

# Big programming exercise, task 16: Arcsine function

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## 1 Introduction

This report aims to implement the arcsine function using numerical root-finding routines from the GNU scientific library (GSL).

## 2 Theory

Root finding routines are mathematical routines that find the roots of functions. A root of a function  $f(x)$  is the value of  $x$  at which  $f(x)$  is equal to 0.

A system of equations can be recast as a root finding problem. Given two functions of  $x$ :  $f(x)$  and  $g(x)$  equal to each other [1].

$$f(x) = g(x) \Rightarrow h(x) = f(x) - g(x). \quad (1)$$

Finding the solution of the equation is the same as finding the roots of  $h(x)$ . In this project I specifically look at the arcsine function.

$$a(x) = \arcsin(x). \quad (2)$$

This function can be recast as a root finding problem by taking the sine on both sides and subtracting one side:

$$\sin(a(x)) = \sin(\arcsin(x)) = x \Rightarrow 0 = h(x) = x - \sin(a(x)) \quad (3)$$

The value of  $a(x)$  can now be computed numerically by finding the values of  $a(x)$  that correspond to the roots of  $h(x)$  for different values of  $x$ .

## 3 Numerical solution

In order to find the roots of equation 3 I use the multidimensional root finding algorithms from GSL [2].

Note that this is a one-dimensional problem and can be treated using the one-dimensional root-finding algorithms from GSL. I chose to work with the multidimensional algorithms as I am more familiar with them.

I used the "Hybrid" algorithm, casting equation 3 with  $x$  as a parameter and  $a(x)$  as the unknown to be found. The values of  $a(x)$  corresponding to the roots of the equation were then found for different values of  $x$  ranging from  $x = -1$  to  $x = 1$  with step size  $dx = 0.05$ . The found values of  $a(x)$  are plotted as function of  $x$ . I also plot the built-in arcsin function from the math library "math.h" as reference and compare it to the numerical solution. The plot can be seen on figure 3

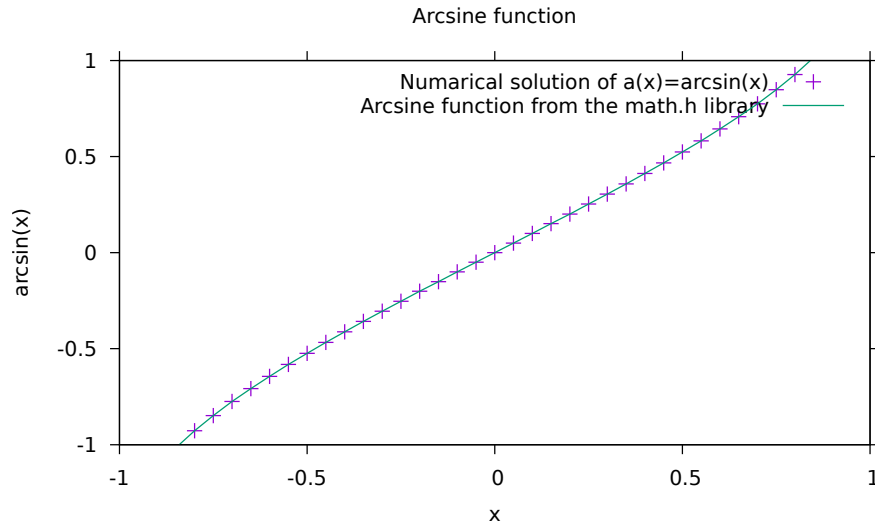


Figure 1: This figure shows the numerical solution of the arcsine function (plusses) together with the math.h arcsine function (line).

It can be seen on figure 3 that the found numerical solution closely follow the math.h function. This result illustrates the power of the root-finding algorithms as tools for numerical computation.

## 4 References

### References

- [1] Wikipedia article on the root finding algorythms  
*WIKIPEDIA: ROOT-FINDING ALGORYTHM*,  
[https://en.wikipedia.org/wiki/Root-finding\\_algorithm](https://en.wikipedia.org/wiki/Root-finding_algorithm)
  
- [2] Gnu Scientific Library page for the multidimensional root finding.  
*GSL: MULTIDIMENSIONAL ROOT FINDING*,  
<https://www.gnu.org/software/gsl/doc/html/multiroots.html>