

Student: Jeferson Morales Mariciano <jmorale@ethz.ch>

Assignment 8

Due date: Thursday, 14 November 2024, 23:59

Exercise 8.5, Inner Direct Products (★)

(8 Points)

a) Let $\langle G; *, \hat{}, e \rangle$ be a commutative group. Let H and K be subgroups of G such that

(i) $G = \{h * k \mid h \in H, k \in K\},$

(ii) $H \cap K = \{e\}.$

Prove that G is isomorphic to the direct product $H \times K$. In this case, G is called the *inner* direct product of H and K .

b) Use the previous subtask to prove that $\langle \mathbb{Z}_{15}^*, \odot_{15} \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_4$. You can use the subtask even if you have not proved it. **Do not** prove the isomorphism directly.

a)

b)