ETH zürich

**Discrete Mathematics** **2024**

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## Assignment 8 **Due date:** Thursday, 14 November 2024, 23:59

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## Exercise 8.5, Inner Direct Products $(\star)$ (8 Points)

**a)** Let $\langle G; *, ^\frown, e\rangle$ be a commutative group. Let $H$ and $K$ be subgroups of $G$ such that

(i) $G = \{h * k \mid h \in H, k \in K\}$,

(ii) $H \cap K = \{e\}$.

Prove that $G$ is isomorphic to the direct product $H \times K$. In this case, $G$ is called the *inner* direct product of $H$ and $K$.

**b)** Use the previous subtask to prove that $\langle \mathbb{Z}_{15}^*, \odot_{15}\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_4$. You can use the subtask even if you have not proved it. **Do not** prove the isomorphism directly.

**a)**

**b)**