convergence, $x_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$



Convergence history with $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ of objective function values comparing Cauchy Point and Dogleg methods:

N	**Dogleg**	**Cauchy**
1	1.0000	1
2	1.0000	1
3	0.7838	0.783752441406250
4	0.5165	0.667859425835307
5	0.3669	0.667859425835307
6	0.2193	0.611562564038043
7	0.1636	0.591324573691497
8	0.0621	0.573386256785240
9	0.0621	0.554558208423006
10	0.0398	0.537912629398300
11	0.0194	0.520195576039467
12	0.0044	0.504566856105441
13	0.0010	0.487682517002878
14	0.0000	0.472818613394980
15	0.0000	0.456491435839246
16	0.0000	0.442147412520641
...	/	...
13972	/	7.8324e-13