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1 ubuntu

1.1 run

1 | ~\$ bash cp.sh PA

1.2 cp.sh

```

1 #!/bin/bash
2 clear
3 g++ $1.cpp -DDBG -o $1
4 if [[ "$?" == "0" ]]; then
5     echo Running
6     ./$1 < $1.in > $1.out
7     echo END
8 fi

```

2 Basic

2.1 ascii

| int | char | int | char | int | char |
|-----|------|-----|------|-----|------|
| 32 | | 64 | @ | 96 | ` |
| 33 | ! | 65 | A | 97 | a |
| 34 | " | 66 | B | 98 | b |
| 35 | # | 67 | C | 99 | c |
| 36 | \$ | 68 | D | 100 | d |
| 37 | % | 69 | E | 101 | e |
| 38 | & | 70 | F | 102 | f |
| 39 | ' | 71 | G | 103 | g |
| 40 | (| 72 | H | 104 | h |
| 41 |) | 73 | I | 105 | i |
| 42 | * | 74 | J | 106 | j |
| 43 | + | 75 | K | 107 | k |
| 44 | , | 76 | L | 108 | l |
| 45 | - | 77 | M | 109 | m |
| 46 | . | 78 | N | 110 | n |
| 47 | / | 79 | O | 111 | o |
| 48 | 0 | 80 | P | 112 | p |
| 49 | 1 | 81 | Q | 113 | q |
| 50 | 2 | 82 | R | 114 | r |
| 51 | 3 | 83 | S | 115 | s |
| 52 | 4 | 84 | T | 116 | t |
| 53 | 5 | 85 | U | 117 | u |
| 54 | 6 | 86 | V | 118 | v |
| 55 | 7 | 87 | W | 119 | w |
| 56 | 8 | 88 | X | 120 | x |
| 57 | 9 | 89 | Y | 121 | y |
| 58 | : | 90 | Z | 122 | z |
| 59 | ; | 91 | [| 123 | { |
| 60 | < | 92 | \ | 124 | |
| 61 | = | 93 |] | 125 | } |
| 62 | > | 94 | ^ | 126 | ~ |
| 63 | ? | 95 | _ | | |

2.2 limits

| [Type] | [size] | [range] |
|--------------------|---|-------------------------------|
| char | 1 | 127 to -128 |
| signed char | 1 | 127 to -128 |
| unsigned char | 1 | 0 to 255 |
| short | 2 | 32767 to -32768 |
| int | 4 | 2147483647 to -2147483648 |
| unsigned int | 4 | 0 to 4294967295 |
| long | 4 | 2147483647 to -2147483648 |
| unsigned long | 4 | 0 to 18446744073709551615 |
| long long | 8 | |
| | 9223372036854775807 to -9223372036854775808 | |
| double | 8 | 1.79769e+308 to 2.22507e-308 |
| long double | 16 | 1.18973e+4932 to 3.3621e-4932 |
| float | 4 | 3.40282e+38 to 1.17549e-38 |
| unsigned long long | 8 | 0 to 18446744073709551615 |
| string | 32 | |

3 字串

3.1 最長迴文子字串

```

1 #include<bits/stdc++.h>
2 #define T(x) ((x)%2 ? s[(x)/2] : '. ')
3 using namespace std;
4
5 string s;
6 int n;
7
8 int ex(int l,int r){
9     int i=0;
10    while(l-i>=0&&r+i<n&&T(l-i)==T(r+i)) i++;
11    return i;
12 }
13
14 int main(){
15     cin>>s;
16     n=2*s.size()+1;
17     int mx=0;
18     int center=0;
19     vector<int> r(n);
20     int ans=1;
21     r[0]=1;
22     for(int i=1;i<n;i++){
23         int ii=center-(i-center);
24         int len=mx-i+1;
25         if(i>mx){
26             r[i]=ex(i,i);
27             center=i;
28             mx=i+r[i]-1;
29         }
30         else if(r[ii]==len){
31             r[i]=len+ex(i-len,i+len);
32             center=i;
33             mx=i+r[i]-1;
34         }
35         else r[i]=min(r[ii],len);
36         ans=max(ans,r[i]);
37     }
38     cout<<ans-1<<"\n";
39     return 0;
40 }

```

3.2 stringstream

```

1 string s,word;
2 stringstream ss;
3 getline(cin,s);
4 ss<<s;
5 while(ss>>word) cout<<word<<endl;

```

4 STL

4.1 BIT

```

1 template <class T> class BIT {
2 private:
3     int size;
4     vector<T> bit;
5     vector<T> arr;
6
7 public:
8     BIT(int sz=0): size(sz), bit(sz+1), arr(sz) {}
9
10    /** Sets the value at index idx to val. */
11    void set(int idx, T val) {
12        add(idx, val - arr[idx]);
13    }

```

```

14
15    /** Adds val to the element at index idx. */
16    void add(int idx, T val) {
17        arr[idx] += val;
18        for (++idx; idx<=size; idx+=(idx & -idx))
19            bit[idx] += val;
20    }
21
22    /** @return The sum of all values in [0, idx]. */
23    T pre_sum(int idx) {
24        T total = 0;
25        for (++idx; idx>0; idx--=(idx & -idx))
26            total += bit[idx];
27        return total;
28    }
29 };

```

4.2 priority_queue

```

1 priority_queue: 優先隊列，資料預設由大到小排序。
2
3 讀取優先權最高的值：
4     x = pq.top();
5     pq.pop(); //讀取後刪除
6 判斷是否為空的priority_queue：
7     pq.empty() //回傳 true
8     pq.size() //回傳 0
9 如需改變priority_queue的優先權定義：
10    priority_queue<T> pq; //預設由大到小
11    priority_queue<T, vector<T>, greater<T>> pq; //改成由小到大
12
13    priority_queue<T, vector<T>, cmp> pq; //cmp

```

4.3 deque

```

1 deque 是 C++ 標準模板函式庫
2 (Standard Template Library, STL)
3 中的雙向佇列容器 (Double-ended Queue) ,
4 跟 vector 相似，不過在 vector
5 中若是要添加新元素至開端，
6 其時間複雜度為 O(N)，但在 deque 中則是 O(1)。
7 同樣也能在我們需要儲存更多元素的時候自動擴展空間，
8 讓我們不必煩惱佇列長度的問題。
9 dq.push_back() //在 deque 的最尾端新增元素
10 dq.push_front() //在 deque 的開頭新增元素
11 dq.pop_back() //移除 deque 最尾端的元素
12 dq.pop_front() //移除 deque 最開頭的元素
13 dq.back() //取出 deque 最尾端的元素
14 dq.front() //回傳 deque 最開頭的元素
15 dq.insert()
16 dq.insert(position, n, val)
17     position: 插入元素的 index 值
18     n: 元素插入次數
19     val: 插入的元素值
20 dq.erase()
21 //刪除元素，需要使用迭代器指定刪除的元素或位置，
22 //同時也會返回指向刪除元素下一元素的迭代器。
23 dq.clear() //清空整個 deque 佇列。
24 dq.size() //檢查 deque 的尺寸
25 dq.empty() //如果 deque 佇列為空返回 1；
26 //若是存在任何元素，則返回 0
27 dq.begin() //返回一個指向 deque 開頭的迭代器
28 dq.end() //指向 deque 結尾，
29 //不是最後一個元素，
30 //而是最後一個元素的下一個位置

```

4.4 map

```

1 map：存放 key-value pairs 的映射資料結構，
2   會按 key 由小到大排序。
3 元素存取
4 operator[]：存取指定的[i]元素的資料
5
6 迭代器
7 begin()：回傳指向map頭部元素的迭代器
8 end()：回傳指向map末尾的迭代器
9 rbegin()：回傳一個指向map尾部的反向迭代器
10 rend()：回傳一個指向map頭部的反向迭代器
11
12 遍歷整個map時，利用iterator操作：
13 取key：it->first 或 (*it).first
14 取value：it->second 或 (*it).second
15
16 容量
17 empty()：檢查容器是否為空，空則回傳true
18 size()：回傳元素數量
19 max_size()：回傳可以容納的最大元素個數
20
21 修改器
22 clear()：刪除所有元素
23 insert()：插入元素
24 erase()：刪除一個元素
25 swap()：交換兩個map
26
27 查找
28 count()：回傳指定元素出現的次數
29 find()：查找一個元素
30
31 //實作範例
32 #include <bits/stdc++.h>
33 using namespace std;
34 int main(){
35     //declaration container and iterator
36     map<string, string> mp;
37     map<string, string>::iterator iter;
38     map<string, string>::reverse_iterator iter_r;
39
40     //insert element
41     mp.insert(pair<string, string>
42               ("r000", "student_zero"));
43     mp["r123"] = "student_first";
44     mp["r456"] = "student_second";
45
46     //traversal
47     for(iter=mp.begin(); iter!=mp.end(); iter++){
48         cout<<iter->first<<" "
49              <<iter->second<<endl;
50     }
51     for(iter_r=mp.rbegin(); iter_r!=mp.rend(); iter_r++){
52         cout<<iter_r->first<<" "
53              <<iter_r->second<<endl;
54     }
55
56     //find and erase the element
57     iter=mp.find("r123");
58     mp.erase(iter);
59     iter=mp.find("r123");
60     if(iter!=mp.end())
61         cout<<"Find, the value is "
62              <<iter->second<<endl;
63     else cout<<"Do not Find"<<endl;
64     return 0;
65 }

```

4.5 unordered_map

```

1 unordered_map：存放 key-value pairs
2   的「無序」映射資料結構。
3 用法與map相同

```

4.6 set

```

1 set： 集合，去除重複的元素，資料由小到大排序。
2
3 取值： 使用iterator
4     x = *st.begin();
5         // set中的第一個元素(最小的元素)。
6     x = *st.rbegin();
7         // set中的最後一個元素(最大的元素)。
8
9 判斷是否為空的set：
10     st.empty() 回傳true
11     st.size() 回傳零
12
13 常用來搭配的member function：
14     st.count(x);
15     auto it = st.find(x);
16         // binary search, O(log(N))
17     auto it = st.lower_bound(x);
18         // binary search, O(log(N))
19     auto it = st.upper_bound(x);
20         // binary search, O(log(N))

```

4.7 multiset

```

1 與 set 用法雷同，但會保留重複的元素。
2 資料由小到大排序。
3 宣告：
4     multiset<int> st;
5 刪除資料：
6     st.erase(val);
7         //會刪除所有值為 val 的元素。
8     st.erase(st.find(val));
9         //只刪除第一個值為 val 的元素。

```

4.8 unordered_set

```

1 unordered_set 的實作方式通常是用雜湊表(hash table)，
2 資料插入和查詢的時間複雜度很低，為常數級別O(1)，
3 相對的代價是消耗較多的記憶體，空間複雜度較高，
4 無自動排序功能。
5
6 unordered_set 判斷元素是否存在
7 unordered_set<int> myunordered_set;
8 myunordered_set.insert(2);
9 myunordered_set.insert(4);
10 myunordered_set.insert(6);
11 cout << myunordered_set.count(4) << "\n"; // 1
12 cout << myunordered_set.count(8) << "\n"; // 0

```

4.9 單調隊列

```

1 //單調隊列
2 "如果一個選手比你小還比你強，你就可以退役了。"--單調隊列
3
4 example
5
6 給出一個長度為 n 的數組，
7 輸出每 k 個連續的數中的最大值和最小值。
8
9 #include <bits/stdc++.h>
10 #define maxn 1000100
11 using namespace std;
12 int q[maxn], a[maxn];
13 int n, k;
14
15 void getmin() {
16     // 得到這個隊列裡的最小值，直接找到最後的就行了

```

```

17 |     int head=0,tail=0;
18 |     for(int i=1;i<k;i++) {
19 |         while(head<=tail&&a[q[tail]]>=a[i]) tail--;
20 |         q[++tail]=i;
21 |     }
22 |     for(int i=k;i<=n;i++) {
23 |         while(head<=tail&&a[q[tail]]>=a[i]) tail--;
24 |         q[++tail]=i;
25 |         while(q[head]<=i-k) head++;
26 |         cout<<a[q[head]]<<" ";
27 |     }
28 |     cout<<endl;
29 | }
30 |
31 | void getmax() { // 和上面同理
32 |     int head=0,tail=0;
33 |     for(int i=1;i<k;i++) {
34 |         while(head<=tail&&a[q[tail]]<=a[i])tail--;
35 |         q[++tail]=i;
36 |     }
37 |     for(int i=k;i<=n;i++) {
38 |         while(head<=tail&&a[q[tail]]<=a[i])tail--;
39 |         q[++tail]=i;
40 |         while(q[head]<=i-k) head++;
41 |         cout<<a[q[head]]<<" ";
42 |     }
43 |     cout<<endl;
44 | }
45 |
46 | int main(){
47 |     cin>>n>>k; //每k個連續的數
48 |     for(int i=1;i<=n;i++) cin>>a[i];
49 |     getmin();
50 |     getmax();
51 |     return 0;
52 | }

```

5 sort

5.1 大數排序

```

1 | #python大數排序
2 |
3 | while True:
4 |     try:
5 |         n = int(input())           # 有幾筆數字需要排序
6 |         arr = []                   # 建立空串列
7 |         for i in range(n):
8 |             arr.append(int(input())) # 依序將數字存入串列
9 |             arr.sort()              # 串列排序
10 |        for i in arr:
11 |            print(i)                # 依序印出串列中每個項目
12 |    except:
13 |        break

```

6 math

6.1 質數與因數

```

1 | 埃氏篩法
2 | int n;
3 | vector<int> isprime(n+1,1);
4 | isprime[0]=isprime[1]=0;
5 | for(int i=2;i<=n;i++){
6 |     if(isprime[i])
7 |         for(int j=i*i;j<=n;j+=i) isprime[j]=0;
8 | }
9 |
10 | 歐拉篩O(n)
11 | #define MAXN 47000 //sqrt(2^31)=46,340...

```

```

12 | bool isPrime[MAXN];
13 | int prime[MAXN];
14 | int primeSize=0;
15 | void getPrimes(){
16 |     memset(isPrime, true, sizeof(isPrime));
17 |     isPrime[0]=isPrime[1]=false;
18 |     for(int i=2;i<MAXN;i++){
19 |         if(isPrime[i]) prime[primeSize++]=i;
20 |         for(int
21 |             j=0;j<primeSize&&i*prime[j]<=MAXN;++j){
22 |             isPrime[i*prime[j]]=false;
23 |             if(i%prime[j]==0) break;
24 |         }
25 |     }
26 | }
27 | 最大公因數 0(log(min(a,b)))
28 | int GCD(int a,int b){
29 |     if(b==0) return a;
30 |     return GCD(b,a%b);
31 | }
32 |
33 | 質因數分解
34 | void primeFactorization(int n){
35 |     for(int i=0;i<(int)p.size();++i){
36 |         if(p[i]*p[i]>n) break;
37 |         if(n%p[i]) continue;
38 |         cout<<p[i]<<' ';
39 |         while(n%p[i]==0) n/=p[i];
40 |     }
41 |     if(n!=1) cout<<n<<' ';
42 |     cout<<'\n';
43 | }
44 |
45 | 擴展歐幾里得算法
46 | //ax+by=GCD(a,b)
47 | #include <bits/stdc++.h>
48 | using namespace std;
49 |
50 | int ext_euc(int a,int b,int &x,int &y){
51 |     if(b==0){
52 |         x=1,y=0;
53 |         return a;
54 |     }
55 |     int d=ext_euc(b,a%b,y,x);
56 |     y-=a/b*x;
57 |     return d;
58 | }
59 |
60 | int main(){
61 |     int a,b,x,y;
62 |     cin>>a>>b;
63 |     ext_euc(a,b,x,y);
64 |     cout<<x<<' '<<y<<endl;
65 |     return 0;
66 | }
67 |
68 |
69 |
70 | 歌德巴赫猜想
71 | solution : 把偶數 N (6≤N≤10^6) 寫成兩個質數的和。
72 | #include <iostream>
73 | using namespace std;
74 | #define N 20000000
75 | int ox[N],p[N],pr;
76 | void PrimeTable(){
77 |     ox[0]=ox[1]=1;
78 |     pr=0;
79 |     for(int i=2;i<N;i++){
80 |         if(!ox[i]) p[pr++]=i;
81 |         for(int j=0;i*p[j]<N&&j<pr;j++)
82 |             ox[i*p[j]]=1;
83 |     }
84 | }
85 |
86 | int main(){
87 |     PrimeTable();

```

```

88     int n;
89     while(cin>>n,n){
90         int x;
91         for(x=1;;x+=2)
92             if(!ox[x]&&!ox[n-x]) break;
93         printf("%d = %d + %d\n",n,x,n-x);
94     }
95 }
96 problem : 給定整數 N ,
97           求 N 最少可以拆成多少個質數的和。
98 如果 N 是質數,則答案為 1。
99 如果 N 是偶數(不包含2),則答案為 2 (強歌德巴赫猜想)。
100 如果 N 是奇數且 N-2 是質數,則答案為 2 (2+質數)。
101 其他狀況答案為 3 (弱歌德巴赫猜想)。
102 #include<bits/stdc++.h>
103 using namespace std;
104
105 bool isPrime(int n){
106     for(int i=2;i<n;++i){
107         if(i*i>n) return true;
108         if(n%i==0) return false;
109     }
110     return true;
111 }
112
113 int main(){
114     int n;
115     cin>>n;
116     if(isPrime(n)) cout<<"1\n";
117     else if(n%2==0||isPrime(n-2)) cout<<"2\n";
118     else cout<<"3\n";
119 }

```

6.2 快速幂

```

1 計算 a^b
2 #include<iostream>
3 #define ll long long
4 using namespace std;
5
6 const ll MOD=1000000007;
7 ll fp(ll a, ll b) {
8     int ans=1;
9     while(b>0){
10         if(b&1) ans=ans*a%MOD;
11         a=a*a%MOD;
12         b>>=1;
13     }
14     return ans;
15 }
16
17 int main() {
18     int a,b;
19     cin>>a>>b;
20     cout<<fp(a,b);
21 }

```

6.3 歐拉函數

```

1 //計算閉區間 [1,n] 中的正整數與 n 互質的個數
2
3 int phi(){
4     int ans=n;
5     for(int i=2;i*i<=n;i++){
6         if(n%i==0){
7             ans=ans-ans/i;
8             while(n%i==0) n/=i;
9         }
10    }
11    if(n>1) ans=ans-ans/n;
12    return ans;
13 }

```

6.4 atan

```

1 說明
2     atan() 和 atan2() 函數分別計算 x 和 y/x 的反正切。
3
4 回覆值
5     atan() 函數會傳回介於範圍 - /2 到 /2 弧度之間的值。
6     atan2() 函數會傳回介於 - 至 弧度之間的值。
7     如果 atan2() 函數的兩個引數都是零,
8     則函數會將 errno 設為 EDOM, 並傳回值 0。
9
10 範例
11 #include <math.h>
12 #include <stdio.h>
13
14 int main(void){
15     double a,b,c,d;
16
17     c=0.45;
18     d=0.23;
19
20     a=atan(c);
21     b=atan2(c,d);
22
23     printf("atan(%lf)=%lf\n",c,a);
24     printf("atan2(%lf,%lf)=%lf\n",c,d,b);
25 }
26
27 /*
28 atan(0.450000)=0.422854
29 atan2(0.450000,0.230000)=1.098299
30 */

```

6.5 大步小步

```

1 題意
2 給定 B,N,P, 求出 L 滿足 B^L ≡ N(mod P)。
3
4 題解
5 餘數的循環節長度必定為 P 的因數, 因此
6   B^0, B^1, B^2, ..., B^(P-1), ...
7 也就是說如果有解則 L<N, 枚舉 0, 1, 2, ..., L-1
8 能得到結果, 但會超時。
9
10 將 L 拆成 mx+y, 只要分別枚舉 x,y 就能得到答案,
11 設 m=√P 能保證最多枚舉 2√P 次。
12
13 B^(mx+y) ≡ N(mod P)
14 B^(mx) B^y ≡ N(mod P)
15 B^y ≡ N(B^(-m))^x (mod P)
16
17 先求出 B^0, B^1, B^2, ..., B^(m-1),
18 再枚舉 N(B^(-m)), N(B^(-m))^2, ... 查看是否有對應的 B^y。
19 這種算法稱為大步小步演算法,
20 大步指的是枚舉 x (一次跨 m 步),
21 小步指的是枚舉 y (一次跨 1 步)。
22
23 複雜度分析
24 利用 map/unorder_map 存放 B^0, B^1, B^2, ..., B^(m-1),
25 枚舉 x 查詢 map/unorder_map 是否有對應的 B^y,
26 存放和查詢最多 2√P 次, 時間複雜度為 O(√P log √P)/O(√P)。
27
28 #include <bits/stdc++.h>
29 using namespace std;
30 using LL = long long;
31 LL B, N, P;
32
33 LL fpow(LL a, LL b, LL c){
34     LL res=1;

```

```

35     for(;b; b >=>1){
36         if(b&1)
37             res=(res*a)%c;
38         a=(a*a)%c;
39     }
40     return res;
41 }
42
43 LL BSGS(LL a, LL b, LL p){
44     a%=p, b%=p;
45     if(a==0)
46         return b==0?1:-1;
47     if(b==1)
48         return 0;
49     map<LL, LL> tb;
50     LL sq=ceil(sqrt(p-1));
51     LL inv=fpow(a, p-sq-1, p);
52     tb[1]=sq;
53     for(LL i=1, tmp=1; i<sq; ++i){
54         tmp=(tmp*a)%p;
55         if(!tb.count(tmp))
56             tb[tmp]=i;
57     }
58     for(LL i=0; i<sq; ++i){
59         if(tb.count(b)){
60             LL res=tb[b];
61             return i*sq+(res==sq?0:res);
62         }
63         b=(b*inv)%p;
64     }
65     return -1;
66 }
67
68 int main(){
69     ios::sync_with_stdio(false);
70     cin.tie(0), cout.tie(0);
71     while(cin>>P>>B>>N){
72         LL ans=BSGS(B, N, P);
73         if(ans!=-1)
74             cout<<"no solution\n";
75         else
76             cout<<ans<<"\n";
77     }
78 }

```

7 algorithm

7.1 basic

```

1 min_element: 找尋最小元素
2 min_element(first, last)
3 max_element: 找尋最大元素
4 max_element(first, last)
5 sort: 排序, 預設由小排到大。
6 sort(first, last)
7 sort(first, last, cmp): 可自行定義比較運算子 cmp。
8 find: 尋找元素。
9 find(first, last, val)
10 lower_bound: 尋找第一個小於 x 的元素位置,
11     如果不存在, 則回傳 last。
12 lower_bound(first, last, val)
13 upper_bound: 尋找第一個大於 x 的元素位置,
14     如果不存在, 則回傳 last。
15 upper_bound(first, last, val)
16 next_permutation: 將序列順序轉換成下一個字典序,
17     如果存在回傳 true, 反之回傳 false。
18 next_permutation(first, last)
19 prev_permutation: 將序列順序轉換成上一個字典序,
20     如果存在回傳 true, 反之回傳 false。
21 prev_permutation(first, last)

```

7.2 二分搜

```

1 int binary_search(int target) {
2     // For range [ok, ng) or (ng, ok], "ok" is for the
3     // index that target value exists, with "ng" doesn't.
4     int ok = maxn, ng = -1;
5     // For first lower_bound, ok=maxn and ng=-1,
6     // for last lower_bound, ok = -1 and ng = maxn
7     // (the "check" funtion
8     // should be changed depending on it.)
9     while(abs(ok - ng) > 1) {
10         int mid = (ok + ng) >> 1;
11         if(check(mid)) ok = mid;
12         else ng = mid;
13     }
14     // Be careful, "arr[mid]>=target" for first
15     // lower_bound and "arr[mid]<=target" for
16     // last lower_bound. For range (ng, ok],
17     // convert it into (ng, mid] and (mid, ok] than
18     // choose the first one, or convert [ok, ng) into
19     // [ok, mid) and [mid, ng) and then choose
20     // the second one.
21     return ok;
22 }
23
24 lower_bound(arr, arr + n, k); //最左邊 ≥ k 的位置
25 upper_bound(arr, arr + n, k); //最左邊 > k 的位置
26 upper_bound(arr, arr + n, k) - 1; //最右邊 ≤ k 的位置
27 lower_bound(arr, arr + n, k) - 1; //最右邊 < k 的位置
28 (lower_bound, upper_bound) //等於 k 的範圍
29 equal_range(arr, arr+n, k);

```

7.3 三分搜

```

1 題意
2 給定兩射線方向和速度, 問兩射線最近距離。
3
4 題解
5 假設 F(t) 為兩射線在時間 t 的距離, F(t) 為二次函數,
6 可用三分搜找二次函數最小值。
7
8 #include <bits/stdc++.h>
9 using namespace std;
10
11 struct Point{
12     double x, y, z;
13     Point() {}
14     Point(double _x, double _y, double _z):
15         x(_x), y(_y), z(_z){}
16     friend istream& operator>>(istream& is, Point& p)
17     {
18         is >> p.x >> p.y >> p.z;
19         return is;
20     }
21     Point operator+(const Point &rhs) const{
22         return Point(x+rhs.x, y+rhs.y, z+rhs.z);
23     }
24     Point operator-(const Point &rhs) const{
25         return Point(x-rhs.x, y-rhs.y, z-rhs.z);
26     }
27     Point operator*(const double &d) const{
28         return Point(x*d, y*d, z*d);
29     }
30     Point operator/(const double &d) const{
31         return Point(x/d, y/d, z/d);
32     }
33     double dist(const Point &rhs) const{
34         double res = 0;
35         res+=(x-rhs.x)*(x-rhs.x);
36         res+=(y-rhs.y)*(y-rhs.y);
37         res+=(z-rhs.z)*(z-rhs.z);
38         return res;
39     }
40 };

```



```

40
41 int main(){
42     ios::sync_with_stdio(false);
43     cin.tie(0),cout.tie(0);
44     int T;
45     cin>>T;
46     for(int ti=1;ti<=T;++ti){
47         double time;
48         Point x1,y1,d1,x2,y2,d2;
49         cin>>time>>x1>>y1>>x2>>y2;
50         d1=(y1-x1)/time;
51         d2=(y2-x2)/time;
52         double L=0,R=1e8,m1,m2,f1,f2;
53         double ans = x1.dist(x2);
54         while(abs(L-R)>1e-10){
55             m1=(L+R)/2;
56             m2=(m1+R)/2;
57             f1=((d1*m1)+x1).dist((d2*m1)+x2);
58             f2=((d1*m2)+x1).dist((d2*m2)+x2);
59             ans = min(ans,min(f1,f2));
60             if(f1<f2) R=m2;
61             else L=m1;
62         }
63         cout<<"Case "<<ti<<" : ";
64         cout<<fixed<<setprecision(4)<<sqrt(ans)<<"\n";
65     }
66 }

```

7.4 prefix sum

```

1 // 前綴和
2 陣列前n項的和。
3 b[i]=a[0]+a[1]+a[2]+ ... +a[i]
4 區間和 [l, r]: b[r]-b[l-1] (要保留b[l]所以-1)
5
6 #include<bits/stdc++.h>
7 using namespace std;
8 int main(){
9     int n;
10    cin>>n;
11    int a[n],b[n];
12    for(int i=0;i<n;i++) cin>>a[i];
13    b[0]=a[0];
14    for(int i=1;i<n;i++) b[i]=b[i-1]+a[i];
15    for(int i=0;i<n;i++) cout<<b[i]<<" ";
16    cout<<"\n";
17    int l,r;
18    cin>>l>>r;
19    cout<<b[r]-b[l-1]; //區間和
20 }

```

7.5 差分

```

1 // 差分
2 用途：在區間 [l, r] 加上一個數字v。
3 b[l] += v; (b[0~l] 加上v)
4 b[r+1] -= v; (b[r+1~n] 減去v (b[r] 仍保留v) )
5 給的 a[] 是前綴和數列，建構 b[]，
6 因為 a[i] = b[0] + b[1] + b[2] + ... + b[i]，
7 所以 b[i] = a[i] - a[i-1]。
8 在 b[l] 加上 v，b[r+1] 減去 v，
9 最後再從 0 跑到 n 使 b[i] += b[i-1]。
10 這樣一來，b[] 是一個在某區間加上v的前綴和。
11
12 #include <bits/stdc++.h>
13 using namespace std;
14 int a[1000], b[1000];
15 // a: 前綴和數列, b: 差分數列
16 int main(){
17     int n, l, r, v;
18     cin >> n;
19     for(int i=1; i<=n; i++){

```

```

20         cin >> a[i];
21         b[i] = a[i] - a[i-1]; //建構差分數列
22     }
23     cin >> l >> r >> v;
24     b[l] += v;
25     b[r+1] -= v;
26
27     for(int i=1; i<=n; i++){
28         b[i] += b[i-1];
29         cout << b[i] << " ";
30     }
31 }

```

7.6 greedy

1 //貪心
2 貪心演算法的核心為，
3 採取在目前狀態下最好或最佳（即最有利）的選擇。
4 貪心演算法雖然能獲得當前最佳解，
5 但不保證能獲得最後（全域）最佳解，
6 提出想法後可以先試圖尋找有沒有能推翻原本的想法的反例，
7 確認無誤再實作。

8
9 刪數字問題

10 //problem
11 給定一個數字 $N(\leq 10^{100})$ ，需要刪除 K 個數字，
12 請問刪除 K 個數字後最小的數字為何？

13 //solution
14 刪除滿足第 i 位數大於第 i+1 位數的最左邊第 i 位數，
15 扣除高位數的影響較扣除低位數的大。

```

16 //code
17 int main(){
18     string s;
19     int k;
20     cin>>s>>k;
21     for(int i=0;i<k;++i){
22         if((int)s.size()==0) break;
23         int pos =(int)s.size()-1;
24         for(int j=0;j<(int)s.size()-1;++j){
25             if(s[j]>s[j+1]){
26                 pos=j;
27                 break;
28             }
29         }
30         s.erase(pos,1);
31     }
32     while((int)s.size()>0&&s[0]=='0')
33         s.erase(0,1);
34     if((int)s.size()) cout<<s<<"\n";
35     else cout<<0<<"\n";
36 }

```

37 最小區間覆蓋長度

38 //problem
39 給定 n 條線段區間為 [Li,Ri]，
40 請問最少要選幾個區間才能完全覆蓋 [0,S]？

41 //solution
42 先將所有區間依照左界由小到大排序，
43 對於當前區間 [Li,Ri]，要從左界 >Ri 的所有區間中，
44 找到有著最大的右界的區間，連接當前區間。

45 //problem
46 長度 n 的直線中有數個加熱器，
47 在 x 的加熱器可以讓 [x-r,x+r] 內的物品加熱，
48 問最少要幾個加熱器可以把 [0,n] 的範圍加熱。

49 //solution
50 對於最左邊沒加熱的點a，選擇最遠可以加熱a的加熱器，

59 更新已加熱範圍，重複上述動作繼續尋找加熱器。

```

60
61 //code
62 int main(){
63     int n, r;
64     int a[1005];
65     cin>>n>>r;
66     for(int i=1;i<=n;++i) cin>>a[i];
67     int i=1,ans=0;
68     while(i<=n){
69         int R=min(i+r-1,n),L=max(i-r+1,0)
70         int nextR=-1;
71         for(int j=R;j>=L;--j){
72             if(a[j]){
73                 nextR=j;
74                 break;
75             }
76         }
77         if(nextR==-1){
78             ans=-1;
79             break;
80         }
81         ++ans;
82         i=nextR+r;
83     }
84     cout<<ans<<'\n';
85 }

```

88 最多不重疊區間

89 //problem

90 給你 n 條線段區間為 $[Li, Ri]$ ，
91 請問最多可以選擇幾條不重疊的線段(頭尾可相連)?

92 //solution

93 依照右界由小到大排序，
94 每次取到一個不重疊的線段，答案 +1。

```

95
96 //code
97 struct Line{
98     int L,R;
99     bool operator<(const Line &rhs)const{
100         return R<rhs.R;
101     }
102 };
103
104
105 int main(){
106     int t;
107     cin>>t;
108     Line a[30];
109     while(t--){
110         int n=0;
111         while(cin>>a[n].L>>a[n].R,a[n].L||a[n].R)
112             ++n;
113         sort(a,a+n);
114         int ans=1,R=a[0].R;
115         for(int i=1;i<n;i++){
116             if(a[i].L>=R){
117                 ++ans;
118                 R=a[i].R;
119             }
120         }
121         cout<<ans<<'\n';
122     }
123 }

```

126 最小化最大延遲問題

127 //problem

128 給定 N 項工作，每項工作的需要處理時長為 T_i ，
129 期限是 D_i ，第 i 項工作延遲的時間為 $Li = \max(0, Fi - Di)$ ，
130 原本 Fi 為第 i 項工作的完成時間，
131 求一種工作排序使 $\max Li$ 最小。

132 //solution

134 按照到期時間從早到晚處理。

```

135
136 //code
137 struct Work{
138     int t, d;
139     bool operator<(const Work &rhs)const{
140         return d<rhs.d;
141     }
142 };
143
144 int main(){
145     int n;
146     Work a[10000];
147     cin>>n;
148     for(int i=0;i<n;++i)
149         cin>>a[i].t>>a[i].d;
150     sort(a,a+n);
151     int maxL=0,sumT=0;
152     for(int i=0;i<n;++i){
153         sumT+=a[i].t;
154         maxL=max(maxL,sumT-a[i].d);
155     }
156     cout<<maxL<<'\n';
157 }

```

160 最少延遲數量問題

161 //problem

162 給定 N 個工作，每個工作的需要處理時長為 T_i ，
163 期限是 D_i ，求一種工作排序使得逾期工作數量最小。

164 //solution

165 期限越早到期的工作越先做。將工作依照到期時間從早到晚排序，
166 依序放入工作列表中，如果發現有工作預期，
167 就從目前選擇的工作中，移除耗時最長的工作。

168 上述方法為 Moore-Hodgson's Algorithm。

171 //problem

172 給定烏龜的重量和可承受重量，問最多可以疊幾隻烏龜？

173 //solution

174 和最少延遲數量問題是相同的問題，只要將題敘做轉換。

175 工作處理時長 → 烏龜重量

176 工作期限 → 烏龜可承受重量

177 多少工作不延期 → 可以疊幾隻烏龜

```

178
179 //code
180 struct Work{
181     int t, d;
182     bool operator<(const Work &rhs)const{
183         return d<rhs.d;
184     }
185 };
186
187
188 int main(){
189     int n=0;
190     Work a[10000];
191     priority_queue<int> pq;
192     while(cin>>a[n].t>>a[n].d)
193         ++n;
194     sort(a,a+n);
195     int sumT=0,ans=n;
196     for(int i=0;i<n;++i){
197         pq.push(a[i].t);
198         sumT+=a[i].t;
199         if(a[i].d<sumT){
200             int x=pq.top();
201             pq.pop();
202             sumT-=x;
203             --ans;
204         }
205     }
206     cout<<ans<<'\n';
207 }

```

210 任務調度問題


```

211 //problem
212 給定 N 項工作，每項工作的需要處理時長為  $T_i$ ，
213 期限是  $D_i$ ，如果第  $i$  項工作延遲需要受到  $p_i$  單位懲罰，
214 請問最少會受到多少單位懲罰。

```

```

215 //solution
216 依照懲罰由大到小排序，
217 每項工作依序嘗試可不可以放在  $D_i - T_i + 1, D_i - T_i, \dots, 1, 0$ ，
218 如果有空閒就放進去，否則延後執行。

```

```

219 //problem
220 給定 N 項工作，每項工作的需要處理時長為  $T_i$ ，
221 期限是  $D_i$ ，如果第  $i$  項工作在期限內完成會獲得  $a_i$ 
222 單位獎勵，
223 請問最多會獲得多少單位獎勵。

```

```

224 //solution
225 和上題相似，這題變成依照獎勵由大到小排序。

```

```

226 //code
227 struct Work{
228     int d,p;
229     bool operator<(const Work &rhs)const{
230         return p>rhs.p;
231     }
232 };
233 int main(){
234     int n;
235     Work a[100005];
236     bitset<100005> ok;
237     while(cin>>n){
238         ok.reset();
239         for(int i=0;i<n;++i)
240             cin>>a[i].d>>a[i].p;
241         sort(a,a+n);
242         int ans=0;
243         for(int i=0;i<n;++i){
244             int j=a[i].d;
245             while(j--){
246                 if(!ok[j]){
247                     ans+=a[i].p;
248                     ok[j]=true;
249                     break;
250                 }
251             }
252         }
253         cout<<ans<<"\n";
254     }
255 }

```

7.7 floyd warshall

```

1 int w[n][n];
2 int d[n][n];
3 int p[n][n];
4 // 由 i 點到 j 點的路徑，其中繼點為 p[i][j]。
5 void floyd_warshall(){ //O(V^3)
6     for(int i=0;i<n;i++){
7         for(int j=0;j<n;j++){
8             d[i][j]=w[i][j];
9             p[i][j]=-1; // 預設為沒有中繼點
10        }
11    }
12    for(int i=0;i<n;i++) d[i][i]=0;
13    for(int k=0;k<n;k++){
14        for(int i=0;i<n;i++){
15            for(int j=0;j<n;j++){
16                if(d[i][k]+d[k][j]<d[i][j]){
17                    d[i][j]=d[i][k]+d[k][j];
18                    p[i][j]=k; // 由 i 點走到 j 點經過了 k 點
19                }
20            }
21        }
22    }
23    // 這支函式並不會印出起點和終點，必須另行印出。

```

```

23 void find_path(int s,int t){ // 印出最短路徑
24     if(p[s][t]==-1) return; // 沒有中繼點就結束
25     find_path(s,p[s][t]); // 前半段最短路徑
26     cout<<p[s][t]; // 中繼點
27     find_path(p[s][t],t); // 後半段最短路徑
28 }

```

7.8 dinic

```

1 const int maxn = 1e5 + 10;
2 const int inf = 0x3f3f3f3f;
3
4 struct Edge {
5     int s, t, cap, flow;
6 };
7
8 int n, m, S, T;
9 int level[maxn], dfs_idx[maxn];
10 vector<Edge> E;
11 vector<vector<int>> G;
12
13 void init() {
14     S = 0;
15     T = n + m;
16     E.clear();
17     G.assign(maxn, vector<int>());
18 }
19
20 void addEdge(int s, int t, int cap) {
21     E.push_back({s, t, cap, 0});
22     E.push_back({t, s, 0, 0});
23     G[s].push_back(E.size()-2);
24     G[t].push_back(E.size()-1);
25 }
26
27 bool bfs() {
28     queue<int> q({S});
29
30     memset(level, -1, sizeof(level));
31     level[S] = 0;
32
33     while(!q.empty()) {
34         int cur = q.front();
35         q.pop();
36
37         for(int i : G[cur]) {
38             Edge e = E[i];
39             if(level[e.t]==-1 && e.cap>e.flow) {
40                 level[e.t] = level[e.s] + 1;
41                 q.push(e.t);
42             }
43         }
44     }
45     return level[T];
46 }
47
48 int dfs(int cur, int lim) {
49     if(cur==T || lim==0) return lim;
50
51     int result = 0;
52     for(int& i=dfs_idx[cur]; i<G[cur].size() && lim; i++) {
53         Edge& e = E[G[cur][i]];
54         if(level[e.s]+1 != level[e.t]) continue;
55
56         int flow = dfs(e.t, min(lim, e.cap-e.flow));
57         if(flow <= 0) continue;
58
59         e.flow += flow;
60         result += flow;
61         E[G[cur][i]^1].flow -= flow;
62         lim -= flow;
63     }
64     return result;
65 }

```

```

66
67 int dinic() {          // O((V^2)E)
68     int result = 0;
69     while(bfs()) {
70         memset(dfs_idx, 0, sizeof(dfs_idx));
71         result += dfs(S, inf);
72     }
73     return result;
74 }

```

7.9 SegmentTree

```

1 #define MAXN 1000
2 int data[MAXN]; //原數據
3 int st[4 * MAXN]; //線段樹
4 int tag[4 * MAXN]; //懶標
5
6 inline int pull(int l, int r) {
7     // 隨題目改變sum、max、min
8     // l、r是左右樹的index
9     return st[l] + st[r];
10 }
11
12 void build(int l, int r, int i) {
13     // 在[l, r]區間建樹，目前根的index為i
14     if (l == r) {
15         st[i] = data[l];
16         return;
17     }
18     int mid = l + ((r - l) >> 1);
19     build(l, mid, i * 2);
20     build(mid + 1, r, i * 2 + 1);
21     st[i] = pull(i * 2, i * 2 + 1);
22 }
23
24 int query(int ql, int qr, int l, int r, int i) {
25     // [ql, qr]是查詢區間,[l, r]是當前節點包含的區間
26     if (ql <= l && r <= qr)
27         return st[i];
28     int mid = l + ((r - l) >> 1);
29     if (tag[i]) {
30         //如果當前懶標有值則更新左右節點
31         st[i * 2] += tag[i] * (mid - l + 1);
32         st[i * 2 + 1] += tag[i] * (r - mid);
33         tag[i * 2] += tag[i]; //下傳懶標至左節點
34         tag[i * 2 + 1] += tag[i]; //下傳懶標至右節點
35         tag[i] = 0;
36     }
37     int sum = 0;
38     if (ql <= mid)
39         sum += query(ql, qr, l, mid, i * 2);
40     if (qr > mid)
41         sum += query(ql, qr, mid + 1, r, i * 2 + 1);
42     return sum;
43 }
44
45 void update(int ql, int qr, int l, int r, int i, int c) {
46     // [ql, qr]是查詢區間,[l, r]是當前節點包含的區間
47     // c是變化量
48     if (ql <= l && r <= qr) {
49         st[i] += (r - l + 1) * c;
50         //求和,此需乘上區間長度
51         tag[i] += c;
52         return;
53     }
54     int mid = l + ((r - l) >> 1);
55     if (tag[i] && l != r) {
56         //如果當前懶標有值則更新左右節點
57         st[i * 2] += tag[i] * (mid - l + 1);
58         st[i * 2 + 1] += tag[i] * (r - mid);
59         tag[i * 2] += tag[i]; //下傳懶標至左節點
60         tag[i * 2 + 1] += tag[i]; //下傳懶標至右節點
61         tag[i] = 0;
62     }

```

```

62     if (ql <= mid) update(ql, qr, l, mid, i * 2, c);
63     if (qr > mid) update(ql, qr, mid + 1, r, i * 2 + 1, c);
64     st[i] = pull(i * 2, i * 2 + 1);
65 }
66 //如果是直接改值而不是加值，query與update中的tag與st的
67 //改值從+=改成=

```

7.10 Nim Game

```

1 //兩人輪流取銅板，每人每次需在某堆取一枚以上的銅板，
2 //但不能同時在兩堆取銅板，直到最後，
3 //將銅板拿光的人贏得此遊戲。
4
5 #include <bits/stdc++.h>
6 #define maxn 23+5
7 using namespace std;
8
9 int SG[maxn];
10 int visited[1000+5];
11 int pile[maxn], ans;
12
13 void calculateSG(){
14     SG[0]=0;
15     for(int i=1;i<=maxn;i++){
16         int cur=0;
17         for(int j=0;j<i;j++){
18             for(int k=0;k<=j;k++){
19                 visited[SG[j]^SG[k]]=i;
20             }
21             while(visited[cur]==i) cur++;
22             SG[i]=cur;
23         }
24     }
25
26 int main(){
27     calculateSG();
28     int Case=0,n;
29     while(cin>>n,n){
30         ans=0;
31         for(int i=1;i<=n;i++) cin>>pile[i];
32         for(int i=1;i<=n;i++){
33             if(pile[i]&1) ans^=SG[n-i];
34             cout<<"Game "<<Case<<": ";
35             if(!ans) cout<<"-1 -1 -1\n";
36             else{
37                 bool flag=0;
38                 for(int i=1;i<=n;i++){
39                     if(pile[i]){
40                         for(int j=i+1;j<=n;j++){
41                             for(int k=j;k<=n;k++){
42                                 if((SG[n-i]^SG[n-j]^SG[n-k])==ans){
43                                     cout<<i-1<<" "<<j-1<<" "<<k-1<<endl;
44                                     flag=1;
45                                     break;
46                                 }
47                             }
48                             if(flag) break;
49                         }
50                     }
51                     if(flag) break;
52                 }
53             }
54             return 0;
55         }
56     }
57     /*
58     input
59     4 1 0 1 100
60     3 1 0 5
61     2 2 1
62     0
63     output
64     Game 1: 0 2 3
65     Game 2: 0 1 1
66     Game 3: -1 -1 -1

```

67 */

7.11 Trie

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int maxn = 300000 + 10;
5 const int mod = 20071027;
6
7 int dp[maxn];
8 int mp[4000*100 + 10][26];
9 char str[maxn];
10
11 struct Trie {
12     int seq;
13     int val[maxn];
14
15     Trie() {
16         seq = 0;
17         memset(val, 0, sizeof(val));
18         memset(mp, 0, sizeof(mp));
19     }
20
21     void insert(char* s, int len) {
22         int r = 0;
23         for(int i=0; i<len; i++) {
24             int c = s[i] - 'a';
25             if(!mp[r][c]) mp[r][c] = ++seq;
26             r = mp[r][c];
27         }
28         val[r] = len;
29         return;
30     }
31
32     int find(int idx, int len) {
33         int result = 0;
34         for(int r=0; idx<len; idx++) {
35             int c = str[idx] - 'a';
36             if(!(r = mp[r][c])) return result;
37             if(val[r])
38                 result = (result + dp[idx + 1]) % mod;
39         }
40         return result;
41     }
42 };
43
44 int main() {
45     int n, tc = 1;
46
47     while(~scanf("%s%d", str, &n)) {
48         Trie tr;
49         int len = strlen(str);
50         char word[100+10];
51
52         memset(dp, 0, sizeof(dp));
53         dp[len] = 1;
54
55         while(n--) {
56             scanf("%s", word);
57             tr.insert(word, strlen(word));
58         }
59
60         for(int i=len-1; i>=0; i--)
61             dp[i] = tr.find(i, len);
62         printf("Case %d: %d\n", tc++, dp[0]);
63     }
64     return 0;
65 }
66
67 /*****
68 ****Input****
69 * abcd
70 * 4
71 * a b cd ab
72 ****

```

73 ****Output***

74 * Case 1: 2

75 ****

76 */

7.12 SPFA

```

1 struct Edge
2 {
3     int t;
4     long long w;
5     Edge(){};
6     Edge(int _t, long long _w) : t(_t), w(_w) {}
7 };
8
9 bool SPFA(int st) // 平均O(V + E) 最糟O(VE)
10 {
11     vector<int> cnt(n, 0);
12     bitset<MXV> inq(0);
13     queue<int> q;
14     q.push(st);
15     dis[st] = 0;
16     inq[st] = true;
17     while (!q.empty())
18     {
19         int cur = q.front();
20         q.pop();
21         inq[cur] = false;
22         for (auto &e : G[cur])
23         {
24             if (dis[e.t] <= dis[cur] + e.w)
25                 continue;
26             dis[e.t] = dis[cur] + e.w;
27             if (inq[e.t])
28                 continue;
29             ++cnt[e.t];
30             if (cnt[e.t] > n)
31                 return false; // negative cycle
32             inq[e.t] = true;
33             q.push(e.t);
34         }
35     }
36     return true;
37 }

```

7.13 dijkstra

```

1 #include<bits/stdc++.h>
2 #define maxn 50000+5
3 #define INF 0x3f3f3f3f
4 using namespace std;
5
6 struct edge{
7     int v,w;
8 };
9
10 struct Item{
11     int u,dis;
12     bool operator< (const Item &rhs) const{
13         return dis>rhs.dis;
14     }
15 };
16
17 vector<edge> G[maxn];
18 int dist[maxn];
19
20 void dijkstra(int s){ // O((V + E)log(E))
21     memset(dist,INF,sizeof(dist));
22     dist[s]=0;
23     priority_queue<Item> pq;
24     pq.push({s,0});
25     while(!pq.empty()){
26         Item now=pq.top();

```

```

27     pq.pop();
28     if(now.dis>dist[now.u]) continue;
29     for(edge e:G[now.u]){
30         if(dist[e.v]>dist[now.u]+e.w){
31             dist[e.v]=dist[now.u]+e.w;
32             pq.push({e.v,dist[e.v]});
33         }
34     }
35 }
36 }
37
38 int main(){
39     int t,cas=1;
40     cin>>t;
41     while(t--){
42         int n,m,s,t;
43         cin>>n>>m>>s>>t;
44         for(int i=0;i<=n;i++) G[i].clear();
45         int u,v,w;
46         for(int i=0;i<m;i++){
47             cin>>u>>v>>w;
48             G[u].push_back({v,w});
49             G[v].push_back({u,w});
50         }
51         dijkstra(s);
52         cout<<"Case #"<<cas++<<" ";
53         if(dist[t]==INF) cout<<"unreachable\n";
54         else cout<<dist[t]<<endl;
55     }
56 }

```

7.14 SCC Tarjan

```

1 //Strongly Connected Components
2 //Tarjan O(V + E)
3 int dfn[N], low[N], dfncnt, sk[N], in_stack[N], tp;
4 //dfn[u]: dfs時u被visited的順序
5 //low[u]: 在u的dfs子樹中能回到最早已在stack中的節點
6 int scc[N], sc;//節點 u 所在 SCC 的編號
7 int sz[N];//強連通 u 的大小
8
9 void tarjan(int u) {
10     low[u] = dfn[u] = ++dfncnt, s[++tp] = u,
11     in_stack[u] = 1;
12     for (int i = h[u]; i; i = e[i].nex) {
13         const int &v = e[i].t;
14         if (!dfn[v]) {
15             tarjan(v);
16             low[u] = min(low[u], low[v]);
17         } else if (in_stack[v]) {
18             low[u] = min(low[u], dfn[v]);
19         }
20     }
21     if (dfn[u] == low[u]) {
22         ++sc;
23         while (s[tp] != u) {
24             scc[s[tp]] = sc;
25             sz[sc]++;
26             in_stack[s[tp]] = 0;
27             --tp;
28         }
29         scc[s[tp]] = sc;
30         sz[sc]++;
31         in_stack[s[tp]] = 0;
32         --tp;
33     }
34 }

```

7.15 SCC Kosaraju

```

1 //做兩次dfs, O(V + E)
2 //g 是原圖, g2 是反圖
3 //s是dfs離開的節點

```

```

4 void dfs1(int u) {
5     vis[u] = true;
6     for (int v : g[u])
7         if (!vis[v]) dfs1(v);
8     s.push_back(u);
9 }
10
11 void dfs2(int u) {
12     group[u] = sccCnt;
13     for (int v : g2[u])
14         if (!group[v]) dfs2(v);
15 }
16
17 void kosaraju() {
18     sccCnt = 0;
19     for (int i = 1; i <= n; ++i)
20         if (!vis[i]) dfs1(i);
21     for (int i = n; i >= 1; --i)
22         if (!group[s[i]]) {
23             ++sccCnt;
24             dfs2(s[i]);
25         }
26 }

```

7.16 ArticulationPoints Tarjan

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 vector<vector<int>>> G;
5 int N;
6 int timer;
7 bool visited[105];
8 int visTime[105]; // 第一次visit的時間
9 int low[105];
10 // 最小能回到的父節點(不能是自己的parent)的visTime
11 int res;
12 //求割點數量
13 void tarjan(int u, int parent) {
14     int child = 0;
15     bool isCut = false;
16     visited[u] = true;
17     visTime[u] = low[u] = ++timer;
18     for (int v: G[u]) {
19         if (!visited[v]) {
20             ++child;
21             tarjan(v, u);
22             low[u] = min(low[u], low[v]);
23             if (parent != -1 && low[v] >= visTime[u])
24                 isCut = true;
25         }
26         else if (v != parent)
27             low[u] = min(low[u], visTime[v]);
28     }
29     //If u is root of DFS tree->有兩個以上的children
30     if (parent == -1 && child >= 2)
31         isCut = true;
32     if (isCut)
33         ++res;
34 }
35
36 int main()
37 {
38     char input[105];
39     char* token;
40     while (scanf("%d", &N) != EOF && N)
41     {
42         G.assign(105, vector<int>());
43         memset(visited, false, sizeof(visited));
44         memset(low, 0, sizeof(low));
45         memset(visTime, 0, sizeof(visTime));
46         timer = 0;
47         res = 0;
48         getchar(); // for \n
49         while (fgets(input, 105, stdin))

```

```

50 {
51     if (input[0] == '0')
52         break;
53     int size = strlen(input);
54     input[size - 1] = '\0';
55     --size;
56     token = strtok(input, " ");
57     int u = atoi(token);
58     int v;
59     while (token = strtok(NULL, " "))
60     {
61         v = atoi(token);
62         G[u].emplace_back(v);
63         G[v].emplace_back(u);
64     }
65 }
66 tarjan(1, -1);
67 printf("%d\n", res);
68 }
69 return 0;
70 }

```

7.17 最小樹狀圖

```

1  定義
2  有向圖上的最小生成樹 (Directed Minimum Spanning Tree)
3  稱為最小樹形圖。
4
5  const int maxn = 60 + 10;
6  const int inf = 0x3f3f3f3f;
7
8  struct Edge {
9      int s, t, cap, cost;
10 }; // cap 為頻寬 (optional)
11
12 int n, m, c;
13 int inEdge[maxn], idx[maxn], pre[maxn], vis[maxn];
14
15 // 對於每個點，選擇對它入度最小的那條邊
16 // 找環，如果沒有則 return;
17 // 進行縮環並更新其他點到環的距離。
18 int dirMST(vector<Edge> edges, int low) {
19     int result = 0, root = 0, N = n;
20
21     while(true) {
22         memset(inEdge, 0x3f, sizeof(inEdge));
23
24         // 找所有點的 in edge 放進 inEdge
25         // optional: low 為最小 cap 限制
26         for(const Edge& e : edges) {
27             if(e.cap < low) continue;
28             if(e.s!=e.t && e.cost<inEdge[e.t]) {
29                 inEdge[e.t] = e.cost;
30                 pre[e.t] = e.s;
31             }
32         }
33
34         for(int i=0; i<N; i++) {
35             if(i!=root && inEdge[i]==inf)
36                 return -1; //除了 root 還有點沒有 in edge
37         }
38
39         int seq = inEdge[root] = 0;
40         memset(idx, -1, sizeof(idx));
41         memset(vis, -1, sizeof(vis));
42
43         // 找所有的 cycle，一起編號為 seq
44         for(int i=0; i<N; i++) {
45             result += inEdge[i];
46             int cur = i;
47             while(vis[cur]!=i && idx[cur]==-1) {
48                 if(cur == root) break;
49                 vis[cur] = i;
50                 cur = pre[cur];

```

```

51     }
52     if(cur!=root && idx[cur]==-1) {
53         for(int j=pre[cur]; j!=cur; j=pre[j])
54             idx[j] = seq;
55         idx[cur] = seq++;
56     }
57 }
58
59 if(seq == 0) return result; // 沒有 cycle
60
61 for(int i=0; i<N; i++)
62     // 沒有被縮點的點
63     if(idx[i] == -1) idx[i] = seq++;
64
65 // 縮點並重新編號
66 for(Edge& e : edges) {
67     if(idx[e.s] != idx[e.t])
68         e.cost -= inEdge[e.t];
69     e.s = idx[e.s];
70     e.t = idx[e.t];
71 }
72 N = seq;
73 root = idx[root];
74 }
75 }
76
77 =====
78
79 Tarjan 的DMST 演算法
80 Tarjan 提出了一種能夠在
81 O(m+nlog n)時間內解決最小樹形圖問題的演算法。
82
83 流程
84 Tarjan 的演算法分為收縮與伸展兩個過程。
85 接下來先介紹收縮的過程。
86 我們要假設輸入的圖是滿足強連通的，
87 如果不滿足那就加入 O(n) 條邊使其滿足，
88 並且這些邊的邊權是無窮大的。
89
90 我們需要一個堆存儲結點的入邊編號，入邊權值，
91 結點總代價等相關信息，由於後續過程中會有堆的合併操作，
92 這裡採用左偏樹 與並查集實現。
93 演算法的每一步都選擇一個任意結點v，
94 需要保證v不是根節點，並且在堆中沒有它的入邊。
95 再將v的最小入邊加入到堆中，
96 如果新加入的這條邊使堆中的邊形成了環，
97 那麼將構成環的那些結點收縮，
98 我們不妨將這些已經收縮的結點命名為超級結點，
99 再繼續這個過程，如果所有的頂點都縮成了超級結點，
100 那麼收縮過程就結束了。
101 整個收縮過程結束後會得到一棵收縮樹，
102 之後就會對它進行伸展操作。
103
104 堆中的邊總是會形成一條路徑v0 <- v1<- ... <- vk，
105 由於圖是強連通的，這個路徑必然存在，
106 並且其中的 vi 可能是最初的單一結點，
107 也可能是壓縮後的超級結點。
108
109 最初有 v0=a，其中 a 是圖中任意的一個結點，
110 每次都選擇一條最小入邊 vk <- u，
111 如果 u 不是v0,v1,...,vk中的一個結點，
112 那麼就將結點擴展到 v k+1=u。
113 如果 u 是他們其中的一個結點 vi，
114 那麼就找到了一個關於 vi <- ... <- vk <- vi的環，
115 再將他們收縮為一個超級結點c。
116
117 向隊列 P 中放入所有的結點或超級結點，
118 並初始選擇任一節點 a，只要佇列不為空，就進行以下步驟：
119
120 選擇 a 的最小入邊，保證不存在自環，
121 並找到另一頭的結點 b。
122 如果結點b沒有被記錄過說明未形成環，
123 令 a <- b，繼續目前操作尋找環。

```

```

124 如果 b 被記錄過了，就表示出現了環。
125 總結點數加一，並將環上的所有結點重新編號，對堆進行合併，
126 以及結點/超級結點的總權值的更新。
127 更新權值操作就是將環上所有結點的入邊都收集起來，
128 並減去環上入邊的邊權。
129
130 typedef long long ll;
131 #define maxn 102
132 #define INF 0x3f3f3f3f
133
134 struct UnionFind {
135     int fa[maxn << 1];
136     UnionFind() { memset(fa, 0, sizeof(fa)); }
137     void clear(int n) {
138         memset(fa + 1, 0, sizeof(int) * n);
139     }
140     int find(int x) {
141         return fa[x] ? fa[x] = find(fa[x]) : x;
142     }
143     int operator[](int x) { return find(x); }
144 };
145
146 struct Edge {
147     int u, v, w, w0;
148 };
149
150 struct Heap {
151     Edge *e;
152     int rk, constant;
153     Heap *lch, *rch;
154
155     Heap(Edge *_e):
156         e(_e), rk(1), constant(0), lch(NULL), rch(NULL){}
157
158     void push() {
159         if (lch) lch->constant += constant;
160         if (rch) rch->constant += constant;
161         e->w += constant;
162         constant = 0;
163     }
164 };
165
166 Heap *merge(Heap *x, Heap *y) {
167     if (!x) return y;
168     if (!y) return x;
169     if (x->e->w + x->constant > y->e->w + y->constant)
170         swap(x, y);
171     x->push();
172     x->rch = merge(x->rch, y);
173     if (!x->lch || x->lch->rk < x->rch->rk)
174         swap(x->lch, x->rch);
175     if (x->rch)
176         x->rk = x->rch->rk + 1;
177     else
178         x->rk = 1;
179     return x;
180 }
181
182 Edge *extract(Heap *&x) {
183     Edge *r = x->e;
184     x->push();
185     x = merge(x->lch, x->rch);
186     return r;
187 }
188
189 vector<Edge> in[maxn];
190 int n, m, fa[maxn << 1], nxt[maxn << 1];
191 Edge *ed[maxn << 1];
192 Heap *Q[maxn << 1];
193 UnionFind id;
194
195 void contract() {
196     bool mark[maxn << 1];
197     //將圖上的每一個節點與其相連的那些節點進行記錄
198     for (int i = 1; i <= n; i++) {
199         queue<Heap *> q;

```

```

200         for (int j = 0; j < in[i].size(); j++)
201             q.push(new Heap(&in[i][j]));
202         while (q.size() > 1) {
203             Heap *u = q.front();
204             q.pop();
205             Heap *v = q.front();
206             q.pop();
207             q.push(merge(u, v));
208         }
209         Q[i] = q.front();
210     }
211     mark[1] = true;
212     for(int a=1,b=1,p;Q[a];b=a,mark[b]=true){
213         //尋找最小入邊以及其端點，保證無環
214         do {
215             ed[a] = extract(Q[a]);
216             a = id[ed[a]->u];
217         } while (a == b && Q[a]);
218         if (a == b) break;
219         if (!mark[a]) continue;
220         //對發現的環進行收縮，以及環內的節點重新編號，
221         //總權值更新
222         for (a = b, n++; a != n; a = p) {
223             id.fa[a] = fa[a] = n;
224             if (Q[a]) Q[a]->constant -= ed[a]->w;
225             Q[n] = merge(Q[n], Q[a]);
226             p = id[ed[a]->u];
227             nxt[p == n ? b : p] = a;
228         }
229     }
230 }
231
232 ll expand(int x, int r);
233 ll expand_iter(int x) {
234     ll r = 0;
235     for(int u=nxt[x];u!=x;u=nxt[u]){
236         if (ed[u]->w0 >= INF)
237             return INF;
238         else
239             r+=expand(ed[u]->v,u)+ed[u]->w0;
240     }
241     return r;
242 }
243
244 ll expand(int x, int t) {
245     ll r = 0;
246     for (; x != t; x = fa[x]) {
247         r += expand_iter(x);
248         if (r >= INF) return INF;
249     }
250     return r;
251 }
252
253 void link(int u, int v, int w) {
254     in[v].push_back({u, v, w, w});
255 }
256
257 int main() {
258     int rt;
259     scanf("%d %d %d", &n, &m, &rt);
260     for (int i = 0; i < m; i++) {
261         int u, v, w;
262         scanf("%d %d %d", &u, &v, &w);
263         link(u, v, w);
264     }
265     //保證強連通
266     for (int i = 1; i <= n; i++)
267         link(i > 1 ? i - 1 : n, i, INF);
268     contract();
269     ll ans = expand(rt, n);
270     if (ans >= INF)
271         puts("-1");
272     else
273         printf("%lld\n", ans);
274     return 0;
275 }
276

```


8 geometry

8.1 intersection

```

1 using LL = long long;
2
3 struct Point2D {
4     LL x, y;
5 };
6
7 struct Line2D {
8     Point2D s, e;
9     LL a, b, c; // L: ax + by = c
10    Line2D(Point2D s, Point2D e): s(s), e(e) {
11        a = e.y - s.y;
12        b = s.x - e.x;
13        c = a * s.x + b * s.y;
14    }
15 };
16
17 // 用克拉馬公式求二元一次解
18 Point2D intersection2D(Line2D l1, Line2D l2) {
19     LL D = l1.a * l2.b - l2.a * l1.b;
20     LL Dx = l1.c * l2.b - l2.c * l1.b;
21     LL Dy = l1.a * l2.c - l2.a * l1.c;
22
23     if(D) { // intersection
24         double x = 1.0 * Dx / D;
25         double y = 1.0 * Dy / D;
26     } else {
27         if(Dx || Dy) // Parallel lines
28             else // Same line
29     }
30 }

```

8.2 半平面相交

```

1 // Q: 給定一張凸包(已排序的點),
2 // 找出圖中離凸包外最遠的距離
3
4 const int maxn = 100 + 10;
5 const double eps = 1e-7;
6
7 struct Vector {
8     double x, y;
9     Vector(double x=0.0, double y=0.0): x(x), y(y) {}
10
11    Vector operator+(Vector v) {
12        return Vector(x+v.x, y+v.y);
13    }
14    Vector operator-(Vector v) {
15        return Vector(x-v.x, y-v.y);
16    }
17    Vector operator*(double val) {
18        return Vector(x*val, y*val);
19    }
20    double dot(Vector v) { return x*v.x + y*v.y; }
21    double cross(Vector v) { return x*v.y - y*v.x; }
22    double length() { return sqrt(dot(*this)); }
23    Vector unit_normal_vector() {
24        double len = length();
25        return Vector(-y/len, x/len);
26    }
27 };
28
29 using Point = Vector;
30
31 struct Line {
32     Point p;
33     Vector v;
34     double ang;
35     Line(Point p={}, Vector v={}): p(p), v(v) {
36         ang = atan2(v.y, v.x);
37     }

```

```

38    bool operator<(const Line& l) const {
39        return ang < l.ang;
40    }
41    Point intersection(Line l) {
42        Vector u = p - l.p;
43        double t = l.v.cross(u) / v.cross(l.v);
44        return p + v*t;
45    }
46 };
47
48 int n, m;
49 Line narrow[maxn]; // 要判斷的直線
50 Point poly[maxn]; // 能形成半平面交的凸包邊界點
51
52 // return true if point p is on the left of line l
53 bool onLeft(Point p, Line l) {
54     return l.v.cross(p-l.p) > 0;
55 }
56
57 int halfplaneIntersection() {
58     int l, r;
59     Line L[maxn]; // 排序後的向量隊列
60     Point P[maxn]; // s[i] 跟 s[i-1] 的交點
61
62     L[l=r=0] = narrow[0]; // notice: narrow is sorted
63     for(int i=1; i<n; i++) {
64         while(l<r && !onLeft(P[r-1], narrow[i])) r--;
65         while(l<r && !onLeft(P[l], narrow[i])) l++;
66
67         L[++r] = narrow[i];
68         if(l < r) P[r-1] = L[r-1].intersection(L[r]);
69     }
70
71     while(l<r && !onLeft(P[r-1], L[l])) r--;
72     if(r-l <= 1) return 0;
73
74     P[r] = L[r].intersection(L[l]);
75
76     int m=0;
77     for(int i=1; i<=r; i++) {
78         poly[m++] = P[i];
79     }
80
81     return m;
82 }
83
84 Point pt[maxn];
85 Vector vec[maxn];
86 Vector normal[maxn]; // normal[i] = vec[i] 的單位法向量
87
88 double bsearch(double l=0.0, double r=1e4) {
89     if(abs(r-l) < 1e-7) return l;
90
91     double mid = (l + r) / 2;
92
93     for(int i=0; i<n; i++) {
94         narrow[i] = Line(pt[i]+normal[i]*mid, vec[i]);
95     }
96
97     if(halfplaneIntersection())
98         return bsearch(mid, r);
99     else return bsearch(l, mid);
100 }
101
102 int main() {
103     while(~scanf("%d", &n) && n) {
104         for(int i=0; i<n; i++) {
105             double x, y;
106             scanf("%lf%lf", &x, &y);
107             pt[i] = {x, y};
108         }
109         for(int i=0; i<n; i++) {
110             vec[i] = pt[(i+1)%n] - pt[i];
111             normal[i] = vec[i].unit_normal_vector();
112         }
113
114         printf("%.6lf\n", bsearch());

```

```

115     }
116     return 0;
117 }

```

8.3 凸包

```

1 // q: 平面上給定多個區域，由多個座標點所形成，再給定
2 // 多點(x,y)，判斷有落點的區域(destroyed)的面積總和。
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 const int maxn = 500 + 10;
7 const int maxCoordinate = 500 + 10;
8
9 struct Point {
10     int x, y;
11 };
12
13 int n;
14 bool destroyed[maxn];
15 Point arr[maxn];
16 vector<Point> polygons[maxn];
17
18 void scanAndSortPoints() {
19     int minX = maxCoordinate, minY = maxCoordinate;
20     for(int i=0; i<n; i++) {
21         int x, y;
22         scanf("%d%d", &x, &y);
23         arr[i] = (Point){x, y};
24         if(y < minY || (y == minY && x < minX)) {
25             // If there are floating points, use:
26             // if(y<minY || (abs(y-minY)<eps && x<minX)) {
27                 minX = x, minY = y;
28             }
29         }
30     }
31     sort(arr, arr+n, [minX, minY](Point& a, Point& b){
32         double theta1 = atan2(a.y - minY, a.x - minX);
33         double theta2 = atan2(b.y - minY, b.x - minX);
34         return theta1 < theta2;
35     });
36     return;
37 }
38
39 // returns cross product of u(AB) x v(AC)
40 int cross(Point& A, Point& B, Point& C) {
41     int u[2] = {B.x - A.x, B.y - A.y};
42     int v[2] = {C.x - A.x, C.y - A.y};
43     return (u[0] * v[1]) - (u[1] * v[0]);
44 }
45
46 // size of arr = n >= 3
47 // st = the stack using vector, m = index of the top
48 vector<Point> convex_hull() {
49     vector<Point> st(arr, arr+3);
50     for(int i=3, m=2; i<n; i++, m++) {
51         while(m >= 2) {
52             if(cross(st[m], st[m-1], arr[i]) < 0)
53                 break;
54             st.pop_back();
55             m--;
56         }
57         st.push_back(arr[i]);
58     }
59     return st;
60 }
61
62 bool inPolygon(vector<Point>& vec, Point p) {
63     vec.push_back(vec[0]);
64     for(int i=1; i<vec.size(); i++) {
65         if(cross(vec[i-1], vec[i], p) < 0) {
66             return false;
67         }
68     }
69     vec.pop_back();

```

```

70     return true;
71 }
72
73     1 | x1    x2    x3    x4    x5          xn |
74     A = - | x      x      x      x      x ... x |
75           2 | y1    y2    y3    y4    y5          yn |
76 double calculateArea(vector<Point>& v) {
77     v.push_back(v[0]); // make v[n] = v[0]
78     double result = 0.0;
79     for(int i=1; i<v.size(); i++)
80         result += v[i-1].x*v[i].y - v[i-1].y*v[i].x;
81     v.pop_back();
82     return result / 2.0;
83 }
84
85 int main() {
86     int p = 0;
87     while(~scanf("%d", &n) && (n != -1)) {
88         scanAndSortPoints();
89         polygons[p++] = convex_hull();
90     }
91
92     int x, y;
93     double result = 0.0;
94     while(~scanf("%d%d", &x, &y)) {
95         for(int i=0; i<p; i++) {
96             if(inPolygon(polygons[i], (Point){x, y}))
97                 destroyed[i] = true;
98         }
99     }
100     for(int i=0; i<p; i++) {
101         if(destroyed[i])
102             result += calculateArea(polygons[i]);
103     }
104     printf("%.2lf\n", result);
105     return 0;
106 }

```

9 動態規劃

9.1 LCS 和 LIS

```

1 //最長共同子序列(LCS)
2 給定兩序列 A,B，求最長的序列 C，
3 C 同時為 A,B 的子序列。
4
5 //最長遞增子序列 (LIS)
6 給你一個序列 A，求最長的序列 B，
7 B 是一個（非）嚴格遞增序列，且為 A 的子序列。
8
9 //LCS 和 LIS 題目轉換
10 LIS 轉成 LCS
11     1. A 為原序列， B=sort(A)
12     2. 對 A,B 做 LCS
13 LCS 轉成 LIS
14     1. A, B 為原本的兩序列
15     2. 最 A 序列作編號轉換，將轉換規則套用在 B
16     3. 對 B 做 LIS
17     4. 重複的數字在編號轉換時後要變成不同的數字，
18        越早出現的數字要越小
19     5. 如果有數字在 B 裡面而不在 A 裡面，
20        直接忽略這個數字不做轉換即可

```

10 Section2

10.1 thm

- 中文測試

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

- $\binom{x}{y} = \frac{x!}{y!(x-y)!}$
- $\int_0^\infty e^{-x} \, dx$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

11 DP

11.1 DP 公式

- Edit distance

S_1 經過增、刪或換字變成 S_2

$$dp[i][j] = \begin{cases} dp[i-1][j-1] & \text{if } S_1[i] = S_2[j] \\ \min \begin{Bmatrix} dp[i][j-1] \\ dp[i-1][j] \\ dp[i-1][j-1] \end{Bmatrix} + 1 & \text{if } S_1[i] \neq S_2[j] \end{cases}$$

11.2 DP 表格

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