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## Optimization Methods

2024

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**Discussed with:**

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## Assignment 4

**Due date:** Monday, 3 June 2024, 12:00 AM

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### 1. Exercise (20/100)

Consider the quadratic function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  defined as:

$$f(\mathbf{x}) = 7x^2 + 4xy + y^2 \quad (1)$$

where  $\mathbf{x} = (x, y)^T$ .

1. Write this function in canonical form, i.e.  $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c$ , where  $A$  is a symmetric matrix.
2. Describe briefly how the Conjugate Gradient (CG) Method works and discuss whether it is suitable to minimize  $f$  from equation 1. Explain your reasoning in detail (max. 30 lines).

### 2. Exercise (20/100)

Consider the following constrained minimization problem for  $\mathbf{x} = (x, y, z)^T$

$$\begin{aligned} \min_{\mathbf{x}} f(\mathbf{x}) &:= -3x^2 + y^2 + 2z^2 + 2(x + y + z) \\ \text{subject to } c(\mathbf{x}) &= x^2 + y^2 + z^2 - 1 = 0 \end{aligned} \quad (2)$$

Write down the Lagrangian function and derive the KKT conditions for 2

### 3. Exercise (60/100)

1. Read the chapter on Simplex method, in particular the section 13.3 The Simplex Method, in Numerical Optimization, Nocedal and Wright. Explain how the method works, with a particular attention to the search direction.
2. Consider the following constrained minimization problem,  $\mathbf{x} = (x_1, x_2)^T$ ;

$$\min_{\mathbf{x}} f(\mathbf{x}) := 4x_1 + 3x_2 \quad (3)$$

subject to:

$$\begin{aligned}
6 - 2x_1 - 3x_2 &\geq 0 \\
3 + 3x_1 - 2x_2 &\geq 0 \\
5 - 2x_2 &\geq 0 \\
4 - 2x_1 - x_2 &\geq 0 \\
x_2 &\geq 0 \\
x_1 &\geq 0
\end{aligned} \tag{4}$$

- a) Sketch the feasible region for this problem.
- b) Which are the basic feasible points of the problem 3? Compute them by hand using the geometrical interpretation and find the optimal point  $\mathbf{x}^*$  that minimizes  $f$  subject to the constraints.
- c) Prove that the first order necessary conditions holds for the optimal point.