

93201Q





Scholarship 2018 Statistics

2.00 p.m. Monday 12 November 2018 Time allowed: Three hours Total marks: 40

QUESTION BOOKLET

There are FIVE questions in this booklet. Answer ALL questions.

Pull out Formulae and Tables Booklet S-STATF from the centre of this booklet.

Write your answers in Answer Booklet 93201A.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

QUESTION ONE (8 marks)

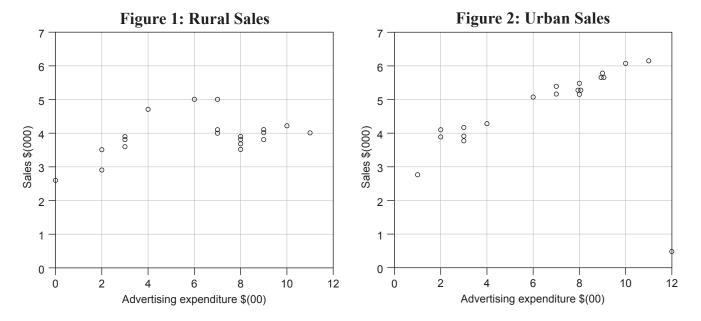
An investigation was carried out by a clothing retailer regarding the impact of advertising expenditure on the total value of sales over two different types of outlet; rural and urban. The following data were obtained:

Table 1: Advertising and Sales Data

Advertising Expenditure (A) (\$00)	Rural Sales (S) (\$000)
0	2.6
2	3.5
2	2.9
3	3.9
3	3.8
3	3.6
4	4.7
6	5.0
7	4.0
7	5.0
8	3.5
9	4.1
10	4.2
9	4.0
8	3.7
7	4.1
11	4.0
8	3.9
9	3.8
8	3.8

Advertising Expenditure (A) (\$00)	Urban Sales (S) (\$000)
1	2.8
2	3.9
2	4.1
3	3.9
3	3.8
3	4.2
4	4.3
6	5.1
7	5.2
12	0.5
8	5.5
9	5.8
10	6.1
9	5.7
8	5.3
7	5.4
11	6.2
9	5.7
8	5.3
8	5.2

Figures 1 and 2 below show a scatterplot for the data for each type of outlet.



The following are possible models to fit to these data.

$$S = 0.0825A + 3.3955$$

$$S = -0.0356A^2 + 0.4783A + 2.6392$$

$$S = 0.1263A + 3.8788$$

$$S = 0.29A + 3.12$$

- (a) Describe the relationship between the advertising expenditure, A, and sales, S, for each type of outlet.
- (b) By selecting an appropriate model for each type of outlet, obtain a sales prediction for an advertising expenditure of \$500, one for urban outlets and one for rural outlets.
- (c) Discuss any reservations there may be with using your sales predictions in (b).
- (d) Suggest two further variables that would possibly influence sales.

Describe the expected relationship between each of your suggested variables and sales.

(e) It was claimed that increasing the advertising expenditure to \$1400 for each type of outlet would not lead to any increase in sales.

Discuss this claim using the given data.

QUESTION TWO (8 marks)

Two new varieties of cheese, V1 and V2, are to be compared. In order to establish if there was a significant/real overall difference in consumer response to the two varieties, a study was conducted. Fifty consumers were randomly assigned into two equal groups, Group 1 to taste V1, and Group 2 to taste V2.

Each variety of cheese was scored over ten characteristics, with each characteristic being given a rating between 1 and 9 (1=strongly dislike, 9 = strongly like). Consumers in Group 1 scored variety V1, and consumers in Group 2 scored variety V2. The ratings for each consumer were then added to give a total score for that variety. The total scores from the consumers in each group are given in Table 2 below.

Table 2: Total Scores

Total Scores for V1		
66	57	
43	56	
68	65	
83	74	
62	54	
84	41	
88	53	
45	90	
32	63	
42	53	
41	67	
39	63	
65		

Total Scores for V2			
48	61		
82	62		
83	71		
67	63		
78	75		
60	74		
80	79		
68	67		
67	61		
79	59		
65	83		
71	68		
71			

A randomisation test was carried out on the total scores. Table 3 below gives some statistics, and Figure 3 opposite gives some output from this test.

Table 3: Summary of Test Scores by Group

	Min	1st Qu	Median	Mean	3rd Qu	Max	Std Dev	Sample Size
Group 1	32.00	44.00	62.00	59.76	67.50	90.00	16.01	25
Group 2	48.00	62.50	68.00	69.68	78.50	83.00	8.87	25

5

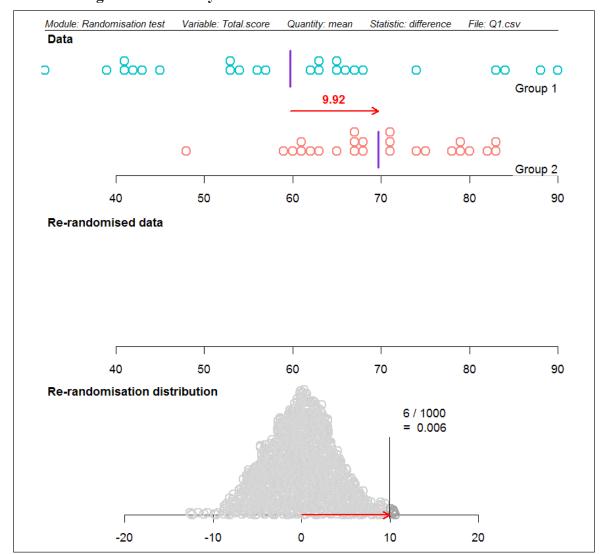


Figure 3: Summary of Scores and Re-randomisation Distribution

- (a) What can you conclude from Table 3 and Figure 3? Justify your answer.
- (b) Why were the consumers randomly assigned into two equal groups?
- (c) Suppose one consumer did not score one of the ten characteristics.

 Suggest how this situation could be dealt with when processing the results.
- (d) Describe how you could graphically present the scores for each characteristic separately in order to show any differences between V1 and V2 across all the characteristics.
- (e) A different study is to be designed to compare the initial taste of V1 with its taste as it ages over time.
 - Describe a possible design for this study.
- (f) How could the design of your suggested study in (e) be modified if now V1 and V2 are to be compared with respect to possible changes in taste as the cheeses age?

QUESTION THREE (8 marks)

- (a) A random sample of 400 articles published during 2017 was taken from a news website to investigate the nature of the words used in the headlines.
 - (i) The headlines for these articles were analysed using sentiment analysis. Sentiment analysis assigns a score on a continuous scale between 0 and 1 that measures the overall sentiment or emotion of a piece of text. A sentiment score of 0 indicates a completely negative sentiment, and a sentiment score of 1 indicates a completely positive sentiment.

The mean sentiment score of the headlines was found to be 0.538, and the standard deviation 0.287. 74.9% of the headlines had a sentiment score of less than 0.8.

Explain why a normal distribution may not be a good model for the sentiment scores of all headlines for articles published on this news website.

Support your answer with at least one calculation.

(ii) The distribution of the first word length of the first word of each headline of these 400 articles is shown in the following graph.

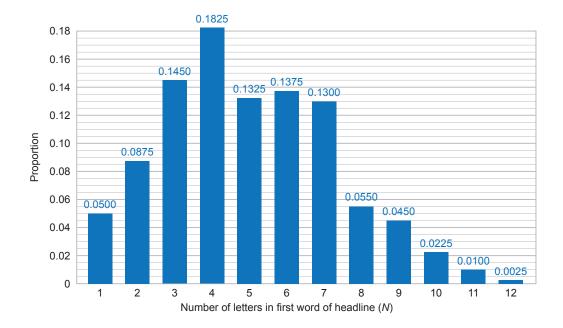


Figure 4: Lengths of First Word of Each Headline

Let the random variable N be the number of letters in the first word of each headline in the sampled articles.

Investigate whether a Poisson distribution would be a good model for N-1.

Support your answer with statistical reasoning and calculations.

(b) Some news websites display how many minutes a reader is expected to take to read an article. A random sample of 200 news articles was taken from each of two different news websites, Website A and Website B, that displayed expected reading times. For each article, the reading time displayed and the number of words for each article were used to calculate the reading speed in words per minute for the article. The table below shows an excerpt of the sample data from Website A.

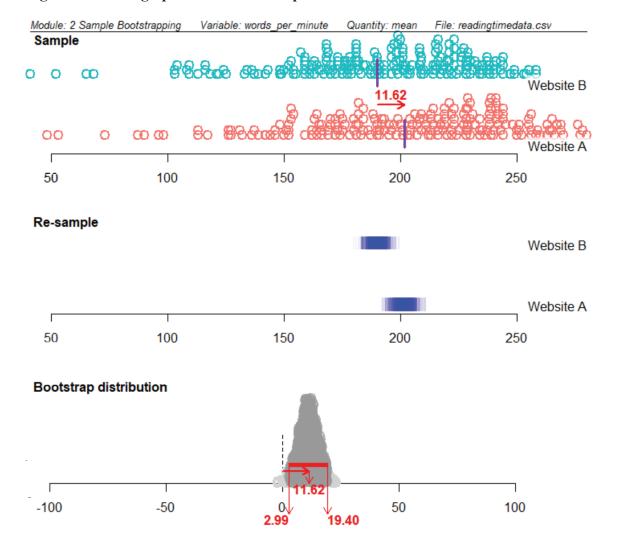
Table 4: Sample Data Excerpt from Website A

Website	Article	Number of words	Reading time in minutes	Reading speed (words per minute)
A	1	684	3.6	190
A	2	299	1.3	230
A	3	187	1.0	187

Figure 5 below gives a summary of the reading speeds for the sample from each website and the bootstrap distribution of the difference between the means of the reading speeds for the two websites.

What can be concluded about the mean reading speeds of news websites A and B? Use Figure 5 to support your answer.

Figure 5: Reading Speeds and Bootstrap Distribution of Difference between Means



QUESTION FOUR (8 marks)

- (a) The developer of a particular pregnancy test claims that 98% of women who are pregnant will test positive for pregnancy using the test, and only 4% of women who are not pregnant will test positive for pregnancy using the test (referred to as a "false positive").
 - (i) Explain why the proportion of women using this test who are actually pregnant when the test is positive for pregnancy is not necessarily 98%.
 - Support your answer with statistical reasoning and calculations.
 - (ii) A study was conducted to investigate the accuracy of the pregnancy test. The study found that:
 - 94% of the women who were pregnant tested positive for pregnancy
 - 81% of the women who were not pregnant tested negative for pregnancy
 - 44 of the 55 women who tested positive for pregnancy were pregnant.

What proportion of the women in this study were pregnant?

Support your answer with statistical reasoning and calculations.

(b) Read the following excerpts from report on Attitudes and Behaviour towards Alcohol Survey (ABAS) 2013/14 to 2015/16: Attitudes to drinking in pregnancy.

Purpose

This report presents descriptive results about New Zealanders' attitudes to drinking alcohol during pregnancy. Results from the survey are used to inform the planning and development of alcohol activities, policies, and programmes that aim to reduce alcohol-related harm in New Zealand.

Method

For each survey in 2013/14, 2014/15 and 2015/16, approximately 4000 people aged 15 years and over were surveyed over four months (November, December, January, and February). Households were stratified into telephone directory regions. A random sample of telephone numbers was generated from all number ranges found in the White Pages using a Random Digit Dialling (RDD) approach. The mode of the interview was Computer-Assisted Telephone Interviewing (CATI).

Analysis

This report presents the analysis of five questions from ABAS that assessed New Zealanders' attitudes towards drinking in pregnancy. Responses to these attitude statements were on a five-point scale of 'strongly agree', 'agree, 'neither agree nor disagree', 'disagree' and 'strongly disagree'. The data have been weighted (adjusted) so that the sample reflects the makeup of the New Zealand population at the last Census (2013). Results are presented as weighted estimates with error bars representing the 95% confidence intervals. The confidence level for comparing estimates by sub-group was set at 95%.

Drinking in pregnancy is OK

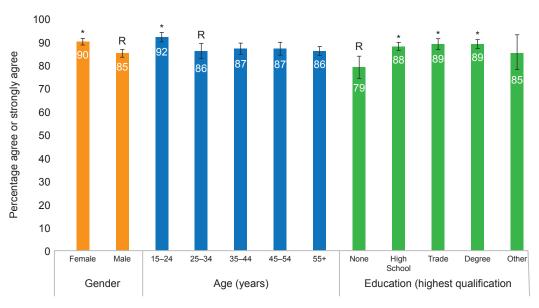
General attitude to drinking in pregnancy was assessed by asking all respondents to indicate their level of agreement with the statement 'During pregnancy drinking small amounts of alcohol is OK'. Overall, 84% [95% CI: 82, 85] of respondents disagreed with this statement in 2015/16. This question was also asked of respondents in the 2013/14 and 2014/15 surveys. Overall, there were no significant changes in level of disagreement across the three survey years.

Encourage others to stop drinking if pregnant

To assess the level of support from others to encourage pregnant women not to drink, all respondents were asked to indicate their level of agreement with the statement 'I would

encourage a friend or family member to stop drinking completely if she was pregnant'. Overall, 88% [95% CI: 86, 89] of respondents agreed with this statement. As shown in Figure 6, agreement was higher among: females (compared with males), 15 to 24-year-olds (compared with 25 to 34-year-olds) and those with a formal qualification (compared with no formal qualifications).

Figure 6: Percentage of Respondents who agreed with the statement 'I would encourage a friend or family member to stop drinking conpletely if she was pregnant' in 2015/16, by gender, age, and education level



Base = All respondents (ABAS 2015 / 2016)

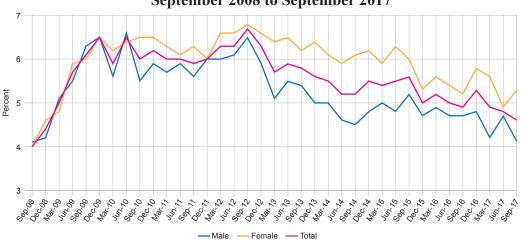
- (i) Describe two strengths in respect to the method used for this survey.
- (ii) Describe two potential non-sampling errors for this survey.
- (iii) In the paragraph titled **Drinking in pregnancy is OK**, explain what was meant by the expression "no significant changes".
- (iv) Figure 6 has error bars (vertical lines) to represent 95% confidence intervals.Give two possible reasons why the error bars for the "Other" sub-group are longer than those for the "Degree" subgroup.

^{*} Significantly different from the reference group (indicated with 'R')

QUESTION FIVE (8 marks)

(a) The graphs of the seasonally adjusted New Zealand unemployment rates for each quarter from September 2008 to September 2017 inclusive are shown in Figure 7 below.

Figure 7: Unemployment Rate by Sex, Seasonally Adjusted September 2008 to September 2017



- (i) Write a short paragraph discussing features of these data for these unemployment rates for males and females.
- (ii) Suggest a reason for each of the features described in (i).
- (b) Figure 8 below shows the total numbers in thousands who were unemployed each quarter from 2008 to 2017 (on the horizontal axis, 1 represents the first (March) quarter 2008, 2 represents the second (June) quarter 2008 and so on).

Figure 8: Total Unemployed versus Time

160

140

120

80

0

4

8

12

16

20

24

83

32

36

40

44

Number of quarters since Dec quarter 2007

(i) Two possible models for the trend component of the time series data are:

Model A:
$$y = 0.3515x + 124.14$$
 and

Model B:
$$y = -0.9351x + 161.56$$
,

where y is the trend component of the total number unemployed, and x is the number of the quarter after the December Quarter 2007.

Select which of these models would be better to use in estimating a forecast for the total number unemployed for the first quarter in 2019 and calculate this estimate. Justify the selection of the model you used.

- (ii) Give one reservation there may be in using your forecast in (i).
- (c) Figure 9 below shows the actual number of people unemployed in NZ in each quarter for the years 2008 to 2017 inclusive, as raw data. Forecasts for the numbers of people unemployed in each quarter of 2018 and 2019 were obtained using the Holt-Winters Additive Model and these are given in the graph and table below.

Figure 9: Number of People Unemployed in New Zealand

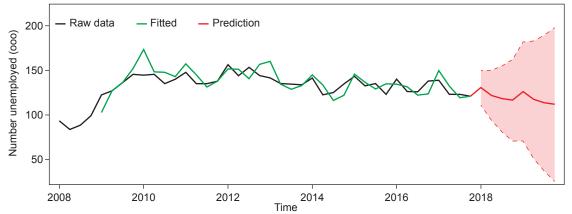


Table 5: Number of People Unemployed in New Zealand

		fit	upr	lwr
2018	Q1	130.9003	150.1705	111.63009
2018	Q2	121.9377	149.9295	93.94592
2018	Q3	118.2623	155.0386	81.48603
2018	Q4	116.5011	162.3640	70.63815
2019	Q1	126.3196	182.1055	70.53363
2019	Q2	117.3570	182.9506	51.76335
2019	Q3	113.6816	189.5140	37.84914
2019	Q4	111.9204	198.4153	25.42542

Compare your forecast made in (b) with the forecast made from the Holt-Winters Additive Model. Explain why they are different.

Question Five continues on the following page.

(d) It was claimed that the annual percentage unemployment rate for females had dropped at a faster rate than that for males over the years 2014 to 2017 inclusive.

Using values given in Table 6 below, investigate if this claim is justified.

Table 6: Mean Quarterly % Unemployment

Year	Male	Female
2014	4.725	6.075
2015	4.925	6.025
2016	4.775	5.500
2017	4.333	5.267