Contents

1 字串

1.1 最長迴文子字串

```cpp
#include<bits/stdc++.h>
#define T(x) ((x)%2 ? s[(x)/2] : '.')
using namespace std;

string s;
int n;

int ex(int l,int r){
  int i=0;
  while(l-i>=0&&r+i<n&&T(l-i)==T(r+i)) i++;
  return i;
}

int main(){
  cin>>s;
  n=2*s.size()+1;
  int mx=0;
  int center=0;
  vector<int> r(n);
  int ans=1;
  r[0]=1;
  for(int i=1;i<n;i++){
    int ii=center-(i-center);
    int len=mx-i+1;
    if(i>mx){
      r[i]=ex(i,i);
      center=i;
      mx=i+r[i]-1;
    }
    else if(r[ii]==len){
      r[i]=len+ex(i-len,i+len);
      center=i;
      mx=i+r[i]-1;
    }
    else r[i]=min(r[ii],len);
    ans=max(ans,r[i]);
  }
  cout<<ans-1<<"\n";
  return 0;
}
```

1.2 Manacher

s: 增長為兩倍的字串，以'@'為首，以'$'為間隔，以'\0'節尾

p: 以 s[i] 為中心，半徑為 p[i] 是迴文

return: 最長的迴文長度

```cpp
const int maxn = 1e5 + 10;

char s[maxn<<1] = "@$";
int p[maxn<<1];

int manacher(char* str, int n) {
  for(int i=1; i<=n; i++) {
    s[i<<1] = str[i-1];
    s[i<<1|1] = '$';
  }

  int cur = 0, r = 0, res = 0;
  s[n = (n+1) << 1] = 0;
  for(int i=1; i<n; i++) {
    p[i] = (i>r) ? 1 : min(p[cur*2-i], r-i);
    for(; s[i-p[i]]==s[i+p[i]]; p[i]++);
    if(i+p[i] > r) {
      r = i + p[i];
      cur = i;
    }
    res = max(res, p[i]);
  }
  return res - 1;
}
```

1.3 KMP

```cpp
const int maxn = 1e6 + 10;

int n, m;             // len(a), len(b)
int f[maxn];          // failure function
char a[maxn], b[maxn];

void failureFuntion() { // f[0] = 0
    for(int i=1, j=0; i<m; ) {
        if(b[i] == b[j]) f[i++] = ++j;
        else if(j) j = f[j-1];
        else f[i++] = 0;
    }
}

int kmp() {
    int i = 0, j = 0, res = 0;
    while(i < n) {
        if(a[i] == b[j]) i++, j++;
        else if(j) j = f[j-1];
        else i++;
        if(j == m) {
            res++; // 找到答案
            j = 0; // non-overlapping
        }
    }
    return res;
}

// Problem: 所有在b裡，前後綴相同的長度
// b = ababcababababcabab
// f = 0012012341234567 89
// 前9 = 後9
// 前4 = 前9的後4 = 後4
// 前2 = 前4的後2 = 前9的後2 = 後2
for(int j=m; j; j=f[j-1]) {
    // j 是答案
}
```

1.4 Z Algorithm

```cpp
const int maxn = 1e6 + 10;

int z[maxn];   // s[0:z[i]) = s[i:i+z[i])
string s;

void makeZ() { // z[0] = 0
  for(int i=1, l=0, r=0; i<s.length(); i++) {
    if(i<=r && z[i-l]<r-i+1) z[i] = z[i-l];
    else {
      z[i] = max(0, r-i+1);
      while(i+z[i]<s.length() &&
            s[z[i]]==s[i+z[i]]) z[i]++;
    }
    if(i+z[i]-1 > r) l = i, r = i+z[i]-1;
  }
}
```

1.5 Suffix Array

- $O(n \log(n))$
- SA：後綴數組
- HA：相鄰後綴的共同前綴長度 (Longest Common Prefix)
- maxc：可用字元的最大 ASCII 值
- maxn >= maxc
- 記得先取 n 的值 (strlen(s))

```cpp
const int maxn = 2e5 + 10;
const int maxc = 256 + 10;

int n;
int SA[maxn], HA[maxn];
int rk[maxn], cnt[maxn], tmp[maxn];
char s[maxn];

void getSA() {
  int mx = maxc;
  for(int i=0; i<mx; cnt[i++]=0);

  // 第一次 stable counting sort，編 rank 和 sa
  for(int i=0; i<n; i++) cnt[rk[i]=s[i]]++;
  for(int i=1; i<mx; i++) cnt[i] += cnt[i-1];
  for(int i=n-1;i>=0;i--) SA[--cnt[s[i]]]=i;

  // 倍增法運算
  for(int k=1, r=0; k<n; k<<=1, r=0) {
    for(int i=0; i<mx; cnt[i++]=0);
    for(int i=0; i<n; i++) cnt[rk[i]]++;
    for(int i=1; i<mx; i++) cnt[i]+=cnt[i-1];
    for(int i=n-k; i<n; i++) tmp[r++] = i;
    for(int i=0; i<n; i++) {
      if(SA[i] >= k) tmp[r++] = SA[i] - k;
```

```
    }

    // 計算本回 SA
    for(int i=n-1; i>=0; i--) {
      SA[--cnt[rk[tmp[i]]]] = tmp[i];
    }

    // 計算本回 rank
    tmp[SA[0]] = r = 0;
    for(int i=1; i<n; i++) {
      if((SA[i-1]+k >= n) ||
         (rk[SA[i-1]] != rk[SA[i]]) ||
         (rk[SA[i-1]+k] != rk[SA[i]+k])) r++;
      tmp[SA[i]] = r;
    }
    for(int i=0; i<n; i++) rk[i] = tmp[i];
    if((mx=r+1) == n) break;
  }
}

void getHA() {  // HA[0] = 0
  for(int i=0; i<n; i++) rk[SA[i]] = i;
  for(int i=0, k=0; i<n; i++) {
    if(!rk[i]) continue;
    if(k) k--;
    while(s[i+k] == s[SA[rk[i]-1]+k]) k++;
    HA[rk[i]] = k;
  }
}
```

2 math

2.1 公式

1. Most Divisor Number

Range	最多因數數	因數個數
10^9	735134400	1344
2^{31}	2095133040	1600
10^{18}	897612484786617600	103680
2^{64}	9200527969062830400	161280

2. Catlan Number

$$C_n = \frac{1}{n}\binom{2n}{n}, C_{n+1} = \frac{2(2n+1)}{n+2}C_n$$

$$C = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, \ldots$$

3. Lagrange Polynomial

拉格朗日插值法：找出 n 次多項函數 $f(x)$ 的點 $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$

$$L(x) = \sum_{j=0}^{n} y_j l_j(x)$$

$$l_j(x) = \prod_{i=0, i\neq j}^{n} \frac{x - x_i}{x_j - x_i}$$

4. Fibonacci

$$\begin{bmatrix} f_{n-1} & f_n \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} f_n & f_{n+1} \end{bmatrix}$$
$$\begin{bmatrix} f_n & f_{n+1} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^p = \begin{bmatrix} f_{n+p} & f_{n+p+1} \end{bmatrix}, p \in \mathbb{N}$$

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

5. Pick's Theorem

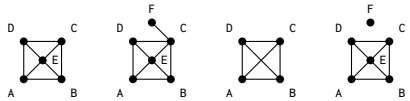
給定頂點座標均是整點（或正方形格子點）的簡單多邊形，其面積 A 和內部格點數目 i、邊上格點數目 b 的關係為

$$A = i + \frac{b}{2} - 1$$

6. Euler's Formula

對於有 V 個點、E 條邊、F 個面（含外部）的連通平面圖

$$F + V - E = 2$$



(1)、(2)○；(3)×，$\overline{AC}$ 與 $\overline{BD}$ 相交；(4)×，非連通圖

7. Simpson Integral

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

2.2 Rational

```
const char sep = '/';   // 分數的分隔符
bool div0;              // 要記得適時歸零
using ll = long long;

struct Rational {
  ll p, q;

  Rational(ll a=0, ll b=1) {
    p = a, q = b;
    reduce();
  }

  Rational(string s) {
    if(s.find(sep) == string::npos) {
      p = stoll(s);
      q = 1;
    } else {
      p = stoll(s.substr(0, s.find(sep)));
      q = stoll(s.substr(s.find(sep)+1));
    }
    reduce();
  }

  void reduce() {
    ll t = abs(__gcd(p, q));
    if(t == 0) {
      div0 = true;
      return;
    }
    p /= t, q /= t;
    if(q < 0) p = -p, q = -q;
    return;
  }

  string toString() {
    if(q == 0) {
      div0 = true;
      return "INVALID";
    }
    if(p%q == 0) return to_string(p/q);
    return to_string(p) + sep + to_string(q);
  }

  friend istream& operator>>(
    istream& i, Rational& r) {
    string s;
    i >> s;
    r = Rational(s);
    return i;
  }

  friend ostream& operator<<(
    ostream& o, Rational r) {
    o << r.toString();
    return o;
  }
};

Rational operator+(Rational x, Rational y) {
  ll t = abs(__gcd(x.q, y.q));
  if(t == 0) return Rational(0, 0);
  return Rational(
    y.q/t*x.p + x.q/t*y.p, x.q/t*y.q);
}

Rational operator-(Rational x, Rational y) {
  return x + Rational(-y.p, y.q);
}

Rational operator*(Rational x, Rational y) {
  return Rational(x.p*y.p, x.q*y.q);
}

Rational operator/(Rational x, Rational y) {
  return x * Rational(y.q, y.p);
}
```

2.3 乘法逆元、組合數

$$x^{-1} mod\ m$$
$$= \begin{cases} 1, & \text{if } x = 1 \\ -\lfloor \frac{m}{x} \rfloor (m\ mod\ x)^{-1}, & \text{otherwise} \end{cases} \quad (mod\ m)$$
$$= \begin{cases} 1, & \text{if } x = 1 \\ (m - \lfloor \frac{m}{x} \rfloor)(m\ mod\ x)^{-1}, & \text{otherwise} \end{cases} \quad (mod\ m)$$

若 $p \in prime$, 根據費馬小定理, 則

$$\because \quad ax \equiv 1 \ (mod\ p)$$
$$\therefore \quad ax \equiv a^{p-1} \ (mod\ p)$$
$$\therefore \quad x \equiv a^{p-2} \ (mod\ p)$$

```
using ll = long long;
const int maxn = 2e5 + 10;
const int mod = 1e9 + 7;

int fact[maxn] = {1, 1};// x! % mod
int inv[maxn] = {1, 1}; // x^(-1) % mod
int invFact[maxn] = {1, 1};// (x!)^(-1) % mod

void build() {
  for(int x=2; x<maxn; x++) {
    fact[x] = (ll)x * fact[x-1] % mod;
    inv[x] = (ll)(mod-mod/x)*inv[mod%x]%mod;
    invFact[x] = (ll)invFact[x-1]*inv[x]%mod;
  }
}

// 前提：mod 為質數
void build() {
  auto qpow = [&](ll a, int b) {
    ll res = 1;
    for(; b; b>>=1) {
      if(b & 1) res = res * a % mod;
      a = a * a % mod;
    }
    return res;
  };

  for(int x=2; x<maxn; x++) {
    fact[x] = (ll)x * fact[x-1] % mod;
    invFact[x] = qpow(fact[x], mod-2);
  }
}

// C(a, b) % mod
int comb(int a, int b) {
  if(a < b) return 0;
  ll x = fact[a];
  ll y = (ll)invFact[b] * invFact[a-b] % mod;
  return x * y % mod;
}
```

2.4 歐拉函數

```cpp
//計算閉區間 [1,n] 中有幾個正整數與 n 互質

int phi(){
    int ans=n;
    for(int i=2;i*i<=n;i++)
        if(n%i==0){
            ans=ans-ans/i;
            while(n%i==0) n/=i;
        }
    if(n>1) ans=ans-ans/n;
    return ans;
}
```

2.5 質數與因數

```cpp
歐拉篩O(n)
#define MAXN 47000 //sqrt(2^31)=46,340...
bool isPrime[MAXN];
int p[MAXN];
int pSize=0;
void getPrimes(){
  memset(isPrime,true,sizeof(isPrime));
  isPrime[0]=isPrime[1]=false;
  for(int i=2;i<MAXN;i++){
    if(isPrime[i]) p[pSize++]=i;
    for(int j=0;j<pSize&&i*p[j]<=MAXN;++j){
      isPrime[i*p[j]]=false;
      if(i%p[j]==0) break;
    }
  }
}
problem :
給定整數 N，求N最少可以拆成多少個質數的和。
如果N是質數，則答案為 1。
如果N是偶數(N!=2)，則答案為2(強歌德巴赫猜想)。
如果N是奇數且N-2是質數，則答案為2(2+質數)。
其他狀況答案為 3 (弱歌德巴赫猜想)。
bool isPrime(int n){
  for(int i=2;i<n;++i){
    if(i*i>n) return true;
    if(n%i==0) return false;
  }
  return true;
}
int main(){
  int n;
  cin>>n;
  if(isPrime(n)) cout<<"1\n";
  else if(n%2==0||isPrime(n-2)) cout<<"2\n";
  else cout<<"3\n";
}
```

2.6 高斯消去

計算 $AX = B$

傳入：

$M =$ 增廣矩陣 $[A|B]$
$equ =$ 有幾個 equation
$var =$ 有幾個 variable

回傳：$X = (x_0, \ldots, x_{n-1})$ 的解集

>>無法判斷無解或無限多組解<<

```cpp
using DBL = double;
using mat = vector<vector<DBL>>;

vector<DBL> Gauss(mat& M, int equ, int var) {
  auto dcmp = [](DBL a, DBL b=0.0) {
    return (a > b) - (a < b);
  };

  for(int r=0, c=0; r<equ && c<var; ) {
    int mx = r;  // 找絕對值最大的 M[i][c]
    for(int i=r+1; i<equ; i++) {
      if(dcmp(abs(M[i][c]),abs(M[mx][c]))==1)
        mx = i;
    }
    if(mx != r) swap(M[mx], M[r]);

    if(dcmp(M[r][c]) == 0) {
      c++;
      continue;
    }

    for(int i=r+1; i<equ; i++) {
      if(dcmp(M[i][c]) == 0) continue;
      DBL t = M[i][c] / M[r][c];
      for(int j=c; j<M[c].size(); j++) {
        M[i][j] -= t * M[r][j];
      }
    }
    r++, c++;
  }

  vector<DBL> X(var);
  for(int i=var-1; i>=0; i--) {
    X[i] = M[i][var];
    for(int j=var-1; j>i; j--) {
      X[i] -= M[i][j] * X[j];
    }
    X[i] /= M[i][i];
  }
  return X;
}
```

2.7 Extended GCD

```cpp
ll exgcd(ll a, ll b, ll& x, ll& y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    ll gcd = exgcd(b, a % b, x, y);
    ll y1 = y;
    y = x - (a / b) * y;
    x = y1;
    return gcd;
}
int main() {
    ll n;
    ll x, y;
    ll c1, c2, a, b;
    while (~scanf("%lld", &n) && n) {
        scanf("%lld %lld", &c1, &a);
        scanf("%lld %lld", &c2, &b);
        ll gcd = exgcd(a, b, x, y);
        if (n % gcd != 0) {
            printf("failed\n");
            continue;
        }
        ll l = ceil((double)(-n) * x / b);
        ll r = floor((double)(n) * y / a);
        if (l > r) {
            printf("failed\n");
            continue;
        }
        if (c1 * b < c2 * a) { //斜率正or負
            //斜率負，帶入k的上界
            x = n * x / gcd + b / gcd * r;
            y = n * y / gcd - a / gcd * r;
        }
        else {
            //斜率正，帶入k的下界
            x = n * x / gcd + b / gcd * l;
            y = n * y / gcd - a / gcd * l;
        }
        printf("%lld %lld\n", x, y);
    }
    return 0;
}
```

2.8 Pisano Period

```cpp
/*Pisano Period:
  費氏數列在mod n的情況下會有循環週期，
  且週期的結束判斷會在
  fib[i - 1] == 0 && fib[i] == 1時，
  此時循環週期長度是i - 1
  Pisano period可證一個週期的長度會在[n, n ^
      n]之間
  mod 1都等於0，沒有週期*/
```

2.9 矩陣快速冪

```cpp
using ll = long long;
using mat = vector<vector<ll>>;
const int mod = 1e9 + 7;
mat operator*(mat A, mat B) {
  mat res(A.size(), vector<ll>(B[0].size()));
  for(int i=0; i<A.size(); i++) {
    for(int j=0; j<B[0].size(); j++) {
      for(int k=0; k<B.size(); k++) {
        res[i][j] += A[i][k] * B[k][j] % mod;
        res[i][j] %= mod;
      }
    }
  }
  return res;
}
// 迴圈版本
mat mpow(mat M, int n) {
  mat res(M.size(), vector<ll>(M[0].size()));
  for(int i=0; i<res.size(); i++)
    res[i][i] = 1;
  for(; n; n>>=1) {
    if(n & 1) res = res * M;
    M = M * M;
  }
  return res;
}
```

3 algorithm

3.1 JosephusProblem

```cpp
//JosephusProblem，只是規定要先砍1號
//所以當作有n - 1個人，目標的13順移成12
//再者從0開始比較好算，所以目標12順移成11

// O(n)
int getWinner(int n, int k) {
    int winner = 0;
    for (int i = 1; i <= n; ++i)
        winner = (winner + k) % i;
    return winner;
}

int main() {
    int n;
    while (scanf("%d", &n) != EOF && n){
        --n;
        for (int k = 1; k <= n; ++k){
            if (getWinner(n, k) == 11){
                printf("%d\n", k);
                break;
            }
        }
    }
    return 0;
}

// O(k log(n))
int josephus(int n, int k) {
  if (n == 1) return 0;
  if (k == 1) return n - 1;
  if (k > n) return (josephus(n-1,k)+k)%n;
```

```
  int res = josephus(n - n / k, k);
  res -= n % k;
  if (res < 0) res += n; // mod n
  else res += res / (k - 1); // 还原位置
  return res;
}
```

3.2 二分搜

```
// 以下經過check()後 . 為false，o 為true
//皆為[l, r]區間
//.....vooooooo 即答案左邊界，符合條件最小的
int bsearch(int l, int r) {
    while (l < r) {
        int mid = (l + r) >> 1;
        if (check(mid)) r = mid;
        else l = mid + 1;
    }
    return l;
}

//ooooov..... 即答案右邊界，符合條件最大的
int bsearch(int l, int r) {
    while (l < r) {
        int mid = (l + r + 1) >> 1;
        if (check(mid)) l = mid;
        else r = mid - 1;
    }
    return l;
}
```

3.3 三分搜

```
//只要是單峰函數，三分可找最大或最小，以下為最小化
//計算lmid以及rmid時要避免數字溢出
while (r - l > eps) { // [l, r]
  mid = (l + r) / 2;
  lmid = mid - eps;
  rmid = mid + eps;
  if (f(lmid) < f(rmid)) r = mid;
  else l = mid;
}
```

3.4 dinic

```
const int maxn = 1e5 + 10;
const int inf = 0x3f3f3f3f;
struct Edge { int s, t, cap, flow; };
int n, m, S, T;
int level[maxn], dfs_idx[maxn];
vector<Edge> E;
vector<vector<int>> G;
void init() {
    S = 0;
    T = n + m;
    E.clear();
    G.assign(maxn, vector<int>());
}
void addEdge(int s, int t, int cap) {
    E.push_back({s, t, cap, 0});
    E.push_back({t, s, 0, 0});
    G[s].push_back(E.size()-2);
    G[t].push_back(E.size()-1);
}
bool bfs() {
    queue<int> q({S});
    memset(level, -1, sizeof(level));
    level[S] = 0;
    while(!q.empty()) {
        int cur = q.front();
        q.pop();
        for(int i : G[cur]) {
            Edge e = E[i];
            if(level[e.t]==-1 &&
                 e.cap>e.flow) {
                level[e.t] = level[e.s] + 1;
                q.push(e.t);
            }
        }
    }
    return ~level[T];
}
int dfs(int cur, int lim) {
  if(cur==T || lim<=0) return lim;
  int result = 0;
  for(int& i=dfs_idx[cur]; i<G[cur].size()
       && lim>0; i++) {
    Edge& e = E[G[cur][i]];
    if(level[e.s]+1 != level[e.t]) continue;
    int flow = dfs(e.t, min(lim,
         e.cap-e.flow));
    if(flow <= 0) continue;
    e.flow += flow;
    result += flow;
    E[G[cur][i]^1].flow -= flow;
    lim -= flow;
  }
  return result;
}
int dinic() { // O((V^2)E)
    int result = 0;
    while(bfs()) {
        memset(dfs_idx, 0, sizeof(dfs_idx));
        result += dfs(S, inf);
    }
    return result;
}
```

3.5 SCC Tarjan

```
//單純考SCC，每個SCC中找成本最小的蓋，如果有多個一樣小
//的要數出來，因為題目要方法數
//注意以下程式有縮點，但沒存起來，
//存法就是開一個array -> ID[u] = SCCID
#define maxn 100005
#define MOD 1000000007
long long cost[maxn];
vector<vector<int>> G;
int SCC = 0;
stack<int> sk;
int dfn[maxn];
int low[maxn];
bool inStack[maxn];
int dfsTime = 1;
long long totalCost = 0;
long long ways = 1;
void dfs(int u) {
  dfn[u] = low[u] = dfsTime;
  ++dfsTime;
  sk.push(u);
  inStack[u] = true;
  for (int v: G[u]) {
    if (dfn[v] == 0) {
        dfs(v);
        low[u] = min(low[u], low[v]);
    }
    else if (inStack[v]) {
        //屬於同個SCC且是我的back edge
        low[u] = min(low[u], dfn[v]);
    }
  }
  //如果是SCC
  if (dfn[u] == low[u]) {
    long long minCost = 0x3f3f3f3f;
    int currWays = 0;
    ++SCC;
    while (1) {
        int v = sk.top();
        inStack[v] = 0;
        sk.pop();
        if (minCost > cost[v]) {
            minCost = cost[v];
            currWays = 1;
        }
        else if (minCost == cost[v]) {
            ++currWays;
        }
        if (v == u) break;
    }
    totalCost += minCost;
    ways = (ways * currWays) % MOD;
  }
}
```

3.6 BCC 邊

```
//oi-wiki，找無向圖的邊雙連通分量個數，
//並輸出每個邊雙連通分量
//對於任意u、v，刪去哪個邊都不會不連通
//-> 邊雙連通(V + E)
constexpr int N = 5e5 + 5, M = 2e6 + 5;
int n, m, ans;
int tot = 1, hd[N];
struct edge {int to, nt;} e[M << 1];
void add(int u, int v) {e[++tot].to = v,
     e[tot].nt = hd[u], hd[u] = tot;}
void uadd(int u, int v) {add(u,v),add(v,u);}
bool bz[M << 1];
int bcc_cnt, dfn[N], low[N], vis_bcc[N];
vector<vector<int>> bcc;
void tarjan(int x, int in) {
  dfn[x] = low[x] = ++bcc_cnt;
  for (int i = hd[x]; i; i = e[i].nt) {
    int v = e[i].to;
    if (dfn[v] == 0) {
      tarjan(v, i);
      if (dfn[x] < low[v])
        bz[i] = bz[i ^ 1] = true;
      low[x] = min(low[x], low[v]);
    } else if (i != (in ^ 1))
      low[x] = min(low[x], dfn[v]);
  }
}
void dfs(int x, int id) {
  vis_bcc[x] = id, bcc[id - 1].push_back(x);
  for (int i = hd[x]; i; i = e[i].nt) {
    int v = e[i].to;
    if (vis_bcc[v] || bz[i]) continue;
    dfs(v, id);
  }
}
```

3.7 BCC 點

```
//oi-wiki，找無向圖的點雙連通分量個數，
//並輸出每個點雙連通分量
//對於任意u、v，刪去哪個點(只能刪一個)都不會不連通
//-> 點雙連通(V + E)
constexpr int N = 5e5 + 5, M = 2e6 + 5;
int n, m;

struct edge { int to, nt; } e[M << 1];

int hd[N], tot = 1;

void add(int u, int v) { e[++tot] = edge{v,
     hd[u]}, hd[u] = tot; }

void uadd(int u, int v) {add(u,v),add(v,u);}

int ans;
int dfn[N], low[N], bcc_cnt;
int sta[N], top, cnt;
```

```cpp
bool cut[N];
vector<int> dcc[N];
int root;

void tarjan(int u) {
  dfn[u]=low[u] = ++bcc_cnt, sta[++top] = u;
  if (u == root && hd[u] == 0) {
    dcc[++cnt].push_back(u);
    return;
  }
  int f = 0;
  for (int i = hd[u]; i; i = e[i].nt) {
    int v = e[i].to;
    if (!dfn[v]) {
      tarjan(v);
      low[u] = min(low[u], low[v]);
      if (low[v] >= dfn[u]) {
        if (++f > 1 || u != root)
          cut[u] = true;
        cnt++;
        do dcc[cnt].push_back(sta[top--]);
        while (sta[top + 1] != v);
        dcc[cnt].push_back(u);
      }
    } else
      low[u] = min(low[u], dfn[v]);
  }
}
```

3.8 ArticulationPoints Tarjan

```cpp
vector<vector<int>> G;
int N, timer;
bool visited[105];
int dfn[105]; // 第一次visit的時間
int low[105];
//最小能回到的父節點
//(不能是自己的parent)的visTime
int res;
//求割點數量
void tarjan(int u, int parent) {
  int child = 0;
  bool isCut = false;
  visited[u] = true;
  dfn[u] = low[u] = ++timer;
  for (int v: G[u]) {
    if (!visited[v]) {
      ++child;
      tarjan(v, u);
      low[u] = min(low[u], low[v]);
      if (parent != -1 && low[v] >= dfn[u])
        isCut = true;
    }
    else if (v != parent)
      low[u] = min(low[u], dfn[v]);
  }
//If u is root of DFS tree->有兩個以上的children
  if (parent == -1 && child >= 2)
    isCut = true;
  if (isCut) ++res;
}
```

3.9 最小樹狀圖

```cpp
const int maxn = 60 + 10;
const int inf = 0x3f3f3f3f;
struct Edge {
  int s, t, cap, cost;
}; // cap 為頻寬 (optional)
int n, m, c;
int inEdge[maxn], idx[maxn], pre[maxn],
    vis[maxn];
// 對於每個點，選擇對它入度最小的那條邊
// 找環，如果沒有則 return;
// 進行縮環並更新其他點到環的距離。
int dirMST(vector<Edge> edges, int low) {
  int result = 0, root = 0, N = n;
  while(true) {
    memset(inEdge, 0x3f, sizeof(inEdge));
    // 找所有點的 in edge 放進 inEdge
    // optional: low 為最小 cap 限制
    for(const Edge& e : edges) {
      if(e.cap < low) continue;
      if(e.s!=e.t && e.cost<inEdge[e.t]) {
        inEdge[e.t] = e.cost;
        pre[e.t] = e.s;
      }
    }
    for(int i=0; i<N; i++) {
      if(i!=root && inEdge[i]==inf)
        return -1;//除了root 還有點沒有in edge
    }
    int seq = inEdge[root] = 0;
    memset(idx, -1, sizeof(idx));
    memset(vis, -1, sizeof(vis));
    // 找所有的 cycle，一起編號為 seq
    for(int i=0; i<N; i++) {
      result += inEdge[i];
      int cur = i;
      while(vis[cur]!=i && idx[cur]==-1) {
        if(cur == root) break;
        vis[cur] = i;
        cur = pre[cur];
      }
      if(cur!=root && idx[cur]==-1) {
        for(int j=pre[cur]; j!=cur; j=pre[j])
          idx[j] = seq;
        idx[cur] = seq++;
      }
    }
    if(seq == 0) return result; // 沒有 cycle
    for(int i=0; i<N; i++)
      // 沒有被縮點的點
      if(idx[i] == -1) idx[i] = seq++;
    // 縮點並重新編號
    for(Edge& e : edges) {
      if(idx[e.s] != idx[e.t])
        e.cost -= inEdge[e.t];
      e.s = idx[e.s];
      e.t = idx[e.t];
    }
    N = seq;
    root = idx[root];
  }
}
```

3.10 KM

```cpp
#define maxn 505
int W[maxn][maxn];
int Lx[maxn], Ly[maxn];
bool S[maxn], T[maxn];
//L[i] = j -> S_i配給T_j, -1 for 還沒匹配
int L[maxn];
int n;
bool match(int i) {
  S[i] = true;
  for (int j = 0; j < n; ++j) {
    // KM重點
    // Lx + Ly >= selected_edge(x, y)
    // 要想辦法降低Lx + Ly
    // 所以選Lx + Ly == selected_edge(x, y)
    if (Lx[i] + Ly[j] == W[i][j] && !T[j]) {
      T[j] = true;
      if ((L[j] == -1) || match(L[j])) {
        L[j] = i;
        return true;
      }
    }
  }
  return false;
}
//修改二分圖上的交錯路徑上點的權重
//此舉是在通過調整vertex labeling看看
//能不能產生出新的增廣路
//(KM的增廣路要求Lx[i] + Ly[j] == W[i][j])
//在這裡優先從最小的diff調調看，才能保證最大權重匹配
void update() {
  int diff = 0x3f3f3f3f;
  for (int i = 0; i < n; ++i) {
    if (S[i]) {
      for (int j = 0; j < n; ++j) {
        if (!T[j]) diff = min(diff, Lx[i] +
            Ly[j] - W[i][j]);
      }
    }
  }
  for (int i = 0; i < n; ++i) {
    if (S[i]) Lx[i] -= diff;
    if (T[i]) Ly[i] += diff;
  }
}
void KM() {
  for (int i = 0; i < n; ++i) {
    L[i] = -1;
    Lx[i] = Ly[i] = 0;
    for (int j = 0; j < n; ++j)
      Lx[i] = max(Lx[i], W[i][j]);
  }
  for (int i = 0; i < n; ++i) {
    while(1) {
      memset(S, false, sizeof(S));
      memset(T, false, sizeof(T));
      if (match(i)) break;
      else update(); //去調整vertex
          labeling以增加增廣路徑
    }
  }
}
int main() {
  while (scanf("%d", &n) != EOF) {
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j)
        scanf("%d", &W[i][j]);
    KM();
    int res = 0;
    for (int i = 0; i < n; ++i) {
      if (i != 0) printf(" %d", Lx[i]);
      else printf("%d", Lx[i]);
      res += Lx[i];
    }
    puts("");
    for (int i = 0; i < n; ++i) {
      if (i != 0) printf(" %d", Ly[i]);
      else printf("%d", Ly[i]);
      res += Ly[i];
    }
    puts("");
    printf("%d\n", res);
  }
  return 0;
}
```

3.11 二分圖最大匹配

```cpp
/* 核心: 最大點獨立集 = |V| -
     |最大匹配數|，用匈牙利演算法找出最大匹配數 */
vector<Student> boys;
vector<Student> girls;
vector<vector<int>> G;
bool used[505];
int p[505];
bool match(int i) {
    for (int j: G[i]) {
        if (!used[j]) {
            used[j] = true;
            if (p[j] == -1 || match(p[j])) {
                p[j] = i;
```

```
                return true;
            }
        }
    }
    return false;
}
void maxMatch(int n) {
    memset(p, -1, sizeof(p));
    int res = 0;
    for (int i = 0; i < boys.size(); ++i) {
        memset(used, false, sizeof(used));
        if (match(i)) ++res;
    }
    cout << n - res << '\n';
}
```

3.12 差分

```
用途：在區間 [l, r] 加上一個數字v。
b[l] += v; (b[0~l] 加上v)
b[r+1] -= v; (b[r+1~n] 減去v (b[r] 仍保留v) )
給的 a[] 是前綴和數列，建構 b[]，
因為 a[i] = b[0] + b[1] + b[2] + … + b[i]，
所以 b[i] = a[i] - a[i-1]。
在 b[l] 加上 v，b[r+1] 減去 v，
最後再從 0 跑到 n 使 b[i] += b[i-1]。
這樣一來，b[] 是一個在某區間加上v的前綴和。
int a[1000], b[1000];
// a: 前綴和數列, b: 差分數列
int main(){
    int n, l, r, v;
    cin >> n;
    for(int i=1; i<=n; i++){
        cin >> a[i];
        b[i] = a[i] - a[i-1]; //建構差分數列
    }
    cin >> l >> r >> v;
    b[l] += v;
    b[r+1] -= v;
    for(int i=1; i<=n; i++){
        b[i] += b[i-1];
        cout << b[i] << ' ';
    }
}
```

3.13 MCMF

```
#define maxn 225
#define INF 0x3f3f3f3f
struct Edge {
  int u, v, cap, flow, cost;
};
//node size, edge size, source, target
int n, m, s, t;
vector<vector<int>> G;
vector<Edge> edges;
bool inqueue[maxn];
long long dis[maxn];
int parent[maxn];
long long outFlow[maxn];
void addEdge(int u,int v,int cap,int cost) {
  edges.emplace_back(Edge{u,v,cap,0,cost});
  edges.emplace_back(Edge{v,u,0,0,-cost});
  m = edges.size();
  G[u].emplace_back(m - 2);
  G[v].emplace_back(m - 1);
}
//一邊求最短路的同時一邊MaxFLow
bool SPFA(long long& maxFlow, long long&
     minCost) {
  // memset(outFlow, 0x3f, sizeof(outFlow));
  memset(dis, 0x3f, sizeof(dis));
  memset(inqueue, false, sizeof(inqueue));
  queue<int> q;
  q.push(s);
  dis[s] = 0;
  inqueue[s] = true;
  outFlow[s] = INF;
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    inqueue[u] = false;
    for (const int edgeIndex: G[u]) {
      const Edge& edge = edges[edgeIndex];
      if ((edge.cap > edge.flow) &&
        (dis[edge.v] > dis[u] + edge.cost)) {
        dis[edge.v] = dis[u] + edge.cost;
        parent[edge.v] = edgeIndex;
        outFlow[edge.v] = min(outFlow[u],
          (long long)(edge.cap - edge.flow));
        if (!inqueue[edge.v]) {
          q.push(edge.v);
          inqueue[edge.v] = true;
        }
      }
    }
  }
  //如果dis[t] > 0代表根本不賺還倒賠
  if (dis[t] > 0) return false;
  maxFlow += outFlow[t];
  minCost += dis[t] * outFlow[t];
  //一路更新回去這次最短路流完後要維護的
  //MaxFlow演算法相關(如反向邊等)
  int curr = t;
  while (curr != s) {
    edges[parent[curr]].flow += outFlow[t];
    edges[parent[curr]^1].flow -= outFlow[t];
    curr = edges[parent[curr]].u;
  }
  return true;
}
long long MCMF() {
  long long maxFlow = 0, minCost = 0;
  while (SPFA(maxFlow, minCost));
  return minCost;
}
int main() {
  int T;
  scanf("%d", &T);
  for (int Case = 1; Case <= T; ++Case){
    //總共幾個月，囤貨成本
    int M, I;
    scanf("%d %d", &M, &I);
    //node size
    n = M + M + 2;
    G.assign(n + 5, vector<int>());
    edges.clear();
    s = 0;
    t = M + M + 1;
    for (int i = 1; i <= M; ++i) {
      int produceCost, produceMax,
          sellPrice, sellMax, inventoryMonth;
      scanf("%d %d %d %d %d", &produceCost,
           &produceMax, &sellPrice,
           &sellMax, &inventoryMonth);
      addEdge(s, i, produceMax, produceCost);
      addEdge(M + i, t, sellMax, -sellPrice);
      for (int j=0; j<=inventoryMonth; ++j) {
        if (i + j <= M)
          addEdge(i, M + i + j, INF, I * j);
      }
    }
    printf("Case %d: %lld\n", Case, -MCMF());
  }
  return 0;
}
```

3.14 LCA 樹鍊剖分

```
#define maxn 5005
//LCA，用來練習樹鍊剖分
//題意：給定樹，找任兩點的中點，
//若中點不存在(路徑為even)，就是中間的兩個點
int dfn[maxn];
int parent[maxn];
int depth[maxn];
int subtreeSize[maxn];
int top[maxn]; //樹鍊的頂點
int dfnToNode[maxn]; //將dfn轉成node編碼
int hson[maxn]; //重兒子
int dfsTime = 1;
vector<vector<int>> G; //tree
//處理parent、depth、subtreeSize、dfnToNode
void dfs1(int u, int p) {
  parent[u] = p;
  hson[u] = -1;
  subtreeSize[u] = 1;
  for (int v: G[u]) {
    if (v != p) {
      depth[v] = depth[u] + 1;
      dfs1(v, u);
      subtreeSize[u] += subtreeSize[v];
      if (hson[u] == -1 ||
        subtreeSize[hson[u]]<subtreeSize[v]){
        hson[u] = v;
      }
    }
  }
}
//實際剖分 <- 參數t是top的意思
//t初始應為root本身
void dfs2(int u, int t) {
  top[u] = t;
  dfn[u] = dfsTime;
  dfnToNode[dfsTime] = u;
  ++dfsTime;
  //葉子點 -> 沒有重兒子
  if (hson[u] == -1) return;
  //優先對重兒子dfs，才能保證同一重鍊dfn連續
  dfs2(hson[u], t);
  for (int v: G[u]) {
    if (v != parent[u] && v != hson[u])
      dfs2(v, v);
  }
}
//不斷跳鍊，當跳到同一條鍊時，深度小的即為LCA
//跳鍊時優先鍊頂深度大的跳
int LCA(int u, int v) {
  while (top[u] != top[v]) {
    if (depth[top[u]] > depth[top[v]])
      u = parent[top[u]];
    else
      v = parent[top[v]];
  }
  return (depth[u] > depth[v]) ? v : u;
}
int getK_parent(int u, int k) {
  while (k-- && (u != -1)) u = parent[u];
  return u;
}
int main() {
  int n;
  while (scanf("%d", &n) && n) {
    dfsTime = 1;
    G.assign(n + 5, vector<int>());
    int u, v;
    for (int i = 1; i < n; ++i) {
      scanf("%d %d", &u, &v);
      G[u].emplace_back(v);
      G[v].emplace_back(u);
    }
    dfs1(1, -1);
    dfs2(1, 1);
    int q;
    scanf("%d", &q);
    for (int i = 0; i < q; ++i) {
      scanf("%d %d", &u, &v);
      //先得到LCA
      int lca = LCA(u, v);
```

```
      //計算路徑長(經過的邊)
      int dis = depth[u] + depth[v] - 2 *
           depth[lca];
      //讓v比u深或等於
      if (depth[u] > depth[v]) swap(u, v);
      if (u == v) {
        printf("The fleas meet at %d.\n", u);
      }
      else if (dis % 2 == 0) {
        //路徑長是even -> 有中點
        printf("The fleas meet at %d.\n",
             getK_parent(v, dis / 2));
      }
      else {
        //路徑長是odd -> 沒有中點
        if (depth[u] == depth[v]) {
          int x = getK_parent(u, dis / 2);
          int y = getK_parent(v, dis / 2);
          if (x > y) swap(x, y);
          printf("The fleas jump forever
               between %d and %d.\n", x, y);
        }
        else {
          //技巧: 讓深的點v往上dis / 2步 = y,
          //這個點的parent設為x
          //此時的x、y就是答案要的中點兩點
          //主要是往下不好找,所以改用深的點用parent往
          int y = getK_parent(v, dis / 2);
          int x = getK_parent(y, 1);
          if (x > y) swap(x, y);
          printf("The fleas jump forever
               between %d and %d.\n", x, y);
        }
      }
    }
  }
}
```

4 DataStructure

4.1 帶權併查集

val[x] 為 x 到 p[x] 的距離 (隨題目變化更改)

merge(u, v, w)

$u \xrightarrow{w} v$

$pu = pv$ 時，$val[v] - val[u] \neq w$ 代表有誤

若 $[l, r]$ 的總和為 w，則應呼叫 merge(l-1, r, w)

```
const int maxn = 2e5 + 10;

int p[maxn], val[maxn];

int findP(int x) {
    if(p[x] == -1) return x;
    int par = findP(p[x]);
    val[x] += val[p[x]];  //依題目更新val[x]
    return p[x] = par;
}

void merge(int u, int v, int w) {
    int pu = findP(u);
    int pv = findP(v);
    if(pu == pv) {
        // 理論上 val[v]-val[u] == w
        // 依題目判斷 error 的條件
        return;
    }
    val[pv] = val[u] - val[v] + w;
    p[pv] = pu;
}
```

4.2 Trie

```
const int maxc = 26;      // 單字字符數
const char minc = 'a';    // 首個 ASCII
struct TrieNode {
  int cnt;
  TrieNode* child[maxc];
  TrieNode() {
    cnt = 0;
    for(auto& node : child)
      node = nullptr;
  }
};
struct Trie {
  TrieNode* root;
  Trie() { root = new TrieNode(); }
  void insert(string word) {
    TrieNode* cur = root;
    for(auto& ch : word) {
      int c = ch - minc;
      if(!cur->child[c])
        cur->child[c] = new TrieNode();
      cur = cur->child[c];
    }
    cur->cnt++;
  }
  void remove(string word) {
    TrieNode* cur = root;
    for(auto& ch : word) {
      int c = ch - minc;
      if(!cur->child[c]) return;
      cur = cur->child[c];
    }
    cur->cnt--;
  }
  // 字典裡有出現 word
  bool search(string word, bool prefix=0) {
    TrieNode* cur = root;
    for(auto& ch : word) {
      int c = ch - minc;
      if(!(cur=cur->child[c])) return false;
    }
    return cur->cnt || prefix;
  }
  // 字典裡有 word 的前綴為 prefix
  bool startsWith(string prefix) {
    return search(prefix, true);
  }
};
```

4.3 AC Trie

```
const int maxn = 1e4 + 10; // 單字字數
const int maxl = 50 + 10; // 單字字長
const int maxc = 128;     // 單字字符數
const char minc = ' ';    // 首個 ASCII

int trie[maxn*maxl][maxc]; // 原字典樹
int val[maxn*maxl];       // 結尾(單字編號)
int cnt[maxn*maxl];       // 結尾(重複個數)
int fail[maxn*maxl];      // failure link
bool vis[maxn*maxl];      // 同單字不重複

struct ACTrie {
  int seq, root;
  ACTrie() {
    seq = 0;
    root = newNode();
  }
  int newNode() {
    for(int i=0; i<maxc; trie[seq][i++]=0);
    val[seq] = cnt[seq] = fail[seq] = 0;
    return seq++;
  }
  void insert(char* s, int wordId=0) {
    int p = root;
    for(; *s; s++) {
      int c = *s - minc;
      if(!trie[p][c]) trie[p][c] = newNode();
      p = trie[p][c];
    }
    val[p] = wordId;
    cnt[p]++;
  }
  void build() {
    queue<int> q({root});
    while(!q.empty()) {
      int p = q.front();
      q.pop();
      for(int i=0; i<maxc; i++) {
        int& t = trie[p][i];
        if(t) {
          fail[t] = p?trie[fail[p]][i]:root;
          q.push(t);
        } else {
          t = trie[fail[p]][i];
        }
      }
    }
  }
  // 要存 wordId 才要 vec
  // 同單字重複match要把所有vis取消掉
  int match(char* s, vector<int>& vec) {
    int res = 0;
    memset(vis, 0, sizeof(vis));
    for(int p=root; *s; s++) {
      p = trie[p][*s-minc];
      for(int k=p; k && !vis[k]; k=fail[k]) {
        vis[k] = true;
        res += cnt[k];
        if(cnt[k]) vec.push_back(val[k]);
      }
    }
    return res;    // 匹配到的單字量
  }
};

ACTrie ac;     // 建構,初始化
ac.insert(s); // 加字典單字
// 加完字典後
ac.build();    // !!! 建 failure link !!!
ac.match(s);   // 多模式匹配(傳入vec可以存編號)
```

4.4 線段樹 1D

```
#define MAXN 1000
int data[MAXN]; //原數據
int st[4 * MAXN]; //線段樹
int tag[4 * MAXN]; //懶標

inline int pull(int l, int r) {
// 隨題目改變sum、max、min
// l、r是左右樹的index
    return st[l] + st[r];
}
void build(int l, int r, int i) {
// 在[l, r]區間建樹,目前根的index為i
    if (l == r) {
        st[i] = data[l];
        return;
    }
    int mid = l + ((r - l) >> 1);
    build(l, mid, i * 2);
    build(mid + 1, r, i * 2 + 1);
    st[i] = pull(i * 2, i * 2 + 1);
}
int qry(int ql, int qr, int l, int r, int i){
// [ql,qr]是查詢區間, [l,r]是當前節點包含的區間
    if (ql <= l && r <= qr)
        return st[i];
    int mid = l + ((r - l) >> 1);
    if (tag[i]) {
        //如果當前懶標有值則更新左右節點
        st[i * 2] += tag[i] * (mid - l + 1);
        st[i * 2 + 1] += tag[i] * (r - mid);
        tag[i * 2] += tag[i];
```

```
        tag[i*2+1] += tag[i];
        tag[i] = 0;
    }
    int sum = 0;
    if (ql <= mid)
        sum+=query(ql, qr, l, mid, i * 2);
    if (qr > mid)
        sum+=query(ql, qr, mid+1, r, i*2+1);
    return sum;
}
void update(
    int ql,int qr,int l,int r,int i,int c) {
// [ql,qr]是查詢區間, [l,r]是當前節點包含的區間
// c是變化量
    if (ql <= l && r <= qr) {
        st[i] += (r - l + 1) * c;
              //求和,此需乘上區間長度
        tag[i] += c;
        return;
    }
    int mid = l + ((r - l) >> 1);
    if (tag[i] && l != r) {
        //如果當前懶標有值則更新左右節點
        st[i * 2] += tag[i] * (mid - l + 1);
        st[i * 2 + 1] += tag[i] * (r - mid);
        tag[i * 2] += tag[i];//下傳懶標至左節點
        tag[i*2+1] += tag[i];//下傳懶標至右節點
        tag[i] = 0;
    }
    if (ql <= mid)
        update(ql, qr, l, mid, i * 2, c);
    if (qr > mid)
        update(ql, qr, mid+1, r, i*2+1, c);
    st[i] = pull(i * 2, i * 2 + 1);
}
//如果是直接改值而不是加值，query與update中的tag與st
//改值從+=改成=
```

4.5 線段樹 2D

```
#define maxn 2005 //500 * 4 + 5 //純2D
     segment tree 區間查詢單點修改最大最小值
int maxST[maxn][maxn], minST[maxn][maxn];
int N;
void modifyY(int index, int l, int r,int val,
  int yPos, int xIndex, bool xIsLeaf) {
  if (l == r) {
    if (xIsLeaf) {
      maxST[xIndex][index] =
           minST[xIndex][index] = val;
      return;
    }
    maxST[xIndex][index] =
         max(maxST[xIndex*2][index],
         maxST[xIndex*2 + 1][index]);
    minST[xIndex][index] =
         min(minST[xIndex*2][index],
         minST[xIndex*2 + 1][index]);
  }
  else {
    int mid = (l + r) / 2;
    if (yPos <= mid)
      modifyY(index*2, l, mid, val, yPos,
           xIndex, xIsLeaf);
    else
      modifyY(index*2 + 1, mid + 1, r, val,
           yPos, xIndex, xIsLeaf);
    maxST[xIndex][index] =
         max(maxST[xIndex][index*2],
         maxST[xIndex][index*2 + 1]);
    minST[xIndex][index] =
         min(minST[xIndex][index*2],
         minST[xIndex][index*2 + 1]);
  }
}
void modifyX(int index, int l, int r, int
     val, int xPos, int yPos) {
  if (l == r) {
    modifyY(1, 1, N, val, yPos, index, true);
  }
  else {
    int mid = (l + r) / 2;
    if (xPos <= mid)
      modifyX(index*2,l,mid,val,xPos,yPos);
    else
      modifyX(index*2 + 1, mid + 1, r, val,
           xPos, yPos);
    modifyY(1, 1, N, val, yPos, index,false);
  }
}
void queryY(int index, int l, int r,int yql,
  int yqr, int xIndex, int& vmax, int &vmin){
  if (yql <= l && r <= yqr) {
    vmax = max(vmax, maxST[xIndex][index]);
    vmin = min(vmin, minST[xIndex][index]);
  }
  else {
    int mid = (l + r) / 2;
    if (yql <= mid)
      queryY(index*2, l, mid, yql, yqr,
           xIndex, vmax, vmin);
    if (mid < yqr)
      queryY(index*2 + 1, mid + 1, r, yql,
           yqr, xIndex, vmax, vmin);
  }
}
void queryX(int index, int l, int r, int
     xql, int xqr, int yql, int yqr, int&
     vmax, int& vmin) {
  if (xql <= l && r <= xqr) {
    queryY(1,1,N,yql,yqr,index,vmax,vmin);
  }
  else {
    int mid = (l + r) / 2;
    if (xql <= mid)
      queryX(index*2, l, mid, xql, xqr, yql,
           yqr, vmax, vmin);
    if (mid < xqr)
      queryX(index*2 + 1, mid + 1, r, xql,
           xqr, yql, yqr, vmax, vmin);
  }
}
int main() {
  while (scanf("%d", &N) != EOF) {
    int val;
    for (int i = 1; i <= N; ++i) {
      for (int j = 1; j <= N; ++j) {
        scanf("%d", &val);
        modifyX(1, 1, N, val, i, j);
      }
    }
    int q;
    int vmax, vmin;
    int xql, xqr, yql, yqr;
    char op;
    scanf("%d", &q);
    while (q--) {
      getchar(); //for \n
      scanf("%c", &op);
      if (op == 'q') {
        scanf("%d %d %d %d", &xql, &yql,
             &xqr, &yqr);
        vmax = -0x3f3f3f3f;
        vmin = 0x3f3f3f3f;
        queryX(1, 1, N, xql, xqr, yql, yqr,
             vmax, vmin);
        printf("%d %d\n", vmax, vmin);
      }
      else {
        scanf("%d %d %d", &xql, &yql, &val);
        modifyX(1, 1, N, val, xql, yql);
      }
    }
  }
  return 0;
}
```

5 Geometry

5.1 公式

1. Circle and Line

點 $P(x_0, y_0)$

到直線 $L : ax + by + c = 0$ 的距離

$$d(P, L) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

兩平行直線 $L_1 : ax + by + c_1 = 0$

與 $L_2 : ax + by + c_2 = 0$ 的距離

$$d(L_1, L_2) = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

2. Triangle

設三角形頂點為 $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$

點 A, B, C 的對邊長分別為 a, b, c

三角形面積為 Δ

重心為 (G_x, G_y)，內心為 (I_x, I_y)，

外心為 (O_x, O_y) 和垂心為 (H_x, H_y)

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$G_x = \frac{1}{3}(x_1 + x_2 + x_3)$$

$$G_y = \frac{1}{3}(y_1 + y_2 + y_3)$$

$$I_x = \frac{ax_1 + bx_2 + cx_3}{a + b + c}$$

$$I_y = \frac{ay_1 + by_2 + cy_3}{a + b + c}$$

$$O_x = \frac{1}{4\Delta}\begin{vmatrix} x_1^2 + y_1^2 & y_1 & 1 \\ x_2^2 + y_2^2 & y_2 & 1 \\ x_3^2 + y_3^2 & y_3 & 1 \end{vmatrix}$$

$$O_y = \frac{1}{4\Delta}\begin{vmatrix} x_1 & x_1^2 + y_1^2 & 1 \\ x_2 & x_2^2 + y_2^2 & 1 \\ x_3 & x_3^2 + y_3^2 & 1 \end{vmatrix}$$

$$H_x = -\frac{1}{2\Delta}\begin{vmatrix} x_2x_3 + y_2y_3 & y_1 & 1 \\ x_1x_3 + y_1y_3 & y_2 & 1 \\ x_1x_2 + y_1y_2 & y_3 & 1 \end{vmatrix}$$

$$H_y = -\frac{1}{2\Delta}\begin{vmatrix} x_1 & x_2x_3 + y_2y_3 & 1 \\ x_2 & x_1x_3 + y_1y_3 & 1 \\ x_3 & x_1x_2 + y_1y_2 & 1 \end{vmatrix}$$

任意三角形，重心、外心、垂心共線

$$G_x = \frac{2}{3}O_x + \frac{1}{3}H_x$$

$$G_y = \frac{2}{3}O_y + \frac{1}{3}H_y$$

3. Quadrilateral

任意凸四邊形 $ABCD$ 的四邊長分別為 a, b, c, d

且已知 $\angle A + \angle C$，則四邊形 $ABCD$ 的面積為

$$\sqrt{(s-a)(s-b)(s-c)(s-d) - \Delta}$$

where

$$s = \frac{a + b + c + d}{2}$$

$$\Delta = abcd\cos^2\left(\frac{A + C}{2}\right)$$

特例：若 $ABCD$ 為圓內接四邊形，則 $\Delta = 0$

若只知道其中一角，則可用餘弦定理

$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

求出對角線長，再用海龍計算兩個三角形面積即可。

5.2 Template

Predefined Variables

```cpp
using DBL = double;
using Tp = DBL;  // 存點的型態

const DBL pi = acos(-1);
const DBL eps = 1e-9;
const Tp inf = 1e30;
const int maxn = 5e4 + 10;
```

Vector、Point

```cpp
struct Vector {
  Tp x, y;
  Vector(Tp x=0, Tp y=0): x(x), y(y) {}
  DBL length();
};

using Point = Vector;
using Polygon = vector<Point>;

Vector operator+(Vector a, Vector b)
{return Vector(a.x+b.x, a.y+b.y);}
Vector operator-(Vector a, Vector b)
{return Vector(a.x-b.x, a.y-b.y);}
Vector operator*(Vector a, DBL b)
{return Vector(a.x*b, a.y*b);}
Vector operator/(Vector a, DBL b)
{return Vector(a.x/b, a.y/b);}
Tp dot(Vector a, Vector b)
{return a.x*b.x + a.y*b.y;}
Tp cross(Vector a, Vector b)
{return a.x*b.y - a.y*b.x;}
DBL Vector::length()
{return sqrt(dot(*this, *this));}
Vector unit_normal_vector(Vector v) {
  DBL len = v.length();
  return Vector(-v.y/len, v.x/len);
}
```

Line

```cpp
struct Line {
  Point p;
  Vector v;
  DBL ang;
  Line(Point _p={}, Vector _v={}) {
    p = _p;
    v = _v;
    ang = atan2(v.y, v.x);
  }
  bool operator<(const Line& l) const
  {return ang < l.ang;}
};
```

Segment

```cpp
struct Segment {
  Point s, e;
  Vector v;
  Segment(): s(0, 0), e(0, 0), v(0, 0) {}
  Segment(Point s, Point e): s(s), e(e) {
    v = e - s;
  }
  DBL length() { return v.length(); }
};
```

Circle

```cpp
struct Circle {
  Point o;
  DBL r;
  Circle(): o({0, 0}), r(0) {}
  Circle(Point o, DBL r=0): o(o), r(r) {}
  Circle(Point a, Point b) { // ab 直徑
    o = (a + b) / 2;
    r = dis(o, a);
  }
  Circle(Point a, Point b, Point c) {
    Vector u = b-a, v = c-a;
    DBL c1=dot(u, a+b)/2, c2=dot(v, a+c)/2;
    DBL dx=c1*v.y-c2*u.y, dy=u.x*c2-v.x*c1;
    o = Point(dx, dy) / cross(u, v);
    r = dis(o, a);
  }
  bool cover(Point p) {return dis(o,p) <= r;}
};
```

5.3 旋轉卡尺

```cpp
// 回傳凸包內最遠兩點的距離^2
int longest_distance(Polygon& p) {
  auto test = [&](Line l, Point a, Point b) {
   return cross(l.v,a-l.p)<=cross(l.v,b-l.p);
  };
  if(p.size() <= 2) {
    return cross(p[0]-p[1], p[0]-p[1]);
  }
  int mx = 0, n = p.size();
  for(int i=0, j=1; i<n; i++) {
    Line l(p[i], p[(i+1)%n] - p[i]);
    for(;test(l,p[j],p[(j+1)%n]);j=(j+1)%n);
    mx = max({
      mx,
      dot(p[(i+1)%n]-p[j], p[(i+1)%n]-p[j]),
      dot(p[i]-p[j], p[i]-p[j])
    });
  }
  return mx;
}
```

5.4 半平面相交

Template

```cpp
using DBL = double;
using Tp = DBL;              // 存點的型態
const int maxn = 5e4 + 10;
const DBL eps = 1e-9;
struct Vector;
using Point = Vector;
using Polygon = vector<Point>;
Vector operator+(Vector, Vector);
Vector operator-(Vector, Vector);
Vector operator*(Vector, DBL);
Tp cross(Vector, Vector);
struct Line;
Point intersection(Line, Line);
int dcmp(DBL, DBL);          // 不見得會用到
```

Halfplane Intersection

```cpp
// Return: 能形成半平面交的凸包邊界點
Polygon halfplaneIntersect(vector<Line>&nar){
  sort(nar.begin(), nar.end());
  // p 是否在 l 的左半平面
  auto lft = [&](Point p, Line l) {
    return dcmp(cross(l.v, p-l.p)) > 0;
  };

  int ql = 0, qr = 0;
  Line L[maxn] = {nar[0]};
  Point P[maxn];

  for(int i=1; i<nar.size(); i++) {
    for(; ql<qr&&!lft(P[qr-1],nar[i]); qr--);
    for(; ql<qr&&!lft(P[ql],nar[i]); ql++);
    L[++qr] = nar[i];
    if(dcmp(cross(L[qr].v,L[qr-1].v))==0) {
      if(lft(nar[i].p,L[--qr])) L[qr]=nar[i];
    }
    if(ql < qr)
      P[qr-1] = intersection(L[qr-1], L[qr]);
  }
  for(; ql<qr && !lft(P[qr-1], L[ql]); qr--);
  if(qr-ql <= 1) return {};
  P[qr] = intersection(L[qr], L[ql]);
  return Polygon(P+ql, P+qr+1);
}
```

5.5 Polygon

```cpp
// 判斷點 (point) 是否在凸包 (p) 內
bool pointInConvex(Polygon& p, Point point) {
  // 根據 Tp 型態來寫，沒浮點數不用 dblcmp
  auto dblcmp=[](DBL v){return (v>0)-(v<0);};
  // 不包含線上，改 '>=' 為 '>'
  auto test = [&](Point& p0, Point& p1) {
    return dblcmp(cross(p1-p0, point-p0))>=0;
  };
  p.push_back(p[0]);
  for(int i=1; i<p.size(); i++) {
    if(!test(p[i-1], p[i])) {
      p.pop_back();
      return false;
    }
  }
  p.pop_back();
  return true;
}

// 計算簡單多邊形的面積
// ! p 為排序過的點 !
DBL polygonArea(Polygon& p) {
  DBL sum = 0;
  for(int i=0, n=p.size(); i<n; i++)
    sum += cross(p[i], p[(i+1)%n]);
  return abs(sum) / 2.0;
}
```

5.6 凸包

- Tp 為 Point 裡 x 和 y 的型態
- struct Point 需要加入並另外計算的 variables:
  1. ang, 該點與基準點的 atan2 值
  2. d2, 該點與基準點的 (距離)^2
- 注意計算 d2 的型態範圍限制

Template

```cpp
using DBL = double;
using Tp = long long;          // 存點的型態
const DBL eps = 1e-9;
const Tp inf = 1e9;            // 座標極大值
struct Vector;
using Point = Vector;
using Polygon = vector<Point>;
Vector operator-(Vector, Vector);
Tp cross(Vector, Vector);
int dcmp(DBL, DBL);
```

Convex Hull

```cpp
Polygon convex_hull(Point* p, int n) {
  auto rmv = [](Point a, Point b, Point c) {
    return cross(b-a, c-b) <= 0; // 非浮點數
    return dcmp(cross(b-a, c-b)) <= 0;
  };

  // 選最下裡最左的當基準點，可在輸入時計算
  Tp lx = inf, ly = inf;
  for(int i=0; i<n; i++) {
```

```
    if(p[i].y<ly || (p[i].y==ly&&p[i].x<lx)){
      lx = p[i].x, ly = p[i].y;
    }
  }

  for(int i=0; i<n; i++) {
    p[i].ang=atan2(p[i].y-ly,p[i].x-lx);
    p[i].d2 = (p[i].x-lx)*(p[i].x-lx) +
              (p[i].y-ly)*(p[i].y-ly);
  }
  sort(p, p+n, [&](Point& a, Point& b) {
    if(dcmp(a.ang, b.ang))
      return a.ang < b.ang;
    return a.d2 < b.d2;
  });

  int m = 1;    // stack size
  Point st[n] = {p[n] = p[0]};
  for(int i=1; i<=n; i++) {
    for(;m>1&&rmv(st[m-2],st[m-1],p[i]);m--);
    st[m++] = p[i];
  }
  return Polygon(st, st+m-1);
}
```

5.7 最小圓覆蓋

```
vector<Point> p(3);   // 在圓上的點
Circle MEC(vector<Point>& v, int n, int d=0){
  Circle mec;
  if(d == 1) mec = Circle(p[0]);
  if(d == 2) mec = Circle(p[0], p[1]);
  if(d == 3) return Circle(p[0], p[1], p[2]);
  for(int i=0; i<n; i++) {
    if(mec.cover(v[i])) continue;
    p[d] = v[i];
    mec = MEC(v, i, d+1);
  }
  return mec;
}
```

5.8 交點、距離

```
int dcmp(DBL a, DBL b=0.0) {
  if(abs(a-b) < eps) return 0;
  return a<b ? -1 : 1;
}
bool hasIntersection(Point p, Segment s) {
  if(dcmp(cross(s.s-p, s.e-p))) return false;
  return dcmp(dot(s.s-p, s.e-p)) <= 0;
}
bool hasIntersection(Point p, Line l)
{return dcmp(cross(p-l.p, l.v)) == 0;}
bool hasIntersection(Segment a, Segment b) {
  // 判斷在 X 軸 Y 軸的投影是否相交
  auto intr1D=[](DBL w, DBL x, DBL y, DBL z){
    if(w > x) swap(w, x);
    if(y > z) swap(y, z);
    return dcmp(max(w, y), min(x, z)) <= 0;
  };

  DBL a1 = cross(a.v, b.s-a.s);
  DBL a2 = cross(a.v, b.e-a.s);
  DBL b1 = cross(b.v, a.s-b.s);
  DBL b2 = cross(b.v, a.e-b.s);

  return intr1D(a.s.x, a.e.x, b.s.x, b.e.x)
      && intr1D(a.s.y, a.e.y, b.s.y, b.e.y)
      && dcmp(a1) * dcmp(a2) <= 0
      && dcmp(b1) * dcmp(b2) <= 0;
}
Point intersection(Segment a, Segment b) {
  Vector v = b.s - a.s;
  DBL c1 = cross(a.v, b.v);
  DBL c2 = cross(v, b.v);
  DBL c3 = cross(v, a.v);

  if(dcmp(c1) < 0) c1=-c1, c2=-c2, c3=-c3;
  if(dcmp(c1) && dcmp(c2)>=0 && dcmp(c3)>=0
    && dcmp(c1, c2)>=0 && dcmp(c1, c3)>=0)
    return a.s + (a.v * (c2 / c1));
  return Point(inf, inf); // a 和 b 共線
}
Point intersection(Line a, Line b) {
  // cross(a.v, b.v) == 0 時平行
  Vector u = a.p - b.p;
  DBL t = 1.0*cross(b.v, u)/cross(a.v, b.v);
  return a.p + a.v*t;
}
DBL dis(Point a, Point b)
{return sqrt(dot(a-b, a-b));}
DBL dis(Point p, Line l)
{return abs(cross(p-l.p, l.v))/l.v.length();}
DBL dis(Point p, Segment s) {
  Vector u = p - s.s, v = p - s.e;
  if(dcmp(dot(s.v, u))<=0) return u.length();
  if(dcmp(dot(s.v, v))>=0) return v.length();
  return abs(cross(s.v, u)) / s.length();
}
DBL dis(Segment a, Segment b) {
  if(hasIntersection(a, b)) return 0;
  return min({
    dis(a.s, b), dis(a.e, b),
    dis(b.s, a), dis(b.e, a)
  });
}
DBL dis(Line a, Line b) {
  if(dcmp(cross(a.v, b.v)) == 0) return 0;
  return dis(a.p, b);
}
Point getPedal(Line l, Point p) {
// 返回 p 在 l 上的垂足(投影點)
  DBL len = dot(p-l.p, l.v) / dot(l.v, l.v);
  return l.p + l.v * len;
}
```

6 DP

6.1 Deque 最大差距

```
/*定義dp[l][r]是l ~ r時與先手最大差異值
  轉移式: dp[l][r] = max{a[l] - solve(l + 1,
        r), a[r] - solve(l, r - 1)}
  裡面用減的主要是因為求的是相減且會一直換手,
  所以正負正負...*/
#define maxn 3005
bool vis[maxn][maxn];
long long dp[maxn][maxn];
long long a[maxn];
long long solve(int l, int r) {
    if (l > r) return 0;
    if (vis[l][r]) return dp[l][r];
    vis[l][r] = true;
    long long res = a[l] - solve(l + 1, r);
    res = max(res, a[r] - solve(l, r - 1));
    return dp[l][r] = res;
}
int main() {
    ...
    printf("%lld\n", solve(1, n));
}
```

6.2 string DP

Edit distance S_1 最少需要經過幾次增、刪或換字變成 S_2

$$dp[i,j] = \begin{cases} i+1, & \text{if } j=-1 \\ j+1, & \text{if } i=-1 \\ dp[i-1,j-1], & \text{if } S_1[i]=S_2[j] \\ \min\begin{Bmatrix} dp[i,j-1] \\ dp[i-1,j] \\ dp[i-1,j-1] \end{Bmatrix}+1, & \text{if } S_1[i]\neq S_2[j] \end{cases}$$

Longest Palindromic Subsequence

$$dp[l,r] = \begin{cases} 1 & \text{if} \quad l=r \\ dp[l+1,r-1] & \text{if} \quad S[l]=S[r] \\ \max\{dp[l+1,r], dp[l,r-1]\} & \text{if} \quad S[l]\neq S[r] \end{cases}$$

6.3 LCS 和 LIS

```
//LCS 和 LIS 題目轉換
LIS 轉成 LCS
    1. A 為原序列， B=sort(A)
    2. 對 A,B 做 LCS
LCS 轉成 LIS
    1. A, B 為原本的兩序列
    2. 最 A 序列作編號轉換，將轉換規則套用在 B
    3. 對 B 做 LIS
    4. 重複的數字在編號轉換時後要變成不同的數字，
       越早出現的數字要越小
    5. 如果有數字在 B 裡面而不在 A 裡面，
       直接忽略這個數字不做轉換即可
```

6.4 樹 DP 有幾個 path 長度為 k

```
#define maxn 50005
#define maxk 505
//dp[u][u的child且距離u長度k的數量]
long long dp[maxn][maxk];
vector<vector<int>> G;
int n, k;
long long res = 0;
void dfs(int u, int p) {
  //u自己
  dp[u][0] = 1;
  for (int v: G[u]) {
    if (v == p) continue;
    dfs(v, u);
    for (int i = 1; i <= k; ++i) {
      //子樹v距離i - 1的等於對於u來說距離i的
      dp[u][i] += dp[v][i - 1];
    }
  }
  //統計在u子樹中距離u為k的數量
  res += dp[u][k];
  long long cnt = 0;
  for (int v: G[u]) {
    if (v == p) continue; //重點算法
    for (int x = 0; x <= k - 2; ++x) {
      cnt +=
        dp[v][x]*(dp[u][k-x-1]-dp[v][k-x-2]);
    }
  }
  res += cnt / 2;
}
int main() {
  ...
  dfs(1, -1);
}
```