ETH zürich

Discrete Mathematics **2024**

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Assignment 11 **Due date:** Thursday, 5 December 2024, 23:59

Exercise 11.4, Combining Proof Systems $(\star)$ **(8 Points)**

Let

$$\Sigma = (\mathcal{S}, \mathcal{P}, \tau, \phi)$$

be a complete and sound proof system.

a) Define $\mathcal{P}'$ and ϕ' so that

$$\Sigma' = (\mathcal{S} \times \mathcal{S} \times \mathcal{S}, \mathcal{P}', \tau', \phi')$$

is a complete and sound proof system (and prove it!), where

$$\tau'((s_1, s_2, s_3)) = 1 \iff \text{at least 2 among } \tau(s_1), \tau(s_2), \tau(s_3) \text{are equal to} 1$$

b) Let

$$\overline{\Sigma} = (\mathcal{S}^{\in}, \overline{\mathcal{P}}, \overline{\tau}, \overline{\phi})$$

be a complete and sound proof system with

$$\begin{aligned} \overline{\tau}((s_1, s_2)) = 1 \iff & \text{exactly 1 of the statements is true in } \Sigma, \\ & \text{that is, } \tau(s_1) = 1 \text{ or } \tau(s_2) = 1, \text{but not both.} \end{aligned} \tag{1}$$

Define $\mathcal{P}^*$ and ϕ^* so that $\Sigma^* = (\mathcal{S}, \mathcal{P}^*, \tau^*, \phi^*)$ is a complete and sound proof system (and prove it!), where

$$\tau^*(s) = 1 \iff \tau(s) = 0$$

a)