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Assignment 3

Due date: Thursday, 10 October 2024, 23:59

Exercise 3.2, From Natural Language to a Formula (★) (4 Points)

Consider the universe $U = \mathbb{N} \setminus \{0\}$. Express each of the following statements with a formula in predicate logic, in which the only predicates appearing are *divides*(x, y), *equals*(x, y) and *prime*(x) (instead of *divides*(x, y) and *equals*(x, y) you can write $x|y$ and $x = y$ accordingly). You can also use the symbols $+$ and \cdot to denote the addition and multiplication functions, and you can use constants (e.g., 0, 1, . . .). You can also use \Leftrightarrow . No justification is required.

- (i) (★) If a number divides two numbers, then it also divides their sum.
- (ii) (★) The only divisors of a prime number are 1 and the number itself.
- (iii) (★) 1 is the only natural number which has an inverse.
- (iv) (★) A prime number divides the product of two natural numbers if and only if it divides at least one of them.

Exercise 3.8, Proof by Contradiction (★) (4 Points)

Let $n, m \in \mathbb{N}$ be arbitrary. We say “ n divides m ” and write $n|m$ if there exists a $k \in \mathbb{N}$ such that $k \cdot n = m$. Prove that the following statement is true, using a proof by contradiction:

$$n|m \text{ and } n|(m+1) \implies n=1.$$

You are allowed to invoke the statement 3.2 iii) from above to justify one step. You must use the same notation as in the lecture notes, i.e. precisely state what your statements S and T are, and justify each of your proof steps.