

PART 1. DESCRIPTIVE STATISTICS

NOTE: A calculator with statistical functions may be used for the following calculations

- Sample Mean \bar{x}
- Sample Variance s^2
- Sample Standard Deviation s

For all other measures (median, quartiles etc.) all relevant work and calculations must be shown.

SAMPLE MEDIAN (M)

- Arrange all observations in order $x^{(1)}, x^{(2)}, \dots, x^{(n)}$.
- The median is observation number $(n+1)/2$ in the ordered list. (If n is even M is the average of the two observations closest to number $(n+1)/2$)

FIVE NUMBER SUMMARY

MIN: The smallest observation.

Q_1 : The median of the observations located to the left of (below) the median*

M : The median (see above).

Q_3 : The median of the observations located to the right of (above) the median*

MAX: The largest observation.

* not including the median itself.

1.5 IQR CRITERION

- Observations greater than $Q_3 + 1.5IQR$ or less than $Q_1 - 1.5IQR$ are considered (suspected) outliers. ($IQR = Q_3 - Q_1$)

PART 2. PROBABILITY THEORY

SAMPLE SPACE, EVENTS

- The sample space S is the collection (set) of all sample points, s_i representing the basic outcomes that the events are composed from.

$$S = \{s_1, s_2, \dots\}$$

NOTE: The sample space can be described using roster notation (like the above) or using a Venn-Diagram.

- An event A is a collection of one or more sample points.
(except the *impossible event* $\emptyset = \{\}$)
- A *simple event* is an event that consists of exactly one sample point, $E_i = \{s_i\}$.
NOTE: The simple events need not be equally likely, although it is desirable to construct a sample space for which that is the case.

BASIC PROBABILITY RULES

Rule 1 : For any event $0 \leq P(A) \leq 1$

Rule 2 : $P(S) = \sum_i P(E_i) = 1$

Rule 3 : For disjoint (mutually exclusive) events $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$

Rule 4 : For any event $P(A^c) = 1 - P(A)$

Rule 5 : For independent events $P(A \cap B) = P(A \text{ and } B) = P(A) \cdot P(B)$

SAMPLE SPACE WITH n EQUALLY LIKELY SIMPLE EVENTS

$$P(E_i) = \frac{1}{n}$$

If the event A consists of m of the n equally likely simple events then

$$P(A) = \frac{m}{n}$$

GENERAL SAMPLE SPACE (WITH EQUALLY OR UNEQUALLY LIKELY SIMPLE EVENTS)

$$P(E_i) = \text{Long Term Relative Frequency of } E_i$$

If the event A consists of (is the union of) the simple events E_1, E_2, \dots, E_m

$$P(A) = \sum_{i=1}^m P(E_i)$$

CONDITIONAL PROBABILITY

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \quad \text{if } P(B) > 0$$

BAYES' RULE/LAW OF TOTAL PROBABILITY

$$P(A | B) = P(B | A) \cdot \frac{P(A)}{P(B)} \quad \text{if } P(A) > 0 \text{ and } P(B) > 0$$

$$P(B) = P(B | A) \cdot P(A) + P(B | A^c) \cdot P(A^c) \quad \text{if } 0 < P(A) < 1$$

GENERAL RULES FOR UNION/INTERSECTION PROBABILITIES

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B | A) = P(B) \cdot P(A | B)$$

DISCRETE RANDOM VARIABLE (FINITE CASE)

X can assume values x_1, x_2, \dots, x_k

$$P(X = x_i) = p_i \quad 0 \leq p_i \leq 1$$

$$\sum_i p_i = p_1 + p_2 + \dots + p_k = 1$$

CONTINUOUS RANDOM VARIABLE

X can assume all real values over an interval

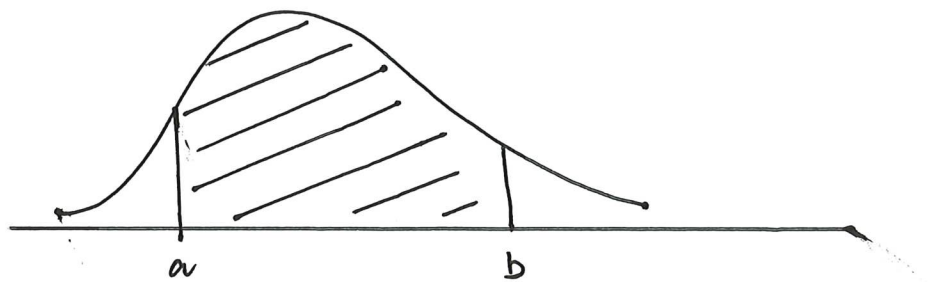
$P(X \leq a) = P(X < a) = \text{area under density curve to the left of } a.$

$P(X \geq a) = P(X > a) = \text{area under density curve to the right of } a.$

$P(a \leq X \leq b) = \text{area under density curve between } a \text{ and } b.$

$P(X = a) = 0$

$P(-\infty < X < \infty) = \text{total area under density curve} = 1$



POPULATION MEAN/VARIANCE (FINITE DISCRETE RANDOM VARIABLE)

$$\mu_X = \sum_i x_i p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k$$

$$\sigma_X^2 = \sum_i (x_i - \mu_X)^2 p_i = \mu_{X^2} - (\mu_X)^2 = \left(\sum_i x_i^2 p_i \right) - (\mu_X)^2$$

$$\sigma_X = \sqrt{\sigma_X^2}$$

RULES FOR MEANS

$$\text{If } Z = a + bX \quad \text{then} \quad \mu_Z = a + b\mu_X$$

$$\text{If } Z = X + Y \quad \text{then} \quad \mu_Z = \mu_X + \mu_Y$$

$$\text{If } Z = X - Y \quad \text{then} \quad \mu_Z = \mu_X - \mu_Y$$

RULES FOR VARIANCES

$$\text{If } Z = a + bX \quad \text{then} \quad \sigma_Z^2 = b^2\sigma_X^2$$

For the following two rules, assume X and Y are independent random variables :

$$\text{If } Z = X + Y \quad \text{then} \quad \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$$

$$\text{If } Z = X - Y \quad \text{then} \quad \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$$

THE STANDARD NORMAL DISTRIBUTION

$Z \sim N(0, 1)$: Z is normally distributed with mean 0 and standard deviation 1.

$$P(Z \leq z) = \Phi(z) \quad (\text{table value of } z \text{ using Table A})$$

THE NON-STANDARD NORMAL DISTRIBUTION

$X \sim N(\mu, \sigma)$: X is normally distributed with mean μ and standard deviation σ

$$\text{If } Z = \frac{X - \mu}{\sigma} \quad \text{then} \quad Z \sim N(0, 1)$$

$$P(X \leq a) = \Phi(z_a) \quad (\text{table value of } z_a \text{ from Table A})$$

$$P(a \leq X \leq b) = \Phi(z_b) - \Phi(z_a)$$

$$\text{where } z_a = \frac{a - \mu}{\sigma} \quad z_b = \frac{b - \mu}{\sigma} \quad (z\text{-scores})$$

BINOMIAL EXPERIMENT

- The experiment consists of a fixed number (n) of trials (Bernoulli Trials).
- Each trial has two possible outcomes labeled S=Success and F=Failure.
- The probability of success, p , is the same for all trials.
- The n trials are independent.
- The random variable of interest, X , counts the number of *successes*.

BINOMIAL FORMULA

$X \sim B(n, p)$: X is binomial distributed with n trials and probability of success p

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0, 1, \dots, n$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

MEAN AND VARIANCE FOR THE BINOMIAL DISTRIBUTION

$$\mu_X = np$$

$$\sigma_X^2 = np(1-p) \quad \sigma_X = \sqrt{np(1-p)}$$

NORMAL APPROXIMATION TO THE BINOMIAL DISTRIBUTION*

$$P(X \leq k) \approx \Phi(z_k) \quad \text{where}$$

$$z_k = \frac{k - \mu_X}{\sigma_X}$$

*) Without using continuity correction (valid if $np \geq 10$ and $n(1-p) \geq 10$).

PART 3. INFERENCE STATISTICS

SAMPLING DISTRIBUTION OF THE SAMPLE MEAN \bar{X}

ASSUMPTIONS : The sample mean is calculated from n independent observations with mean μ and standard deviation σ . Furthermore, the observations are either normal distributed OR the sample size n is large.

$$\bar{X} \sim N(\mu_{\bar{X}}, \sigma_{\bar{X}})$$

where

$$\mu_{\bar{X}} = \mu \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

SAMPLING DISTRIBUTION OF THE SAMPLE PROPORTION \hat{p}

ASSUMPTIONS : The sample proportion is calculated from a large sample binomial experiment such that $np \geq 10$ and $n(1-p) \geq 10$.

$$\hat{p} = \frac{X}{n} \quad X \sim B(n, p)$$

$$\hat{p} \sim N(\mu_{\hat{p}}, \sigma_{\hat{p}}) \quad \text{where}$$

$$\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

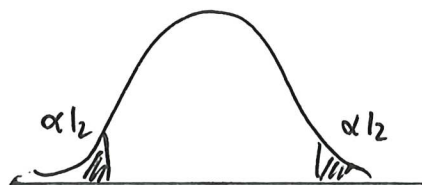
TESTING OF HYPOTHESES

- State the appropriate null- and alternative hypotheses.
- Select the desired significance level α (optional).
- Determine the appropriate test statistic and calculate the value.
- Determine the rejection region for the test statistic, and check whether the observed test statistic lies in the rejection region or not, OR calculate the P -value of the observed test statistic.

- State the appropriate conclusion and interpret the result in the context of the question originally posed.

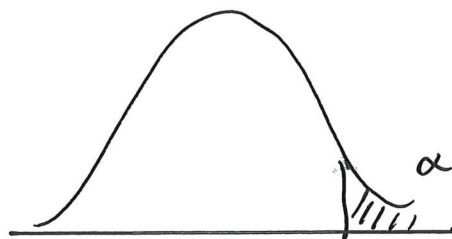
$$H_0: \theta = \theta_0$$

$$H_a: \theta \neq \theta_0$$



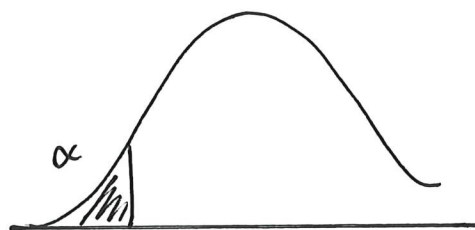
$$H_0: \theta = \theta_0$$

$$H_A: \theta > \theta_0$$



$$H_0: \theta = \theta_0$$

$$H_A: \theta < \theta_0$$



P-VALUES

The P -value of a test statistic is equal to the *smallest significance level α at which the null-hypothesis is rejected*.

For a normal distributed test statistic, the P -value can be found using the observed value of the test statistic z_{obs} as

$$P = 2 P(Z > |z_{\text{obs}}|) \quad (\text{two-sided alternative})$$

$$P = P(Z > z_{\text{obs}}) \quad (\text{upper one-sided } (>) \text{ alternative})$$

$$P = P(Z < z_{\text{obs}}) \quad (\text{lower one-sided } (<) \text{ alternative})$$

When the test statistic is t -distributed replace all z 's with t 's and use the t -table to find a bracket for the P -value.

SAMPLE SIZE FOR DESIRED MARGIN OF ERROR m

a) When estimating μ

$$n = \left(\frac{z^* \sigma}{m} \right)^2$$

NOTE: σ (or at least an estimate or “guessed” value of σ) must be provided in order to calculate the sample size.

b) When estimating p

$$n = \tilde{p}(1 - \tilde{p}) \left(\frac{z^*}{m} \right)^2$$

NOTE: \tilde{p} is an estimate or “guessed” value of p . If not available you may use the conservative value $\tilde{p} = 0.5$.

TABLE OF CONFIDENCE INTERVALS

Unknown Target Parameter	Assumptions/ Special Conditions	Confidence Interval	Degrees of Freedom for Critical Value (Table E)
μ	σ is given (1)	$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$	∞
μ	σ is unknown	$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$	$n - 1$
μ_d	Matched Pairs Difference σ is unknown	$\bar{d} \pm t^* \frac{s_d}{\sqrt{n}}$	$n - 1$
$\mu_1 - \mu_2$	Independent Samples σ 's are given	$\bar{x}_1 - \bar{x}_2 \pm z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	∞
$\mu_1 - \mu_2$	Independent Samples σ 's are unknown	$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$\min(n_1 - 1, n_2 - 1)$
$\mu_1 - \mu_2$	Independent Samples Pooled $\sigma_1 = \sigma_2$ (unknown)	$\bar{x}_1 - \bar{x}_2 \pm t^* s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$	$n_1 + n_2 - 2$
p	Large Sample $n\hat{p} \geq 10$ $n(1 - \hat{p}) \geq 10$	$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	∞
$p_1 - p_2$	Large Sample $n_1\hat{p}_1 \geq 10$ $n_1(1 - \hat{p}_1) \geq 10$ $n_2\hat{p}_2 \geq 10$ $n_2(1 - \hat{p}_2) \geq 10$	$\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	∞

(1) For large samples, say $n \geq 30$, we may use the approximation $\sigma \approx s$.

TABLE OF TEST-STATISTICS

Null Hypothesis H_0	Alternative Hypothesis H_A	Assumptions/ Special Conditions	Test Statistic	Distribution of Test Statistic (H_0 true)
$\mu = \mu_0$	$\mu \neq \mu_0$ $\mu > \mu_0$ $\mu < \mu_0$	σ is given (1)	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$N(0,1)$
$\mu = \mu_0$	$\mu \neq \mu_0$ $\mu > \mu_0$ $\mu < \mu_0$	σ is unknown	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t(df)$ $df = n - 1$
$\mu_d = 0$	$\mu_d \neq 0$ $\mu_d > 0$ $\mu_d < 0$	Matched Pairs σ is unknown	$t = \frac{\bar{d}}{s_d/\sqrt{n}}$	$t(df)$ $df = n - 1$
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	Independent Samples σ 's are given	$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$N(0,1)$
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	Independent Samples σ 's are unknown	$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$	$t(df)$ (approx) $df = \min(n_1 - 1, n_2 - 1)$
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	Independent Samples Pooled $\sigma_1 = \sigma_2$ (unknown)	$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2(1/n_1 + 1/n_2)}}$ $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$	$t(df)$ $df = n_1 + n_2 - 2$
$p = p_0$	$p \neq p_0$ $p > p_0$ $p < p_0$	Large Sample $np_0 \geq 10$ $n(1 - p_0) \geq 10$	$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$	$N(0,1)$ (approx)
$p_1 = p_2$	$p_1 \neq p_2$ $p_1 > p_2$ $p_1 < p_2$	Large Sample $n_1\hat{p} \geq 10$ $n_1(1 - \hat{p}) \geq 10$ $n_2\hat{p} \geq 10$ $n_2(1 - \hat{p}) \geq 10$	$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}}$ $\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$	$N(0,1)$ (approx)

Table entry for z is the area under the standard Normal curve to the left of z .

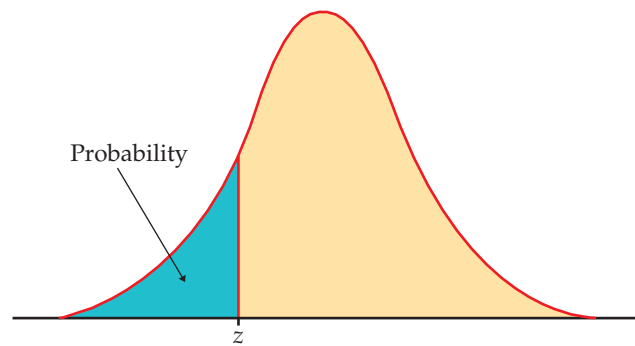


TABLE A Standard Normal Probabilities										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard Normal curve to the left of z .

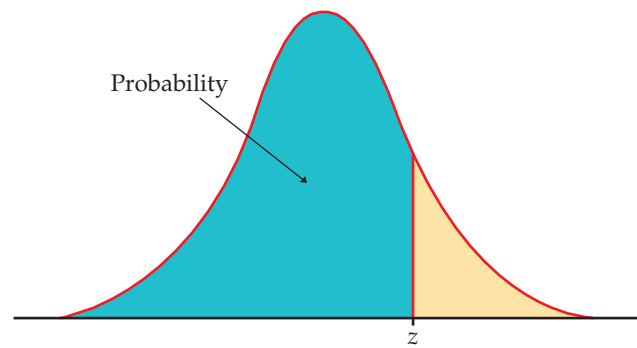


TABLE A Standard Normal Probabilities (continued)										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE C		Binomial Probabilities								
		Entry is $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$								
		<i>p</i>								
<i>n</i>	<i>k</i>	.01	.02	.03	.04	.05	.06	.07	.08	.09
2	0	.9801	.9604	.9409	.9216	.9025	.8836	.8649	.8464	.8281
	1	.0198	.0392	.0582	.0768	.0950	.1128	.1302	.1472	.1638
	2	.0001	.0004	.0009	.0016	.0025	.0036	.0049	.0064	.0081
3	0	.9703	.9412	.9127	.8847	.8574	.8306	.8044	.7787	.7536
	1	.0294	.0576	.0847	.1106	.1354	.1590	.1816	.2031	.2236
	2	.0003	.0012	.0026	.0046	.0071	.0102	.0137	.0177	.0221
	3				.0001	.0001	.0002	.0003	.0005	.0007
4	0	.9606	.9224	.8853	.8493	.8145	.7807	.7481	.7164	.6857
	1	.0388	.0753	.1095	.1416	.1715	.1993	.2252	.2492	.2713
	2	.0006	.0023	.0051	.0088	.0135	.0191	.0254	.0325	.0402
	3			.0001	.0002	.0005	.0008	.0013	.0019	.0027
	4									.0001
5	0	.9510	.9039	.8587	.8154	.7738	.7339	.6957	.6591	.6240
	1	.0480	.0922	.1328	.1699	.2036	.2342	.2618	.2866	.3086
	2	.0010	.0038	.0082	.0142	.0214	.0299	.0394	.0498	.0610
	3		.0001	.0003	.0006	.0011	.0019	.0030	.0043	.0060
	4						.0001	.0001	.0002	.0003
	5									
6	0	.9415	.8858	.8330	.7828	.7351	.6899	.6470	.6064	.5679
	1	.0571	.1085	.1546	.1957	.2321	.2642	.2922	.3164	.3370
	2	.0014	.0055	.0120	.0204	.0305	.0422	.0550	.0688	.0833
	3		.0002	.0005	.0011	.0021	.0036	.0055	.0080	.0110
	4					.0001	.0002	.0003	.0005	.0008
	5									
	6									
7	0	.9321	.8681	.8080	.7514	.6983	.6485	.6017	.5578	.5168
	1	.0659	.1240	.1749	.2192	.2573	.2897	.3170	.3396	.3578
	2	.0020	.0076	.0162	.0274	.0406	.0555	.0716	.0886	.1061
	3		.0003	.0008	.0019	.0036	.0059	.0090	.0128	.0175
	4				.0001	.0002	.0004	.0007	.0011	.0017
	5								.0001	.0001
	6									
	7									
8	0	.9227	.8508	.7837	.7214	.6634	.6096	.5596	.5132	.4703
	1	.0746	.1389	.1939	.2405	.2793	.3113	.3370	.3570	.3721
	2	.0026	.0099	.0210	.0351	.0515	.0695	.0888	.1087	.1288
	3	.0001	.0004	.0013	.0029	.0054	.0089	.0134	.0189	.0255
	4			.0001	.0002	.0004	.0007	.0013	.0021	.0031
	5							.0001	.0001	.0002
	6									
	7									
	8									

TABLE C		Binomial Probabilities (continued)								
		Entry is $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$								
		<i>p</i>								
<i>n</i>	<i>k</i>	.10	.15	.20	.25	.30	.35	.40	.45	.50
2	0	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500
	1	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000
	2	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
3	0	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250
	1	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750
	2	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750
	3	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250
4	0	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625
	1	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500
	2	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750
	3	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500
	4	.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625
5	0	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313
	1	.3280	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563
	2	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125
	3	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125
	4	.0004	.0022	.0064	.0146	.0284	.0488	.0768	.1128	.1562
	5		.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0312
6	0	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277	.0156
	1	.3543	.3993	.3932	.3560	.3025	.2437	.1866	.1359	.0938
	2	.0984	.1762	.2458	.2966	.3241	.3280	.3110	.2780	.2344
	3	.0146	.0415	.0819	.1318	.1852	.2355	.2765	.3032	.3125
	4	.0012	.0055	.0154	.0330	.0595	.0951	.1382	.1861	.2344
	5	.0001	.0004	.0015	.0044	.0102	.0205	.0369	.0609	.0937
	6			.0001	.0002	.0007	.0018	.0041	.0083	.0156
7	0	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152	.0078
	1	.3720	.3960	.3670	.3115	.2471	.1848	.1306	.0872	.0547
	2	.1240	.2097	.2753	.3115	.3177	.2985	.2613	.2140	.1641
	3	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734
	4	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734
	5	.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641
	6		.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547
	7				.0001	.0002	.0006	.0016	.0037	.0078
8	0	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039
	1	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0313
	2	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094
	3	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188
	4	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734
	5	.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188
	6		.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094
	7			.0001	.0004	.0012	.0033	.0079	.0164	.0312
	8					.0001	.0002	.0007	.0017	.0039

(Continued)

TABLE C		Binomial Probabilities (continued)								
		Entry is $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$								
		<i>p</i>								
<i>n</i>	<i>k</i>	.01	.02	.03	.04	.05	.06	.07	.08	.09
9	0	.9135	.8337	.7602	.6925	.6302	.5730	.5204	.4722	.4279
	1	.0830	.1531	.2116	.2597	.2985	.3292	.3525	.3695	.3809
	2	.0034	.0125	.0262	.0433	.0629	.0840	.1061	.1285	.1507
	3	.0001	.0006	.0019	.0042	.0077	.0125	.0186	.0261	.0348
	4			.0001	.0003	.0006	.0012	.0021	.0034	.0052
	5						.0001	.0002	.0003	.0005
	6									
	7									
	8									
	9									
10	0	.9044	.8171	.7374	.6648	.5987	.5386	.4840	.4344	.3894
	1	.0914	.1667	.2281	.2770	.3151	.3438	.3643	.3777	.3851
	2	.0042	.0153	.0317	.0519	.0746	.0988	.1234	.1478	.1714
	3	.0001	.0008	.0026	.0058	.0105	.0168	.0248	.0343	.0452
	4			.0001	.0004	.0010	.0019	.0033	.0052	.0078
	5					.0001	.0001	.0003	.0005	.0009
	6									.0001
	7									
	8									
	9									
	10									
12	0	.8864	.7847	.6938	.6127	.5404	.4759	.4186	.3677	.3225
	1	.1074	.1922	.2575	.3064	.3413	.3645	.3781	.3837	.3827
	2	.0060	.0216	.0438	.0702	.0988	.1280	.1565	.1835	.2082
	3	.0002	.0015	.0045	.0098	.0173	.0272	.0393	.0532	.0686
	4		.0001	.0003	.0009	.0021	.0039	.0067	.0104	.0153
	5				.0001	.0002	.0004	.0008	.0014	.0024
	6							.0001	.0001	.0003
	7									
	8									
	9									
	10									
	11									
	12									
15	0	.8601	.7386	.6333	.5421	.4633	.3953	.3367	.2863	.2430
	1	.1303	.2261	.2938	.3388	.3658	.3785	.3801	.3734	.3605
	2	.0092	.0323	.0636	.0988	.1348	.1691	.2003	.2273	.2496
	3	.0004	.0029	.0085	.0178	.0307	.0468	.0653	.0857	.1070
	4		.0002	.0008	.0022	.0049	.0090	.0148	.0223	.0317
	5			.0001	.0002	.0006	.0013	.0024	.0043	.0069
	6						.0001	.0003	.0006	.0011
	7								.0001	.0001
	8									
	9									
	10									
	11									
	12									
	13									
	14									
	15									

TABLE C		Binomial Probabilities (continued)								
		Entry is $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$								
		<i>p</i>								
<i>n</i>	<i>k</i>	.10	.15	.20	.25	.30	.35	.40	.45	.50
9	0	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7			.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8				.0001	.0004	.0013	.0035	.0083	.0176
	9						.0001	.0003	.0008	.0020
10	0	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7		.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8			.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9					.0001	.0005	.0016	.0042	.0098
	10							.0001	.0003	.0010
12	0	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7		.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8		.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9			.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10					.0002	.0008	.0025	.0068	.0161
	11						.0001	.0003	.0010	.0029
	12							.0001	.0001	.0002
15	0	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	.0000
	1	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005
	2	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032
	3	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139
	4	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417
	5	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916
	6	.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527
	7	.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964
	8		.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964
	9		.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527
	10			.0001	.0007	.0030	.0096	.0245	.0515	.0916
	11				.0001	.0006	.0024	.0074	.0191	.0417
	12					.0001	.0004	.0016	.0052	.0139
	13						.0001	.0003	.0010	.0032
	14							.0001	.0001	.0005
	15									

(Continued)

TABLE C		Binomial Probabilities (continued)								
<i>n</i>	<i>k</i>	<i>p</i>								
		.01	.02	.03	.04	.05	.06	.07	.08	.09
20	0	.8179	.6676	.5438	.4420	.3585	.2901	.2342	.1887	.1516
	1	.1652	.2725	.3364	.3683	.3774	.3703	.3526	.3282	.3000
	2	.0159	.0528	.0988	.1458	.1887	.2246	.2521	.2711	.2818
	3	.0010	.0065	.0183	.0364	.0596	.0860	.1139	.1414	.1672
	4		.0006	.0024	.0065	.0133	.0233	.0364	.0523	.0703
	5			.0002	.0009	.0022	.0048	.0088	.0145	.0222
	6				.0001	.0003	.0008	.0017	.0032	.0055
	7						.0001	.0002	.0005	.0011
	8								.0001	.0002
	9									
	10									
	11									
	12									
	13									
	14									
	15									
	16									
	17									
	18									
	19									
	20									

<i>n</i>	<i>k</i>	<i>p</i>								
		.10	.15	.20	.25	.30	.35	.40	.45	.50
20	0	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000
	1	.2702	.1368	.0576	.0211	.0068	.0020	.0005	.0001	.0000
	2	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002
	3	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011
	4	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046
	5	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148
	6	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370
	7	.0020	.0160	.0545	.1124	.1643	.1844	.1659	.1221	.0739
	8	.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201
	9	.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602
	10		.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762
	11			.0005	.0030	.0120	.0336	.0710	.1185	.1602
	12			.0001	.0008	.0039	.0136	.0355	.0727	.1201
	13				.0002	.0010	.0045	.0146	.0366	.0739
	14					.0002	.0012	.0049	.0150	.0370
	15						.0003	.0013	.0049	.0148
	16							.0003	.0013	.0046
	17								.0002	.0011
	18									.0002
	19									
	20									

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^* .

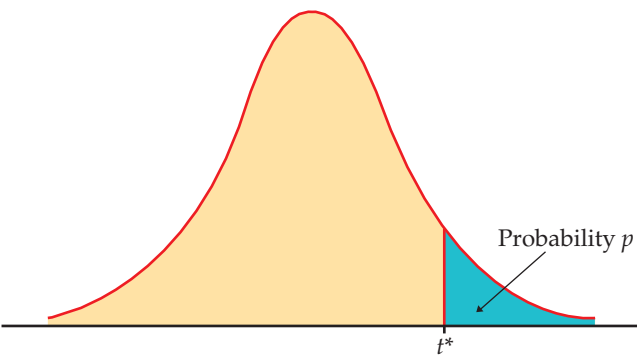


TABLE D t Distribution Critical Values												
df	Upper-tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											