CSCI 4120 Computer-Aided Design of Very Large Scale Integrated Circuits

Homework 1 (Part A)

Due Date: Feb 17, 2025 11:59pm

1. Given that $\forall x\, F = Fx \cdot Fx'$ and $\exists x\, F = Fx + Fx'$:

   (a) Prove that $\forall x\, y\, F = Fxy \cdot Fxy' \cdot Fx'y \cdot Fx'y'$

   (b) Prove that $\exists x\, y\, F = Fxy + Fxy' + Fx'y + Fx'y'$

   (c) What is $\forall x \exists y\, F$? Prove your answer.

2. The Unate Recursive Paradigm (URP) is to determine tautology of a Boolean equation available as a SOP cube list. The idea can be extended to do unate recursive complement since $\bar{f} = x\overline{f_x} + x'\overline{f_{x'}}$

   (a) Show by hand a recursion tree for the algorithm URP Complement, running on the function $F(x, y, z, w) = \bar{x}yz + \bar{x}\bar{z}w + xyz$

   (b) Show with a simple Karnaugh map that your answer for the complement in part (i) is correct.

3. A simple comparator takes two 2-bit unsigned binary numbers a_1a_0 and b_1b_0, and compares their magnitude, and sets the output $z = 1$ when a_1a_0 is larger than or equal to b_1b_0.

   (a) Write the Boolean function for z.

   (b) Construct an ROBDD (Reduced Order Binary Decision Diagram) for z with a good variable ordering.

4. Show how you can build a BDD for the Boolean function $F = \overline{a \oplus b}$ using the ITE recursion.

5. Consider a 2-level circuit C representing the following Boolean function:

   $$F = a'c' + a'bc + ac'd + ab'd$$

   (a) How many literals are there in this 2-level circuit C?
   (b) Draw a Karnaugh map for the function F.
   (c) Which of the cubes (product terms) are prime?
   (d) Which of the cubes are redundant?
   (e) Show how the 3 steps of *Expand*, *Irredundant* and *Reduce* can be applied to F to obtain an equivalent 2-level circuit C' with the minimum number of literals.

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1 (a) $\forall xy\, F = \forall x(\forall y\, F)$
$= \forall x\,(F_y \cdot F_{\bar{y}})$
$= x\,(F_y \cdot F_{\bar{y}}) \cdot \bar{x}\,(F_y \cdot F_{\bar{y}})$
$= F_{xy} \cdot F_{x\bar{y}} \cdot F_{\bar{x}y} \cdot F_{\bar{x}\bar{y}}$

(c) $\forall x \exists y\, F = \forall x\,(F_y + F_{\bar{y}})$
$= x\,(F_y + F_{\bar{y}}) \cdot \bar{x}\,(F_y + F_{\bar{y}})$
$= (F_{xy} + F_{x\bar{y}}) \cdot (F_{\bar{x}y} + F_{\bar{x}\bar{y}})$

(b) $\exists xy\, F = \exists x\,(\exists y\, F)$
$= \exists x\,(F_y + F_{\bar{y}})$
$= x\,(F_y + F_{\bar{y}}) + \bar{x}\,(F_y + F_{\bar{y}})$
$= F_{xy} + F_{x\bar{y}} + F_{\bar{x}y} + F_{\bar{x}\bar{y}}$

2 (a) $F(x,y,z,w) = \bar{x}yz + \bar{x}\bar{z}w + xyz$

	x	y	z	w
$\bar{x}yz$	10	01	01	11
$\bar{x}\bar{z}w$	10	11	10	01
xyz	01	01	01	11

F_x

yz	11	01	01	11

$F_{\bar{x}}$

yz	11	01	01	11
$\bar{z}w$	11	11	10	01

$F_{\bar{x}z}$

y	11	01	11	11

$F_{\bar{x}\bar{z}}$

w	11	11	11	01

It is not a tautology and

$F_x = yz$, $F_{\bar{x}z} = y$, $F_{\bar{x}\bar{z}} = w$, $\forall z\, F_{\bar{x}} = F_{\bar{x}z} \cdot F_{\bar{x}\bar{z}}$

By Shannon Expansion wrt. z, we have
$\overline{F_{\bar{x}}} = z\,\overline{F_{\bar{x}z}} + \bar{z}\,\overline{F_{\bar{x}\bar{z}}}$
$= z\bar{y} + \bar{z}\bar{w}$

And from URP, we have

$\overline{F_x} = \overline{yz} = \bar{y} + \bar{z}$

And
$\overline{F} = x\overline{F_x} + \bar{x}\,\overline{F_{\bar{x}}}$
$= x(\bar{y}+\bar{z}) + \bar{x}\,(z\bar{y} + \bar{z}\bar{w})$
$= x\bar{y} + x\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{z}\bar{w}$

2(b) K-Map for $\bar{F}$ in (a)

$$\bar{F} = x\bar{y} + x\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{z}\bar{w}$$

xy \ zw	00	01	11	10
00	1	0	1	1
01	1	0	0	0
11	1	1	0	0
10	1	1	1	1

K-Map for F

$$F = \bar{x}yz + \bar{x}\bar{z}w + xyz$$

xy \ zw	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	0	0	1	1
10	0	0	0	0

$\therefore$ Complement in (a) is correct.

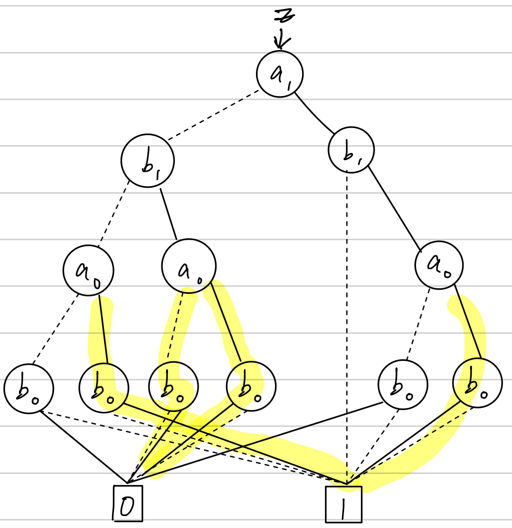
3 (a)

$$a > b \Rightarrow a_1\bar{b}_1 + a_0\bar{b}_0 a_1 + a_0\bar{b}_0\bar{b}_1$$

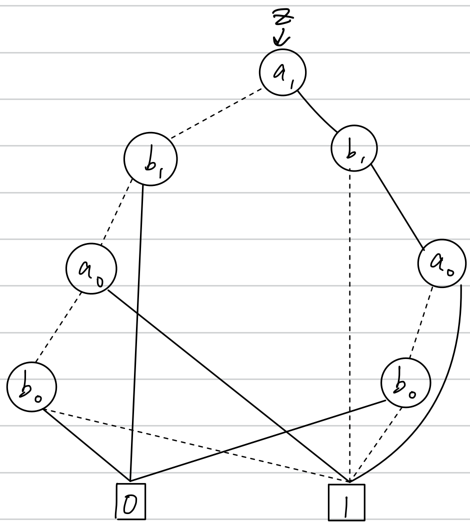
$$a = b \Rightarrow \bar{a}_1\bar{a}_0\bar{b}_1\bar{b}_0 + \bar{a}_1 a_0\bar{b}_1 b_0 + a_1 a_0 b_1 b_0 + a_1\bar{a}_0 b_1\bar{b}_0$$

$$\begin{aligned} z &= \text{"}a>b\text{"} + \text{"}a=b\text{"} \\ &= a_1\bar{b}_1 + a_0\bar{b}_0 a_1 + a_0\bar{b}_0\bar{b}_1 \\ &\quad + \bar{a}_1\bar{a}_0\bar{b}_1\bar{b}_0 + \bar{a}_1 a_0\bar{b}_1 b_0 + a_1 a_0 b_1 b_0 + a_1\bar{a}_0 b_1\bar{b}_0 \\ &= a_1\bar{b}_1 + (a_0\bar{b}_0 a_1 + a_0\bar{b}_0\bar{b}_1) \\ &\quad + \bar{a}_1\bar{b}_1(\bar{a}_0\bar{b}_0 + a_0 b_0) + a_1 b_1(\bar{a}_0\bar{b}_0 + a_0 b_0) \\ &= a_1\bar{b}_1 + (a_0\bar{b}_0 a_1 + a_0\bar{b}_0\bar{b}_1) \\ &\quad + (\bar{a}_1\bar{b}_1 + a_1 b_1)(\bar{a}_0\bar{b}_0 + a_0 b_0) \\ &= a_1\bar{b}_1 + (a_0\bar{b}_0 a_1 + a_0\bar{b}_0\bar{b}_1) + (a_1 \overline{\oplus} b_1)(a_0 \overline{\oplus} b_0) \end{aligned}$$

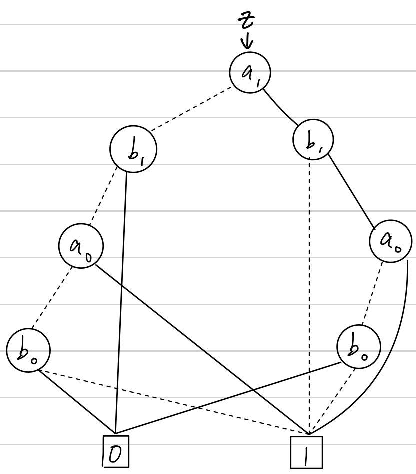
(b)



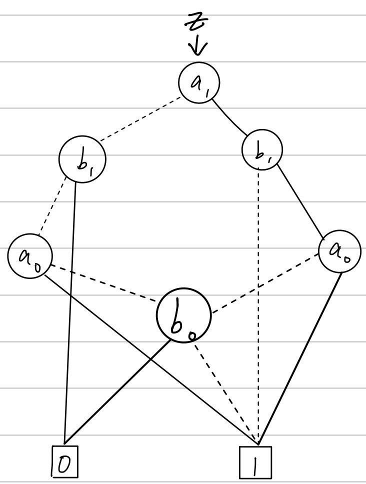
$\Rightarrow$



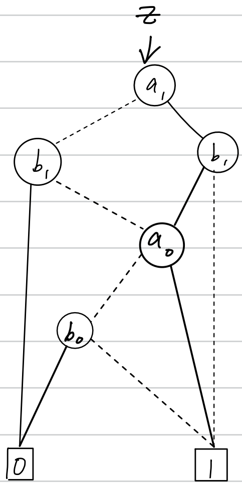
$\Rightarrow$



$\Rightarrow$

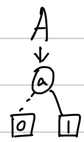


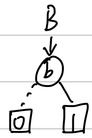
$\Rightarrow$



Q4 $F = a \overline{\oplus} b$

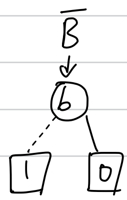
assume A and B are simple BDD of $A = a$ and $B = b$





Then implies $\Downarrow$

$\overline{B} = \overline{b}$



$a \overline{\oplus} b$: $ITE(A, B, \overline{B})$

$\downarrow$

assume $A > B$

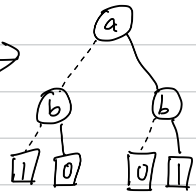
$\downarrow$

let $x = a$ which is root of A

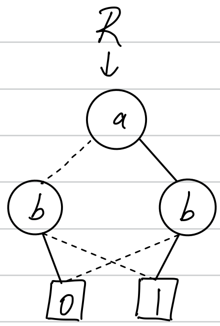
$PosFac = ITE(1, B, \overline{B})$
$NegFac = ITE(0, B, \overline{B})$
R = new node lable by a

$R.low = NegFac$
$= ITE(0, B, \overline{B})$
$\hookrightarrow$ return $\overline{B}$

$R.high = PosFac$
$= ITE(1, B, \overline{B})$
$\hookrightarrow$ return B



Simplify (R)



$\Rightarrow$ return R

J

(a) 11

(b)

$ab \backslash cd$	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

(c) $a'c'$, $ab'd$
(d) $ac'd$, $a'bc$

PTO

(e) By ESPRESSO Algorithm

Expansion

1) Build a OFF set of F

$F = a'c' + a'bc + ac'd + ab'd$

URP

	a	b	c	d
$a'c'$	10	11	10	11
$a'bc$	10	01	01	11
$ac'd$	01	11	10	01
$ab'd$	01	10	11	01

F_c

$a'b$	10	01	11	11
$ab'd$	01	10	11	01

F_{ca}

$b'd$	11	10	11	01

$F_{c\bar{a}}$

b	11	01	11	11

$F_{\bar{c}}$

a'	10	11	11	11
ad	01	11	11	01
$ab'd$	01	10	11	01

$F_{\bar{c}a}$

d	11	11	11	01
$b'd$	11	10	11	01

$F_{\bar{c}\bar{a}}$

11 11 11 11 ⇒ Tautology

$$\overline{F_c} = a\overline{F_{ca}} + \bar{a}\,\overline{F_{c\bar{a}}}$$
$$= a\overline{(\bar{b}d)} + \bar{a}\bar{b}$$
$$= ab + a\bar{d} + \bar{a}\bar{b}$$

$$\overline{F_{\bar{c}}} = a\overline{F_{\bar{c}a}} + \bar{a}\,\overline{F_{\bar{c}\bar{a}}}$$
$$= a\overline{(d + \bar{b}d)} + \bar{a}\overline{(1)}$$
$$= a\overline{(d(1+\bar{b}))}$$
$$= a\bar{d}$$

$$\overline{F} = c\overline{F_c} + \bar{c}\,\overline{F_{\bar{c}}}$$
$$= abc + ac\bar{d} + \bar{a}\bar{b}c + a\bar{c}\bar{d}$$

K-Map of $\overline{F}$

ab \ cd	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

K-Map of F

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

PTO

2) Expand 4 cubes

$F = a'c' + a'bc + ac'd + ab'd$

	abc	$ac\bar{d}$	$\bar{a}\bar{b}c$	$a\bar{c}\bar{d}$
a'	1	1		1
c'	1	1	1	

$\Rightarrow a'c'$

	abc	$ac\bar{d}$	$\bar{a}\bar{b}c$	$a\bar{c}\bar{d}$
a'	1	1		1
b			1	
c				1

$\Rightarrow a'b$

	abc	$ac\bar{d}$	$\bar{a}\bar{b}c$	$a\bar{c}\bar{d}$
a			1	
c'	1	1	1	
d		1		1

$\Rightarrow c'd$

	abc	$ac\bar{d}$	$\bar{a}\bar{b}c$	$a\bar{c}\bar{d}$
a			1	
b'	1			
d		1		1

$\Rightarrow ab'd$

Expansion form would be

$F = a'c' + c'd + a'b + ab'd$

K-Map of F

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

$F = a'c' + c'd + a'b + ab'd$ (9)

$\Rightarrow$

Irredundant K-Map of F

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

Nothing to remove

$\Downarrow$

Reduce K-Map of F

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

$F = a'c'd' + c'd + a'bc + ab'cd$ (12)

$\Leftarrow$

Expand K-Map of F

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

It would not be reduced again as it has reached local optimum(9). Further reducing would increase literal again.