ETH zürich

**Discrete Mathematics** **2024**

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## Assignment 2 — **Due date:** Thursday, 3 October 2024, 23:59

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## 1. Exercise 2.3, Simplifying a Formula ($\star$) (8 Points)

Consider the propositional formula

$$F = ((B \vee C) \to ((A \vee \neg B) \wedge C)) \vee (A \wedge \neg C)$$

Give a formula $G$ that is equivalent to $F$, but in which each atomic formula $A$, $B$, and $C$ appears at most once. Prove that $F \equiv G$ by providing a sequence of equivalence transformations with *at most* 12 steps.

**Expectation.** Your proof should be in the form of a sequence of steps, where each step consists of applying the definition of $\to$ (that is $F \to G \equiv \neg F \vee G$), one of the rules given in Lemma 2.1 of the lecture notes [1], or one of the following rules: $F \wedge \neg F \equiv \bot$, $F \wedge \bot \equiv \bot$, $F \vee \bot \equiv F$, $F \vee \neg F \equiv \top$, $F \wedge \top \equiv F$, and $F \vee \top \equiv \top$. For this exercise, associativity is to be applied as in Lemma 2.1.3. Each step of your proof should apply a *single* rule *once* and state *which* rule was applied.

The following 12 steps show the equivalence between $F$ and $G$, where $G := \neg B \vee A$.

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|---|---|
| $F = ((B \vee C) \to ((A \vee \neg B) \wedge C)) \vee (A \wedge \neg C)$ | |
| $F = \Big(\neg(B \vee C) \vee ((A \vee \neg B) \wedge C)\Big) \vee (A \wedge \neg C)$ | def of $\to$ |
| $F = \Big((\neg B \wedge \neg C) \vee ((A \vee \neg B) \wedge C)\Big) \vee (A \wedge \neg C)$ | def De Morgan |
| $F = \Big((\neg B \wedge \neg C) \vee (C \wedge (A \vee \neg B))\Big) \vee (A \wedge \neg C)$ | def commutative |
| $F = \Big((\neg B \wedge \neg C) \vee (C \wedge A) \vee (C \wedge \neg B)\Big) \vee (A \wedge \neg C)$ | def 1st distributive law |
| $F = \Big((\neg B \wedge \neg C) \vee (C \wedge \neg B) \vee (C \wedge A)\Big) \vee (A \wedge \neg C)$ | def commutative |
| $F = \Big((\neg B \wedge (C \vee \neg C)) \vee (C \wedge A)\Big) \vee (A \wedge \neg C)$ | def 1st distributive law |
| $F = \Big((\neg B \wedge \top) \vee (C \wedge A)\Big) \vee (A \wedge \neg C)$ | $F \vee \neg F \equiv \top$ |
| $F = \Big(\neg B \vee (C \wedge A)\Big) \vee (A \wedge \neg C)$ | $F \wedge \top \equiv F$ |
| $F = \neg B \vee \Big((C \wedge A) \vee (A \wedge \neg C)\Big)$ | def associativity |
| $F = \neg B \vee \Big(A \wedge (C \vee \neg C)\Big)$ | def 1st distributive law |
| $F = \neg B \vee \Big(A \wedge \top\Big)$ | $F \vee \neg F \equiv \top$ |
| $F = \neg B \vee A$ | $F \wedge \top \equiv F$ |

[1] Lemma 2.1 states rules involving propositional symbols, but you may apply those rules at the level of formulas (see Section 2.3.5 of the lecture notes).

Finally, for the formula $G$ defined as $G :\equiv \neg B \vee A$, we have shown that $F \equiv G$ by applying a 12-step sequence of equivalence transformations.