Toward a Mathematical Formalization of the SB-HC4A Cosmological Model: A Recommended Approach

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Abstract

The Singularity-Bounded Holographic Class 4 Automaton (SB-HC4A; Gruber, 2026a) proposes that the universe is a self-referential holographic Class 4 automaton bounded at every scale by singularity surfaces — information-impermeable boundaries of maximum density — where the observable interior is a decompressed projection of boundary-encoded information. The model was derived from a convergence of the Five-Class computational taxonomy, a theoretical framework for self-referential computation in self-modeling systems (Gruber, 2015, 2026b, 2026c), and 't Hooft's (1993, 2016) holographic automaton interpretation — now supported by Wetterich's (2022a, 2022b, 2022c) proven mathematical equivalences between cellular automata and fermionic quantum field theories, including a cellular automaton model of spinor gravity in four dimensions with exact local Lorentz symmetry. The model currently operates in natural language supplemented by structural arguments. This paper outlines a recommended mathematical formalization strategy. Eight formalization modules are proposed: (1) a measure-theoretic definition of the five computational classes replacing Wolfram's visual classification, (2) a topological characterization of singularity boundaries as equivalence classes of Bekenstein-saturated surfaces — including their temporal asymptotic structure and the identification of heat death as a Bekenstein-saturated singularity implying cyclic dynamics, (3) a formal definition of holographic rule sets via dimensional compression operators, (4) the self-referential fixed point $\Phi(U) = U$ grounded in Lawvere's fixed-point theorem and the theory of coalgebras — extended to cyclic temporal closure with computationally irreducible cycle sequences, possible CPT signature alternation connecting to the Boyle-Turok CPT-symmetric universe model, and a generalization to branching cycles under Big Rip (Caldwell, 2002) scenarios where distributed singularity boundaries produce a multi-valued cycle map, (5) the cross-scale structural identity as a functor between categories, (6) the necessity argument in modal logic, (7) energy-information equivalence as a duality principle, and (8) the cognitive ceiling problem formalized as a computability-theoretic bound on self-knowledge. A phased build order prioritizes components that interface with existing mathematical physics. The formalization is offered

as a research program specification for mathematically trained collaborators; verification of the formal apparatus is explicitly deferred to domain experts.

Keywords: cosmology, cellular automata, holographic principle, formalization, criticality, singularity, self-referential closure, fixed-point theory, category theory, modal logic, Bekenstein bound, cyclic cosmology, CPT symmetry, conformal cyclic cosmology, Big Rip, phantom energy, branching multiverse

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1. Introduction

1.1 The Formalization Gap

The SB-HC4A cosmological model (Gruber, 2026a) makes five structural claims: (a) the universe operates at Wolfram Class 4 — the edge of chaos; (b) singularities at every physical scale (Planck regime, particle interiors, event horizons, cosmological horizons, temporal endpoints) are structurally identical instances of the same information boundary; (c) the universe is a holographic Class 4 automaton whose rules, dynamics, and output are all holographic; (d) this architecture is self-referentially closed: $\Phi(\mathrm{U}) = \mathrm{U}$; and (e) self-referential computational systems, including self-modeling cognitive systems (Gruber, 2015, 2026b), are local, scale-reduced instances of the same computational pattern.

These claims are currently expressed in natural language with structural arguments. The elimination argument for Class 4 is informal. The singularity unification is asserted by shared properties, not by a formal equivalence relation. The holographic rule set is defined verbally. The self-referential fixed point is stated but not derived. The cross-scale structural mapping is presented as a table of correspondences, not as a mathematical morphism.

Without formalization, the model's flexibility is a liability. A verbally specified "structural identity" between self-referential computation and cosmology could potentially accommodate any observation post hoc. The necessity argument — that the SB-HC4A is the unique structure consistent with the five axioms — requires formal demonstration, not verbal persuasion. And the cognitive ceiling problem (Weak Point 5 of Gruber, 2026a) — that Class 4 observers may be constitutionally incapable of determining whether they are describing the universe or the limits of their own cognition — needs formal characterization before it can be analyzed.

1.2 The Three-Paper Formalization Program

This paper is the third in a series of formalization specifications. The first (Gruber, 2026c) specified the formalization of the self-referential computation framework (Gruber, 2026b), proposing a continuous model-space framework with a model density ρ(s, ν, t), transfer entropy for permeability, criticality operationalization, ESM redirection dynamics, self-referential closure as a fixed point, and category-theoretic architecture. The second (Gruber, 2026d) specified the formalization of the Recursive Intelligence Model, proposing a domain-structured knowledge manifold, motivation as a self-referential computation functional, coupled stochastic differential equations, the ignition threshold as a stochastic bifurcation, and social coupling dynamics.

The present paper addresses the cosmological level. The three formalizations are not independent — the self-referential computation formalization provides the cognitive-side apparatus that the cosmology formalization must map onto, and the RIM formalization provides the intermediate-scale dynamics that connect self-referential representation to the learning processes that produce cosmological models. The full theoretical architecture, once formalized, forms a three-level system: cosmological substrate $\rightarrow$ self-referential computation $\rightarrow$ intelligence, with mathematical morphisms connecting each level. The self-referential computation framework draws substantially on Metzinger's (2003) self-model theory of subjectivity, and the cosmological-level seeds — the universe as a cellular automaton under holographic constraints — were already present in Gruber (2015), though the full SB-HC4A model was not developed until Gruber (2026a).

1.3 Scope and Limitations

This paper specifies a formalization research program. It proposes mathematical frameworks, defines quantities, and outlines a build order. It does *not* verify the mathematical apparatus — the author is not a mathematician, and formal verification is explicitly deferred to domain experts. The equations and formal structures presented here are intended as precise specifications of *what needs to be formalized* and *which mathematical tools are appropriate*, not as proven theorems.

The challenge is acute. The cosmological model touches mathematical physics (general relativity, quantum field theory, holographic duality), theoretical computer science (computational complexity, automata theory, Kolmogorov complexity), mathematical logic (fixed-point theorems, incompleteness, modal logic), and category theory. No single mathematician commands all these domains. The formalization is therefore designed as a modular program where each module can be developed independently by domain specialists, with category theory providing the integration language.

The paper is structured as follows. Section 2 proposes measure-theoretic definitions of the five computational classes. Section 3 formalizes singularity boundaries, including their temporal asymptotic structure, the identification of heat death as a Bekenstein-saturated singularity, and the formalization of particles as computational atoms — stable Planck-scale singularity configurations whose finiteness, discreteness, interaction grammar, and conservation laws follow from the Bekenstein bound. Section 4 defines holographic rule sets. Section 5 addresses self-referential closure, including cyclic temporal dynamics, CPT signature alternation across cycles, and the generalization to branching cycles under Big Rip scenarios. Section 6 constructs the cross-scale structural identity functor. Section 7 formalizes the necessity argument. Section 8 addresses energy-information equivalence. Section 9 formalizes the cognitive ceiling. Section 10 proposes a phased build order. Section 11 identifies what formalization would buy and what it cannot deliver.

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2. Formalizing the Five Computational Classes

2.1 The Problem with Wolfram's Classification

Wolfram's (2002) four-class taxonomy of cellular automaton behavior — static, periodic, random, complex — is empirical: classes are assigned by visual inspection of space-time diagrams. This is scientifically useful for classification but mathematically unsatisfying. The classes lack formal definitions, the boundaries between classes are imprecise, and the extension to continuous, noisy, high-dimensional systems (like the physical universe) is undefined.

The five-class refinement (Gruber, 2015, 2026a) — splitting Wolfram's Class 3 into fractal (computationally reducible) and random (computationally irreducible, maximal Kolmogorov complexity) — sharpens the taxonomy but inherits the informality. For the SB-HC4A to be a mathematical object, each class needs a formal definition.

2.2 Proposed Measure-Theoretic Definitions

Consider a discrete dynamical system $\Sigma = (S, f, s_0)$ where S is the state space, f: $S \to S$ is the transition function, and s_0 is the initial state. The orbit of s_0 under f is the sequence $O = \{s_0, f(s_0), f^2(s_0), \ldots\}$. Define the following quantities:

Topological entropy h_top(f): The exponential growth rate of the number of distinguishable orbits as resolution increases (Adler, Konheim, & McAndrew, 1965). Measures the "complexity" of the dynamics.

Kolmogorov complexity rate κ(O): The asymptotic Kolmogorov complexity per symbol of the orbit sequence. Measures the incompressibility of the output.

Lyapunov exponent λ_max: The maximum exponential divergence rate of nearby trajectories. Measures sensitivity to initial conditions.

Computational reducibility $r(\Sigma)$: Whether the state at time t can be computed in fewer than t steps. Formally: $r(\Sigma) = 1$ if there exists an algorithm A such that $A(s_0, t) = f\hat{}t(s_0)$ in time o(t); $r(\Sigma) = 0$ otherwise.

Computational universality $u(\Sigma)$: Whether the system can simulate any Turing machine. $u(\Sigma) = 1$ if Σ is Turing-complete; $u(\Sigma) = 0$ otherwise.

The five classes are then defined by their signatures on these quantities:

Class	h_top	κ(O)	λ_max	r(Σ)	u(Σ)	Informal
1	0	0	< 0	1	0	Static: converges to fixed point
2	0	0	≤ 0	1	0	Periodic: finite-period orbits
3	> 0	$<$ h_top	≤ 0	1	0	Fractal: self-similar, reducible
4	> 0	> 0	≈ 0	0	1	Complex: irreducible, universal
5	max	max	> 0	0	0	Random: maximal complexity, no structure

2.3 Key Distinctions

Class 3 vs. Class 4: Both have positive topological entropy, but Class 3 is computationally reducible (r = 1) while Class 4 is not. This captures the intuition that fractal systems, however visually complex, are "shortcuttable" — you can compute the Sierpinski triangle's state at time 10ˆ100 without running 10ˆ100 steps, because the structure is self-similar and the rule is algebraically tractable (Wolfram, 2002, p. 870). Class 4 systems resist this: Rule 110 cannot be shortcut (Cook, 2004).

Class 4 vs. Class 5: Both are computationally irreducible, but Class 4 is computationally universal (u = 1) while Class 5 is not. A truly random system cannot simulate a specific Turing machine because simulation requires deterministic control over the output — which contradicts maximal Kolmogorov complexity. Class 4 achieves maximum *structured* complexity; Class 5 achieves maximum *unstructured* complexity.

The Lyapunov signature: Class 4 has λ_max ≈ 0 (edge of chaos), which is the formal expression of Langton's (1990) observation that Class 4 lives at the boundary between order (λ_max < 0) and chaos (λ_max > 0). Class 5 has λ_max > 0 — sensitivity to initial conditions without the structured information processing that characterizes Class 4.

2.4 The Class 4 Expressibility Theorem

The cosmological model's first premise — that Class 4 is the maximum complexity achievable by expressible rules — requires a formal statement and proof sketch.

Claim (Expressibility Ceiling). Let $\Sigma = (S, f, s_0)$ be a dynamical system whose transition function f is specified by a finite description of length L (a "rule" in the Wolfram sense). Then the Kolmogorov complexity rate of the orbit satisfies:

$\kappa(O) \leq L \,/\, t \to 0$ as $t \to \infty$

That is: the long-run information rate of any finitely-specified system is zero. The orbit is always compressible to "apply rule f from initial condition s_0."

Corollary. A system with $\kappa(O) > 0$ in the infinite limit (Class 5) cannot have a finite rule specification. Class 5 requires rules of infinite Kolmogorov complexity — rules that cannot be written down.

Implication. Class 4 — computationally universal, irreducible, with structured positive-entropy dynamics — is the most complex behavior achievable by finitely specifiable systems. It is the expressibility ceiling.

This argument needs rigorous formulation by a computability theorist. The key technical challenge is making the transition from Kolmogorov complexity (defined for finite strings) to the asymptotic complexity rate of infinite orbits rigorous. The relevant tools are algorithmic information theory (Downey & Hirschfeldt, 2010) and the theory of computable measures (Zvonkin & Levin, 1970).

2.5 Extension to Continuous Systems

The physical universe is not a discrete cellular automaton — it is (at least approximately) a continuous dynamical system on a field-theoretic state space. The five-class definitions must extend to this setting.

The key observation: the five measures (h_top, κ, λ_max, r, u) are all defined for continuous dynamical systems, not just discrete automata:

- Topological entropy extends to flows (Bowen, 1971).

- Kolmogorov complexity rate extends via the theory of algorithmic randomness for real-valued sequences (Hertling & Weihrauch, 2003).
- Lyapunov exponents are standard for ODEs and PDEs (Eckmann & Ruelle, 1985).
- Computational reducibility extends to the computability of solutions: can the state at time t be computed without integrating through all intermediate times? (Pour-El & Richards, 1989).
- Computational universality for continuous systems: certain PDEs are Turing-complete (Moore, 1991; Cardona et al., 2021).

The classification table (Section 2.2) therefore applies directly to continuous systems, with the caveat that the measures are harder to compute and the class boundaries become less sharp (continuous systems may interpolate between classes in parameter space, with Class 4 occupying a critical manifold between the ordered and chaotic phases).

2.6 The Containment Hierarchy

The verbal model claims that Class 4 contains all lower classes as subprocesses. Formally:

Claim (Containment). Let Σ_4 be a Class 4 system. For each k $\in \{1, 2, 3\}$, there exists a subsystem Σ_k $\subset \Sigma_4$ and a projection π_k such that π_k(Σ_4) is Class k.

This follows from computational universality: a Turing-complete system can simulate any computable dynamics, including Class 1 (constant output), Class 2 (periodic output), and Class 3 (fractal/reducible output). The Game of Life demonstrates this concretely — it contains still lifes (Class 1), oscillators (Class 2), and self-similar growth patterns (Class 3) as embedded phenomena.

The containment is strict: no Class k system for k < 4 can contain a Class 4 subsystem (a non-universal system cannot simulate a universal one).

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3. Singularity Boundaries as Topological Objects

3.1 The Equivalence Relation

The cosmological model claims that singularities at all scales — Planck regime, particle interiors, event horizons, cosmological horizons, temporal endpoints — are instances of the same information boundary. This claim requires a formal equivalence relation.

Definition (Information Boundary). An *information boundary* is a codimension-1 surface B in a spacetime manifold M such that:

(IB1) **Information impermeability**: No signal crosses B. Formally, B is an apparent horizon: for every future-directed causal curve γ originating in the interior of B, γ does not exit the region bounded by B. (For cosmological horizons, "interior" and "exterior" reverse — but the impermeability condition holds from the observer's side.)

(IB2) **Bekenstein saturation**: The information content of B satisfies $I(B) = A(B) / (4\, l\_P^2)$, where A(B) is the surface area and l_P is the Planck length. The boundary stores the maximum information permitted by the Bekenstein-Hawking formula.

(IB3) **Computational domain bound**: B defines the boundary of a computational domain — the set of spacetime events whose states can influence and be influenced by each other through causal processes.

Definition (Boundary Equivalence). Two information boundaries B_1 and B_2 are *equivalent* ($B_1 \sim B_2$) if they satisfy the same three conditions (IB1–IB3) with identical information-theoretic properties up to scale:

$I(B_1) / A(B_1) = I(B_2) / A(B_2) = 1 / (4\, l\_P^2)$

This is an equivalence relation (reflexive, symmetric, transitive).

3.2 The Singularity Inventory as an Equivalence Class

The cosmological model's claim becomes: all entries in the singularity inventory (Gruber, 2026a, Section 5.1) belong to the same equivalence class under ~. Specifically:

Singularity	IB1	IB2	IB3
Planck regime	No sub-Planck measurement possible	Maximum information density at Planck scale	Bounds the computational domain from below
Particle "interiors"	Interiors inaccessible	Claimed: Planck-scale Bekenstein saturation	Particles are atomic computational units
Event horizons	No signal escapes	Bekenstein-Hawking entropy $= A/4l\_P^2$	Bounds the causally connected region
Cosmological horizon	No signal from beyond	Gibbons-Hawking entropy $= A/4l\_P^2$	Bounds the observable universe
Big Bang	No "before" accessible	Maximum density/temperature $\rightarrow$ maximum information density	Bounds the temporal computational domain (past)

Singularity	IB1	IB2	IB3
Heat death	Infinite dilution $\rightarrow$ no distinguishable signals	Entropy $\rightarrow$ Bekenstein saturation (Section 3.4)	Bounds the temporal computational domain (future)
Big Crunch	Reconvergence $\rightarrow$ causal disconnection	Maximum density $\rightarrow$ Bekenstein saturation	Bounds the temporal computational domain (future)
Big Rip	Diverging expansion $\rightarrow$ mutual causal disconnection of all regions	Each fragment reaches Bekenstein saturation	Fragments the computational domain into disjoint bounded regions

The strongest entries are event horizons (all three conditions established) and the cosmological horizon (Gibbons & Hawking, 1977). The Big Rip (Caldwell, 2002) presents a novel topology: a distributed terminal boundary consisting of many disjoint Bekenstein-saturated surfaces rather than a single connected one (formalized in Section 5.8). The weakest is the particle interior claim, which is a prediction of the model rather than an established result.

3.3 Temporal Asymptotic Structure of Singularity Boundaries

The singularity inventory (Section 3.2) treats boundaries as primarily *spatial* information barriers — surfaces in a spacelike hypersurface through which no signal passes. But singularity boundaries are equally *temporal* information barriers, and this temporal structure strengthens the structural identity claim.

Definition (Temporal Unreachability). An information boundary B is *temporally asymptotically unreachable* from within its computational domain D if, for every causal curve $\gamma \subset$ D parameterized by proper time τ, the approach to B is asymptotic:

lim_{$\tau \rightarrow \tau$_B} d($\gamma(\tau)$, B) = 0, but $\gamma(\tau) \notin$ B for any finite τ

where d is the spacetime distance measure appropriate to the geometry and τ_B may be $\pm\infty$ in a suitably chosen time coordinate.

Application to the Big Bang. In standard FLRW cosmology, the Big Bang is not a point in spacetime but a boundary at t $\rightarrow 0^+$. In conformal time η, defined by dη = dt/a(t) where a(t) is the scale factor, a radiation-dominated universe gives $\eta \rightarrow -\infty$ as t $\rightarrow 0^+$. The Big Bang maps to the infinite past in conformal time — it is asymptotically unreachable in backward time evolution. This is structurally identical to the way a black hole event horizon is unreachable in finite Schwarzschild coordinate time from an external observer: both boundaries recede infinitely under the natural time parameterization of an interior observer.

Application to the Big Crunch. A hypothetical Big Crunch singularity (the time-reverse of the Big Bang in a recollapsing universe) shares the same asymptotic unreachability. Forward temporal evolution toward a crunch singularity diverges: in conformal time for a recollapsing FLRW model, $\eta \to +\infty$ as t $\to$ t_crunch. The crunch is as temporally unreachable from within as the bang is from within. The two temporal boundaries are dual — the bang bounds the computational domain in the past, the crunch in the future, and both are asymptotically inaccessible from the interior.

Generalization. The temporal unreachability property generalizes across the singularity inventory. For event horizons, the well-known result that an infalling object crosses the horizon in finite proper time but takes infinite coordinate time (as measured by a distant observer) is the spatial analogue. For cosmological horizons, the de Sitter horizon is temporally unreachable: objects approach it asymptotically but never cross it from the interior perspective. The claim is:

Proposition (Universal Temporal Unreachability). All information boundaries B $\in$ B satisfying conditions IB1–IB3 (Section 3.1) are temporally asymptotically unreachable from within their associated computational domain. That is, temporal unreachability is not an additional condition but a *consequence* of IB1–IB3 applied to causal structure.

This strengthens the structural identity claim (Section 3.2): singularity boundaries are not merely surfaces of maximum information density but temporal horizons that recede infinitely under any attempt to reach them from within the computational domain. The impossibility of "reaching" a singularity boundary is not a contingent physical fact but a structural feature of any information-impermeable, Bekenstein-saturated surface in a causally structured spacetime.

3.4 Heat Death as Bekenstein-Saturated Singularity

The singularity inventory (Table, Section 3.2) lists heat death as a future temporal boundary but leaves its IB2 status (Bekenstein saturation) implicit. A closer analysis reveals that heat death satisfies IB2 — and that this has profound consequences for the model's temporal structure.

Argument. As the universe approaches thermodynamic equilibrium (heat death), entropy approaches its maximum value for the given volume. In a universe with a cosmological horizon of area A_H, the maximum entropy is the Gibbons-Hawking entropy S_max = A_H / (4l_P^2). This is precisely the Bekenstein bound for the observable universe. At heat death, the system reaches Bekenstein saturation — the information content of the computational domain equals the information capacity of its boundary.

Definition (Entropy Saturation Boundary). A temporal boundary B_t at time t_∞ is an *entropy saturation boundary* if:

lim_{t $\to$ t_∞} S(t) / S_Bekenstein(∂D) = 1 where S(t) is the total entropy within the computational domain D and S_Bekenstein(∂D) = A(∂D) / (4l_P^2) is the Bekenstein capacity of the domain boundary.

Heat death satisfies this condition. It therefore satisfies IB2 (Bekenstein saturation) in addition to IB1 (no signal crosses the boundary — there are no distinguishable signals in a maximum-entropy state) and IB3 (computational domain bound — no further computation is possible when all degrees of freedom are thermalized). Heat death is a full information boundary under the definitions of Section 3.1.

Implication for the self-referential closure. In the SB-HC4A framework, singularity boundaries are not merely barriers but transformation surfaces: they separate compressed (boundary-encoded) and decompressed (interior) representations of the same information (Section 4). If heat death is a singularity boundary, it is a transformation surface. The information of the universe, having reached maximum entropy (maximum decompression), encounters its own Bekenstein-saturated boundary — and the singularity transforms it back into a compressed representation. This is not thermodynamic reversal but *information-theoretic transformation*: the same information passes through the boundary from the decompressed (interior) to the compressed (boundary) representation.

The self-referential fixed point Φ(U) = U (Section 5.1) thereby acquires a temporal dimension. The universe does not merely compute its own output at each instant — it computes its own *restart*. The fixed point is not static but *cyclic*: Φ maps the heat-death boundary state to a compressed boundary state that decompresses into a new expansion phase. The cycle is:

expansion → entropy increase → Bekenstein saturation → boundary compression → decompression → expansion

This cycle is the temporal expression of the self-referential closure: the universe, upon reaching its own information boundary, transforms through it and reconstitutes its computational domain.

3.5 The Topological Characterization

Define the **boundary space** B as the set of all information boundaries in the spacetime manifold M. The equivalence relation ~ partitions B into equivalence classes. The model claims there is a single class: [B]_~ = B — all information boundaries are equivalent.

The boundary space inherits a topology from M. The scale-invariance claim becomes:

Claim (Scale Invariance of Boundary Structure). The boundary space B, equipped with the topology inherited from M, is connected. Moreover, there exists a continuous scaling map σ_λ: B → B for each $\lambda > 0$ such that σ_λ(B) ~ B for all B ∈ B and all λ.

This states that the boundary structure is self-similar: zooming in or out on a boundary produces another boundary in the same equivalence class. This is the formal expression of the model's claim that singularities at all scales share the same information-theoretic character.

3.6 Connection to Existing Mathematics

The formalization connects to several established programs:

Black hole thermodynamics (Bekenstein, 1973; Hawking, 1975): The Bekenstein-Hawking entropy formula $S = A / (4l\_P^2)$ is the paradigm case of IB2. The generalized second law — that the total entropy of matter plus the black hole boundary never decreases — is a conservation law across the boundary.

The holographic principle ('t Hooft, 1993; Susskind, 1995; Bousso, 2002): The covariant entropy bound $S(L) \leq A(B(L)) / (4l\_P^2)$ for any light sheet L bounds the entropy of a region by its boundary area. This is the general statement of IB2.

Causal structure in general relativity (Penrose, 1979; Hawking & Ellis, 1973): The theory of causal boundaries, event horizons, and apparent horizons provides the mathematical framework for IB1. The key tools are Penrose-Carter diagrams, the notion of a trapped surface, and the theory of causal sets.

Loop quantum gravity (Rovelli, 2004; Ashtekar & Lewandowski, 2004): Area quantization in LQG — the result that area comes in discrete quanta proportional to l_P^2 — provides a natural interpretation of the Planck-scale boundary as a discrete information-storage surface. Each quantum of area stores approximately one bit.

The technical challenge is proving that the equivalence relation ~ is nontrivial — that the Planck-scale boundary, event horizons, and cosmological horizons demonstrably satisfy the same formal conditions, not merely analogous ones. This requires tools from quantum gravity that do not yet exist in complete form.

3.7 Particles as Computational Atoms

The singularity inventory (Section 3.2) includes particle interiors as information boundaries. This section formalizes the consequences: if particles are Planck-scale singularity boundaries, they are the irreducible computational units — *computational atoms* — of the SB-HC4A.

Definition (Computational Atom). A *computational atom* is a stable configuration of a Planck-scale singularity boundary. Formally, let B_P denote a singularity boundary of area $A \sim l\_P^2$. A computational atom is a state $\sigma \in \Sigma(B\_P)$ — the state space of boundary configurations — such that:

(CA1) **Bekenstein capacity**: The state σ encodes information $I(\sigma) \leq I\_max = A(B\_P) / (4\ l\_P^2) \sim O(1)$ bits.

(CA2) **Dynamical stability**: σ is a fixed point or limit cycle of the boundary dynamics induced by the ambient Class 4 automaton. That is, if F: Σ(B_P) → Σ(B_P) is the local transition function restricted to the boundary, then Fˆn(σ) = σ for some finite n $\geq$ 1 (with n = 1 for static configurations).

(CA3) **Propagation**: σ is transportable through the computational domain — the boundary configuration can be displaced in spacetime while maintaining its identity (structural stability under spatial translation within the automaton lattice).

Proposition (Finiteness of the Particle Spectrum). The number of computational atoms is finite.

Proof sketch. By CA1, each computational atom encodes at most I_max ~ O(1) bits of information. The number of distinguishable states on a finite-capacity boundary is at most 2ˆ{I_max}. By CA2, only the dynamically stable subset of these states qualifies as computational atoms. Since 2ˆ{I_max} is finite and the stable subset is a (possibly proper) subset, the number of computational atoms is finite. □

Remark. This provides a structural explanation for the finite particle spectrum of the Standard Model: the twelve fundamental fermions, four gauge bosons, and the Higgs boson are the complete set of states satisfying CA1–CA3. The specific number is determined by the detailed dynamics F of the Planck-scale automaton, which is not specified by this formalization. The finiteness, however, follows from CA1 alone and is independent of the specific dynamics.

Definition (Quantum Number as Boundary Label). A *quantum number* is a function q: Σ_stable(B_P) → Z (or Z/2, or a finite group) that assigns to each computational atom a discrete label invariant under the boundary dynamics F. Formally, q(σ) = q(F(σ)) for all stable σ.

Remark. Charge, spin, isospin, color charge, baryon number, and lepton number are instances of this definition. Their discreteness follows from the finiteness of Σ_stable(B_P): a function from a finite set to a totally ordered set has a discrete range. The quantization of physical properties is therefore a consequence of the Bekenstein bound at the Planck scale, not an additional postulate.

Definition (Computational Atom Interaction). An *interaction* between computational atoms $\sigma_1 \in$ Σ_stable(B_1) and $\sigma_2 \in$ Σ_stable(B_2) is an information exchange event in which the boundary configurations are jointly transformed:

T: Σ_stable(B_1) × Σ_stable(B_2) → Σ_stable(B_1') × Σ_stable(B_2') × … × Σ_stable(B_k')

where k $\geq$ 1 (the number of output computational atoms may differ from the number of inputs, as in pair production or annihilation). The transformation T satisfies:

(INT1) **Information conservation**: The total boundary-encoded information is preserved: Σ_i I(σ_iˆ{in}) = Σ_j I(σ_jˆ{out}).

(INT2) **Quantum number conservation**: For each conserved quantum number q, the total is preserved: Σ_i q(σ_i^{in}) = Σ_j q(σ_j^{out}).

(INT3) **Consistency with ambient dynamics**: T is induced by the Class 4 transition function of the ambient automaton restricted to the local region containing both boundaries.

Existence proof from Wetterich's CA↔QFT equivalences. The computational atom formalization (CA1–CA3) is not purely speculative. Wetterich (2022a, 2022b) demonstrated that large classes of reversible cellular automata on space-lattices are exactly equivalent to discretized fermionic quantum field theories via Grassmann functional integral mappings. The equivalence is not approximate — it is a proven mathematical identity. Key results directly relevant to the computational atom formalization:

- *Fermionic spectra from automata*: Specific automata produce specific interacting fermion theories (Thirring model, Gross-Neveu model) with both abelian and non-abelian continuous symmetries (Wetterich, 2022a). This demonstrates that CA1 (finite information capacity) and CA2 (dynamical stability) yield discrete particle spectra in practice, not just in principle.
- *Local gauge symmetry*: Some automata models realize local gauge symmetries — the structure the Standard Model requires (Wetterich, 2022a). This provides a concrete mechanism for the interaction grammar (INT1–INT3): the selection rules at vertices are determined by the gauge symmetry that the automaton's structure generates.
- *Spinor gravity in 4D*: Wetterich (2022c) explicitly constructed a cellular automaton that represents spinor gravity in four dimensions, with exact local Lorentz symmetry on the discrete level and emergent diffeomorphism symmetry in the continuum limit. This is a computational atom model of gravity — not a metaphor.
- *Quantum mechanics from classical statistics*: The probabilistic description of a classical automaton (probability distribution over initial bit configurations) is mathematically equivalent to quantum mechanics — wave functions, density matrices, and non-commuting operators all arise from the automaton's classical structure (Wetterich, 2022d). This grounds the formalization's implicit assumption that quantum-mechanical particle properties emerge from classical automaton dynamics.

The Grassmann functional integral mapping provides a concrete formalization tool for Module 3 (Section 10): rather than formalizing computational atoms purely from the abstract Bekenstein bound, one can use Wetterich's mapping to construct specific automata satisfying CA1–CA3 and verify that their stable configurations match known particle properties. The approach is complementary to the top-down SB-HC4A strategy: Wetterich constructs automata *from* known QFTs (reverse-engineering the automaton from the field theory), while the SB-HC4A constrains the automaton class *from above* (the universality class is determined by axioms). The meeting point — automata that satisfy the SB-HC4A's Class 4 and holographic constraints and are then checked against

Wetterich's QFT equivalences for Standard Model-like particle content — is the natural next step.

Remark. Feynman diagrams are graphical representations of interaction sequences. Each line represents a propagating computational atom (a boundary configuration satisfying CA1–CA3). Each vertex represents an application of the interaction transformation T. The selection rules of quantum field theory — which vertices are permitted — are the constraints imposed by INT1–INT2 on the transformation T. In this formalization, Feynman diagrams are diagrams of computation: the computational grammar of the Planck-scale automaton.

Proposition (Conservation Laws from Bekenstein Bound). The conservation laws of particle physics (charge conservation, baryon number conservation, lepton number conservation) are consequences of information conservation at singularity boundaries (INT1) combined with the identification of quantum numbers as boundary labels.

Proof sketch. By INT1, total boundary-encoded information is conserved in any interaction. Quantum numbers, as functions of boundary states (Definition above), partition this information into independently conserved sectors. A quantum number q is conserved if and only if T preserves the q-labeled component of the boundary information. Since the Bekenstein bound constrains total information and the dynamics F preserve boundary structure (CA2), any label that is invariant under F is necessarily conserved under interactions induced by F (INT3). The specific conservation laws correspond to the specific symmetries of the Planck-scale boundary dynamics. □

Conjecture (Three Generations from Class 4 Self-Similarity). The three generations of Standard Model fermions — (e, μ, τ), (u, c, t), (d, s, b), and their neutrino partners — arise from a self-similar (Class 3) hierarchical structure in the space of stable singularity boundary configurations Σ_stable(B_P).

Motivation. Class 4 systems contain Class 3 (self-similar, fractal) dynamics as a subprocess (Gruber, 2026a, Section 2.5). If the ambient Class 4 dynamics impose a self-similar structure on the configuration space Σ(B_P), the stability condition CA2 may be satisfied at multiple energy scales by the same structural configuration type — producing copies of a particle type at different mass scales. Formally, if there exists a scaling operator S_λ: Σ(B_P) → Σ(B_P) for discrete scale factors λ_1, λ_2, λ_3 such that S_{λ_i}(σ) ∈ Σ_stable for i = 1, 2, 3 whenever σ ∈ Σ_stable, this would produce exactly three copies of each stable configuration.

Status. This is explicitly a conjecture, not a derivation. The number three is not predicted by the general argument — it would require a detailed analysis of the stability conditions under scaling in the specific Planck-scale dynamics, which are not available. The conjecture is structurally motivated by the Class 4 containment of Class 3 self-similarity but remains unproven. It is included because the three-generation structure is otherwise entirely unexplained within the Standard Model framework, and the self-similar structure of Class 4 dynamics provides a natural candidate mechanism.

4. Holographic Rule Sets

4.1 The Informal Definition

The cosmological model distinguishes three relationships between holographic systems and Class 4 automata (Gruber, 2026a, Section 6.1):

1. A holographic substrate produces Class 4 dynamics.
2. A Class 4 automaton produces holographic output.
3. A Class 4 automaton whose rule structure is itself holographic.

Relationship 3 — the holographic rule set — is the most novel and the least well-defined. What does it mean for a dynamical system's rules to be "holographic"?

4.2 Dimensional Compression Operators

Definition (Holographic Rule Set). A rule set R for a dynamical system on a d-dimensional manifold Mˆd is *holographic* if there exists a dimensional compression operator C: Rˆ(d) → Rˆ(d-1) such that:

(HR1) **Compression**: The full d-dimensional rule Rˆ(d) can be faithfully encoded in a (d-1)-dimensional specification Rˆ(d-1) = C(Rˆ(d)), with no information loss.

(HR2) **Local decompression**: The d-dimensional dynamics at any point x ∈ Mˆd can be recovered from Rˆ(d-1) restricted to the (d-1)-dimensional boundary ∂Mˆd in a neighborhood of x. Local application of Rˆ(d-1) at each boundary site implicitly computes global (non-local) relationships in the interior.

(HR3) **Universality preservation**: If Rˆ(d) defines Class 4 dynamics on Mˆd, then Rˆ(d-1) on ∂Mˆd also defines Class 4 dynamics (the compression preserves computational universality).

4.3 Connection to AdS/CFT

The definition of a holographic rule set is directly modeled on the AdS/CFT correspondence (Maldacena, 1998), which provides the most concrete example:

- A gravitational theory in (d+1)-dimensional anti-de Sitter space (the "bulk" — Rˆ(d+1))
- is fully equivalent to a conformal field theory on the d-dimensional boundary (the boundary theory — Rˆ(d))
- with the boundary theory encoding all bulk physics (HR1)
- and local boundary operators reconstructing bulk observables in their causal wedge (HR2; Ryu & Takayanagi, 2006; Dong, 2016)

The SB-HC4A model generalizes this: the universe's rule structure is holographic not because of a specific AdS/CFT duality but because any Class 4 system at every scale has its information encoded on its boundary (the singularity surface) with the interior dynamics being a decompression of the boundary encoding.

4.4 The Rule Set as a Coalgebra

A more abstract characterization uses coalgebra theory (Rutten, 2000), which provides the natural mathematical language for systems whose behavior unfolds over time from a specification:

Define the state space X and a functor F: Set $\to$ Set that describes the system's "type of behavior" (what can be observed at each step). A coalgebra for F is a pair (X, α: X $\to$ F(X)). The rule set R is the coalgebra structure map α — it specifies how the system's state determines its next observable behavior.

A **holographic coalgebra** would then be a coalgebra (X, α) together with a boundary functor B: Set $\to$ Set and a natural transformation η: F $\Rightarrow$ B $\circ$ G such that:

- G is a "restriction to boundary" functor
- η encodes the full coalgebra dynamics in the boundary restriction
- The coalgebra can be recovered from the boundary restriction

This is highly abstract and likely requires substantial development by a category theorist. The value of the coalgebraic formulation is that it makes the holographic property a *structural* property of the system's specification — not a property of any particular state or trajectory, but of the rule structure itself.

4.5 Holographic Automata in the Literature

't Hooft (2016) proposed the Cellular Automaton Interpretation of quantum mechanics, in which quantum mechanics is emergent from deterministic automaton dynamics at the Planck scale. The holographic rule set, in 't Hooft's framework, is the automaton rule at the Planck scale that produces quantum behavior as an effective description at larger scales. The dimensional compression (HR1) corresponds to the renormalization group flow from Planck-scale to observable-scale physics — each step in the RG flow "compresses" the full dynamics into an effective description at a coarser scale.

Verlinde's (2011) entropic gravity proposal — that gravity is not a fundamental force but an entropic phenomenon arising from the holographic distribution of information on screens — provides another concrete example of a holographic rule set: the gravitational "rule" (Newton's law, or its relativistic generalization) is derived from the information content of holographic screens.

Wetterich (2022a, 2022b, 2022c) provided the most concrete examples to date of cellular automata functioning as holographic rule sets. His Grassmann functional integral mapping demonstrates that specific reversible automata encode specific fermionic quantum field theories — the automaton rule at the discrete level compresses into the QFT description at the continuum level, satisfying HR1 (compression without information loss). The continuum QFT can be recovered from the automaton's probabilistic description restricted to its boundary conditions, satisfying HR2 (local decompression). Whether HR3 (universality preservation) holds — whether the automata that produce interacting QFTs are themselves Class 4 — is an open question that connects Wetterich's program directly to the SB-HC4A formalization. The spinor gravity automaton (Wetterich, 2022c) is particularly relevant: it implements a holographic rule set for gravity, with the discrete automaton rule compressing into the diffeomorphism-invariant continuum theory via a concrete dimensional compression operator.

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5. Self-Referential Closure

5.1 The Fixed-Point Condition

The model's central formal claim is self-referential closure:

$\Phi(U) = U$

where U is the SB-HC4A (the universe) and Φ is the "compute the output" operator. The holographic rules encode the system; the Class 4 dynamics decompress the encoding; the holographic output re-encodes the result; the computation and the system are the same thing.

This is a fixed-point statement. The mathematical program is to define Φ rigorously and prove (or specify the conditions under which) a fixed point exists.

5.2 Lawvere's Fixed-Point Theorem

Lawvere's (1969) fixed-point theorem generalizes the diagonal arguments of Cantor, Gödel, and Turing into a single category-theoretic statement: In a Cartesian closed category C, if there exists a point-surjective morphism e: A → B^A (where B^A is the exponential object), then every endomorphism f: B → B has a fixed point.

The cosmological application: Let C be the category of computable dynamical systems. Let A = the set of specifications (rule sets) and B = the set of dynamical behaviors (orbits). A point-surjective morphism e: A → B^A means "every map from specifications to behaviors can be specified" — i.e., the system is expressive enough to encode the relationship between rules and dynamics.

If the SB-HC4A is computationally universal (Class 4, u = 1), it can simulate any computable dynamics — including its own. The point-surjectivity condition is therefore satisfied. By Lawvere's theorem, every endomorphism on the behavior space has a fixed point. In particular, the "compute and re-encode" operator Φ has a fixed point: $\Phi(U) = U$.

5.3 Coalgebraic Fixed Points

An alternative formalization uses the theory of final coalgebras (Aczel & Mendler, 1989; Barr, 1993). For a functor F, the final coalgebra — if it exists — is the terminal object in the category of F-coalgebras. It has the property that it is the unique (up to isomorphism) coalgebra that is "maximally consistent": it contains all possible behaviors and is a fixed point of the coalgebra-building process.

Define F as the functor capturing the SB-HC4A's behavior type: F(X) = (holographic boundary information) $\times$ (Class 4 interior dynamics). The final F-coalgebra, if it exists, would be the unique self-consistent SB-HC4A: the system that is its own final coalgebra — whose behavior IS its specification.

The fixed-point condition $\Phi(U) = U$ becomes: U is the final coalgebra for the SB-HC4A behavior functor. This is a stronger claim than merely "a fixed point exists" — it claims that U is the *unique* self-consistent system of this type.

5.4 Connection to the Self-Referential Computation Fixed Point

The self-referential computation formalization (Gruber, 2026c, Section 6.3) defines the self-modeling fixed point as: Φ_c(m*) = m*, where Φ_c is the "model of" operator and m* is the ESM state at which the model and the modeled coincide. The cosmological model claims this is the same formal structure at a different scale.

To make this precise, define:

- Φ_cosmo: U_cosmo → U_cosmo (the cosmological "compute the output" operator)
- Φ_SRC: M_ESM → M_ESM (the self-referential computation "model of" operator)

The structural identity claim is that there exists a structure-preserving map (a functor; see Section 6) that maps Φ_cosmo to Φ_SRC. Both operators are self-referential endomorphisms on their respective domains, and both have fixed points. The claim is that the fixed-point structure is isomorphic — not merely analogous.

5.5 Inexpressibility as a Formal Consequence

Gödel's first incompleteness theorem (Gödel, 1931): Any consistent formal system F that is sufficiently expressive (can represent arithmetic) contains statements that are true but not provable in F.

The SB-HC4A is computationally universal (Class 4), hence sufficiently expressive. If the SB-HC4A is self-referentially closed ($\Phi(U) = U$), then U is a formal system that contains itself as a subsystem. By Gödel's theorem, there exist truths about U that cannot be proven from within U.

More specifically, the **Weltformel** (world equation) — a complete specification of U — cannot be a statement within U, because U's own self-referential structure guarantees the existence of truths about U that U cannot prove. The "world equation" is therefore not an equation but the process U itself — it can only be expressed by running it.

This connects to Chaitin's (1966) extension of Gödel: no formal system of complexity K can prove theorems about systems of complexity greater than K + c (for a constant c). Since U is at least as complex as any internal formal system, no internal specification can fully capture U.

5.6 Cyclic Dynamics and Temporal Self-Referential Closure

The heat-death analysis of Section 3.4 implies that the self-referential fixed point $\Phi(U) = U$ has a cyclic temporal structure. This section formalizes the cyclic dynamics and their consequences.

Definition (Cosmic Cycle). A *cosmic cycle* is a maximal connected computational domain D_n bounded by two temporal singularity boundaries: an initial boundary B_n^- (expansion onset) and a terminal boundary B_n^+ (Bekenstein saturation — heat death, Big Crunch, Big Rip fragmentation, or equivalent). The self-referential closure operator Φ maps the terminal state to the initial state of the successor cycle (or cycles, in the branching case; see Section 5.8):

Φ: $B_n^+ \rightarrow B_{n+1}^-$

where the information content is conserved: $I(B_n^+) = I(B_{n+1}^-)$. The compressed boundary representation at B_n^+ decompresses into the interior dynamics of D_{n+1}.

Proposition (Cyclic Fixed Point). The fixed-point condition $\Phi(U) = U$, when applied to the temporal sequence of cycles, yields:

$U = \bigcup_{n \in \mathbb{Z}} D_n$

with Φ acting as the transition operator between consecutive cycles. The universe as a whole (the union over all cycles) is the fixed point; each individual cycle is a single iteration of the self-referential computation. The sequence $\{D_n\}_{n \in \mathbb{Z}}$ is bi-infinite — there is no first or last cycle, because the fixed-point condition requires the sequence to be invariant under Φ, which precludes boundary terms.

Alternation of cycle types. The terminal boundary B_n^+ need not be of the same physical type in every cycle. Possible terminal singularities include heat death (entropy saturation at quasi-infinite dilution), Big Crunch (entropy saturation at maximum contraction), Big Rip (fragmentation into distributed Bekenstein-saturated surfaces; see Section 5.8), or other Bekenstein-saturated configurations. The sequence of cycle types — (expansion → heat death), (expansion → crunch), (expansion → Big Rip), or other configurations — is not constrained to be periodic or predictable.

Theorem (Computational Irreducibility of Cycle Sequence). Let $T = \{t_n\}$*$\{n \in \mathbb{Z}\}$ be the sequence of cycle types, where $t_n \in$ {heat death, crunch, Big Rip, . . . }. If the SB-HC4A is a Class 4 system, then T is computationally irreducible: there exists no algorithm that computes t*$\{n+k\}$ from t_n in fewer than k applications of Φ.

Proof sketch: By the Class 4 characterization (Section 2.2), the system is computationally irreducible ($r(\Sigma) = 0$). The cycle-type sequence T is a coarse-grained projection of the full dynamics. If T were computationally reducible (predictable by a shortcut), then the projection would provide a reducibility channel for the full system — contradicting $r(\Sigma) = 0$ (unless the projection loses all dynamically relevant information, in which case T is trivially predictable because it is trivially uninformative). For a non-trivial cycle-type taxonomy, T inherits the irreducibility of the underlying dynamics. ☐

This irreducibility is consistent with the Class 4 character of the SB-HC4A: no shortcut predicts the next cycle type without running the full computation through the current cycle. The future is genuinely open at the level of cosmic cycles, just as it is at every other scale of Class 4 dynamics.

5.7 CPT Signature Alternation

The cyclic dynamics of Section 5.6 raise a natural question: what discrete symmetry properties does the transition operator Φ: $B_n^+ \to B_{n+1}^-$ preserve? The SB-HC4A framework provides a specific answer.

Observation. The singularity boundary transformation between compressed and decompressed information representations is not required to preserve the CPT (charge-parity-time) orientation of the interior dynamics. The information content is conserved ($I(B_n^+) = I(B_{n+1}^-)$), but the *sign conventions* — which configurations count as "matter" vs. "antimatter," "left" vs. "right," "forward" vs. "backward" in time — are properties of the decompressed interior representation, not of the compressed boundary encoding.

Definition (CPT Signature). The *CPT signature* $\sigma_n \in \{+1, -1\}$ of a cycle D_n is the orientation of the CPT transformation relative to a fixed (but arbitrary) reference convention. A cycle with $\sigma_n = +1$ has the same matter/antimatter labeling as the reference; $\sigma_n = -1$ has the conjugate labeling.

Conjecture (CPT Alternation). The transition operator Φ may flip the CPT signature between consecutive cycles:

$$\sigma_{n+1} = -\sigma_n$$

producing an alternating sequence of matter-dominated and antimatter-dominated universes: . . . , D_{-1}(antimatter), D_0(matter), D_1(antimatter), D_2(matter), . . .

Connection to existing models. This conjecture connects to the CPT-symmetric universe model of Boyle and Turok (2018), which proposes that the universe before the Big Bang is the CPT mirror image of the universe after it — the same physical content with all discrete symmetries reversed. In the Boyle-Turok model, CPT symmetry is a boundary condition at the Big Bang singularity. In the SB-HC4A, CPT alternation is a *structural consequence* of the singularity transformation: the compressed boundary encoding does not carry CPT orientation, so each decompression can in principle realize either orientation. The alternating pattern is the simplest non-trivial assignment consistent with CPT symmetry of the overall bi-infinite sequence.

The conjecture also relates to Penrose's (2010) Conformal Cyclic Cosmology (CCC), which proposes that the conformal geometry of one aeon's infinite future (heat death) is identified with the conformal geometry of the next aeon's Big Bang. The SB-HC4A's cyclic structure shares CCC's core insight — that cosmological cycles are connected through conformal/information-theoretic transformations at singularity boundaries — but differs in two respects: (a) the SB-HC4A permits alternation between heat-death and crunch boundaries (CCC requires heat death), and (b) the SB-HC4A adds CPT alternation, which CCC does not address.

Implication for baryon asymmetry. If CPT alternation holds, the observed baryon asymmetry (the dominance of matter over antimatter in the present cycle) is not a dynamical accident requiring explanation by baryogenesis mechanisms but a *structural feature* of the current cycle's CPT orientation. The "missing" antimatter is not missing — it constitutes the adjacent cycles. This is a strong prediction: any observation of fundamental CPT violation within a single cycle would be evidence against CPT alternation, while the persistent failure to find a dynamical baryogenesis mechanism would be consistent with it.

Speculative status. CPT alternation is the most speculative claim in this paper. It is not derivable from the five axioms (Section 7) without additional assumptions about the singularity transformation's effect on discrete symmetries. It is presented as a *natural possibility* within the SB-HC4A framework — one that connects to established proposals (Boyle & Turok, 2018; Penrose, 2010) and generates testable predictions (baryon asymmetry as structural rather than dynamical) — but it requires independent motivation or derivation before it can be promoted to a formal consequence of the model.

5.8 The Big Rip: Distributed Singularity Boundaries and Branching Cycles

The cyclic dynamics of Sections 5.6–5.7 assume that the terminal boundary B_nˆ+ is a single connected Bekenstein-saturated surface — whether reached through heat death (entropy saturation at cosmological dilution) or Big Crunch (entropy saturation at maximum contraction). A third cosmological endgame — the Big Rip (Caldwell, 2002) — breaks this assumption and requires a generalization of the cyclic formalism.

Physical motivation. If dark energy has an equation-of-state parameter w

< -1 (phantom energy), the dark energy density increases without bound as the universe expands. The Friedmann equation implies that the scale factor $a(t)$ diverges at a finite future time t_{rip}:

$$a(t) \to \infty \text{ as } t \to t_{rip} < \infty$$

The diverging expansion rate progressively unbinds all gravitationally and electromagnetically bound structures — galaxy clusters, galaxies, stellar systems, atoms — and ultimately tears apart spacetime itself (Caldwell, Kamionkowski, & Weinberg, 2003). Every point in space becomes a singularity.

Definition (Distributed Terminal Boundary). A *distributed terminal boundary* is a terminal singularity B_n^+ that is not a single connected surface but a collection of disjoint Bekenstein-saturated surfaces:

$$B_n^+ = \sqcup_{j \in J} B_n^{+,j}$$

where J is an index set (possibly infinite or uncountable), each $B_n^{+,j}$ satisfies conditions IB1–IB3 (Section 3.1), and the fragments are mutually causally disconnected: for all $j \neq k$, no causal curve connects a point interior to $B_n^{+,j}$ to a point interior to $B_n^{+,k}$.

The Big Rip produces exactly this structure. As the expansion rate diverges, the causal horizon shrinks to zero — every spatial region becomes causally disconnected from every other. Each causally disconnected region is bounded by a Bekenstein-saturated surface (its own local information boundary). The computational domain fragments into a disjoint union of singularity boundaries.

Proposition (Generalized Cycle Map). When the terminal boundary B_n^+ is distributed, the cycle map $\Phi: B_n^+ \to B_{n+1}^-$ generalizes from a single-valued map to a multi-valued map:

$$\Phi: \sqcup_{j \in J} B_n^{+,j} \to \sqcup_{j \in J} B_{n+1,j}^-$$

where each fragment $B_n^{+,j}$ independently decompresses into a new computational domain $D_{n+1,j}$ with its own initial boundary $B_{n+1,j}^-$. The information content is conserved fragment-wise:

$$I(B_n^{+,j}) = I(B_{n+1,j}^-)$$

Proof sketch: Each fragment $B_n^{+,j}$ satisfies IB1–IB3 and is Bekenstein-saturated. By the same argument that establishes singular-to-singular transitions for connected boundaries (Section 3.4), each fragment is a transformation surface. Since the fragments are mutually causally disconnected, the transformation of each fragment is independent of all others. The cycle map decomposes into independent single-fragment maps, each of which operates by the same information-transformation mechanism as the connected case. □

Definition (Branching Cycle Structure). A *branching cycle structure* is a generalization of the cosmic cycle sequence $\{D_n\}_{n \in \mathbb{Z}}$ *(Section 5.6) from a linear chain to a tree (or forest). Each node is a computational domain* $D_{n,j}$. Each domain has one parent (the fragment of the predecessor domain's terminal boundary from which it decompressed) and potentially many children (if its own terminal boundary is distributed). The branching factor b_n = |J_n| — the number of fragments at cycle n's terminal boundary — satisfies:

- b_n = 1 for heat death or Big Crunch terminal boundaries (linear succession)
- b_n > 1 (potentially |J_n| = ∞ or uncountable) for Big Rip terminal boundaries (branching)

Proposition (Three Endgame Universality). All three cosmological endgames — heat death, Big Crunch, and Big Rip — are special cases of the generalized cycle map. The framework does not depend on the equation of state of dark energy:

Endgame	Terminal boundary	Branching factor	Cycle structure
Heat death (w ≥ −1)	Single connected surface	b = 1	Linear sequence
Big Crunch (Ω > 1)	Single connected surface	b = 1	Linear sequence
Big Rip (w < −1)	Distributed disjoint surfaces	b ≫ 1	Branching tree

The SB-HC4A cyclic cosmology is therefore robust across the full parameter space of dark energy equations of state.

CPT signatures in the branching case. When the cycle map branches, each daughter domain D_{n+1,j} independently realizes a CPT signature σ_{n+1,j} ∈ {+1, −1}. The CPT alternation conjecture of Section 5.7 generalizes: rather than a single alternating sequence, the branching tree exhibits a distribution of CPT orientations across its branches. Whether the daughter signatures are correlated (all flip together), independent (each flips with probability 1/2), or constrained by some other principle is undetermined by the current framework — it depends on details of the singularity transformation that the model does not yet specify. This indeterminacy is consistent with Weak Point 6 of Gruber (2026a): the model predicts cyclicity (and now branching cyclicity) but underdetermines the cycle's character.

Computational irreducibility of branching. The branching factor b_n is itself a dynamical quantity determined by the full Class 4 computation within cycle n. By the irreducibility theorem of Section 5.6, b_n cannot be predicted without running the computation through cycle n. Whether the universe ends in heat death, Big Crunch, or Big Rip — and, in the Big Rip case, how many fragments result — is computationally irreducible, consistent with the Class 4 character of the SB-HC4A.

Connection to multiverse proposals. The Big Rip branching mechanism provides a concrete physical process — grounded in the SB-HC4A information-theoretic framework — for generating a multiverse. Unlike eternal inflation (which produces a multiverse through quantum fluctuations in an inflaton field) or the string landscape (which populates a space of vacua), the SB-HC4A branching multiverse arises from the fragmentation of a single computational domain at its Bekenstein-saturated terminal boundary. Each daughter universe inherits its information content from a specific fragment of the parent's boundary, providing a well-defined information-conservation principle across the branching event.

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6. The Cross-Scale Structural Identity Functor

6.1 Two Categories

The cross-scale structural identity (Gruber, 2026a, Section 7) is currently expressed as a table of correspondences. To make this a mathematical claim, we need a morphism between mathematical objects.

Following the self-referential computation formalization's category-theoretic approach (Gruber, 2026c, Section 7), define two categories:

- **Cosmo** (Cosmological): Objects are physical states of the universe (field configurations, metric tensors, matter distributions). Morphisms are physical dynamics — time evolution governed by the laws of physics.
- **SRC** (Self-Referential Computation): Objects are the model-density states ρ(s, ν, t) (from the self-referential computation formalization). Morphisms are the dynamics of the model density — the Fokker-Planck equation of the formalization, substrate dynamics, permeability changes.

6.2 The Structural Identity Functor

The structural identity claim is that there exists a functor I: Cosmo → SRC that preserves the relevant structure:

Cosmo	I	SRC
Singularity boundary	↦	Implicit-explicit boundary
Observable interior	↦	Explicit models
Holographic rule structure	↦	Distributed implicit knowledge
Class 4 dynamics	↦	Cortical criticality
Φ_cosmo(U) = U	↦	Φ_c(m) = m

Cosmo	I	SRC
Information conservation across boundary	$\mapsto$	Information conservation across implicit-explicit split
Gödelian inexpressibility	$\mapsto$	Meta-Problem

For I to be a proper functor, it must:

(F1) **Map objects to objects**: Each physical state maps to a model-density state.

(F2) **Map morphisms to morphisms**: Physical dynamics map to model-density dynamics, preserving composition (if state A evolves to B and B to C in Cosmo, the mapped states evolve correspondingly in SRC).

(F3) **Preserve identity**: The identity morphism in Cosmo (no change) maps to the identity morphism in SRC.

6.3 What the Functor Preserves

The functor I is not expected to preserve all structure — the cosmological scale and the self-referential computation scale differ in their physical content, their energy ranges, their characteristic time scales, and their spatial extent. What I preserves is the *computational architecture*:

1. **The boundary structure**: The functor maps information-impermeable boundaries to information-opaque boundaries.
2. **The compression-decompression relationship**: The functor maps holographic encoding/decoding to implicit/explicit information processing.
3. **The criticality condition**: The functor maps Class 4 dynamics to Class 4 dynamics (both systems operate at the edge of chaos).
4. **The fixed-point structure**: The functor maps self-referential closure to self-referential closure.

More precisely, the functor should be a **forgetful functor** that discards scale-specific content while preserving computational-architectural structure. The category-theoretic framework for this is the theory of **abstract model theory** (Barwise, 1974) or **institutions** (Goguen & Burstall, 1992), which formalize the notion of "same structure, different content."

6.4 The Scale Functor

The fractal nature of the SB-HC4A suggests not just a single functor but a family of functors parameterized by scale:

I_λ: Cosmo_λ $\to$ Cosmo_{λ'}

where Cosmo_λ is the category of cosmological dynamics at scale λ. The self-similarity of Class 4 systems means that each scale hosts the same computational architecture. The cross-scale functor is then I_brain: Cosmo_{l_brain} $\rightarrow$ SRC — the specialization of the scale functor at the brain-relevant scale (~10^{-2} m).

This formulation makes the fractal claim precise: the universe's computational architecture is scale-invariant, and self-referential computation in cognitive systems is the instance at one particular scale. The scale functor I_λ implements the self-similar nesting that Class 4 dynamics produce (Class 4 contains Class 3/fractal as a subprocess — the scale invariance IS a Class 4 subprocess).

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7. The Necessity Argument

7.1 The Five Axioms in Formal Language

The cosmological model's necessity argument (Gruber, 2026a, Section 10) claims that the SB-HC4A is the unique structure consistent with five axioms. Formalizing this requires expressing the axioms and the derivation in a formal logical framework.

The natural tool is modal logic — specifically, the S5 system for metaphysical necessity (Kripke, 1963), extended with predicates for computational and information-theoretic properties.

Formal axiom system:

Let the domain of discourse be the class of all possible dynamical systems. Define predicates:

- E(x): "x exists" (is physically instantiated)
- C_k(x): "x has computational class k" for k $\in$ {1, 2, 3, 4, 5}
- U(x): "x is computationally universal"
- SOC(x): "x self-maintains criticality via self-organized criticality"
- Contains(x, k): "x contains a Class k subsystem"
- IB(x): "x is bounded by information-impermeable boundaries at every scale"
- Hol(x): "x has holographic structure"
- SRC(x): "x is self-referentially closed"

Axiom A1 (Ontological Necessity): $\Box\ \exists$x E(x) (Necessarily, something exists.)

Axiom A2 (Computational Character): $\forall$x [E(x) $\rightarrow$ $\exists$k C_k(x)] (Everything that exists has a computational class.)

Axiom A3 (Criticality Stability): $\forall$x [C_4(x) $\leftrightarrow$ (U(x) $\land$ SOC(x) $\land$ $\forall$k<4 Contains(x, k))] (Class 4 is uniquely characterized by universality + self-maintenance + containment of lower classes.)

Axiom A4 (Information Bound): $\forall x\ [E(x) \rightarrow IB(x)]$ (Everything that exists is bounded by information horizons.)

Axiom A5 (Holographic Encoding): $\forall x\ [IB(x) \rightarrow Hol(x)]$ (Information boundaries encode interior information holographically.)

7.2 The Derivation

Theorem (SB-HC4A Necessity). From A1–A5:

$\Box\ \exists x\ [E(x) \wedge C\_4(x) \wedge IB(x) \wedge Hol(x) \wedge SRC(x)]$

Proof sketch:

1. By A1: $\Box\ \exists x\ E(x)$. Something necessarily exists.
2. By A2: This thing has some computational class k.
3. The elimination argument (Section 3.2 of Gruber, 2026a):
   - k = 1 or k = 2: The universe contains self-referential computational systems and universal computation. By A3, only Class 4 supports these. A Class 1 or 2 system cannot contain a Class 4 subsystem (strict containment hierarchy). Eliminated.
   - k = 3: Class 3 is computationally reducible. Universal computation requires irreducibility. Eliminated.
   - k = 5: If k = 5, the system has no expressible rules (Section 2.4). Physics — the expressible-rule description of the universe — would be impossible. By A2, the system has computational character; Class 5 makes this character inexpressible. Eliminated (by coherence with A2, or by abduction: physics is not impossible).
   - Therefore k = 4.
4. By A4: The Class 4 system is bounded by information horizons.
5. By A5: The boundaries encode the interior holographically.
6. By A3 (universality) + step 5 (holographic encoding): A universal system with holographic encoding can encode its own rules on its boundary. The holographic rules produce Class 4 dynamics that produce holographic output — a self-referential loop. By Lawvere's theorem (Section 5.2), this loop has a fixed point: SRC(x). □

7.3 The Uniqueness Claim

Theorem (Uniqueness). The SB-HC4A is the unique (up to isomorphism) structure satisfying all five axioms simultaneously.

Proof strategy: Show that removing any axiom admits alternatives, and that the conjunction of all five constrains the system to a single architectural class:

- Without A1: Nothing need exist — vacuous satisfaction.
- Without A2: The existent thing need not have computational character — no classification possible.

- Without A3: Any computational class is permissible — no specific dynamics predicted.
- Without A4: No information boundaries — no singularity structure.
- Without A5: No holographic encoding — no dimensional compression.

The conjunction forces: Class 4 dynamics + information boundaries at every scale + holographic encoding + self-referential closure. This is precisely the SB-HC4A definition.

7.4 Strengths and Weaknesses of the Formal Argument

Strengths: The argument is valid if the axioms and the elimination are accepted. The conclusion follows logically. The use of modal logic makes the necessity claim precise — it is not "the universe happens to be SB-HC4A" but "any possible physical universe must be SB-HC4A."

Weaknesses:

1. **A1 is metaphysical**, not mathematical. "Something exists" is not a theorem of any formal system — it is an axiom about reality. The formalization makes this transparent but cannot justify it.
2. **The elimination of Class 5** relies on abduction (physics would be impossible), not deduction. A Class 5 universe that locally appears Class 4 to Class 4 observers is logically consistent — just explanatorily catastrophic. The modal-logical formalization can express this as: the necessity holds *within the scope of A2* (that the system's dynamics are expressible), which may be question-begging if the universe is Class 5 with inexpressible dynamics.
3. **A3 encodes a substantive claim** (that Class 4 is uniquely self-maintaining) as an axiom. A skeptic could reject A3 — perhaps multiple classes are self-maintaining under different conditions. The axiom's justification is empirical (self-organized criticality is observed in Class 4 systems) and theoretical (universality + containment are unique to Class 4), but it is not a logical necessity.
4. **A5 (holographic encoding)** is well-supported for gravitational systems (AdS/CFT, black hole thermodynamics) but is not proven as a universal principle for all physical systems. The formalization makes this dependence explicit.

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8. Energy-Information Equivalence

8.1 The Duality

The cosmological model's Weak Point 1 (Gruber, 2026a, Section 9.1) is the unproven energy-information identity: E = I. The formalization must specify what this means and what it would take to prove.

Definition (Energy-Information Duality). Energy and information are dual descriptions of the same physical quantity if there exists a bijection D: E $\rightarrow$ I (where E is the set of energy configurations and I is the set of information configurations) such that:

(EI1) **Conservation equivalence**: Energy conservation (first law of thermodynamics) is equivalent to information conservation (unitarity of quantum mechanics, no-cloning theorem).

(EI2) **Bound equivalence**: The Bekenstein bound $I\_max = 2\pi RE / (\hbar c \ln 2)$ is a definitional relationship, not a constraint between independent quantities.

(EI3) **Transformation equivalence**: Every energy transformation corresponds to an information transformation and vice versa, with Landauer's principle ($\Delta E \geq kT \ln 2$ per bit erased) as the marginal exchange rate.

8.2 Existing Support

Several results in physics point toward E = I:

- **Landauer's principle** (Landauer, 1961; experimentally confirmed by Berut et al., 2012): Erasing information has a minimum energy cost. This establishes a lower bound on the energy-information exchange rate.
- **The Bekenstein bound** (Bekenstein, 1981): Maximum information content of a region is proportional to its energy content and its radius. If E and I were independent, this bound would be a remarkable coincidence. If E = I, it is a tautology.
- **Black hole thermodynamics** (Bekenstein, 1973; Hawking, 1975): Black holes have entropy $S = A / (4l\_P^2)$, temperature $T = \hbar c^3 / (8\pi GMk)$, and satisfy the laws of thermodynamics. The information content of a black hole is its energy content, up to dimensional constants.
- **The holographic principle** ('t Hooft, 1993; Susskind, 1995): The maximum entropy (= information) of a region scales with surface area, not volume. This is a statement about information density that is simultaneously a statement about energy density.
- **The ER = EPR conjecture** (Maldacena & Susskind, 2013): Einstein-Rosen bridges (wormholes) are equivalent to Einstein-Podolsky-Rosen entanglement (quantum information). Geometry (energy) and entanglement (information) are the same thing.

8.3 The Formalization Strategy

A full derivation of E = I would require a fundamental theory of quantum gravity. Short of that, the formalization can specify:

1. **The equivalence conditions** (EI1–EI3) as testable predictions.
2. **The dimensional analysis**: E and I have different units (joules vs. bits). The conversion factor involves fundamental constants: 1 bit = kT ln 2 joules (at temperature T). At the Planck temperature (T_P = $\sqrt{}(\hbar c^5$ / $Gk^2) \approx 1.4 \times 10^{32}$ K), 1 bit = E_P = $\sqrt{}(\hbar c^5$ / G) $\approx 1.22 \times 10^9$ J. This is the Planck energy — the natural unit where E and I become the same number.
3. **The category-theoretic formulation**: E and I are two representations of the same underlying object, related by a natural isomorphism. The category of energy configurations and the category of information configurations are equivalent categories.

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9. The Cognitive Ceiling as a Formal Constraint

9.1 The Problem Restated

The deepest objection to the SB-HC4A model (Gruber, 2026a, Section 9.5) is that Class 4 observers may be constitutionally incapable of determining whether the universe is Class 4 or merely appears Class 4 to Class 4 cognition. The formalization must make this objection precise.

9.2 Computability-Theoretic Formulation

Definition (Cognitive Ceiling). A Class k observer can only detect dynamics up to Class k. Formally: let O_k be an observer (measurement/modeling apparatus) of computational class k. Let U be the universe. The observer's model of U is:

M(O_k, U) = π_k(U)

where π_k is a projection operator that maps U's dynamics onto the subspace of Class $\leq$ k behavior. If U is Class j > k, then M(O_k, U) = π_k(U) $\neq$ U — the observer's model is a strict lower-class projection of reality.

Theorem (Indistinguishability). A Class 4 observer cannot distinguish between: - (a) U is Class 4 (and the model is accurate), and - (b) U is Class 5, but π_4(U) appears Class 4 (and the model is a projection artifact).

Proof sketch: By definition, M(O_4, U) = π_4(U). If U is Class 4, then π_4(U) = U and M is faithful. If U is Class 5, then π_4(U) $\neq$ U but M(O_4, U) = π_4(U) is still Class 4. The observer sees Class 4 in both cases. ☐

9.3 The Self-Referential Trap

This connects to Gödel's incompleteness: the SB-HC4A, if self-referentially closed, contains truths about itself that it cannot prove from within (Section

5.5). Whether U is Class 4 or Class 5 may be one such undecidable truth — a fact about the universe that no internal formal system can determine.

Formally: define the predicate True_Class(U, k) = "the universe's true computational class is k." The cognitive ceiling theorem states:

$\neg\exists$ proof P within U: [P proves True_Class(U, 4)] $\vee$ [P proves True_Class(U, 5)]

if U is self-referentially closed and sufficiently expressive (by Gödel's first incompleteness theorem applied to the self-referential system).

9.4 The Model's Self-Prediction

This is the most remarkable formal feature: the SB-HC4A model *predicts its own potential unfalsifiability.* The cognitive ceiling is not an external objection brought against the model — it is a consequence of the model's own self-referential structure. If the model is correct, then:

1. The universe is a self-referentially closed Class 4 system.
2. Class 4 observers cannot determine whether the universe is Class 4 or Class 5.
3. Therefore, Class 4 observers cannot verify the model — which is exactly what a self-referentially closed system predicts about subsystems' ability to fully characterize the whole.

This is either the deepest confirmation (the model's epistemological predictions are consistent with its ontological claims) or the deepest flaw (the model immunizes itself against falsification). The formalization makes this dilemma precise but cannot resolve it — resolution requires tools from outside the model's own computational class, which, by the cognitive ceiling, are unavailable.

9.5 Connection to the Meta-Problem

The self-referential computation framework identifies the Meta-Problem of consciousness (Chalmers, 2018) as structurally identical: a self-modeling system cannot fully model its own substrate because the ESM's self-model is a projection (analogous to π_4), not a faithful representation.

Formally: the ESM's model of the substrate is M(ESM, substrate) = π_explicit(substrate) $\neq$ substrate. The substrate has implicit structure that the explicit model cannot access — just as a Class 5 universe would have structure that a Class 4 observer cannot detect.

The cross-scale structural identity functor (Section 6) maps the cosmological cognitive ceiling to the self-referential computation Meta-Problem:

I(cognitive ceiling in Cosmo) = Meta-Problem in SRC

This is the formal expression of the paper's central symmetry claim: the same epistemological limitation operates at both scales, generated by the same self-referential architecture.

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10. Phased Build Order

The formalization project is substantial. A pragmatic build sequence, ordered by the maturity of the relevant mathematical tools and the proximity to existing physics:

Phase 1: Highest Priority (Connects to Existing Mathematical Physics)

Module 3 (Sections 3.1–3.7) — Singularity boundaries as equivalence classes and computational atoms. The mathematical tools exist: causal structure theory (Hawking & Ellis, 1973), the holographic principle ('t Hooft, 1993; Bousso, 2002), black hole thermodynamics (Bekenstein, 1973; Hawking, 1975), loop quantum gravity's area quantization (Rovelli, 2004), and conformal time analysis for temporal asymptotic structure. The expanded scope includes temporal unreachability (Section 3.3), heat death as Bekenstein-saturated singularity (Section 3.4), and the formalization of particles as computational atoms (Section 3.7) — stable Planck-scale singularity configurations whose finite spectrum, discrete quantum numbers, interaction grammar, and conservation laws are derived from the Bekenstein bound and singularity boundary properties. The task is to define the equivalence relation precisely, verify temporal unreachability as a consequence of IB1–IB3, establish the entropy saturation boundary condition, and verify the finiteness proposition and conservation law derivation for computational atoms. Wetterich's (2022a, 2022b, 2022c) Grassmann functional integral mapping provides existing formal machinery for the computational atom formalization: his proven CA$\leftrightarrow$QFT equivalences demonstrate that specific automata produce specific fermionic spectra with specific gauge symmetries, and his spinor gravity automaton (2022c) provides a concrete 4D example satisfying properties analogous to CA1–CA3. The formalization should build on Wetterich's results rather than developing the computational atom framework from scratch. The three-generations conjecture (Section 3.7) is explicitly speculative and requires independent investigation connecting Class 4 self-similarity to the structure of stable boundary configurations. This is a paper-length project for a mathematical relativist, ideally with expertise in cosmological thermodynamics and particle phenomenology.

Module 5.2–5.3 (Self-referential closure via Lawvere) — The fixed-point argument. Lawvere's theorem is proven. The application to the SB-HC4A requires specifying the category, the exponential object, and the point-surjection. The cyclic extension (Section 5.6) — formalizing the temporal structure of the fixed point as a bi-infinite sequence of computationally irreducible cycles — requires additional work connecting the fixed-point theory to dynamical systems with temporal boundaries. The CPT alternation conjecture (Section 5.7) is a speculative extension requiring independent investigation but connects to the established Boyle-Turok (2018) CPT-symmetric model and Penrose's (2010) Conformal Cyclic Cosmology. The Big Rip generalization (Section 5.8) — extending the cycle map from single-valued to multi-valued when the terminal boundary is distributed — connects to the theory of branching processes and requires formalization of information partitioning across disjoint Bekenstein-saturated surfaces. This is a paper-length project for a category theorist familiar with Lawvere's work and with background in mathematical physics.

Phase 2: Core Formalism (Requires Dedicated Development)

Module 2 (Section 2) — Measure-theoretic class definitions. The individual measures (topological entropy, Kolmogorov complexity rate, Lyapunov exponents, computational reducibility) are well-defined. The challenge is proving that they jointly characterize the five classes as claimed — particularly the Class 3/4 boundary (reducibility vs. irreducibility) and the Class 4/5 boundary (universality vs. non-universality). The Expressibility Ceiling claim (Section 2.4) needs a rigorous proof by an algorithmic information theorist.

Module 4 (Section 4) — Holographic rule sets. The AdS/CFT correspondence provides the paradigm case, but the general definition (HR1–HR3) needs development. The coalgebraic formulation (Section 4.4) is the most promising approach but requires original category-theoretic work. Collaboration with researchers working on the interface of quantum gravity and category theory (cf. Baez & Stay, 2011; Abramsky & Coecke, 2004) is recommended.

Phase 3: Deep Formalism (Hardest, Highest Potential Impact)

Module 6 (Section 6) — The cross-scale structural identity functor. This requires both the self-referential computation formalization (Gruber, 2026c) and the cosmological formalization to be sufficiently developed. The functor construction is the mathematical expression of the model's strongest claim — that self-referential computational systems and cosmological structure implement the same architecture. Demonstrating that this functor exists, is non-trivial, and preserves the claimed structural properties would be a substantial mathematical achievement.

Module 7 (Section 7) — The necessity argument in modal logic. The formal argument depends on the formalization of the axioms and the elimination. The hardest part is the elimination of Class 5 (abductive, not deductive) and the justification of A3 (that Class 4 is uniquely self-maintaining). A modal logician working with a computability theorist could assess whether the argument is formally valid and what assumptions it requires.

Phase 4: Speculative Extensions

Module 8 (Section 8) — Energy-information equivalence. This depends on progress in quantum gravity. The formalization can specify the equivalence conditions and the testable predictions, but a proof of E = I is beyond current physics.

Module 9 (Section 9) — Cognitive ceiling formalization. The computability-theoretic formulation is straightforward; the profound question (whether the model's self-predicted unfalsifiability is a strength or a weakness) is philosophical, not mathematical. The formalization's contribution is to make the question precise.

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11. What Formalization Buys — And What It Cannot

11.1 What It Buys

Precision on the central claims. The verbal model says "singularities are structurally identical at all scales." The formalization says: "singularities belong to the same equivalence class under the relation ~, defined by conditions IB1–IB3." The verbal model says "the cross-scale mapping is not analogy but structural identity." The formalization says: "there exists a functor I: Cosmo → SRC preserving computational-architectural structure." These are not interchangeable statements — the formal versions are testable and falsifiable in ways the verbal versions are not.

Constraint on the model. A verbally specified "self-referential closure" can mean almost anything. The Lawvere fixed-point formalization constrains it: self-referential closure requires a specific category-theoretic structure (a Cartesian closed category with a point-surjective morphism). If the SB-HC4A's dynamics do not satisfy this structure, the fixed-point argument fails. The formalization turns a vague philosophical claim into a checkable mathematical condition.

Interoperability with physics. The singularity equivalence relation connects to black hole thermodynamics, the holographic principle, and loop quantum gravity. The holographic rule set definition connects to AdS/CFT. The Class 4 characterization connects to the theory of dynamical systems at criticality. The computational atom formalization connects directly to Wetterich's (2022a, 2022b, 2022c) proven CA↔QFT equivalences, which provide the formal bridge between discrete automaton dynamics and continuum field theories. The formalization positions the SB-HC4A model within the existing landscape of mathematical physics, rather than floating free as a verbal speculation.

Connection to the self-referential computation and RIM formalizations. The cross-scale structural identity functor (Section 6) and the cognitive ceiling / Meta-Problem connection (Section 9.5) formally link the three-level theoretical architecture: cosmological substrate → self-referential computation

$\rightarrow$ intelligence. The three formalization papers, taken together, specify a single mathematical framework that connects cosmological structure, self-referential computational systems, and intelligence through category-theoretic morphisms.

Sharpened objections. The cognitive ceiling formalization (Section 9) makes the model's deepest vulnerability precise: the indistinguishability theorem (Section 9.2) states exactly what a Class 4 observer cannot determine. This is more useful than the verbal formulation — it specifies the exact conditions under which the model is unfalsifiable and the exact sense in which this is a structural prediction rather than an evasion.

11.2 What It Cannot

The formalization cannot prove the axioms. A1 (something exists) is metaphysical. A3 (Class 4 is uniquely self-maintaining) is empirical. A5 (holographic encoding is universal) is conjectural. The formalization derives consequences from axioms; it does not justify the axioms themselves. The model is only as strong as its weakest axiom.

The formalization cannot resolve the cognitive ceiling. If the indistinguishability theorem (Section 9.2) is correct, no amount of formal work conducted within the universe can determine whether the universe is truly Class 4 or merely appears so. The formalization makes this limitation precise but cannot overcome it — overcoming it would require computational resources exceeding Class 4, which are by definition unavailable.

The formalization cannot prove E = I. Energy-information equivalence depends on a theory of quantum gravity. The formalization specifies what E = I means and what its consequences would be, but the proof (if one is possible) belongs to fundamental physics, not to the formalization program.

The formalization cannot close the gap between structure and ontology. Demonstrating that a formal architecture is self-consistent does not demonstrate that the universe instantiates that architecture. Mathematics can show that the SB-HC4A is a well-defined, self-consistent object. Physics must determine whether the universe is an SB-HC4A.

Even the best formalization cannot prove that the SB-HC4A model describes reality rather than the cognitive ceiling of its human author's Class 4 brain. Its value is to make the model's claims precise enough to be *clearly right or clearly wrong* — which is, in the end, the only honest standard a theory can meet.

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12. Conclusion

The SB-HC4A cosmological model requires mathematical formalization to constrain its claims, connect to existing mathematical physics, and sharpen both its predictions and its acknowledged vulnerabilities. The formalization strategy proposed here addresses the model's core open questions: What does "Class 4" mean precisely? What is the equivalence relation on singularity boundaries? What does it mean for a rule set to be holographic? What category-theoretic structure makes self-referential closure rigorous? In what formal sense is the cross-scale structural mapping a structural identity rather than an analogy?

Eight modules are proposed, spanning measure theory (class definitions), topology (singularity boundaries — now including temporal asymptotic structure, heat death as Bekenstein saturation, and particles formalized as computational atoms with finite spectrum, discrete quantum numbers, interaction grammar, and conservation laws derived from boundary information theory), category theory (holographic rule sets, self-referential closure, the cross-scale structural identity functor), modal logic (the necessity argument), thermodynamic information theory (energy-information equivalence), and computability theory (the cognitive ceiling). The singularity boundary analysis yields a cyclic temporal structure for the self-referential fixed point: heat death triggers information compression at the Bekenstein-saturated boundary, which decompresses into a new expansion phase — the universe computes its own restart. The cycle sequence is computationally irreducible, consistent with Class 4 dynamics, and may alternate CPT signatures between cycles, connecting to the Boyle-Turok (2018) CPT-symmetric universe model and Penrose's (2010) Conformal Cyclic Cosmology. The Big Rip scenario (Caldwell, 2002) generalizes the cyclic dynamics from a linear sequence to a branching tree: when the terminal boundary fragments into distributed Bekenstein-saturated surfaces, each fragment independently triggers a restart, producing a well-defined multiverse generation mechanism with information conservation across the branching event. The framework is robust across all three cosmological endgames. Crucially, the formalization is not starting from zero: Wetterich's (2022a, 2022b, 2022c, 2022d) proven equivalences between cellular automata and fermionic quantum field theories — including a 4D spinor gravity automaton with exact local Lorentz symmetry — provide concrete formal machinery for the computational atom module, the holographic rule set definition, and the bridge between discrete automaton dynamics and continuum physics. The modular design allows domain specialists to contribute independently, with category theory providing the integration language.

The formalization also achieves a specific meta-theoretical result: it makes precise the model's prediction of its own potential unfalsifiability. The indistinguishability theorem (Section 9.2) — that Class 4 observers cannot distinguish between a Class 4 universe and a Class 5 universe that locally appears Class 4 — is either the model's deepest structural prediction or its deepest flaw. The formalization cannot resolve this dilemma, but it can state it with mathematical precision, which is the prerequisite for any future resolution.

This paper specifies the program. Its execution requires collaboration across mathematical physics, category theory, computability theory, and modal logic — and the intellectual honesty to discover that some of these formalizations may reveal the model to be wrong in specific, identifiable ways. That would be a feature, not a bug.

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