

# Mean of a Discrete Random Variable

## MEAN OF A DISCRETE RANDOM VARIABLE

Suppose that  $X$  is a discrete random variable whose distribution is

Value of $X$	$x_1$	$x_2$	$x_3$	$\cdots$	$x_k$
Probability	$p_1$	$p_2$	$p_3$	$\cdots$	$p_k$

To find the **mean** of  $X$ , multiply each possible value by its probability, then add all the products:

$$\begin{aligned}\mu_X &= x_1p_1 + x_2p_2 + \cdots + x_kp_k \\ &= \sum x_i p_i\end{aligned}$$

# Coin Tosses

- Toss a coin four times
- Define the random variable:

$X$  = Number of heads observed

- Consider the sample:  $x_1, x_2, \dots, x_n$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Sample: 0,2,1,3,2,1,1,3,1,4



- Consider the possible values of the random variable  $x_1, x_2, \dots$

$$\mu_X = \sum x_i p_i$$

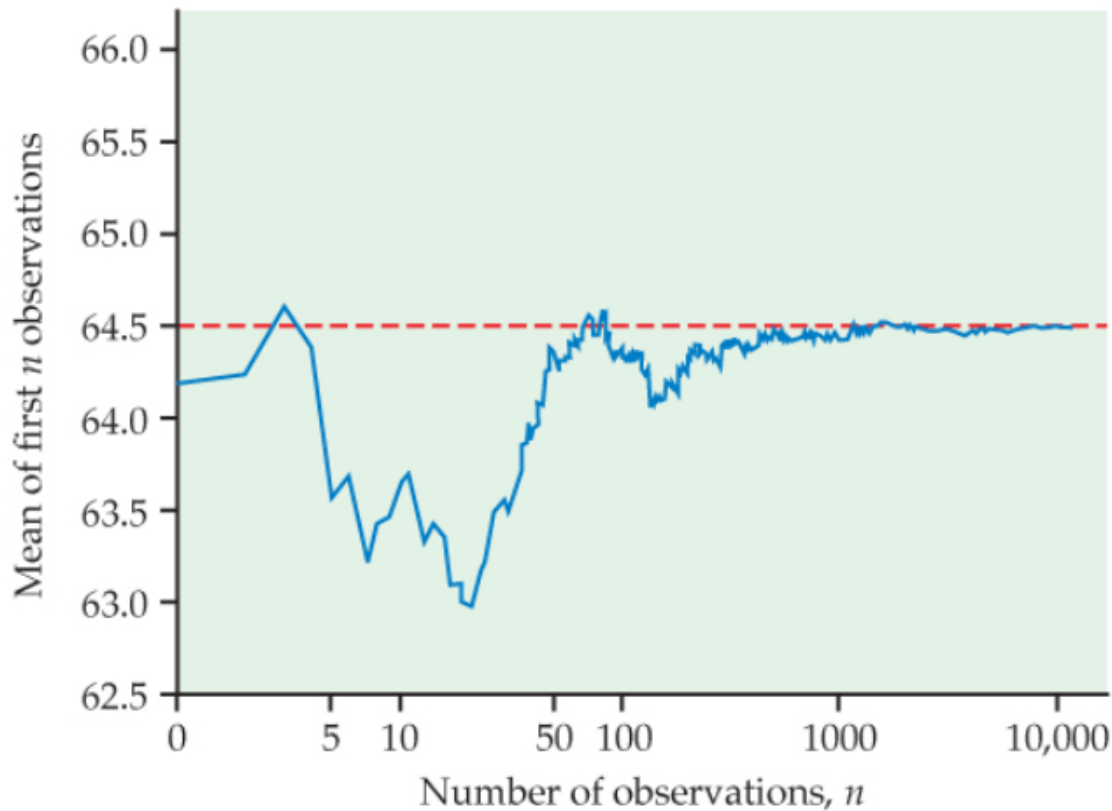
# Law of Large Numbers

## LAW OF LARGE NUMBERS

Draw independent observations at random from any population with finite mean  $\mu$ . Decide how accurately you would like to estimate  $\mu$ . As the number of observations drawn increases, the mean  $\bar{x}$  of the observed values eventually approaches the mean  $\mu$  of the population as closely as you specified and then stays that close.

# Law of Large Numbers

**FIGURE 4.14** The law of large numbers in action, **Example 4.30**. As we take more observations, the sample mean always approaches the mean of the population.



**Figure 4.14**

Moore/McCabe/Craig, *Introduction to the Practice of Statistics*, 9e, © 2017 W. H. Freeman and Company

# Variance of a Discrete Random Variable

## VARIANCE OF A DISCRETE RANDOM VARIABLE

Suppose that  $X$  is a discrete random variable whose distribution is

Value of $X$	$x_1$	$x_2$	$x_3$	$\cdots$	$x_k$
Probability	$p_1$	$p_2$	$p_3$	$\cdots$	$p_k$

and that  $\mu_X$  is the mean of  $X$ . The **variance** of  $X$  is

$$\begin{aligned}\sigma_X^2 &= (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_k - \mu_X)^2 p_k \\ &= \sum (x_i - \mu_X)^2 p_i\end{aligned}$$

The **standard deviation**  $\sigma_X$  of  $X$  is the square root of the variance.

# Roll a Die

## Random Outcome

0.279975	3
0.847286	6
0.996788	6
0.31363	3
0.988031	6
0.812048	5
0.069484	1
0.679286	5
0.904567	6
0.91403	6
0.010568	1
0.984634	6
0.231322	3
0.525632	4
0.805128	5
0.142173	1
0.723179	5
0.352772	3
0.832194	5
0.358609	3

MEAN 4.15  
STD.DEV 1.785173

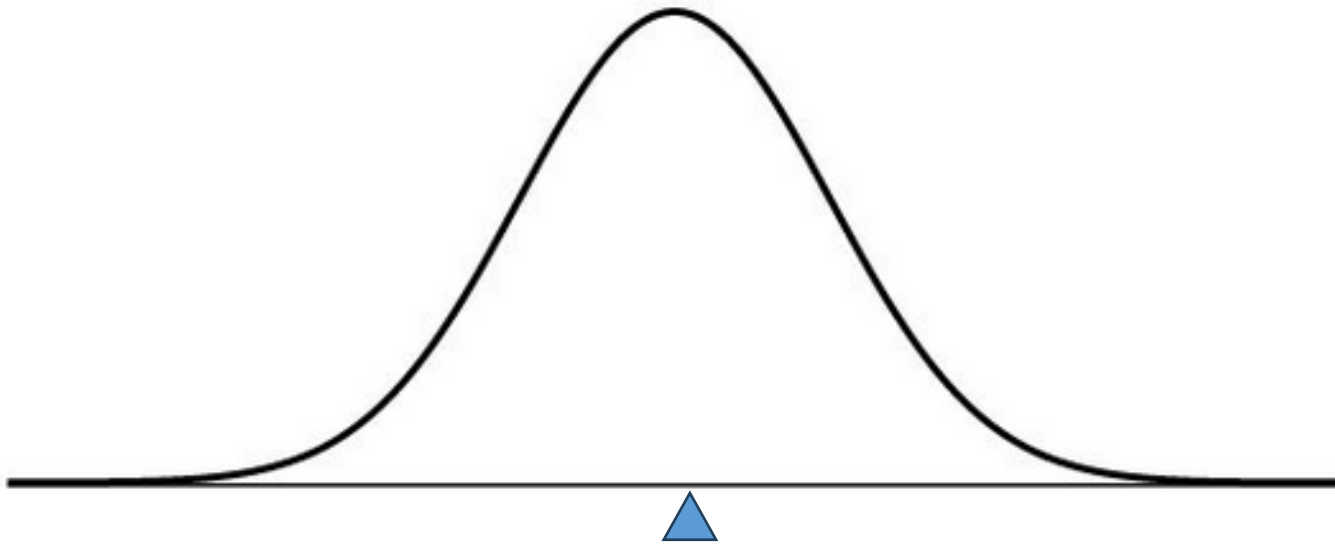
## Theoretical Distribution

$x_i$	$p_i$	$x_i p_i$	$(x_i - \mu)^2 p_i$
1	0.166667	0.166667	1.041667
2	0.166667	0.333333	0.375
3	0.166667	0.5	0.041667
4	0.166667	0.666667	0.041667
5	0.166667	0.833333	0.375
6	0.166667	1	1.041667
SUM		3.5	2.916667

MEAN 3.5  
VAR 2.916667  
STDEV 1.707825



# Mean and Variance of a Continuous Random Variable



$E[X] = \mu_X$  (balance point of probability density function)

$$V[X] = \sigma_X^2 = E[(X - \mu_X)^2]$$

# CASE: Auto Insurance Risk

- An insurance company has assessed the following risks associated with insuring a particular type of driver.

Premium = \$1,000 per year

Minor vehicle damage	2000	0.05
Major vehicle damage	20,000	0.002
Personal injury	100,000	0.002
Death (maximum claim)	1,000,000	0.0001

# CASE: Auto Insurance Risk

Minor vehicle damage	2000	0.05
Major vehicle damage	20,000	0.002
Personal injury	100,000	0.002
Death (maximum claim)	1,000,000	0.0001

X = Annual Cost (in \$\$) to Insurance Co

$x_i$	$p_i$

# CASE: Roulette Wheel

<https://tools-unite.com/tools/random-picker-wheel>

		0		00	
1 to 18	1st 12	1	2	3	
		4	5	6	
		7	\$	9	
EVEN	2nd 12	10	11	12	
		13	14	15	
		16	17	18	
J&M CASINO	3rd 12	19	20	21	
		22	23	24	
		25	26	27	
ODD	2 to 1	28	29	30	
		31	32	33	
		34	35	36	
19 to 36	2 to 1	2 to 1	2 to 1	2 to 1	

Bet	House Odds
single number	35 to 1
two numbers ("split")	17 to 1
three numbers ("street")	11 to 1
four numbers ("square")	8 to 1
five numbers ("line")	6 to 1
six numbers ("line")	5 to 1
twelve numbers (column or section)	2 to 1
low or high (1 to 18 or 19 to 36, respectively)	1 to 1
even or odd (0 and 00 are neither even nor odd)	1 to 1
red or black	1 to 1



# CASE: Roulette Wheel

Example: Bet \$1 on a single number (e.g. 8)

$X = \$ \text{ Amount Gained}$

Find the mean and variance of  $X$

# CASE: Roulette Wheel (cont.)

Bet	House Odds x to 1	Probability of Winning	Expected Gain	Variance	Std.Dev.
Single Number	35	0.02632	-0.05263	33.20776	5.76262
Two Numbers	17	0.05263	-0.05263	16.15512	4.01934
Three Numbers	11	0.07895	-0.05263	10.47091	3.23588
Four Numbers	8	0.10526	-0.05263	7.62881	2.76203
Five Numbers	6	0.13158	-0.07895	5.59903	2.36623
Six Numbers	5	0.15789	-0.05263	4.78670	2.18785
Twelve Number	2	0.31579	-0.05263	1.94460	1.39449
Low or High	1	0.47368	-0.05263	0.99723	0.99861
Even or Odd	1	0.47368	-0.05263	0.99723	0.99861
Red or Black	1	0.47368	-0.05263	0.99723	0.99861