Dr. Maria Giuseppina Nestola	Assignment 2 Optimization Methods – SP24	March 27, 2024 Deadline: April 9, 2024

Please refer to the Assignment rules document.

Exercise 1 (40/100)

Consider the highly non-linear Rosenbrock's function:

$$f(x, y) := (1 - x)^2 + 100(y - x^2)^2 \tag{1}$$

1. Implement in MATLAB two functions:
   - the Newton's method (`Newton.m`)
   - the Steepest descent (Gradient) method (`GD.m`)

   where both methods can be run with backtracking algorithm (`backtracking.m`), for reference see J. Nocedal and S. Wright, *Numerical optimization*, page 37 and with step size $\beta = 1$. You can reuse your previous implementation of Steepest descent method and simply extend it in a way that it can use the backtracking algorithm for computing the step size β. Use the following values for the backtracking parameters: $\tilde{\alpha} = 1, \rho = 0.9$. You can choose the parameter c_1 such that it is in the interval $\{0.5, 10^{-4}\}$.
2. Minimize the Rosenbrock's function (1) by using the Steepest Descent (Gradient) method with backtracking and fixed step size $\beta = 1$. Use starting value $(0, 0)$, maximum number of iterations $N = 50000$ and tolerance TOL$= 10^{-6}$.
3. Minimize the Rosenbrock's function (1) by using Newton method with backtracking and fixed step size $\beta = 1$. Use starting value $(0, 0)$, maximum number of iterations $N = 50000$ and tolerance TOL$= 10^{-6}$.
4. Plot the obtained iterates on the energy landscape in 2D.
5. Analyze convergence behaviour of the methods by plotting the gradient norm and the function value at each iteration.
6. Compare and comment on the the performances of the different methods.

Exercise 2 (40/100)

Consider again the highly non-linear Rosenbrock's function:

$$f(x, y) := (1 - x)^2 + 100(y - x^2)^2 \tag{2}$$

1. Implement the BFGS method (`BFGS.m`) with backtraking for the step size β. For reference see J. Nocedal and S. Wright, *Numerical optimization*, page 141.
2. Test your implementation by minimizing the Rosenbrock's function. Use starting values $x_0 = (0, 0)$, $H_0 = I$, maximum number of iterations $N = 500$ and tolerance TOL$= 10^{-6}$.
3. Plot the obtained iterates on the energy landscape in 2D.
4. Analyze convergence behaviour of the methods by plotting the gradient norm and the function value at each iteration.
5. Produce a table in which you compare the number of iterations required by BFGS, by Newton's method (with backtracking) and by Steepest descent method (with backtracking). You can use the results from the previous exercise. Comment the results by comparing the different methods.

Exercise 3: Steepest Descent (20/100)

Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be given by $f = \frac{1}{2}x^T Ax - b^T x$ with A symmetric positive definite. How many iterations does the SD method take to minimize the function f if we use the optimal step length? Please, prove your answer.