CENG 4120 Computer-Aided Design for Very Large Scale Integrated Circuits

Homework 2 – Written

(Due on 15-3-2025 11:59pm)

1. Consider the following Boolean function:

$$F = ac'd + cd + ad' \qquad G = abc' + bc + acd$$

(i) Rewrite F and G such that algebraic methods can be used to find the common factors between them. Let R = c', S = d', F = aRd + cd+ aS, G = abR + bc + acd

(ii) Find all the kernels and co-kernels of F and G (after rewrite in part (i)), using the recursive algorithm *FindKernels*(). Show the recursion trees.

(iii) Find all the single cube divisors and multiple cube divisors between F and G.

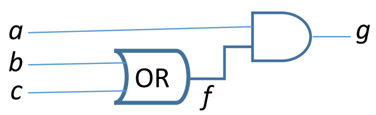
2. Consider the Boolean function:

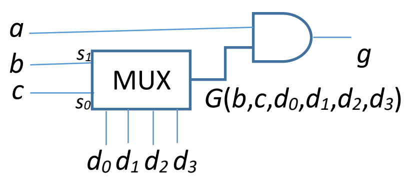
$$H = ab + c'd + a'c'd' + ab'cd'$$

Given that $H' = a'c + ab'c'd' + ab'cd$

(i) Construct the blocking matrix for the 4 cubes in H.
(ii) Expand the 4 cubes in H to primes. Show your steps.

3. Suppose a circuit has an OR gate which is mal-functioned, and we are going to repair it with a 4-input MUX as follows:





(i) Construct the function $G(b, c, d_0, d_1, d_2, d_3)$.
(ii) Construct a function $Z(b, c, d_0, d_1, d_2, d_3)$ such that $Z() \equiv 1$ if and only if $G() \equiv (b + c)$
(iii) Show how to derive the four inputs d_0, d_1, d_2, d_3 to the MUX using the quantification operator on $Z()$ so that the OR gate can be fixed.

4. (i) Consider a normalized Polish expression (NPE) $X = a\ b\ c\ d\ e\ f\ g + \times + \times + \times$. Show how X can be converted to

$$Y = g\ f\ c \times d + \times\ b + e \times a \times$$

inversion: ab, bc, ef, fg, compliment chain,

using a sequence of moves from the set {M1, M2, M3} in lecture 9.

(ii) Prove that the set of moves {M1, M2, M3} can convert any NPE X to any other NPE Y with the same n blocks.

(ii) equivilance : exact same state.

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1(i) Let $R = c'$, $S = d'$

$F = aRd + cd + aS$ $\quad$ $G = abR + bc + acd$

(ii) $F = aRd + cd + aS$

a	c	d	R	S
cube: aRd, aS	X	cube: aRd, cd	X	X
co: a		co: d		
K: $Rd+S$		K: $aR+c$		

cokernal: a $\quad$ d $\quad$ 1 (trivial)

kernal: $Rd+c$ $\quad$ $aR+c$ $\quad$ $aRd+cd+aS$

$c'd+c$ $\quad$ $ac'+c$ $\quad$ $ac'd+cd+ad'$

$G = abR + bc + acd$

a	b	c	d	R
cube: $abR+acd$	cube: $abR+bc$	cube: $bc+acd$	X	X
co: a	co: b	co: c		
k: $bR+cd$	k: $aR+c$	k: $b+ad$		

cokernal: a $\quad$ b $\quad$ c $\quad$ 1 (trivial)

kernal: $bR+cd$ $\quad$ $aR+c$ $\quad$ $b+ad$ $\quad$ $abR+bc+acd$

$bc'+cd$ $\quad$ $ac'+c$ $\quad$ $b+ad$

(iii) cokernal: a $\quad$ d

kernal: $Rd+c$ $\quad$ $aR+c$

F

cokernal: a $\quad$ b $\quad$ c

kernal: $bR+cd$ $\quad$ $aR+c$ $\quad$ $b+ad$

G

$Rd+c \cap bR+cd = \phi$

$Rd+c \cap aR+c = c$

$Rd+c \cap b+ad = \phi$

$aR+c \cap bR+cd = \phi$

$aR+c \cap aR+c = aR+c$

$aR+c \cap b+ad = \phi$

common multicube divisor $= aR+c = ac'+c$

common single cube divisor $= c$

2. (i) $H' = a'c + ab'c'd' + ab'cd$ $\qquad$ $H = ab + c'd + a'c'd' + ab'cd'$

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	1	1
10	0	1	0	1

ab

	$a'c$	$ab'c'd'$	$ab'cd$
a	1		
b		1	1

$c'd$

	$a'c$	$ab'c'd'$	$ab'cd$
c'	1		1
d		1	

$a'c'd'$

	$a'c$	$ab'c'd'$	$ab'cd$
a'		1	1
c'	1		1
d'			1

$ab'cd'$

	$a'c$	$ab'c'd'$	$ab'cd$
a	1		
b'			
c		1	
d'			1

(ii) find min set of row covering all column

ab

	$a'c$	$ab'c'd'$	$ab'cd$
a	1		
b		1	1

$\Rightarrow ab$

$c'd$

	$a'c$	$ab'c'd'$	$ab'cd$
c'	1		1
d		1	

$\Rightarrow c'd$

$H = ab + c'd + a'c' + acd'$

$ab \backslash cd$	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	1	1
10	0	1	0	1

$a'c'd'$

	$a'c$	$ab'c'd'$	$ab'cd$
a'		1	1
c'	1		1
d'			1

$\Rightarrow a'c'$

$ab'cd'$

	$a'c$	$ab'c'd'$	$ab'cd$
a	1		
b'			
c		1	
d'			1

$\Rightarrow acd'$

3(i) $s_1 = b$, $s_0 = c$

$b=0, c=0, G=d_0$

$b=0, c=1, G=d_1$

$b=1, c=0, G=d_2$

$b=1, c=1, G=d_3$

$G(b,c,d_0,d_1,d_2,d_3) = d_0b'c' + d_1b'c + d_2bc' + d_3bc$

(ii) $Z = (d_0b'c' + d_1b'c + d_2bc' + d_3bc) \overline{\oplus} (b+c)$

$Z_{b'c'} = G_{b'c'} \overline{\oplus} f_{b'c'} \rightarrow$ Set $b=0, c=0 \rightarrow (d_0 \cancel{b'c'} + \cancel{d_1b'c} + \cancel{d_2bc'} + \cancel{d_3bc}) \overline{\oplus} \cancel{(b+c)} = d_0 \overline{\oplus} 0 = d_0'$

$Z_{b'c} = G_{b'c} \overline{\oplus} f_{b'c} \rightarrow$ Set $b=0, c=1 \rightarrow (\cancel{d_0b'c'} + d_1\cancel{b'c} + \cancel{d_2bc'} + \cancel{d_3bc}) \overline{\oplus} \cancel{(b+c)} = d_1 \overline{\oplus} 1 = d_1$

$Z_{bc'} = G_{bc'} \overline{\oplus} f_{bc'} \rightarrow$ Set $b=1, c=0 \rightarrow (\cancel{d_0b'c'} + \cancel{d_1b'c} + d_2\cancel{bc'} + \cancel{d_3bc}) \overline{\oplus} \cancel{(b+c)} = d_2 \overline{\oplus} 1 = d_2$

$Z_{bc} = G_{bc} \overline{\oplus} f_{bc} \rightarrow$ Set $b=1, c=1 \rightarrow (\cancel{d_0b'c'} + \cancel{d_1b'c} + \cancel{d_2bc'} + d_3\cancel{bc}) \overline{\oplus} \cancel{(b+c)} = d_3 \overline{\oplus} 1 = d_3$

(iii) $(\forall bc\ Z)[d_0, d_1, d_2, d_3] = d_0'd_1d_2d_3$

To derive four inputs, solve $d_0'd_1d_2d_3 = 1$

$\therefore d_0 = 0, d_1 = 1, d_2 = 1, d_3 = 1$

4(i) $X = abcdefg+x+x+x$

$\Downarrow$ M2

$abcdefg\,x+x+x+$

$\Downarrow$ M1 × 6

$bcdefga\ x+x+x+$

$\Downarrow$ M1 × 2

$bcdfgea\ x+x+x+$

$\Downarrow$ M1 × 4

$cdfgbea\ x+x+x+$

$\Downarrow$ M1 × 3

$gcdfbea\ x+x+x+$

$\Downarrow$ M1 × 2

$gfcdbea\,x+x+x+$

$\Downarrow$ M3 × 5

$gfcdbe\,x+x+xa+$

$\Downarrow$ M2

$gfcdbe\,x+x+xax$

$\Downarrow$ M3 × 4

$gfcdb\,x+x+exax$

$\Downarrow$ M3 × 3

$gfcd\,x+x\,b+exax$

$\Downarrow$ M3

$Y = gfc\,x\,d+xb+exax$

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(ii) with move $M\{M1, M2, M3\}$

① wts $f_i \in M\{f_1 = M1, f_2 = M2, f_3 = M3\}$ are involution

assume a f_i function is performed on X to result Y.

Case $f_1 = M1$ assume X with state $\{\dots\ a\ b\ \dots\}$

$f_1(X \text{ on } a,b) = Y$ Y with state $\{\dots\ b\ a\ \dots\}$

with inverse fn $f_1^{-1}(Y \text{ on } a,b)$ we have state $\{\dots\ a\ b\ \dots\}$

$= X \rightarrow f_1 = f_1^{-1}$

$\therefore$ $f_1^{-1}(f_1(X)) = X$, which means $M1$ is involution

Case $f_2 = M2$ assume X with state $\{\dots\ + *\ \dots\}$

$f_2(X \text{ on } *,+) = Y$ Y with state $\{\dots\ * +\ \dots\}$

with inverse fn $f_2^{-1}(Y \text{ on } a,b)$ we have state $\{\dots\ + *\ \dots\}$

$= X \rightarrow f_2 = f_2^{-1}$

$\therefore$ $f_2^{-1}(f_3(X)) = X$, which means $M2$ is involution

Case $f_3 = M3$ assume X with state $\{\dots\ a+\ \dots\}$

$f_3(X \text{ on } a,+) = Y$ Y with state $\{\dots\ +a\ \dots\}$

f_3 only captures valid output while discarding invalid output,

we can assume f_3 will always output solution with valid state

i.e. Y is of state $\{\dots a+ \dots\}$ which is a valid state

with inverse fn $f_3^{-1}(Y \text{ on } a,+)$, this time we do not need to validate as X is an NPE by definition,

and Y is an NPE inherited by fn property.

i.e. we have state $\{\dots\ +a\ \dots\}$ $= X \rightarrow f_3 = f_3^{-1}$

$\therefore$ $f_3^{-1}(f_3(X)) = X$, which means $M3$ is involution

All 3 moves are involution → There is symmetry between X and Y for any Y resulted by performing M on X.

$$X \underset{M}{\overset{M}{\rightleftharpoons}} Y$$

② wts any X and Y can converge to a common state

let X: $ac+b*d+$ let Y: $adb*+c*$

By performing combinations of $M2$ & $M3$: $acbd+*+$ worst # of move $= (2^3+2^2+2^1$

$adbc*+*$

$\Rightarrow adbc+*+$

By performing combinations of $M1$ to order operand: $abcd+*+$ $abcc+*+$

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both X & Y come to exact same state

Since both X & Y can come to exact same state and
all 3 Moves are involutions

from any NPE X can go to any NPE Y and vice versa

exact same state

$$\begin{tikzcd}
X & \circ \arrow[r, shift left] & \circ \arrow[l, shift left] \arrow[r, shift left] & \circ \arrow[l, shift left] \arrow[dl, shift left] \\
Y & \circ \arrow[r, shift left] & \circ \arrow[l, shift left] \arrow[ur, shift left] &
\end{tikzcd}$$

* arrows are operations in M.