General Form a Test Statistic

- Suppose:
  - θ is an unknown parameter
  - $\hat{\theta}$ is an unbiased estimator for θ
  - Sample size *n* is large
  - $\hat{\theta} \sim N(\mu_{\hat{\theta}}, \sigma_{\hat{\theta}})$ (at least by approximation)

H_0: $\theta = \theta_0$
H_a: $\theta \neq \theta_0$ (or $\theta > \theta_0$ or $\theta < \theta_0$)

Test Statistic:

$$z = \frac{\text{estimate} - \text{hypothesized value}}{\text{standard deviation of the estimate}}$$

A **test statistic** measures compatibility between the null hypothesis and the data. We use it for the probability calculation that we need for our test of significance. It is a **random variable** with a **distribution** that we know.

$$z = \frac{\hat{\theta} - \theta_0}{\sigma_{\hat{\theta}}}$$

Hypothesis Test for a Proportion

$H_0: p = p_0$

$H_a: p \neq p_0$
(or $p > p_0$
or $p < p_0$)

Test Statistic:

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$\sim N(0,1)$ (approx.) if H_0 is true

CASE : Quality Inspection

- A customer who buys large quantities of microchips used in consumer electronics products has been informed by the vendor that *only 5% of their parts are defective.*

Quality Inspection (cont.)

- The customer suspects that the actual percentage is higher than 5%, and decides to inspect an SRS of size 200 before accepting the next batch.
- 26 microchips were found to be defective. Is there sufficient evidence to support the customer’s claim?
- Are the assumptions for the test satisfied?
- How much does the actual percentage of defective chips differ from what the manufacturer is informing customers?

Confidence Interval for the Difference Between Two Proportions

$1-\alpha$ Confidence Interval for $p_1 - p_2$:

$$\hat{p}_1 - \hat{p}_2 \pm m = \hat{p}_1 - \hat{p}_2 \pm z^* \sigma_{\hat{p}_1 - \hat{p}_2}$$

$$= \hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{p_1\,(1-p_1)}{n_1} + \frac{p_2\,(1-p_2)}{n_2}}$$

$$\approx \hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

where:

n_1= sample size of group 1

p_1= proportion of group 1

$X_1 \sim B(n_1, p_1)$

$\hat{p}_1$=$\frac{X_1}{n_1}$

$n_1\hat{p}_1 \geq 10$ $\quad n_1(1-\hat{p}_1) \geq 10$

n_2= sample size of group 2

p_2= proportion of group 2

$X_2 \sim B(n_2, p_2)$

$\hat{p}_2$=$\frac{X_2}{n_2}$

$n_2\hat{p}_2 \geq 10$ $\quad n_2(1-\hat{p}_2) \geq 10$