Degree Distribution

- The probability that k links are present, p^k
- The probabilty that the remaining $N-k-1$ links are missing, $(1-p)^{N-1-k}$
- The number of ways we can select k links from $N-1$ potential links, $\binom{N-1}{k}$

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

In a random network, the probability that vertex i had exactly k links is the product of above terms. Represents a Binomial distribution. Since for every vertex, there is a decision to make. Either connect to another vertex or stay disconnected.

Clustering Coefficient

There are $\binom{k}{2}$ potential edges among the vertex's neighbors, where p denotes such probability of being in the graph. Since t describes the density of links in node i's immediate neighborhood, $t\binom{k}{2}$ is the necessary amount of immediate neighborhood.

$$\begin{aligned}
\Pr\left(\frac{N}{\binom{k}{2}} = t\right) &= \Pr\left(N = t\binom{k}{2}\right) \\
&= \binom{\binom{k}{2}}{t\binom{k}{2}} \cdot p^{t\binom{k}{2}} \cdot (1-p)^{\binom{k}{2} - t\binom{k}{2}}
\end{aligned}$$

Expected Degree

The expectation of a Binomial distribution is given by the formula np, where n is the sample size and p is the probability of success. In our case, p is already given, and n, the sample size, is $N-1$. This concludes that $E[X] = p(N-1)$.

Expected local clustering coefficient

First, note that we assume $C_i = 0$ when $k_i < 2$. Also, the variable p_L represents the probability for a particular realization of a random network with exactly L links. Let's start with:

$$\sum_{k=2}^{N-1} P(k) \cdot \sum_{i=0}^{k(k-1)/2} \frac{i}{k(k-1)/2} \cdot \text{Binom}\left[i; k(k-1)/2, p\right]$$

The fraction $\frac{2}{k(k-1)}$ is independent of i, so it can be treated as a constant during summation.

$$\sum_{k=2}^{N-1} P(k) \cdot \frac{2}{k(k-1)} \sum_{i=0}^{k(k-1)/2} i \cdot \text{Binom}\left[i; k(k-1)/2, p\right]$$

The expression $\sum_{i=0}^{k(k-1)/2} i \cdot \text{Binom}\left[i; k(k-1)/2, p\right]$ represents the expected number of links for a random network. Thus, we can simplify it.

$$\sum_{k=2}^{N-1} P(k) \cdot \frac{2}{k(k-1)} \cdot \frac{k(k-1)}{2} \cdot p_L$$

The total probability $\sum_{k=2}^{N-1} P(k) = 1$. Therefore, we can simplify to:

$$1 \cdot 1 \cdot p_L = p_L$$