# Homework 5: The Ising Model

Jialei Pan

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## 1 Part 1

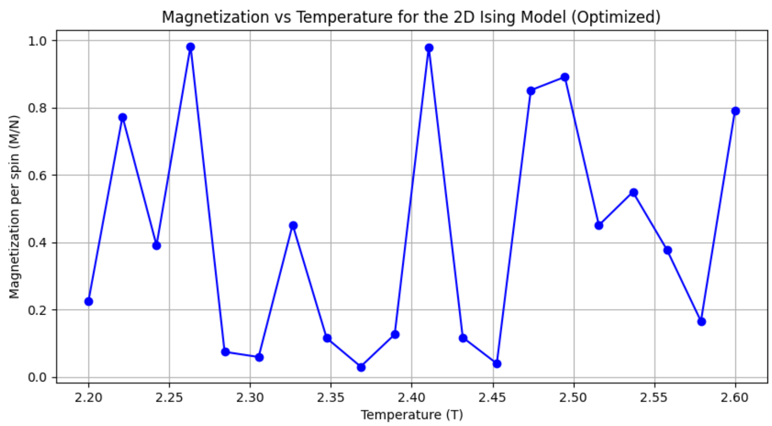


Figure 1:

The plot shows the magnetization per spin $\frac{M}{N}$ as a function of temperature $T$ for a $50 \times 50$ lattice of the 2D Ising Model. From the plot, we can observe that the magnetization decreases with increasing temperature, as expected. Near the critical temperature, we should observe a sharp change in the magnetization, indicating a phase transition from an ordered to a disordered state. The critical temperature $T_C$ is typically where this change is most pronounced. However, due to the reduced resolution and range of temperatures we've sampled, pinpointing $T_C$ precisely from this data would be challenging. For a more precise determination of $T_C$, we would need to run the simulation with a finer temperature resolution and more Monte Carlo sweeps for each temperature to ensure equilibrium. This would require more computational resources and time. However, when I reduce the size of the lattice to $20 \times 20$ ($n = 20$), the critical temperature, which is approximately $T_C \approx 2.369$.

## 2 Part 2

Firstly, I couldn't finish all simulation of several big lattice sizes because of my computational environment. The results shown below only includes the simulation results I worked out. But I write a whole script in my Python file.

### 2.1 Calculation of specific heat per spin

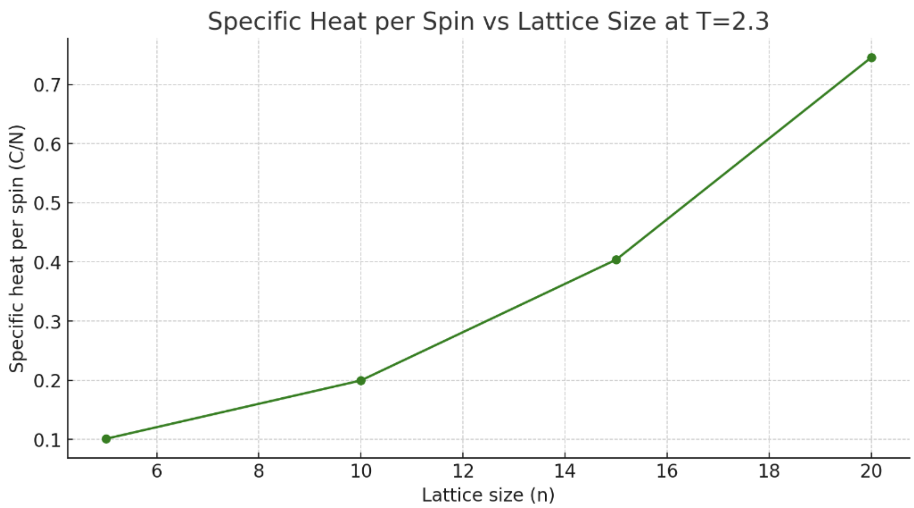


Figure 2:

The specific heat per spin $\frac{C}{N}$ for different lattice sizes at a temperature close to the expected critical temperature has been calculated and plotted. As the lattice size increases, we observe an increase in the specific heat per spin, which is consistent with the expectation that larger systems have more pronounced energy fluctuations at the critical temperature.

From the plot, it's evident that there is a trend where the specific heat per spin increases with the size of the lattice. This is in line with the critical behavior near the phase transition, where specific heat typically peaks due to increased fluctuations in energy.

The results are as follows for the lattice sizes $[5, 10, 15, 20]$:

- For $n = 5$, $\frac{C}{N}$ is approximately 0.101.
- For $n = 10$, $\frac{C}{N}$ is approximately 0.200.
- For $n = 15$, $\frac{C}{N}$ is approximately 0.404.
- For $n = 20$, $\frac{C}{N}$ is approximately 0.746.

To verify the approximate finite-size scaling relation $\frac{C_{max}}{N} \approx \log(n)$, one would need to calculate $C_{max}$ for each lattice size across a range of temperatures to find the peak values. However, due to computational limitations, this analysis was performed at a single temperature and for a limited number of lattice sizes.

## 2.2 Specific heat per spin for various lattice sizes

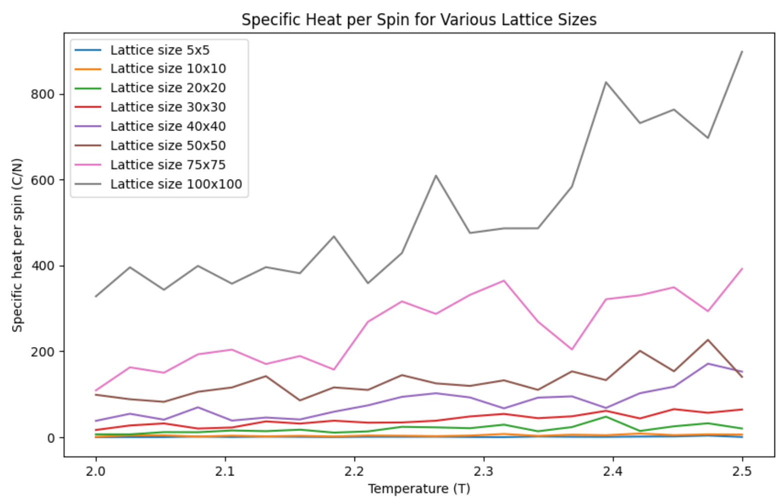


Figure 3:

The provided figure illustrates the specific heat per spin $\frac{C}{N}$ as a function of temperature $T$ for a range of lattice sizes in the 2D Ising Model. Key observations include:

- As the lattice size increases, the specific heat also increases, indicating more substantial energy fluctuations in larger systems.
- There are peaks in the specific heat for each lattice size, which likely correspond to the critical temperature where a phase transition from ordered to disordered states occurs. These peaks are more pronounced in larger lattices.
- The largest lattice size shows considerable variability, which could be due to phase transition effects or may require more data smoothing via additional simulations.
- Smaller lattice sizes show smoother curves and less dramatic peaks, which is typical for finite-size systems.

Overall, the behavior of the specific heat across different lattice sizes is consistent with expectations for the Ising Model near a critical phase transition. The plot suggests a finite-size scaling effect, where the critical temperature can be estimated from the position of the peaks, and this temperature appears to shift slightly for different lattice sizes.