Assignment 9

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Exercise 4

$\triangle_{AB}^{C}$ formed by points $A=(0,0,0)$, $B=(2,0,0)$, $C=\left(0,\frac{1}{\sqrt{2}},0\right)$ rendered with model-view-projection matrix:

$$M=\begin{bmatrix}1&0&0&0\\0&\frac{\sqrt{2}}{2}&-\frac{\sqrt{2}}{2}&0\\0&\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&-0.25\\0&0&0&1\end{bmatrix}$$

$\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$\sin\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

Determine percentage % of rendered image which is covered by triangle:

Apply M matrix to $\triangle_{AB}^{C}$ points using homogeneous coords, hence $\begin{bmatrix}A\\1\end{bmatrix}$ for points, to get from local model coords MC to global world coords WC:

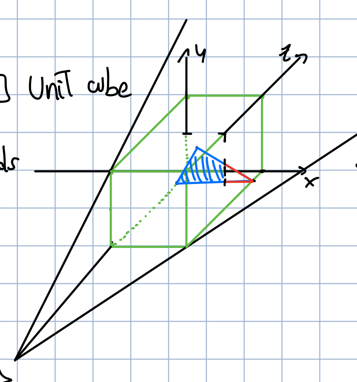
$$A'=M\cdot A=\begin{bmatrix}1&0&0&0\\0&\frac{\sqrt{2}}{2}&-\frac{\sqrt{2}}{2}&0\\0&\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&-0.25\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\-\frac{1}{4}\\1\end{bmatrix}$$

$$B'=M\cdot B=\begin{bmatrix}1&0&0&0\\0&\frac{\sqrt{2}}{2}&-\frac{\sqrt{2}}{2}&0\\0&\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&-0.25\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}2\\0\\0\\1\end{bmatrix}=\begin{bmatrix}2\\0\\-\frac{1}{4}\\1\end{bmatrix}$$

$$C'=M\cdot C=\begin{bmatrix}1&0&0&0\\0&\frac{\sqrt{2}}{2}&-\frac{\sqrt{2}}{2}&0\\0&\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&-0.25\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\\0\\1\end{bmatrix}=\begin{bmatrix}0\\\frac{1}{2}\\\frac{1}{2}-\frac{1}{4}\\1\end{bmatrix}=\begin{bmatrix}0\\\frac{1}{2}\\\frac{1}{4}\\1\end{bmatrix}$$

Since rendering is mapped to cuboidal space $[-1,1]\times[-1,1]\times[-1,1]$ Unit cube

It's reasonable to see the rendered projection in x-y axis coords

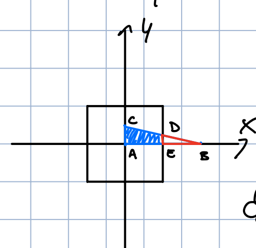


— rendered

— non rendered

axis in the middle of cube

$A''=(0,0)$, $B''=(2,0)$, $C''=\left(0,\frac{1}{2}\right)$



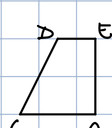
Get $\overline{AB}=\ell_1 \Rightarrow x=0$

Get $\overline{CB}=\ell_2 \Rightarrow \frac{x-2}{2-0}=\frac{y-0}{0-\frac{1}{2}} \Rightarrow \frac{x-2}{2}=\frac{y}{-\frac{1}{2}}$

$\Rightarrow \frac{1}{2}(x-2)=-2y \Rightarrow y=-\frac{1}{4}(x-2) \Rightarrow y=-\frac{1}{4}x+\frac{1}{2}$

Find D_y using ℓ_2, having $D_x=1$ from being at unit cube right most x value:

$D_y=-\frac{1}{4}(1)+\frac{1}{2}=-\frac{1}{4}+\frac{2}{4}=\frac{1}{4}$ $\quad D=\left(1,\frac{1}{4}\right)$



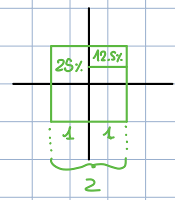
$\frac{(B+b)\cdot h}{2}$

Clearly, $E=(1,0)$, then Area of AEDC being a rectangular trapezoid:

$\overline{CA}=\frac{1}{2}$, $\overline{AE}=1$, $\overline{DE}=\frac{1}{4}$, Area AEDC $=\frac{\left(\frac{1}{2}+\frac{1}{4}\right)\cdot 1}{2}=\frac{\frac{3}{4}}{\frac{2}{1}}=\frac{3}{4}\cdot\frac{1}{2}=\frac{3}{8}$

Area of unit cube in perspective:

$S = 2 \quad A_{TOT} = S^2 = 2^2 = 4$



Percentage of rendered image covered by triangle:

$$\frac{A_{AEDC}}{A_{TOT}} = \frac{\frac{3}{8}}{4} = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32} \simeq 9{,}375\ \%$$