Challenge Deliverable 6
Answer form

4DB00 Dynamics and Control of mechanical systems
2019-2020

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Question a):

Find the set of real-valued parameters K, τ_1 and τ_2 such that the three design requirements are met. Explain briefly how you found these parameters.

Controller parameters:

```
K    =  89.4268
tau1 = ((1+sqrt(2)))/(0.3);
tau2 = (1)/((1+sqrt(2))*0.3);
```

Briefly explain the design steps:

First, I made a Bode plot of $G_l(s)$, to determine whether a phase lag or phase lead is desired to meet the phase margin requirement, stated in the exercise. Since the phase margin of $G_l(s)$ is around 15 degrees, a phase lead is desired.

$$C_{lead}(s) = K_{lead}\frac{\frac{\alpha}{\omega_c}s+1}{\frac{1}{\alpha\omega_c}s+1}$$

To realize this phase lead, I choose for a Lead Filter that creates a phase lead of 45 degrees, which should be sufficient to realize the requirement. To ensure this is indeed the case, I substituted s with $j\omega_c$, and solved $arg\left(C_{lead}(j\omega_c)\right) = 45°$for α, resulting in $\alpha = 1 \pm 2^{.5}$. Here, only the solution bigger zero has a meaning.

Now, since α is known, K can be found with $K_{lead} = \frac{1}{\alpha|G_l(j\omega_c)|}$

Note that $\tau_1$and τ_2 are $\frac{\alpha}{\omega_c}$ and $\frac{1}{\alpha\omega_c}$ respectively.

Question b):

For the open-loop system $L_{l,1}(s)$, determine the gain margin GM in dB and the modulus margin MM in dB. Are these margins sufficiently large in your opinion?

$GM = 7.6797$ dB
$MM = -4.6687$ dB

These margins are sufficiently large in my opinion. A GM > 0 and |MM| <6 dB are required for stability, which are both easily met here.

Question c):

Determine the response of the closed-loop interconnection between the "virtual test setup" and the controller $C_{l,1}(s)$, for a constant velocity reference signal. Utilize the "Sim_Control" request within the DOMS toolbox for this purpose. Use the simulation conditions as described in the deliverable and add the requested plot here.

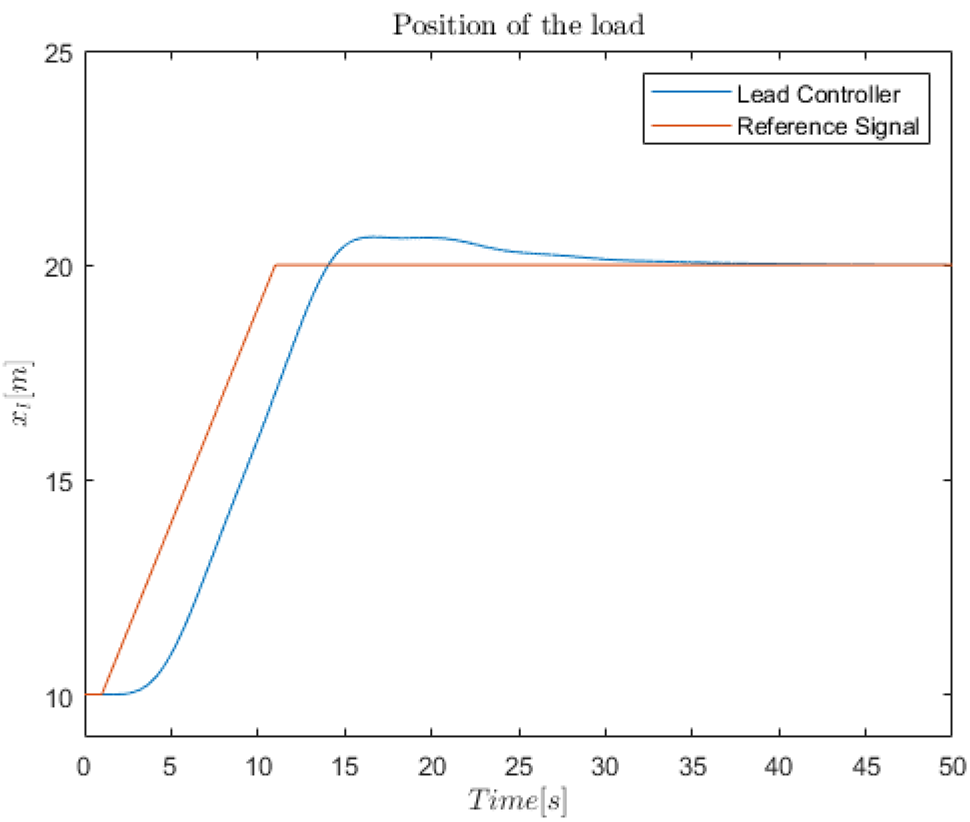

Figure 1: Response with constant velocity signal and lead controller

Question d):

Repeat the simulation in question c with the reduced damping coefficient d_W and add the requested plot here. How robust is your controller to this parameter change?

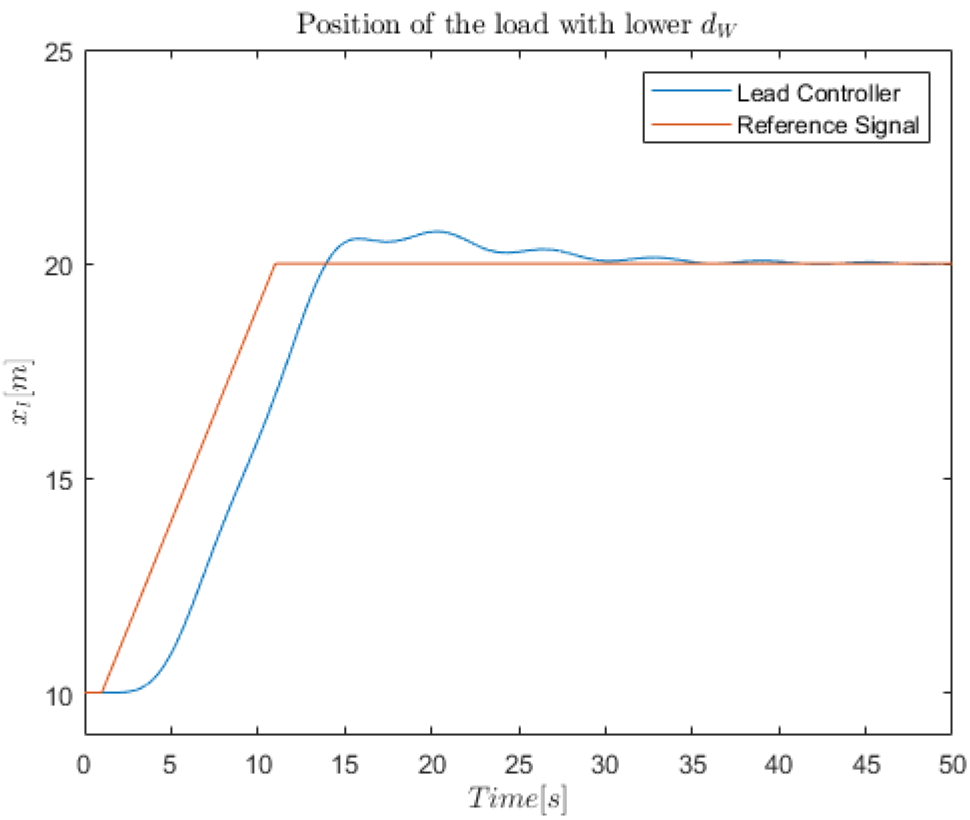

Figure 2: Response with constant velocity signal and lead controller with lower air friction

Regarding robustness, one can conclude that the controller is not robust. The blue trajectory shown in figure 2 is somewhat similar to the one shown in figure 1 in the first 10-15 seconds. However, after this, the lead controller shows a more "jerkier" path when the air friction is lower, and eventually never settles the load at 20 [dB]. This behavior tells us that the closed-loop interaction is unstable when the air friction is low. This, of course, makes the Lead Controller (on its own) not suitable for this application.

Question e):

Find the set of real-valued parameters K, τ_1, τ_2, $\omega_{n,1}$, $\omega_{n,2}$, β_1 and β_2 such that the three design requirements are met. Explain briefly how you found these parameters.

Controller parameters:

```
K      =
tau1   = 0.92
tau2   = 0.2717
wn1    = 1.1068
wn2    = 11.0680
beta1  = 0.2349
beta2  = 0.9
```

(please confirm that these seven lines can *directly* be copied and pasted into Matlab)

Briefly explain the design steps:

Analysing G_l

At ω_c , the phase is around -298 degrees and the magnitude is around -87 dB. Thinking about the required values, ([-125, …, -120] degrees and 0 dB), a total phase increase of around 173 degrees and margin increase of 87 dB.

Designing the Notch

To design the notch filter, I first analyzed the zpk-representation of G_l. From there I could determine ω_{n1} , β_1 and $\omega_{n2} = 10\omega_{n1}$, using that the zpk representation gave the insight that for G_l:

$$s^2 + 2\beta_1\omega_{n1}s + \omega_{n1}^2 = s^2 + 0.5199s + 1.225$$

As first try, I choose $\beta_2 = 0.9$. Later more on this.

Designing the Lead Filter

To do this, the phase increase that is “not covered” by the notch filter that is necessary to meet must be determined. The argument of N (where N is the notch filter) evaluated for $s = j\omega_c$ is 140.87 degrees (rounded at two decimals). The phase increase that the Lead Filter has to add is then $-1 * (140.87 - 298 + 125) = 32.13$ degrees. Using the same steps as in question a), but now for $arg\left(C_{lead}(j\omega_c)\right) = 33°$ (33 degrees to ensure the phase margin is indeed in the correct range of required values). This gives $\alpha = 1.84$. To ensure that the crossover frequency is indeed at 2 rad/s, the following MATLAB code was used:

```
syms K_2

fraq = Gl*C_L*N ;
eqn = K_2*abs(evalfr(fraq, i*wc_REQ_2)) == 1 ;
K2_A = solve(eqn, K_2);
K2 = 36893488147419103232/8583778407751303
C_L_K = K2 * C_L;
L = Gl*C_L_K*N;
```

(The fraction that is defined as K2 is K2_A, but MATLAB does not accept a multiplication between the result of a solve and a transfer function, so this value had to be filled in manually)

Question e) (continued):

When analyzing the open loop, I get a PM of 55.9 at 2 rad/s, $\omega_c = 2\,rad/s$ and GM = 18.9 dB at 11.2 rad/s.

A note on β_2

This value of β_2 , 0.1 below the maximum possible value, seems to be a proper value, but is guessed. Because of this, I tried to lower β_2 after all parameters were determined, to analyze the influence of this on the open loop, and to (maybe) find a more optimal value for β_2. Lowering β_2 resulted in a decreasing Gain Margin, which is less optimal, since this brings the Nyquist contour closer to the -1 point, making the system less robust. However, the PM increased. To ensure the robustness of the system, I choose for a high GM, which is still, after this small analysis for $\beta_2 = 0.9$.

Question f):

Use a similar approach as before, to investigate the controller $C_{l,2}(s)$ that you designed in question e. In addition, compare it to the controller $C_{l,1}(s)$ that you designed in question a. Please comment on stability, tracking performance, robustness and feasibility. You may include *at most* 3 figures in your answer.

With regards to stability both controllers can be defined as stable

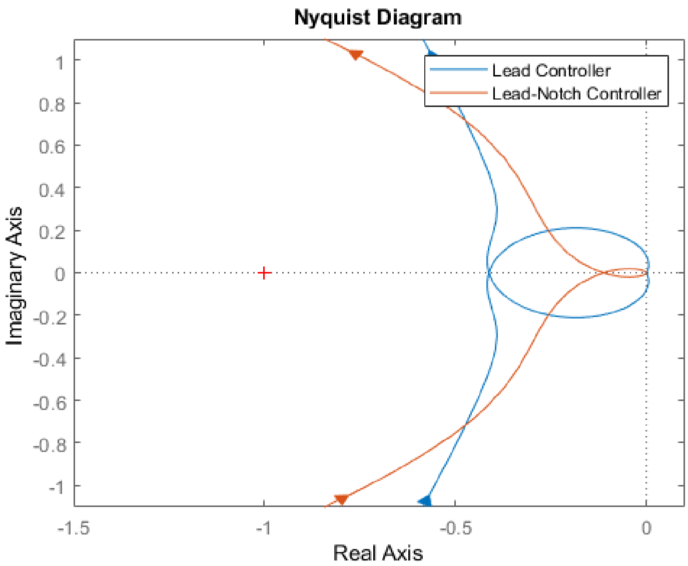

Figure 3: Nyquist contour of the closed loop transfer function of both controllers discussed in this deliverable.

To answer this question, I will use the Nyquist Criterion:

$$Z = N + P$$

Where

- Z is the number of unstable poles of Closed Loop transfer function $T_i(s)$;
- P is the number of unstable poles of the Open Loop transfer function $L_i(s)$;
- N is the number of encirclements in clockwise direction that the Nyquist contour makes around (-1;0).

The closed-loop interconnection of the "lead filter" with $G_l(s)$…

Using the `pole(T_1), pole (L_1)` commands in MATLAB respectively gives me:

Z = 0
P = 0
Looking at the Nyquist contour in figure 1:
N = 0

Since these values agree with the Nyquist Criterion, this closed loop intersection is stable.

Question f) (continued):

The closed-loop interconnection of "lead filter with notch" with $G_l(s)$…

Using the `pole(T_2), pole (L_2)` commands in MATLAB respectively gives me:

Z = 0
P =0
Looking at the Nyquist contour in figure 1 :
N = 0

Since these values agree with the Nyquist Criterion, this closed loop intersection is stable.

When it comes to tracking performance, the controllers perform well

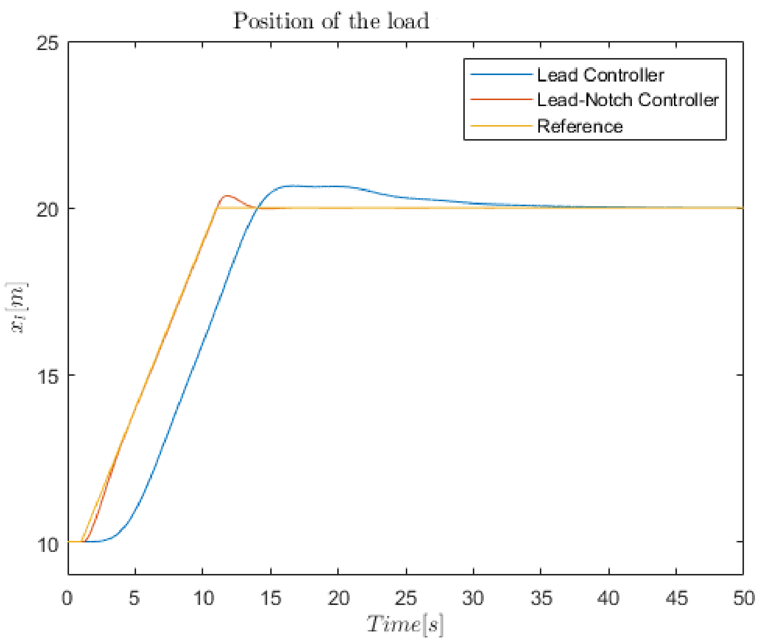

Figure 4: Response with constant velocity signal and both controllers discussed in this deliverable.

The Tracking performance of the Lead-Notch Controller is very good. It follows the reference with an near-to-zero error, except the small overshoot at around t = 10 [s], which, however, stabalizes very quick. A lead only cobtroller has a far worse tracking performance: it gets the same slope as the reference but witha delay of about 4 [s], overshoots a bit more and takes longer to stabalise at the 20 [m] line.

Robustness

When reducing the value of d_W to 100 again, and making a plot like in figure 4, I get:

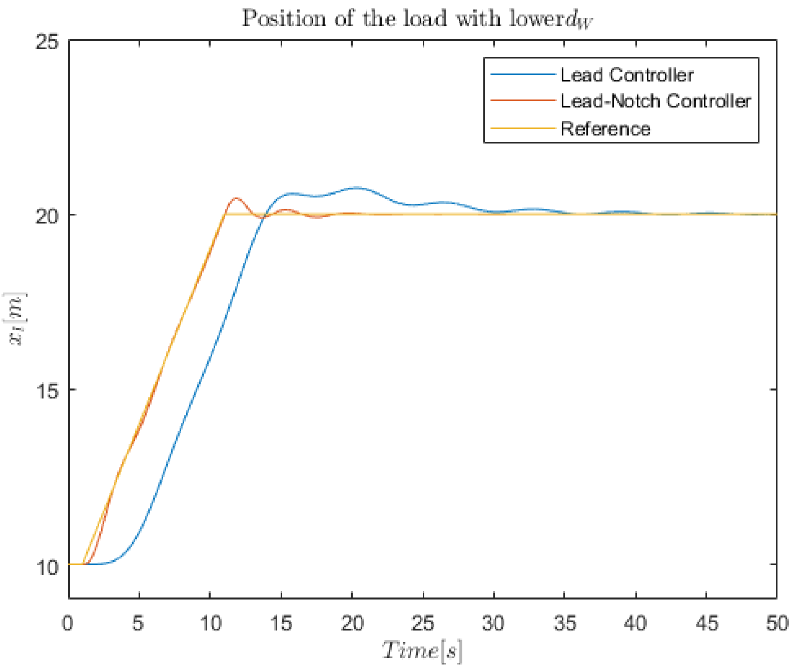

Figure 5: Response with constant velocity signal and both controllers discussed in this deliverable with lower ait friction.

The Tracking performance of the Lead-Notch Controller is similar to the one in figure 4. It follows the slope a bit less perfect, but still quite good and it takes a bit longer to stabilise at 20 [m] than with higher air friction. From this can be concluded that the lead-notch controller is quite robust.

This is in contrast with the lead controller, that fails at being robust: the overshoot is worse when air friction decreases. Furthermore, it does not seem to stabalise around 20 [m] within 50 seconds, while it did at higher air friction. From this can be concluded that the lead controller is not very robust.

Finally, regarding feasibility…

The Lead-Notch controller from exercise e contains both in the nominator and denominator a third term polynomial in *s*. This is expected from a combination of a lead filter and a notch filter since multiplying a second order polynomial (the highest order of a notch filter) with a first order polynomial (the highest order of a lead filter) results in a third order polynomial.

This makes the lead-notch controller quite feasible, since it is not from a very high order (recall C_{aggr} from deliverable 5) and consists out of “common” controller elements, one lead filter and one notch filter.

The lead controller is even more feasible (containing only first order polynomials in *s*), but is less suitable for our luffing crane, as shown in the analysis on robustness and performance.