Choosing Sample Size for a Given Margin of Error

Three items that affect the Margin of Error:

$$m = z^* \frac{\sigma}{\sqrt{n}}$$

- Level of Confidence *C=1-α*
- Standard deviation σ
- Sample Size *n*

For a given confidence level and standard deviation, the sample size necessary to achieve a margin or error *m is:*

$$n = \left(\frac{z^* \sigma}{m}\right)^2$$

Round up to the nearest integer

Sample Size for a One-Sample t-interval

- Margin of Error:

$$m = t^* \frac{s}{\sqrt{n}}$$

Desired margin of error: $m = 2 \quad 1 - \propto = 0.95 \quad t^* = ?$

Initial Sample: $s = 4$ (use z* initially)

$$m \approx z^* \frac{s}{\sqrt{n}}$$

$$n \approx \left(\frac{z^* s}{m}\right)^2 = \left(\frac{1.96 \cdot 4}{2}\right)^2 = 15.37$$

Try n=16

Sample Size for a One-Sample t-interval

Desired margin of error: $m = 2 \quad 1 - \propto = 0.95 \quad t^* = ?$

Initial Sample: $s = 4$ (use z* initially)

Try n=16 df=15 t^*=2.13

$$m = t^* \frac{s}{\sqrt{n}} = 2.13 \frac{4}{\sqrt{16}} = 2.13$$

Try n=17 df=16 t^*=2.12

$$m = t^* \frac{s}{\sqrt{n}} = 2.12 \frac{4}{\sqrt{17}} = 2.06$$

Try n=18 df=17 t^*=2.11

$$m = t^* \frac{s}{\sqrt{n}} = 2.11 \frac{4}{\sqrt{18}} = 1.99$$

Sample Size(s) for a Two-Sample z-Interval

- Margin of Error:

$$m = z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example: $n_1 + n_2 = 100 \quad 1 - \propto = 0.95 \quad z^* = 1.96$

$\sigma_1 = 2 \qquad \sigma_2 = 2$

$$m = 1.96 \sqrt{\frac{4}{n_1} + \frac{4}{100 - n_1}}$$

Smallest margin of error when

$n_1 = 50 \quad n_2 = 50$

Sample Size(s) for a Two-Sample z-Interval

- Margin of Error:

$$m = z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example: $n_1 + n_2 = 100 \quad 1 - \propto = 0.95 \quad z^* = 1.96$

$\sigma_1 = 2 \qquad \sigma_2 = 3$

$$m = 1.96 \sqrt{\frac{4}{n_1} + \frac{9}{100 - n_1}}$$

Smallest margin of error when

$n_1 = 40 \quad n_2 = 60$

In general choose: $n_2 = \frac{\sigma_2}{\sigma_1} n_1$

Sample Size(s) for a Two-Sample t-Interval

- Margin of Error: $m = t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Example: Desired margin of error: $m = 1.50$
$1 - \propto = 0.95 \quad t^* = \ ?$

Initial sample: $s_1 = 2 \qquad s_2 = 3$

$$m \approx 1.96 \sqrt{\frac{4}{n_1} + \frac{9}{\frac{3}{2} n_1}} = 1.96 \sqrt{\frac{4}{n_1} + \frac{6}{n_1}} = \frac{6.198}{\sqrt{n_1}}$$

$n_1 \approx \frac{6.198^2}{m^2} \approx 17.07$ Choose: $n_1 = 18$ and $n_2 = 26$

Sample Size(s) for a Two-Sample t-Interval

- Margin of Error: $$m = t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Example: Desired margin of error: $m = 1.50$
$1 - \propto = 0.95 \quad t^* = \ ?$

Check: $df = 17 \quad t^* = 2.11 \quad m = 2.11\sqrt{\frac{4}{18} + \frac{9}{26}}$ = 1.59

n1	n2	df	t*	m
18	26	17	2.11	1.59
19	29	18	2.10	1.52
20	30	19	2.09	1.48
20	29	19	2.09	1.50

General Form a Confidence Interval

- Suppose:
  - θ is an unknown parameter
  - $\hat{\theta}$ is an unbiased estimator for θ
  - Sample size *n* is large
  - $\hat{\theta} \sim N(\mu_{\hat{\theta}}, \sigma_{\hat{\theta}})$ (at least by approximation)
- $1 - \alpha$ Confidence Interval for θ:

$$\hat{\theta} \pm m = \hat{\theta} \pm z^{*}\sigma_{\hat{\theta}}$$

Estimate $\pm$ Margin of Error

Mean and Standard Deviation for the Binomial Distribution

BINOMIAL MEAN AND STANDARD DEVIATION

If a count X has the binomial distribution $B(n, p)$, then

$$\mu_X = np$$

$$\sigma_X = \sqrt{np(1-p)}$$

$X \sim N(\mu_X, \sigma_X)$ (approx.)
when $np \geq 10, n(1-p) \geq 10$

MEAN AND STANDARD DEVIATION OF A SAMPLE PROPORTION

Let $\hat{p}$ be the sample proportion of successes in an SRS of size n drawn from a large population having population proportion p of successes. The mean and standard deviation of $\hat{p}$ are

$$\hat{p} = \frac{X}{n}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$\hat{p} \sim N(\mu_{\hat{p}}, \sigma_{\hat{p}})$ (approx.)
when $np \geq 10, n(1-p) \geq 10$

Confidence Interval for an Unknown Proportion *p*

- $1-\alpha$ Confidence Interval for p:

$$\hat{p} \pm m = \hat{p} \pm z^*\sigma_{\hat{p}} = \hat{p} \pm z^*\sqrt{\frac{p(1-p)}{n}}$$

$$\approx \hat{p} \pm z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Presidential Approval Ratings

October 24, 2025
According to Gallup an estimated 41% of Americans approve of the way that President Trump is performing in his job (range 37%-47% during second term).

https://news.gallup.com/interactives/507569/presidential-job-approval-center.aspx

X = Number of adults in the sample that say they approve of D. Trump

Survey Methods
The Gallup polls are based on national telephone surveys of typically 1,000 adults conducted over three nights. Margin of sampling error is +/- 3 percentage points with a 95% level of confidence.
Source: www.gallup.com

Confidence Interval for an Unknown Proportion *p*

- 95% Confidence Interval for p
- Example: $n = 1000 \quad \hat{p} = 0.41$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.41 \pm 1.96 \sqrt{\frac{0.41(1-0.41)}{100}}$$

$$= 0.41 \pm 0.03048$$

$$\approx 0.41 \pm 0.03 = (0.38, 0.44)$$

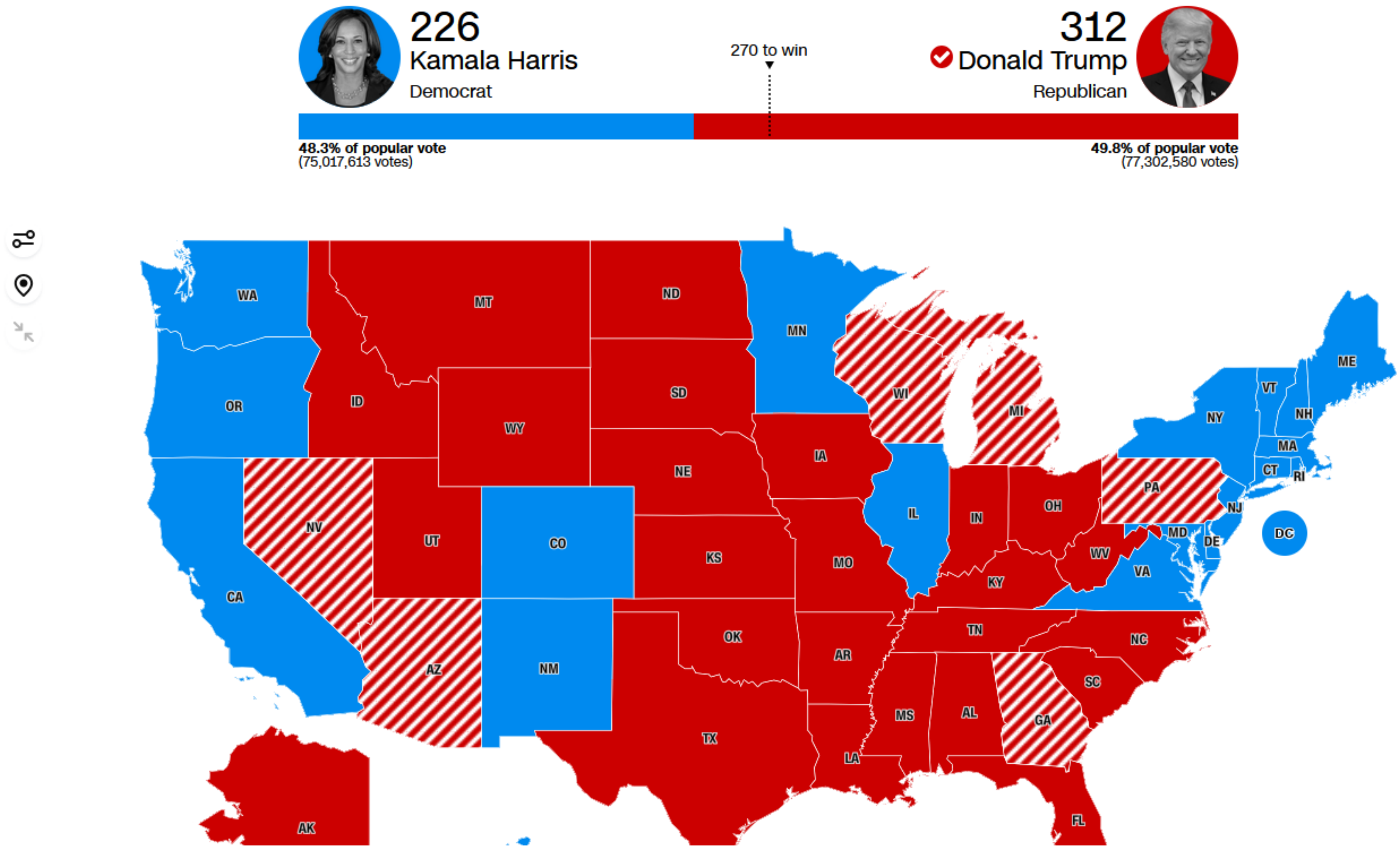
If p=0.38 then m=0.030008 $\approx$ 0.03

If p=0.44 then m=0.030766 $\approx$ 0.03

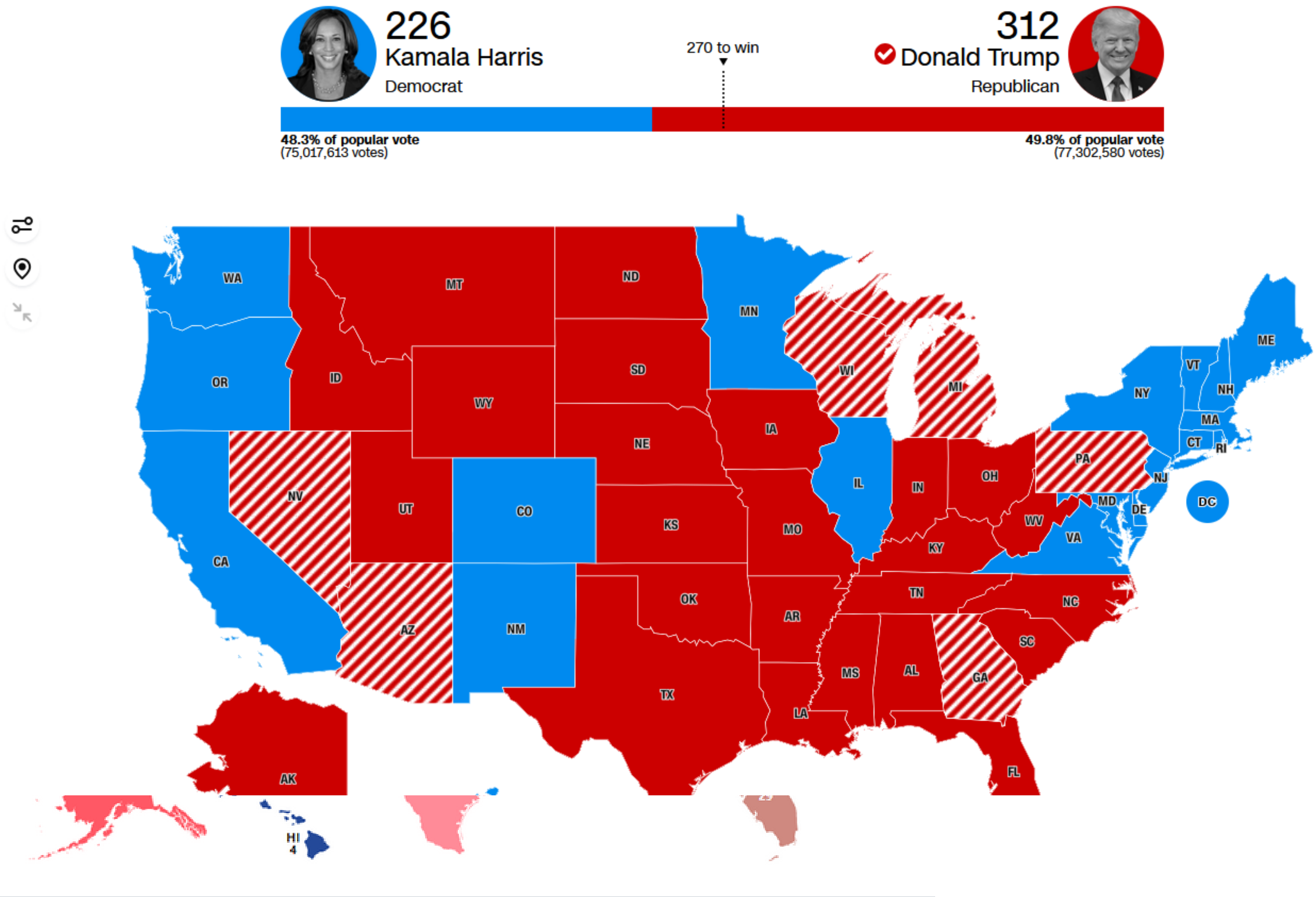
NOTE: *m* is not sensitive to the value of *p* used.

Presidential Election 2024

Final Results



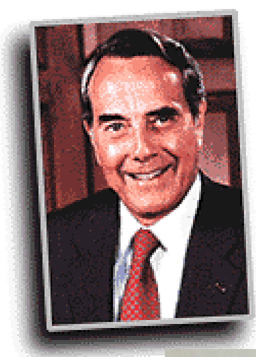
Presidential Election



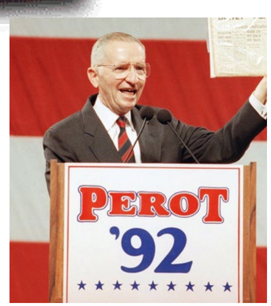
Presidential Election Poll (1996)

(Actuals)

Bill Clinton : 53% (49%)



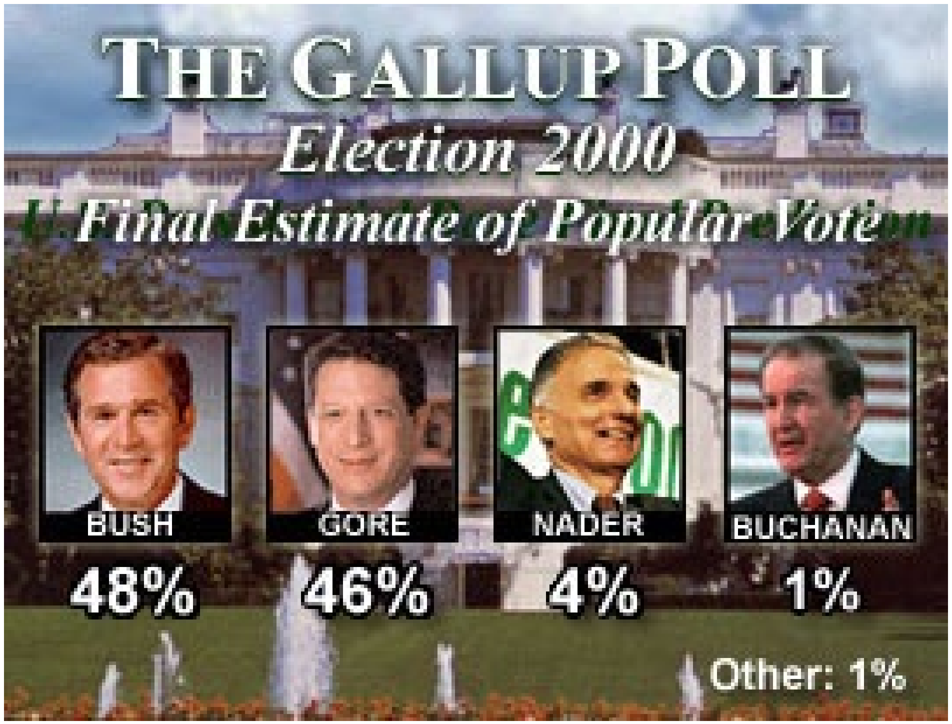
Bob Dole : 35% (41%)



Ross Perot : 9 % (8%)

SOURCE : CBS Evening News Nov. 4, 1996
Margin of Error: $m = \pm 3\%$

Presidential Election Poll (2000)



Popular Vote in Presidential Race Too Close to Call
The 2000 presidential election campaign draws to its finale with the national popular vote essentially too close to call. Gallup's final allocated estimate of the popular vote, based on interviews with **2,350 likely voters** on Sunday and Monday, has Bush at 48%, Gore 46%, Nader 4%, Buchanan 1% and "other" candidates 1%. There is a **2% margin of error** associated with the estimate of each candidate's percentage of the vote.

(47.9%) (48.4%) (2.7%) (<1%)

SOURCE : www.gallup.com Nov. 7, 2000

Presidential Election Poll (2004)

George W. Bush : 46% (51%)

John Kerry : 46% (48%)

Other : 3 % (1%)

Undecided : 6 %

Sample Size :1500

SOURCE : Rasmussen Report March 7, 2004
www.rasmussenreports.com

Presidential Election Poll (2008)

Barack Obama : 53% (53%)

John McCain : 42% (46%)

Other/Undecided : 5 % (1%)

[More historical election results](#)

Sample Size :1500

SOURCE : Gallup Report Nov. 2, 2008
www.gallup.com

Tuesday, November 6, 2012

National '12 President General Election

POLITICO/George Washington University

11/04/2012-11/05/2012

1000 likely voters

D	Barack Obama	47%
R	Mitt Romney	47%
	Undecided	6%

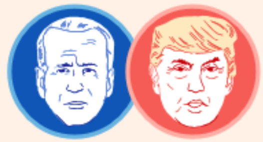
United States presidential election, 2012

2008 ← **November 6, 2012** → 2016

538 electoral votes of the Electoral College
270 electoral votes needed to win

Turnout	58.2% (voting eligible)[1]	
		
Nominee	Barack Obama	Mitt Romney
Party	Democratic	Republican
Home state	Illinois	Massachusetts
Running mate	Joe Biden	Paul Ryan
Electoral vote	332	206
States carried	26 + DC	24
Popular vote	65,899,660[2][3]	60,932,152[2][3]
Percentage	51.1%[2][3]	47.2%[2][3]

Presidential Election Poll (2020)



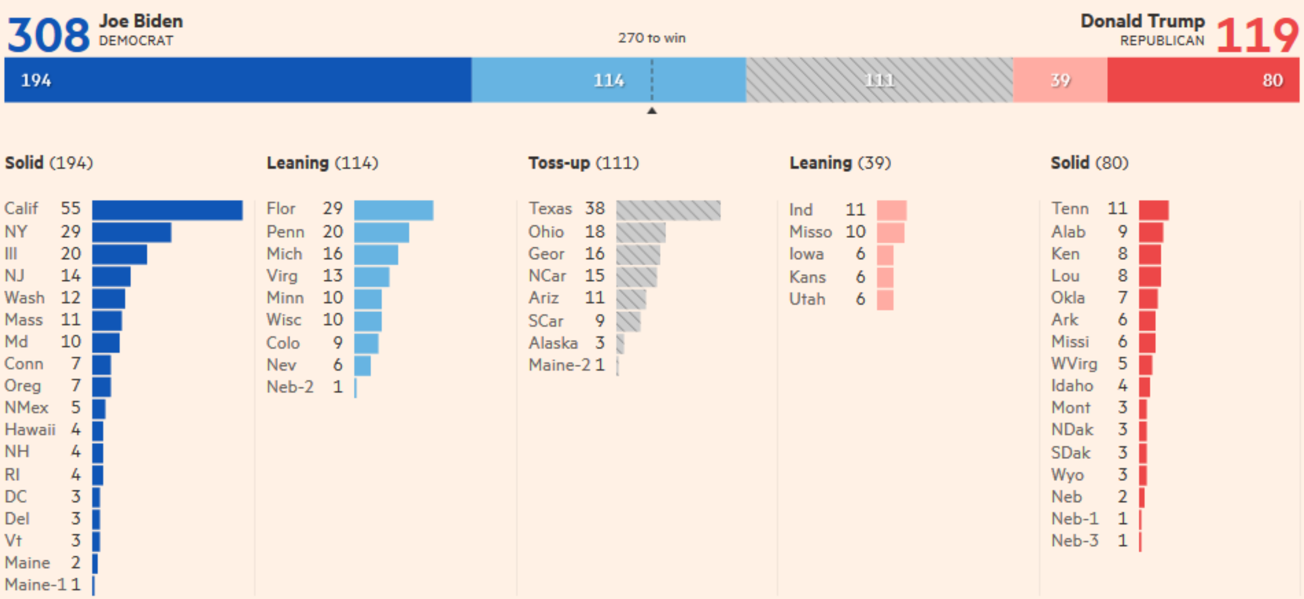
US presidential election 2020

Biden vs Trump: who is leading the 2020 US election polls?

See how the latest state-by-state polling data would translate into electoral college votes and use FT's interactive calculator to zero in on the crucial battleground states

LAST UPDATED 32 MINUTES AGO

If the election were held today, the latest polls suggest this outcome in the electoral college:



States where the difference in poll numbers between Biden and Trump is less than 5 percentage points are classified as 'toss-up' states.

Choosing Sample Size for a Given Margin of Error

Margin of Error:

$$m = z^* \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}}$$

$\tilde{p}$= a "guesstimate" of p

NOTE: If no "guesstimate" is available use p = 0.5

For a given confidence level and standard deviation, the sample size necessary to achieve a margin or error *m is:*

$$n = \tilde{p}(1-\tilde{p})\left(\frac{z^*}{m}\right)^2$$

Round up to the nearest integer

Choosing Sample Size for a Given Margin of Error

Example: $m = \pm 3\%$ $\quad$ $\mathrm{C} = 1 - \alpha = 0.95$ $\quad$ $\tilde{p} = 0.5$

Example: $m = \pm 1\%$ $\quad$ $\mathrm{C} = 1 - \alpha = 0.95$ $\quad$ $\tilde{p} = 0.5$

Example: $m = \pm 1\%$ $\quad$ $\mathrm{C} = 1 - \alpha = 0.95$ $\quad$ $\tilde{p} = 0.03$