Discrete Mathematics **2024**

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Assignment 3

Due date: Thursday, 10 October 2024, 23:59

Exercise 3.2, From Natural Language to a Formula ($\star$) (4 Points)

Consider the universe $U = \mathbb{N} \setminus \{0\}$. Express each of the following statements with a formula in predicate logic, in which the only predicates appearing are $divides(x, y)$, $equals(x, y)$ and $prime(x)$ (instead of divides(x, y) and equals(x, y) you can write $x|y$ and $x = y$ accordingly). You can also use the symbols $+$ and $\cdot$ to denote the addition and multiplication functions, and you can use constants (e.g., 0, 1, . . .). You can also use $\leftrightarrow$. No justification is required.

(i) ($\star$) If a number divides two numbers, then it also divides their sum.

(ii) ($\star$) The only divisors of a prime number are 1 and the number itself.

(iii) ($\star$) 1 is the only natural number which has an inverse.

(iv) ($\star$) A prime number divides the product of two natural numbers if and only if it divides at least one of them.

i)

$$\forall x\, \forall y\, \forall z\ \left(((x \,|\, y) \land (x \,|\, z)) \rightarrow (x \,|\, (y+z))\right)$$

ii)

$$\forall x\, \forall y\ \left((prime(x) \land (y \,|\, x)) \rightarrow ((y = 1) \lor (y = x))\right)$$

iii)

$$\forall x\, \forall y\ \left((x \cdot y = 1) \rightarrow ((x = 1) \land (y = 1))\right)$$

iv)

$$\forall x\ \left(prime(x) \rightarrow \forall y\, \forall z\ \left((x \,|\, (y \cdot z)) \leftrightarrow ((x \,|\, y) \lor (x \,|\, z))\right)\right)$$

Exercise 3.8, Proof by Contradiction ($\star$) (4 Points)

Let $n, m \in \mathbb{N}$ be arbitrary. We say "n divides m" and write $n|m$ if there exists a $k \in \mathbb{N}$ such that $k \cdot n = m$. Prove that the following statement is true, using a proof by contradiction:

$$n|m \ \text{ and } \ n|(m+1) \Longrightarrow n = 1.$$

You are allowed to invoke the statement 3.2 **iii)** from above to justify one step. You must use the same notation as in the lecture notes, i.e. precisely state what your statements S and T are, and justify each of your proof steps.