Università della Svizzera italiana
Institute of Computing CI

Optimization Methods **2024**

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Assignment 2

Due date: Friday, 12 April 2024, 12:00 AM

Exercise 1

Considering the highly non-linear Rosenbrock's function:

$$f(x, y) := (1 - x)^2 + 100(y - x^2)^2 \tag{1}$$

1, 2, 3

Implement in MATLAB two functions: Newton's method (Newton.m), Steepest descent (Gradient) method (GD.m). Both methods can be run with backtracking algorithm (backtracking.m) with step size $\beta = 1$. Use the following values for the backtracking parameters: $\tilde{\alpha} = 1, \rho = 0.9$. You can choose the parameter $c_1 \in [0.5, 10^{-4}]$.

Minimize the Rosenbrock's function 1 by using the Steepest Descent (Gradient) method with backtracking and fixed step size $\beta = 1$. Use starting value $x_0 = (0, 0)$, maximum number of iterations $N = 50000$ and tolerance $TOL = 10^{-6}$.

Minimize the Rosenbrock's function 1 by using Newton method with backtracking and fixed step size $\beta = 1$. Use same parameters as for SD.

Matlab scripts are provided in */code* folder. The 2 main files to run are: *GD.m, Newton.m.*

4, 5, 6

Plot the obtained iterates on the energy landscape in 2D. Analyze convergence behaviour of the methods by plotting the gradient norm and the function value at each iteration. Compare and comment on the the performances of the different methods.

Figure 1: Visualization of Steepest Descent

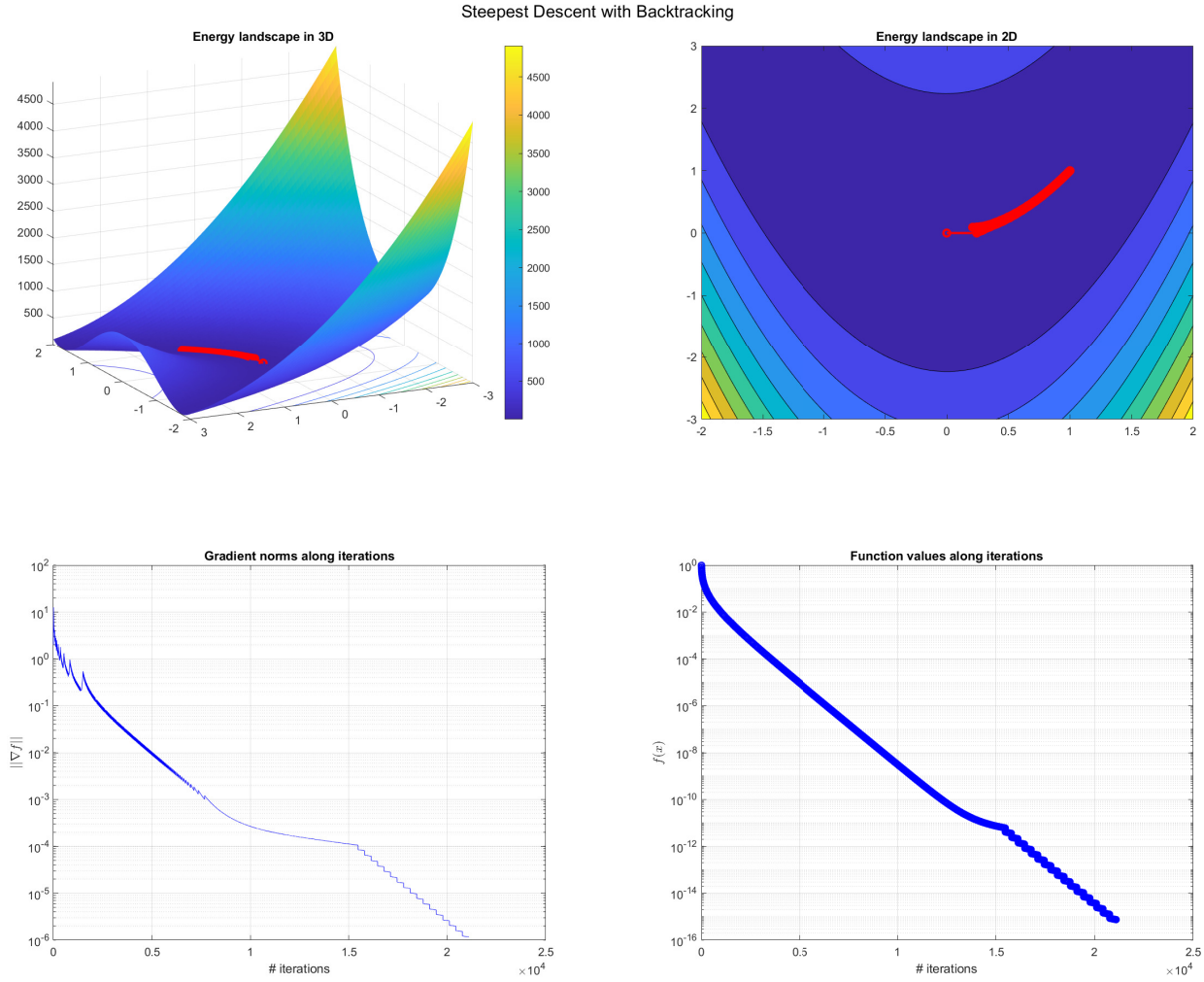


Figure 2: Comparison of convergence

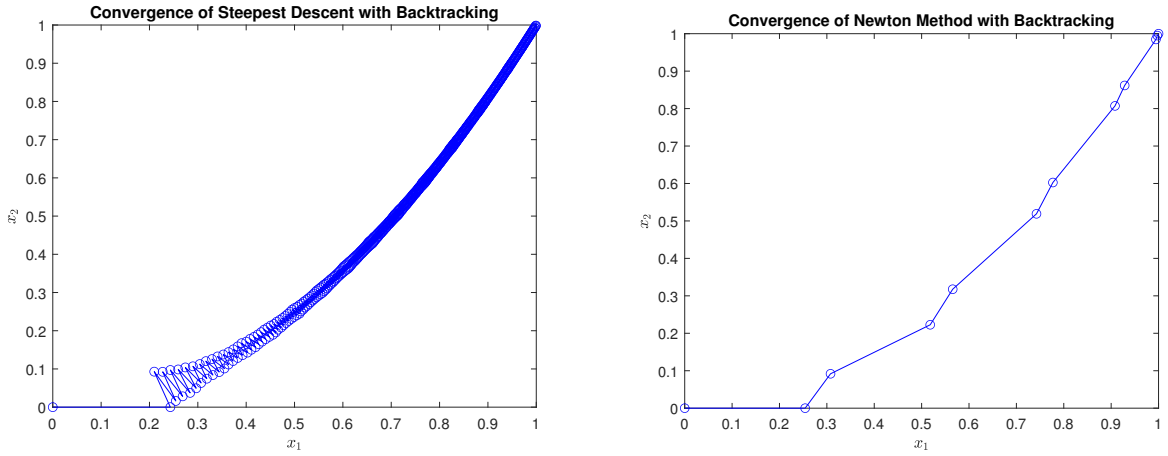
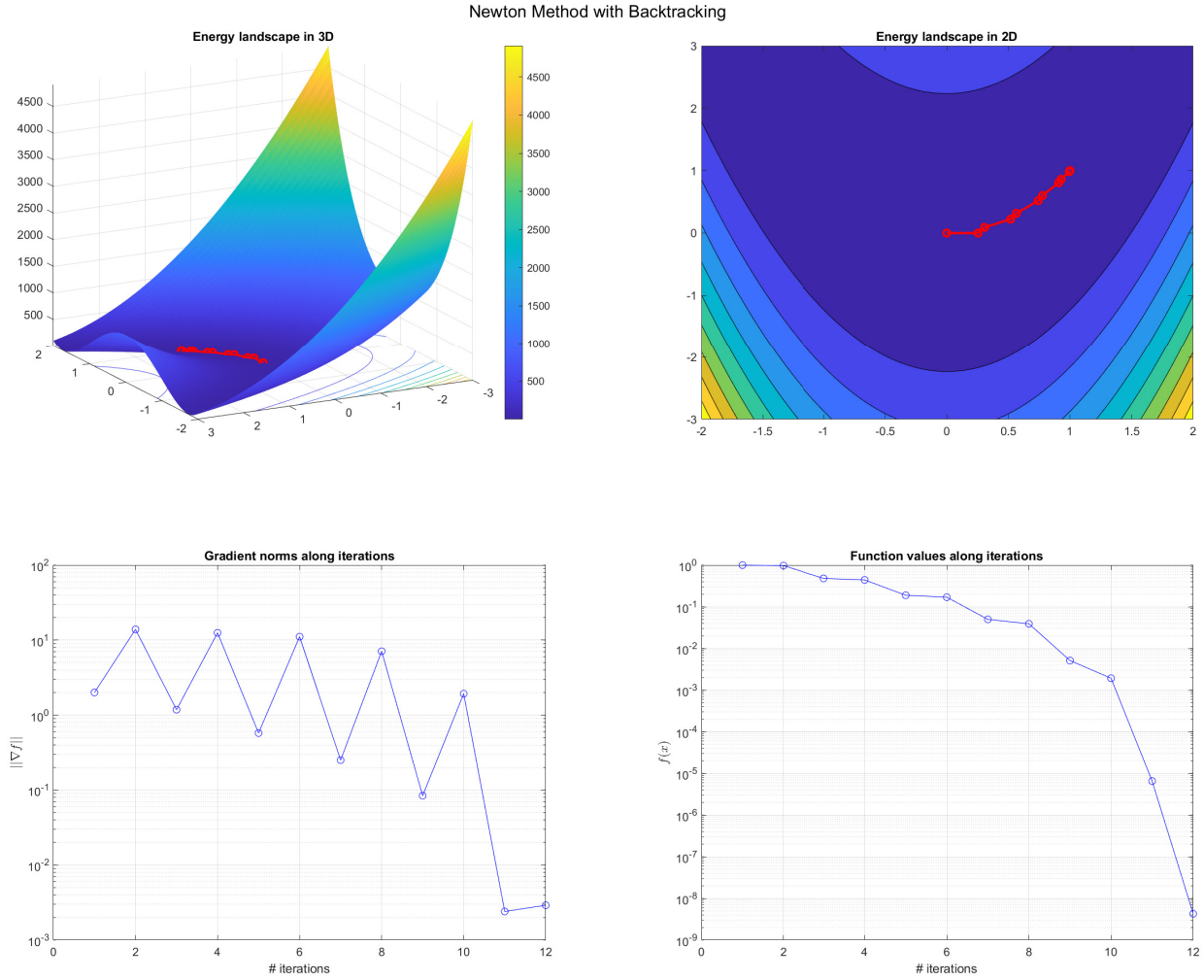


Figure 3: Visualization of Newton Method



Chosen $c_1 = 1e-4$.

Exercise 2

1, 2

Implement the BFGS method (BFGS.m) with backtraking for the step size β. Test your implementation by minimizing the Rosenbrock's function. Use starting values $x_0 = (0, 0)$, $H_0 = I$, maximum number of iterations $N = 500$ and tolerance $TOL = 10^{-6}$.

Matlab scripts are provided in */code* folder. The main file to run is *BFGS.m*.

3, 4

Plot the obtained iterates on the energy landscape in 2D. Analyze convergence behaviour of the methods by plotting the gradient norm and the function value at each iteration.

Figure 4: Visualization of BFGS

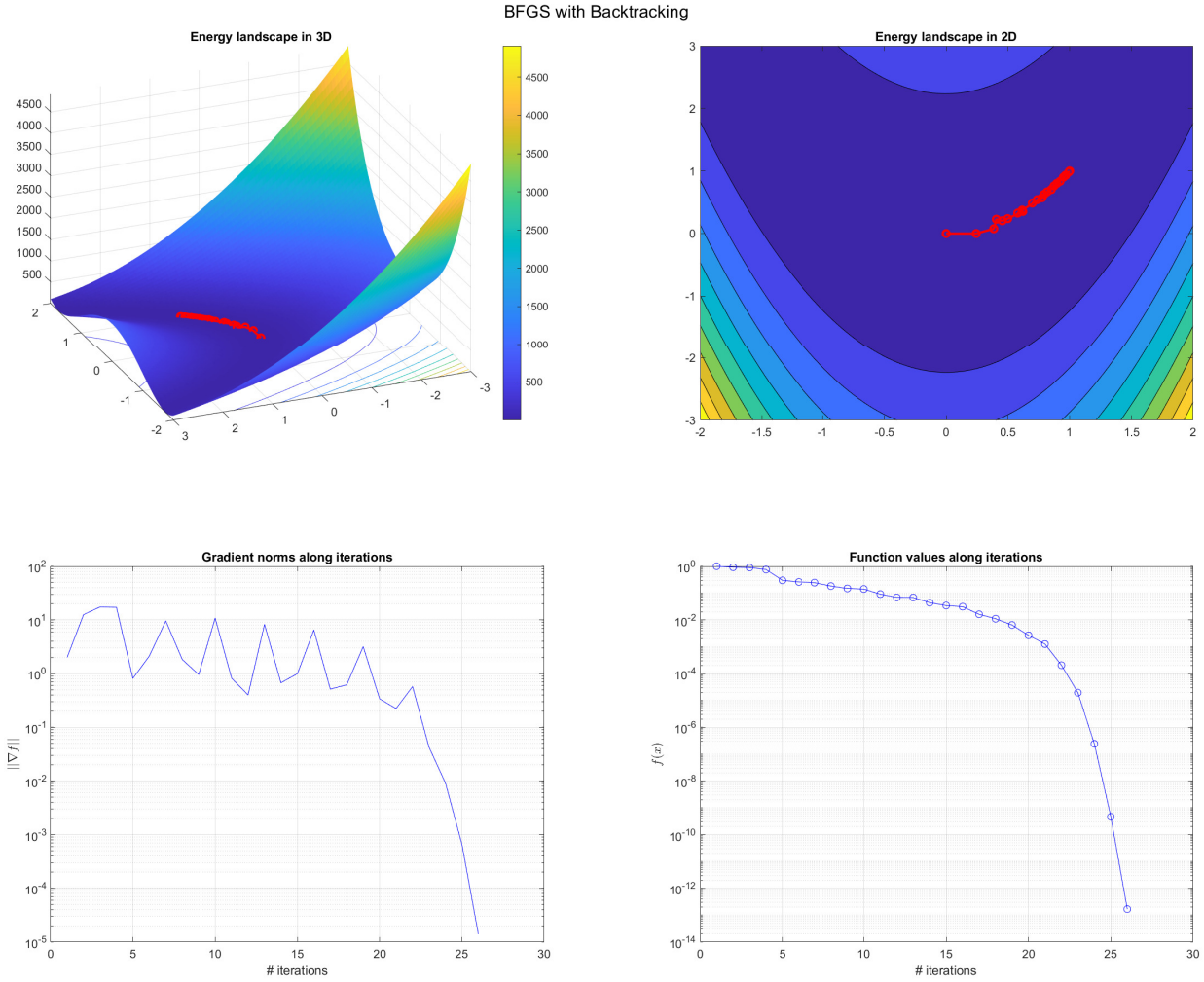
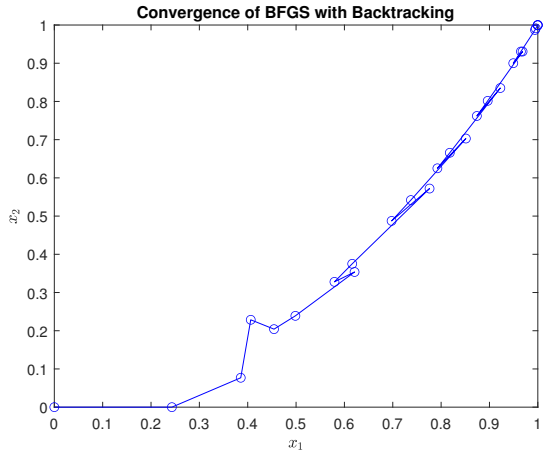


Figure 5: Convergence of BFGS



5

Produce a table in which you compare the number of iterations required by BFGS, by Newton's method (with backtracking) and by Steepest descent method (with backtracking). You can use the results from the previous exercise. Comment the results by comparing the different methods.

Table 1: Comparison of Iterations for Rosenbrock's Function

Steepest Descent	Newton	BFGS
21102	12	26

Exercise 3

Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f = \frac{1}{2}x^T Ax - b^T x$ with A symmetric positive definite. How many iterations does the SD method take to minimize the function f if we use the optimal step length? Please, prove your answer.