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Assignment 9

Due date: Thursday, 21 November 2024, 23:59

Exercise 9.5, More Elementary Properties of Rings (★★) (8 Points)

Note: in a previous version of this exercise the assumption that R is an integral domain was missing. However, the first statement is false in this case. Can you give a counterexample?

Let $\langle R; +, -, 0, \cdot, 1 \rangle$ be a ring, and let $a \in R$ and $b \in R$. Prove the following statements:

- a) If R is an integral domain and if $a^m = b^m$ and $a^n = b^n$ for some positive integers m and n with $\gcd(m, n) = 1$, then $a = b$.
- b) If $1 - ab$ is a unit, then $1 - ba$ is also a unit. *Hint: if $x = (1 - ab)^{-1}$ consider the ring element $1 + bxa$.*

a)

b)