The Central Limit Theorem

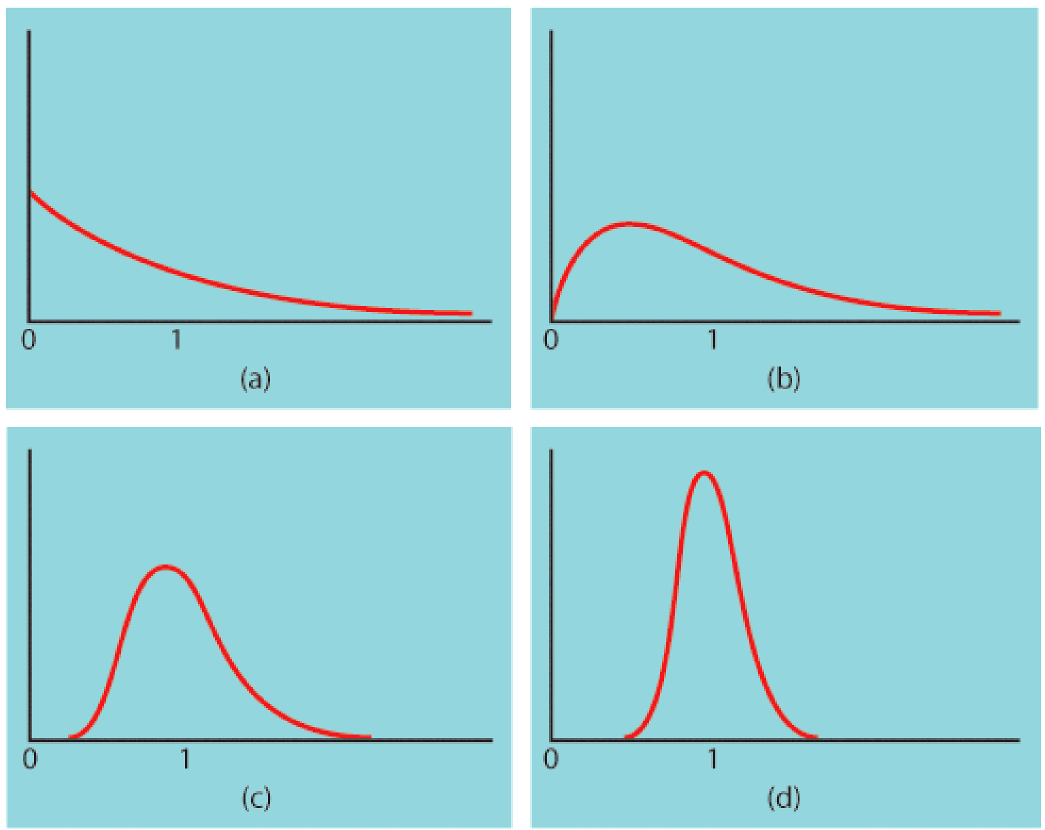
CENTRAL LIMIT THEOREM

Draw an SRS of size n from any population with mean μ and finite standard deviation σ. When n is large, the sampling distribution of the sample mean $\bar{x}$ is approximately Normal:

$$\bar{x} \text{ is approximately } N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Population with strongly skewed distribution

Sampling distribution of $\overline{x}$ for $n = 2$ observations



Sampling distribution of $\overline{x}$ for $n = 10$ observations

Sampling distribution of $\overline{x}$ for $n = 25$ observations

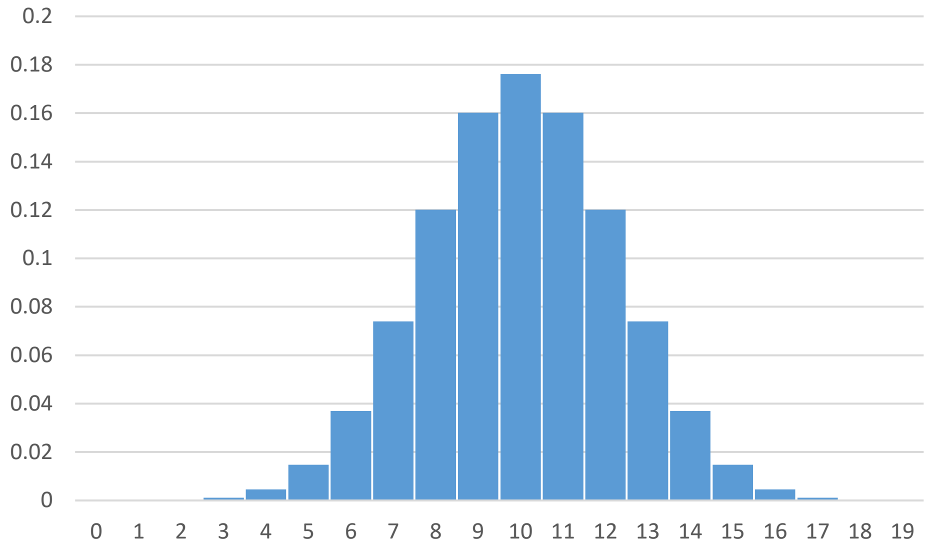
How Large Should *n* Be?

It depends on the population distribution. More observations are required if the population distribution is far from normal.

- A sample size of 25 is generally enough to obtain a normal sampling distribution from a strong skewness or even mild outliers.
- A sample size of 40 will typically be good enough to overcome extreme skewness and outliers.

Normal Approximation to the

Binomial Distribution



$X \sim B(20{,}0.5)$

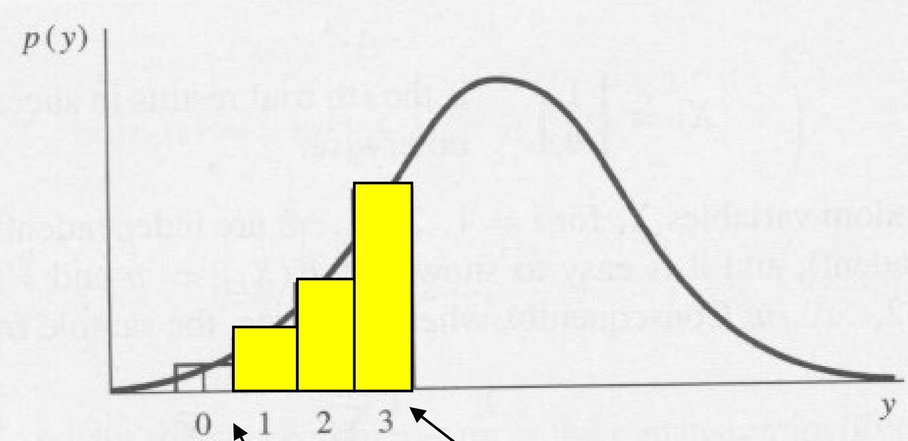
$P(X = 10) =$

Normal Approximation to the

Binomial Distribution

Continuity Correction:

FIGURE 7.7 The normal approximation to the binomial distribution: $n = 10$ and $p = .5$



$$P(1 \le Y_B \le 3) \approx P(0.5 \le Y_N \le 3.5)$$

Application of the Binomial Distribution

- A cellular phone service provider has experienced that on average 10% of the units sold are returned during their three month “money back guarantee” period.
- Out of 100 cell phones sold during a given week, what is the probability that between 5% and 15% of those will be returned within the next three months?