Conclusions from a Hypothesis Test

$CLAIM : H_0$

Reject

There is enough evidence to reject the claim

Do not reject

There is not enough evidence to reject the claim

$CLAIM : H_a$

Reject

There is enough evidence to support the claim

Do not reject

There is not enough evi-
dence to support the claim

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Z-Test and P-Values

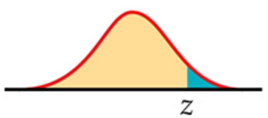
z TEST FOR A POPULATION MEAN

To test the hypothesis $H_0\colon \mu = \mu_0$ based on an SRS of size n from a population with unknown mean μ and known standard deviation σ, compute the test statistic

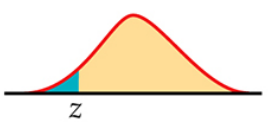
$$z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$$

In terms of a standard Normal random variable Z, the P-value for a test of H_0 against

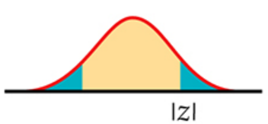
$H_a\colon \mu > \mu_0$ is $P(Z \geq z)$



$H_a\colon \mu < \mu_0$ is $P(Z \leq z)$



$H_a\colon \mu \neq \mu_0$ is $2P(Z \geq |z|)$



These P-values are exact if the population distribution is Normal and are approximately correct for large n in other cases.

Hypothesis Test Decisions Based on the *P*-Value

- Reject H_0 if the *P*-value is small
  (usually 5% or smaller)
- Do not reject H_0 if the *P*-value is large
  (usually: above 10%)
- Grey area: 5-10%
  Decision depends on the particular application.

STATISTICAL SIGNIFICANCE

If the P-value is as small or smaller than α, we say that the data are **statistically significant at level** α.

Hypothesis Testing

- Setup the null- and alternative hypotheses.
- Choose a significance level α (optional).
- Find the appropriate test statistic, and calculate the value using sample data.
- Find the critical (rejection) region.

  OR
- Calculate the *p*-value.
- Make the decision (Reject or do not reject H_0.)
- Interpret/explain the outcome in the context of the question being addressed.

Gas Mileage Data

Statistical Inference: Is μ above 24 [mpg]?

25.3	25.1	29.6	24.6	26.0	26.0
26.3	23.6	26.0	25.4	26.1	23.8
25.1	24.1	25.8	26.4	23.4	24.8
22.6	26.6	25.1	26.6	28.0	23.3
23.8	25.4	26.2	25.1	25.3	21.5

Measured Gas Mileage in [mpg]

$$\bar{x} = 25.23 \text{ [mpg]}$$
$$s = 1.59 \text{ [mpg]}$$

- Setup the null- and alternative hypotheses.

  H_0:

  H_A:

- Choose a significance level α (**skip for now**).

- Find the appropriate test statistic,
  and calculate the value using sample data.

Large Sample Inference Procedures when σ is unknown.

- For large samples (say $n \geq 30$), we may approximate the test statistic using the following approximation.

$$\sigma \approx s$$

Approximate Test Statistic:

$$Z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Approximate 1-α Confidence Interval:

$$\bar{x} \pm z^* \frac{s}{\sqrt{n}}$$

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23.8	25.4	26.2	25.1	25.3	21.5

Measured Gas Mileage in [mpg]

$\bar{x} = 25.23$ [mpg]
$s = 1.59$ [mpg]

- Find the *P*-value.

- Make the decision (Reject or do not reject H_0.)

- Interpret/explain the outcome in the context of the question being addressed.

Water Quality Data

Statistical Inference: Is μ above or below 4.5 [mg/l]?

3.50	3.75	3.00	3.37	3.72	3.96
5.20	5.51	3.46	6.09	1.83	3.75
5.51	3.17	4.68	5.51	5.75	1.60
4.94	1.15	4.46	2.46	2.96	4.20
6.28	3.59	3.21	6.03	4.02	6.95

$\bar{x} = 4.12$ [mg/l]

$\sigma = 1.25$ [mg/l]

Relationship Between Hypothesis Tests and Confidence Intervals

TWO-SIDED SIGNIFICANCE TESTS AND CONFIDENCE INTERVALS

A level α two-sided significance test rejects a hypothesis $H_0: \mu = \mu_0$ exactly when the value μ_0 falls outside a level $1 - \alpha$ confidence interval for μ.