Assignment 2

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Exercise 1

Prove by induction $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1} \quad \forall x \in \mathbb{N}$

1) base case: $n=1$ $\quad \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$ ✓

$\frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2}$

2) inductive step: $n = n+1$ $\quad \sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n+1}{(n+1)+1} = \frac{n+1}{n+2}$ proof? I.H.

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \left(\sum_{k=1}^{n} \frac{1}{k(k+1)}\right) + \frac{\cancel{(n+1)}}{\cancel{(n+1)}((n+1)+1)} = \frac{n}{n+1} + \frac{1}{n+2} = \frac{n(n+2)+1(n+1)}{(n+1)(n+2)} = \frac{n^2+2n+n+1}{(n+1)(n+2)}$$

$$\frac{n^2+3n+1}{n^2+2n+n+2} = \frac{n^2+3n+1}{n^2+3n+2} = \frac{(n^2+3n+1)}{\underbrace{(n^2+3n+1)}_{n+1}+1} \qquad n^2+3n+1 = n+1 \qquad \frac{n+1}{n+2}$$

Exercise 2

Prove by induction n^2+3n+5 is an odd number $\forall x \in \mathbb{N}$. Odd number $= 2m-1$

$P \to n^2+3n+5 = 2m-1$

1) base case: $n=1$ $\quad 1^2+3\cdot 1+5 = 2m-1 \quad 9 = 2m-1 \quad 2m = 10 \quad m = 5$

P is True (odd) with $m=5$

2) induction step: $n = n+1$ $\quad (n+1)^2+3(n+1)+5 \qquad n^2+2n+1+3n+3+5$

$n^2+5n+9 \qquad n^2+5n+10 \qquad \stackrel{\text{I.H.}}{=} \quad n^2 = -3n-6+2m$ then

$-3n-6+2k+5n+9 \qquad 2k+2n+3 \qquad 2\underbrace{(k+n+2)}_{m}-1 \qquad$ true with $m = k+n+2$

Exercise 3

Use only field axioms to prove $(x-y)z = (xz)-(yz) \quad \forall x,y,z \in \mathbb{Q}$

proof. $(x-y)z \stackrel{F_2}{=} z(x-y) \stackrel{F_5}{=} (zx)+(z(-y)) \stackrel{F_2}{=} (xz)+((-y)z) \stackrel{\text{'-' by def.}}{=} (xz)-(yz)$

Bonus exercise

Prove $\sum_{k=1}^{n} F_k = F_{n+2} - 1 \quad \forall x \in \mathbb{N}$, $F_n = F_{n-2} + F_{n+1}$ for $n \geq 2$, $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$

1) base case: $n=2$ $\quad F_0+F_1+F_2 = 0+1+1 = 2$ ✓

$\sum_{k=1}^{n} F_k = F_{n+2}-1 = F_4 - 1 = 3-1 = 2$

2) inductive step: $n = n+1$ $\quad \sum_{k=1}^{n+1} F_k = F_{(n+1)+2} - 1 = F_{n+3} - 1$ $\quad$ proof? I.H.

$$\sum_{k=1}^{n+1} F_k = \left(\sum_{k=1}^{n} F_k\right) + F_{n+1} = F_{n+2} - 1 + F_{n+1} = F_{n+2} + F_{n+1} - 1$$

$$F_n = F_{n-2} + F_{n-1}$$
$$F_{n+3} = F_{n+1} + F_{n+2}$$

$$= F_{n+3} - 1$$