Assessment Schedule – 2023

Mathematics and Statistics: Apply geometric reasoning in solving problems (91031)

Evidence

Do not penalise incorrect rounding if sufficient evidence provided. Accept Grad or Rad in trigonometry questions but only when it does not impact the sensibleness of the answer.

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)	Use Pythagoras to find: $\text{PR} = \sqrt{8^2 + 12^2} = \sqrt{208} = 14.42\text{ m}$ Use Pythagoras to find: $\text{RQ} = \sqrt{45^2 - 14.42^2} = \sqrt{1817}$ $\text{RQ} = x = 42.63\text{ m}$ OR Alternative method.	Showing, with evidence of relevant working, RQ = x = 42.63 m. Do not accept C.A.O.		
(b)	Use trigonometry to find: $\text{AB} = \dfrac{22}{\sin 28} = 46.86\text{ m}$ (accept 81.21rads) Use trigonometry to find: $\text{BC} = \dfrac{22}{\tan 28} = 41.38\text{ m}$ (do **not** accept –78.17 rads) $\text{GH} = \dfrac{50}{\sin 20} = 146.19\text{ m}$ (accept 54.76 rads) BF = 50 – 41.38 = 8.62 m Use Pythagoras to find: $\text{BG} = \sqrt{4^2 + 8.62^2} = \sqrt{90.37}$ $\text{BG} = 9.51\text{ m}$ Cable length = 46.86 + 9.51 + 146.19 = 202.56 m So the advertisement claim is correct. OR Alternative method.	• Showing, with evidence of relevant working, any of the lengths: AB = 46.86 m. BC = 41.38 m. GH = 146.19 m. BG = 9.51 m. Do not accept C.A.O.	• Showing, with evidence of relevant working, that the total length of the cable is 202.56 m. AND Conclusion that the zipline is greater than 200m or state claim is correct. (Answers in radians not acceptable for r)	

(c)	$\angle$FBC = 180 – 64 – 48 = 68° (angle sum of triangle BCF) If the lines ABCD and HGFE are parallel then $\angle$CBG + $\angle$FGB = 180° (co-interior angles between parallel lines add up to 180°) Then $\angle$CBG + $\angle$FGB = 68 + 35 + 58 = 161° i.e. not 180° so not parallel. i.e. the designer's claim is false. OR Alternative method.	• Calculation of at least one appropriate angle, with reasoning: $\angle$CFE =29° $\angle$BFG =87° $\angle$CBF =68° OR Some valid evidence to show that the lines ABCD and HGFE are not parallel, but lacking a clearly drawn conclusion.	• Valid evidence to show that the lines ABCD and HGFE are not parallel, with at least one appropriate geometric reason. AND Statement that the lines are not parallel **or** claim is false.	
(d)	$\angle$PQC = 90° (angle between tangent and radius is 90°) $\angle$SQC = 90 – 42 = 48° $\angle$QSC = 48° (base angles of the isosceles triangle QSC) So x = $\angle$SCQ = 180 – 48 – 48 (angle sum of triangle QSC) i.e. x = 84° OR Alternative method.	• Finding angle $\angle$SCQ = x = 84° with some evidence of working (Reasons not necessary but do not accept C.A.O.)		

(e)	**Method 1: Assumes AB is a straight line** $\angle ACB = 180°$ (Angle at the centre is twice the angle at the circumference) Then $\angle APB = 90°$ As required. **Method 2: Assumes AB is a straight line** Include in the diagram the radius PC. Let $\angle CPB = p$ then $\angle CBP = p$ (base angles of isos triangle BCP are =) Let $\angle CPA = q$, then $\angle CAP = q$ (base angles of isos triangle ACP are =) Then $\angle CAP + \angle APB + \angle PBA = 180°$ (angle sum of triangle ABP) i.e. $q + q + p + p = 180°$ $2q + 2p = 180°$ $q + p = 90°$ $y = 90°$ As required. **Method 3: Assumes AB is a straight line** Include in the diagram the radius PC. Let $\angle ACP = p$ Then $\angle APC = \frac{180-p}{2} = 90 - \frac{p}{2}$ (base angles of isos triangle ACP are =) Also $\angle CPB = \frac{180-(180-p)}{2} = \frac{p}{2}$ (base angles of isos triangle BCP are =) Then $\angle APB = \angle APC + \angle BPC$ $= 90 - \frac{p}{2} + \frac{p}{2}$ Then $\angle APB = 90°$ As required. **OR** **Method 4:** Given A, P, B lie on circumference of a circle, and $\angle APB = y$ For other positions of P IN THE SAME arc, the angle formed at the circumference will also $= y$ Eg If Q is another point on the same arc, then $\angle AQB = y$ (angles on same arc are equal) For other positions of P IN THE OTHER arc Eg. If S, is a point in the other arc, a cyclic quadrilateral AQBS is formed. $\angle ASB = 180 - y$ (opposite angles of cyclic quad add to 180°)	• States a rule as to why y=90° eg angle in semicircle is always 90° OR relationship between angles at the centre and the circumference.	• States that AB is a diameter or straight line and gives clear geometric reasoning that $\angle APB = 90°$. OR Considers other positions of p would also be 90°, with geometric reasons.	**E7** States clearly that they have made an assumption that line ACB is a straight line or diameter. This cannot be implied, there must be some doubt. AND Using geometric reasons to clearly show that $\angle APB = 90°$. **E8** States clearly that they have made an assumption that line ACB is a straight line or diameter. This cannot be implied, there must be some doubt. AND Using geometric reasons to clearly show that $\angle APB = 90°$. AND Considers other positions of p would also be 90°, with reasons.

	For ALL such angles at the circumference to be equal in BOTH arcs, they would need to be semicircles and therefore $\angle$ASB $= y$ $y = 180 - y$ $2y = 180$ $y = 90$ The student is only correct when AB is a diameter. **OR** Alternative method.			

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One point made incompletely.	1 of u	2 of u	3 of u	1 of r	2 of r	Q 1(e) incomplete	Q 1(e) complete

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)(i)	$\angle STR = 110°$ (vertically opposite angles are equal) $\angle TRS = 180 - 110 - 40 = 30°$ (angle sum of triangle RST) $v = \angle PQS = 30°$ (angles in the same segment are equal) OR Alternative method.	• Finding angle $v = \angle PQS = 30°$ with some evidence of working. (Reasons not necessary but do not accept C.A.O.)		
(ii)	$\angle PSR = 90°$ (angle in a semi-circle is 90°) $\angle PSQ = 90 - 40 = 50°$ $\angle PRQ = w = 50°$ (angles in the same segment are equal) OR Alternative method.	• Calculation of at least one appropriate angle, with reasons. OR Any two angles at the circumference shown on the diagram.	• Finding angle $w = \angle PRQ = 50°$, with at least one valid reason.	
(b)(i)	$\angle KJN + \angle KLN = 180°$ (opp angles cyclic quad add to 180) i.e. $75 + 65 + x = 180°$ $x = 180 - 75 - 65$ $x = 40°$ OR Alternative method.	• Finding angle $x = \angle CLN = 40°$ • with some evidence of working • • (Reasons not necessary but do not accept C.A.O.)		
(ii)	$\angle CLM = \angle CNM = 90°$ (angle btwn tgt and rad is 90) $\angle CNL = 40°$ (base angles of isos triangle CLN) $\angle NCL = 180 - 40 - 40 = 100°$ (angle sum of triangle CLN) i.e. $y = \angle LMN = 360 - 100 - 90 - 90$ $y = 80°$ (angle sum of quad CLMN) Alternative method: $\angle CLM = \angle CNM = 90°$ (angle between tgt and rad is 90°) $\angle NLM = 90 - 40 = 50°$ $\angle LNM = 90 - 40 = 50°$ i.e. $y = \angle LMN = 180 - 50 - 50$ $y = 80°$ (angle sum of triangle LMN) OR Alternative method.	• Calculation of at least one appropriate angle, with reasons.	• Finding angle $y = \angle LMN = 80°$ with at least one valid reason.	
(c)(i)	Use of method of Pythagoras: $x = AE = \sqrt{53^2 - 45^2} = \sqrt{784} = 28$ cm	• Showing $AE = x = 28$ cm with evidence of relevant working,		

(ii)	Use of trigonometry in triangle ABE: $\sin A = \frac{45}{53} = 41.38\text{ m}$ $A = 58.1°$ Use of Trigonometry in triangle ACD: $BC = \frac{p-45}{\sin 58.1} = 1.1778p - 53$ Use of Trigonometry in triangle ACD: $\frac{p-45}{\tan 58.1} = 0.6222p - 28$ Perimteter = 45 + p + 1.778p – 53 + 0.6222p – 28 Perimeter = 2.8p – 36 **Alternative method:** Using similar triangles: $\frac{p}{45} = \frac{AC}{53}$ $BC = \frac{53p}{45} - 53$ $\frac{p}{45} = \frac{AD}{28}$ $DE = \frac{28p}{45} - 28$ $P = 45 + p + \frac{53p}{45} - 53 + \frac{28p}{45} - 28$ $\text{Perimeter} = \frac{14p}{5} - 36$ OR Alternative method.	• Finding, with evidence of working, any ONE of: • BC =1.1778p-53 • DE =0.6222p-28 • ∠BAE = 58.1° • OR • Finding ALL of the lengths of BC, AE, DE having substituted a numerical value for p or having omitted p. • OR • Finding a correct ratio involving p of similar triangles. • OR. C.A.O.	• Finding, with evidence of working, any TWO of: BC =1.1778p –53 DE =0.6222p – 28 ∠BAE = 58.1° OR Finding the perimeter of the shaded region BCDE having substituted a numerical value for p or having omitted p, with clear justification. Correct unsimplified expression given for the perimeter with unevaluated trig expressions. OR Finding, with evidence of working, the correct coefficient of p, from correct expressions for both ED and BC.	**E7** Finding the perimeter of the shaded region BCDE in terms of p, with clear justification but with a minor error. A minor error could be e.g. an arithmetic error or error in one of the calculations OR Correct unsimplified expression given for the perimeter . **E8** Finding the perimeter of the shaded region BCDE in terms of p: $\frac{14p}{5} - 36$ or equivalent, with clear justification.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One point made incompletely.	1 of u	2 of u	3 of u	1 of r	2 of r	Q 2(c)(ii) incomplete	Q 2(c)(ii)

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)	$\angle HGF = \frac{(8-2)\times 180°}{8} = 135°$ (angles in a polygon) $\angle GHE\ \frac{360-135-135}{2} = \frac{90}{2} = 45°$ OR alternative method.	Finding angle $\angle GHE = 45°$ with some evidence of working		
(b)	Use of trigonometry in triangle QRS: QR = 25 × sin32 = 12.25 cm Use of trigonometry in triangle QRS: SR = 25 × cos32 = 21.20 cm Use of trigonometry in triangle PRS: $\tan a = \frac{21.20}{20.05}$ $a = \tan^{-1}\left(\frac{21.20}{20.05}\right) = 46.6°$ OR alternative method.	• Finding, with evidence of relevant working any of the lengths: QR = 13.25 cm. SR = 21.20 cm. OR C.A.O.	• Finding, with evidence of relevant working, that $\angle SPR = 46.6°$.	
(c)(i)	Use of Trigonometry in triangle ADP: $PD = \frac{5}{\tan 23} = 11.78\text{ m}$	• Finding length PD = 11.78 m with evidence of relevant working		
(ii)	Use of Trigonometry in triangle BCP: $PC = \frac{5}{\tan 19} = 14.52\text{ m}$ Use of trigonometry in triangle CDP: $g = \cos^{-1}\left(\frac{11.78}{14.52}\right) = 35.78°$ Allow consistency from part (c)(i).	• Finding length PC = 14.52 m with evidence of relevant working OR C.A.O.	• Finding, with evidence of relevant working, that $\angle DPC = g = 35.78°$.	

(d)	**Method 1:** $\angle PTC = 90°$ (angle between tangent and radius is 90°) $\angle PCT = 2x$ (angle at centre is twice that at the circumference) $\angle CPT = y = 180 - 90 - 2x$ (angle sum of triangle PCT) i.e. $y = 90 - 2x$ **Method 2:** $\angle PTC = 90°$ (angle between tangent and radius is 90°) $\angle CTR = x$ (base angles of isosceles triangle CTR) $\angle RPT = y = 180 - 90 - x - x$ (angle sum of triangle PRT) i.e. $y = 90 - 2x$ **Method 3:** $\angle RTQ = 90°$ (angle in a semi-circle is 90°) $\angle RQT = 180 - 90 - x = 90 - x$ (angle sum of triangle QTR) $\angle PTC = 90°$ (angle between tangent and radius is 90°) $\angle CTQ = 90° - x$ (base angles of isosceles triangle QTC) $\angle QTP = 90° - (90 - x) = x$ (angle between tangent and radius is 90°) $\angle RPT = y = 180° - (90 + x) - x$ (angle sum of triangle PQT) i.e. $y = 180 - 90 - x - x$ i.e. $y = 90 - 2x$ OR alternative method.	• Finding, with evidence of relevant working any ONE of the following angles: $\angle RQT = 90 - x$ $\angle RCT = 180 - 2x$ $\angle CQT = 90 - x$ $\angle TCQ = 2x$ $\angle PQT = 90 + x$ $\angle QTP = x$ OR Angle y found by substituting a numerical value for the angle x. OR C.A.O.	• Finding, with evidence of relevant working any TWO of the following angles: $\angle RQT = 90 - x$ $\angle RCT = 180 - 2x$ $\angle CQT = 90 - x$ $\angle TCQ = 2x$ $\angle PQT = 90 + x$ $\angle QTP = x$ OR Angle y found, with geometric reasons, by substituting at least two different numerical values for the angle x. OR The justification and geometric reasoning are not clear and complete, but the result has been obtained that $\angle RPT = y = 90 - 2x$.	**E7** Clear evidence, with clear justification and geometric reasoning, but with a minor arithmetic or algebraic error that $\angle RPT = y = 90 - 2x$. OR Correct but unsimplified expression for y. **E8** Clear evidence, with clear justification and geometric reasoning, that $\angle RPT = y = 90 - 2x$

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One point made incompletely.	1 of u	2 of u	3 of u	1 of r	2 of r	Q 3(d) incomplete	Q 3(d)

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24