Phase Transitions in Artificial Intelligence Systems

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ABSTRACT

We predict that large-scale artificial intelligence systems and cognitive models will undergo sudden phase transitions from disjointed parts into coherent structures as their topological connectivity increases beyond a critical value. These situations, ranging from production systems to semantic net computations, are characterized by event horizons in space-time that determine the range of causal connections between processes. At transition, these event horizons undergo explosive changes in size. This phenomenon, analogous to phase transitions in nature, provides a new paradigm with which to analyze the behavior of large-scale computation and determine its generic features.

1. Introduction

Most problems in artificial intelligence and cognitive modeling involve symbolic computation and search in large networks. The examples of semantic nets, spreading activation, and heuristic search in problem spaces, to name a few, point to the need for a theory which predicts their generic properties. This requires understanding the relationship between the global dynamics of any of these computational systems and the nature of the underlying processes. Furthermore, present technology is making feasible the construction of massively parallel computational systems [2, 9, 13] which are specialized for AI computational tasks. This raises the general question of what kinds of new behaviors emerge when scaling up from ten processors to, say, a million, as well as how to program them by determining overall behavior from local specifications.

Statistical mechanics, based on the law of large numbers, has taught us that many universal and generic features of large systems can be quantitatively understood as approximations to the average behavior of infinite systems. Although such infinite models can be difficult to solve in detail, their overall qualitative features can be determined with a surprising degree of accuracy. Since these features are universal in character and depend only on a few

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general properties of the system, they can be expected to apply to a wide range of actual configurations. This is the case when the number of relevant degrees of freedom specified by constraints, as well as the number of interesting parameters, is much fewer than the actual number of freedoms in the system. In this situation, it becomes useful to treat the unspecified internal degrees of freedom as if they are given by a probability distribution. This implies assuming a lack of correlations between the unspecified and specified degrees of freedom. This assumption has been extremely successful in statistical mechanics. It implies that although degrees of freedom may change according to purely deterministic algorithms, the fact that they are unspecified makes them appear to an outside observer as effectively random.

In this paper we discuss the applicability of statistical mechanics to large-scale computational systems and show that they can have sudden and dramatic changes in overall behavior which cannot be foreseen by examining smaller-scale systems. Moreover, this phenomenon, which is similar to the phase transitions encountered in many physical situations, is largely independent of local details and has important consequences both at the conceptual and the implementation level. The resulting phase changes can be quantitatively inferred from generic parameters of the system. The universal nature of phase transitions allows us to predict its occurrence in a large variety of situations.

Consider, for instance, massively parallel systems which are desired to be robust and adaptable. They should work in the presence of unexpected errors and with changes in the environment in which they are embedded (i.e. *fail soft*). This implies that many of the system's internal degrees of freedom will be allowed to adjust by taking on a range of possible configurations. Furthermore, their large size will necessarily enforce a perspective which concentrates on a few relevant variables. Although these considerations suggest that the assumptions necessary for a statistical description hold for these systems, experiments will be necessary for deciding their applicability. Additional examples include heuristic searches, production systems, disambiguation strategies in linguistics, and spreading activation networks. Since these approaches are not only methodologies with which AI tries to build systems, but also constitute cognitive models [10, 21], our results might also have implications in fields other than artificial intelligence and computation.

After a brief description of phase transitions in nature, and how they emerge from the collective behavior of systems with a large number of interacting components, we discuss a number of computational examples which can be expected to show analogous phenomena.

1.1. Phase transitions

Matter commonly undergoes dramatic changes in its qualitative properties when certain parameters pass through critical values. The most common

everyday example is the melting of a solid with increasing temperature. As the temperature is increased above absolute zero the vibrations of the atoms making up the solid become gradually more violent, leading to the well known phenomenon of expansion. Since this increase in vibration amplitude is gradual, one would naively expect that the macroscopic properties of the substance would accordingly undergo a smooth change. However, there exists a well defined temperature for which something much more dramatic does take place: a sudden change in the properties of the substance over a small temperature range, and the appearance of a qualitatively different phase, which in this case is a liquid. This melting transition involves, among other things, an abrupt softening of the solid even though its average atomic energy changes only slightly. The ensuing liquid can in turn undergo another phase transition into a gas phase, where once again other thermodynamic quantities display discontinuous changes.

Other examples of phase transitions appear in magnets when heated, superconductors which become normal metals, inflationary models of the early evolution of the universe, the creation of self-replicating biochemicals above a critical reactive mass, and many other systems composed of a large number of components.

Another type of interesting phase transition is that displayed by percolation processes. They can be easily visualized by considering a network of channels connecting sites and water flow between them. Given an initial concentration of fluid in a given site and a pressure gradient, a typical problem consists in determining how many sites, on the average, get wet after a long time. This percolation problem, which has been extensively studied in contexts ranging from the spread of an epidemic to the conductivity of electrical networks [6, 11], has been shown to possess a phase transition independent of the detailed geometry of the system. When the average number of paths connecting the sites is below a certain value, the probability of finding a wet site at an arbitrarily large distance from the source is vanishingly small. This means that only a finite number of sites are connected to the source. As the number of paths increases, however, there is a singularity in the probability that distant sites get wet, and above a critical value the connected cluster becomes infinite. In this context, one can define event horizons which quantify the range of causal connections between sites.

These phase changes are characterized by the appearance of mathematical singularities in observables such as the density, viscosity and specific heat of matter. Depending on the nature of the transition, one might also observe phenomena such as hysteresis, sluggish response to external stimuli, or the process of nucleation of a new phase.

Recent results in statistical mechanics have shown that despite the apparent diversity in the composition and underlying structure of these systems, phase transitions take place with universal quantitative characteristics, independent

of the detailed nature of the interactions between individual components. This implies that the singular behavior of observables near the transition point becomes identical for many systems, defining universality classes which only depend on the range of interaction of the forces at play and the dimensionality of the problem [24].

The theoretical treatment of phase transitions relies on models of matter which in the limit of very many components can be considered stochastic in nature, an approximation which works exceedingly well in characterizing their generic properties. In this so-called thermodynamic limit, extensive quantities such as the number of particles and the volume they occupy become infinitely large, while the average number per unit volume remains constant. The theory can also describe the corrections to the infinite model needed to describe the large, but finite, systems encountered in actual experiments.

In the following, we discuss the applicability of these techniques to computational problems. We also show how phase transitions, particularly those analogous to percolation but without the physical restriction to Euclidean space, take place in computational problems involving many interacting processes. We also elucidate their implications for concrete AI applications. In Section 2 we consider a simple model of a hierarchical calling structure and explicitly show that it displays a phase transition. Moreover, we discuss how this model is modified by real-world constraints such as finite size and fluctuations, and describe its implications for heuristic searches. In Section 3 we deal with the more interesting situation provided by spreading activation networks. In addition to connectivity, these nets also involve dynamical processes at the node level so that the ensuing phase diagram contains several novel and interesting phases. We show how collective effects near the transition lead to a critical slowing down of the whole network and how the event horizons behave in the different phases. A final section summarizes our findings and outlines possible generalizations of this new paradigm.

2. Interacting Processes

2.1. A hierarchical model

Consider a large set of interacting processes and assume a local connectivity between them, which means that each process directly invokes only a few others. For simplicity, we initially assume that the processes form a hierarchical calling structure. This can be represented by a set of trees, their sizes determined by the number of processes called by each one. This collection of trees provides a simple, exactly solvable model which displays a percolation phase transition. We focus on the simple global observable given by the number of activated processes, or, more specifically, by the number of processes reachable from a given initial process. Since more general models exhibit the behavior derived below, a reflection of the universal nature of these

phase transitions, these simplifications are not as restrictive as they may first appear.

The trees we study have nodes connected by links and an average branching ratio z. We now determine the average size of the trees in the forest. This average size determines the event horizons in space-time, which are the range of causal interactions between processes. Depending on the actual value of z, we will show below that there are two qualitatively different regimes, separated by a sharp phase transition at a critical value of the branching ratio $z_c = 1$. In the first regime, in which $z < z_c$, the average size of the trees is finite, implying that activation will be localized. In the second regime, for $z > z_c$, the average tree size is infinite. Notice that the location of the transition can be easily understood from the fact that if the expected number of links from a node is more than one it will, on the average, have infinitely many descendants, and otherwise a finite number. The following detailed derivation illustrates how phase transitions appear in this very simple case. Unlike the simple intuitive argument we just gave, these techniques generalize to more complicated cases, such as graphs, in which intuition can fail.

For this system, the average number of nodes which are a distance u from the root, $N(u)$, can be readily computed. In this context, the distance between a node and one of its descendants is just the number of links or steps between them. Thus, $N(u)$ is the average number of descendants from the initial node after u steps.

In a tree, the path from the root to any node is unique so the average number of nodes that are reachable in u steps is given by $N(1)^u$, since each node has an average of $N(1)$ children and all of these are distinct. Since $N(1)$ is the average number of links from a node, it is given by z. We therefore obtain

$$N(u) = z^u .$$

From this result one can compute all the average properties of the trees. For example, the average number of nodes in a tree, A, is given by

$$A = \sum_{u=0}^{\infty} N(u) = \frac{1}{1-z} , \tag{1}$$

when $z < 1$. This quantity is depicted in Fig. 1 and clearly diverges at the critical value $z_c = 1$. This divergence marks the existence of a phase transition characterized by an explosive increase in the number of nodes belonging to each tree as z_c is approached from below. Similarly, the average correlation length, which determines the horizon of any given node, is

$$\langle u \rangle = \frac{1}{A} \sum_{u=0}^{\infty} uN(u) = \frac{z}{1-z} , \tag{2}$$

when $z < 1$. This also diverges at z_c.

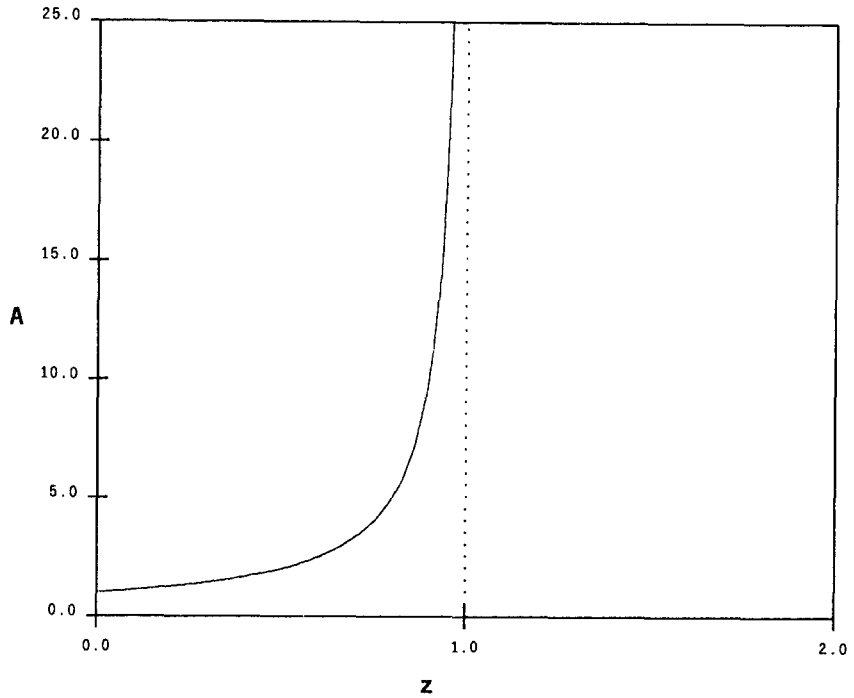

FIG. 1. The average cluster size versus z. In this case $A = (1 - z)^{-1}$ is plotted up to $z = 0.96$.

2.2. Finite size systems

To apply the above ideas to actual computational systems it is important to consider both the effects of finite size and the statistical fluctuations that are unavoidable in real situations.

Strictly speaking, the singular behavior associated with a phase transition takes place only in the limit of infinite size systems. Nevertheless, the signature of a phase transition can be observed in finite size systems. What is required is that the size of the system be larger than a typical correlation length, which in our case is given by (2). Since the correlation length grows as the transition is approached, this requirement will not hold very near the transition point, leading to a smoothing of the singularity, as shown in Fig. 2.

2.3. Heuristic search

As an application of these ideas, we consider a simple example of heuristically guided search [17, 23] in a tree of depth d and uniform branching ratio b. The problem consists of finding the single desired leaf starting from the root. Perfect knowledge of how to proceed at each node would require examining d nodes. At the other extreme, an exhaustive search will, on the average,

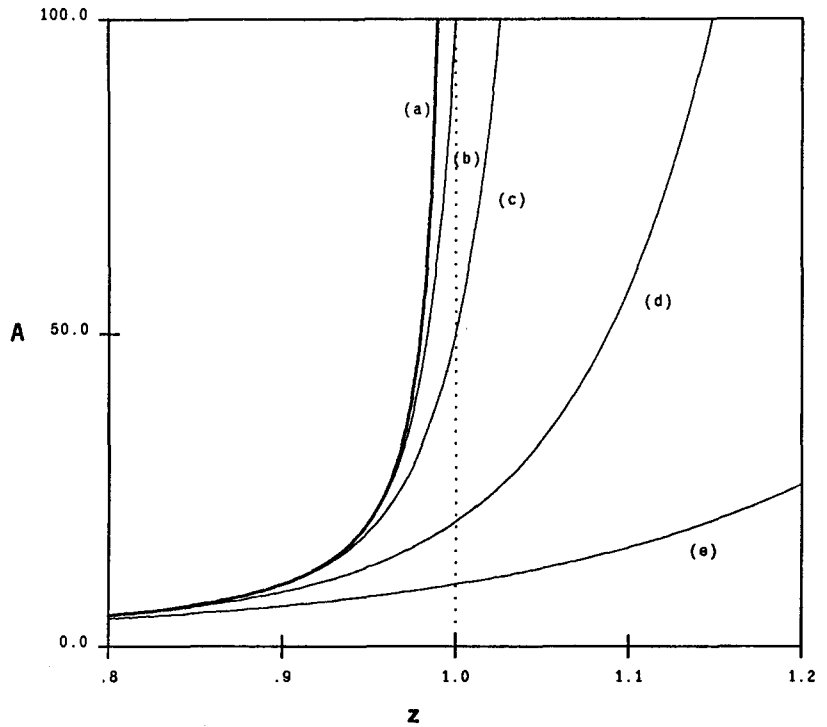

FIG. 2. Average cluster size for trees with finite depth d as a function of z. Here $A = (1 - z)^d/(1 - z)$ is shown for values of z near the transition at $z_c = 1$. The curves represent (a) infinite tree, (b) $d = 100$, (c) $d = 50$, (d) $d = 20$, and (e) $d = 10$.

examine one half of the nodes in the tree, a quantity which grows exponentially with d. A convenient way to distinguish these kinds of behavior is to determine the average number of nodes that must be examined to move one step closer to the desired node, i.e. the total number required to find the solution, divided by d.

We assume that a heuristic applied at each node is able to eliminate each unproductive branch with probability $1 - p$. The above extremes correspond to $p = 0$ (perfect knowledge) and $p = 1$ (no knowledge) respectively. Thus the effective branching ratio $z = bp$. This produces a search tree as shown in Fig. 3. Except for this heuristic, the search is done randomly. Note that for each node along the unique path from the root to the desired leaf, there are $b - 1$ correct choices of how to proceed and one correct choice. For the other nodes, all b choices are incorrect. This nonuniformity requires a slight modification to the arguments presented above in order to determine the average number of nodes that are examined before the desired leaf is found.

During an average search, before trying the correct branch from the root,

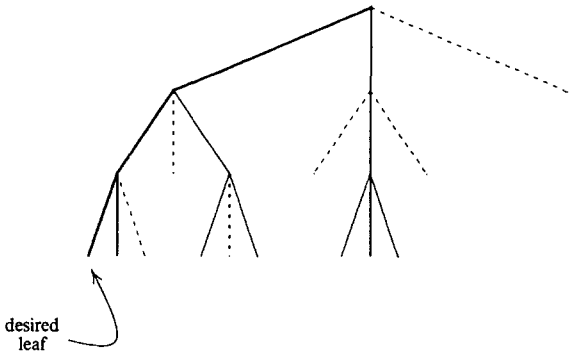

Fig. 3. A search tree in which the dashed links have been eliminated by a heuristic.

both the root itself and one half of the subtree formed by the other branches will be examined. Let $A(d)$ be the average size of the tree restricted to depth d. It can be computed by restricting the upper limit in the sum of (1) to $u = d - 1$. Since there are, on the average, $(b-1)p$ such branches, each with $A(d-1)$ nodes for an average subtree, this consists of $1 + \frac{1}{2}(b-1)pA(d-1)$ nodes. The next step will use $A(d-2)$ instead of $A(d-1)$ and so on. Thus the average number of nodes N examined before the desired leaf is found is

$$N = d + \tfrac{1}{2}(b-1)p \sum_{s=1}^{d-1} A(s) . \tag{3}$$

This can be explicitly evaluated but here we focus on $n = N/d$, the number of nodes per step, when the tree is very deep, i.e. when d is large. When $z < 1$, (3) gives

$$n = \frac{2 - z - p}{2(1 - z)} .$$

This quantity becomes infinite at $z = 1$ and beyond, indicating a sudden transition from linear to exponential search. Thus, near the transition, a small change in the local effectiveness of the heuristic has a major impact on the global behavior of large-scale search problems.

Finally, we should note that this behavior can also occur with more complex model heuristics. For example, the probability to eliminate the unproductive branches could be larger for nodes near the leaves than for nodes near the root. Once again, this detailed behavior, while changing the exact location and details of the transitions, does not affect the qualitative existence of the transition.

2.4. More general models

The tree model we have just discussed assumes that the connectivity is strictly

hierarchical. More realistic models can be based on general graphs which contain arbitrary cycles. Although the direct analytic determination of properties such as the average size of connected clusters is more difficult, the general features of the phase transition seen in the case of trees still appear.

Specifically, for an undirected graph with n nodes in which each of the $\frac{1}{2}n(n-1)$ possible edges exists with probability p, so that the average number of edges leaving a given node is $\mu = (n-1)p$, a transition occurs when this average number becomes unity. Below this value the graph typically consists of small clusters, mainly isolated points and trees. At $\mu = 1$ a *giant cluster* appears which grows rapidly when μ is increased further [3, 8, 16]. As in the case of the trees, this is the point where the average cluster size becomes infinite. Note however, that the simple intuitive understanding of the result no longer applies. Intuitions suggest that when there is one link per node, one obtains a graph consisting mostly of isolated pairs of joined nodes, therefore suggesting that the transition requires a larger number of links per node. Instead the transition is rigorously known to occur when there is one link per node on the average i.e. $\mu = 1$.

Furthermore, other characteristics of the graph, such as the length of paths, diameters of the clusters, connectivity and planarity, also exhibit sharp transitions. These transitions can also be seen in other combinatorial objects such as directed graphs, colored or weighted graphs, multigraphs and lattices of various sorts when discussing their average properties. This is another reflection of the very general nature of this phenomenon in systems with many components.

3. Spreading Activation

A more complicated and interesting case is provided by phase transitions in large network based systems. These are used both in modeling human cognitive processes [1] and in artificial intelligence applications such as semantic nets [18, 20], constraint propagation and truth maintenance [7]. By considering the simple example of spreading activation nets [1] we will illustrate the kinds of phase transitions to be expected in these systems.

These spreading activation networks consist of a set of nodes representing various potentially active states, with weighted links between them. These weights determine how much the activation of a given node directly affects others. The activity of a node is used to encode either the current belief in the validity of its contents, or its importance for further analysis. The behavior of these networks is thus controlled by two parameters, one specifying their topology, and the other describing local interactions, which are the weights on the links.

In a typical application, some nodes are initially activated by external inputs. These nodes in turn cause others to become active with varying intensities,

leading to complicated dynamics characterized by a spatio-temporal modulation of the total net activity. This raises two general questions concerning the behavior of the net. The first one concerns its general dynamical behavior and its approach to equilibrium. In this context one important consideration is the rate at which equilibrium is reached and how it compares with the characteristic times with which inputs change at the sources. The second question deals with the fraction of the net that partakes in its activity, which determines the extent to which the far regions of the net influence the parts under consideration. It turns out that there exist several operating regimes which are separated by sharp boundaries. These phase transitions, which depend on both the topological connectivity and the ratio of excitation to relaxation in the networks, lead to a phase diagram which we now elucidate.

3.1. Activation net dynamics

In order to exhibit the type of phase transition expected in spreading activation nets, we consider the simple network with undirected weighted links. The net consists of n nodes connected by a set of undirected links and it is characterized by three parameters. The first one is the average number of links per node, μ, which specifies its topology. The second is the relaxation rate with which the activity of an isolated node decays to zero, denoted by γ, which ranges between 0 and 1. Finally, there is the amplification, α, which controls the relative amount of activity that flows from a node to all of its neighbors per unit time.

We specify the dynamics in discrete time steps. Let $\boldsymbol{A}(N)$ be a vector whose ith element is the activation of the ith node at time step N, and let $\boldsymbol{C}(N)$ be a vector whose elements specify the external source at the same time step. Using the standard model of activation plus relaxation, one can write the time evolution of the net as

$$\boldsymbol{A}(N) = \boldsymbol{C}(N) + \boldsymbol{M}\boldsymbol{A}(N-1)\,, \tag{4}$$

where $\boldsymbol{M}$ is a matrix determined by the net connectivity. It can be written as

$$\boldsymbol{M} = (1-\gamma)\boldsymbol{I} + \alpha\boldsymbol{R}\,,$$

where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{R}$ is a matrix with zero diagonal elements and whose off-diagonal element R_{ij} is the weight of the link from node j to node i, which is defined to be zero for nonexisting links. This weight determines how much of the activation at node j spreads to node i in a single step. Since the links are undirected, R_{ij} is nonzero if and only if R_{ji} is nonzero. Note the R_{ij} need not be symmetric. We can also define the total activation of the net, $T(N)$, by the sum of the elements of the vector $\boldsymbol{A}(N)$.

In general, the net will consist of a number of distinct connected components

and the activity in each component will be independent of the others. This implies that by suitably relabeling the nodes the matrix $\boldsymbol{M}$ can be cast into block-diagonal form, where each block $\boldsymbol{m}^{(k)}$ corresponds to a single cluster. For example, if there were three independent components of the net, one would obtain a matrix of the form

$$\boldsymbol{M} = \begin{pmatrix} \boldsymbol{m}^{(1)} & & \\ & \boldsymbol{m}^{(2)} & \\ & & \boldsymbol{m}^{(3)} \end{pmatrix},$$

where all elements outside the blocks are equal to zero. Notice that the matrix $\boldsymbol{R}$ will also have this form with blocks $\boldsymbol{r}^{(k)}$. We consider the case where the activation from any node is divided among the attached nodes according to the weight of their connections. For those blocks $\boldsymbol{r}^{(k)}$ which have more than one element, this implies that the sum of the elements in each column equals 1, i.e.,

$$\sum_i r_{ij}^{(k)} = 1\,, \tag{5}$$

for every j, where the sum extends over all elements of the jth column.

Equation (4) completely determines the dynamics of the spreading activation net if the sources and the initial conditions are specified. In order to answer the two questions posed above, we will consider the specific case of a constant source, $\boldsymbol{C}(N) = \boldsymbol{C}$, applied to a single source node (e.g. node 1 so only the first element of $\boldsymbol{C}$ is nonzero) and determine the consequent spread of activity through the net. In doing so, the two relevant questions are (a) the existence of an asymptotic regime and the time if takes to reach it, and (b) the number of nodes that are significantly activated in the long time limit.

The asymptotic behavior of the net is determined by an interplay between activation and decay. For values of the activation larger than the decay per node, one expects an explosive growth in the activity. Conversely in a regime where the decay per node always exceeds its activation, the activity of the network will settle down to a finite value. A transition is then expected when the two parameters becomed equal. This argument be made mathematically precise by considering the fixed point $\boldsymbol{A}^*$ in (4), corresponding to the component of the net which contains the source. This is obtained when $\boldsymbol{A}(N) = \boldsymbol{A}(N-1)$ and leads to a linear equation for $\boldsymbol{A}^*$, namely

$$\boldsymbol{A}^* = \boldsymbol{C} + \boldsymbol{M}\boldsymbol{A}^*\,.$$

Since the vector C has only one nonzero entry, elements of A^* will be nonzero only for nodes which are in the same cluster as the source node. From (4) and (5) the total activation corresponding to a source applied to a nonisolated node evolves according to

$$T(N) = C + (1 - \gamma + \alpha)T(N-1)\,, \tag{6}$$

where C is the total source activation. Note that when the decay is larger than the amplification this approaches a fixed limit $T^* = C/(\gamma - \alpha)$. Conversely, if the decay is smaller than the amplification, the total activation grows without bound and the fixed point A^* is unstable. Equation (6) also determines the relaxation time, τ, with which this component of the net relaxes towards its asymptotic state like $\exp(-N/\tau)$ after N time steps. It is given by

$$\tau = -1/\ln(1 - \gamma + \alpha)\,. \tag{7}$$

When the system is unstable, the activity grows exponentially like $\exp(N/\tau)$, where τ is given by $\tau = +1/\ln(1 - \gamma + \alpha)$.

3.2. Phase diagram

A typical network can be simply modeled as a random graph with the constraint that μ, the average number of edges leaving a node, remains constant as the size of the net increases, i.e., it is a finite quantity. For simplicity we will consider the uniform case in which the weights on all the links leaving a node are equal. From (5) this implies that $R_{ij} = 1/\deg j$ whenever the nodes i and j are linked and where $\deg j$ is the number of links leaving node j. In this case the matrix $\boldsymbol{R}$ entering the dynamical equations is a random one whose off-diagonal elements are nonzero with probability p, which is given by $\mu/(n-1)$. The blocks $\boldsymbol{m}^{(s)}$ will correspond to the connected components of the random graph. Note that in this model any two nodes are equally likely to be connected, which contrasts with analogous physical situations where the strength of their connection weakens as the Euclidean distance between them increases.

The phase transitions that occur in these networks are shown in Fig. 4, which we now discuss. From (6) one sees that the dynamics undergoes a sharp transition when the amplification equals the decay, i.e. $\alpha = \gamma$. Above this line, the fixed point is unstable and the activation grows without bound, whereas below it the activity always reaches a stable fixed point. As in the case of random graphs [8], when $\alpha > \gamma$ there is a phase transition at $\mu = 1$ where the topology of the network suddenly changes from small isolated clusters to a giant one containing very many nodes. The consequent spread of activation can be characterized by the number of nodes whose activity is above some specified

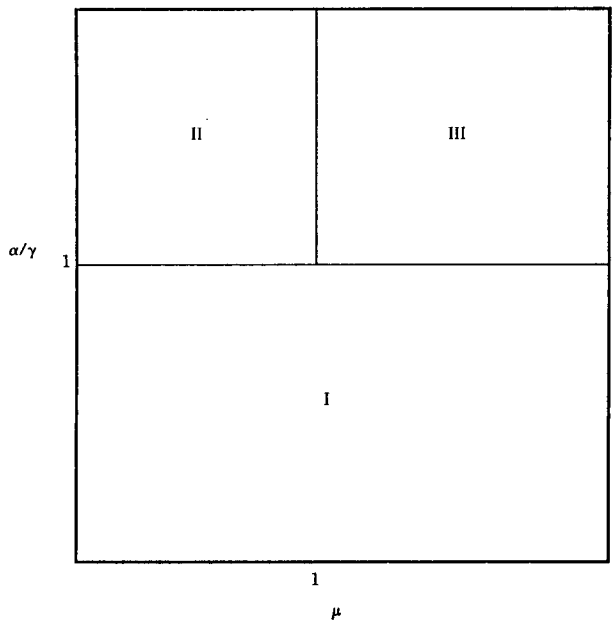

FIG. 4. Phase diagram of spreading activation net. The vertical axis represents the dynamical parameter α/γ (ratio of activation to decay) and the horizontal axis represents the connectivity parameter μ.

positive threshold. The existence of these giant clusters allows the activation to reach arbitrarily remote regions of the network.

Several regimes of this phase diagram, shown in Fig. 4, are worth pointing out. First consider the case where α/γ are small (i.e. region I). In this regime, the activation of the net relaxes quickly to its asymptotic value and it is localized in space. Furthermore, the topological transition at $\mu = 1$ has no observable effect on the spread of activation, since even for $\mu > 1$ the number of nodes activated above threshold in the infinite cluster dies out with distance from the source. This implies that events at the source node do not significantly affect the activation of the far regions of the net. Moreover, they will have little effect on the source itself at later times. Because of its limited extent in time, this dynamics can be thought of as taking place in a finite temporal cluster.

For computational purposes, the activity in this phase implies that one can effectively ignore a priori both far regions of the net as well as its ancient history. Thus, in this regime the event horizon affecting a given node at a given time is localized in both space and time. This leads to an effective equilibration process so that a net with time-varying sources will always remain near the corresponding fixed point, provided the sources do not change much in time intervals comparable to τ.

As α/γ increases towards the value 1, the overall relaxation of the net towards its stable fixed point becomes increasingly sluggish, with its characteristic relaxation time diverging to infinity as the transition is approached. This divergence, determined by (7), is given by

$$\tau = 1/\gamma\varepsilon ,$$

where the parameter ε, given by $1 - \alpha/\gamma$, measures the relative distance below the transition line. This leads to sharp phase transitions into phases II and III.

In phase II, where activity is amplified but connectivity remains small, one encounters a regime where the event horizon grows indefinitely in time but remains localized in space. This means that unlike phase I, ancient history matters in determining the activation of any node. The net reverberations never settle down (i.e., the activity keeps increasing) and the assumption of an equilibrium between the net and the time variations at the source no longer holds.

Finally in region III not only is the event horizon extended in time but also in space. This means that the amount of spreading grows indefinitely and therefore far regions of the net can significantly affect each other. In other words, everything matters. This region is separated from phase II by a sharp transition in which the number of nodes with activation values above a given positive threshold, grows explosively. This can be seen by a mean field theory argument which shows that although the activation decays with distance from the source, the individual activities of the other nodes grow like $(1 - \gamma + \alpha)^N$, showing that arbitrarily remote nodes eventually exceed threshold.

We should point out that the finite size of any real network will introduce corrections analogous to the ones discussed for the hierarchical model. In addition to the spatial smoothing out of the transition due to the finite number of nodes in the net, one would also observe a smoothing of the dynamical transition due to observations over a finite number of time steps. In particular, since near the transition the network is sluggish, meaning that observations might not last long enough to distinguish between continued activity growth and the eventual reaching of the fixed point.

The phase diagram of Fig. 4 has interesting implications for production systems which operate on spreading activation nets. As productions dynamically modify the net, changes in topology and couplings could cause it to cross any of the phase boundaries considered. This may lead to sudden changes in the operating characteristics of the whole system. Moreover, these systems have many other topological parameters [12] which suggests that a much richer phase diagram may exist in them. Examples of such parameters are provided by the number of rules affected by a change in working memory in production systems, and the number of changes to working memory made when a rule fires.

3.3. Nonlinearities, delays, and constraints

There are several ways in which our results can be generalized. The first obvious case is where the links of the network have different weights distributed according to a given probability distribution. Alternatively, the probability for two nodes to be connected could depend on some characteristic property of the nodes, or the links could be directed. Since these cases also resemble percolation processes discussed above there will appear phase transitions as the parameters are varied.

In many scenarios the physical world unavoidably introduces constraints which dominate the design of computing systems which operate at the limits of technology. A common situation in which this arises is in distributed computing systems, which are becoming pervasive due to present advances in networking techniques. In this case, the system pushes towards physical communication limits which strongly interacting parts to be placed physically close. This introduces in turn a dependence on Euclidean space and leads to models consisting of hierarchical levels with local loops. Thus the probability of finding links would depend on the physical distance between nodes. This leads to models which are intermediate between the random trees and graphs considered above and which have similar phase transitions. Moreover, the corresponding clusters will exhibit a noticeable clumpiness in real space. This is an instance of the more general issue of the consequences that physical constraints impose on information systems and remains as an unsolved problem.

Another interesting situation involves nonlinear dynamics, as well as the existence of delays. In these cases, the dynamical equations analogous to (4) will depend *explicitly* on the state of the net at much earlier times or contain nonlinear terms. Furthermore, inhibitory links (i.e., negative weights) are often used to stabilize and sharpen node activations [4]. Studies of such models show that they exhibit complicated oscillatory cycles and moving wavefronts in addition to the fixed points discussed above, as well as chaotic reverberations of the network [5]. These behaviors have time-dependent asymptotic dynamics. As a consequence, the commonly used criterion which infers equilibrium from an unchanging activity is no longer valid.

Finally we should mention novel connectionist architectures which have been used for a number of constraint satisfaction, pattern classification, and optimization problems [14, 15, 22]. Their learning algorithms, which dynamically modify the weights on the links, can lead to bifurcations from one phase to another which in turn can have qualitatively different asymptotic behavior. These nonlinear effects, which result from competition between excitation and inhibition in the network, can also lead to extremely slow collective dynamics and hence complicate the determination of the equilibrium points.

4. Conclusion

In this paper we have pointed out that by considering infinite systems and their collective behavior, the laws of large numbers naturally lead to the emergence of a small number of globally relevant parameters. This drastic collapse of so many degrees of freedom into a few new ones is conceptually similar to the appearance of quasiparticles as descriptors of collective phenomena in matter, or the even more familiar scenario of forests emerging from trees. In all these cases, it is the intermediate scale which presents the most difficult problems, and if our analogy is correct one can expect it to become more relevant as systems do indeed become extremely large.

We have also shown that large-scale AI systems can behave very differently from their small-scale counterparts. In particular, as local parameters are varied they may show sudden changes in overall performance which cannot be generally inferred from investigations of corresponding small-scale systems. Although an explicit analytic determination of the precise location of phase boundaries may be very difficult for real systems, the generic qualitative behavior we have predicted will be pervasive in many systems and observables.

A major underlying theme is the universal character of the phase transitions that we have discussed. Despite vast differences in the local interactions, and details such as the precise nature of the controlling parameters, there are many qualitative similarities between all these systems. This concept of universality also extends to other appropriately scaled variables, such as the power law behavior of the mathematical singularities near transitions. Universality is an essential feature of this study since it allows predictions to be made about complicated models based on the behavior of large but simple counterparts. This is a manifestation of a more general phenomenon in which deep underlying similarities are masked by diversity at the surface level. This approach is to be contrasted with canonical bottom-up methods, which predict the behavior of the larger system as an extrapolation from its smaller version. This methodology becomes increasingly difficult due to the rapid growth in the number of variables needed to describe the system in detail. Moreover, even if such a detailed description were obtained, one would still face the formidable task of interpreting the multitude of results in terms, of a few interesting global parameters.

A final general question concerns the possibility of other macroscopic phenomena taking place in large-scale systems and the range of problems to which they might be applicable. The existence of first-order transitions, hysteresis, and exotic phases of matter, to name a few, suggests possibilities for their application to artificial systems. Moreover, the emergence of universal phenomena such as the ones we discussed in this paper provide an alternative approach to the study and design of cognitive models and artificial intelligence.

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