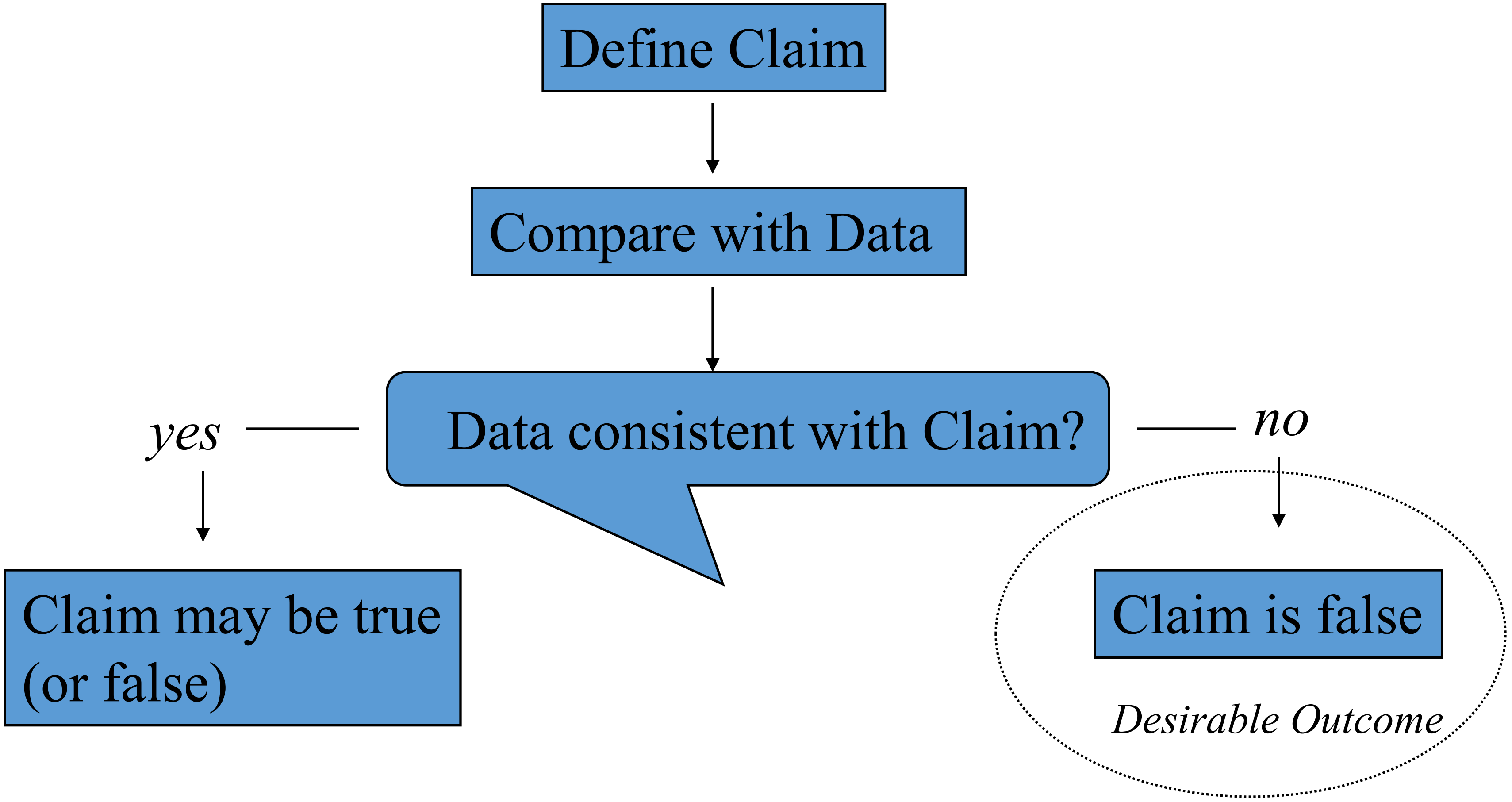
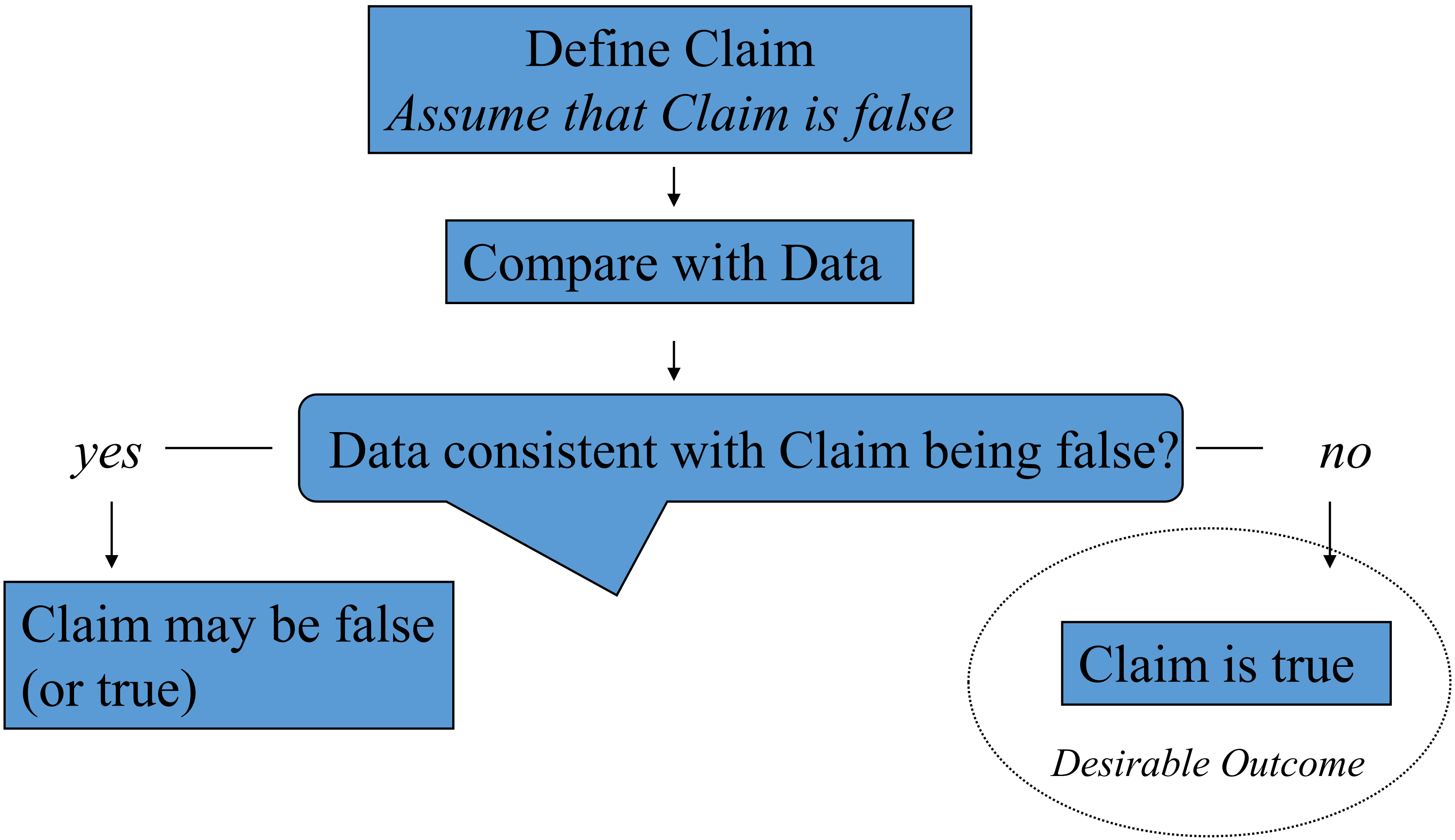
Hypothesis Testing

Scenario 1: Show that a claim is false



Hypothesis Testing

Scenario 2: Show that a claim is true



Hypothesis Testing

- Setup the null- and alternative hypotheses.
- Choose a significance level α.
- Find the appropriate test statistic, and calculate the value using sample data.
- Find the critical (rejection) region.
- Make the decision (Reject or do not reject H_0.)
- Interpret/explain the outcome in the context of the question being addressed.

Null & Alternative Hypotheses

There are two hypotheses involved in making a decision:

Null Hypothesis, H_o: The hypothesis to be tested. Assumed to be true. Usually a statement that a population parameter has a specific value. The "starting point" for the investigation.

Alternative Hypothesis, H_a: A statement about the same population parameter that is used in the null hypothesis. Generally this is a statement that specifies the population parameter has a value different, in some way, from the value given in the null hypothesis. The rejection of the null hypothesis will imply the likely truth of this alternative hypothesis.

Test Statistic

Test Statistic: Measures the difference between what has been observed and what was assumed and is used to determine if the null hypothesis is true.

$$z = \frac{\text{estimate} - \text{hypothesized value}}{\text{standard deviation of the estimate}}$$

A **test statistic** measures compatibility between the null hypothesis and the data. We use it for the probability calculation that we need for our test of significance. It is a **random variable** with a **distribution** that we know.

$H_0: \mu = \mu_0$

$H_a: \mu \neq \mu_0$

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

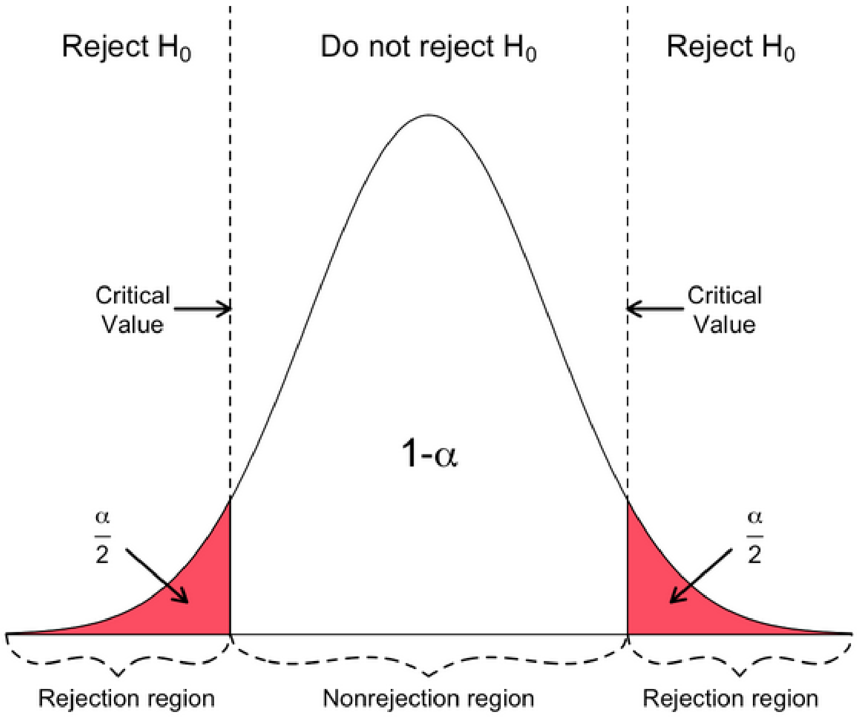
$Z \sim N(0,1)$ if H_0 is true

Critical Region

Critical Region: Identifies values of the test statistic that are inconsistent with the null hypothesis. The null hypothesis is rejected if the test statistic is in the critical region.

$H_0: \mu = \mu_0$

$H_a: \mu \neq \mu_0$



Critical Region

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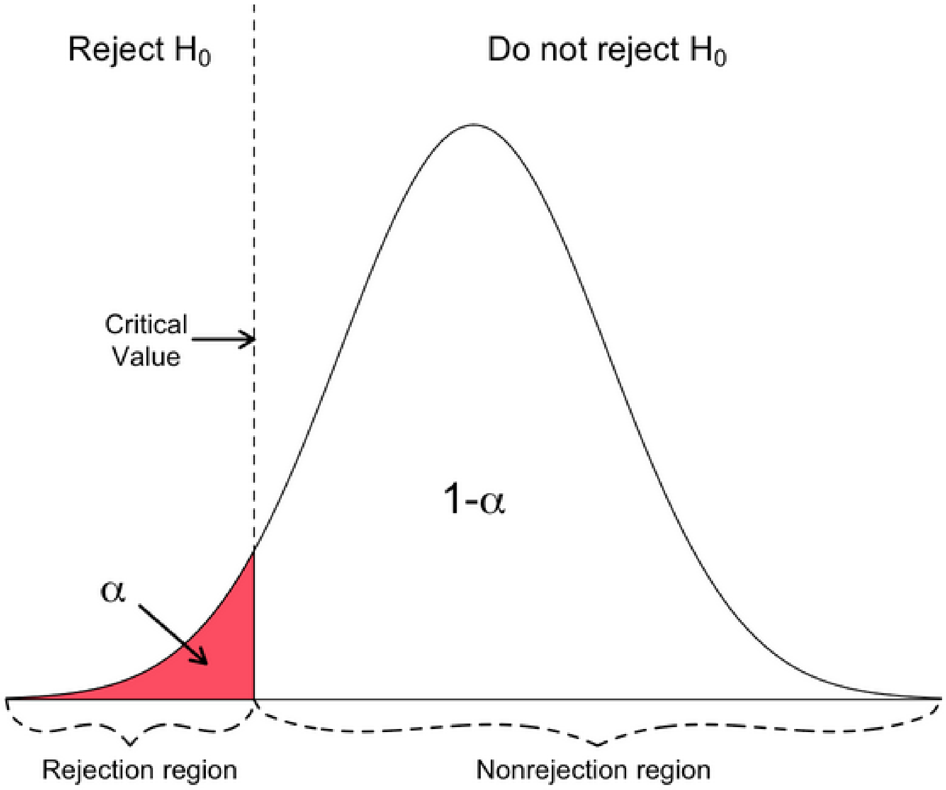
$H_0: \mu = \mu_0$

$H_a: \mu < \mu_0$



Critical Region

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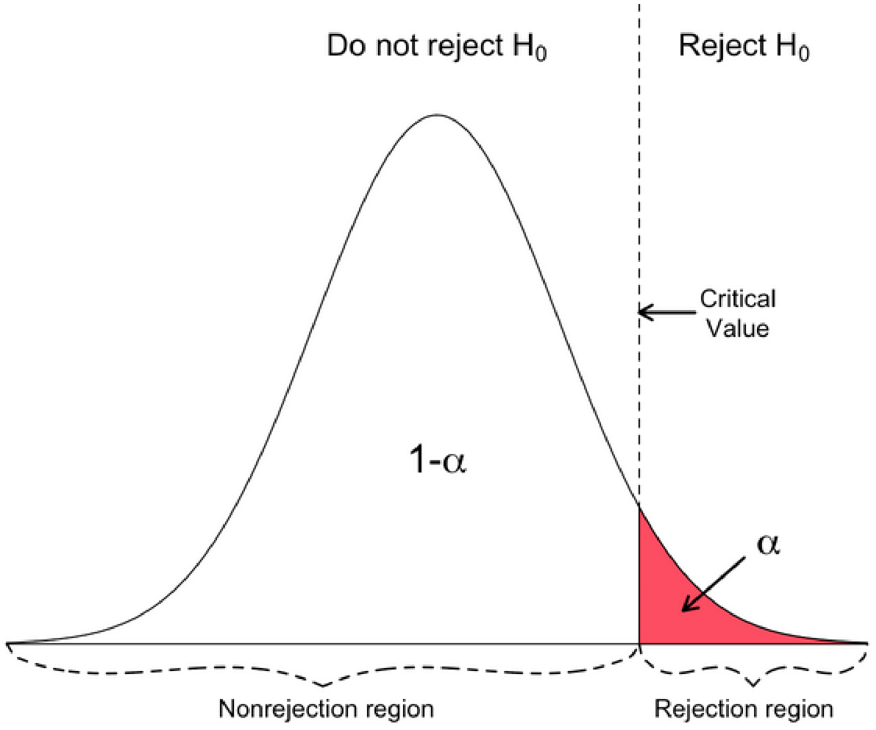
$H_0: \mu = \mu_0$

$H_a: \mu > \mu_0$



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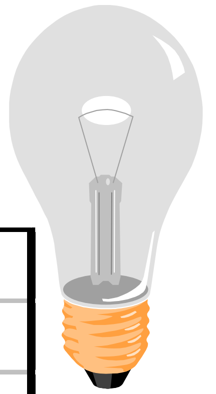
CASE : Lifetimes of Light Bulbs



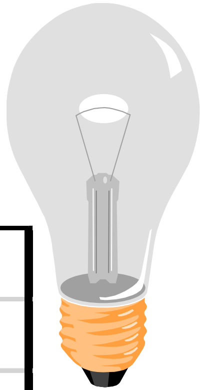
- A company claims that their new long lasting light bulbs have a mean lifetime of 10,000 hours (of continued operation).
- In a laboratory test 10 randomly selected light bulbs were tested and the following lifetimes were observed. The standard deviation is known from similar experiments to be approximately σ=1 (x 1,000 hours).
- Are the results of the laboratory experiment consistent with the claim of 10,000 hours mean lifetime? If not, can we conclude that the mean is more or less than claimed?

10.3
11.2
10.5
9.1
10.3
12.2
12.9
11.3
8.7
12.4

Lifetimes in Thousands of Hours

$\bar{x} = 10.89$ [Thousand Hours]

CASE : Lifetimes of Light Bulbs



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  H_0:

  H_A:

- Choose a significance level α.

- Find the appropriate test statistic,
  and calculate the value using sample data.

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$\bar{x} = 10.89$ [x 1000 Hours]

$\sigma = 1$ [x 1000 Hours]

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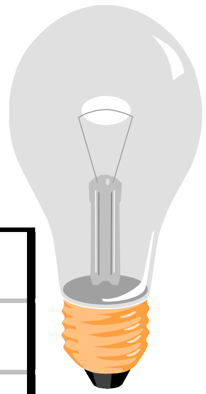

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P-Value

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The probability, assuming H_0 is true, that the test statistic would take a value as extreme or more extreme than that actually observed is called the ***P*-value** of the test. The smaller the *P*-value, the stronger the evidence against H_0 provided by the data.

- The *P*-value is the smallest value of α under which H_0 can be rejected
- *P*-value is the tail area (above or below) the test statistic if H_a is one sided
- *P*-value is two times the tail area (above or below) the test statistic if H_a is two sided