ETH zürich

Discrete Mathematics **2024**

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Assignment 1 **Due date:** Thursday, 26 September 2024, 23:59

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1. Exercise 1.5, Two New Logical Operators

We define two binary logical operators $\heartsuit$ and $\diamondsuit$ as follows:

A	B	$A \heartsuit B$
0	0	1
0	1	0
1	0	1
1	1	1

A	B	$A \diamondsuit B$
0	0	1
0	1	0
1	0	0
1	1	1

1. a) ($\star$)

Are $\heartsuit$ and $\diamondsuit$ commutative, i.e., does it hold

$$A \heartsuit B \equiv B \heartsuit A \quad \text{and} \quad A \diamondsuit B \equiv B \diamondsuit A?$$

Argue by comparing function tables.

2. b) ($\star$)

Prove or disprove that

$$(\neg A \heartsuit B) \diamondsuit (B \diamondsuit C) \equiv \neg (A \diamondsuit B) \heartsuit \neg (A \diamondsuit C)$$

by computing and comparing the function tables of the left-hand-side and the right-hand-side formulas.

3. c) ($\star\star$)

Let F be a formula with the following function table:

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Find a formula G containing only the logical operators $\heartsuit$ and $\diamondsuit$, in which the propositional symbols A, B, and C all appear exactly once, and such that $G \equiv F$. No justification is required.