ETH zürich

Discrete Mathematics **2024**

Student: Jeferson Morales Mariciano <jmorale@ethz.ch>

Assignment 1 **Due date:** Thursday, 26 September 2024, 23:59

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1. Exercise 1.5, Two New Logical Operators

We define two binary logical operators $\heartsuit$ and $\diamondsuit$ as follows:

A	B	$A \heartsuit B$
0	0	1
0	1	0
1	0	1
1	1	1

A	B	$A \diamondsuit B$
0	0	1
0	1	0
1	0	0
1	1	1

1. a) ($\star$)

Are $\heartsuit$ and $\diamondsuit$ commutative, i.e., does it hold

$$A \heartsuit B \equiv B \heartsuit A \quad \text{and} \quad A \diamondsuit B \equiv B \diamondsuit A?$$

Argue by comparing function tables.

2. b) ($\star$)

Prove or disprove that

$$(\neg A \heartsuit B) \diamondsuit (B \diamondsuit C) \equiv \neg (A \diamondsuit B) \heartsuit \neg (A \diamondsuit C)$$

by computing and comparing the function tables of the left-hand-side and the right-hand-side formulas.

From the truth table of the 3 propositional symbols A, B, and C,

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

we compute the left-hand-side formula as follows,

$\neg A$	$\neg A \heartsuit B$	$B \heartsuit C$	$(\neg A \heartsuit B) \diamondsuit (B \heartsuit C)$
1	1	1	1
1	1	0	0
1	1	1	1
1	1	1	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	1	0

and the right-hand-side formula as follows,

$A \diamondsuit B$	$\neg(A \diamondsuit B)$	$A \diamondsuit C$	$\neg(A \diamondsuit C)$	$\neg(A \diamondsuit B) \heartsuit \neg(A \diamondsuit C)$
1	0	1	0	1
1	0	0	1	0
0	1	1	0	1
0	1	0	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1

by comparing the left-hand-side and right-hand-side function tables, we can see that they are not equivalent, i.e.

$(\neg A \heartsuit B) \diamondsuit (B \diamondsuit C)$	$\neg(A \diamondsuit B) \heartsuit \neg(A \diamondsuit C)$
1	1
0	0
1	1
1	1
1	1
0	1
0	0
0	1

$$(\neg A \heartsuit B) \diamondsuit (B \diamondsuit C) \not\equiv \neg(A \diamondsuit B) \heartsuit \neg(A \diamondsuit C)$$

3. c) (⋆⋆)

Let F be a formula with the following function table:

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Find a formula G containing only the logical operators $\heartsuit$ and $\diamondsuit$, in which the propositional symbols A, B, and C all appear exactly once, and such that $G \equiv F$. No justification is required.

Let G be defined as $G \coloneqq (C \diamondsuit A) \heartsuit B$. Then, the truth table of G is as follows,

$C \diamondsuit A$	$(C \diamondsuit A) \heartsuit B$		G	F
1	1		1	1
0	1		1	1
1	1		1	1
0	0	$\longrightarrow$	0	0
0	1		1	1
1	1		1	1
0	0		0	0
1	1		1	1

by comparing the function tables of F and G, we can see that $G \equiv F$. All constraints are satisfied: only logical operators $\heartsuit, \diamondsuit$ and propositional symbols A, B, C appear exactly once.