Discrete Mathematics **2024**

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Assignment 2 **Due date:** Thursday, 3 October 2024, 23:59

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1. Exercise 2.3, Simplifying a Formula ($\star$) (8 Points)

Consider the propositional formula

$$F = ((B \vee C) \to ((A \vee \neg B) \wedge C)) \vee (A \wedge \neg C)$$

Give a formula G that is equivalent to F, but in which each atomic formula A, B, and C appears at most once. Prove that $F \equiv G$ by providing a sequence of equivalence transformations with *at most* 12 steps.

Expectation. Your proof should be in the form of a sequence of steps, where each step consists of applying the definition of $\to$ (that is $F \to G \equiv \neg F \vee G$), one of the rules given in Lemma 2.1 of the lecture notes [1], or one of the following rules: $F \wedge \neg F \equiv \bot$, $F \wedge \bot \equiv \bot$, $F \vee \bot \equiv F$, $F \vee \neg F \equiv \top$, $F \wedge \top \equiv F$, and $F \vee \top \equiv \top$. For this exercise, associativity is to be applied as in Lemma 2.1.3. Each step of your proof should apply a *single* rule *once* and state *which* rule was applied.

The following 12 steps show the equivalence between F and G, where $G := \neg B \vee A$.

$$F = ((B \vee C) \to ((A \vee \neg B) \wedge C)) \vee (A \wedge \neg C)$$

$$F = \Big(\neg (B \vee C) \vee ((A \vee \neg B) \wedge C)\Big) \vee (A \wedge \neg C) \qquad \text{def of } \to$$

$$F = \Big((\neg B \wedge \neg C) \vee ((A \vee \neg B) \wedge C)\Big) \vee (A \wedge \neg C) \qquad \text{def De Morgan}$$

$$F = [(\neg B \wedge \neg C) \vee (C \wedge (A \vee \neg B))] \vee (A \wedge \neg C)$$

$$F = [(\neg B \wedge \neg C) \vee (C \wedge A) \vee (C \wedge \neg B)] \vee (A \wedge \neg C)$$

$$F = (\neg B \wedge \neg C) \vee (C \wedge A) \vee (C \wedge \neg B) \vee (A \wedge \neg C)$$

$$F = [\neg B \wedge (\neg C \vee C)] \vee [A \wedge (C \vee \neg C)]$$

$$F = (\neg B \wedge T) \vee (A \wedge (C \vee \neg C))$$

$$F = (\neg B \wedge T) \vee (A \wedge T)$$

$$F = \neg B \vee (A \wedge T)$$

$$F = \neg B \vee A$$

$$G := \neg B \vee A \quad \text{Q.E.D.}$$

[1] Lemma 2.1 states rules involving propositional symbols, but you may apply those rules at the level of formulas (see Section 2.3.5 of the lecture notes).