Independent Events

Definition:

Two events *A* and *B* are said to be **independent** iff knowing that *A* will occur does not impact the probability that *B* will occur.

$$P(\mathrm{B} \mid \mathrm{A}) = P(\mathrm{B})$$
$$P(\mathrm{A} \mid \mathrm{B}) = P(\mathrm{A})$$

Notes:

- If *A* is independent of *B* then also *B* is independent of *A*.
- *A* and *B* are independent iff:

$$\boxed{P(A \cap B) = P(A) \cdot P(B) \text{ (Rule 5)}}$$

Generalizing Rules 3 and 5

General Rule 3: If *A, B, C* … are (pairwise) disjoint then

$$P(A \cup B \cup C \ldots) = P(A) + P(B) + P(C) + \ldots$$

General Rule 5: If *A, B, C* … are (pairwise) independent then

$$P(A \cap B \cap C \ldots) = P(A) \cdot P(B) \cdot P(C) \cdot \ldots$$

General Multiplication Rule
(for two events not necessarily independent)

General Multiplication Rule: Let A and B be two events defined in sample space *S*.

$$P(A \cap B) = P(B) \cdot P(A|B) \qquad \text{or}$$

$$P(A \cap B) = P(B \cap A) = P(A) \cdot P(B|A)$$

Note: If the events are independent then the rules simplify to Rule 5

$$P(A \cap B) = P(A) \cdot P(B)$$

General Multiplication Rule
(for two or more events not necessarily independent)

General Multiplication Rule: Let A, B, C ... be events defined in sample space *S*.

$$P(A \cap B \cap C \cap \cdots) = P(A) \cdot P(B|A) \cdot P(C|A \cap B) \cdots$$

Note: If all events are independent then

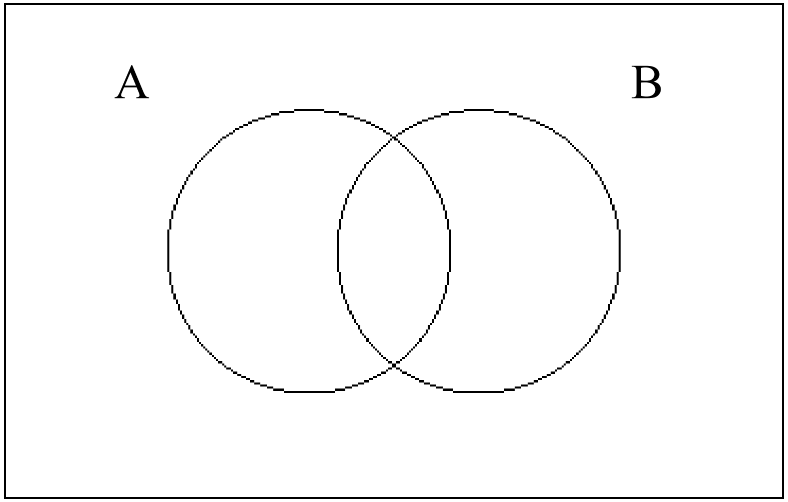
$$P(A \cap B \cap C \cap \cdots) = P(A) \cdot P(B) \cdot P(C) \cdots$$

General Addition Rule
(for two events not necessarily disjoint)

General Addition Rule: Let A and B be two events defined in a sample space *S*:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Illustration:



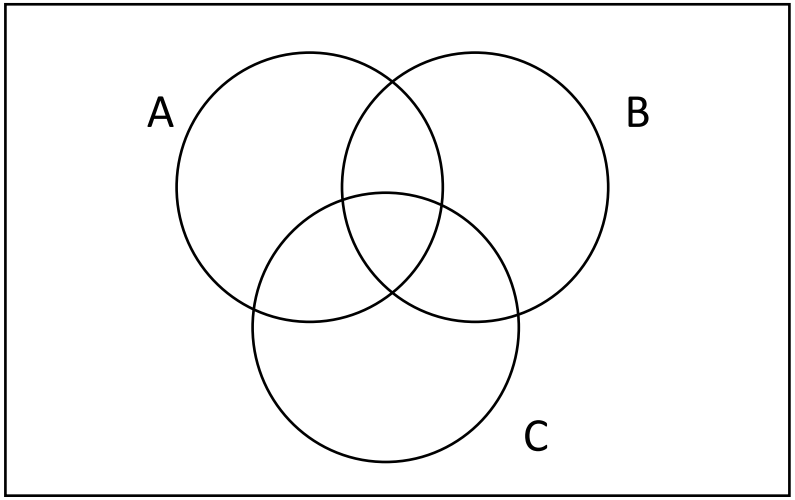
Note: If two events A and B are disjoint then the rule simplifies to Rule 3

$$P(A \cup B) = P(A) + P(B)$$

General Addition Rule
(for three events not necessarily disjoint)

General Addition Rule: Let A, B and C be events defined in a sample space *S*:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$-P(A \cap B) - P(A \cap C) - P(B \cap C)$$
$$+P(A \cap B \cap C)$$



Note: If all events are disjoint then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

Let’s Make a Bet

- You are offered a bet on whether you can roll a six in <u>no more than</u> three attempts using a fair die.

- Would you be better off betting against this outcome? What is the probability of winning?

Families and Health Insurance Coverage.

- 40% of all families with children have no health insurance.
- 30% of all families have children and no health insurance.
- 95% of all families have children or health insurance.

Let A = The family has children

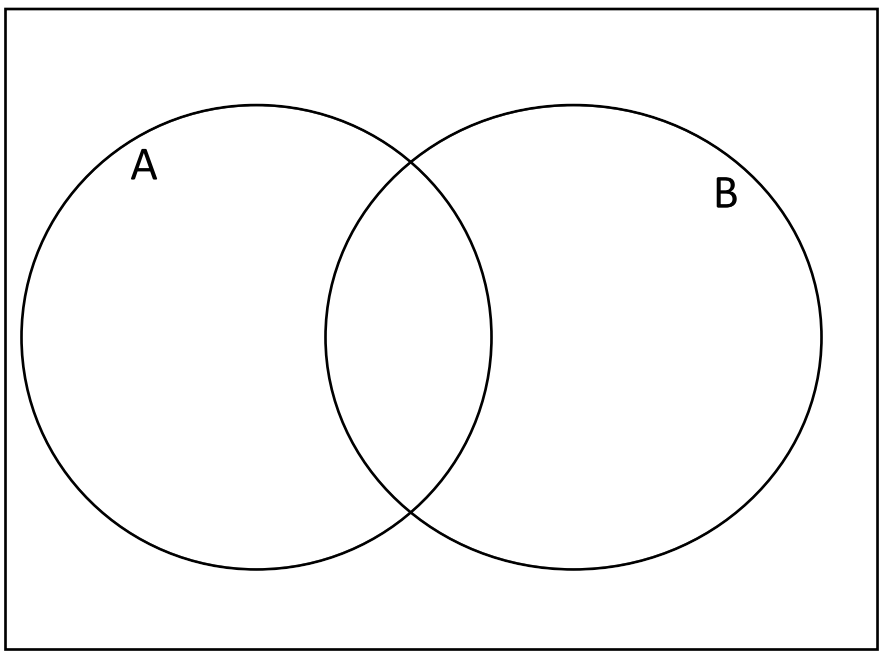
B = The family has health insurance.

Find P(A) and P(B)

Families and Health Insurance Coverage.

Let A = The family has children

B = The family has health insurance.

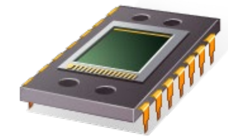


$P(B^c|A) = 0.4$

$P(B^c \cap A) = 0.3$

$P(A \cup B) = 0.95$

CASE: Parts from Two Vendors



- A manufacturer buys parts from two vendors A and B.
- 25% of the parts come from Vendor A and 75% come from Vendor B.
- 10% of the parts from Vendor A are defective.
- 5% of the parts from Vendor B are defective.
- What is the overall percentage of parts that are defective?
- If a part was found to be defective what is the probability it came from Vendor A?