Assessment Schedule – 2019
Scholarship Statistics (93201)
Evidence Statement

General Principles:

1. Ignore incorrect answers if alongside correct answers. The exception is contradictory statements.
2. Ignore minor copying errors.
3. When required in evidence, answers need to be contextual.

QUESTION ONE

Tasks Q1(a)
Evidence:

- Most of these buses have diesel motors, and most of these buses are new when first registered with NZTA.
- Around 90% of these buses that were new when first registered with NZTA have diesel motors, and around 70% of these buses that were used when first registered with NZTA have diesel motors.
- Nearly 80% of these buses with diesel motors were new when first registered with NZTA, but only 40% of these buses with petrol motors were new when first registered with NZTA. All of the buses with some other type of motor were new when first registered with NZTA.
- The distribution of current ages for these buses that were new when first registered with NZTA is positively skewed, whereas the distribution of current ages for these buses that were used when first registered with NZTA is more symmetric, possibly bimodal.
- The current age of these used buses tends to be higher than the current age of these new buses, by around 15 years (using medians).

*Note: Accept other comparisons that are based on a **key** feature of the data displayed in a plot.*

Task Q1(b)(i)
Evidence:

- Service coaches: Linear trend, positive direction, moderate strength, increasing variation / scatter for increasing bus masses.

Task Q1(b)(ii)
Evidence:

- Removing the minibus with mass of 5800 kg and engine capacity of 5400 cc would lower the value of the correlation coefficient.

Task Q1(c)(i)
Evidence:

- Using Model 1: Years registered with NZTA = $-0.655 + 0.6706 \times 40 = 26.2$ years
- Using Model 2: Years registered with NZTA = $-2.796 + 0.8441 \times 40 = 31.0$ years

Accept any rounding.

Task Q1(c)(ii)
Evidence:

- Model 2 should be used.
- Figure 4 shows that, if the type of bus is taken into account, the relationship between age and years registered with NZTA can be modelled with quite different lines (for example, the slope appears steeper for minibuses).
- Figure 4 also shows that the amount of scatter around the fitted line reduces when the type of bus is taken into account.

Task Q1(c)(iii)
Evidence:

Obtaining prediction using Model 2:

- The data used was a random sample of all the buses currently registered with NZTA. There were no minibuses in this sample above 35 years old, so a minibus of this age (40 years) could be unusually old and the model may not hold for ages outside the data the model was fitted to.
- Additionally, from about 29 years onwards, all of the minibuses have years registered with NZTA that sit above the fitted line, indicating issues with the model fitted e.g. a linear model may not be appropriate.

QUESTION TWO
Task Q2(a)
Evidence:

- Both time series (total usage of Auckland Transport and estimated population of Auckland Region) show a positive trend over 2006 to 2018 which, due to the use of y-axis scales, looks like similar rates of increase for both series. However, in terms of comparing *rates of increase* for the two series, we need to use relative measures.
- Total usage has increased from around 52500(000) in 2006 to around 95100(000) in 2018, an increase of around 81%. Estimated population has increased from around 1370(000) in 2006 to around 1700(000) in 2018, an increase of around 24%. So total usage of Auckland Transport has increased at a faster rate than the estimated population of the Auckland Region, over the period 2006 to 2018.

  Note: Alternatively, in 2006, the rate of usage per person (based on estimated population) was around 52500(000) / 1370(000) = 38.3 but in 2018, the rate of usage per person was around 96000(000) / 1695(000) = 56.6. If the rates of increase were similar for the two series, then the rate of usage per person would be similar. Accept piecewise analysis considering separate trends before and after 2013 (both usage and population increase more steeply after 2013).

Task Q2(b)
Evidence:

General:

- More people use trains than ferries, with an average of around 5100(000) uses per quarter for trains by the end of 2018 and an average of around 1550(000) uses per quarter for ferries by the end of 2018.

Trends:

- Over the period 2006 to 2018, usage of both trains and ferries displays an overall positive trend.
- (Although more obvious for trains), after increasing over 2006 to 2011, the usage of trains and ferries plateaued during the period 2011 to 2013, before increasing again from 2014.

Seasonality:

- The usage of both ferries and trains displays a seasonal pattern.
- For trains, there are regular peaks in the 3rd quarter of each year and regular lows in the 4th quarter of each year, whereas for ferries there are regular peaks in the 1st quarter of each year and regular lows in the 3rd quarter of each year.
- The seasonal fluctuations for trains are similar in size between 2006 and 2018, however for ferries the seasonal fluctuations have increased in size from 2015 onwards.

Note: At least one of the points must contain numerical evidence.

Task Q2(c)(i)
Evidence:

- Model 1 forecast 2020 Q2: 19549.90 (17850.78, 21249.01).
- Model 2 forecast 2020 Q2: 20543.03 (19680.01, 21406.06).
- Forecast from Model 1 is lower than forecast from Model 2.
- Forecast interval from Model 1 is wider than forecast interval from Model 2.

Task Q2(c)(ii)
Evidence:

- Trend: The short-term trend for the last three years of bus usage was steeper than previous years, hence Model 2 gave a higher forecast than Model 1.
- Variation in data: Model 1 was based on more data which varied more than the data on which Model 2 was based, which has contributed to the wider forecast interval for Model 1.

QUESTION THREE
Task Q3(a)
Evidence:

Data distribution-based comments:

- These UK students have provided more "continuous" time values for their travel times compared with these NZ students which have been recorded as either 5, 15, 25 or 45 minutes.
- A higher percentage of these 400 UK students sampled travel to school by bus or walking (287 / 400 vs 198 / 400).
- The sample distributions of walking and bus times are positively skewed for these UK students.
- The variability for walking and bus times is greater for these UK students compared to these NZ students, particularly for bus times.

Confidence interval-based comments:

- For UK school students, it's a fairly safe bet that the mean time to travel to school by bus is between 7.9 and 15.2 minutes longer than the mean time to travel to school by walking.
- For NZ school students, it's a fairly safe bet that the mean time to travel to school by bus is between 10.4 and 17.3 minutes longer than the mean time to travel to school by walking.
- Based on these samples we can claim that it takes longer, on average, to travel to school by bus than by walking for both UK and NZ students, as both confidence intervals contain positive values for the estimated differences.

Task Q3(b)(i)
Evidence:

- Overall design: Comparison of two independent groups.
- Units / participants: 136 university students from an environment club.
- Treatment groups: Received daily messages, did not receive daily messages (control group).
- Response variable: Number of days each student walked or cycled to university.
- Random allocation used to allocate treatments to units / participants.

Task Q3(b)(ii)
Evidence:

- The tail proportion is large (0.266), which does not give evidence to support a claim that the daily text messages encouraged the students to cycle or walk to university more often.
- Looking at the data collected, the results look similar both in terms of centre and spread (the difference between the group means is 1.2 days).
- As all participants were tracking each day whether they walked or cycled to university, we could expect to see similar behaviour irrespective of the applied treatment.

Task Q3(b)(iii)
Evidence:

Study:

- Did participants give informed consent?
  It is unethical to carry out an experiment without informed consent of the participants.
- Did participants know the treatments for the experiment?
  Participants may change their behaviour because they know they are in the treatment group; blinding should be used.
- How were participants recruited for the experiment? Had they already expressed interest in cycling or walking more?
  If participants were already keen to increase their walking or cycling to university, this limits the scope of generalisation of a “treatment effect”.
- What was the wording of the messages sent to participants?
  The treatment applied was the wording of the message sent to participants, so it is important to know what the exact wording used was.

Data:

- Did all participants track data for every day during the study period? How were missing days managed by the researchers?
  It is important that the data used for each participant is consistently measured, e.g. perhaps the percentage of days tracked where a participant walked or cycled to university might be a better measure.

Participants:

- How many days did each participant typically walk or cycle to university before the study? For each participant, how did this change during the study period?
  If the study wants to conclude that the treatment increased the number of days participants walked or cycled to university, then they should compare each participant’s walking and cycling rate before the experiment to those observed during the experiment.
- What were the reasons people didn’t walk or cycle before the study?
  The reasons people didn’t walk or cycle before the study might be related to practical reasons like needing to drop children off at school or the distance they live from the university.

  Note: Accept other questions about study, data or participants, with valid statistical or contextual justification.

QUESTION FOUR

Task Q4(a)
Evidence:

Strengths of the study design:

- Selecting a variety of bus and train routes, based on length (long / short), location (downtown / suburban, wealthier / poorer), times (morning / midday / evening / night).
- Consistency and range with data collection e.g. four-minute periods using established 12 pre-set codes for behaviour.

Challenges with how the data was collected:

- Activity measured in terms of "ever-observed" rather than how long the passenger spent.
- No data collecting on late night trips, due to safety reasons for the two researchers.

Note: Accept other strengths and other challenges based on information provided in the report.

Task Q4(b)(i)
Evidence:

For confidence interval 1: The relevant percentages are 76.5 and 56.6 (a point estimate for the difference of 19.9 percentage points). The respective sample sizes are 353 and 459 (e.g. $\frac{270}{0.765} \approx 353$).

For confidence interval 2: The relevant percentages are 76.5 and 12.5 (a point estimate for the difference of 64.0 percentage points). The sample size is 353.

Task Q4(b)(ii)
Evidence:

Confidence interval 1 involves estimating the difference between two percentages from independent groups.

Confidence interval 2 involves estimating the difference between two percentages from within the same group.

Task Q4(b)(iii)
Evidence:

- The study was conducted sometime before 2011, which is over eight years ago, and so the behaviour of passengers may have changed and findings would not be applicable to current bus and train passengers.
- Due to the way the data was collected (non-random samples), we would not be able to assume that the behaviour of the passengers observed in this study is representative of the behaviour of all NZ bus and train passengers.
- Wellington may have a higher percentage of commuters travelling by train, so the passengers observed in this study may not be representative of all NZ.

  Note: Accept other reservations based on valid statistical or contextual points.

Task Q4(c)
Evidence:

Pr(observed talking | female) / Pr(observed talking | male) = 2.1
125 passengers observed talking
402 women, so 410 men (total passengers observed 812)

Let a be the number of women talking and b be the number of men talking

$$\frac{a}{402} \bigg/ \frac{b}{410} = 2.1$$

$a + b = 125$

$a = 84$, $b = 41$

Pr(male | observed talking) = $\frac{41}{125} = 0.328$

Note, due to similar group sizes (402 / 410), an incorrect method can generate an almost correct partial answer e.g. 125 / 3.1 = 40 males. The answer needs to be supported by a correct method. [credit may be given for this technique if evidence of student's understanding of the equal proportion of men and women]

QUESTION FIVE

Task Q5(a)(i)
Evidence:

- the number of stops the bus stops at – the more stops, the longer the time
- the number of passengers at each stop – the more passengers who get on, the longer the time to load passengers
- the number of red lights at each of the five intersections – the more red lights, the longer the time
- the amount of traffic on the road – the more traffic, the longer the time
- the speed the bus travels at – the faster the average speed, the shorter the time
- the time of day she leaves, during rush hour – the more traffic, the longer the time
- how full the bus is – the more full when she gets on, the fewer stops made, the shorter the time.

Note: Accept other realistic reasons based on information provided in question or related to prior contextual knowledge.

Task Q5(a)(ii)
Evidence:

- Normal distribution appears to be an appropriate model for Amelia's future bus commute times.
- The distribution of the sample data is unimodal, symmetric, bell shaped, the variable (time) is continuous.
- There are many possible factors for why the times vary and the interactions between these factors are complex; times can be viewed as unpredictable and suitable for modelling by a probability model.

Task Q5(b)(i)
Evidence:

Parameters of triangular distribution appear to be $a = 10$, $b = 30$, $c = 20$

Jacob appears to have added:

- the minimum values of both distributions to get a
- the maximum values of both distributions to get b
- the mean / median of the uniform distribution (3) with the modal value of the triangular distribution (17) to get c.

These parameters are consistent with the simulated data.

Task Q5(b)(ii)
Evidence:

Using a triangular distribution with $a = 10$, $b = 30$, $c = 20$, (height = 0.1)
$P(X > 25) = 0.125$
$P(X > 28) = 0.02$
$P(X > 28 \mid X > 25) = 0.16$

Task Q5(b)(iii)
Evidence:

- Jacob's model is a triangular distribution. However, the data collected over three years does not have a clear triangular shape.
- Due to the amount of data Jacob has collected, the non-triangular shape cannot be explained only by sampling variation.
- Jacob's model assumes independence of the two random variables used, which is invalid as the length of time he waits for the bus will not be independent from the length of the time for the bus commute for many reasons, including traffic conditions, number of buses on the same route, number of passengers, etc. e.g. if traffic is heavy then this could delay buses arriving at Jacob's bus stop and also cause buses to take longer to get to his destination from his stop.

Note: Jacob's model doesn't take into account whether the resulting distribution when combining a uniform and a triangular distribution, assuming independence, is triangular. However, candidates are not expected to combine continuous random variables, nor have knowledge of combinations of non-uniform discrete random variables in terms of resultant distributions, so this is not expected evidence (but could be accepted if given).

Sufficiency Statement

For each question:

Score 1 – 4 No award	5 – 6 Scholarship level	7 – 8 Outstanding Scholarship level
Shows understanding of relevant statistical and probability concepts and methods, and some progress towards applying this in context.	Application of high-level statistical analysis and critical thinking, knowledge and skills, to complex situations. Shows logical development, precision and clarity of ideas.	In addition to the requirements of Scholarship, demonstration of perception and insight, sophisticated integration and abstraction of ideas, independent reflection and extrapolation, and convincing communication.

Cut Scores

Scholarship	Outstanding Scholarship
22 – 30	31 – 40