Maths Solution

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Question

(Please show your workings). Over all real numbers, find the minimum value of a positive real number, y such that

$$y = \sqrt{((x+6)^2+25)} + \sqrt{((x-6)^2+121)}$$

Solution

Recall that y is said to have a minimum value at the point x where $y^{'}(x) = 0$.

Let $u = \sqrt{((x+6)^2+25)}$ and $v = \sqrt{((x-6)^2+121)}$. Hence, $y^{'} = u^{'} + v^{'}$.

Also let $u = \sqrt{a}$, where $a = (x+6)^2 + 25$. $u^{'} = \frac{du}{da} \times \frac{da}{dx}$

$u^{'} = -2(x+6) \times \frac{1}{2} a^{-\frac{1}{2}}$. Similarly, $v^{'} = -2(x-6) \times \frac{1}{2} b^{-\frac{1}{2}}$, where $b = (x-6)^2+121$.

$$\implies y^{'} = \frac{(x+6)}{((x+6)^2+25)^{\frac{1}{2}}} + \frac{(x-6)}{((x-6)^2+121))^{\frac{1}{2}}} = 0$$
$$\implies (x+6)((x-6)^2+121))^{\frac{1}{2}} + (x-6)((x+6)^2+25)^{\frac{1}{2}} = 0$$
$$\implies (x+6)((x-6)^2+121))^{\frac{1}{2}} = -(x-6)((x+6)^2+25)^{\frac{1}{2}}$$
$$\implies (x+6)^2((x-6)^2+121) = (x-6)^2((x+6)^2+25)$$
$$\implies 121(x+6)^2 = 25(x-6)^2$$
$$\implies 11(x+6) = 5(x-6)$$
$$\implies 11x + 66 - 5x + 30 \implies 6x = 96$$
$$\implies x = 16$$

Hence, y is at a minimum when $x = 16$. So, we have the minimum value of $y = \sqrt{((16+6)^2+25)} + \sqrt{((16-6)^2+121)} = 37.43$