MATH 380 **HOMEWORK SOLUTION SET # 7**

6.78. Finding something to be "statistically significant" is not really useful unless the significance level is sufficiently small. Although there is some freedom to decide what "sufficiently small" means, $\alpha = 0.20$ would lead the student to incorrectly reject H_0 one-fifth of the time, so it is clearly a bad choice.

6.83. **(a)** Power = 0.57. **(b)** Power = 0.81. **(c)** Power = 0.92.

6.84. **(a)** 22.009.

6.88. As the difference between μ_0 and the μ_a increases, so does power.

6.89. **(a)** Changing from the one-sided to the two-sided alternative decreases power (because more evidence is required to reject H_0; consider that the critical value for a one-sided H_a is 1.645, whereas it is 1.96. for a two-sided H_a). **(b)** Decreasing σ increases power (there is less variability in the population, so changes are easier to detect). **(c)** Power increases (larger sample sizes have narrower sampling distributions and hence more power to detect change).

7.6. **(a)** df = $n - 1 = 21$. **(b)** $2.189 < t < 2.518$. **(c)** $0.01 < P\text{-value} < 0.02$. **(d)** $t = 2.24$ is significant at the 5% level but not at the 1% level. **(e)** From software, P-value = 0.0180.

7.7. **(a)** df = $n - 1 = 12$. **(b)** $2.681 < t < 3.055$. **(c)** Because the alternative is two-sided, we double the upper-tail probability to find the P-value: $0.01 < P\text{-value} < 0.02$. **(d)** $t = 2.78$ is significant at the 5% level but not at the 1% level. **(e)** From software, P-value = 0.0167.

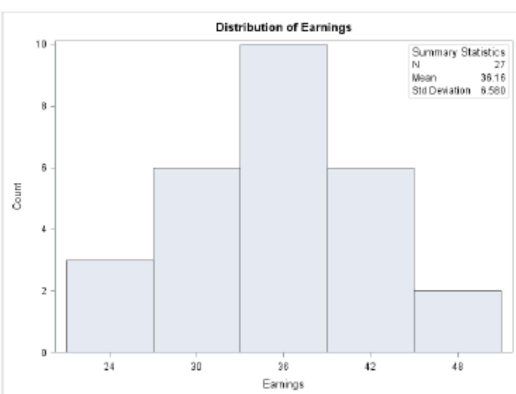
7.8. **(a)** df = $n - 1 = 8$. **(b)** Because $1.397 < |t| < 1.860$, the P-value is between $0.05 < P\text{-value} < 0.10$. **(c)** From software, P-value = 0.0507.

7.10. **(a)** (4.121, 6.518). **(b)** Because $n \geq 40$, we can still use the t procedure for even strongly skewed distributions.

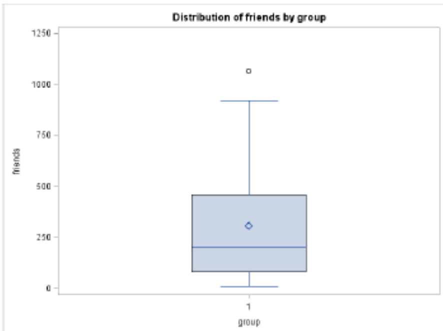
7.11. (17.37, 18.95).

7.12. H_0: $\mu = 19$, H_a: $\mu \neq 19$. $t = \frac{18.16 - 19}{\frac{1.489}{\sqrt{16}}} = -2.27$. Using df = 15, P-value < 0.04 (0.0385 from software). The data are significant at the 5% level; there is evidence that the actual number of miles per gallon is different from the sticker value of 19 mpg.

7.13. **(a)** The histogram below shows the data are Normally distributed, so t procedures are appropriate. **(b)** The 95% confidence interval is 36.157 ± 2.603. **(c)** (33.55, 38.76). **(d)** Inference from the confidence interval applies only to the mean, not to the median. Other tools would need to be used for inference on the median.



7.14. **(a)** The distribution is right-skewed, and the largest observation (1065) stands out from the rest in a boxplot and is an outlier using the 1.5 × *IQR* rule. **(b)** We should hesitate to use *t* procedures with these data because of the skew and outlier (we have a sample size of only n = 30). **(c)** $\bar{x}$ = 305.8 and s = 294.645, so the standard error is $s/\sqrt{30}$ = 53.795. The critical value for 95% confidence is t^* = 2.045, so the margin of error is 110.01. **(d)** With 95% confidence, the mean number of Facebook friends at this university is between 195.79 and 415.81; however, due to the skew and outlier, we hesitate to put much faith in this confidence interval.



7.24. **(a)** For the differences, $\bar{x}$ = \$114 and s = \$114.402. **(b)** We wish to test H_0: $\mu = 0$ versus H_a: $\mu > 0$, where μ is the mean difference between Jocko's estimates and those of the other garage. (The alternative hypothesis is one-sided because the insurance adjusters suspect that Jocko's estimates are too high.) For this test, we find $t = \frac{114 - 0}{114.4/\sqrt{10}} = 3.15$ with df = 9, for which 0.005 < P-value < 0.01 (software gives 0.0059). This is significant evidence against H_0—that is, we have good reason to believe that Jocko's estimates are higher. **(c)** Using df = 9, t^* = 2.262, and the 95% confidence interval is 114 ± 81.83 = (\$32.17, \$195.83). **(d)** Student answers may vary; based on the confidence interval, one could justify any answer in the range \$32.17 to \$195.83.

7.25. **(a)** H_0: $\mu = 0$ versus H_a: $\mu \neq 0$. **(b)** With mean difference $\bar{x} = 2.73$ and standard deviation s = 2.8015, the test statistic is $t = \frac{2.73 - 0}{2.8015/\sqrt{20}} = 4.358$ with df = 19, for which P-value < 0.001 (software gives 0.0003). We have strong evidence that the results of the two computations are different.

7.38. For all tests we have H_0: $\mu_1 = \mu_2$ versus H_a: $\mu_1 \neq \mu_2$. For sprint speed:
$t = \frac{27.3 - 26.0}{\sqrt{\frac{(0.7)^2}{16} + \frac{(1.5)^2}{13}}} = 2.88.$ df = 12, 0.01 < P-value < 0.02. There is evidence of a significant difference in sprint speed between the elite players and the university players. For peak heart rate: $t = \frac{192.0 - 193.0}{\sqrt{\frac{(6.0)^2}{16} + \frac{(6.0)^2}{13}}} = -0.45.$ df = 12, P-value > 0.50. There is not enough evidence to show a difference in peak heart rate between the elite players and the university players. For the intermittent recovery test: $t = \frac{1160 - 781}{\sqrt{\frac{(191)^2}{16} + \frac{(129)^2}{13}}} = 6.35.$ df = 12, P-value < 0.001. There is evidence of a significant difference in the intermittent recovery test between the elite players and the university players.

7.41. (a) The t procedure is robust. Because $n_1 + n_2 = 40$, we can use the t procedures on skewed data. **(b)** H_0: $\mu_{\text{days}} = \mu_{\text{month}}$, H_a: $\mu_{\text{days}} \neq \mu_{\text{month}}$. $t = -2.42$. df = 89; 0.01 < P-value < 0.02.

7.42. H_0: $\mu_{\text{weeks}} = \mu_{\text{year}}$, H_a: $\mu_{\text{weeks}} \neq \mu_{\text{year}}$. $t = -3.31$. 0.002 < P-value < 0.005. The data are significant at the 5% level; there is evidence that the means of the two groups are different. Those who are told 52 weeks have a smaller expectation interval on average than those who are told 1 year.

7.43. H_0: $\mu_{\text{Brown}} = \mu_{\text{Blue}}$, H_a: $\mu_{\text{Brown}} > \mu_{\text{Blue}}$. $t = \dfrac{0.55-(-0.38)}{\sqrt{\dfrac{(1.68)^2}{40}+\dfrac{(1.53)^2}{40}}} = 2.59$. 0.005 < P-value < 0.01 (0.0058 from software). The data show that brown-eyed students appear more trustworthy compared with their blue-eyed counterparts.

7.44. (a) With $n = 1839$, $t^* = 1.961$ (software, df = 1838) or 1.962 (Table D, df = 1000). The confidence interval is $706 \pm t^*(526/\sqrt{1839}) = 681.95$ and 730.05. At 95% confidence, students spend an average of between 681.95 and 730.05 minutes per week preparing for class. **(b)** To convert minutes per day to minutes per week, we multiply the mean by 7 but add the variance 7 times (each day is a separate realization of a random "experiment"). This gives $\bar{x} = 7(106) = 742$ and $s = \sqrt{7(93)^2} = 246.0549$. The 95% confidence interval for the average minutes per week on Facebook is $742 \pm t^*(246.0549/\sqrt{1839}) = 730.75$ to 753.25. **(c)** If these distributions were Normal, the 68–95–99.7 rule would have to hold true. However, in the distribution of study times, minutes per week becomes negative (impossible) 1.34 standard deviations below the mean. In the Facebook daily distribution, the time spent on Facebook becomes negative 1.14 standard deviations below the mean.

7.58. (a) The east distribution is right-skewed, whereas the west distribution is left-skewed. **(b)** The methods of this section seem to be appropriate in spite of the skewness, because the sample sizes are relatively large, and there are no outliers in either distribution. **(c)** We test H_0: $\mu_E = \mu_W$ versus H_a: $\mu_E \neq \mu_W$; we should use a two-sided alternative because we have no reason (before looking at the data) to expect a difference in a particular direction. **(d)** The means and standard deviations are $\bar{x}_E = 21.716$, $s_E = 16.0743$, $\bar{x}_W = 30.283$, and $s_W = 15.3314$ cm. Then SE_D = 4.0556, so $t = -2.112$ with df = 29, 0.04 < P-value < 0.05 (0.0434 from software). We conclude that the means are different. **(e)** The 95% confidence interval is (−16.8604, −0.2730) [software

gives (−16.6852, −0.4481)]. The interval tells us not only that a difference exists but also that the eastern trees are, on average, between about 0.3 and 16.8 cm smaller in dbh than the trees in the western part of the tract.