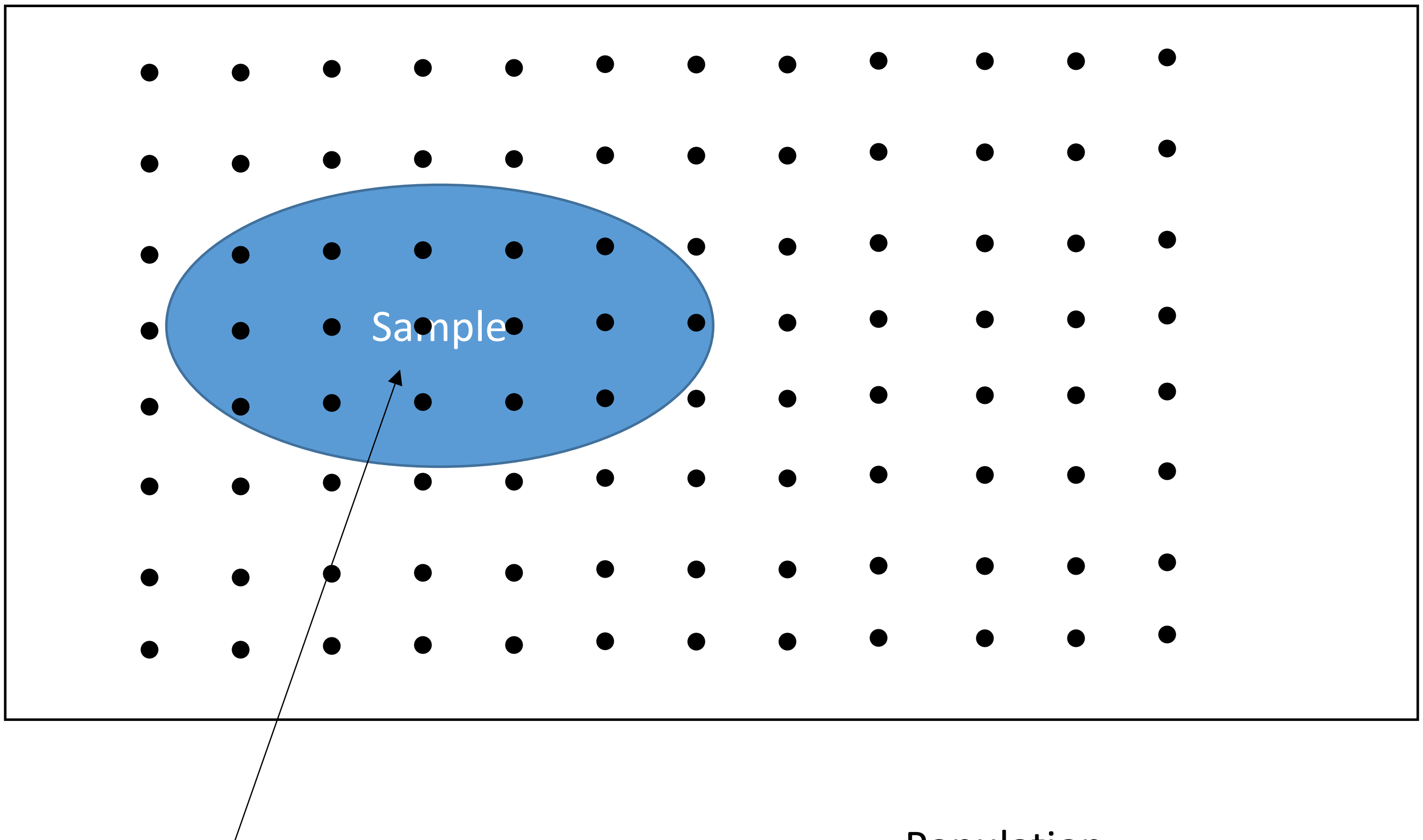
Estimation of Parameters



Parameter Estimate $\hat{\theta}$
(calculated from sample)

Population

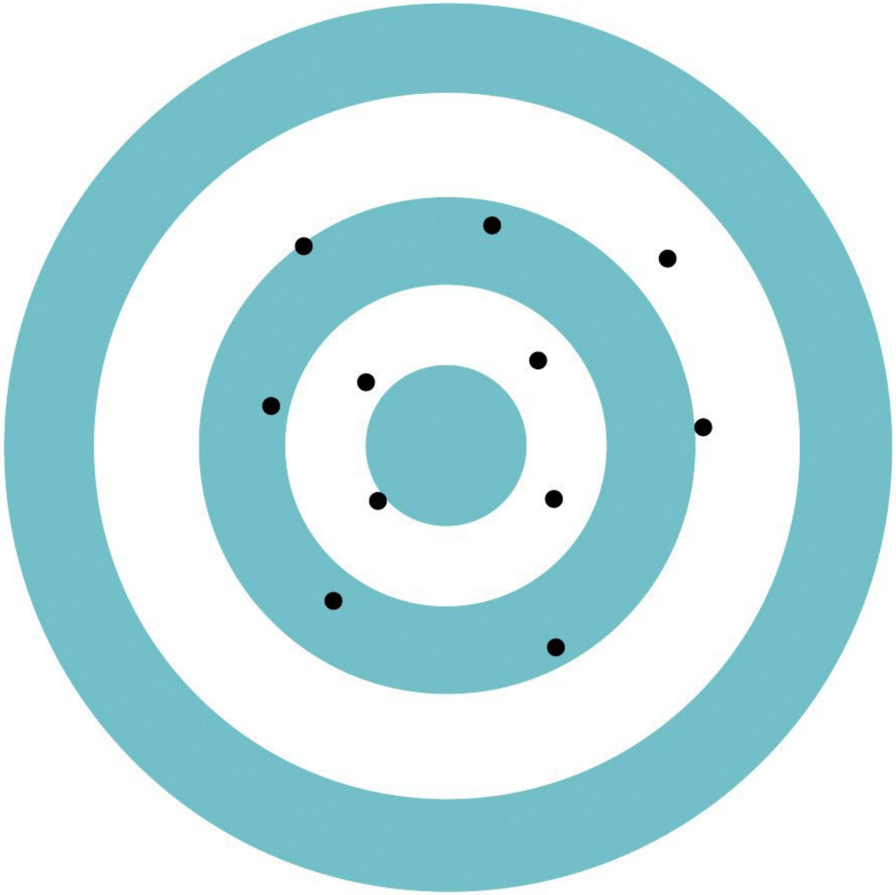
- Case value x_i

Parameter θ (unknown)

Bias and Variability

High bias, low variability

Bias and Variability

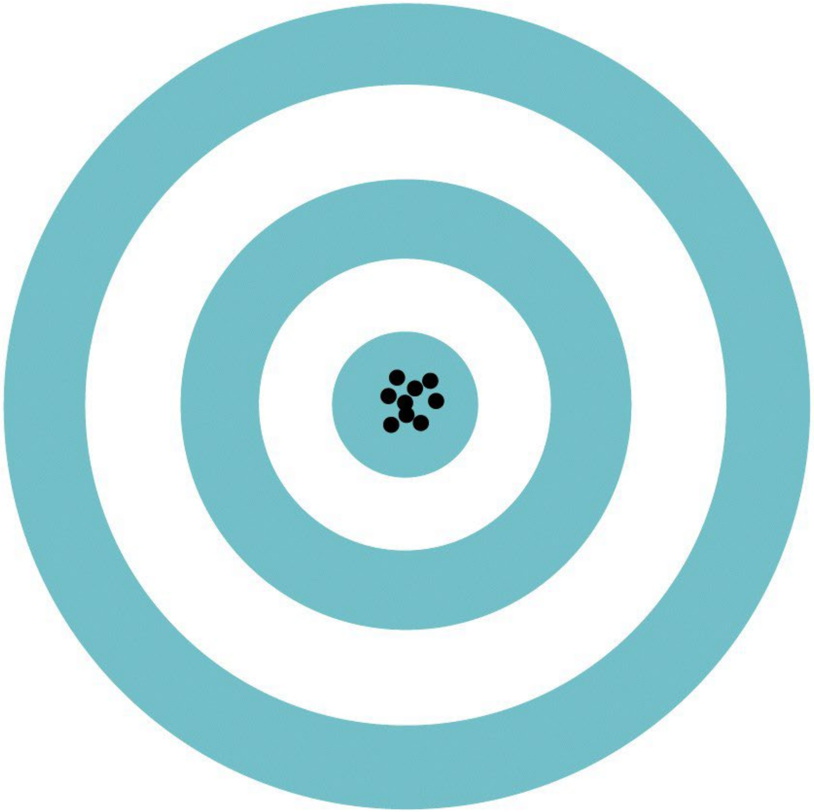


Low bias, high variability

Bias and Variability

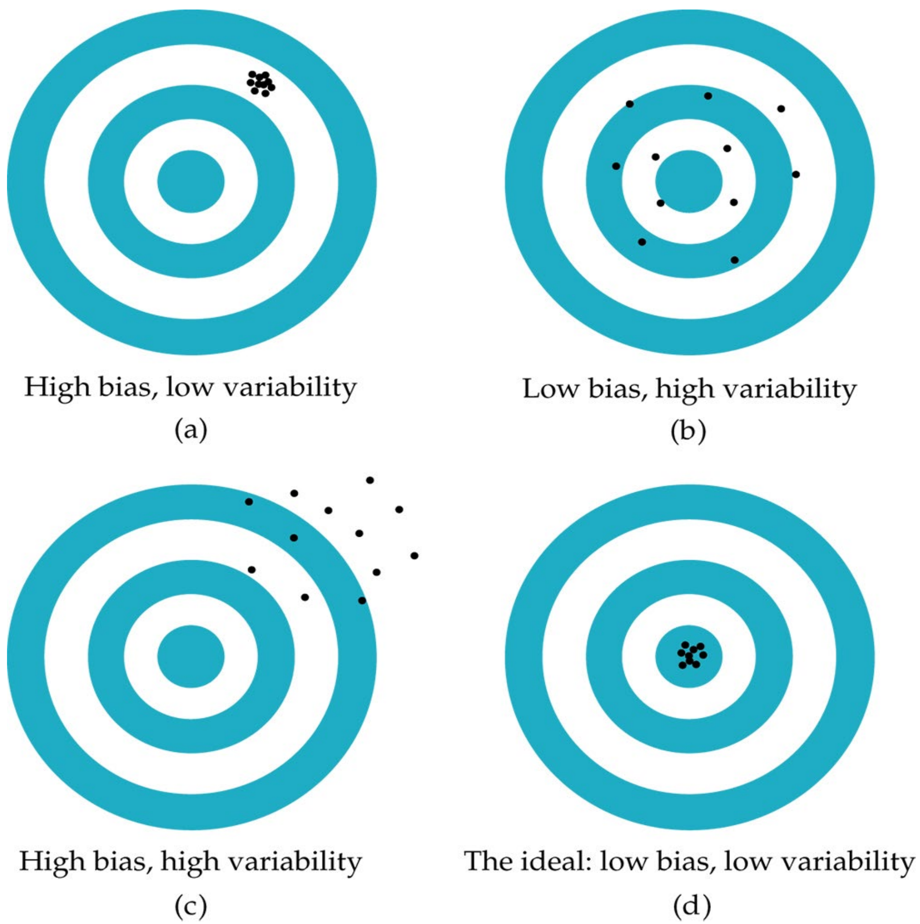
High bias, high variability

Bias and Variability



The ideal: low bias, low variability

Bias and Variability



Point Estimation

A ***point estimator*** $\hat{\theta}$ is a statistic whose value is used to estimate an ***unknown parameter*** θ

Desirable Properties: A point estimator should

- be unbiased . $E[\hat{\theta}] = \theta$
- have a small variance.

Examples

- $\overline{X}$ is an unbiased estimator for μ.
- s^2 is an unbiased estimator for σ^2
- $\hat{p}$ is an unbiased estimator for p.

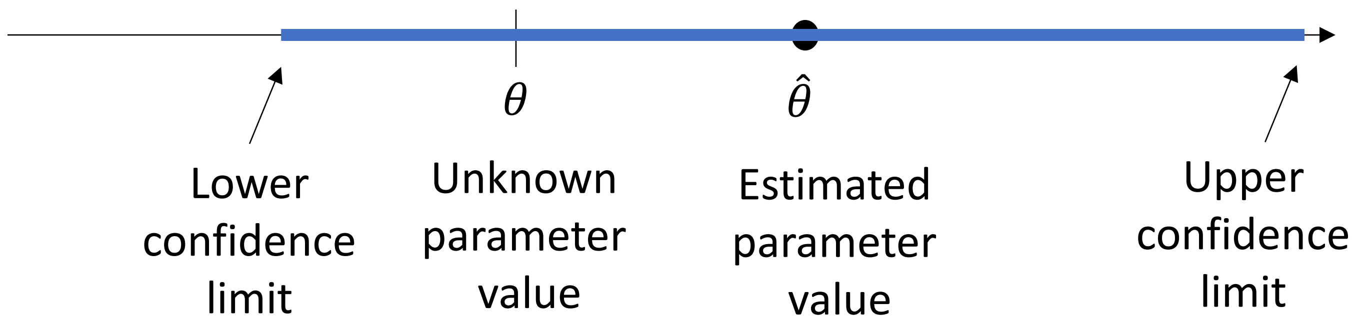
Confidence Interval

A *1-α confidence interval* for an unknown parameter θ is an interval calculated based on the sample data, such that it contains the true value of θ with probability (confidence level) *C=1-α.*

NOTES:

- Usual form: $\hat{\theta} \pm m$ m *= margin of error*
- The confidence intervals gives a *range of likely values* of the unknown parameter

Confidence Interval

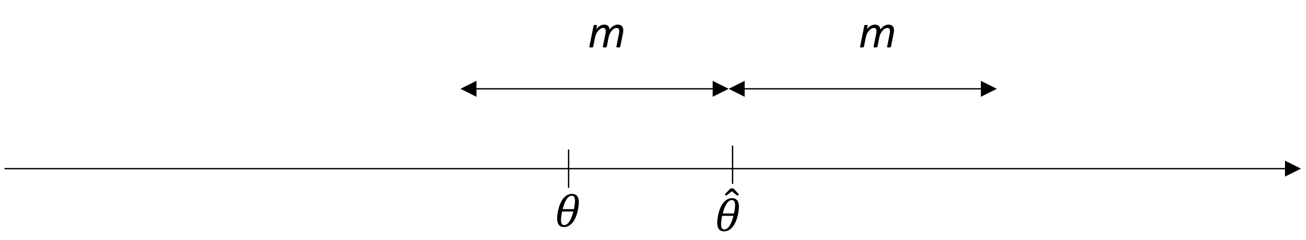


NOTES:

- We never know for sure the true value of θ or whether the confidence interval contains this value.
- There is a high probability $\mathrm{C} = 1 - \alpha$ (usually 95-99%) that the interval captures the unknown value of θ.

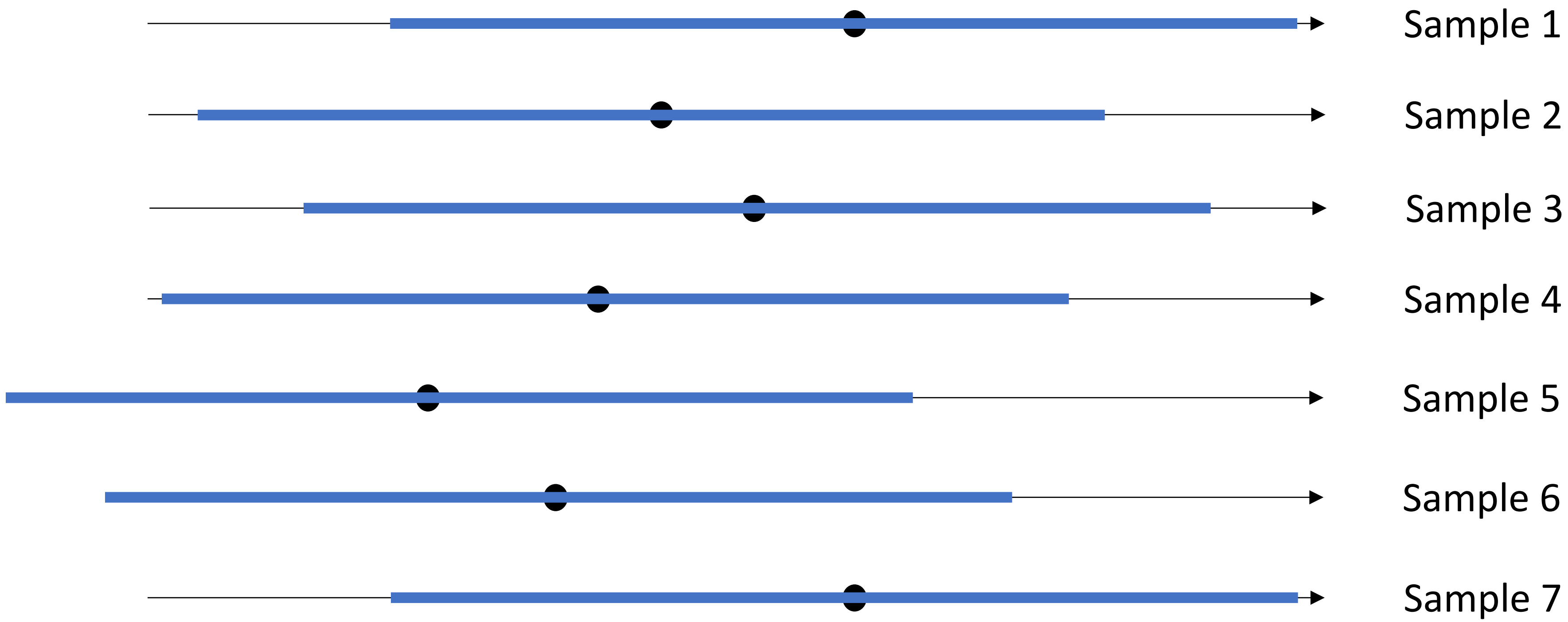
Margin of Error

When estimating an unknown parameter using an unbiased estimator, the margin of error measures the likely maximum difference between the estimate and the true value of the parameter.

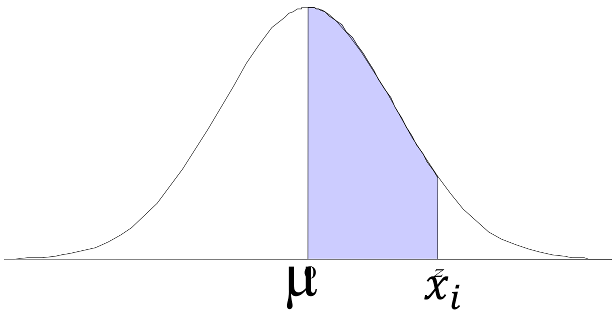


$$P(|\bar{X}\text{-}\mu| < m) = C = 1 - \alpha$$

Confidence Interval



1-α Confidence Interval for μ



Data given:

$$x_1, x_2 \ldots, x_n$$

It is assumed (for now) that the *n* observations are from a simple random sample of *n* normally distributed random variables:

$$X_1, X_2, \ldots, X_n$$

We write: $X_i \sim N(\mu, \sigma)$ (i.i.d.)

i.i.d. = independent and identically distributed.

We also assume that μ is unknown, but σ is given (for now).

1-α Confidence Interval for μ

$$\underbrace{\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}}}$$

$$\bar{X} \pm \underbrace{z^* \frac{\sigma}{\sqrt{n}}}_{\text{Margin of error}}$$

- $\bar{X}$ is the *point estimate* of the population mean

- z^*: *critical value*, the z-value that cuts off a tail area $\alpha/2$
- $\bar{X}$ is assumed to be approximately normal (Central Limit Theorem)