The error function

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1 Normal distribution

A random variable Y that is normally distributed over a range in x can be described using a gaussian curve [1].

A gaussian curve is desctribed by two parameters: the mean and the variance.

The mean μ, also called the expectation value, is the average value of a large sample af Y.

The variance σ^2 is a measure of the dispersion in the distribution of Y. A distribution with low σ^2 will be much more localized than a distribution with high σ^2.
The formula describing the normal distribution is:

$$f(x, \mu\sigma^2) = \frac{1}{2\pi\sigma^2}\exp(-\frac{(x-\mu)^2}{2\sigma^2}). \tag{1}$$

The gaussian curve with different parameters is shown in figure 1.

2 Error function

The gaussian curve defined in equation 1 contains the properties of the normal distribution. In order to extract some of these properties, auxillary functions, such as the error function, are implemented [2].
Considering a normally distributed varaible Y with a mean $\mu_Y = 1$ and variance $\sigma_Y = 1/2$ the errorfunction erf(x) will describe the probability of Y falling in the range [-x x].
The error function is defined as:

$$erf(x) = \int_0^x \frac{2}{\sqrt{\pi}}\exp(-x^2)dx \tag{2}$$

The value of the error function can be evaluated by numerically solving the above integral. The result of this evaluation is shown in figure 2.

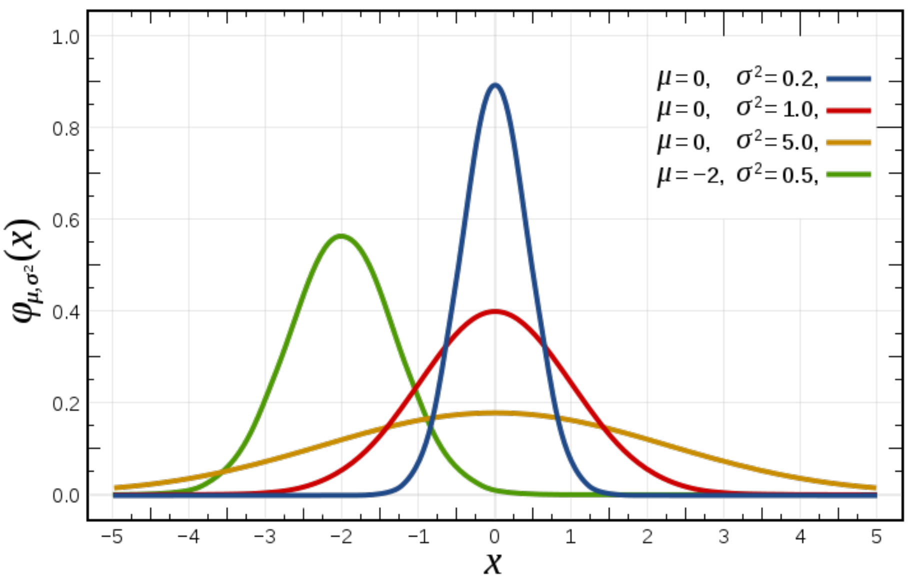

Figure 1: Gaussian curves with different values of the mean (μ) and variance (σ^2)

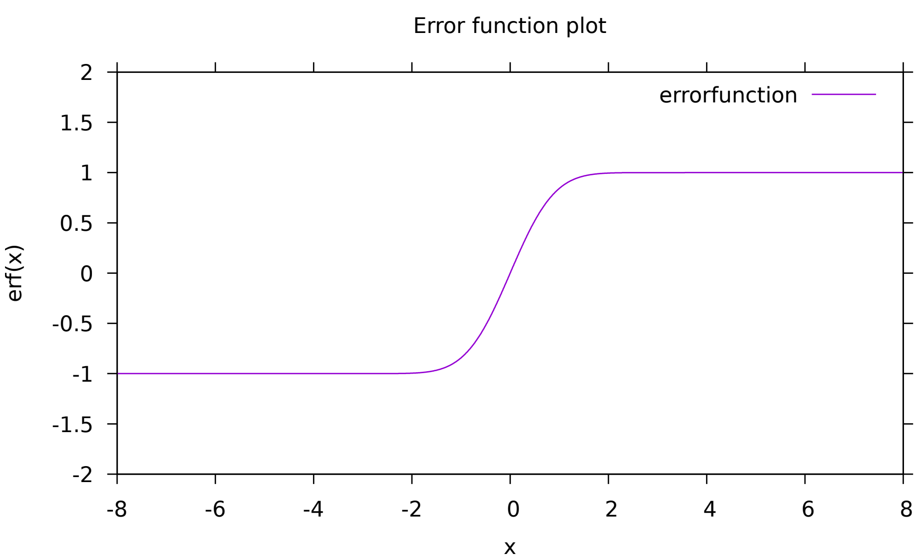

Figure 2: Numerical solution of the error function

3 References

References

[1] Wikipedia article on the gaussian distribution
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[2] Wikipedia article on the error function
WIKIPEDIA: ERROR FUNCTION,
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