

Chapter 1: Image Manipulation and Transformation



Identity

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reflection

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

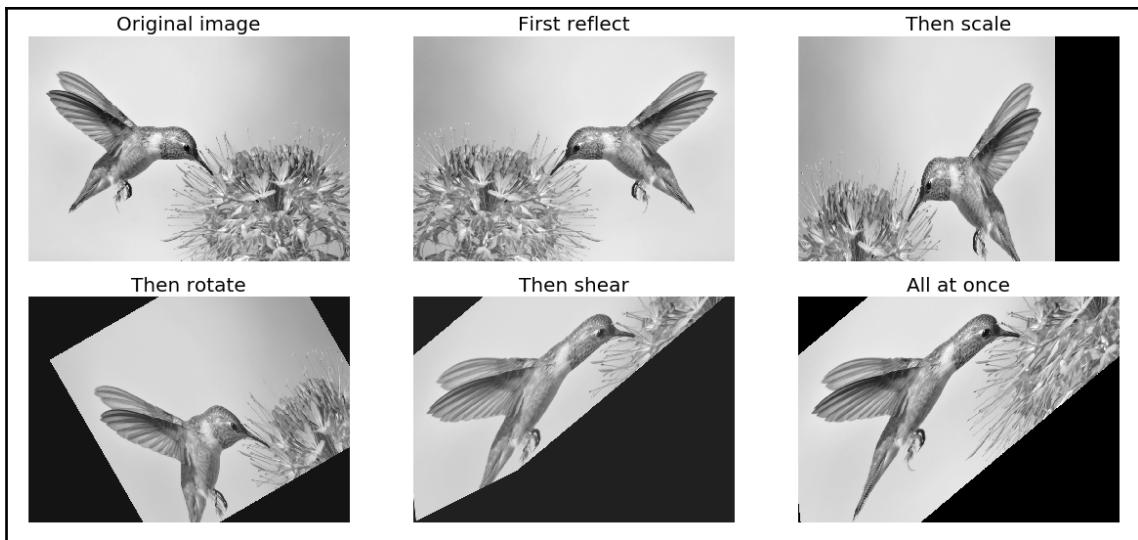
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear-X

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \lambda_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear-Y

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \lambda_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



2D homography (projective transformation)

Definition:

A 2D **homography** is an invertible mapping h from P^2 to itself such that three points x_1, x_2, x_3 lie on the same line if and only if $h(x_1), h(x_2), h(x_3)$ do.

Theorem:

A mapping $h: P^2 \rightarrow P^2$ is a homography if and only if there exist a non-singular 3×3 matrix H such that for any point in P^2 represented by a vector x it is true that $h(x) = Hx$

Definition: Homography

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{or} \quad x' = Hx$$

Line preserving

$$\begin{array}{l} n \text{ pairs of points} \\ \left[\begin{array}{ccccccc} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{array} \right] = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} \\ \begin{array}{c} A \\ 2n \times 9 \\ h \\ 9 \end{array} \end{array}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix} h = 0 \Leftrightarrow Ah = 0$$

Find approximate solution

- Additional constraint needed to avoid 0, e.g. $\|h\| = 1$
- $Ah = 0$ not possible, so minimize $\|Ah\|$

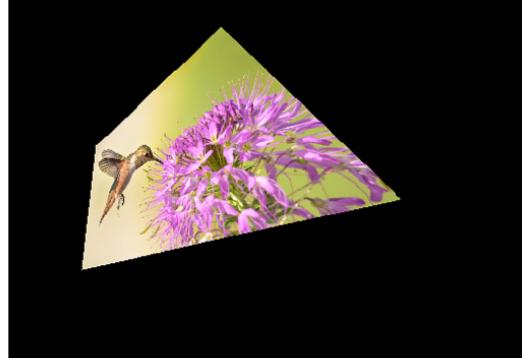
Defines a least squares problem: minimize $\|Ah - 0\|^2$

- Since h is only defined up to scale, solve for unit vector \hat{h}
- Obtain SVD of A
- Solution for h is last column of V = singular value of A
- Solution: $\hat{h} = \text{eigenvector of } A^T A \text{ with smallest eigenvalue}$
- Works with 4 or more points
- Determine H from \hat{h}

ref https://ags.cs.uni-kil.de/fileadmin/inf_aggs/3dcv-ws11-12/3DCV_WS11-12_lec04.pdf

ref <http://6.869.csail.mit.edu/fa12/lectures/lecture13transac/lecture13transac.pdf>

Source image



Destination image



Source image



Destination image



Output Image



www.shutterstock.com • 146430692

Original Gray-scale image



Sketch with anisotropic diffusion



Sketch with DOG



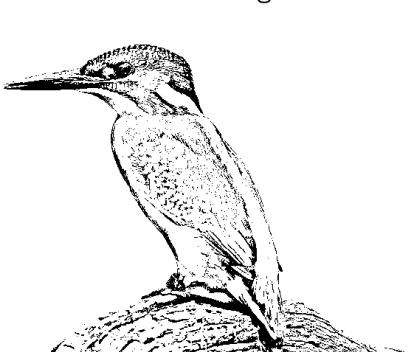
Sketch with XDOG

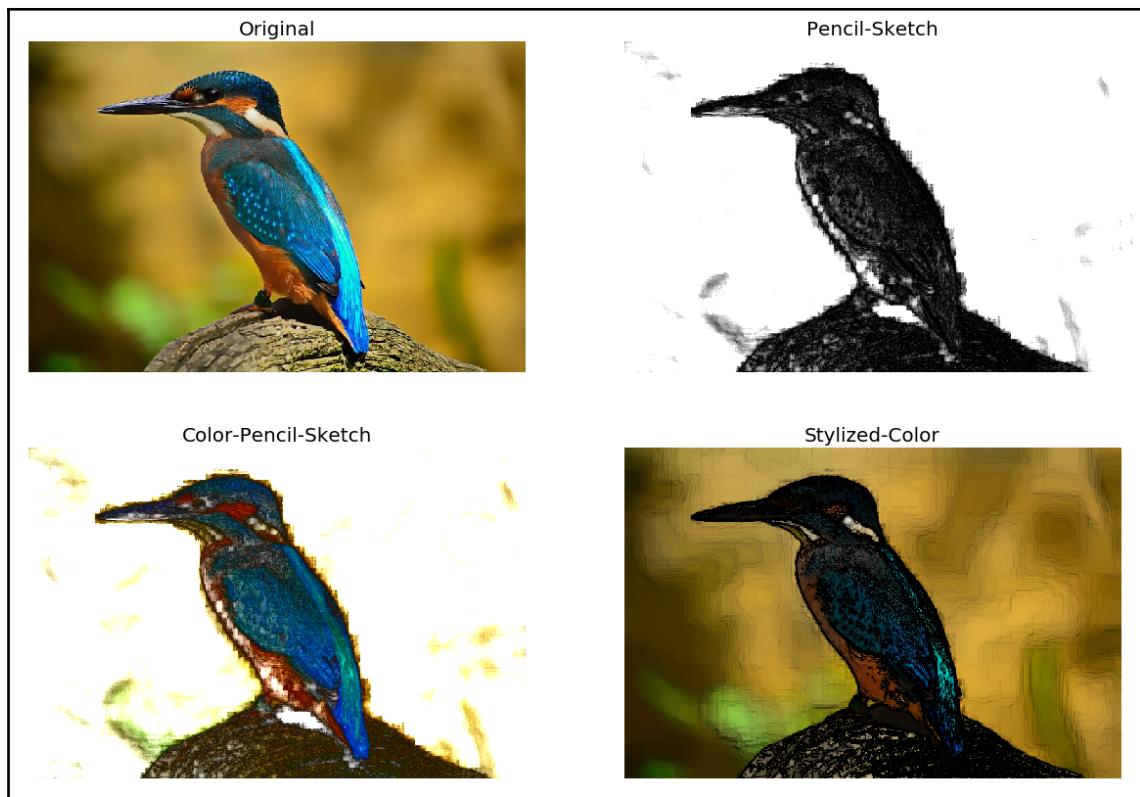


Sketch with Dodge



Sketch with Dodge2





Original Image



Cartoonized Image





An input image



Output image

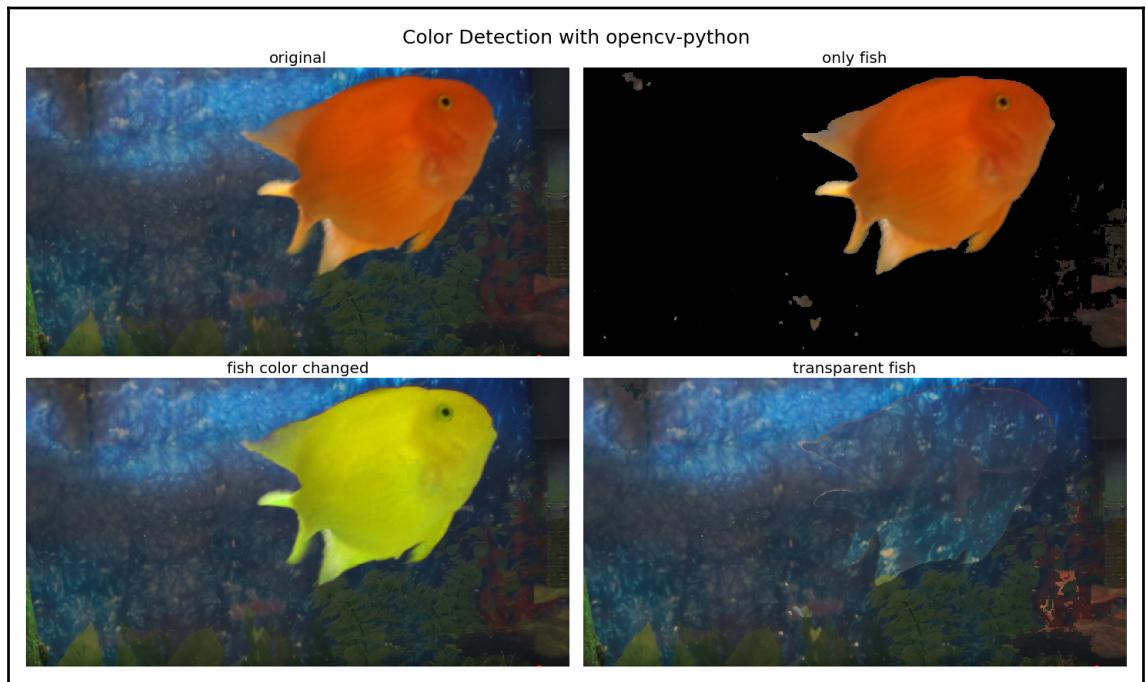
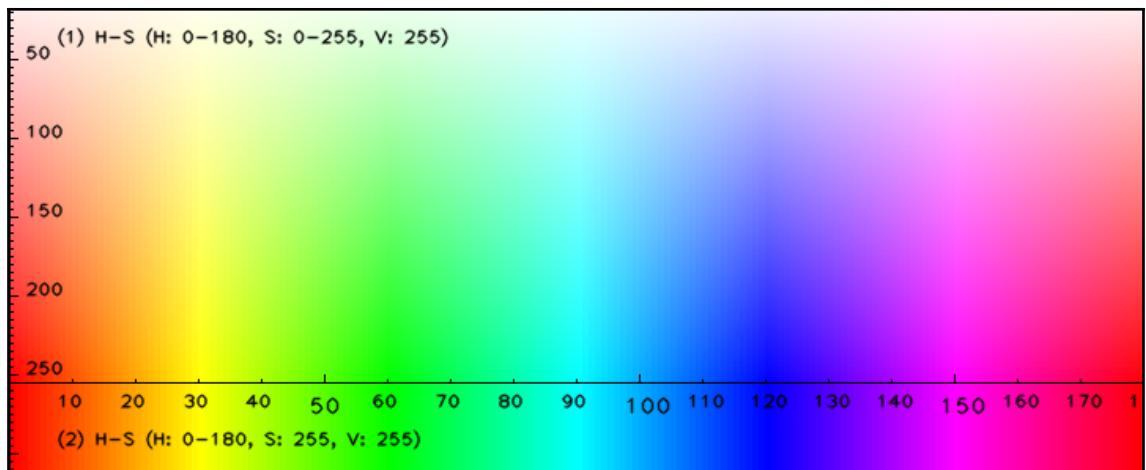


An input image

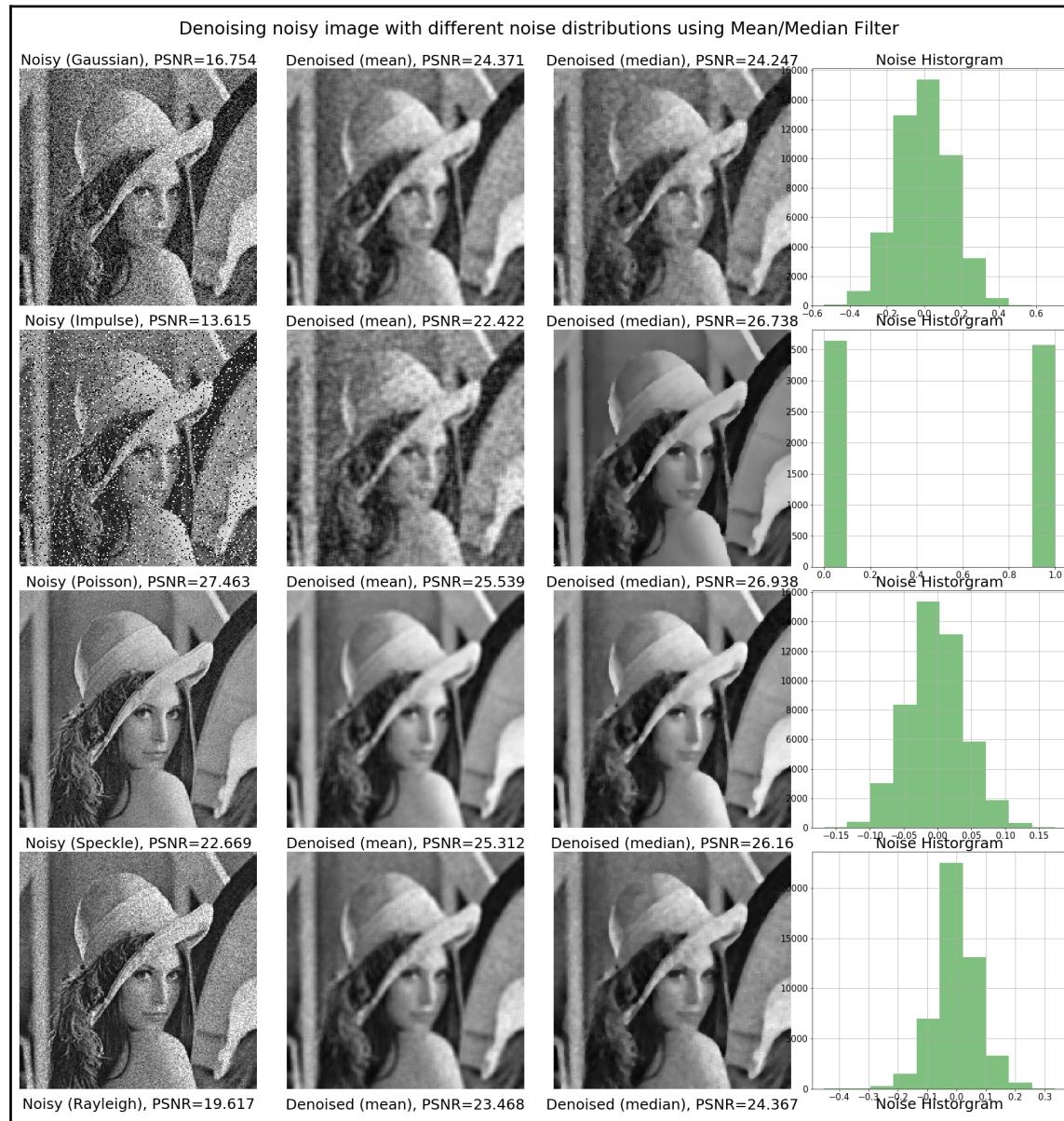


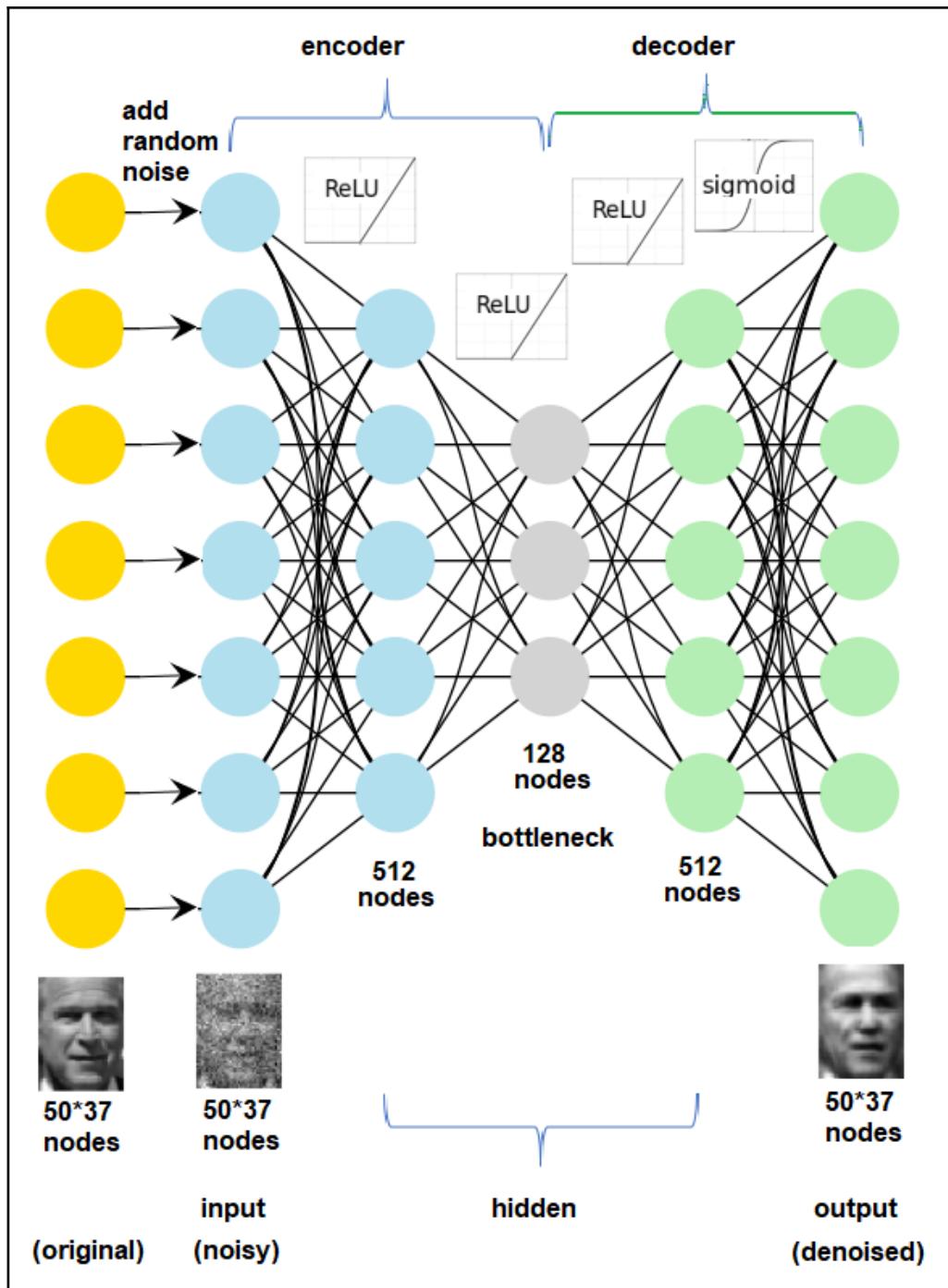
Output image





Chapter 2: Image Enhancement





Original (Row 1), Noisy input (Row 2), DAE output (Row 3) images
and some features (Rows 4-8) learnt by the DAE in Epoch 20



Original (Row 1), Noisy input (Row 2), DAE output (Row 3) images
and some features (Rows 4-8) learnt by the DAE in Epoch 60





Original



Noisy



Anisotropic Diffusion (Perona Malik eq 1, iter=20, $\kappa = 20$)



Anisotropic Diffusion (Perona Malik eq 2, iter=50, $\kappa = 50$)



Anisotropic Heat Diffusion Equation

$$I_t = \operatorname{div}(c(x, y, t) \nabla I) = c(x, y, t) \cdot \Delta I + \nabla c \cdot \nabla I$$

Image divergence Gradient conductivity Laplacian
at time t function

$$= \nabla \cdot (c \nabla I) = \frac{\partial}{\partial x} (c I_x) + \frac{\partial}{\partial y} (c I_y)$$

calculate the conductivity function c every iteration based on the current image I

$$c(x, y, t) = g(\|\nabla I(x, y, t)\|)$$

Perona-Malik edge-stopping function

$$g(\nabla I) = e^{-(\|\nabla I\|/K)^2} \quad (\text{equation 1})$$

$$g(\nabla I) = \frac{1}{1 + \left(\frac{\|\nabla I\|}{K}\right)^2} \quad (\text{equation 2})$$

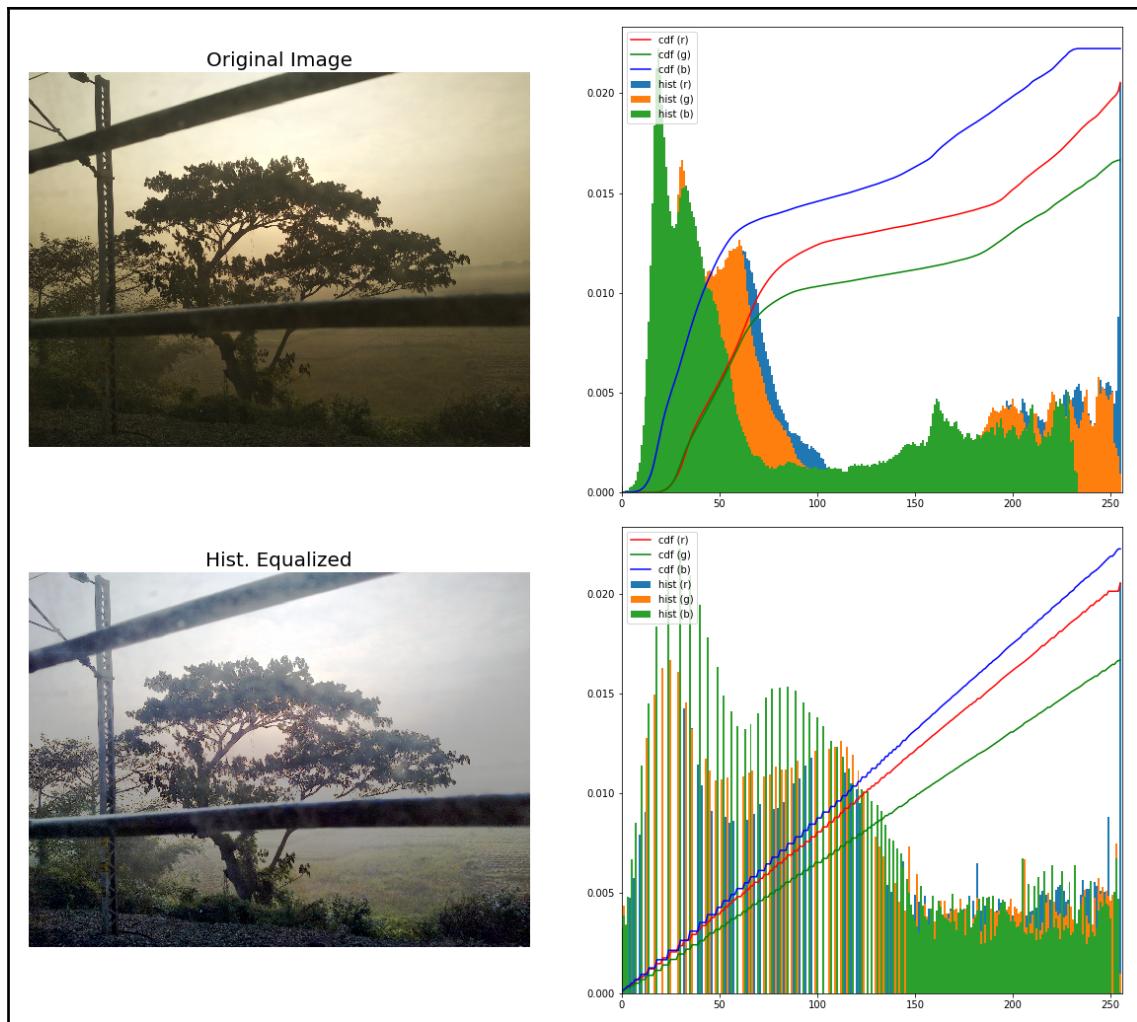
Taken from <http://image.diku.dk/imagecanon/material/PeronaMalik1990.pdf>

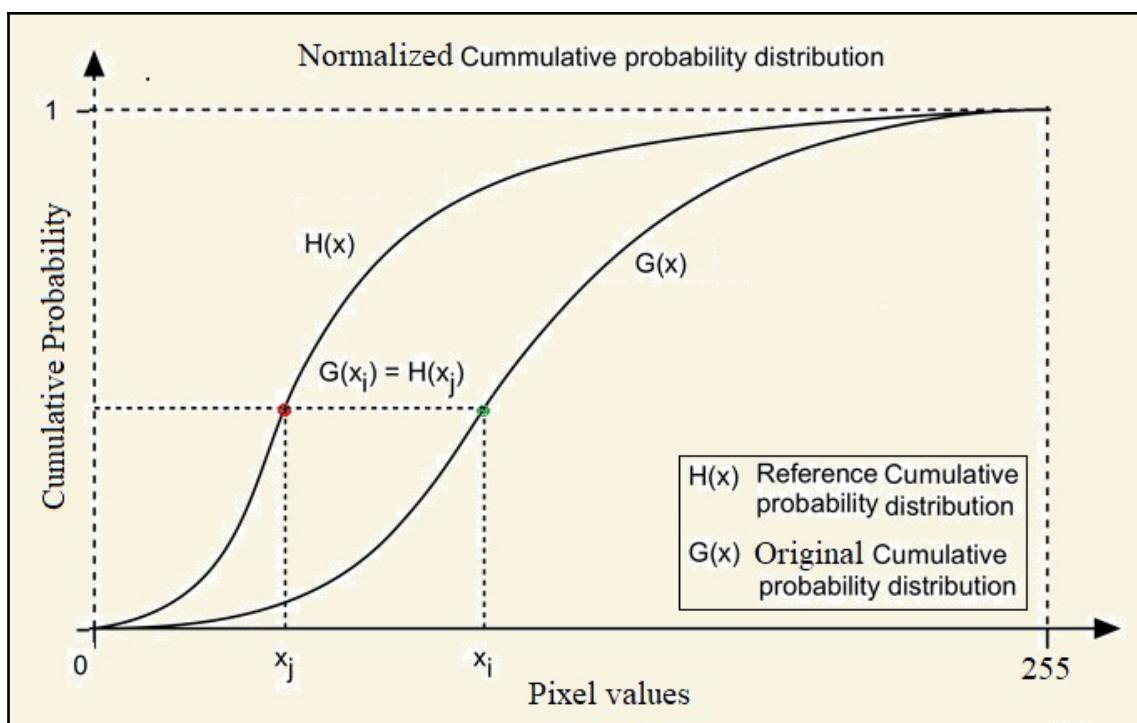
$$s_k = T(r_k) = \sum_{j=0}^k P_r(r_j) = \sum_{j=0}^k n_j / N$$

$$0 \leq r_k \leq 1, \quad k = 0, 1, 2, \dots, 255$$

N: total number of pixels

n_j : frequency of pixel with gray-level j





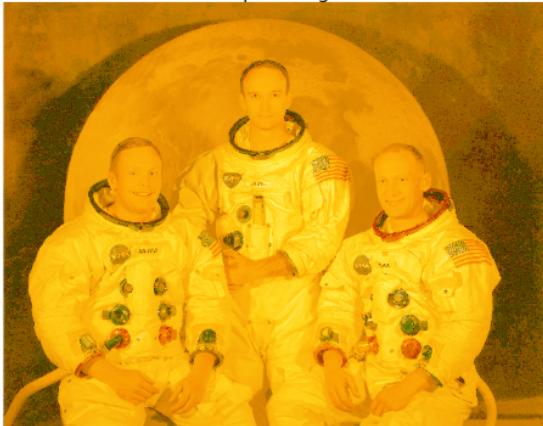
Input Image



Template Image



Output Image



Input Image



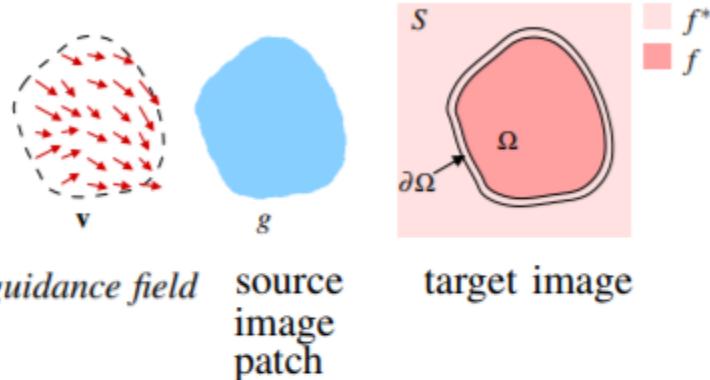
Template Image



Output Image



Guided Interpolation



Solve $\min_f \iint_{\Omega} |\nabla f - \mathbf{v}|^2$ with $f|_{\partial\Omega} = f^*|_{\partial\Omega}$

By Euler-Lagrange equation

$$\Delta f = \nabla \cdot \mathbf{v} \text{ over } \Omega, \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega},$$

Poisson equationDirichlet boundary conditions

Taken from

https://www.cs.virginia.edu/~connelly/class/2014/comp_photo/proj2/poisson.pdf

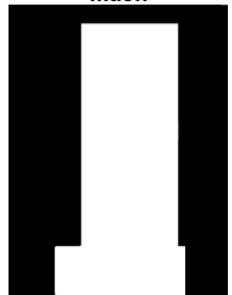
Target

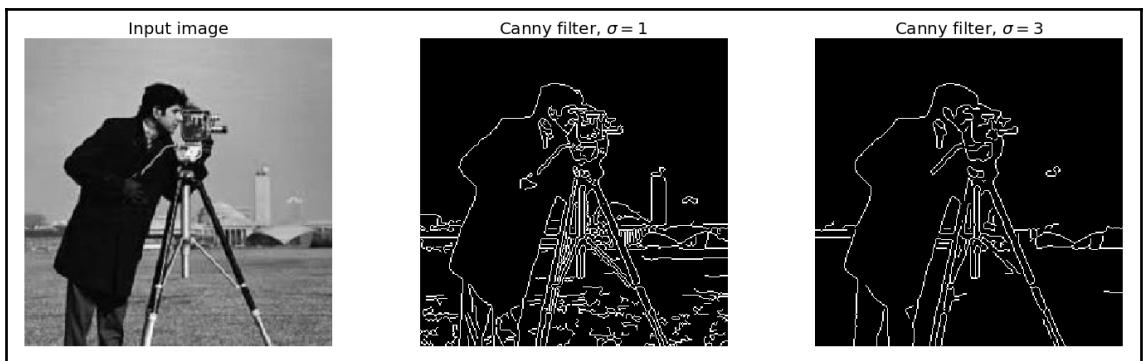


Source



Mask







LoG with zero-crossing, sigma=2



LoG with zero-crossing, sigma=4

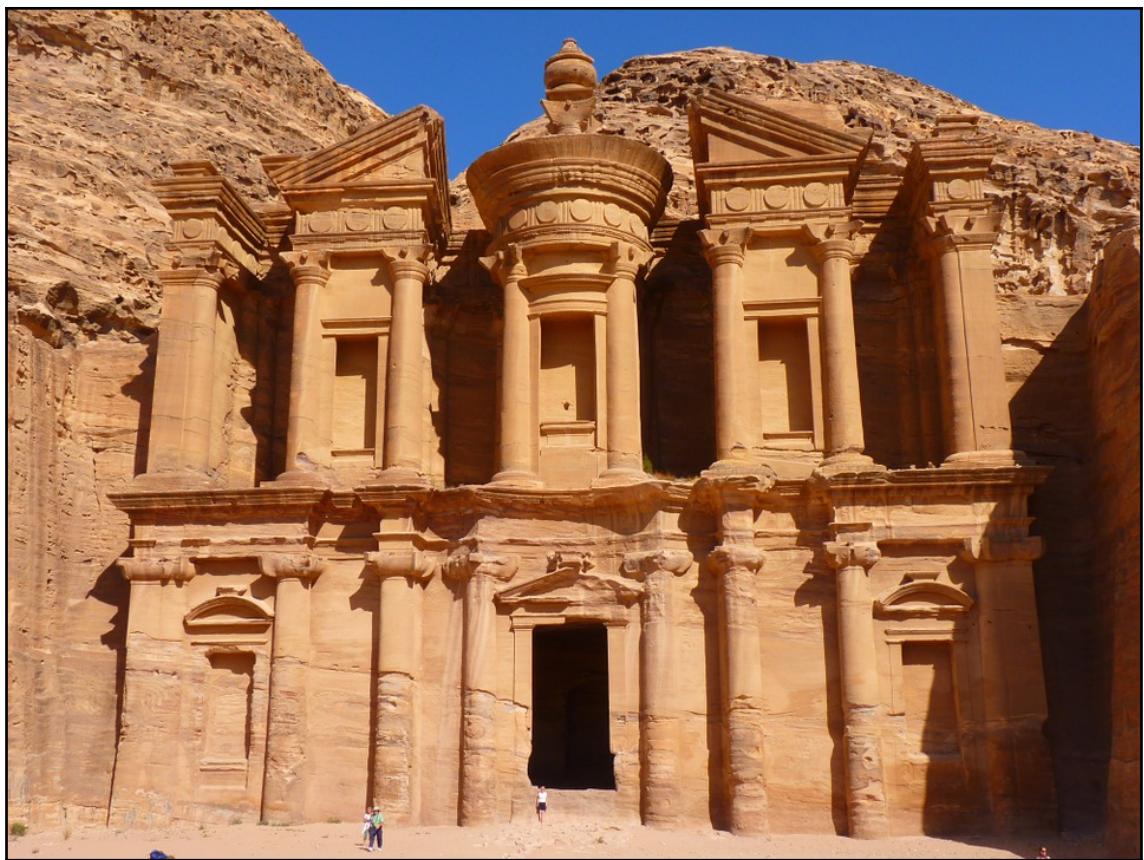


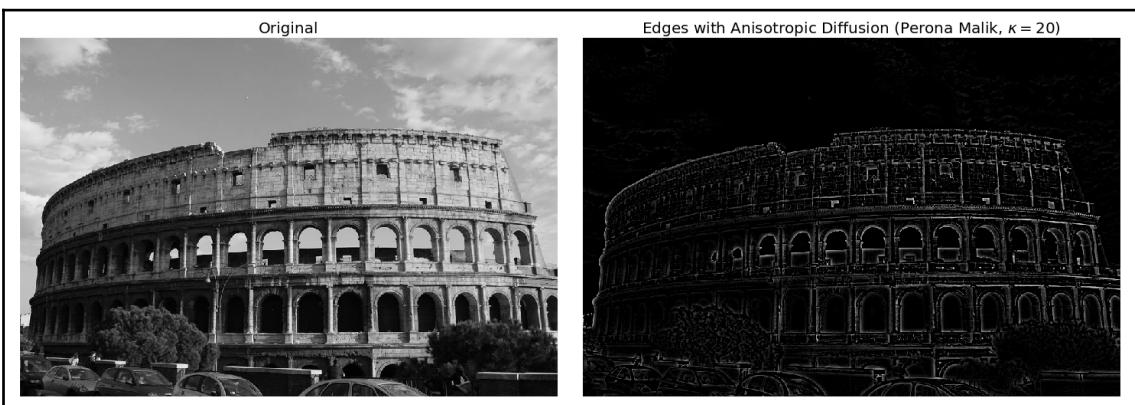
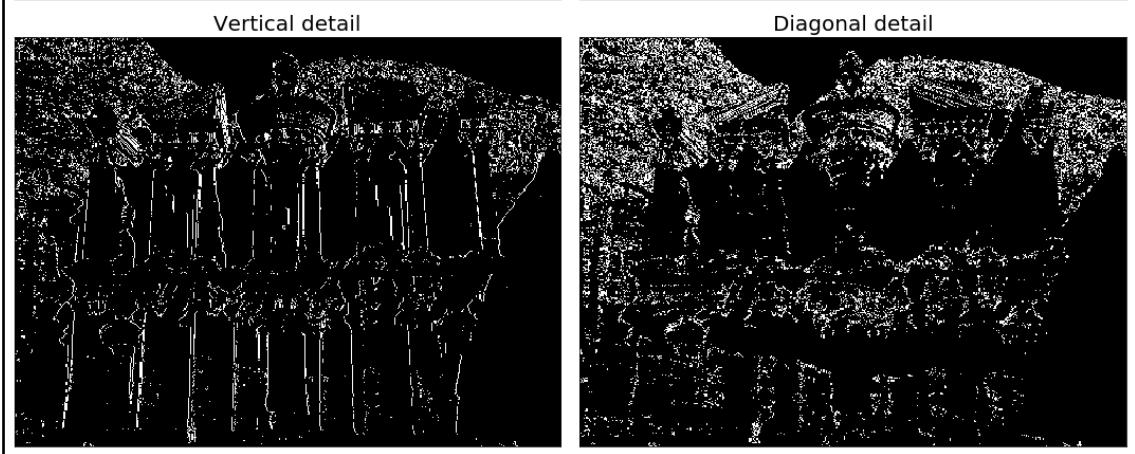
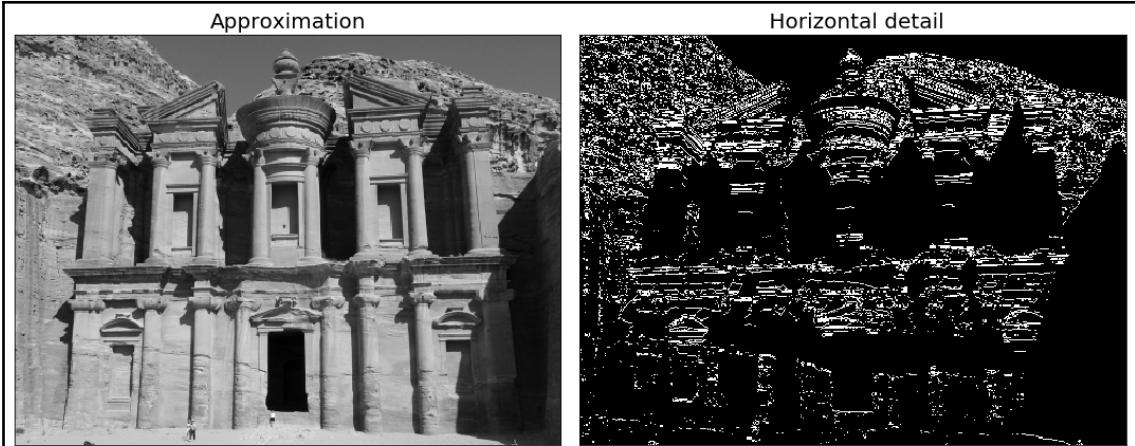
LoG with zero-crossing, sigma=6



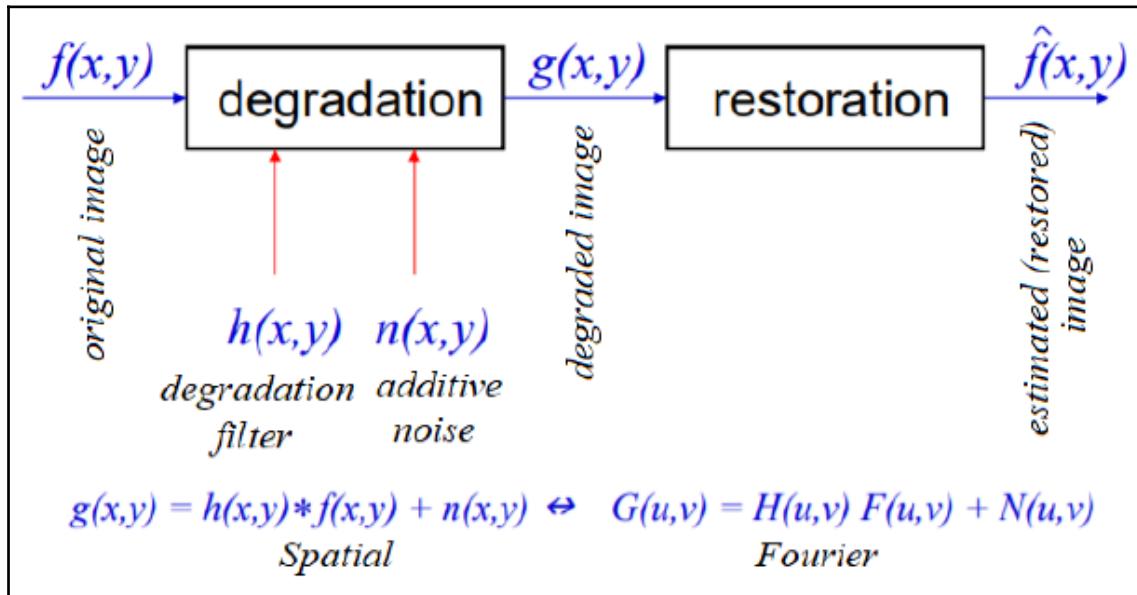
LoG with zero-crossing, sigma=8







Chapter 3: Image Restoration



Original image



Noisy blurred image: PSNR=24.099



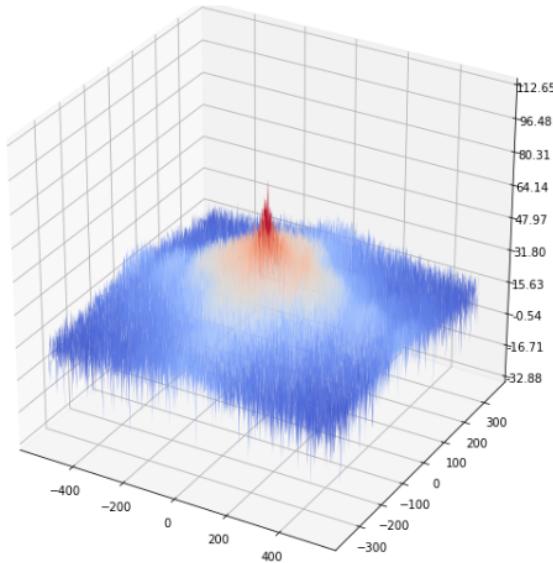
Self tuned Wiener restoration: PSNR=24.664



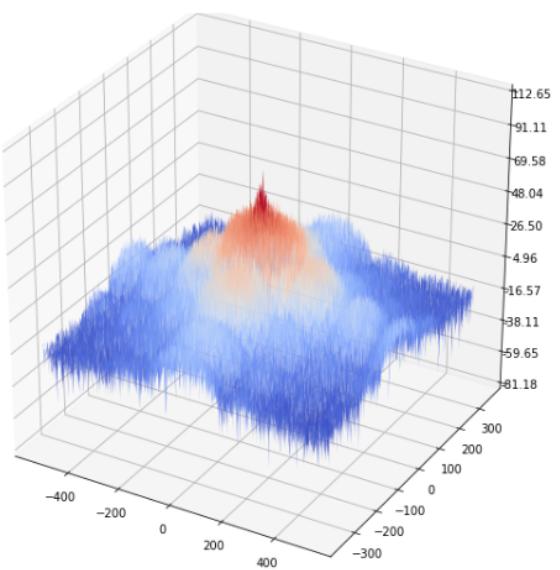
Wiener restoration: PSNR=26.036



Restored Image Spectrum (unsupervised Wiener)



Restored Image Spectrum (Wiener)



Wiener Filter

objective $\min_W E[(f - \hat{f})^2]$
 $= \min_W E[|F(u, v) - \hat{F}(u, v)|^2]$ by Parseval's Theorem
s.t. $\hat{F}(u, v) = G(u, v)W(u, v)$

solution:

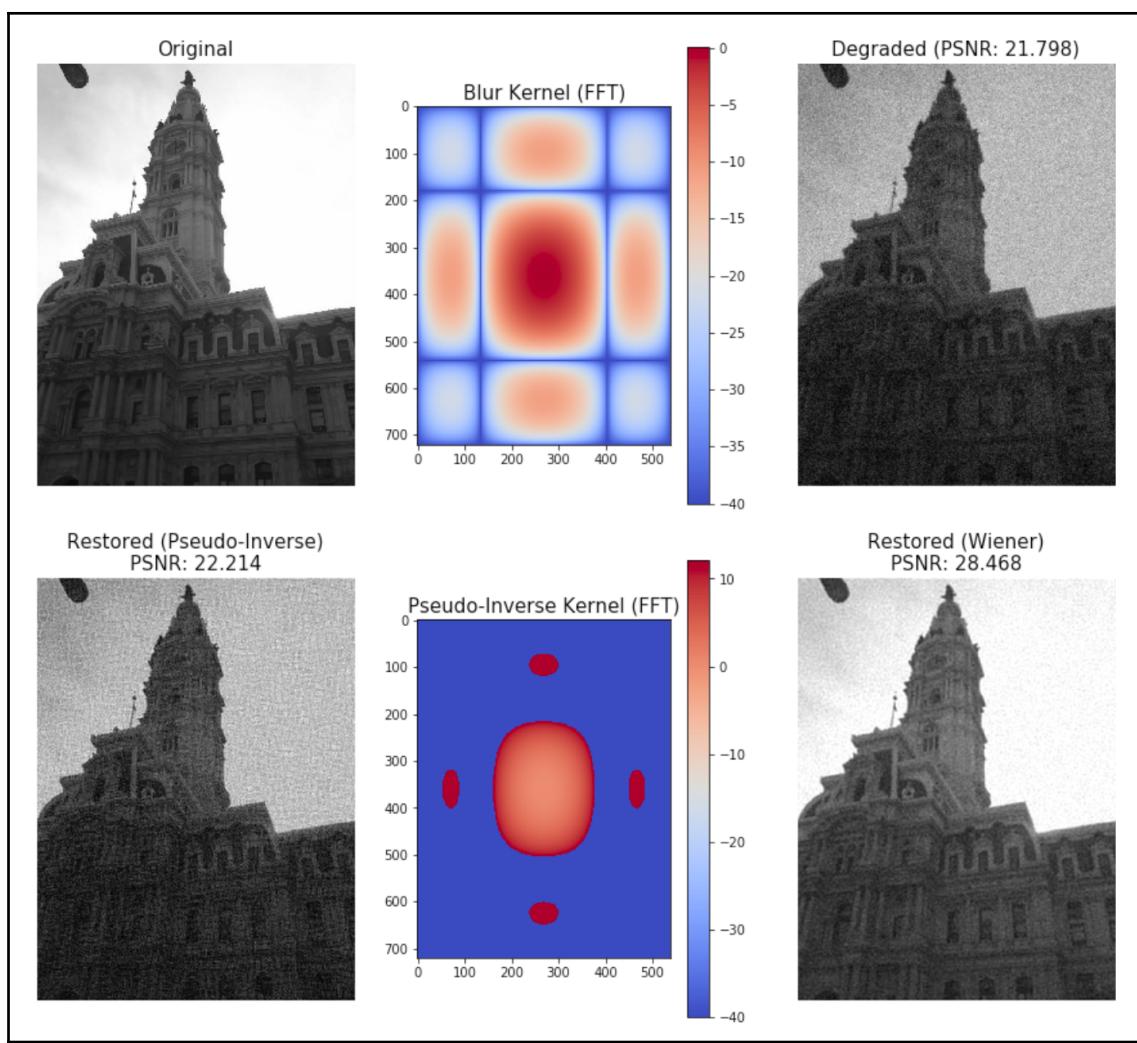
$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + \frac{|N(u, v)|^2}{|F(u, v)|^2}} = \frac{H^*(u, v)}{|H(u, v)|^2 + K}$$
$$= \boxed{\frac{1}{H(u, v)}} \cdot \frac{|H(u, v)|^2}{|H(u, v)|^2 + \frac{|N(u, v)|^2}{|F(u, v)|^2}}$$

inverse filter 1/SNR

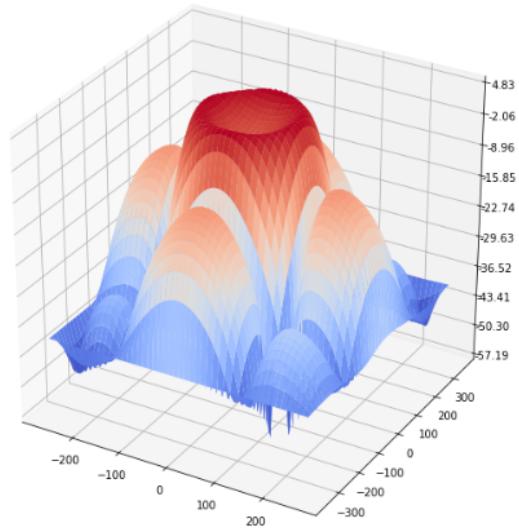
$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + \lambda |\Lambda_D|^2}$$

balance Freq. response
of Laplacian

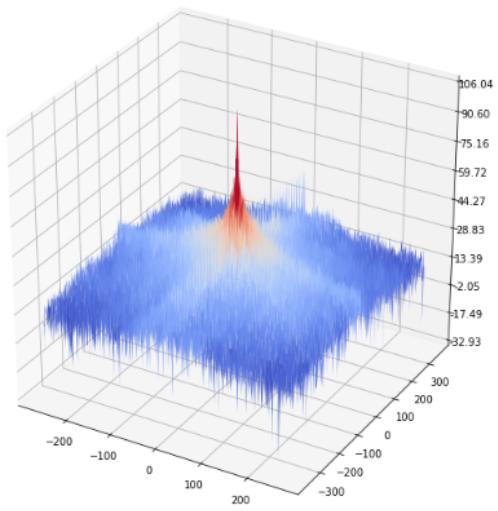
*scikit-image
restoration's
wiener()*



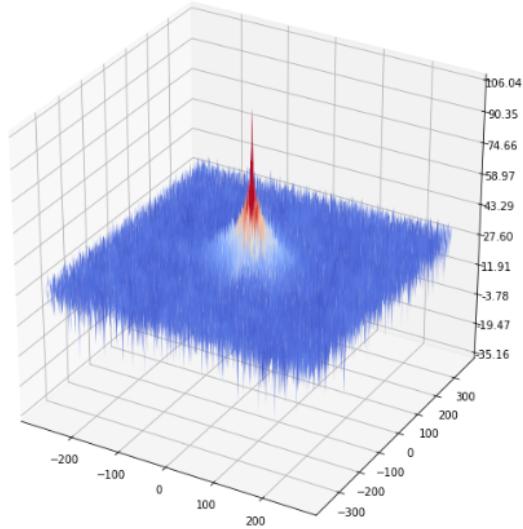
CLS Filter Spectrum



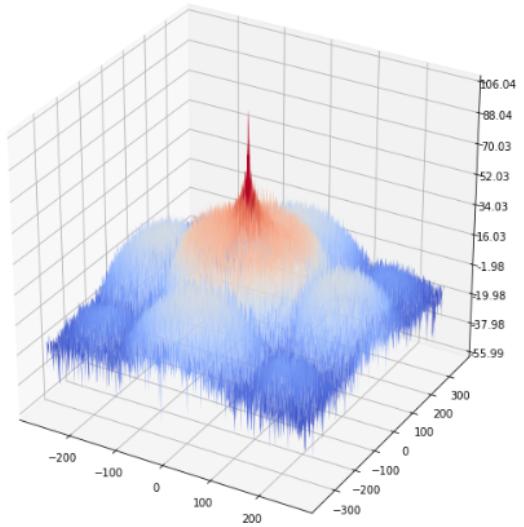
Original Image Spectrum



Corrupted Image Spectrum



Restored Image Spectrum (CLS)



Constrained Least-Squares Filter

$$g = Hf + n \text{ (degradation equation)}$$

$$\text{objective: } \min_f \|g - Hf\|_2^2$$

$$\text{s.t. } \|Cf\|_2^2 < \varepsilon \quad (\text{smoothness constraint})$$

$$\text{solution: } \hat{f} = \underset{\substack{\text{restored} \\ \text{(regularization parameter)}}}{(H^T H + \lambda C^T C)^+} \underset{\substack{\text{Lagrange} \\ \text{multiplier}}}{(H^T g)}$$

$$\text{objective: } \min_f (\|g - Hf\|_2^2 + \lambda \|Cf\|_2^2)$$

Lagrange
multiplier

$$\hat{F}(u, v) = \frac{H^*(u, v) G(u, v)}{\|H(u, v)\|^2 + \lambda \|C(u, v)\|^2}$$

(in freq. domain)

Original Image

Hands-On Image Processing with Python

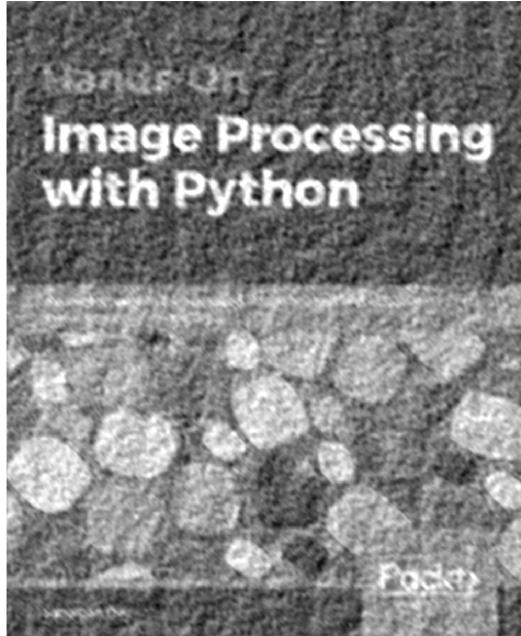
Expert techniques for advanced image analysis and effective interpretation of image data



Degraded Image (with Motion-blur + Noise)
PSNR: 16.592



Restored Image (with Wiener) PSNR: 19.045



Restored Image (with CLS, $\lambda = 7.5$) PSNR: 20.916

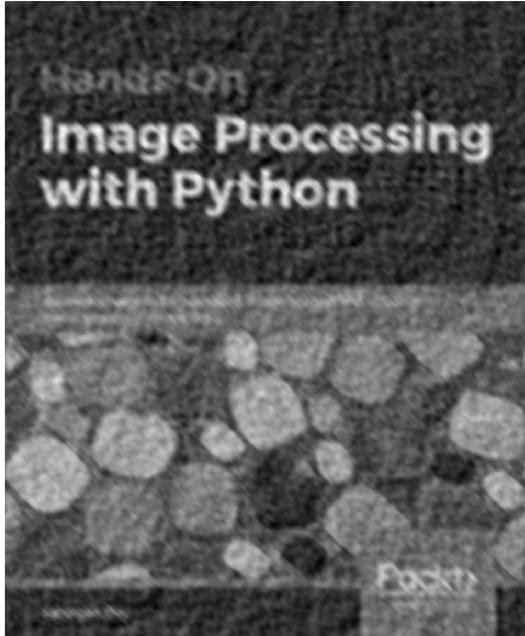
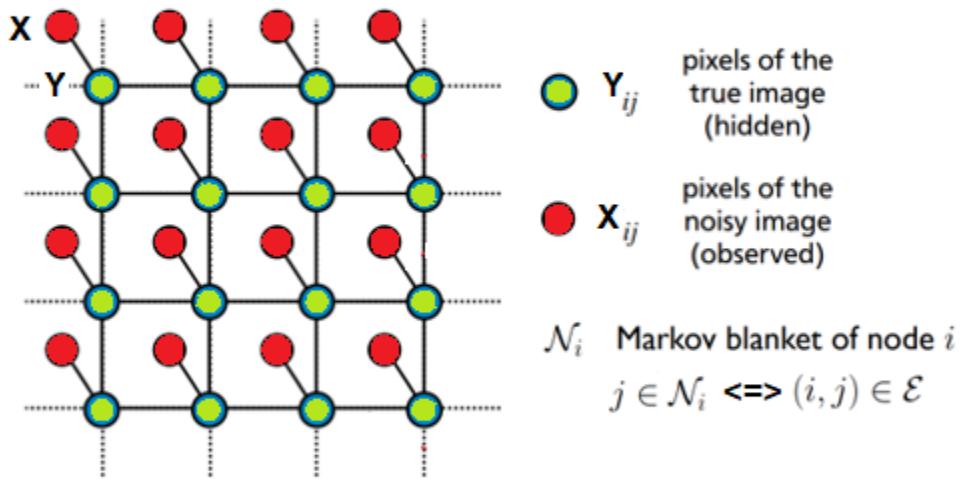


Image Denoising with MRF

Pixel is a node in \mathcal{G} with 4-connected nbds

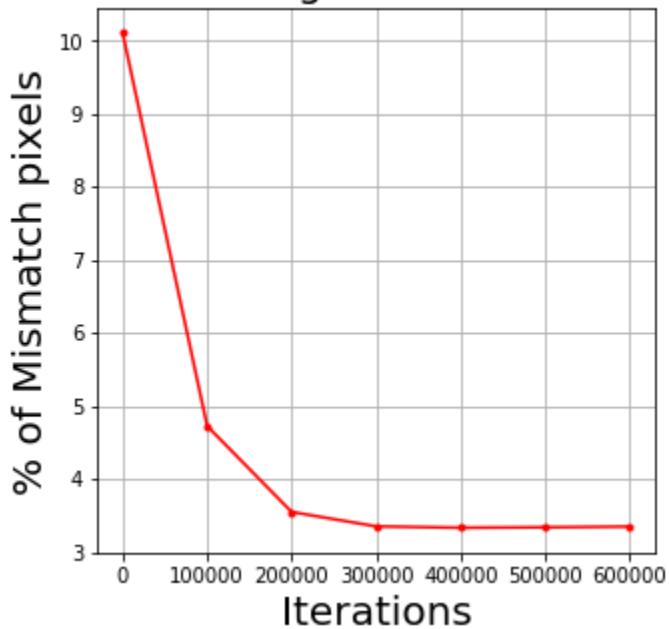
$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$



The energy function (to minimize)

$$E_{\text{total}}(X, Y) = -\zeta \sum_{i,j}^N X_{i,j} \cdot X_{i\pm 1, j\pm 1} - \eta \sum_{i,j=1}^N X_{i,j} \cdot Y_{i,j}$$

% Mismatch of Original and Denoised Image



original image



noisy image



denoised image with MRF

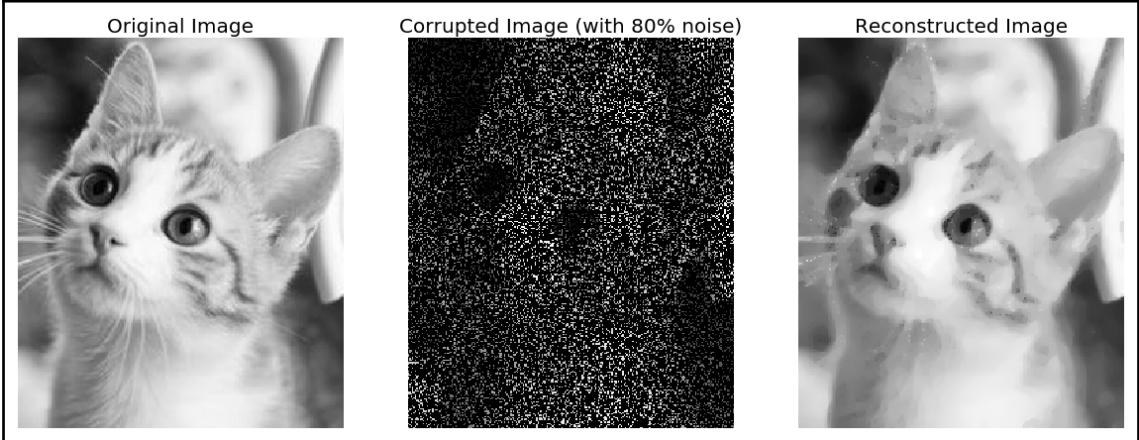


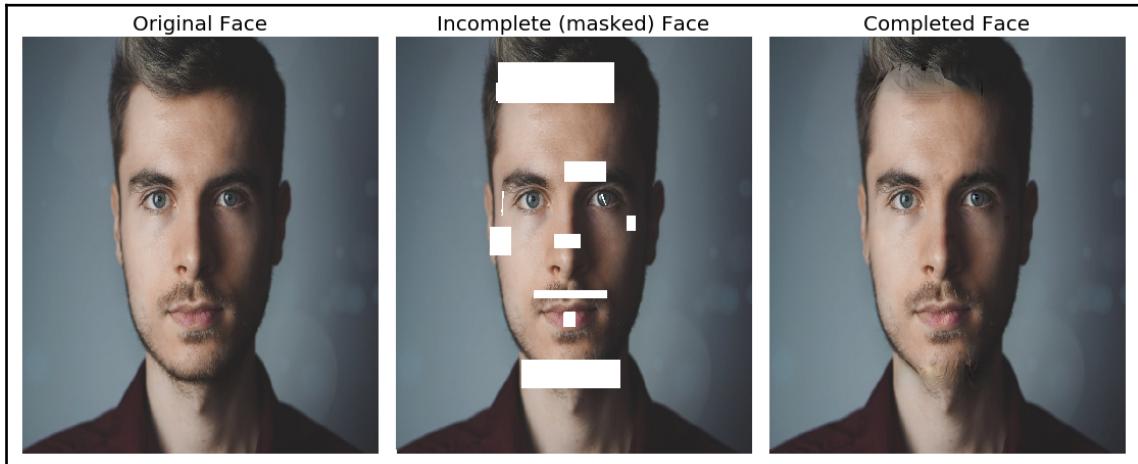


Forward-Backward Splitting with Total Variation

$$\min_x \tau \|g(x) - y\|_2^2 + \|x\|_{\text{TV}}$$

regularization $\left\{ \begin{array}{l} \text{fidelity} \\ \text{L2-norm} \end{array} \right.$ $\left\{ \begin{array}{l} f \\ \text{prior} \end{array} \right.$





Dictionary learned from face patches
Train time 26.1s on 22256 patches

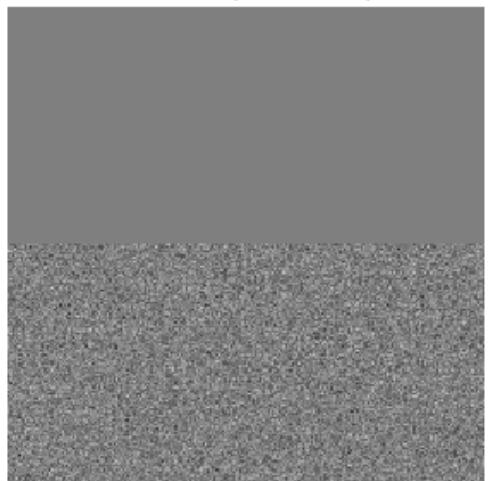


Distorted image

Image



Difference (norm: 13.24)



Orthogonal Matching Pursuit
2 atoms (time: 5.3s)

Image

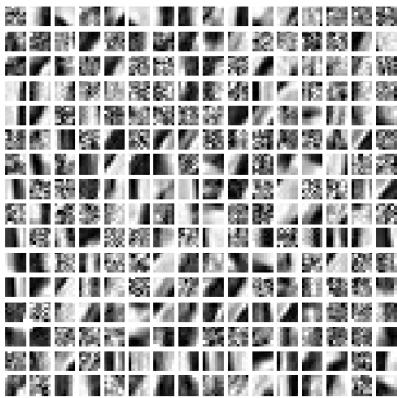


Difference (norm: 6.97)



Orthogonal Matching Pursuit with 2 atoms 3.1s
Minibatch Dictionary Learning: epoch=1, batch=100

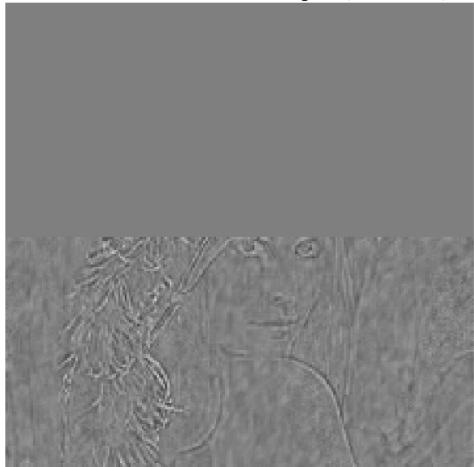
Dictionary
Dictionary learned from lena patche:
Train time 3.1s on 22256 patches



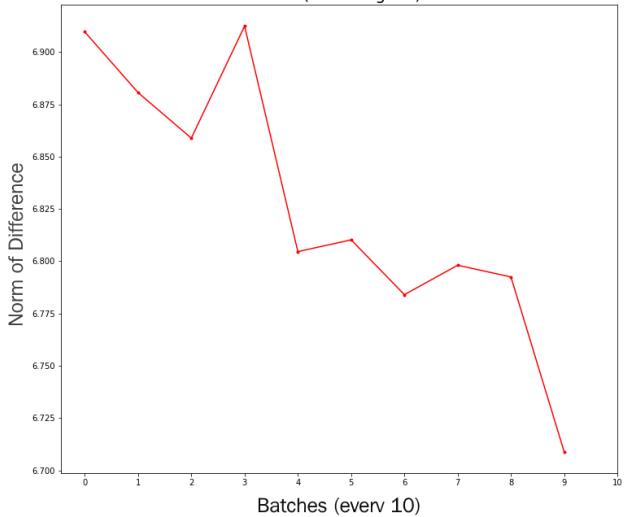
Reconstructed Image



Difference = Reconstructed - Original (norm: 6.71)



Error (w.r.t Original)



original image, zeros=0.215%



image (div 8), zeros=1.788%

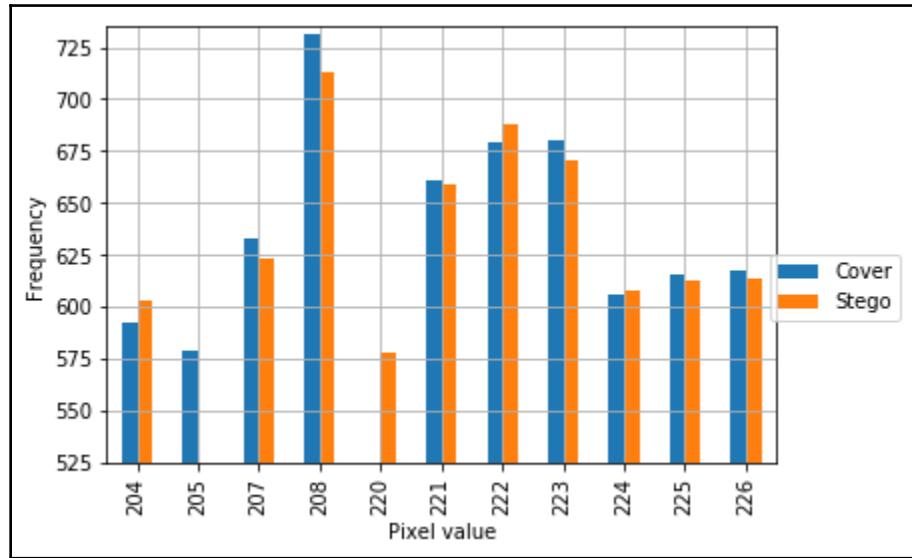
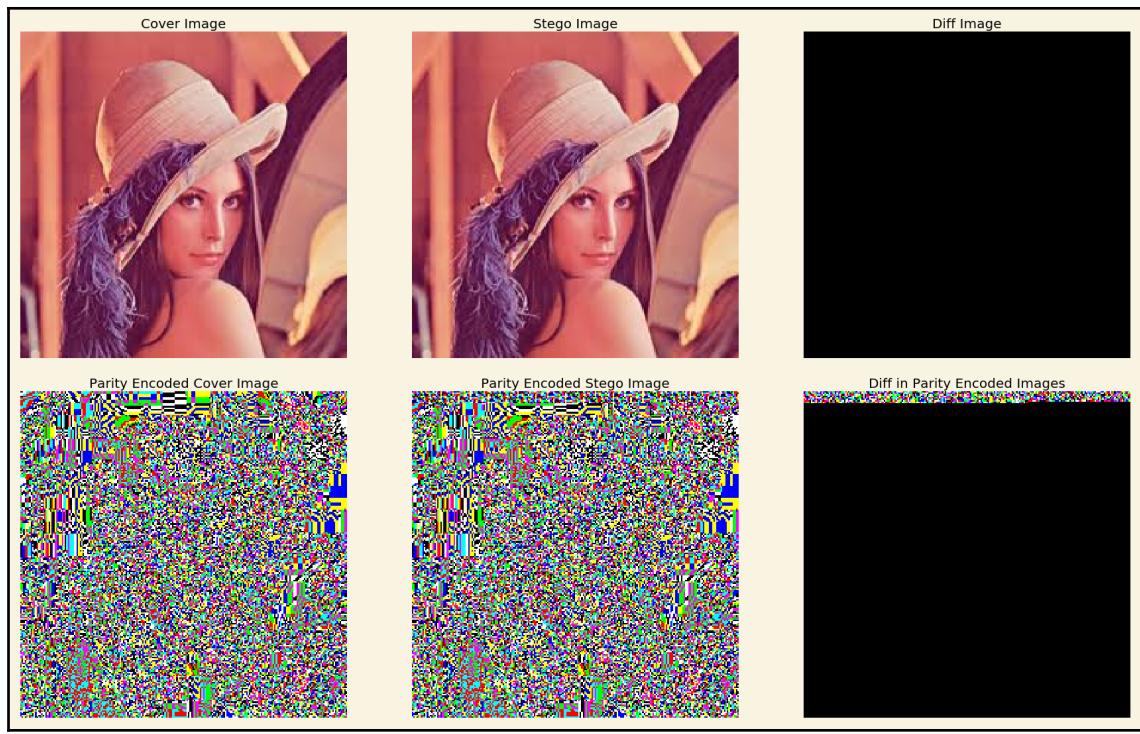


image with DWT (D8)
zeros in transform=88.924%

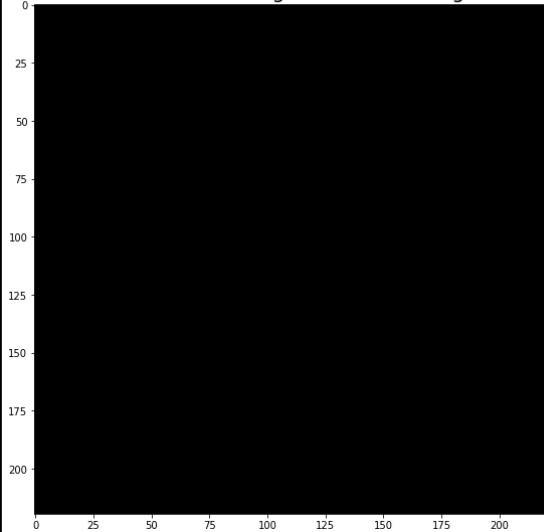


image with DWT (D8) + soft thresholding
zeros in transform=94.547%

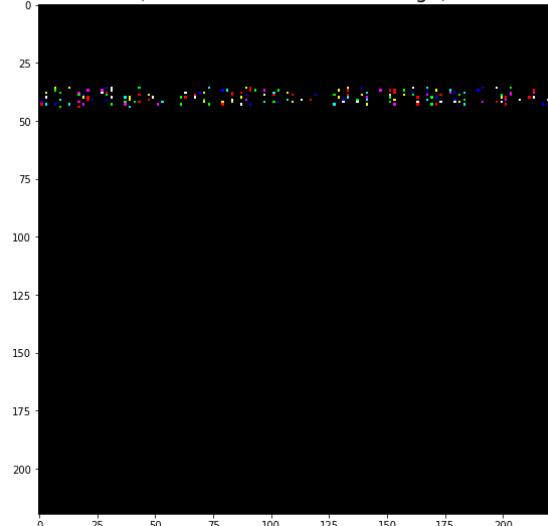




Difference of Stego and Cover Image



Steganalysis with Parity
(Locations of hidden message)



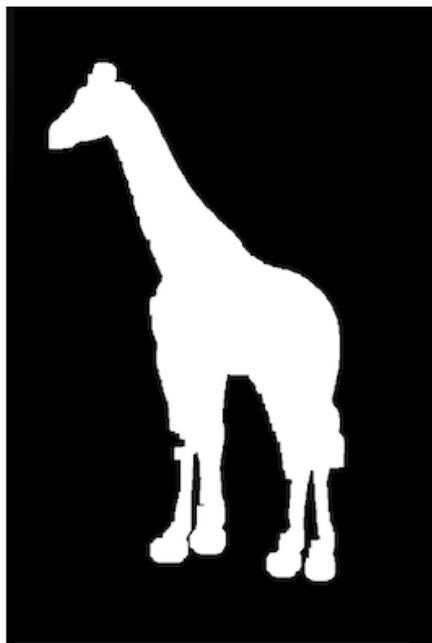
Chapter 4: Binary Image Processing

Erosion	$A \ominus B = \{z (B)_z \subseteq A\}$ shrink the (foreground) object	$\left. \begin{array}{l} \\ \end{array} \right\}$	<i>Fundamental Morphological Operations</i>
Dilation	$A \oplus B = \{z (\hat{B})_z \cap A \neq \emptyset\}$ grow the (foreground) object		
Opening	$A \circ B = (A \ominus B) \oplus B$	$\left. \begin{array}{l} \\ \end{array} \right\}$	<i>Compound Morphological Operations</i>
Closing	$A \bullet B = (A \oplus B) \ominus B$		
Hit or Miss Transform	$A \circledast \{B_1, B_2\} = (A \ominus B_1) \cap (A^c \ominus B_2)$	$\left. \begin{array}{l} \\ \end{array} \right\}$	<i>Compound Morphological Operations</i>
Duality	$(A \bullet B)^c = (A^c \circ \hat{B})$		
	  Binary Image (Set) Structuring Element (SE)		

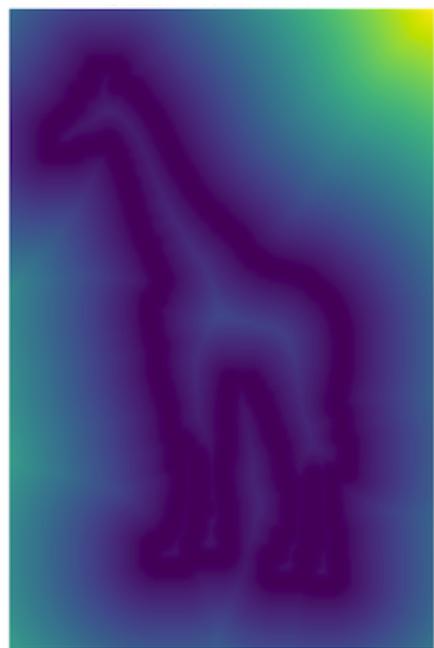
thresholded original image (otsu)



inverse eroded binary image (2x2 square) ...



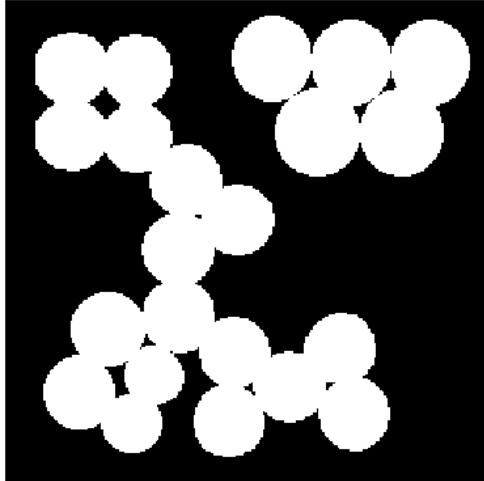
Euclidean distance transform



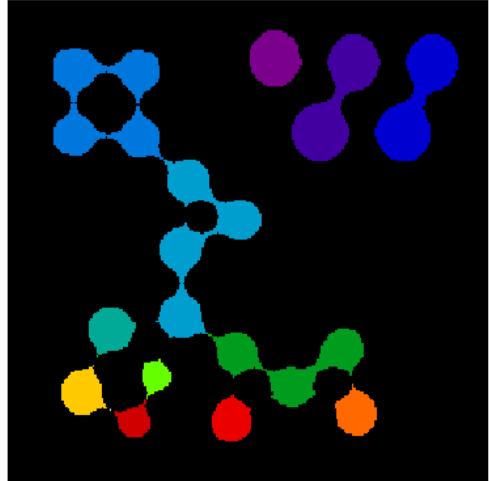
Morphological edges (size 3)



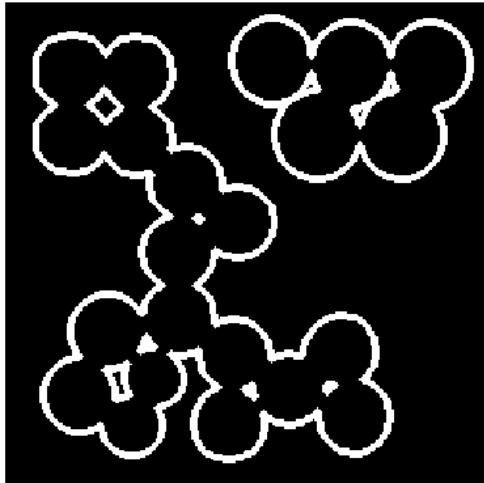
original binary image



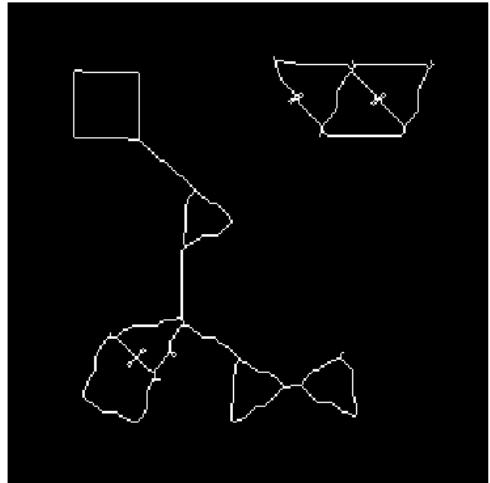
eroded with connected components (radius 8)



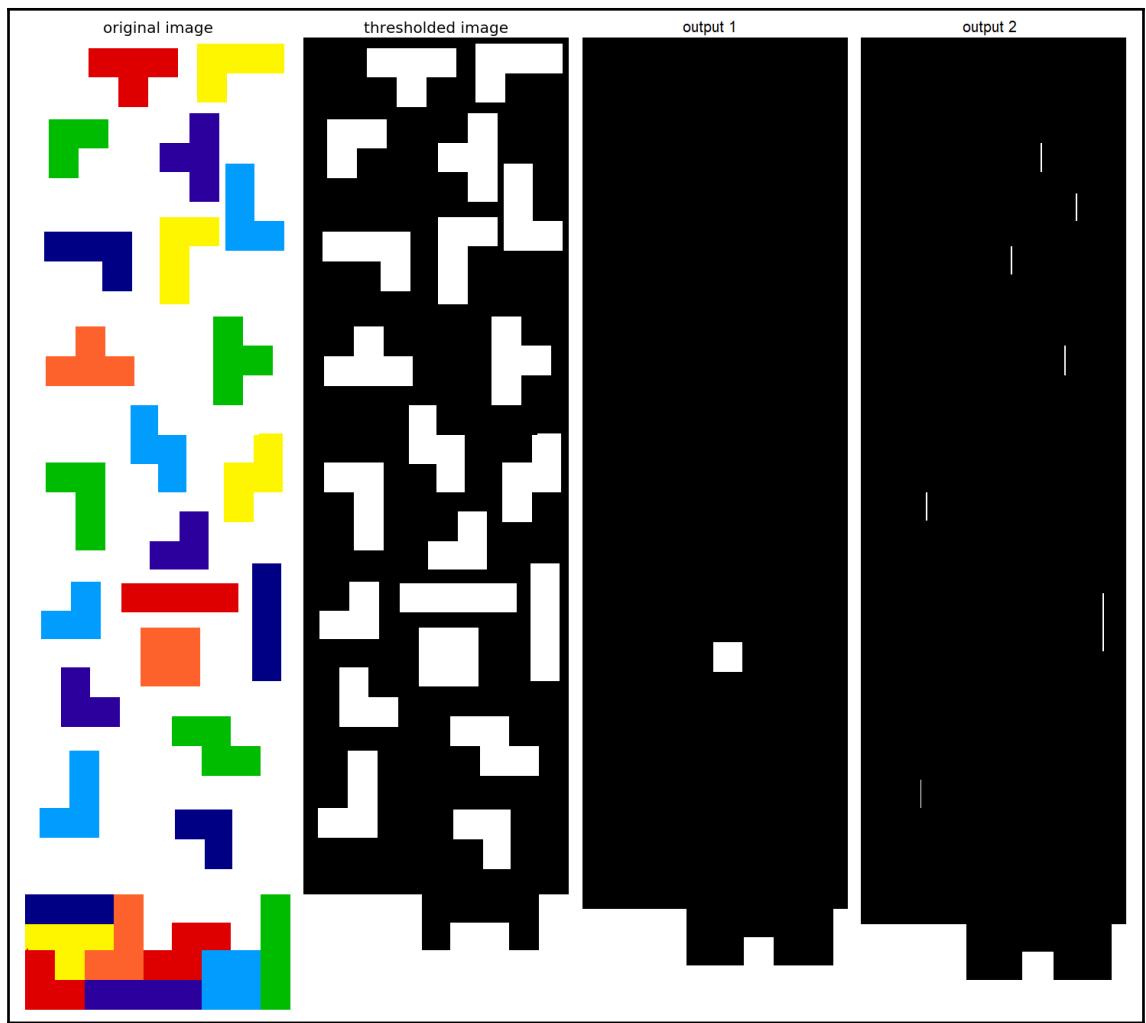
edges (radius 2)

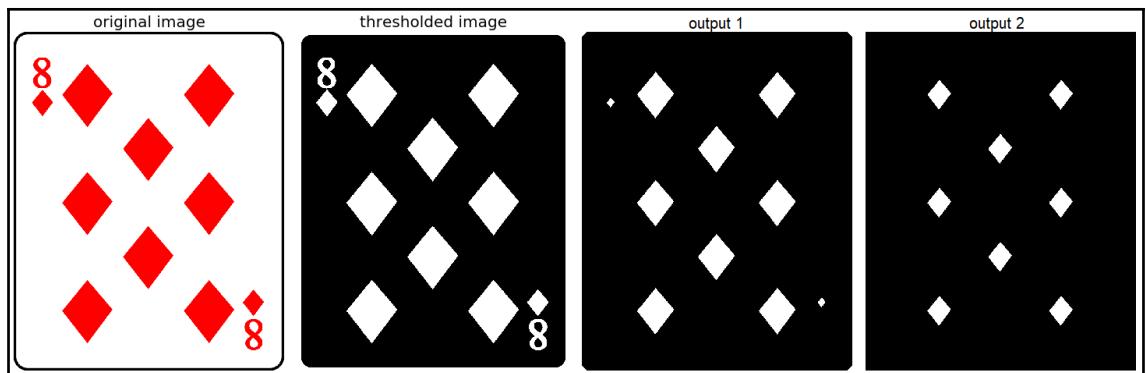


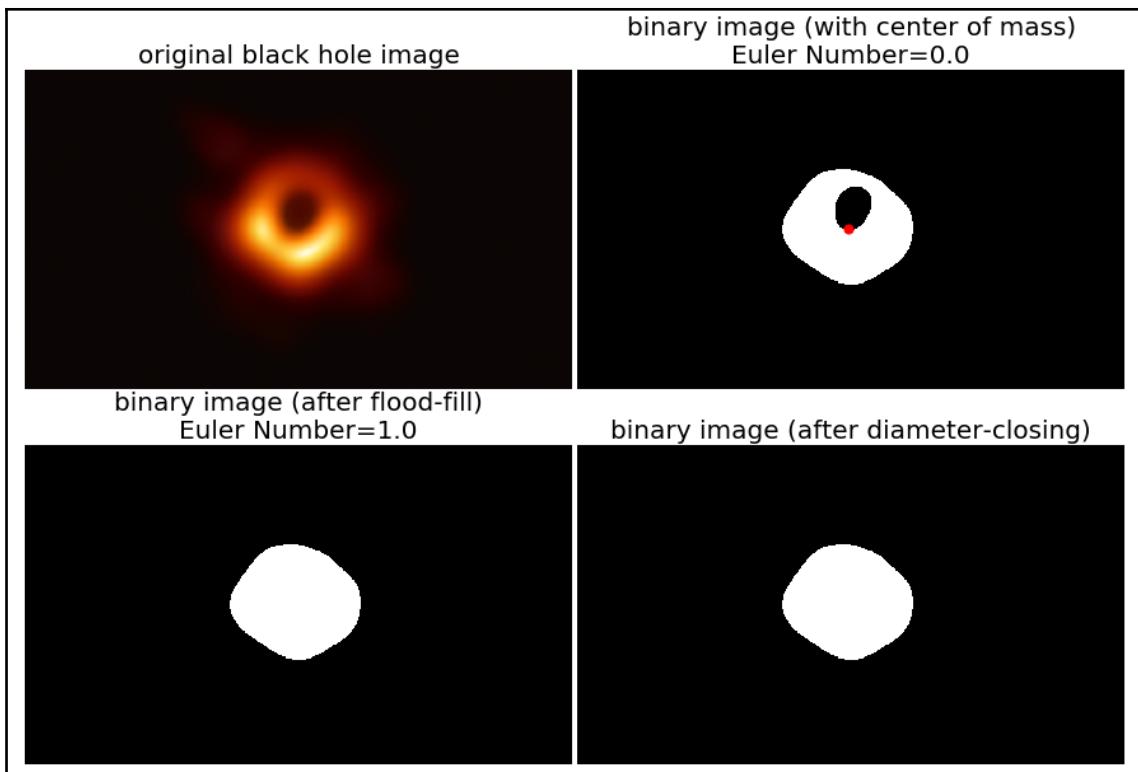
skeleton binary image

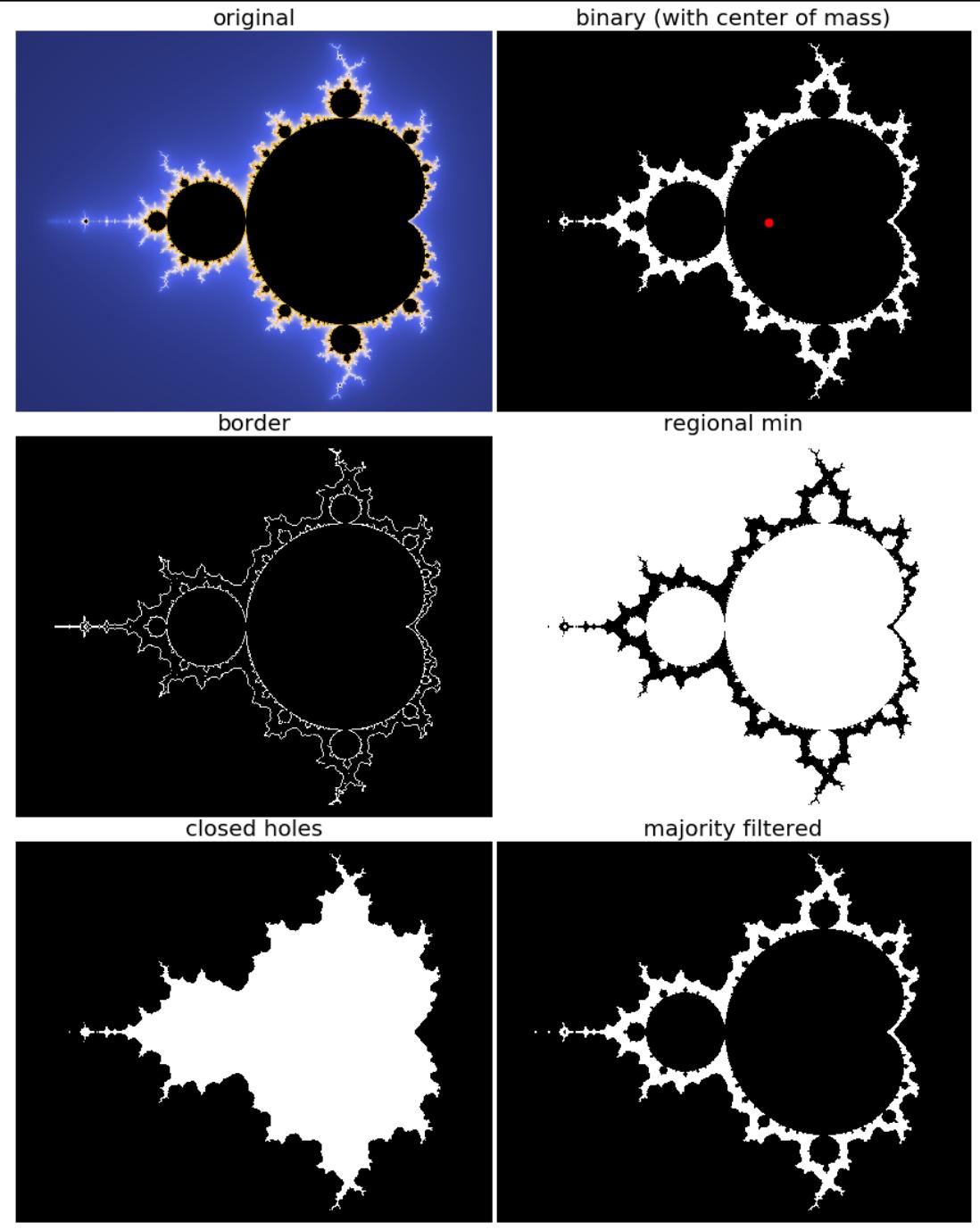


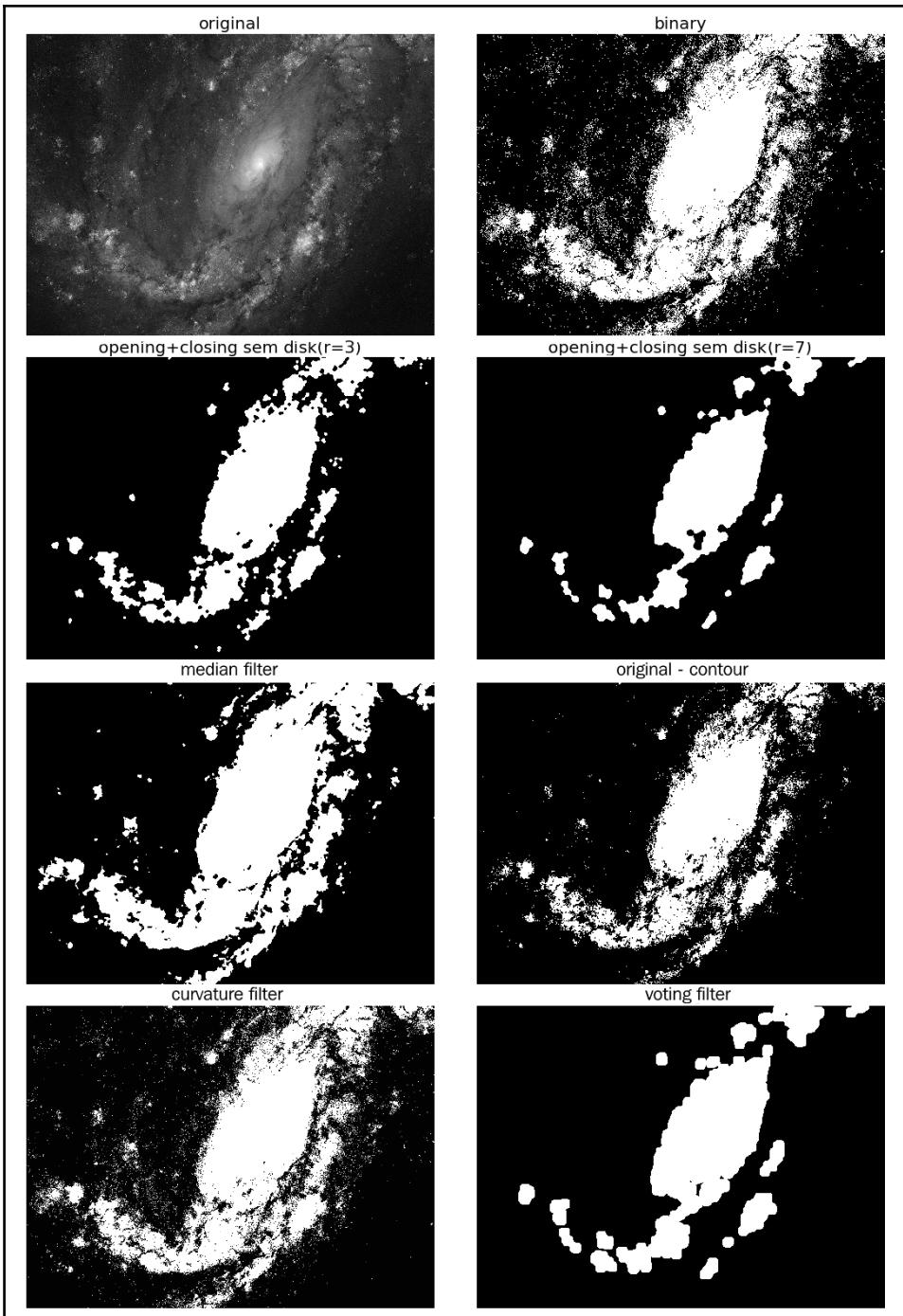




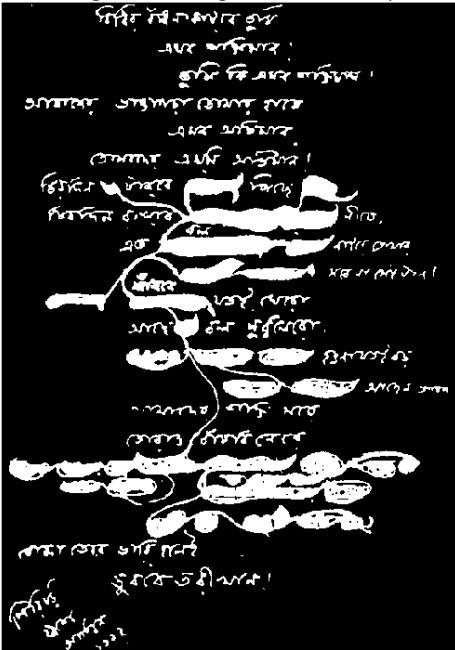








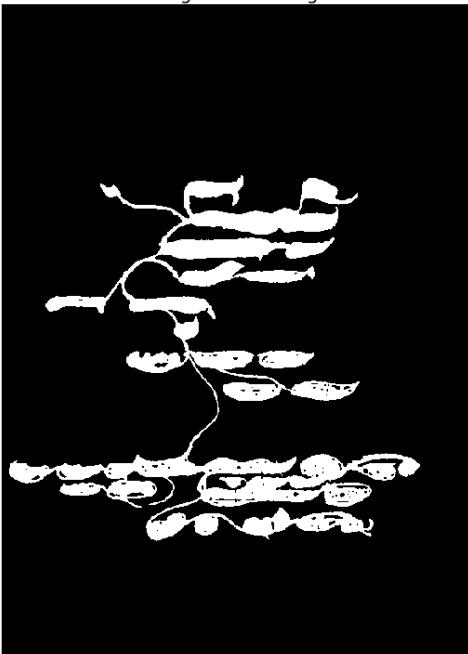
Tagore's drawing-ridden manuscript



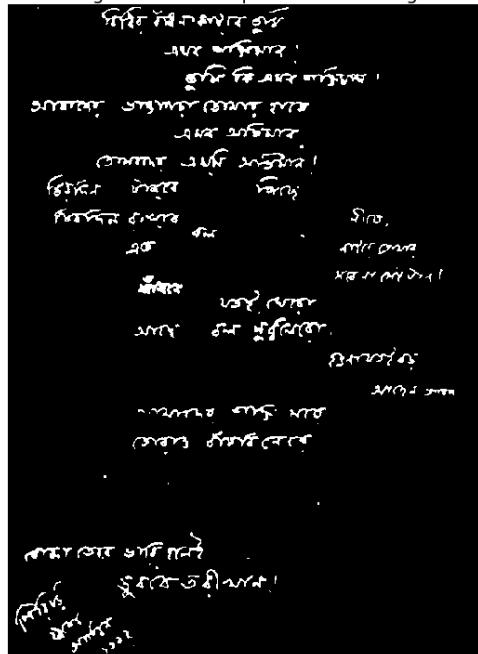
Seed for the reconstruction



Tagore's drawing



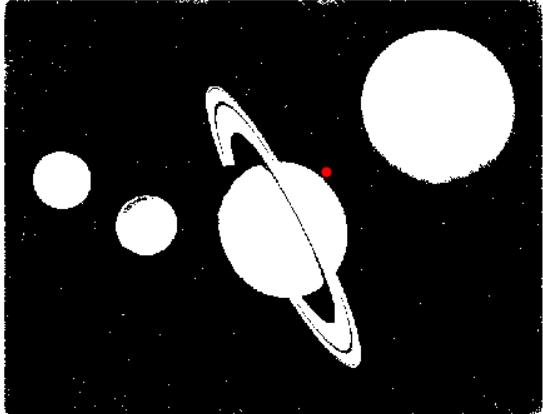
Tagore's manuscript without drawing



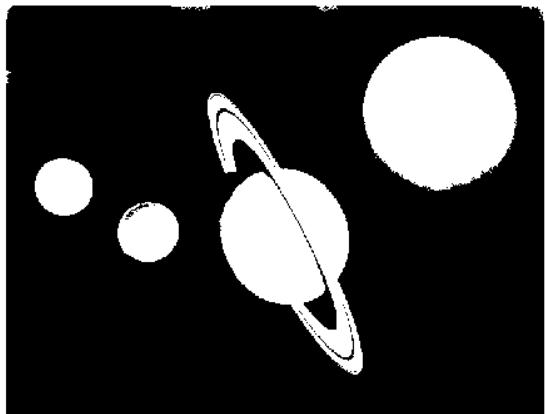
original planets image



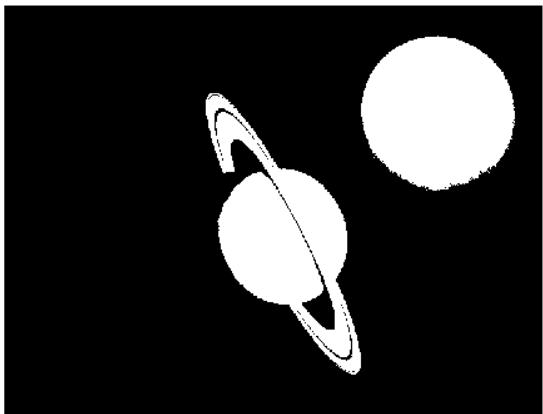
binary image (with center of mass)



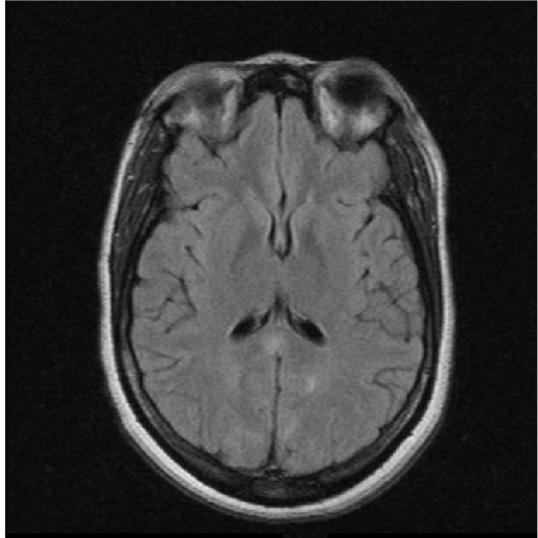
output 1



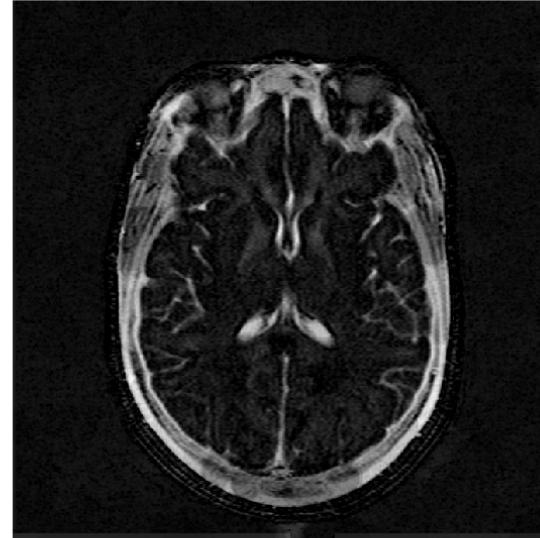
output 2



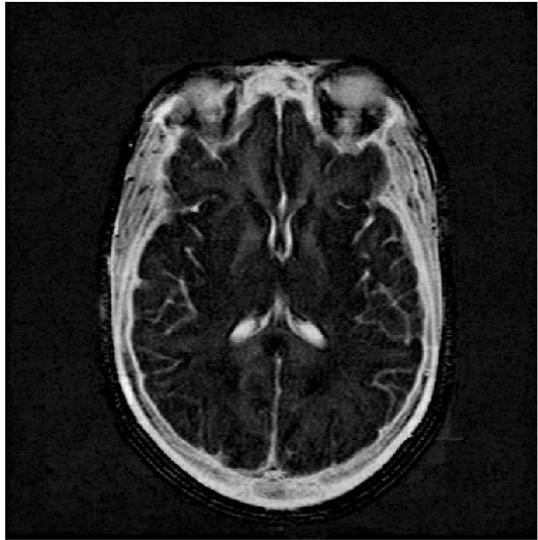
MRI image



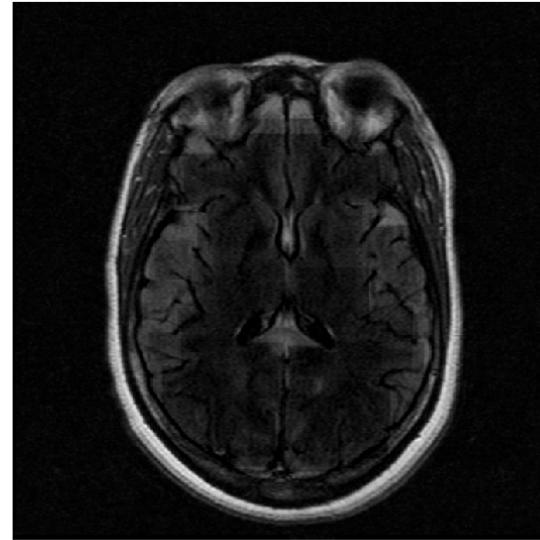
black top hat (small SE)



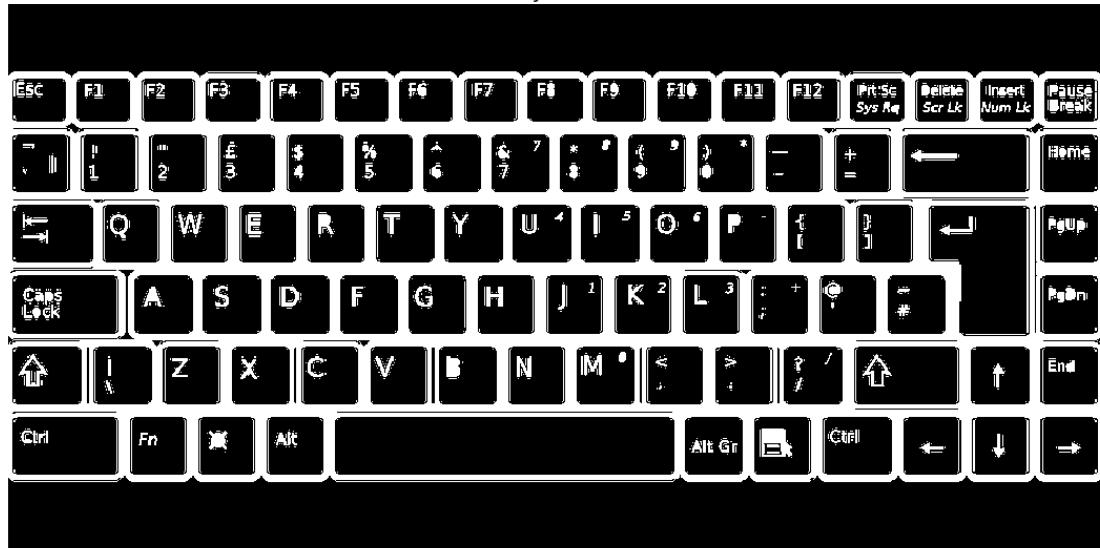
black top hat (large SE)



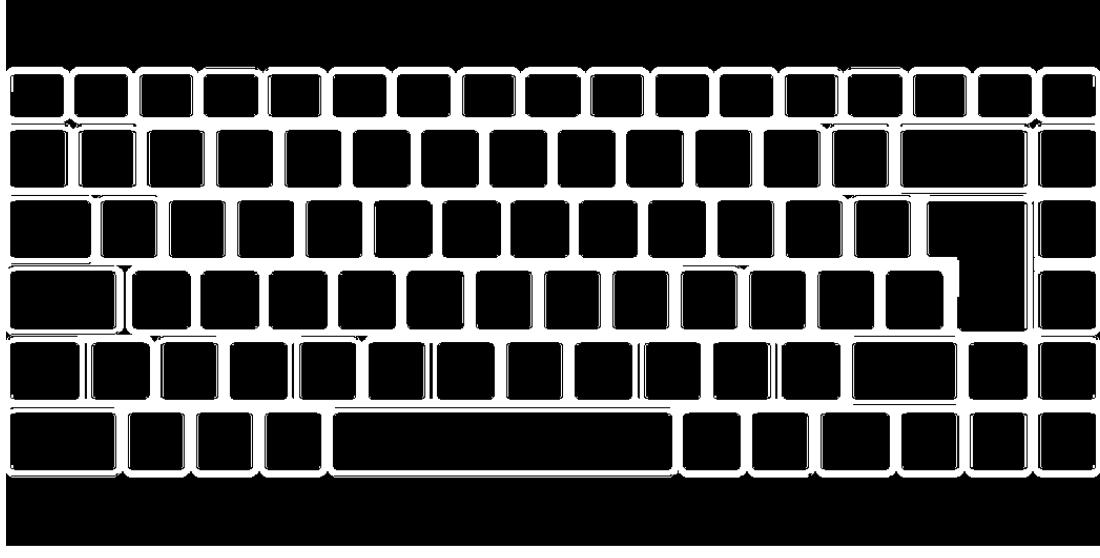
white top hat



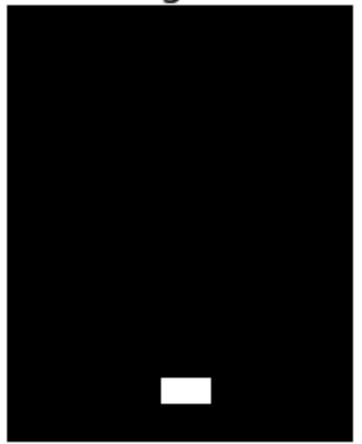
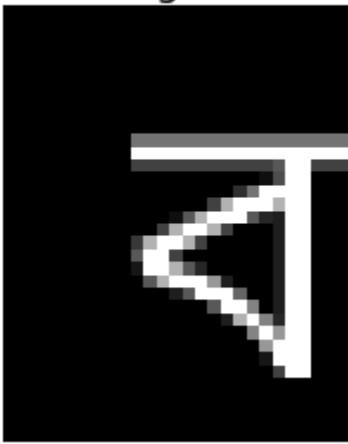
Keyboard



Keyboard without alphabets (with reconstruction)



Structuring element 1 Structuring element 2



Original Image

ভারতীয়

হে মোর চিত্ত, পুণ্য তীর্থে জাগো রে ধীরে
এই ভারতের মহামানবের সাগরতীরে ।
হেথায় দাঁড়ায়ে দু বাহু বাড়ায়ে নমি নরদেবতারে—
উদার ছল্দে, পরমানন্দে বল্দন করি তাঁরে ।
ধ্যানগঙ্গীর এই-যে ভূধর, নদী-জপমালা-ধৃত প্রান্তর,
হেথায় নিত হেরো পরিত্ব ধরিত্বারে—
এই ভারতের মহামানবের সাগরতীরে ॥

কেহ নাহি জানে কার আহ্বানে কত মানুষের ধারা
দুর্বার শ্বেতে এল কোথা হতে, সমুদ্রে হল হারা ।
হেথায় আর্য, হেথা অনার্য, হেথায় দ্বাবিড় চীন—
শক-ছন-দল পাঠান-মোগল এক দেহে হল লীন ।
পশ্চিমে আজি খুলিয়াছে দ্বার, সেখা হতে সবে আনে উপহার,
দিবে আর নিবে, মিলাবে মিলিবে, যাবে না ফিরে—
এই ভারতের মহামানবের সাগরতীরে ॥

এসো হে আর্য, এসো অনার্য, হিন্দু-মুসলমান ।
এসো এসো আজ তুমি ইংরাজ, এসো এসো খুস্টান ।
এসো ব্রাক্ষণ, শুচি করি মন ধরো হাত সবাকার ।
এসো হে পতিত, হোক অপনীত সব অপমানভার ।
মার অভিষেকে এসো এসো ভরা, মঙ্গলঘট হয় নি যে ভরা
সবার-পরম্পর-পরিত্ব-করা তীর্থনীরে—
আজি ভারতের মহামানবের সাগরতীরে ॥

শ্ৰীবিজ্ঞপ্তিপুঁজু

Output with Hit-or-Miss Transform

ভারতীয়

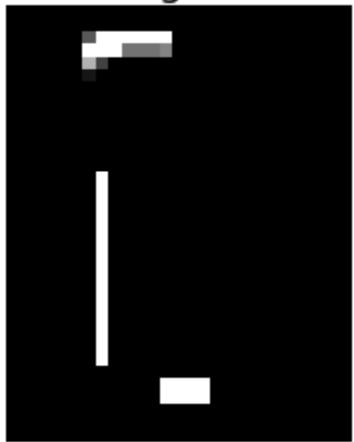
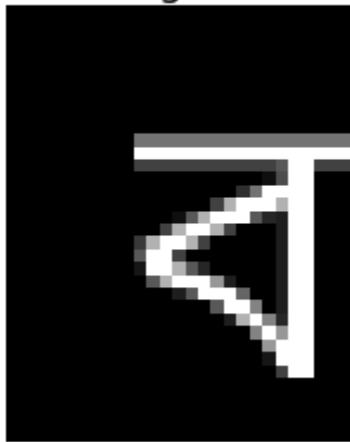
হে মোর চিত্ত, পুণ্য তীর্থে জাগো রে ধীরে
এই ভারতের মহামানবের সাগরতীরে ।
হেথায় দাঁড়ায়ে দু বাহু বাড়ায়ে নমি নরদেবতারে—
উদার ছল্দে, পরমানন্দে বল্দন করি তাঁরে ।
ধ্যানগঙ্গীর এই-যে ভূধর, নদী-জপমালা-ধৃত প্রান্তর,
হেথায় নিত হেরো পরিত্ব ধরিত্বারে—
এই ভারতের মহামানবের সাগরতীরে ॥

কেহ নাহি জানে কার আহ্বানে কত মানুষের ধারা
দুর্বার শ্বেতে এল কোথা হতে, সমুদ্রে হল হারা ।
হেথায় আর্য, হেথা অনার্য, হেথায় দ্বাবিড় চীন—
শক-ছন-দল পাঠান-মোগল এক দেহে হল লীন ।
পশ্চিমে আজি খুলিয়াছে দ্বার, সেখা হতে সবে আনে উপহার,
দিবে আর নিবে, মিলাবে মিলিবে, যাবে না ফিরে—
এই ভারতের মহামানবের সাগরতীরে ॥

এসো হে আর্য, এসো অনার্য, হিন্দু-মুসলমান ।
এসো এসো আজ তুমি ইংরাজ, এসো এসো খুস্টান ।
এসো ব্রাক্ষণ, শুচি করি মন ধরো হাত সবাকার ।
এসো হে পতিত, হোক অপনীত সব অপমানভার ।
মার অভিষেকে এসো এসো ভরা, মঙ্গলঘট হয় নি যে ভরা
সবার-পরম্পর-পরিত্ব-করা তীর্থনীরে—
আজি ভারতের মহামানবের সাগরতীরে ॥

শ্ৰীবিজ্ঞপ্তিপুঁজু

Structuring element 1 Structuring element 2



Original Image

ভারতজীর্ধ

হে মোর চিত্ত, পুণ্য তীর্থে জাগো রে ধীরে
 এই ভারতের মহামানবের সাগরতীরে ।
 হেথায় দাঁড়ায়ে দু বাল্ক বাড়ায়ে নমি নরদেবতারে—
 উদার ছল্দে, পরমানন্দে বন্দন করি তাঁরে ।
 ধ্যানগঙ্গীর এই-যে ভূধর, নদী-জপমালা-ধৃত প্রান্তর,
 হেথায় নিত্য হেরো পবিত্র ধরিত্রীরে—
 এই ভারতের মহামানবের সাগরতীরে ॥

কেহ নাহি জানে কার আশ্বানে কত মানুষের ধারা
 দুর্বার শ্রোতে এল কোথা হতে, সমুদ্রে হল হারা ।
 হেথায় আর্য, হেথা অনার্য, হেথায় দ্বাবিড় চীন—
 শক-ছন-দল পাঠান-মোগল এক দেহে হল লীন ।
 পশ্চিমে আজি খুলিয়াছে দ্বার, সেথা হতে সবে আনে উপহার,
 দিবে আর নিবে, মিলাবে মিলিবে, যাবে না ফিরে—
 এই ভারতের মহামানবের সাগরতীরে ॥

এসো হে আর্য, এসো অনার্য, হিন্দু-মুসলমান ।
 এসো এসো আজ তুমি ইংরাজ, এসো এসো খুষ্টান ।
 এসো ব্রাহ্মণ, শুচি করি মন ধরো হাত সবাকার ।
 এসো হে পতিত, হোক অপনীত সব অপমানভার ।
 মার অভিষেকে এসো এসো ভরা, মঙ্গলঘট হয় নি যে ভরা
 সবার-পরশে-পবিত্র-করা তীর্থনীরে—
 আজি ভারতের মহামানবের সাগরতীরে ॥

শ্ৰীবিজ্ঞপ্তিপুঁজু

Output with Hit-or-Miss Transform

ভারতজীর্ধ

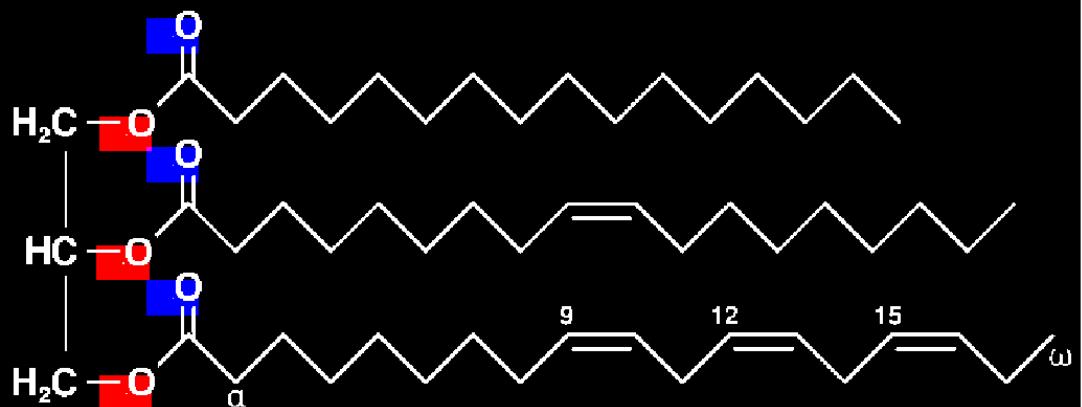
হে মোর চিত্ত, পুণ্য তীর্থে জাগো রে ধীরে
 এই ভারতের মহামানবের সাগরতীরে ।
 হেথায় দাঁড়ায়ে দু ~~বাল্ক~~^{বাল্ক} বাড়ায়ে নমি নরদেবতারে—
 উদার ছল্দে, পরমানন্দে ~~বন্দন~~^{বন্দন} করি তাঁরে ।
 ধ্যানগঙ্গীর এই-যে ভূধর, নদী-জপমালা-ধৃত প্রান্তর,
 হেথায় নিত্য হেরো পবিত্র ধরিত্রীরে—
 এই ভারতের মহামানবের সাগরতীরে ॥

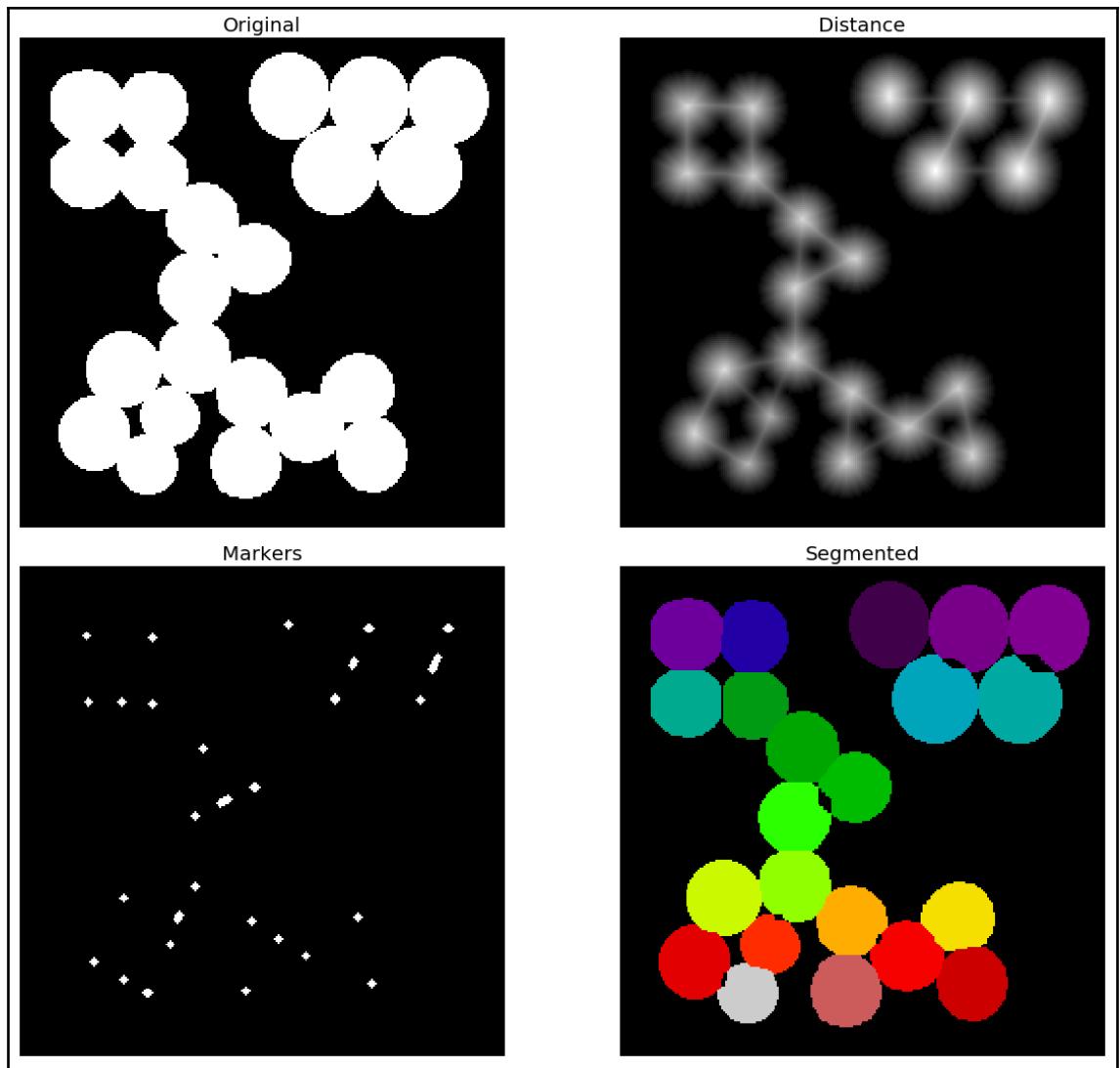
কেহ নাহি জানে কার আশ্বানে কত মানুষের ধারা
 দুর্বার শ্রোতে এল কোথা হতে, সমুদ্রে হল হারা ।
 হেথায় আর্য, হেথা অনার্য, হেথায় দ্বাবিড় চীন—
 শক-ছন-দল পাঠান-মোগল এক দেহে হল লীন ।
 পশ্চিমে আজি খুলিয়াছে দ্বার, সেথা হতে সবে আনে উপহার,
 দিবে আর নিবে, মিলাবে মিলিবে, যাবে না ফিরে—
 এই ভারতের মহামানবের সাগরতীরে ॥

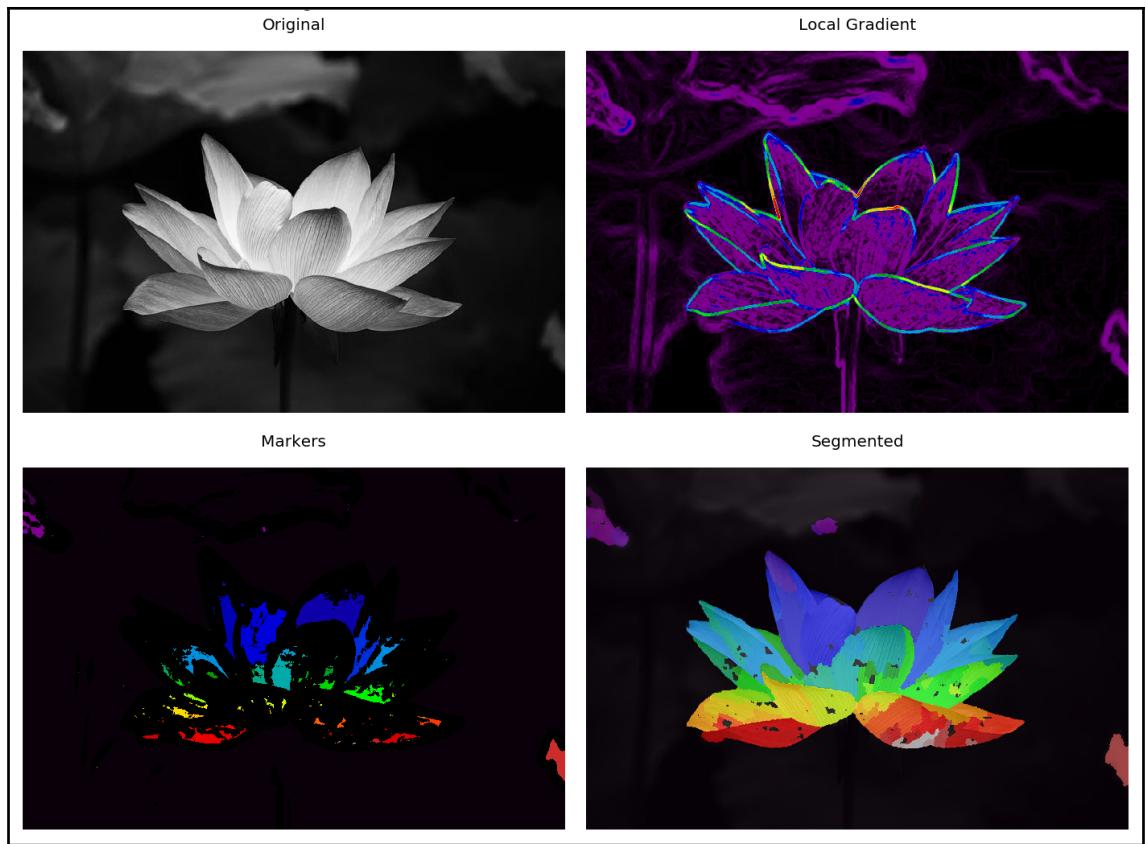
এসো হে আর্য, এসো অনার্য, হিন্দু-মুসলমান ।
 এসো এসো আজ তুমি ইংরাজ, এসো এসো খুষ্টান ।
 এসো ব্রাহ্মণ, শুচি করি মন ধরো হাত সর্বাকার ।
 এসো হে পতিত, হোক অপনীত সর্ব অপমানভার ।
 মার অভিষেকে এসো এসো ভরা, মঙ্গলঘট হয় নি যে ভরা
 সর্বার-পরশে-পবিত্র-করা তীর্থনীরে—
 আজি ভারতের মহামানবের সাগরতীরে ॥

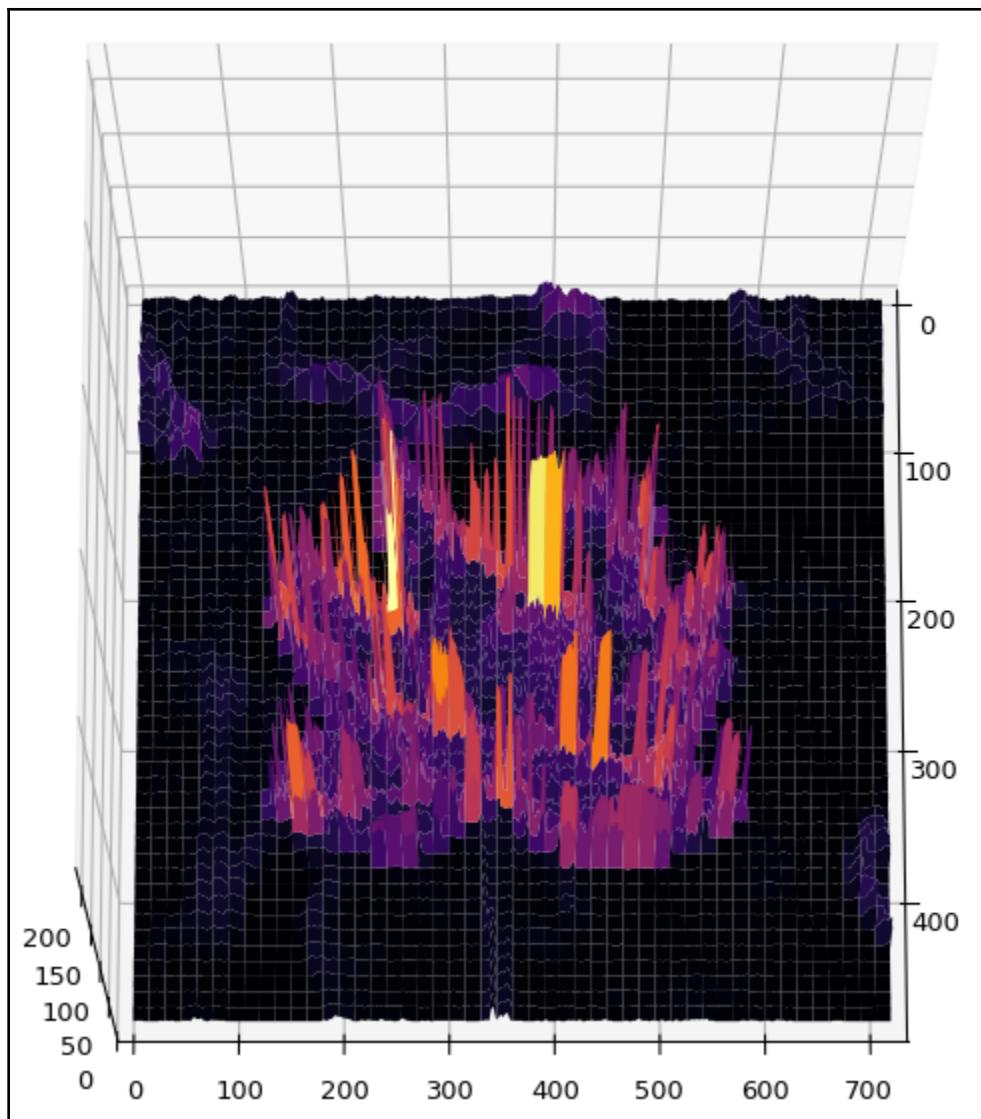
শ্ৰীবিজ্ঞপ্তিপুঁজু

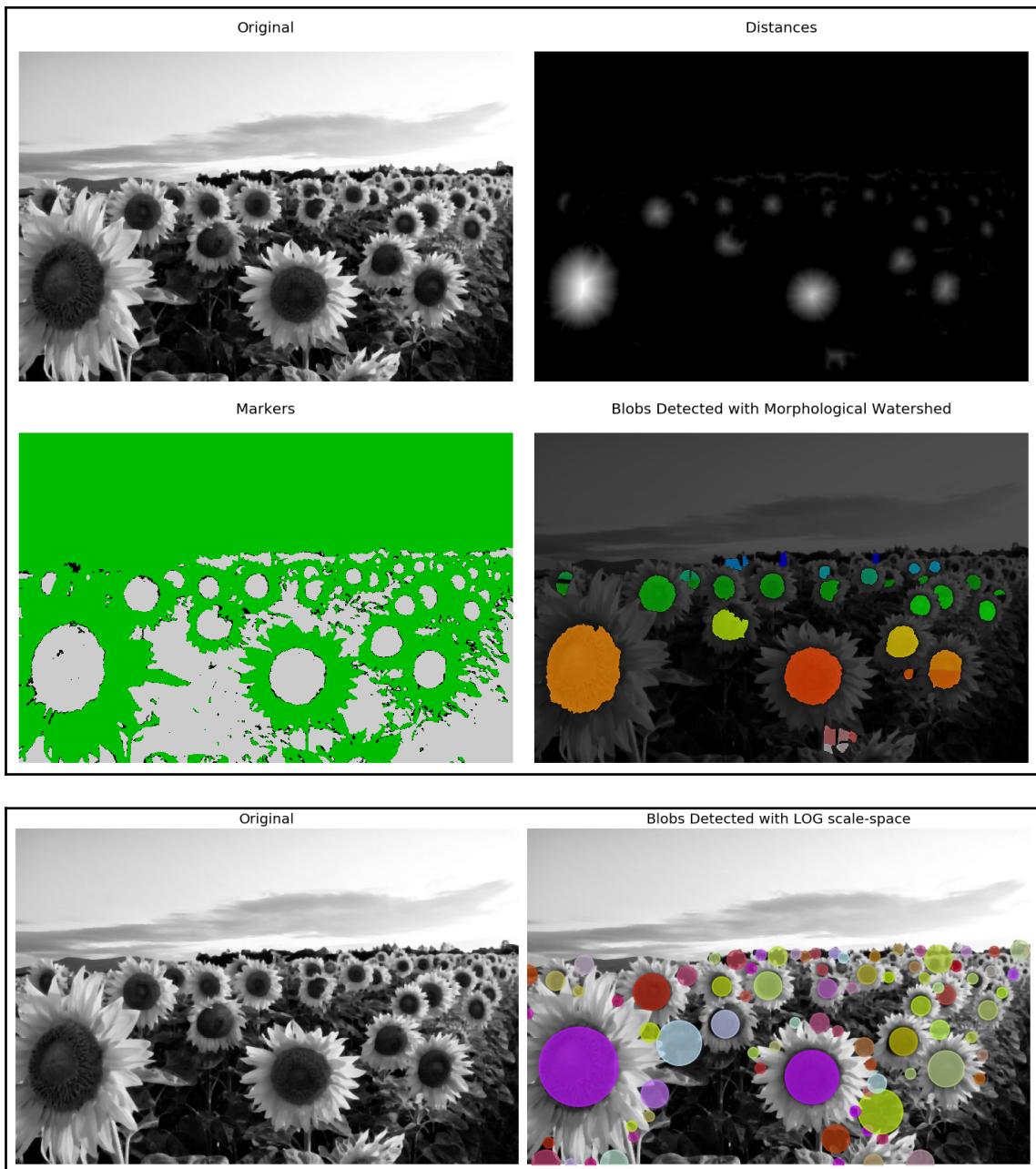
Finding -O- and =O in Triglyceride molecule



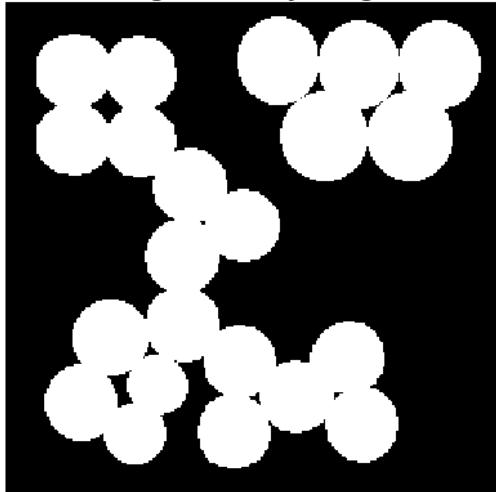




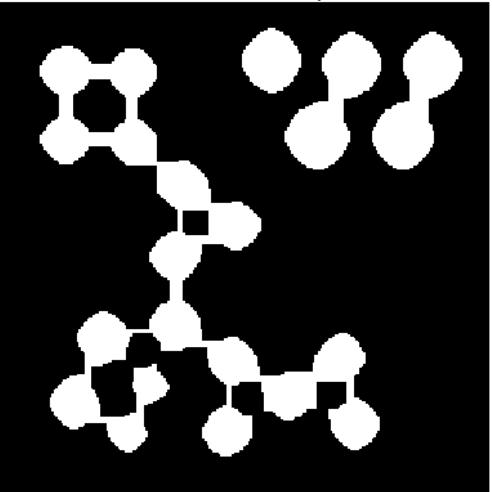




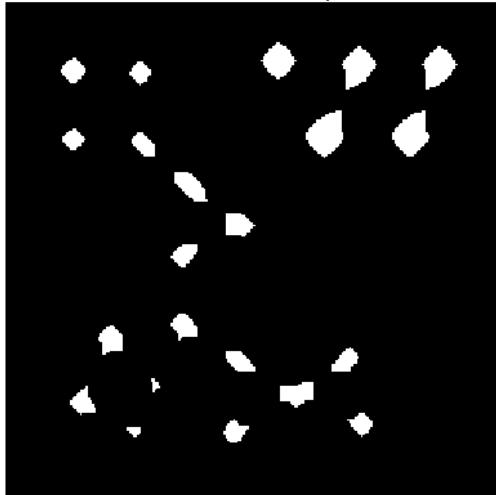
Original binary image



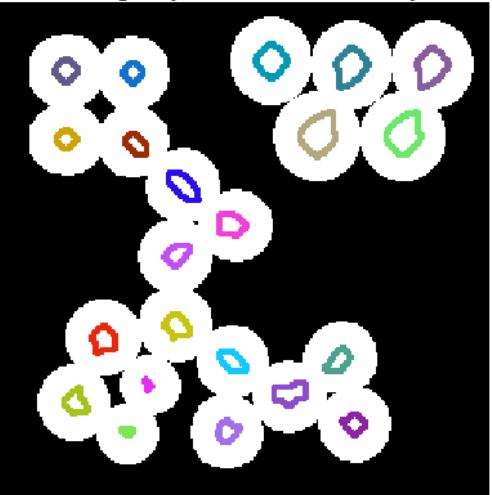
Eroded (11x11 square)



Eroded (21x21 square)



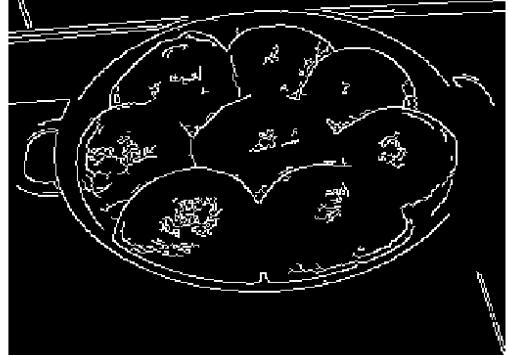
Counting objects: Found 22 objects



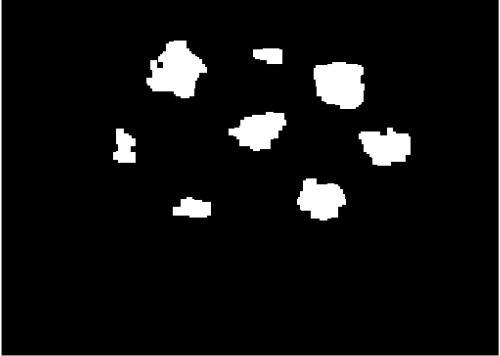
Original image



Edges

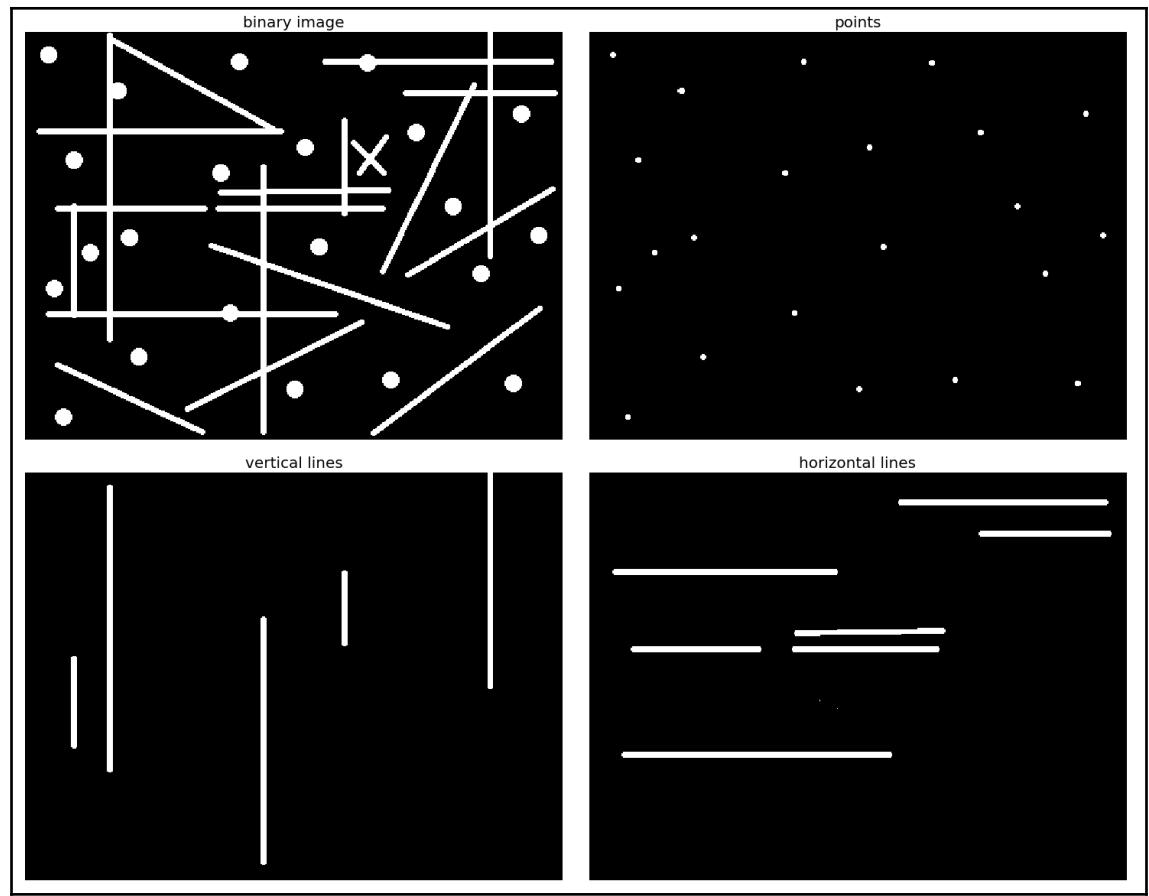


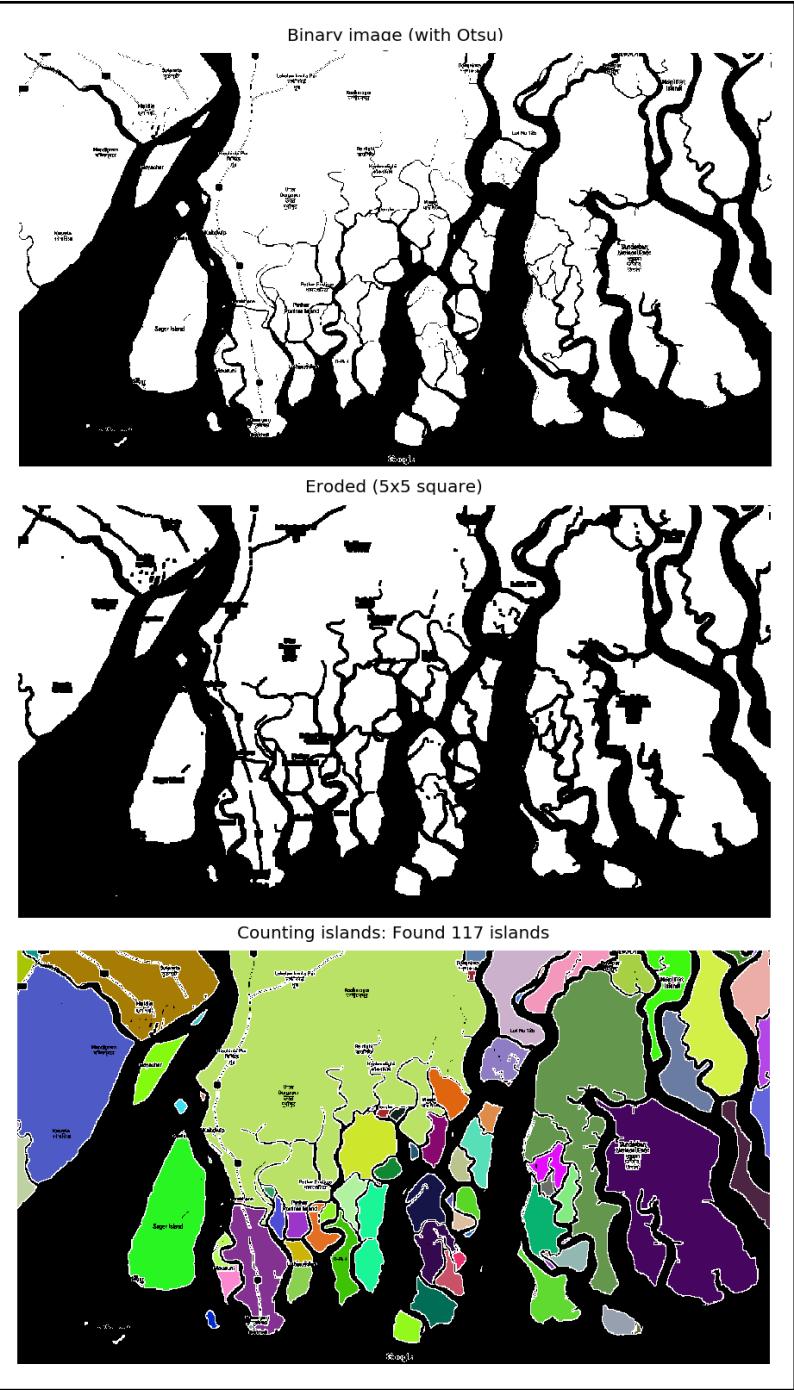
Binary image



Counting objects: Found 8 objects







Chapter 5: Image Registration

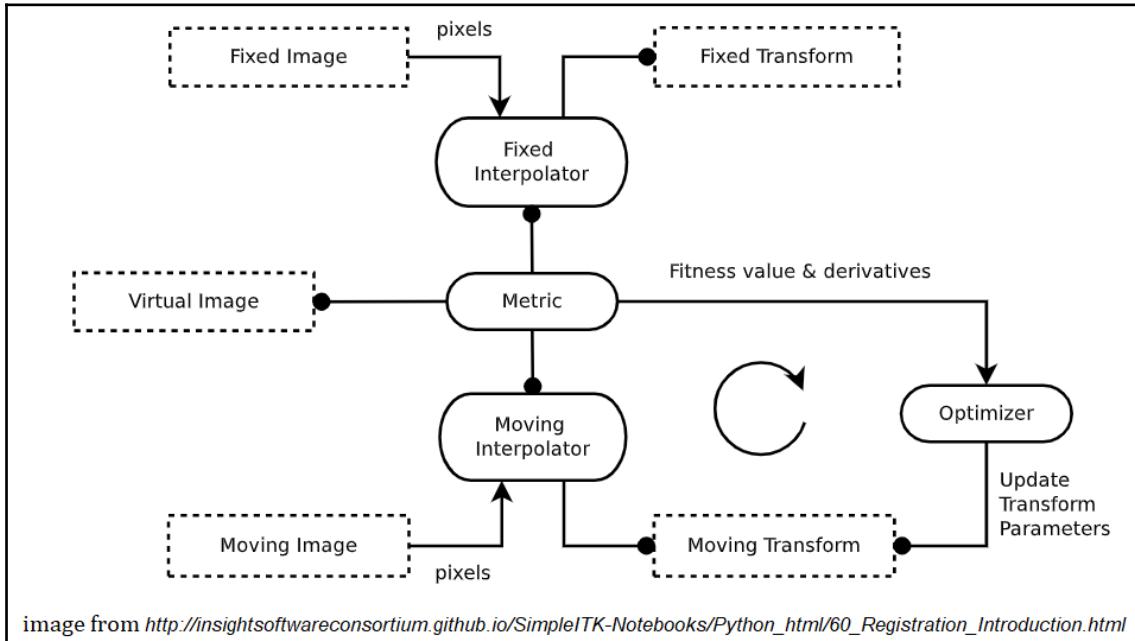
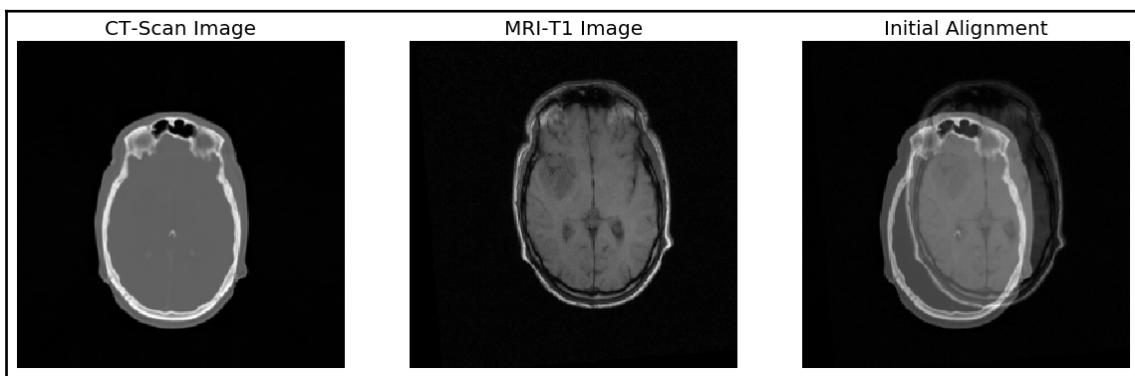
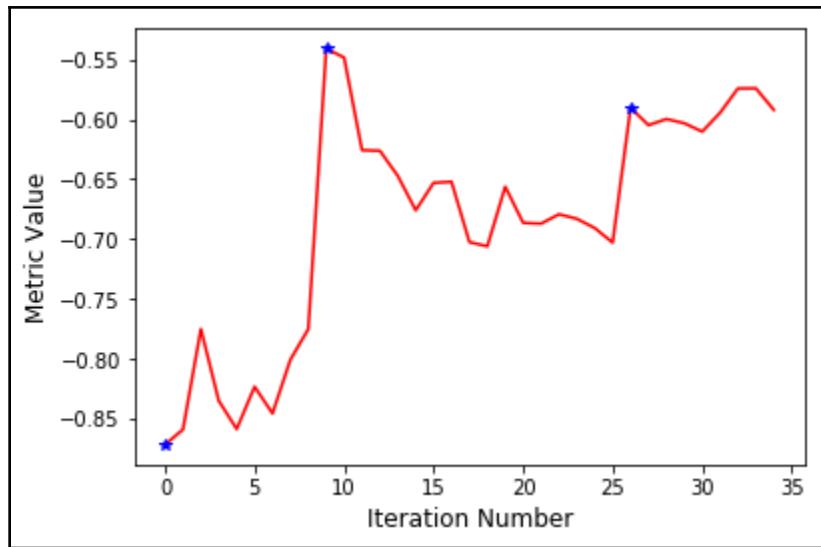
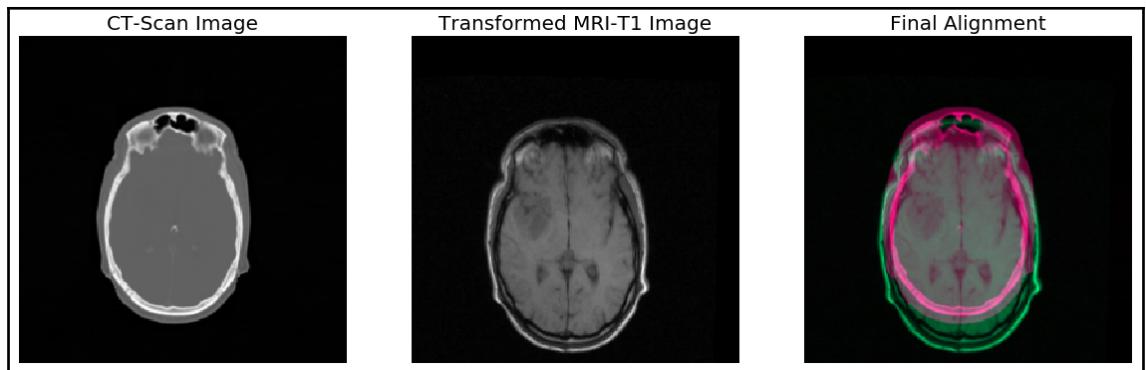
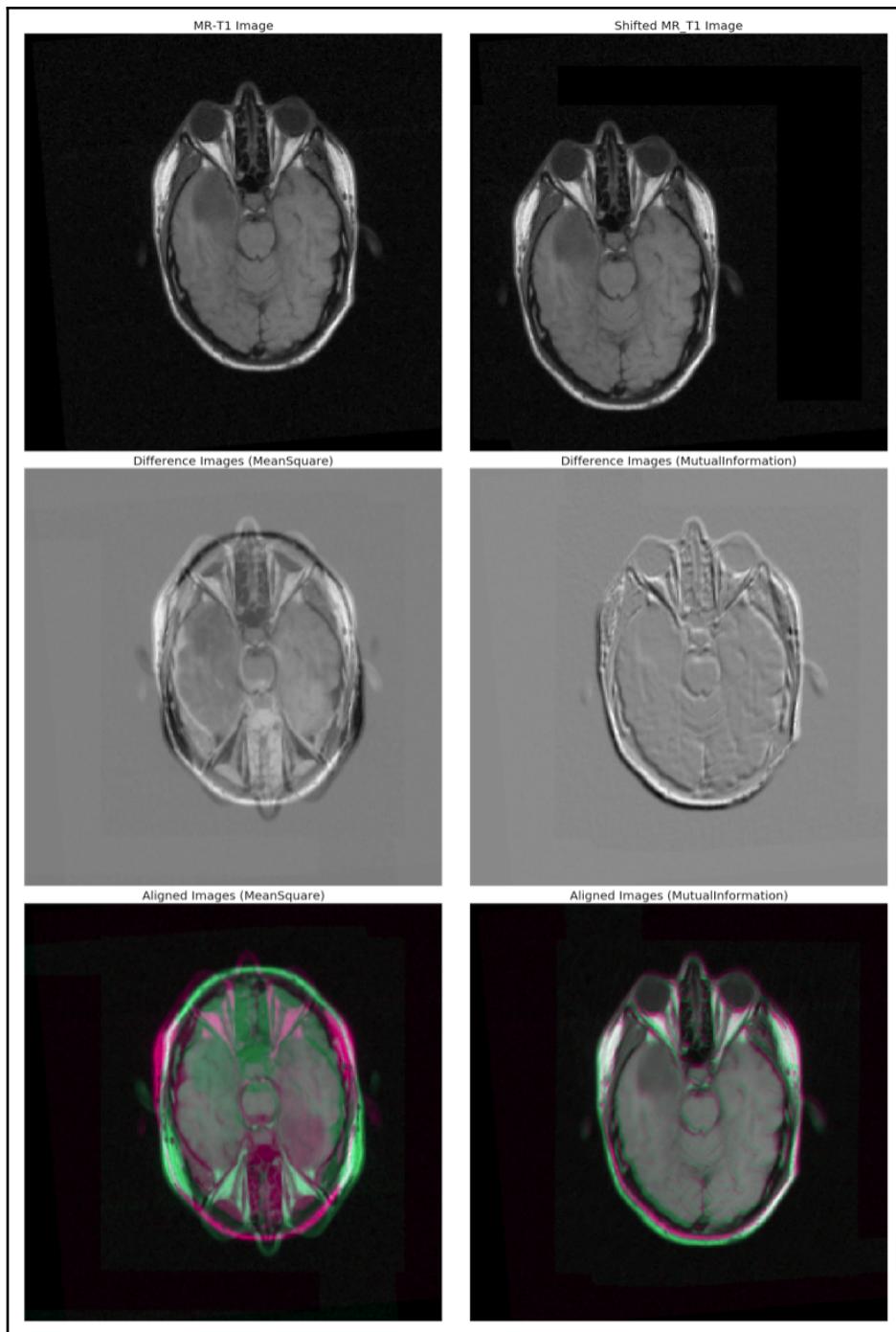
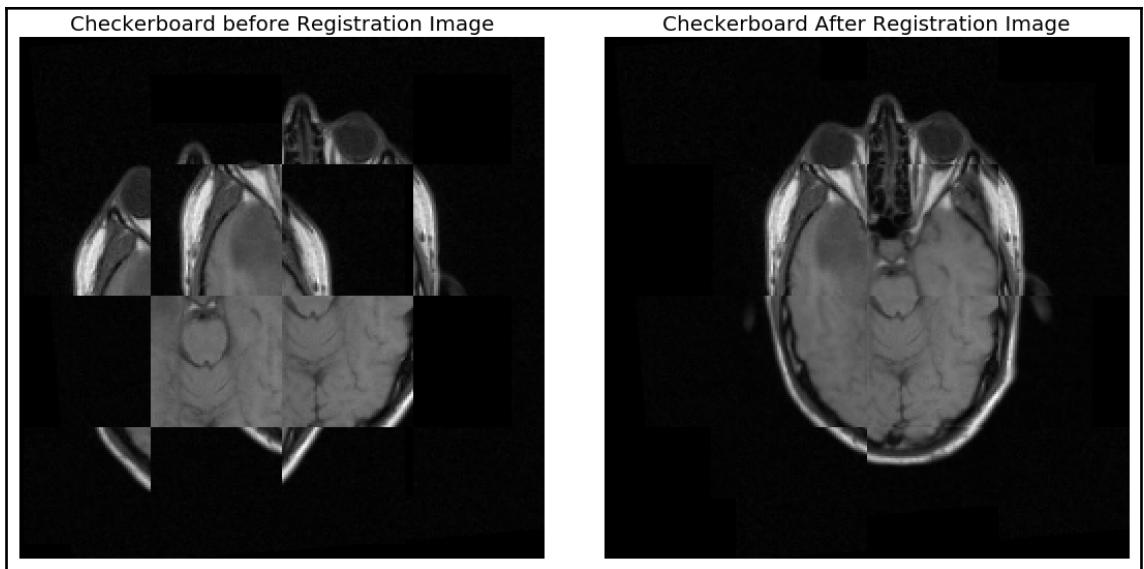


image from http://insightsoftwareconsortium.github.io/SimpleITK-Notebooks/Python_html/60_Registration_Introduction.html









Estimate p : $I_r(\mathbf{x}) = I_w(\phi(\mathbf{x}; \mathbf{p})), \quad \forall \mathbf{x} \in \mathcal{T}$

Optimization problem: $\min_{\mathbf{p}, \alpha} E(\mathbf{p}, \alpha) = \min_{\mathbf{p}, \alpha} \sum_{\mathbf{x} \in \mathcal{T}} |I_r(\mathbf{x}) - \Psi(I_w(\phi(\mathbf{x}; \mathbf{p})), \alpha)|^p$

reference vector: $\bar{\mathbf{i}}_r = [I_r(\mathbf{x}_1) \ I_r(\mathbf{x}_2) \ \dots \ I_r(\mathbf{x}_K)]^t$

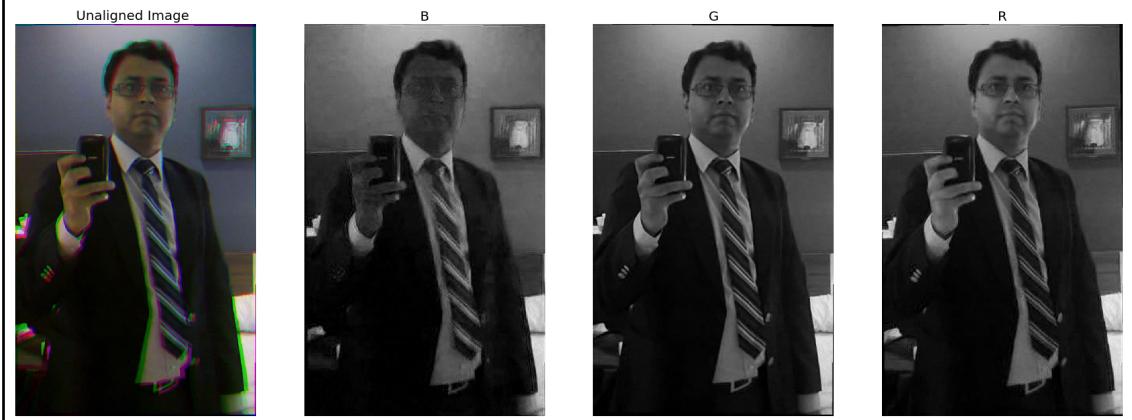
warped vector: $\bar{\mathbf{i}}_w(\mathbf{p}) = [I_w(\mathbf{y}_1(\mathbf{p})) \ I_w(\mathbf{y}_2(\mathbf{p})) \ \dots \ I_w(\mathbf{y}_K(\mathbf{p}))]^t$

(minimize) ECC criterion: $E_{\text{ECC}}(\mathbf{p}) = \left\| \frac{\bar{\mathbf{i}}_r}{\|\bar{\mathbf{i}}_r\|} - \frac{\bar{\mathbf{i}}_w(\mathbf{p})}{\|\bar{\mathbf{i}}_w(\mathbf{p})\|} \right\|^2$

(maximize) enhanced correlation coefficient: $\rho(\mathbf{p}) = \frac{\bar{\mathbf{i}}_r^t \bar{\mathbf{i}}_w(\mathbf{p})}{\|\bar{\mathbf{i}}_r\| \|\bar{\mathbf{i}}_w(\mathbf{p})\|} = \hat{\mathbf{i}}_r^t \frac{\bar{\mathbf{i}}_w(\mathbf{p})}{\|\bar{\mathbf{i}}_w(\mathbf{p})\|}$

Taken from http://xanthippi.ceid.upatras.gr/people/evangelidis/george_files/PAMI_2008.pdf

Unaligned Image and Color Channels



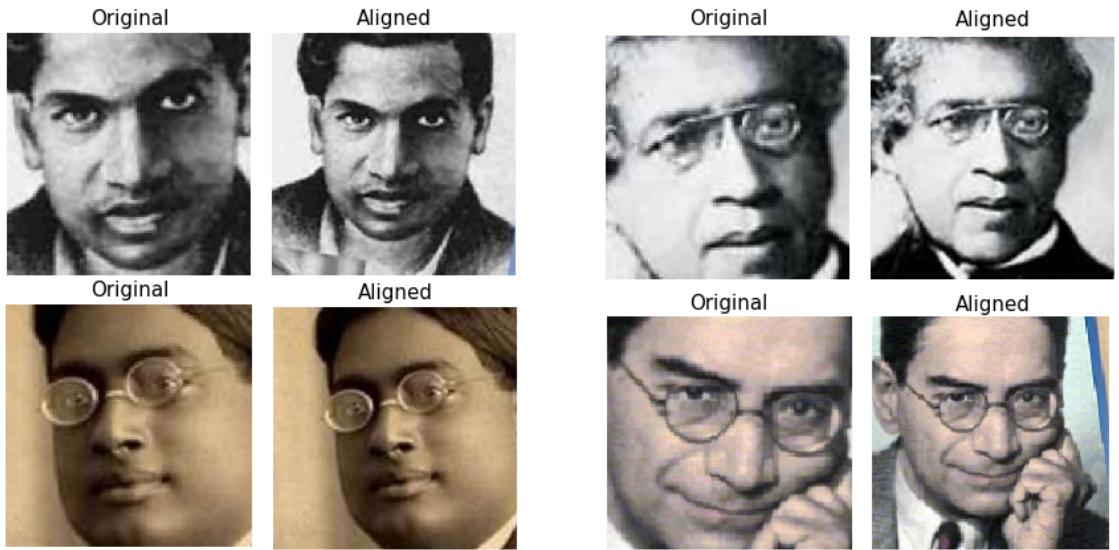
Aligned Image and Color Channels

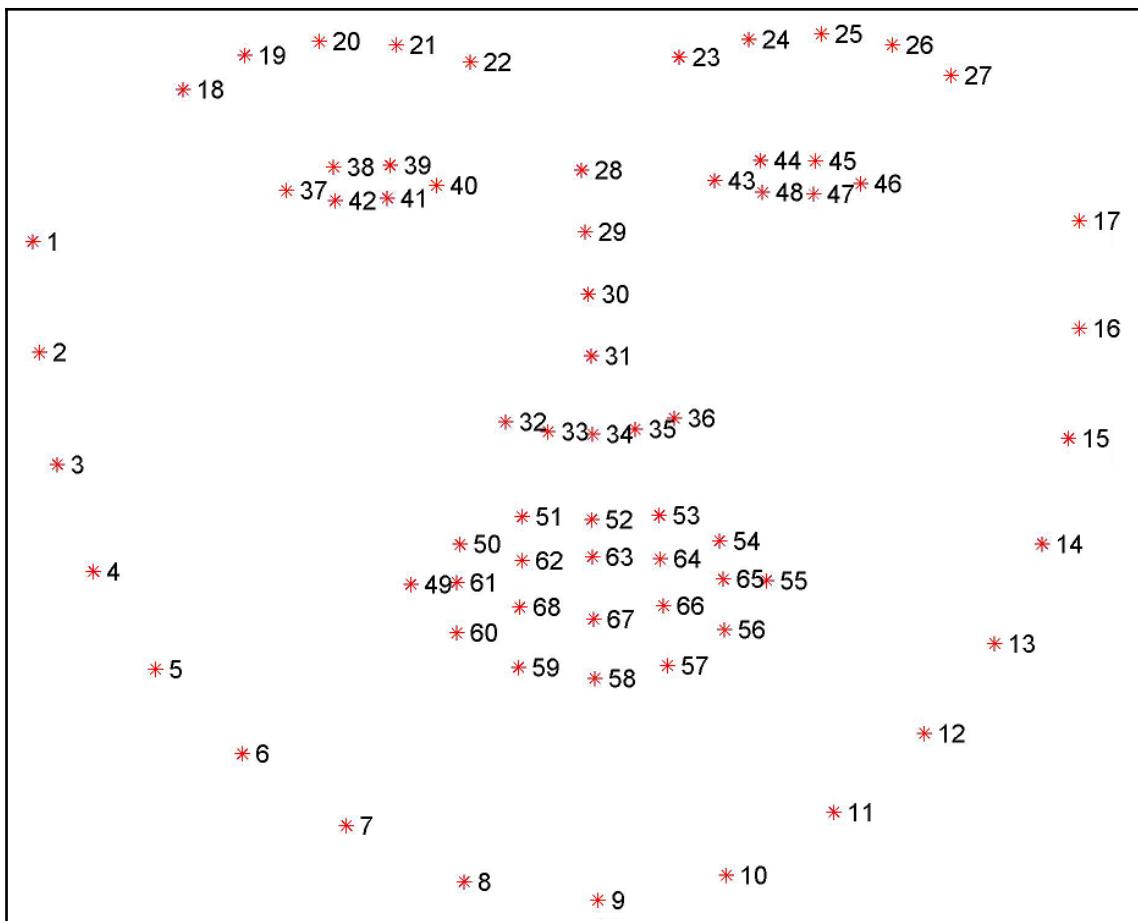


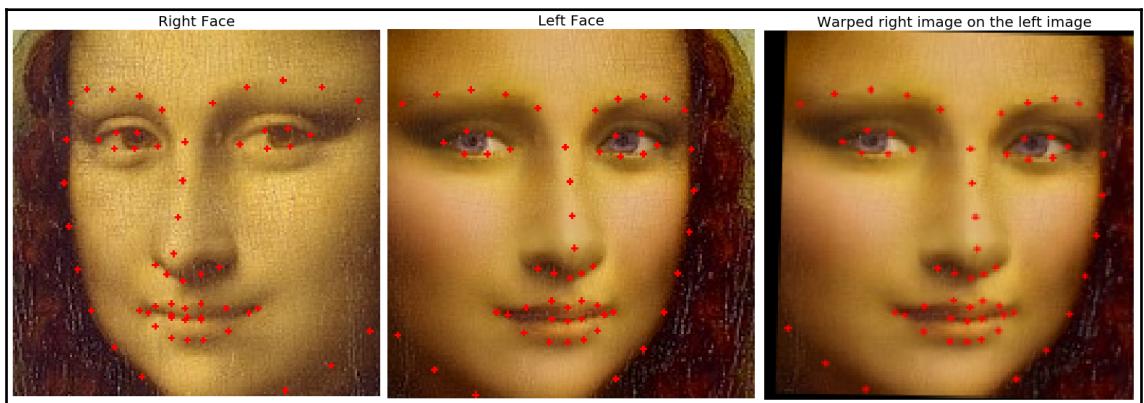
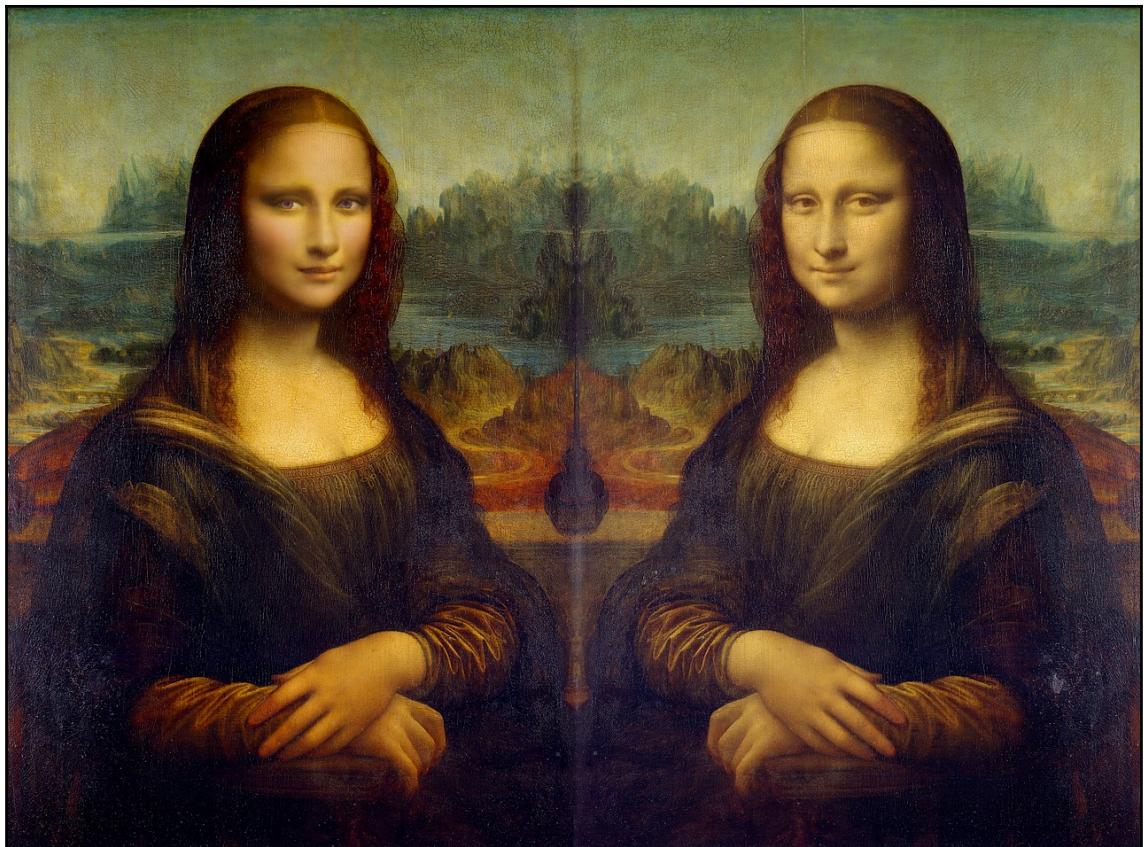
Original Image: Famous Indian Scientists

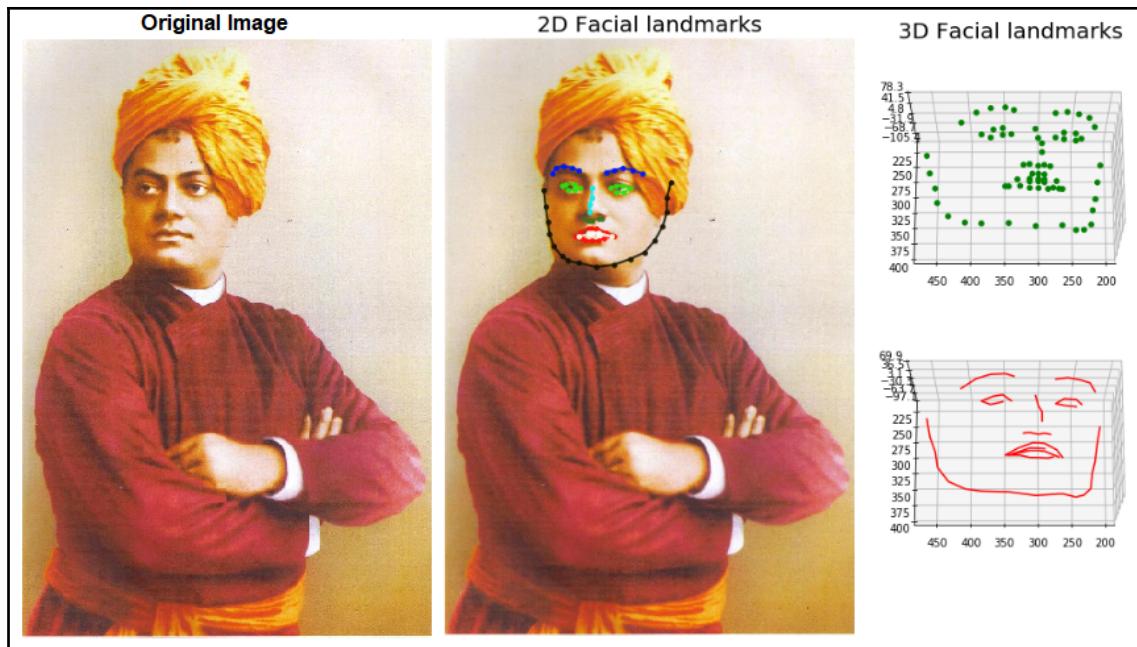


Some of the Faces Detected and Aligned









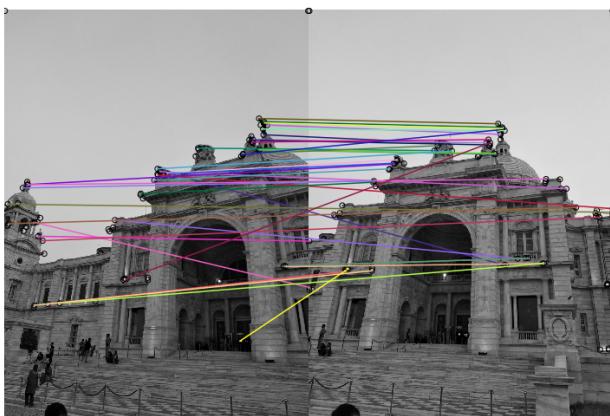
RANSAC Algorithm

$$N = \frac{\log(1 - p)}{\log(1 - (1 - \epsilon)^s)}$$

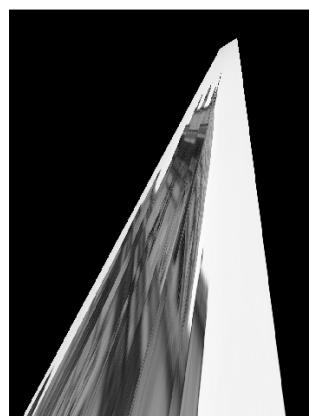
number of trials needed
 probability of an outlier sample size
 (= 4 for 8-point algo)

probability that there is at least one sample free from outliers

Matching without RANSAC



Homography without RANSAC



Robust Matching with RANSAC



Robust Homography with RANSAC

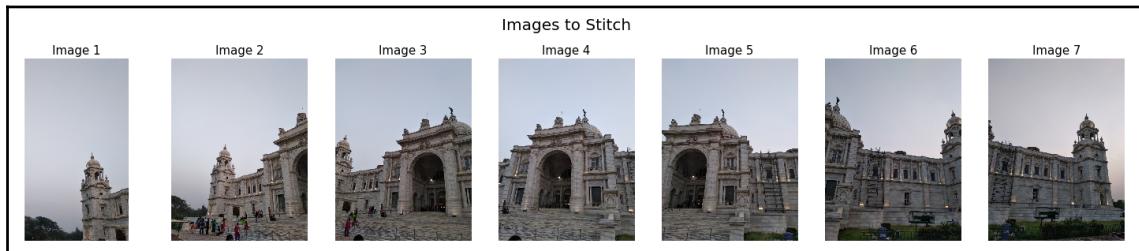
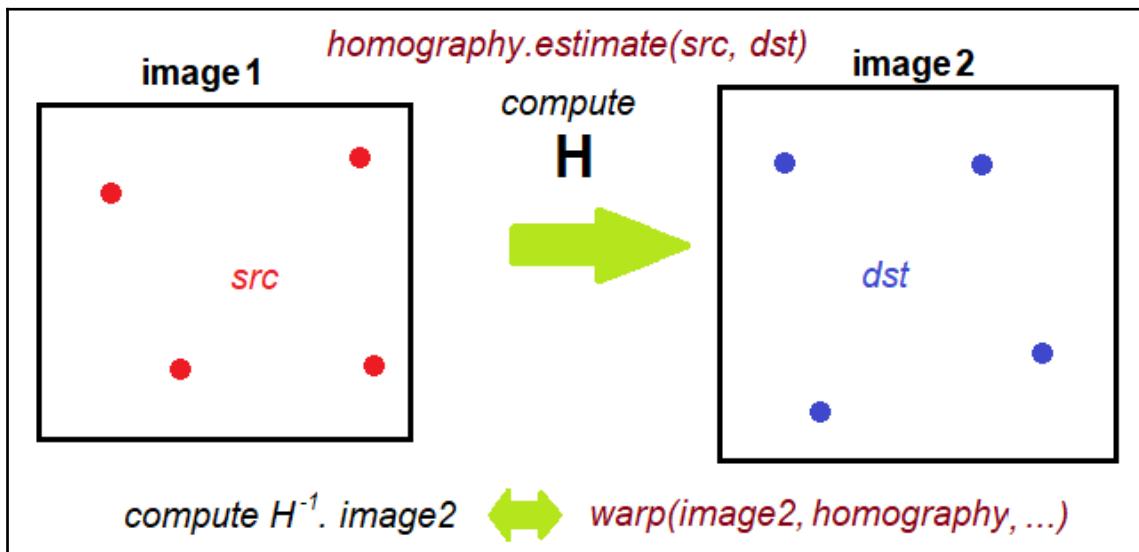


$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

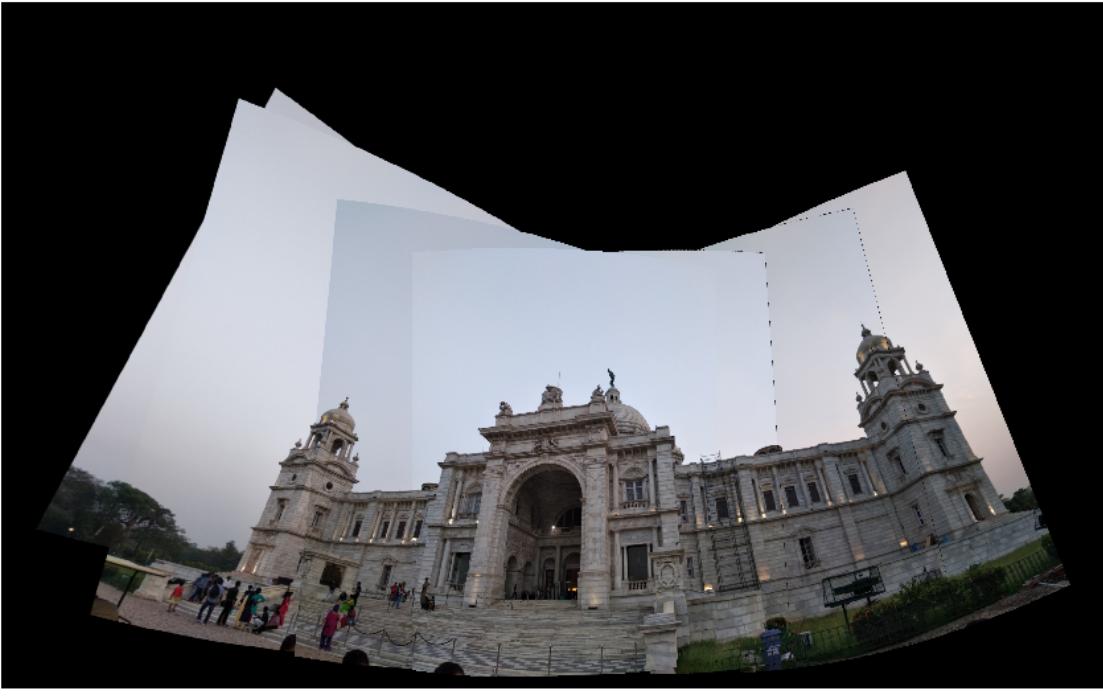
window function image derivatives

$$R = \frac{\det M}{\text{Trace } M}$$

corner response

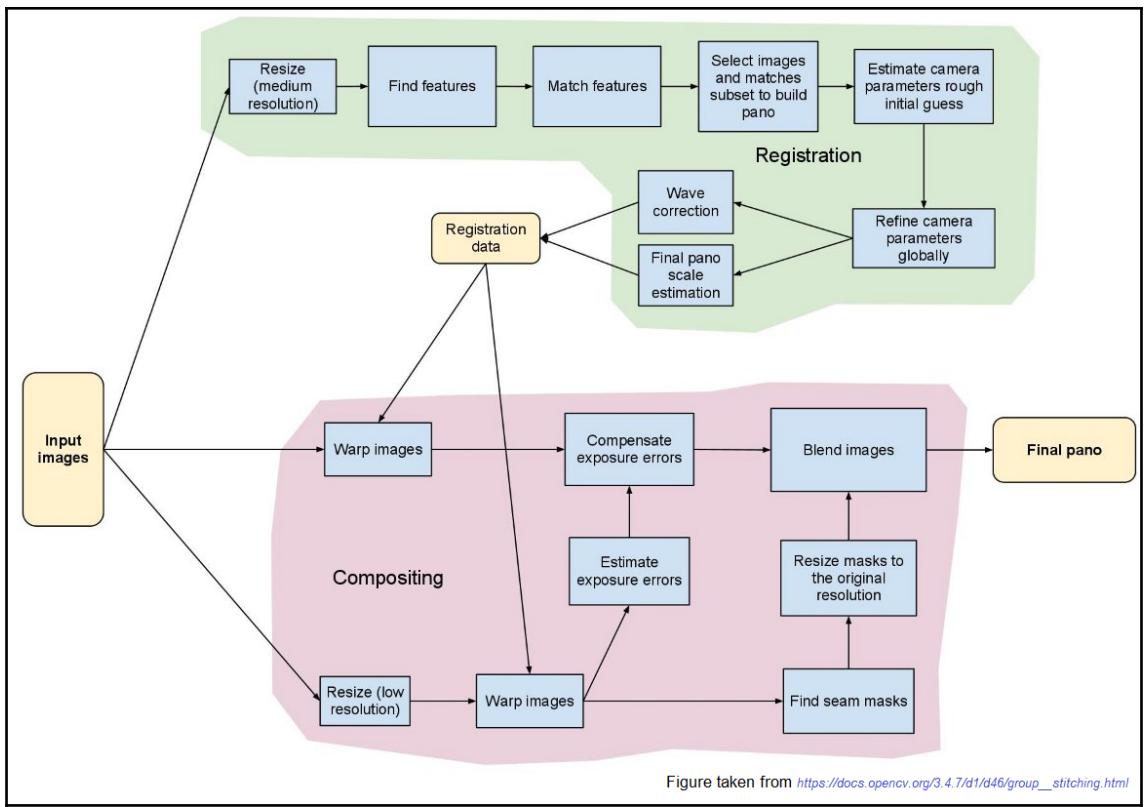


Final Panorama Image



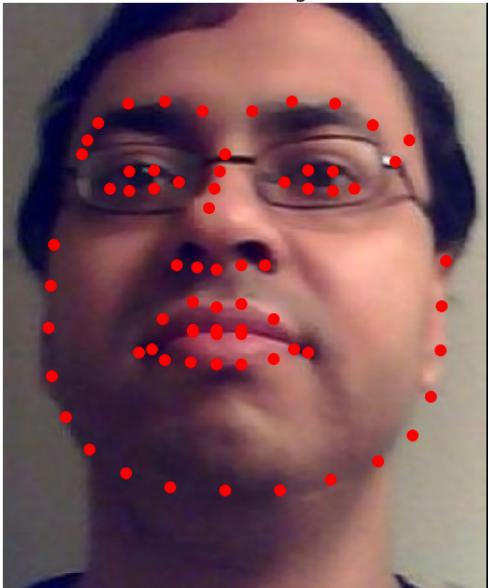
Final Panorama Image



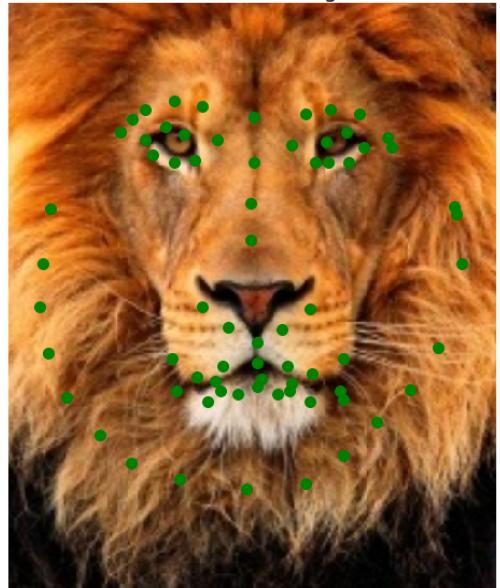


Facial Landmarks computed for the images

Source image

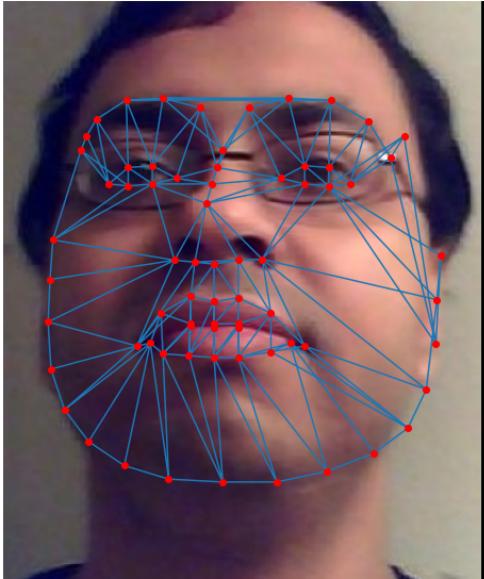


Destination image

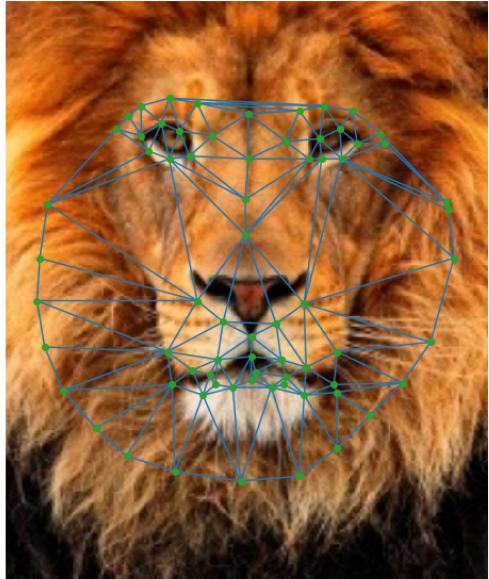


Delaunay triangulation of the images

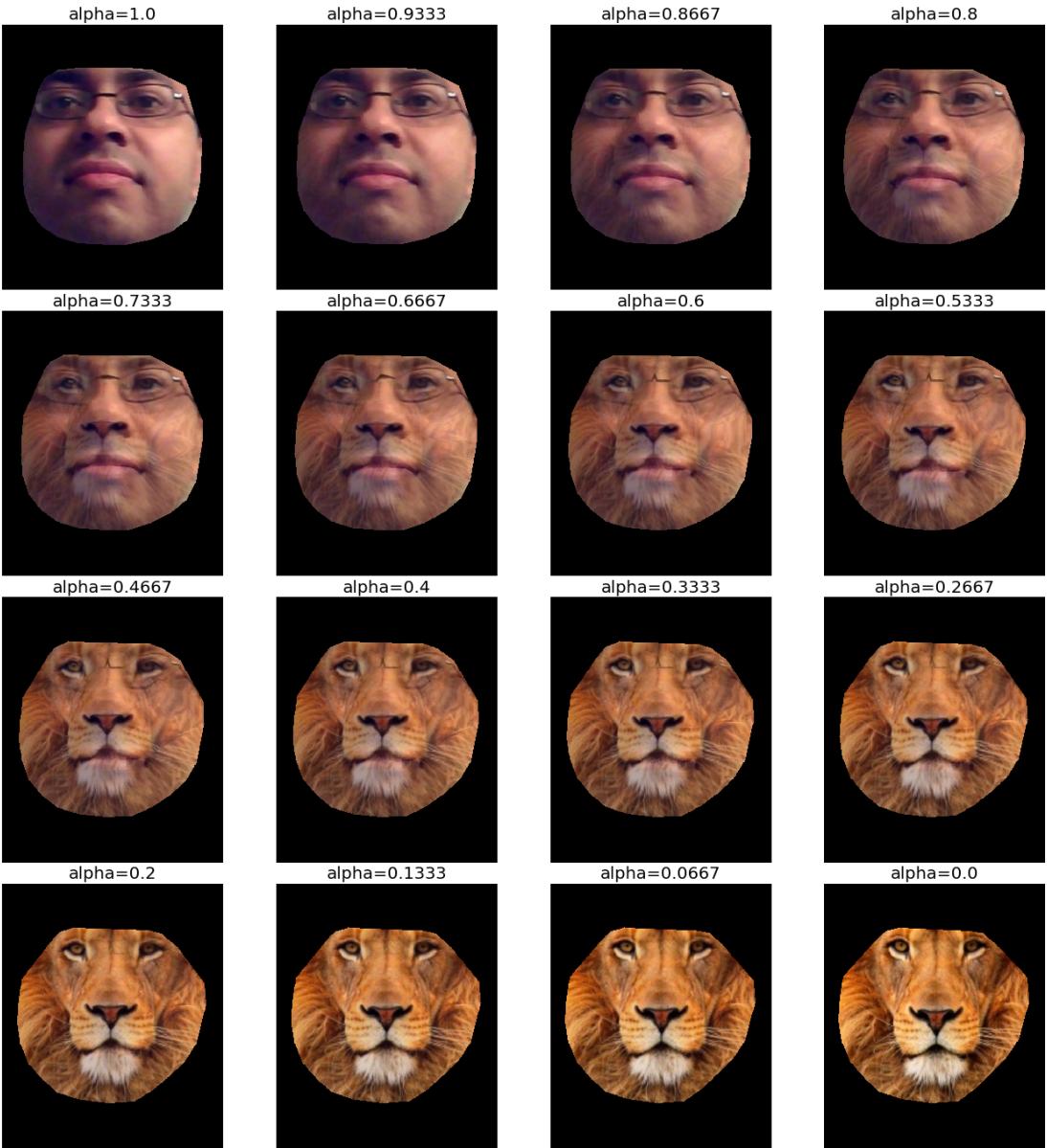
Source image



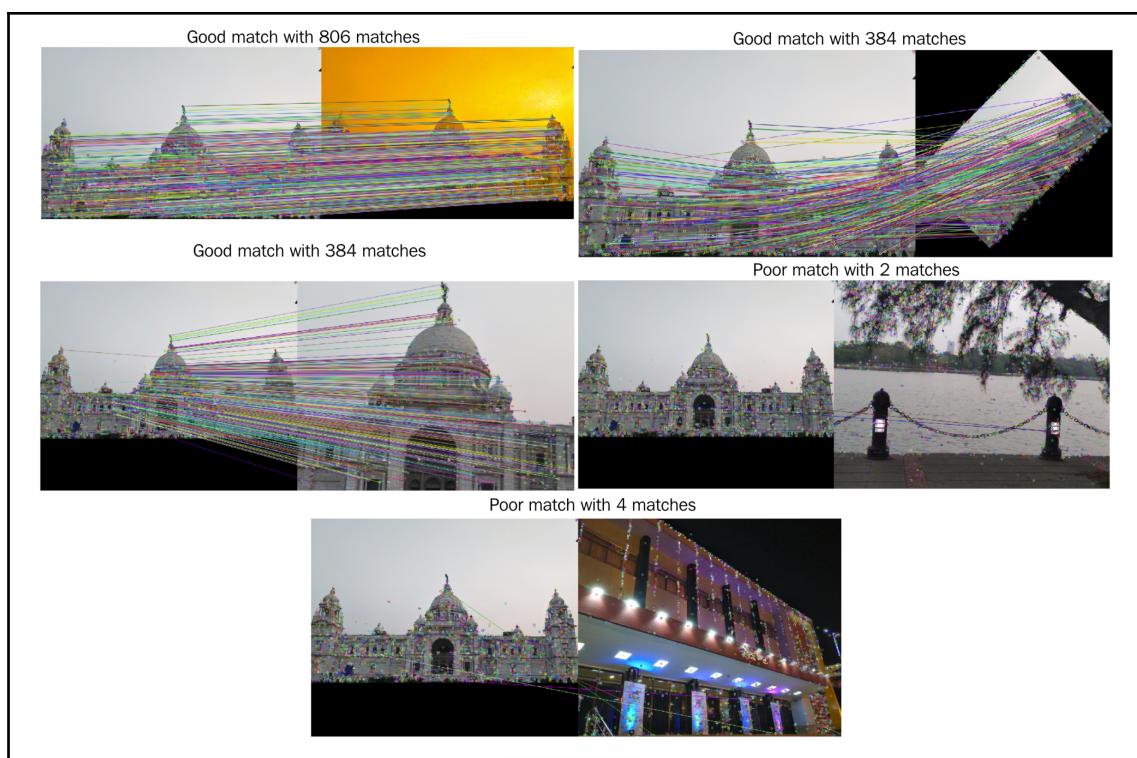
Destination image



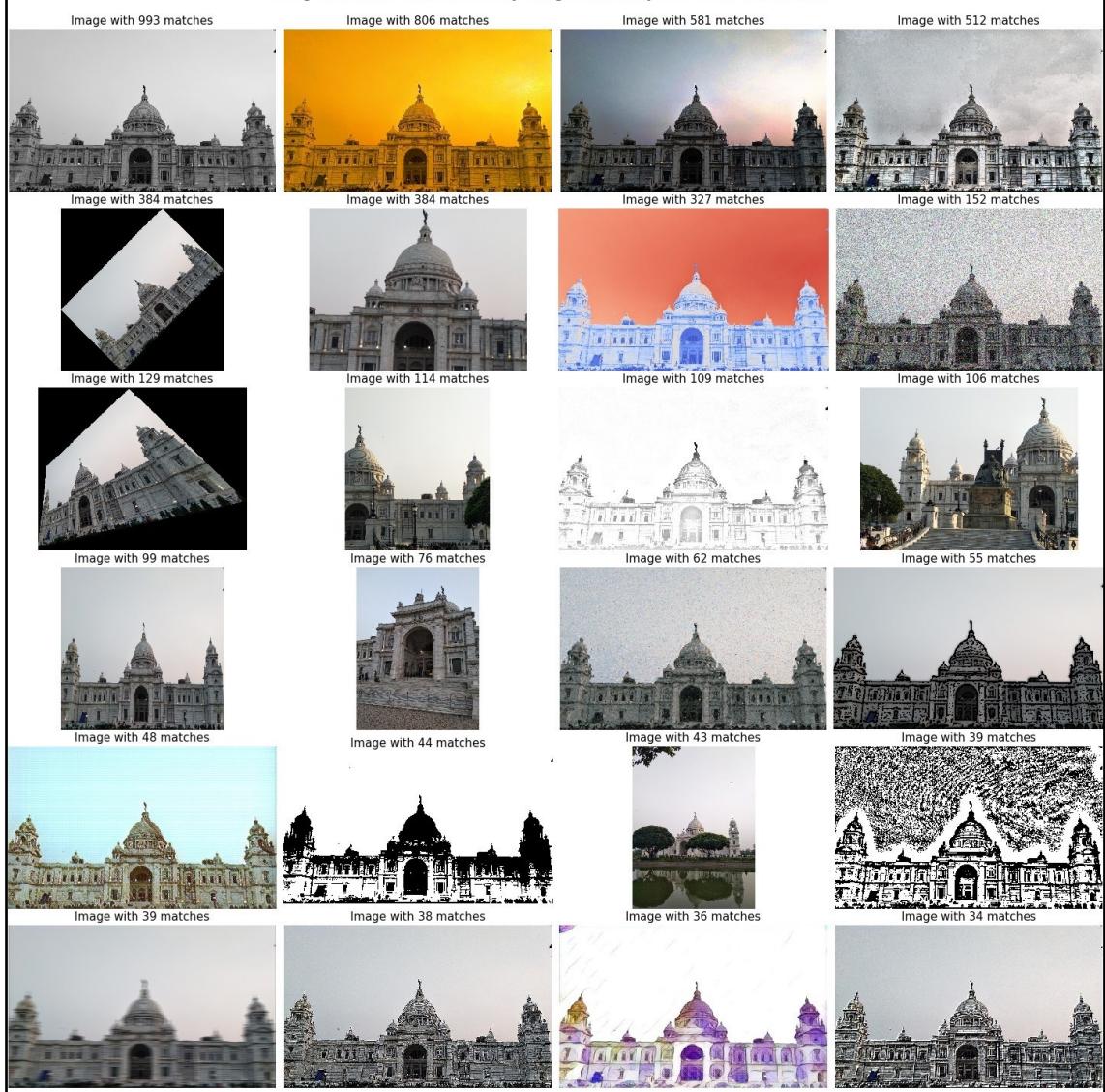
Face morphing

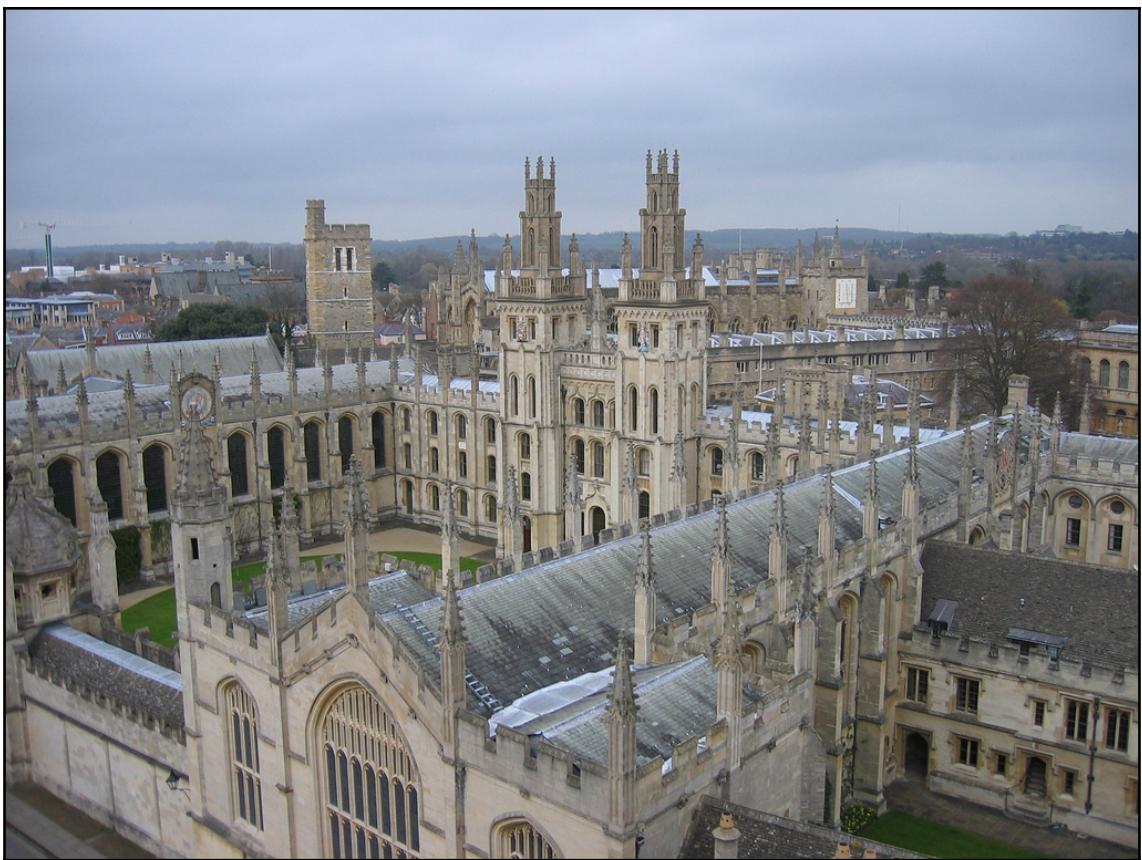




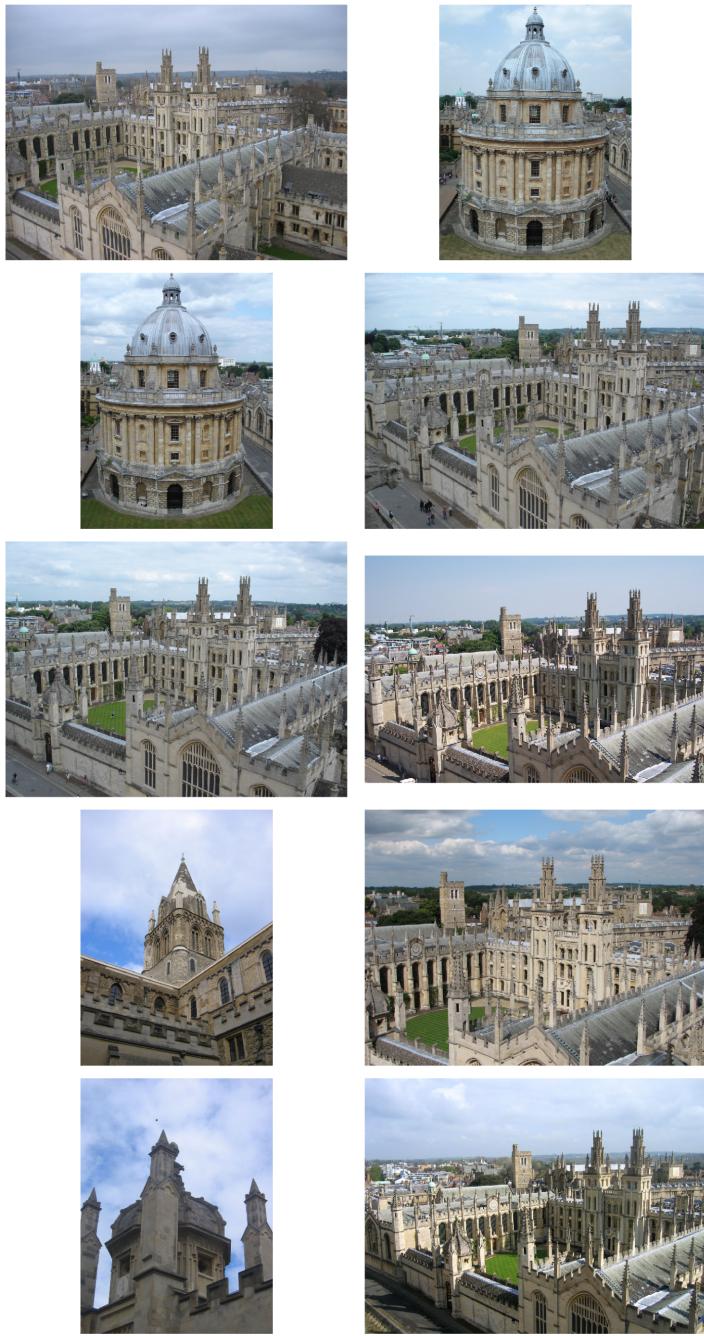


Images matched with the Query Image ranked by the number of matches





Top matched Images found by the Search Engine (with color histogram features)



Chapter 6: Image Segmentation

Otsu Thresholding

$$q_1(t) = \sum_{i=0}^t P(i) \quad q_2(t) = \sum_{i=t+1}^{L-1} P(i)$$

$$\mu_1(t) = \sum_{i=0}^t \frac{iP(i)}{q_1(t)} \quad \mu_2(t) = \sum_{i=t+1}^{L-1} \frac{iP(i)}{q_2(t)}$$

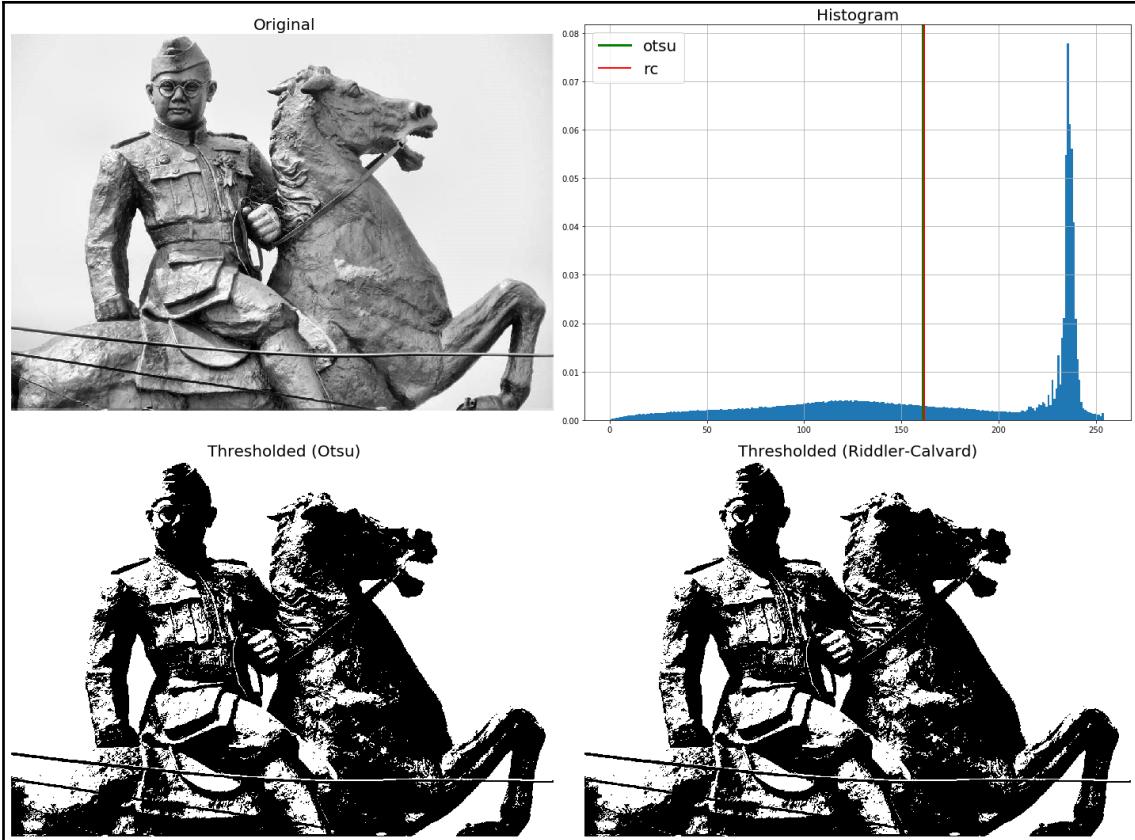
$$\sigma_1^2(t) = \sum_{i=0}^t [i - \mu_1(t)]^2 \frac{P(i)}{q_1(t)} \quad \sigma_2^2(t) = \sum_{i=t+1}^{L-1} [i - \mu_2(t)]^2 \frac{P(i)}{q_2(t)}$$

variance of class with pixel values $\leq t$ variance of class with pixel values $> t$

$$\sigma_w^2(t) = q_1(t) \color{red}\downarrow\color{black} \sigma_1^2(t) + q_2(t) \color{green}\downarrow\color{black} \sigma_2^2(t)$$

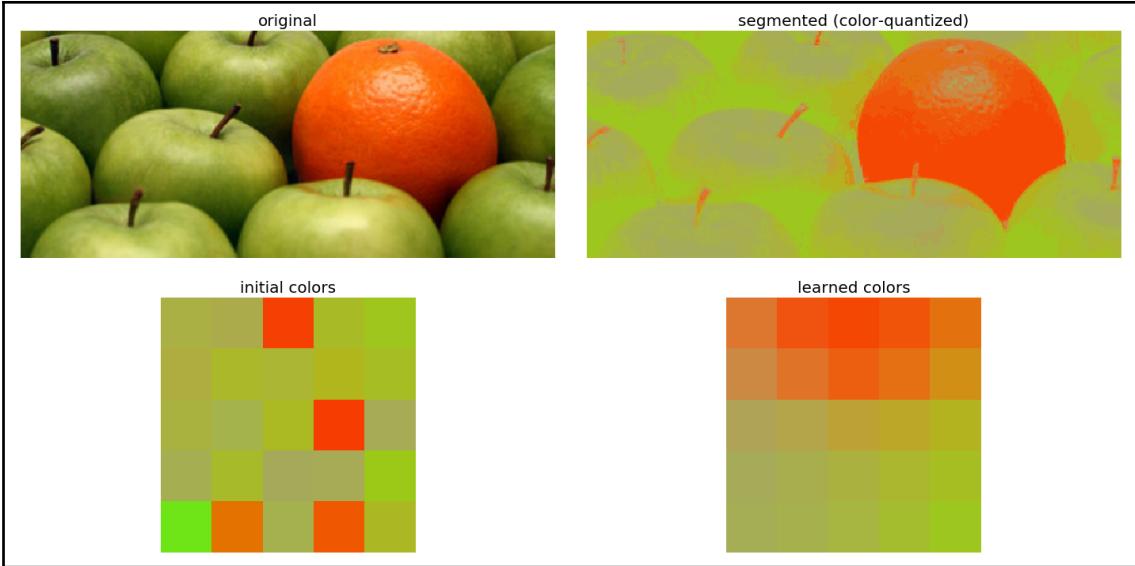
Weighted within-class variance

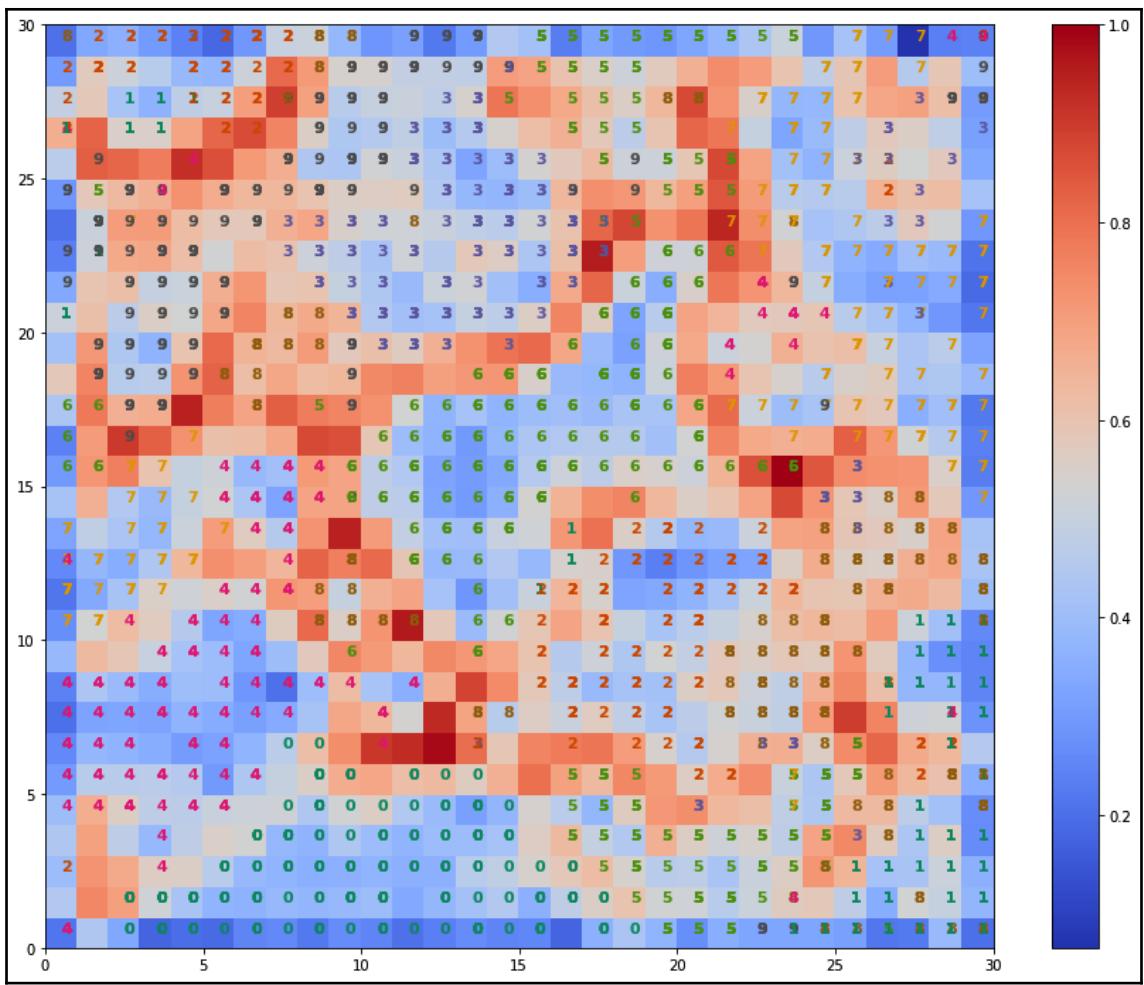
$$t_{otsu} = \underset{0 \leq t \leq L-1}{\operatorname{argmin}} \sigma_w^2(t)$$

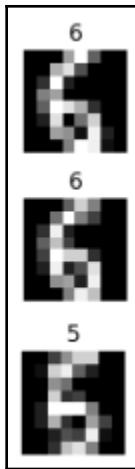


<p style="text-align: center;">Original</p> $\ell(\theta) = \log L_i(X; \theta)$ $\nabla \ell(\theta) \in \mathbb{R}^d \text{ if } \theta \in \mathbb{R}^d$ <p>$\text{I}(\theta) = \text{Cov}(\nabla \ell(\theta)) = E[\nabla \ell(\theta) \nabla \ell(\theta)^T] - E[\nabla \ell(\theta)]E[\nabla \ell(\theta)]^T$ $\text{I}(\theta)$ is a $d \times d$ matrix called Fisher information</p> <p><u>Theorem</u>: $\text{I}(\theta) = -E[H \ell(\theta)]$</p> <p>$\begin{aligned} g(z_n) - g(\theta) &= (\theta, \theta)g'(\theta) + \frac{\partial}{\partial \theta} g''(\theta) \\ &\approx (\theta, \theta)g'(\theta) \quad w \in [z_n, \theta] \end{aligned}$ Taken from the blackboard of MITx 18.6501x Fundamentals of Statistics class lectures</p> <p>$\begin{aligned} \nabla(g(z_n) - g(\theta)) &\approx \nabla(z_n - \theta)g'(\theta) \\ &\stackrel{(a)}{\sim} N(0, \sigma^2) \quad \sim N(0, \sigma^2) \end{aligned}$</p> <p>CLT $\sqrt{n}(\bar{X}_n^k - m_k(\theta)) \xrightarrow{D} N(0, \text{var}(X_i^k))$</p> <p>$\begin{aligned} f_b(w) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-\theta)^2} \quad \text{Test} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((m_k(x_i^k) - \theta)^2)} \quad \text{Type I} \\ P(\bar{\theta}_n - \theta > \epsilon) &\rightarrow 0 \\ n(\bar{\theta}_n - \theta) &\xrightarrow{D} N(0, P(1-p)) \end{aligned}$</p> <p>LI</p> $\ell(\theta) = \log L_i(X; \theta)$ $\nabla \ell(\theta) \in \mathbb{R}^d \text{ if } \theta \in \mathbb{R}^d$ <p>$\text{I}(\theta) = \text{Cov}(\nabla \ell(\theta)) = E[\nabla \ell(\theta) \nabla \ell(\theta)^T] - E[\nabla \ell(\theta)]E[\nabla \ell(\theta)]^T$ $\text{I}(\theta)$ is a $d \times d$ matrix called Fisher information</p> <p><u>Theorem</u>: $\text{I}(\theta) = -E[H \ell(\theta)]$</p> <p>$\begin{aligned} g(z_n) - g(\theta) &= (\theta, \theta)g'(\theta) + \frac{\partial}{\partial \theta} g''(\theta) \\ &\approx (\theta, \theta)g'(\theta) \quad w \in [z_n, \theta] \end{aligned}$ Taken from the blackboard of MITx 18.6501x Fundamentals of Statistics class lectures</p> <p>$\begin{aligned} \nabla(g(z_n) - g(\theta)) &\approx \nabla(z_n - \theta)g'(\theta) \\ &\stackrel{(a)}{\sim} N(0, \sigma^2) \quad \sim N(0, \sigma^2) \end{aligned}$</p> <p>CLT $\sqrt{n}(\bar{X}_n^k - m_k(\theta)) \xrightarrow{D} N(0, \text{var}(X_i^k))$</p> <p>$\begin{aligned} f_b(w) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-\theta)^2} \quad \text{Test} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((m_k(x_i^k) - \theta)^2)} \quad \text{Type I} \\ P(\bar{\theta}_n - \theta > \epsilon) &\rightarrow 0 \\ n(\bar{\theta}_n - \theta) &\xrightarrow{D} N(0, P(1-p)) \end{aligned}$</p>	<p style="text-align: center;">Isodata</p> $\ell(\theta) = \log L_i(X; \theta)$ $\nabla \ell(\theta) \in \mathbb{R}^d \text{ if } \theta \in \mathbb{R}^d$ <p>$\text{I}(\theta) = \text{Cov}(\nabla \ell(\theta)) = E[\nabla \ell(\theta) \nabla \ell(\theta)^T] - E[\nabla \ell(\theta)]E[\nabla \ell(\theta)]^T$ $\text{I}(\theta)$ is a $d \times d$ matrix called Fisher information</p> <p><u>Theorem</u>: $\text{I}(\theta) = -E[H \ell(\theta)]$</p> <p>$\begin{aligned} g(z_n) - g(\theta) &= (\theta, \theta)g'(\theta) + \frac{\partial}{\partial \theta} g''(\theta) \\ &\approx (\theta, \theta)g'(\theta) \quad w \in [z_n, \theta] \end{aligned}$ Taken from the blackboard of MITx 18.6501x Fundamentals of Statistics class lectures</p> <p>$\begin{aligned} \nabla(g(z_n) - g(\theta)) &\approx \nabla(z_n - \theta)g'(\theta) \\ &\stackrel{(a)}{\sim} N(0, \sigma^2) \quad \sim N(0, \sigma^2) \end{aligned}$</p> <p>CLT $\sqrt{n}(\bar{X}_n^k - m_k(\theta)) \xrightarrow{D} N(0, \text{var}(X_i^k))$</p> <p>$\begin{aligned} f_b(w) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-\theta)^2} \quad \text{Test} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((m_k(x_i^k) - \theta)^2)} \quad \text{Type I} \\ P(\bar{\theta}_n - \theta > \epsilon) &\rightarrow 0 \\ n(\bar{\theta}_n - \theta) &\xrightarrow{D} N(0, P(1-p)) \end{aligned}$</p> <p>Mean</p> $\ell(\theta) = \log L_i(X; \theta)$ $\nabla \ell(\theta) \in \mathbb{R}^d \text{ if } \theta \in \mathbb{R}^d$ <p>$\text{I}(\theta) = \text{Cov}(\nabla \ell(\theta)) = E[\nabla \ell(\theta) \nabla \ell(\theta)^T] - E[\nabla \ell(\theta)]E[\nabla \ell(\theta)]^T$ $\text{I}(\theta)$ is a $d \times d$ matrix called Fisher information</p> <p><u>Theorem</u>: $\text{I}(\theta) = -E[H \ell(\theta)]$</p> <p>$\begin{aligned} g(z_n) - g(\theta) &= (\theta, \theta)g'(\theta) + \frac{\partial}{\partial \theta} g''(\theta) \\ &\approx (\theta, \theta)g'(\theta) \quad w \in [z_n, \theta] \end{aligned}$ Taken from the blackboard of MITx 18.6501x Fundamentals of Statistics class lectures</p> <p>$\begin{aligned} \nabla(g(z_n) - g(\theta)) &\approx \nabla(z_n - \theta)g'(\theta) \\ &\stackrel{(a)}{\sim} N(0, \sigma^2) \quad \sim N(0, \sigma^2) \end{aligned}$</p> <p>CLT $\sqrt{n}(\bar{X}_n^k - m_k(\theta)) \xrightarrow{D} N(0, \text{var}(X_i^k))$</p> <p>$\begin{aligned} f_b(w) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-\theta)^2} \quad \text{Test} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((m_k(x_i^k) - \theta)^2)} \quad \text{Type I} \\ P(\bar{\theta}_n - \theta > \epsilon) &\rightarrow 0 \\ n(\bar{\theta}_n - \theta) &\xrightarrow{D} N(0, P(1-p)) \end{aligned}$</p>
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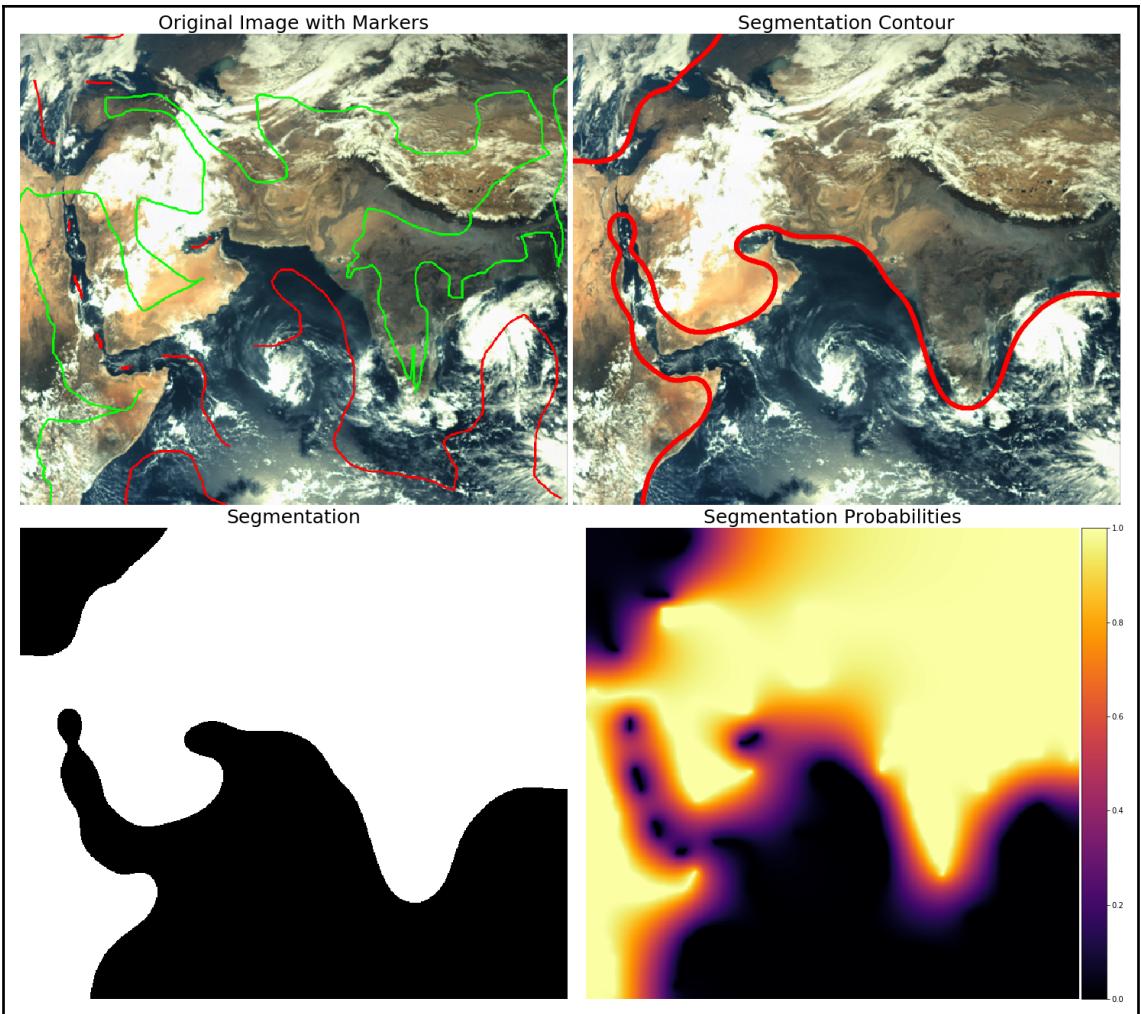


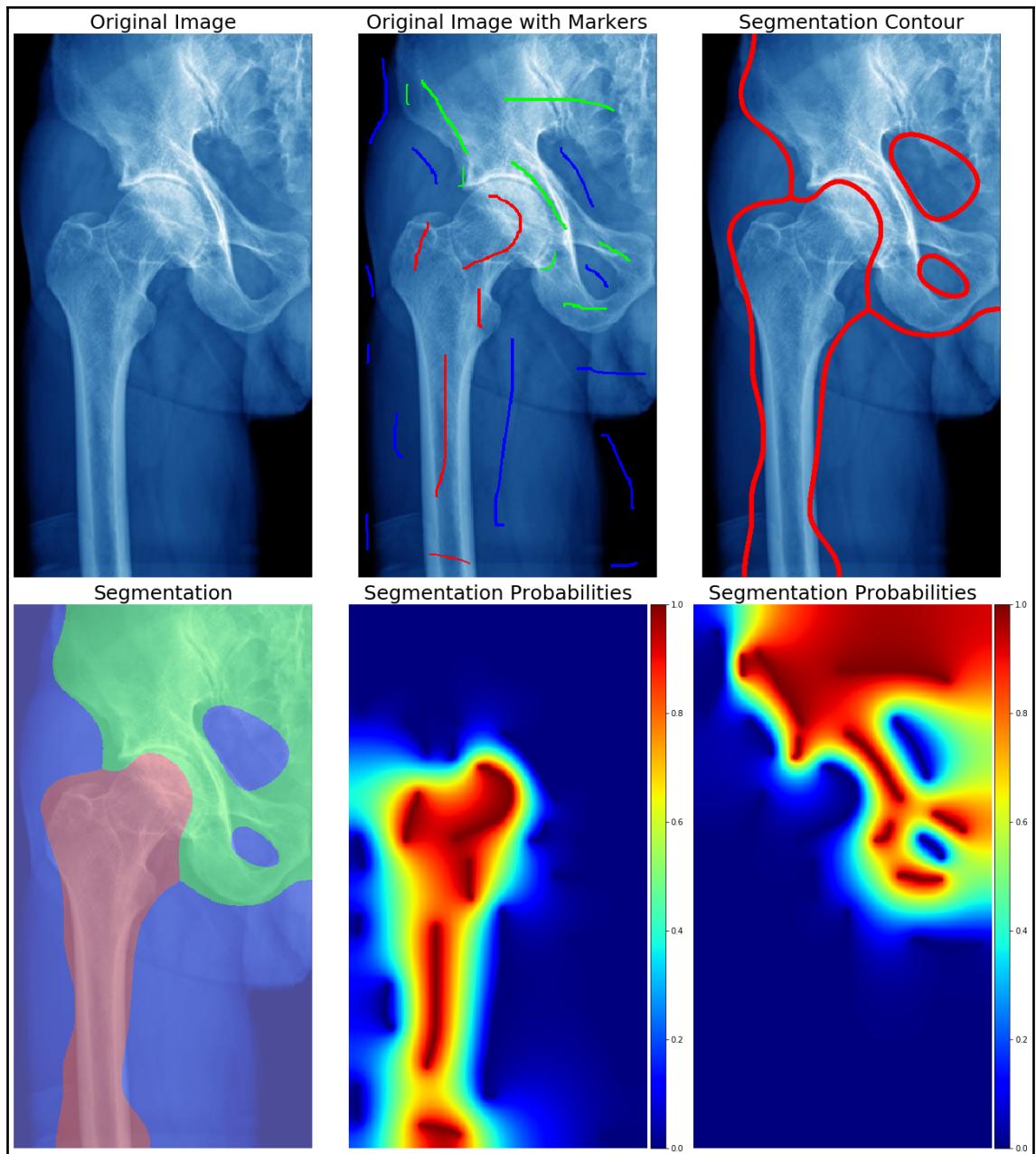




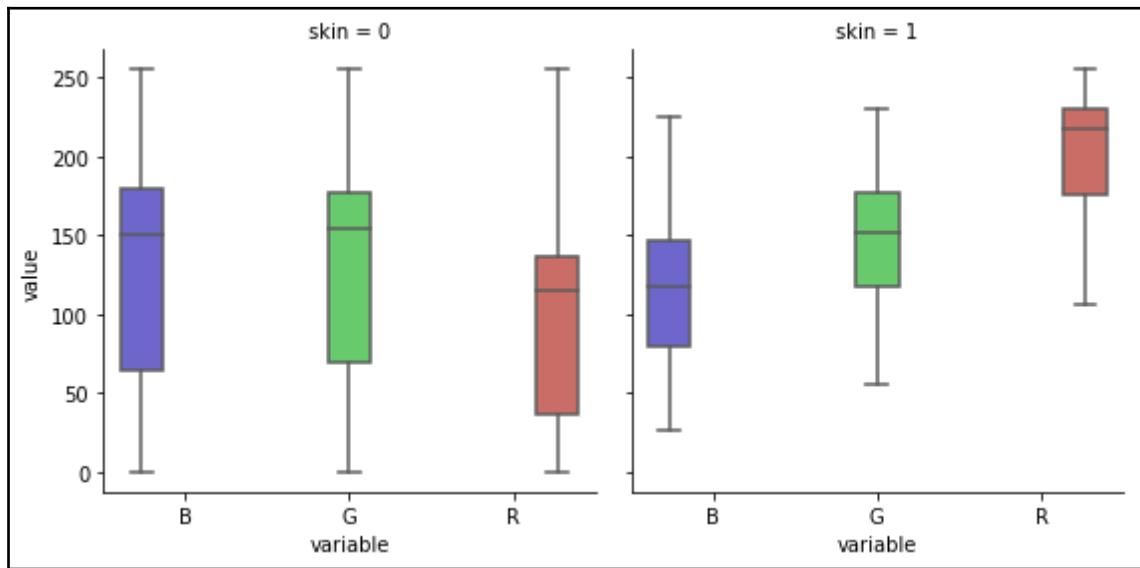
Random Walk for Image Segmentation	Combinatorial Dirichlet problem				
<p><i>Algorithm</i></p> <ol style="list-style-type: none"> 1) Map the image intensities to edge weights in the lattice $w_{ij} = \exp(-\beta(g_i - g_j)^2),$ g_i = image intensity at pixel i 2) Obtain a set, V_M, of marked (labeled) pixels with K labels, either interactively or automatically. 3) Solve for the potentials $L_U x^s = -B^T m^s$ for each label except the final one, f (for computational efficiency). Set $x_i^f = 1 - \sum_{s < f} x_i^s$. 4) Obtain a final segmentation by assigning to each node, v_i, the label corresponding to $\max_s (x_i^s)$. 	<p>The Dirichlet integral $D[u] = \frac{1}{2} \int_{\Omega} \nabla u ^2 d\Omega$.</p> <p>harmonic function $\nabla^2 u = 0$ Laplace equation combinatorial Laplacian matrix</p> $L_{ij} = \begin{cases} d_i & \text{if } i = j, \\ -w_{ij} & \text{if } v_i \text{ and } v_j \text{ are adjacent nodes,} \\ 0 & \text{otherwise,} \end{cases}$ <p>x_i^s = probability (potential) at node, v_i, for label, s $Q(v_j) = s$, $\forall v_j \in V_M$, $\sum_s x_i^s = 1$, $\forall v_i \in V$</p> <table style="margin-left: 200px;"> <tr> <td>set of labels</td> <td>seed</td> </tr> <tr> <td>for seed points</td> <td>points</td> </tr> </table> $m_j^s = \begin{cases} 1 & \text{if } Q(v_j) = s, \\ 0 & \text{if } Q(v_j) \neq s. \end{cases} \quad s \in \mathbb{Z}, 0 < s \leq K$	set of labels	seed	for seed points	points
set of labels	seed				
for seed points	points				

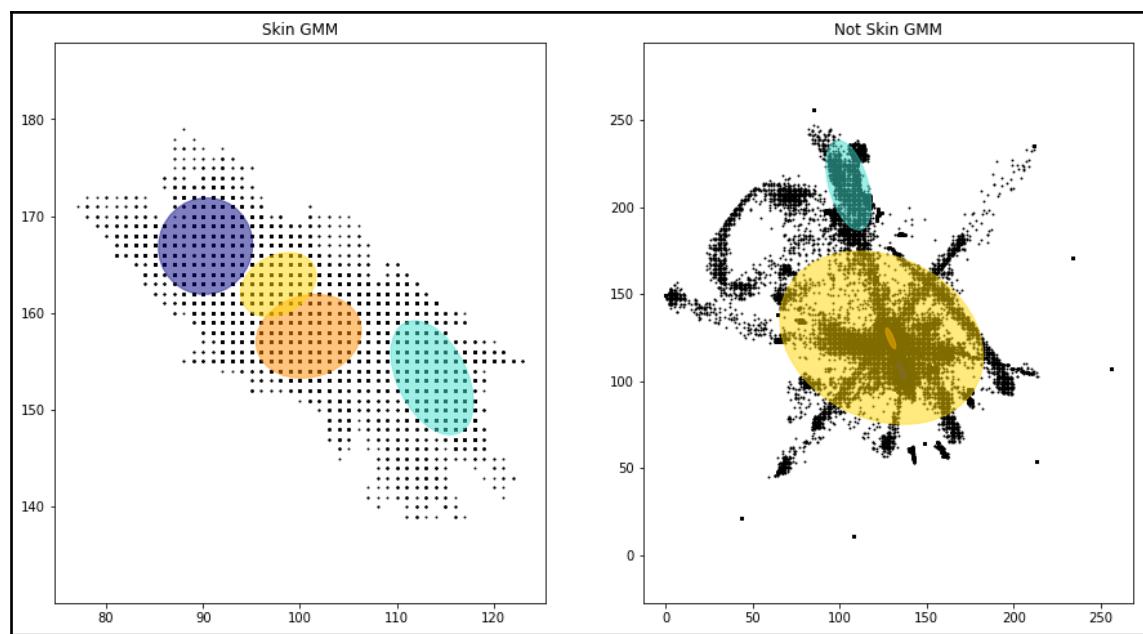
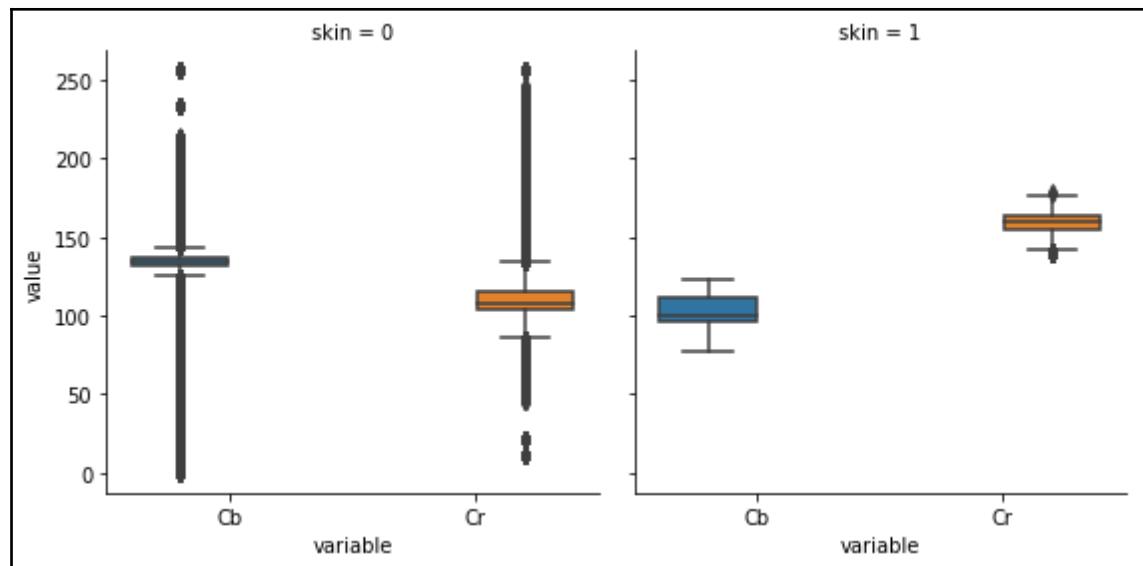
Ref: <http://vision.cse.psu.edu/people/chenpingY/paper/grady2006random.pdf>

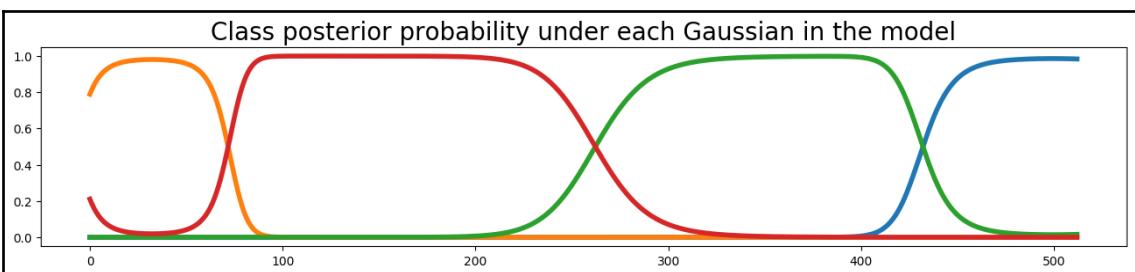
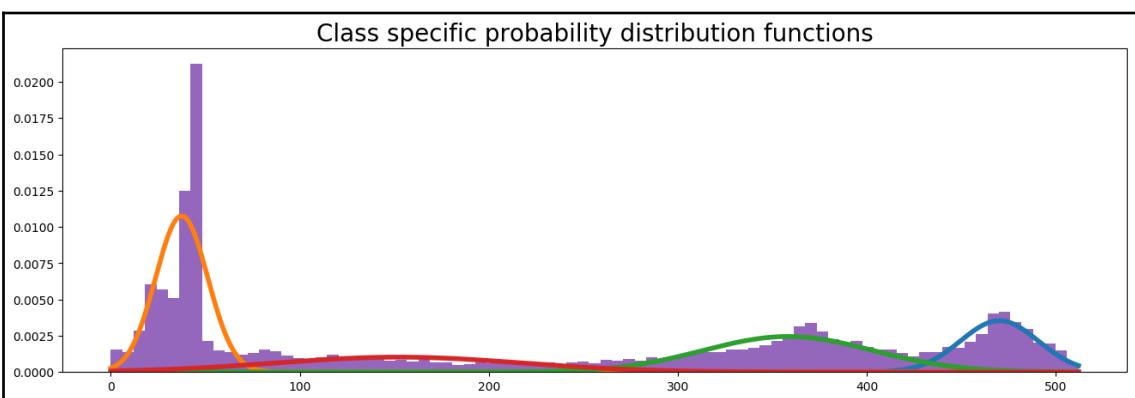
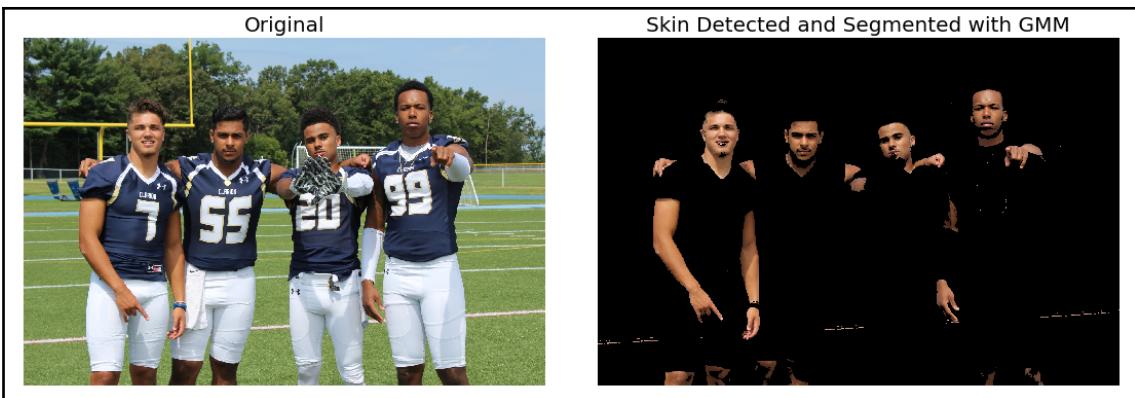




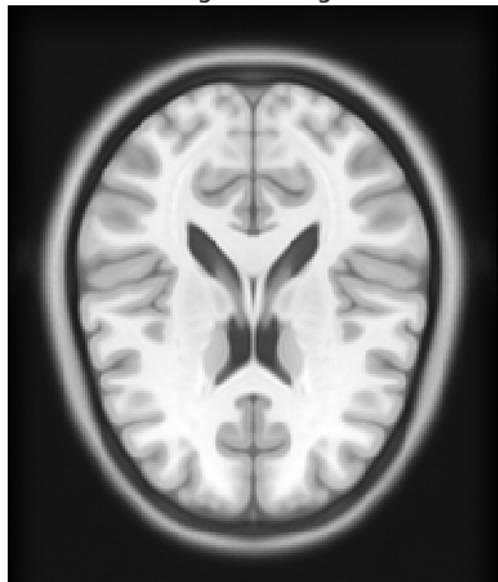
B	G	R	skin
74	85	123	1
73	84	122	1
72	83	121	1
70	81	119	1
70	81	119	1



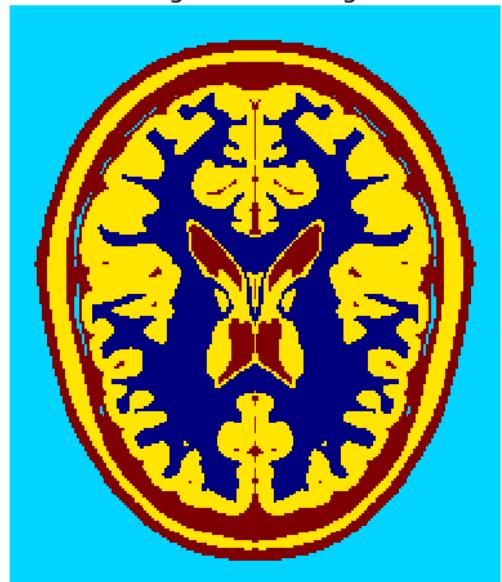


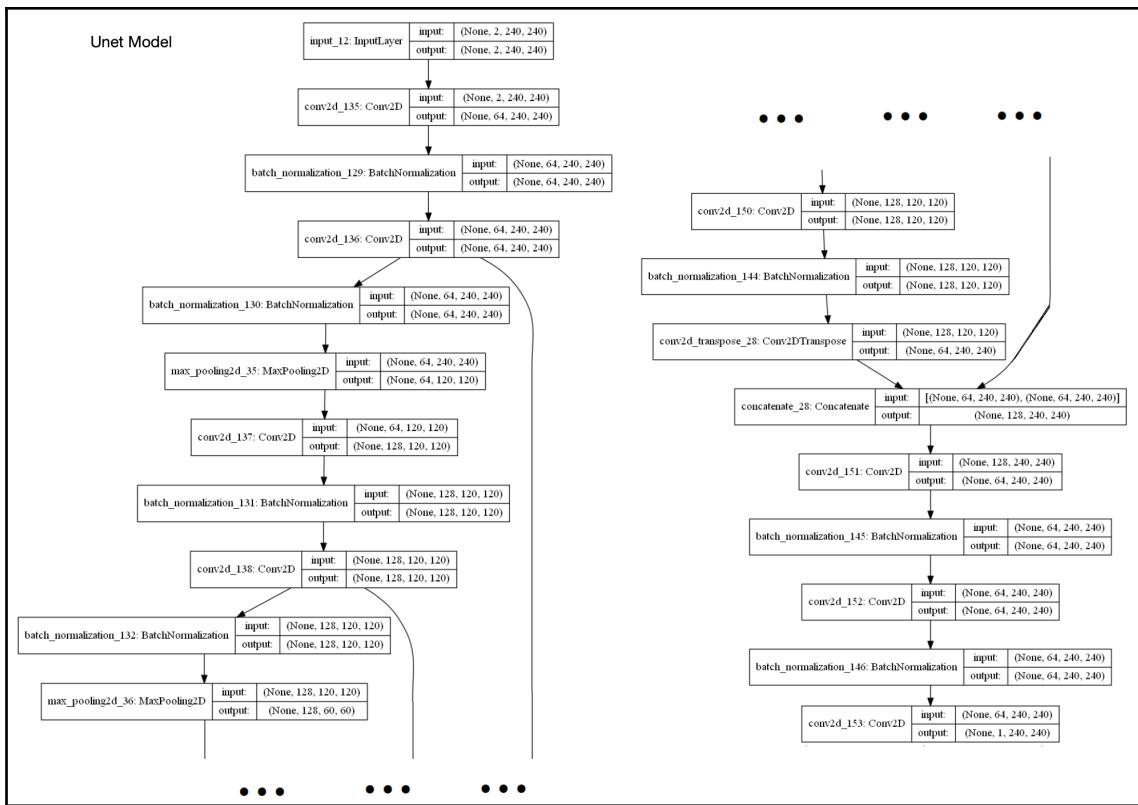


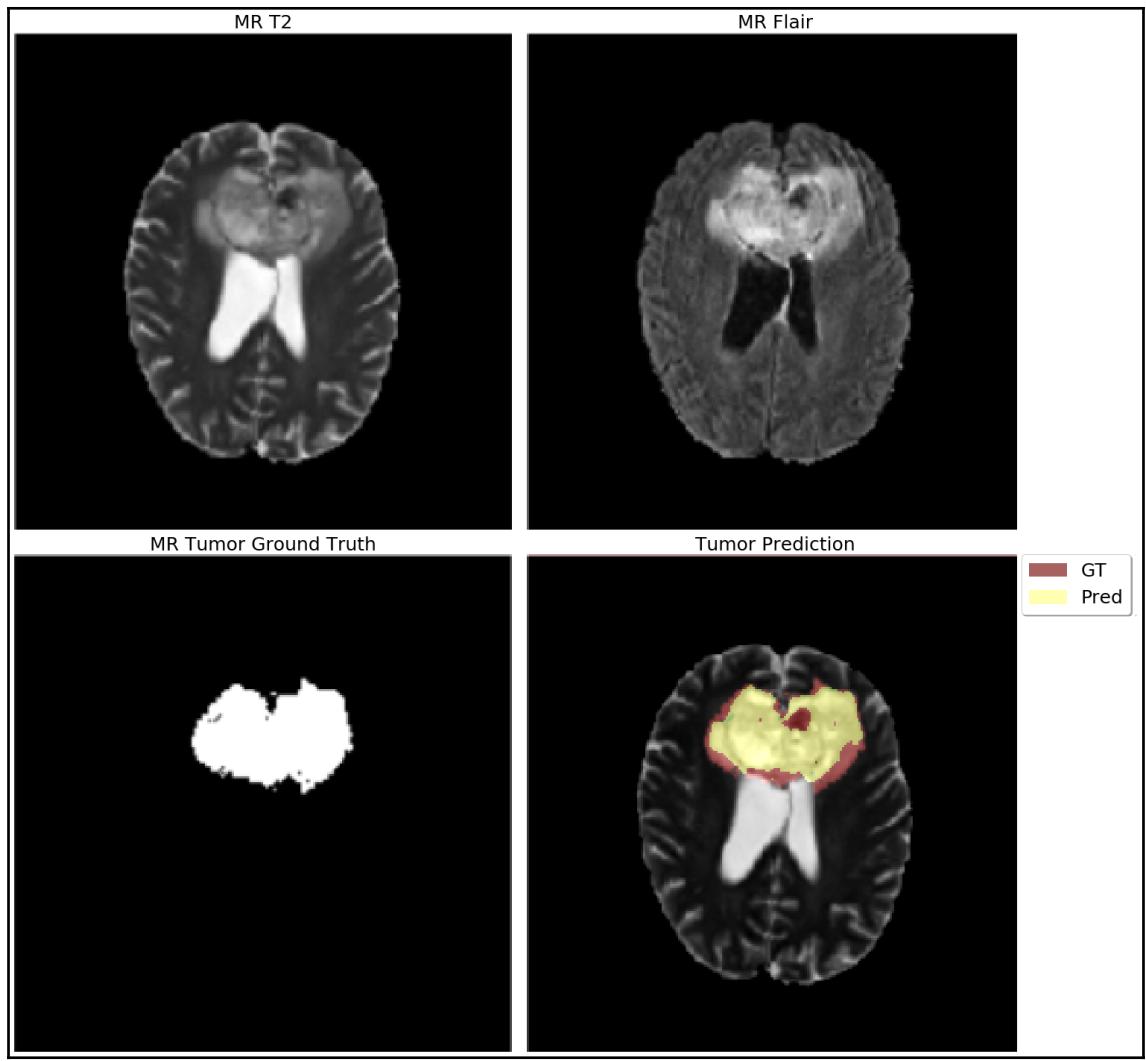
Original image

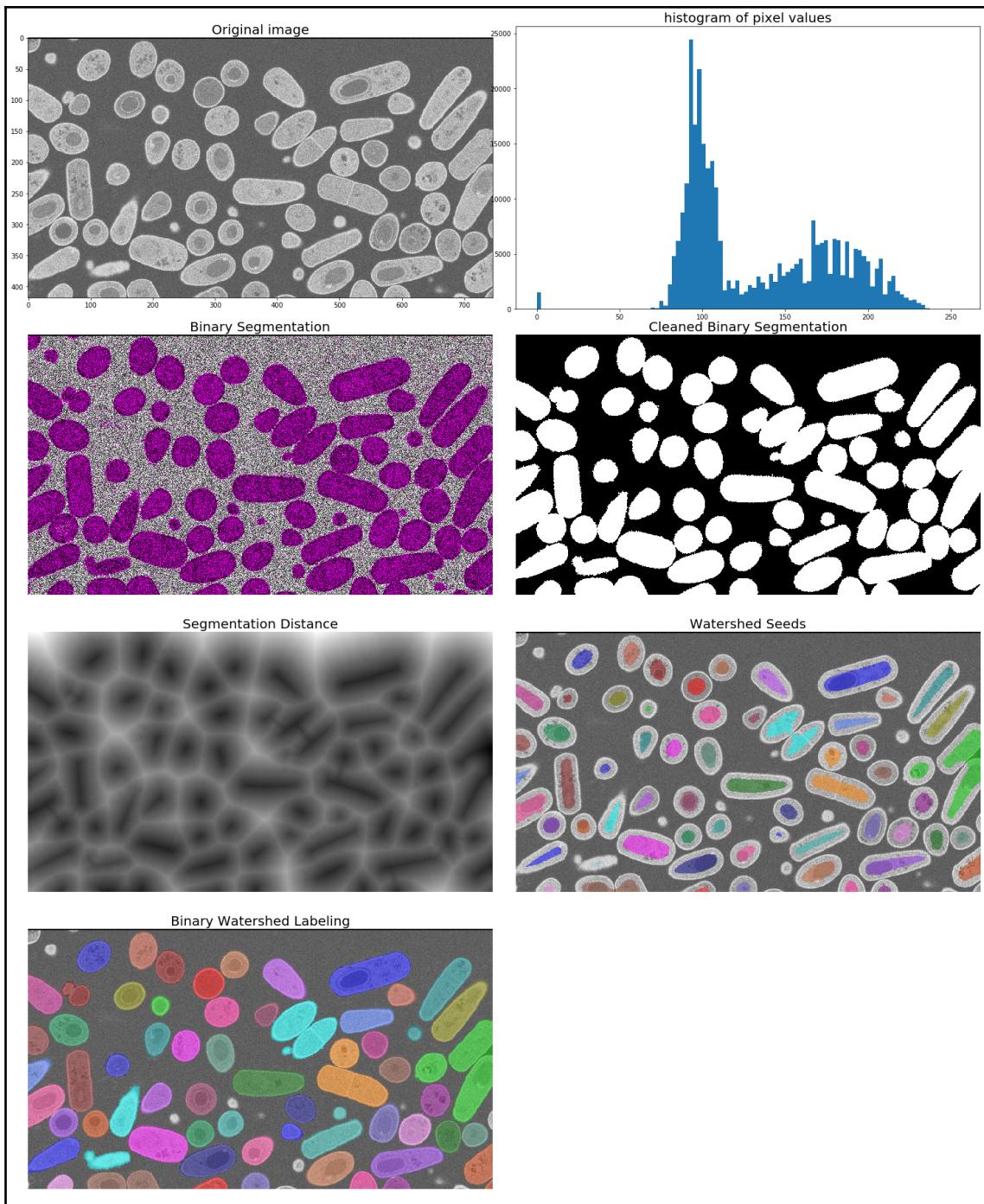


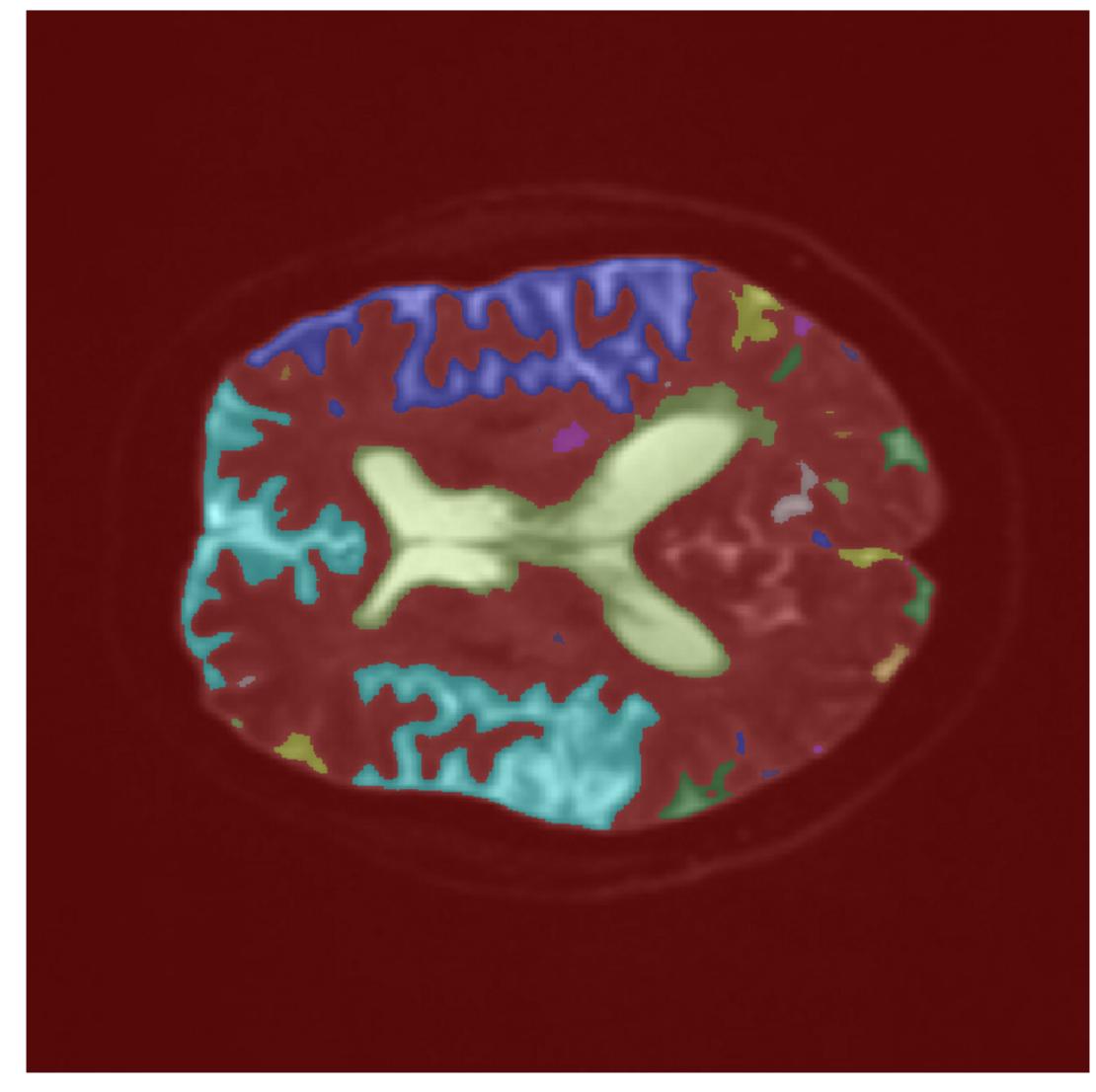
Segmented image











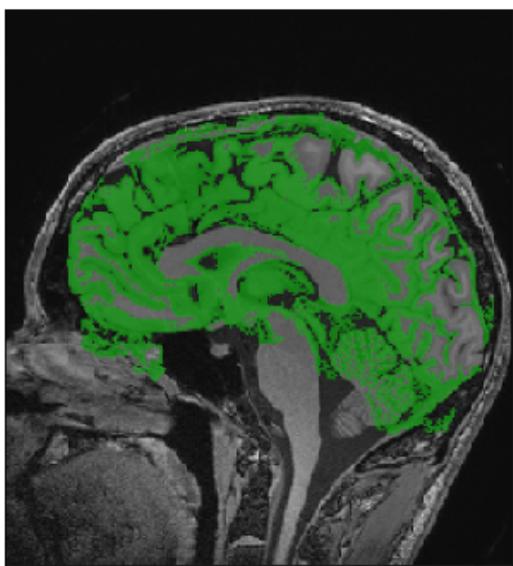
Original Image

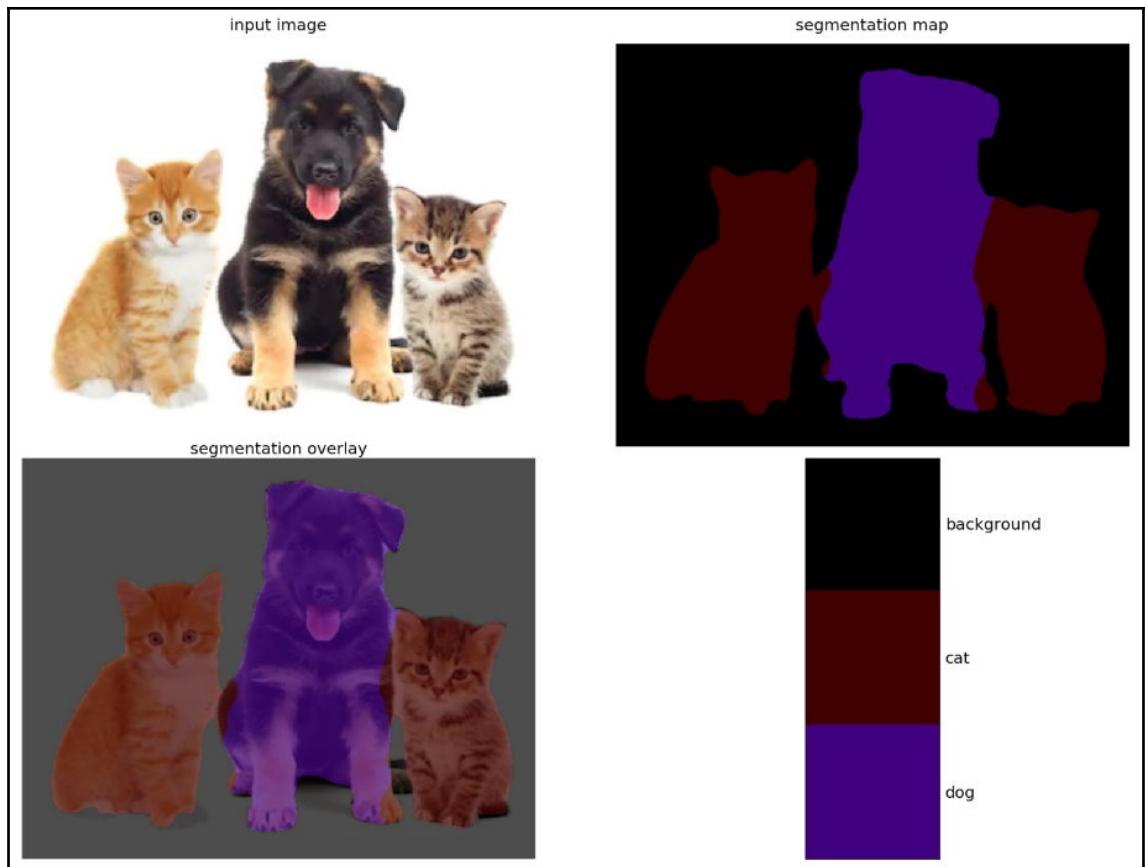


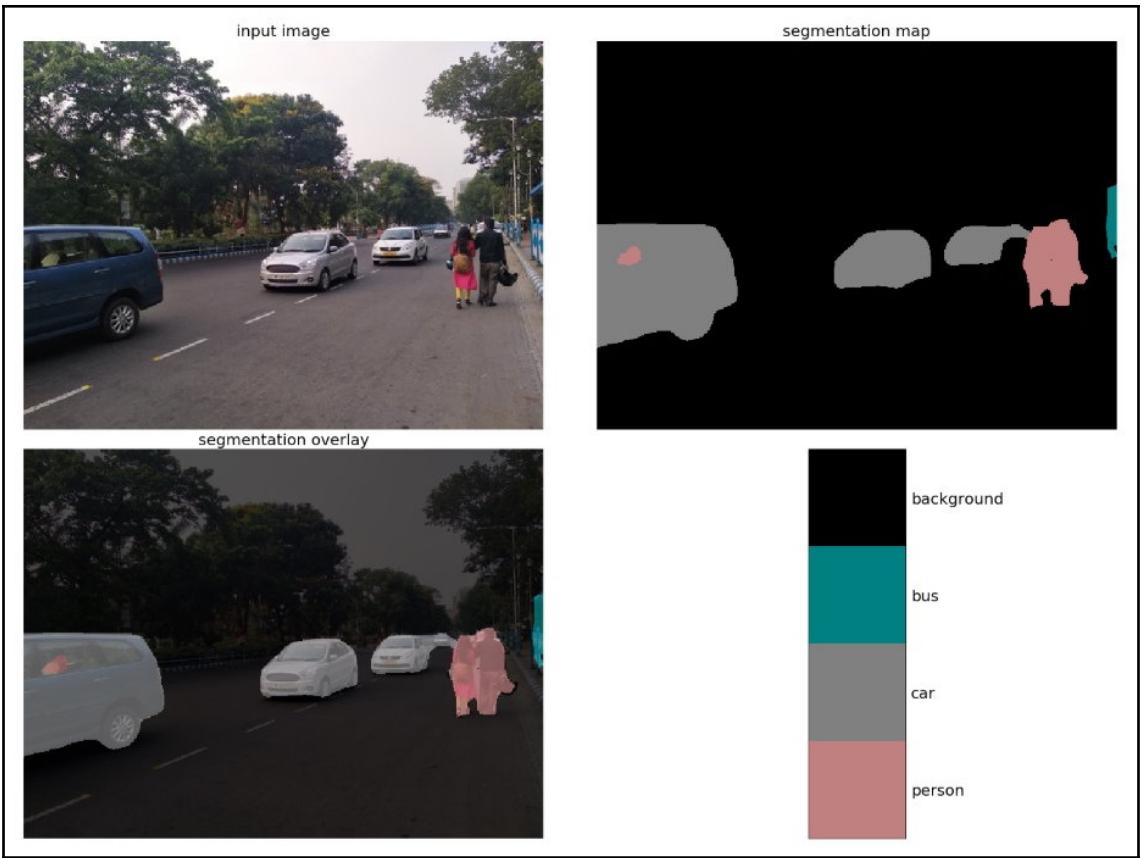
Region Growing



Connected Threshold







Original Image



Segmentation map

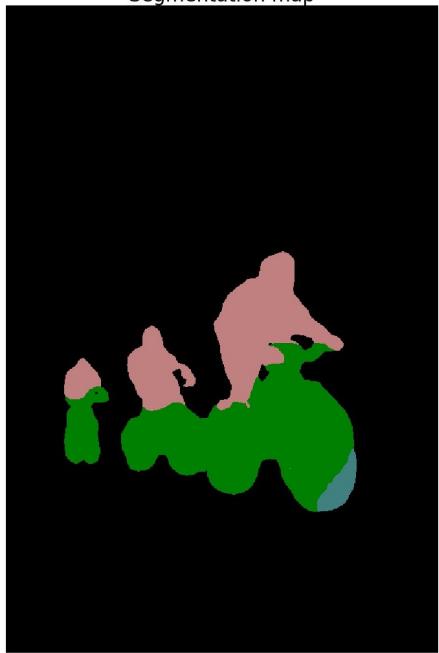
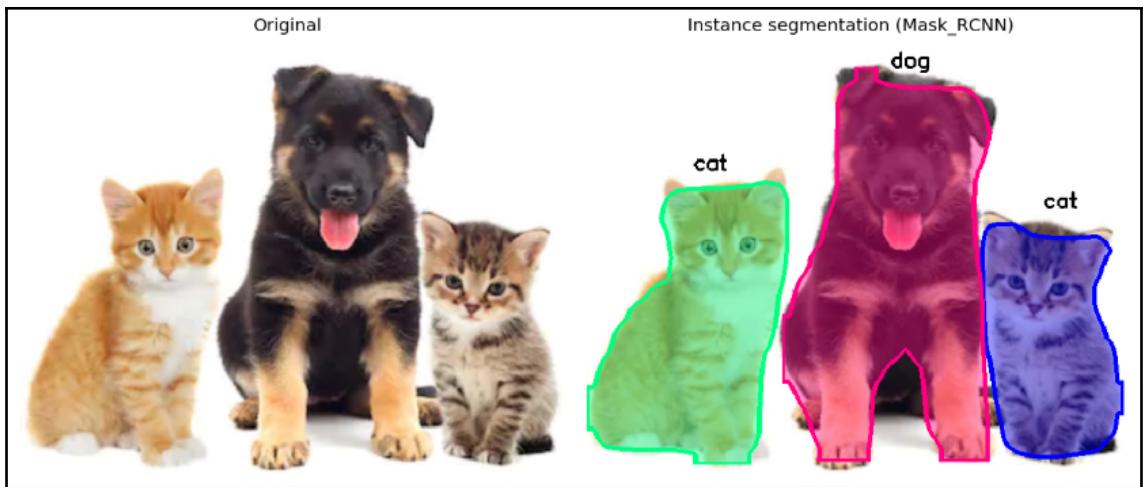
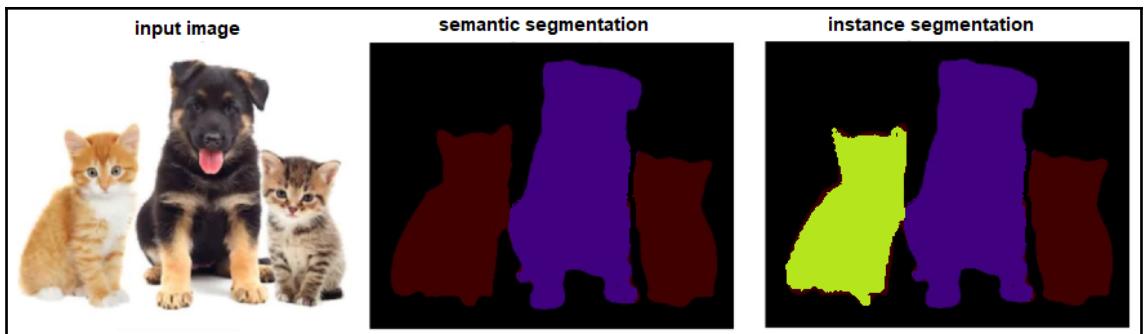


Image with Segmentation overlay



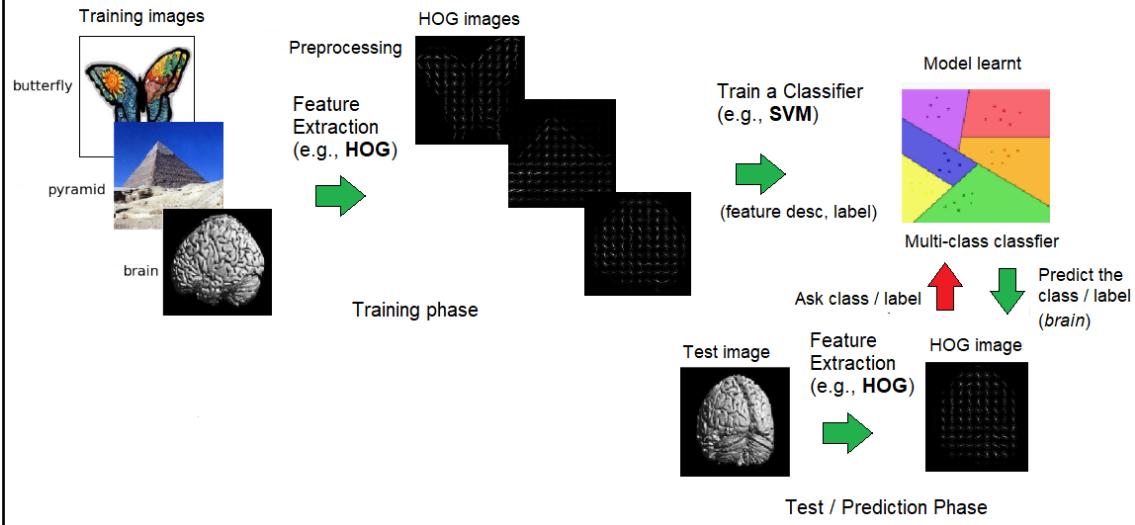
legends

background	
aeroplane	
bicycle	
bird	
boat	
bottle	
bus	
car	
cat	
chair	
cow	
diningtable	
dog	
horse	
motorbike	
person	
pottedplant	
sheep	
sofa	
train	
tvmonitor	

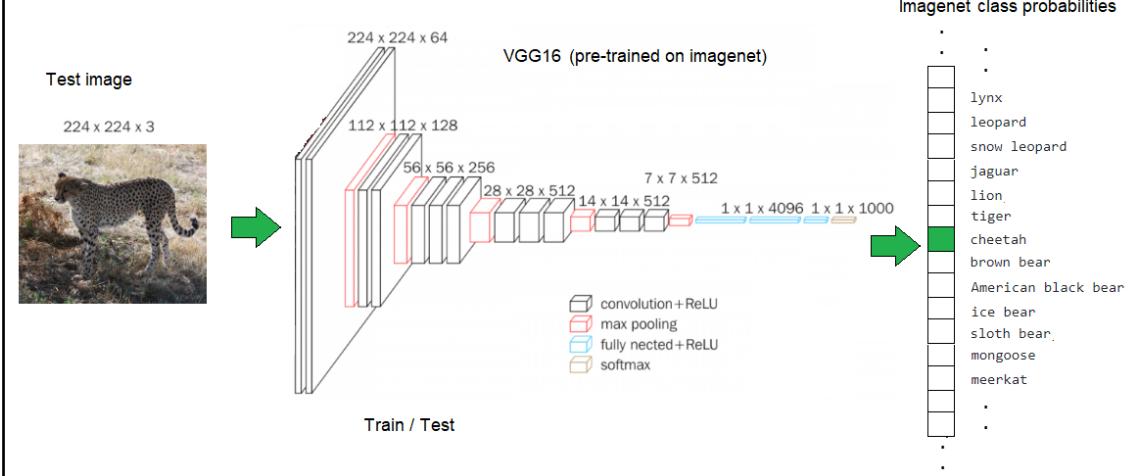


Chapter 7: Image Classification

Feature-based Image Classification



Deep learning-based (end-to-end) Image Classification



Mulitnomial logit Classifier (Softmax Regression)

Training (parameter estimation)

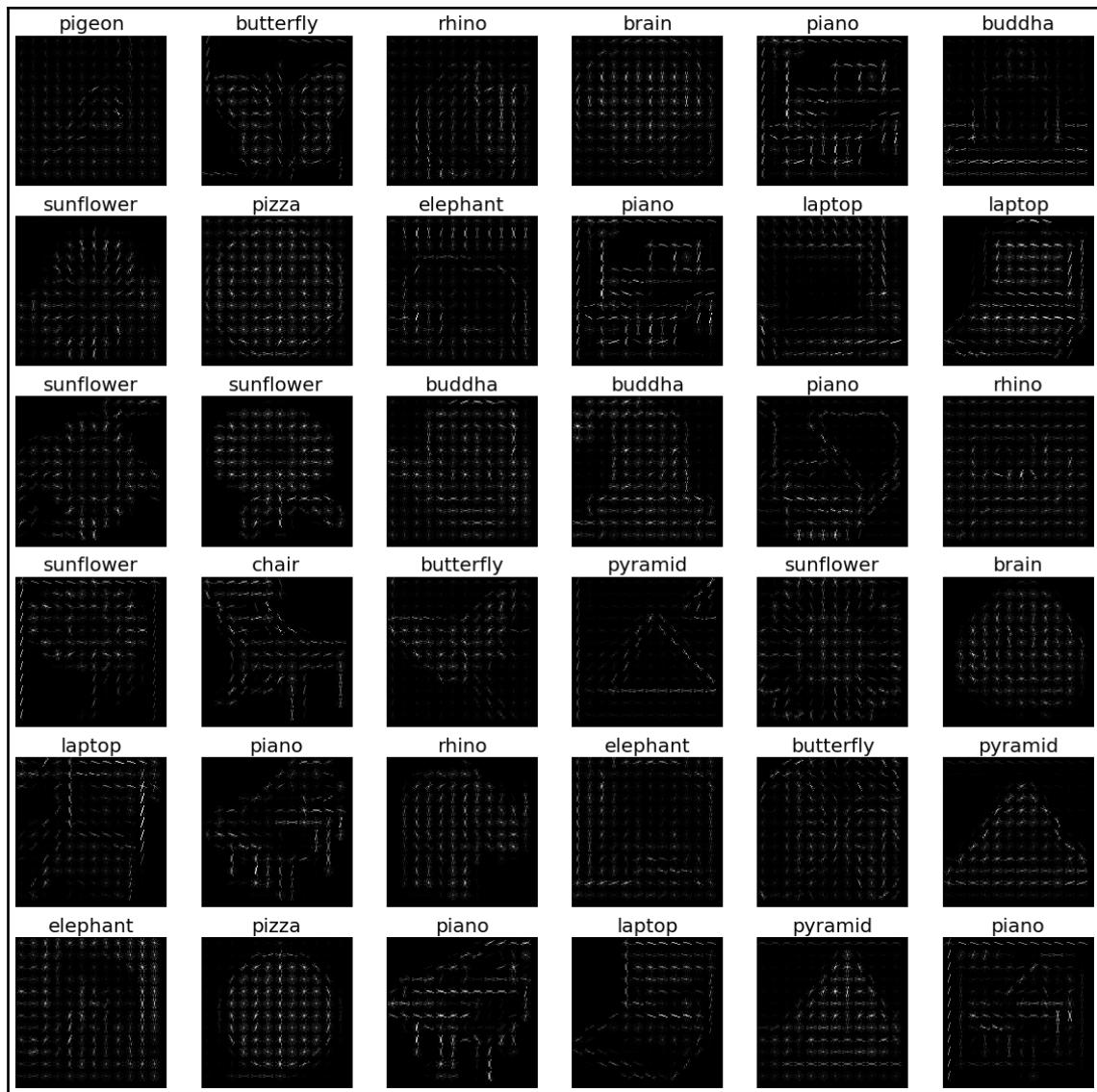
$$J(\theta) = - \left[\sum_{i=1}^m \sum_{k=1}^K 1\{y^{(i)} = k\} \log \frac{\exp(\theta^{(k)\top} x^{(i)})}{\sum_{j=1}^K \exp(\theta^{(j)\top} x^{(i)})} \right]$$
$$\nabla_{\theta^{(k)}} J(\theta) = - \sum_{i=1}^m \left[x^{(i)} \left(1\{y^{(i)} = k\} - P(y^{(i)} = k | x^{(i)}; \theta) \right) \right]$$

$$\text{SGD: } \theta = \theta - \alpha \nabla_{\theta} J(\theta; x^{(i)}, y^{(i)})$$

Prediction

$$P(y^{(i)} = k | x^{(i)}; \theta) = \frac{\exp(\theta^{(k)\top} x^{(i)})}{\sum_{j=1}^K \exp(\theta^{(j)\top} x^{(i)})}$$

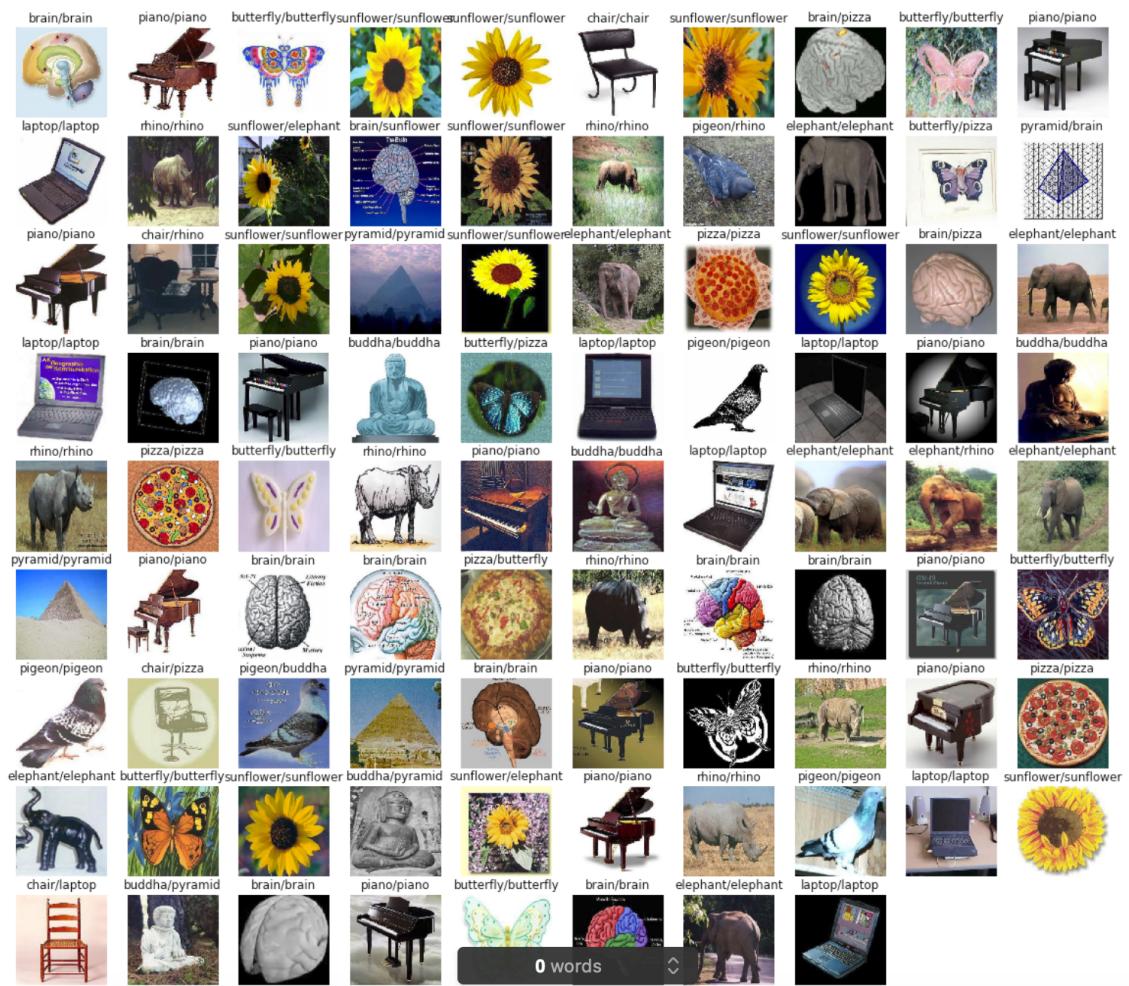




Accuracy: 0.8068181818181818

	precision	recall	f1-score	support
brain	0.90	0.75	0.82	12
buddha	0.75	0.60	0.67	5
butterfly	0.88	0.78	0.82	9
chair	1.00	0.25	0.40	4
elephant	0.78	0.88	0.82	8
laptop	0.88	1.00	0.93	7
piano	1.00	1.00	1.00	12
pigeon	1.00	0.60	0.75	5
pizza	0.38	0.75	0.50	4
pyramid	0.60	0.75	0.67	4
rhino	0.70	1.00	0.82	7
sunflower	0.90	0.82	0.86	11
accuracy			0.81	88
macro avg	0.81	0.76	0.76	88
weighted avg	0.85	0.81	0.81	88

Actual vs. Predicted Class Labels



$$g(x, y; \lambda, \theta, \psi, \sigma, \gamma) = \exp\left(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}\right) \exp\left(i\left(2\pi\frac{x'}{\lambda} + \psi\right)\right)$$

where $x' = x \cos \theta + y \sin \theta$

$$y' = -x \sin \theta + y \cos \theta$$

λ wavelength of the sinusoidal factor

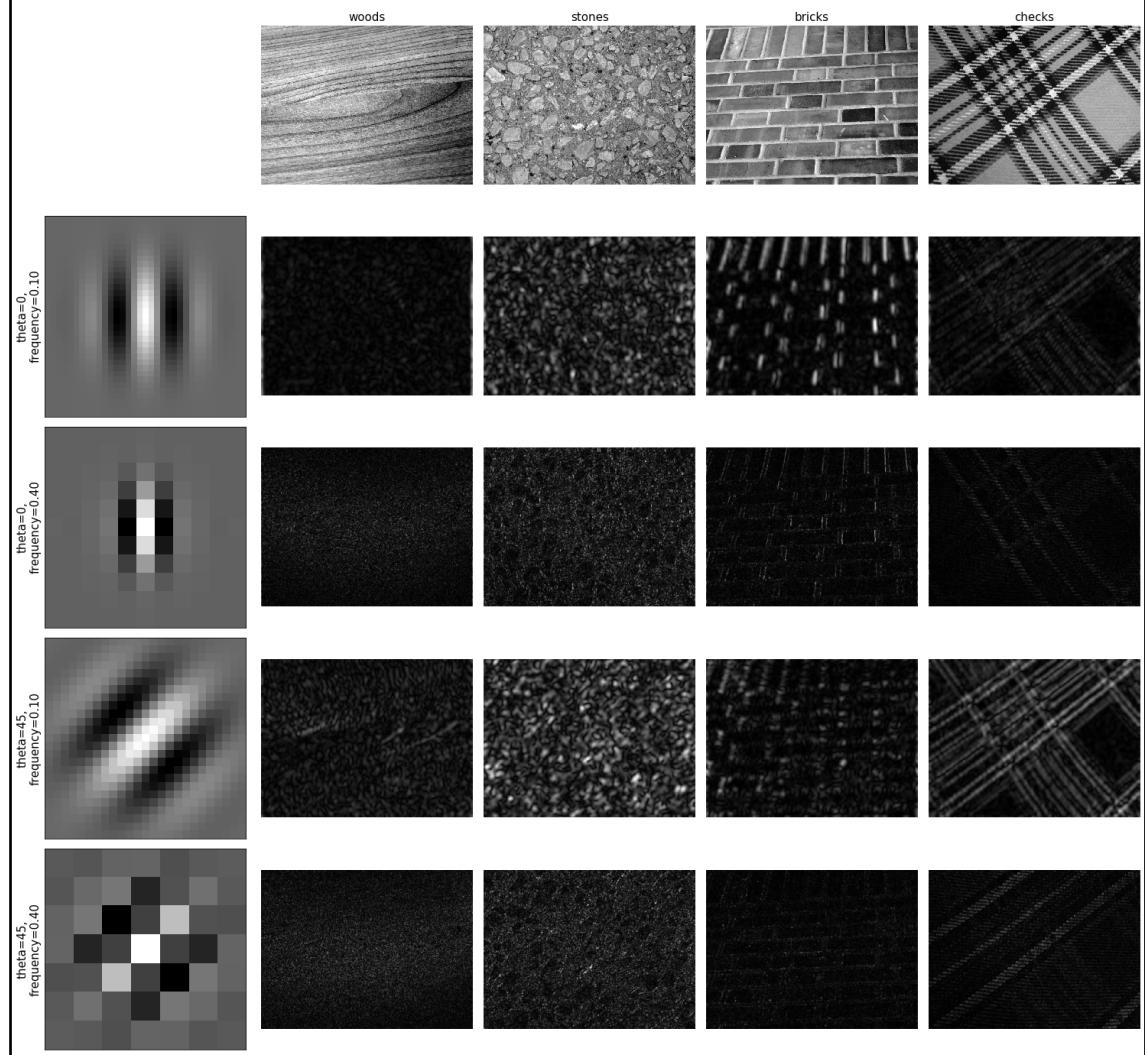
θ orientation of the normal to the parallel stripes of a Gabor function

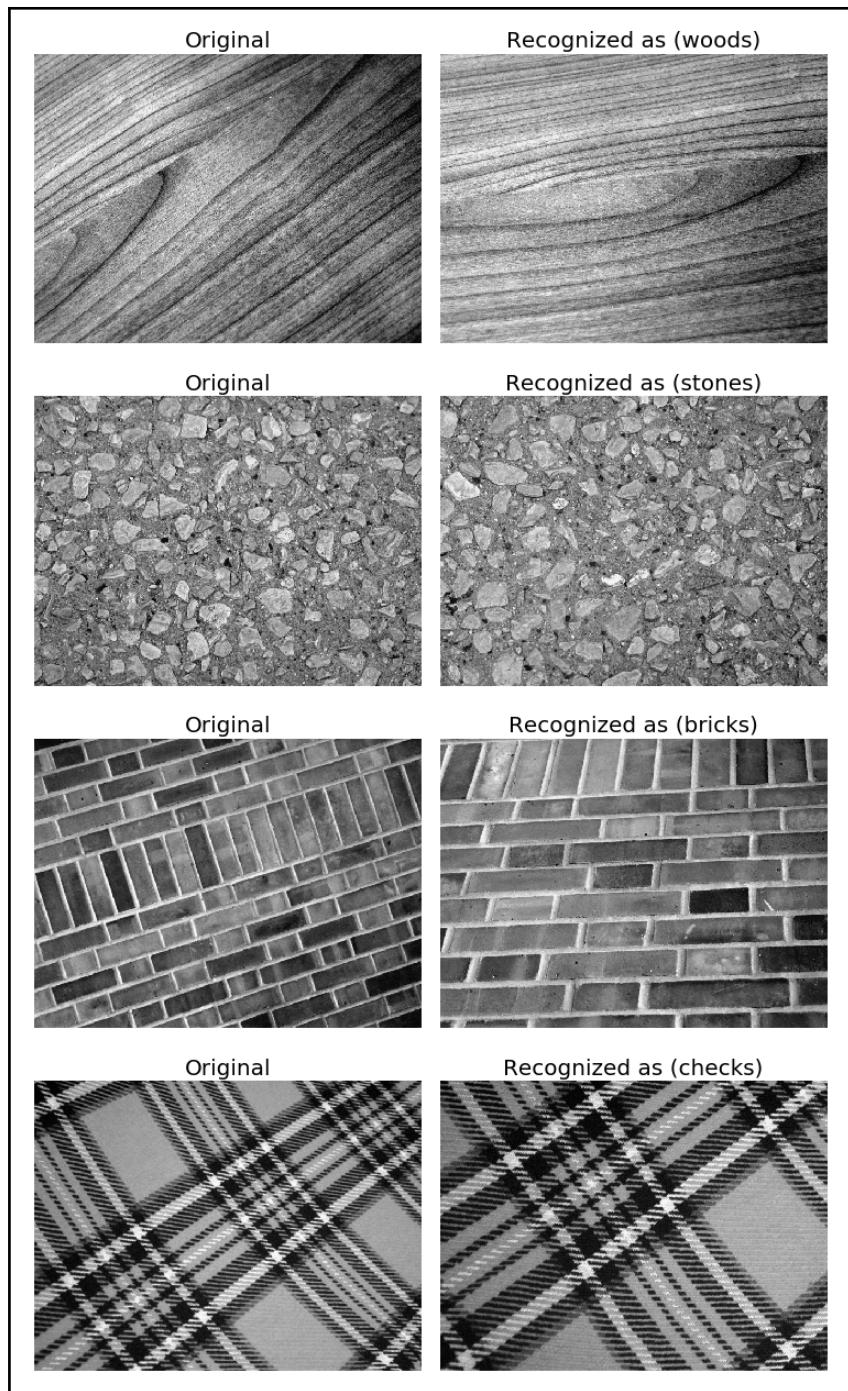
ψ phase offset

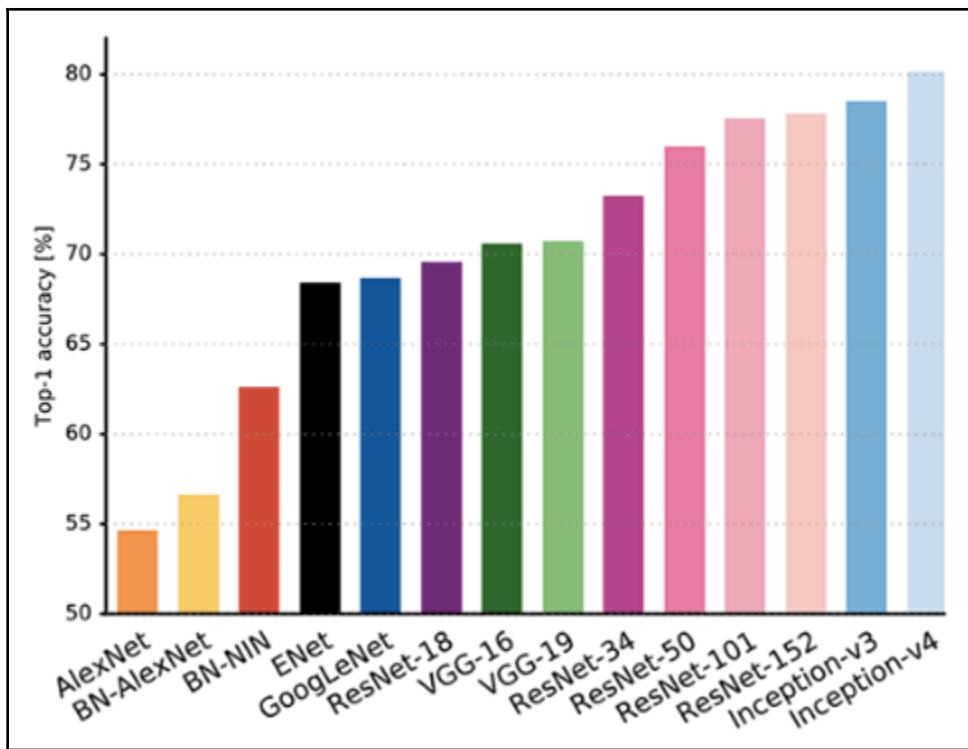
σ standard deviation of the Gaussian envelope

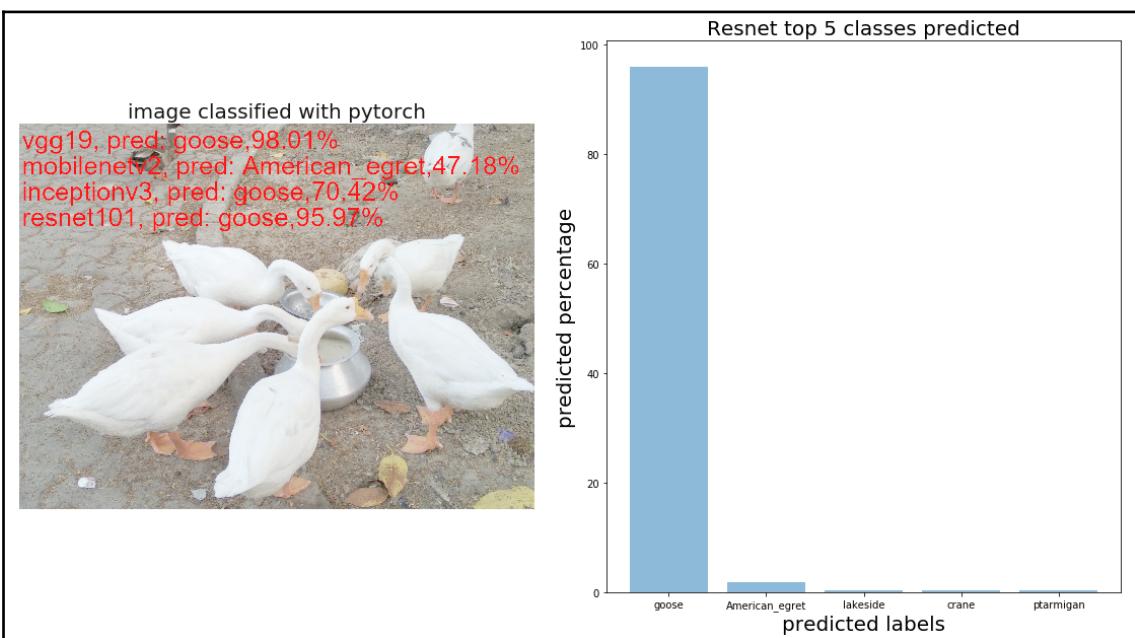
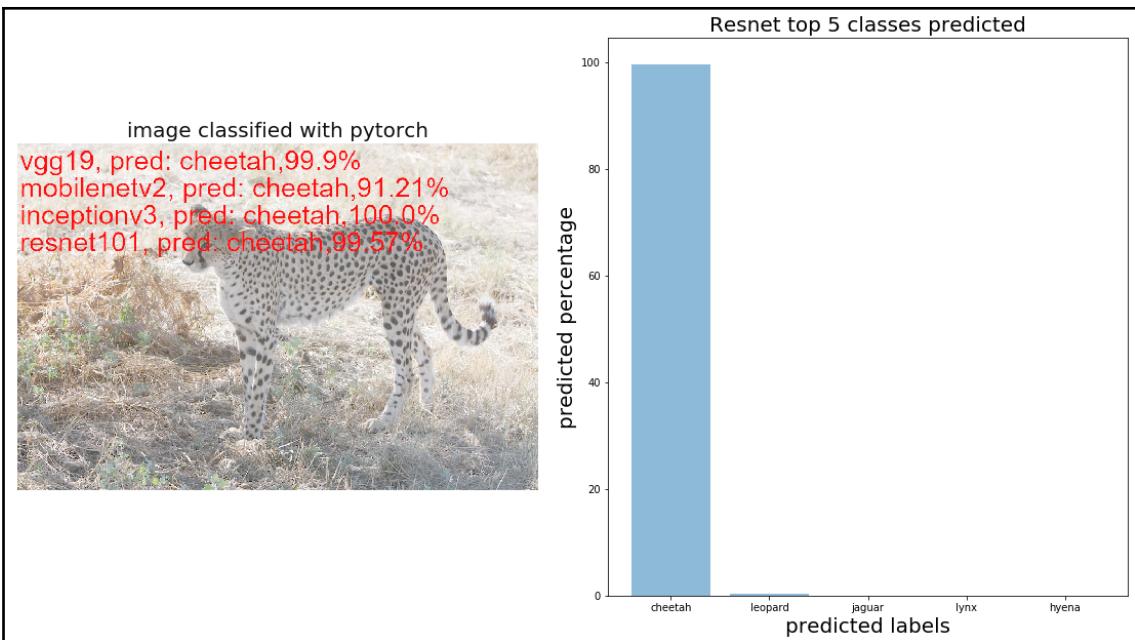
γ spatial aspect ratio

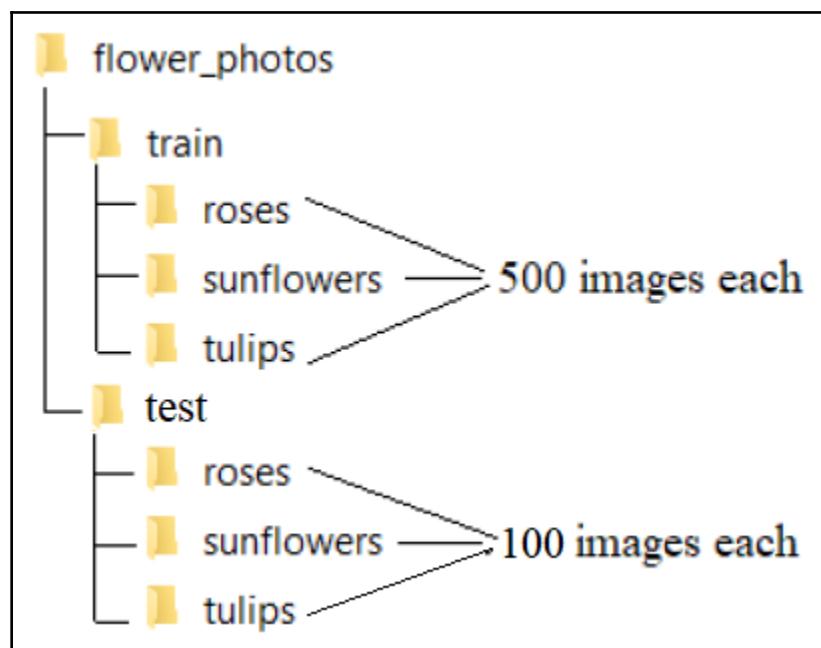
Image responses for Gabor filter kernels

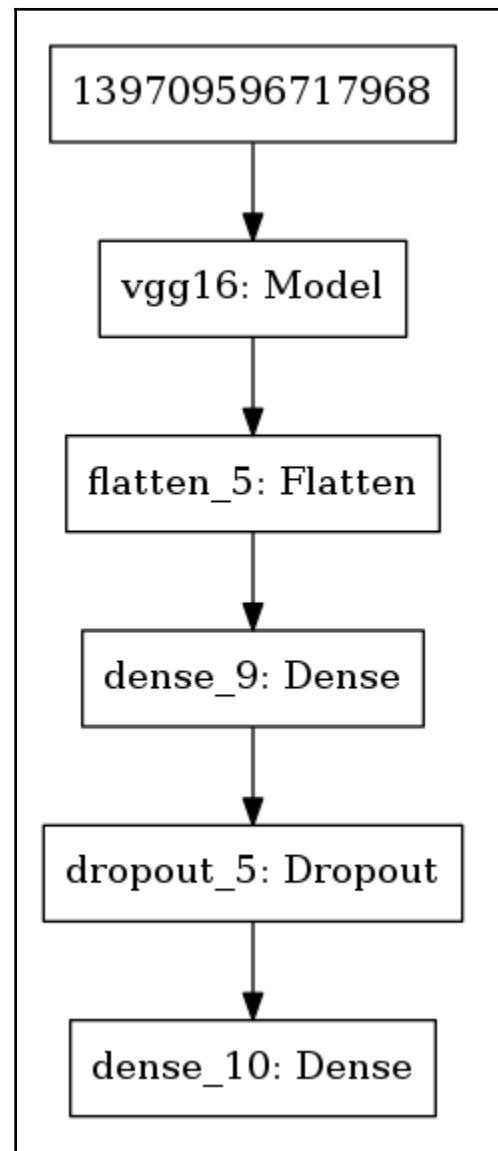


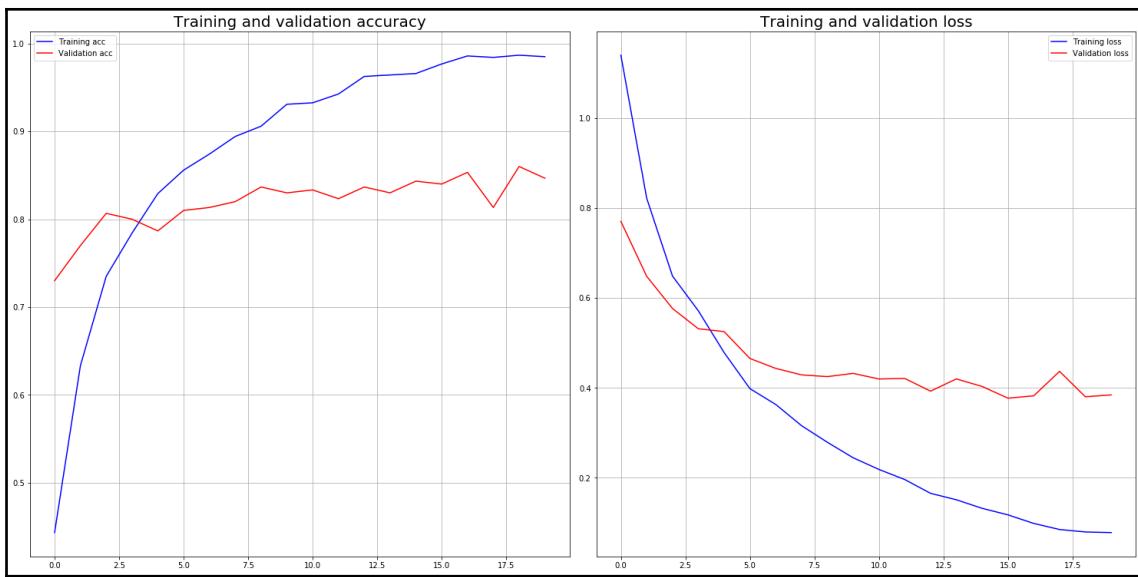


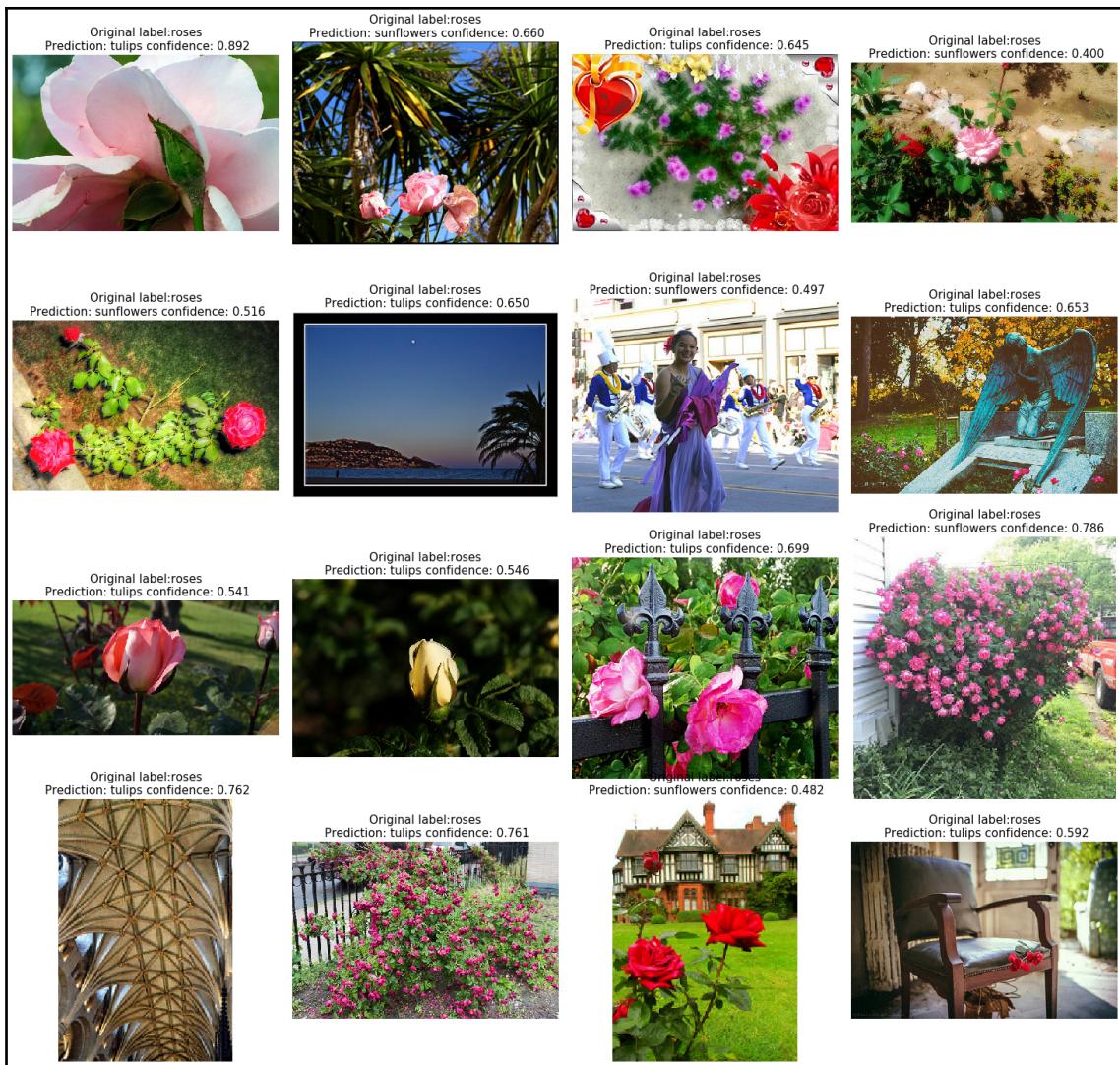




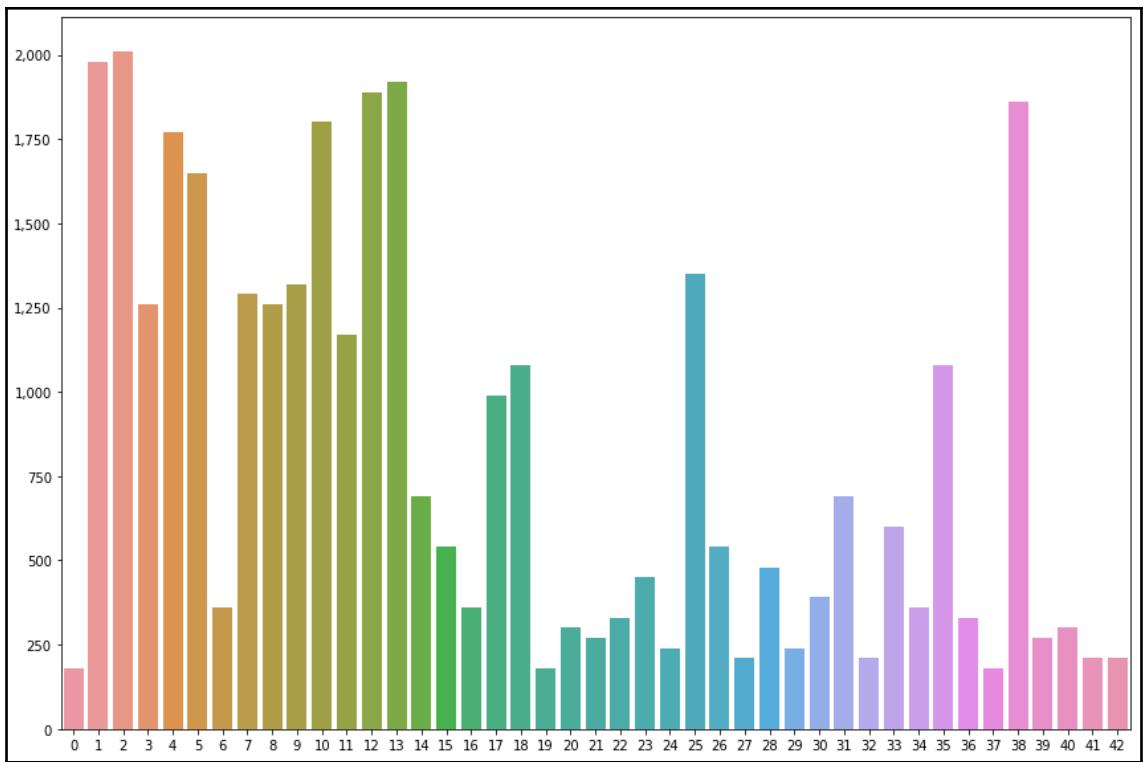




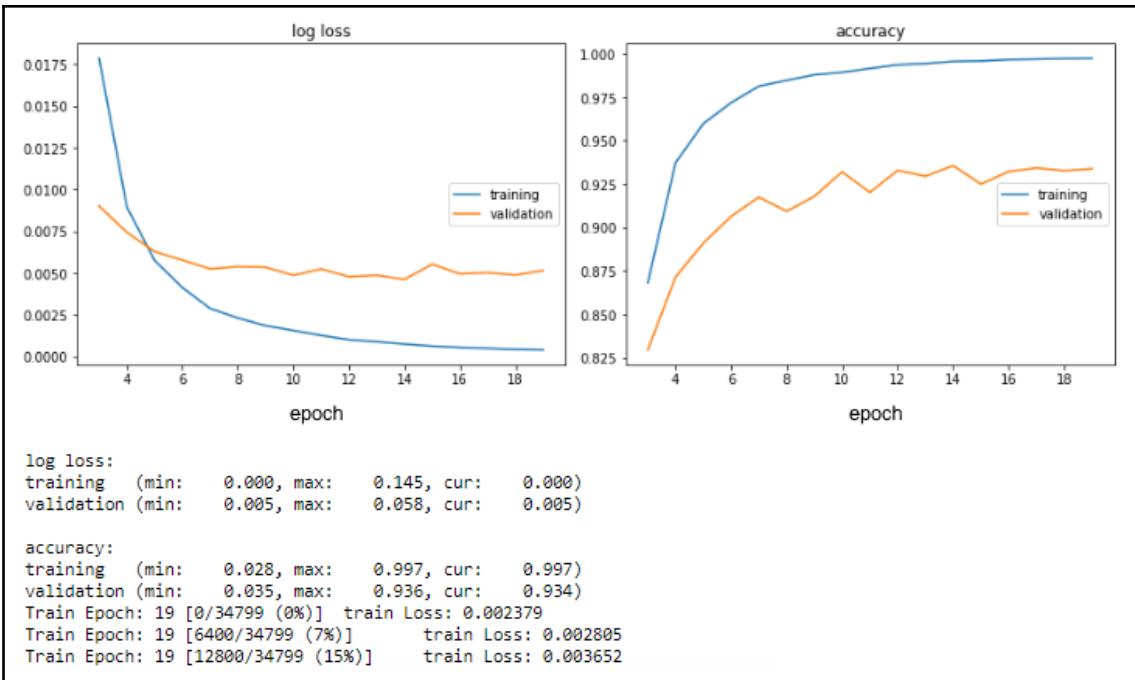




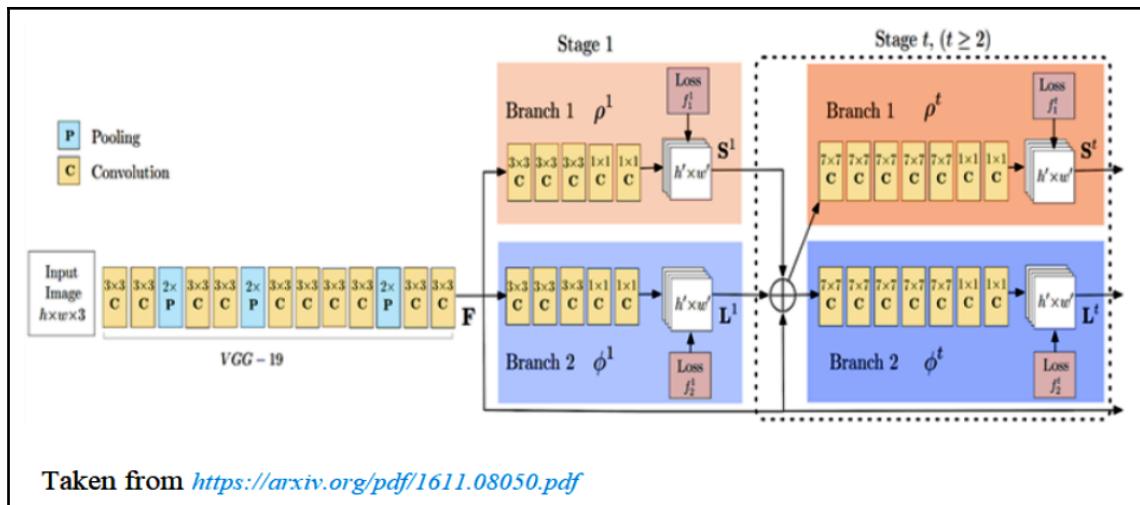
ClassId	SignName
0	0 Speed limit (20km/h)
1	1 Speed limit (30km/h)
2	2 Speed limit (50km/h)
3	3 Speed limit (60km/h)
4	4 Speed limit (70km/h)





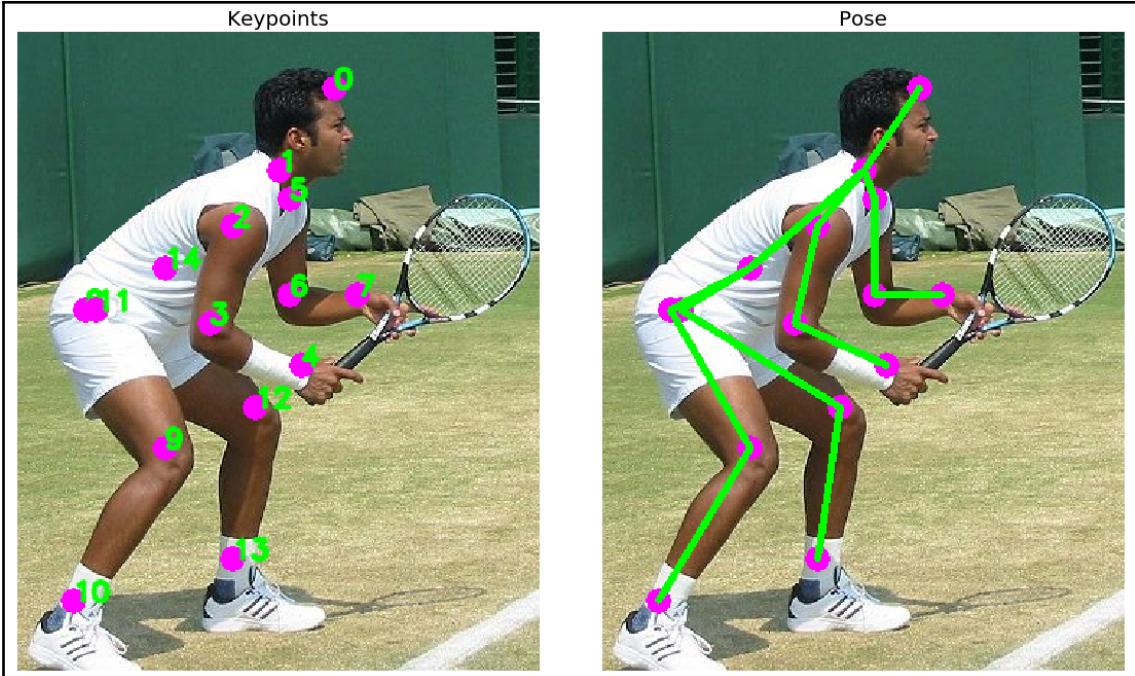




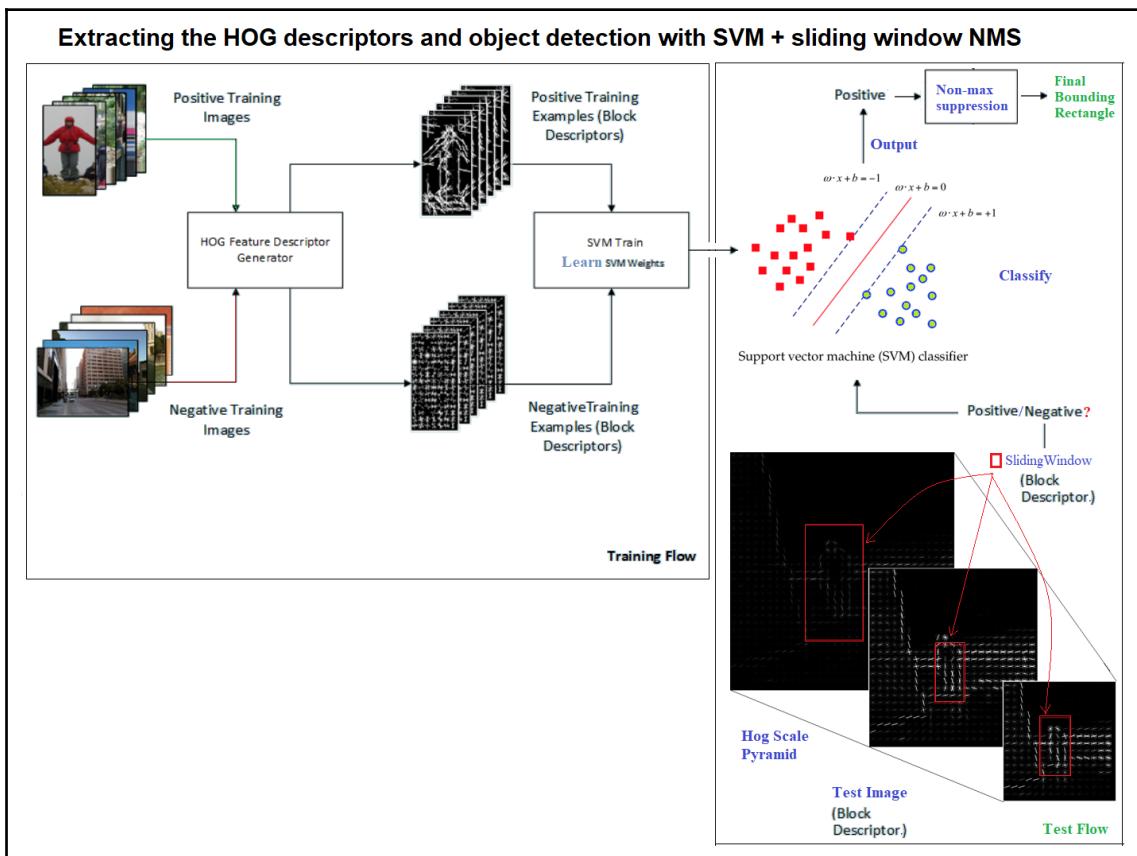
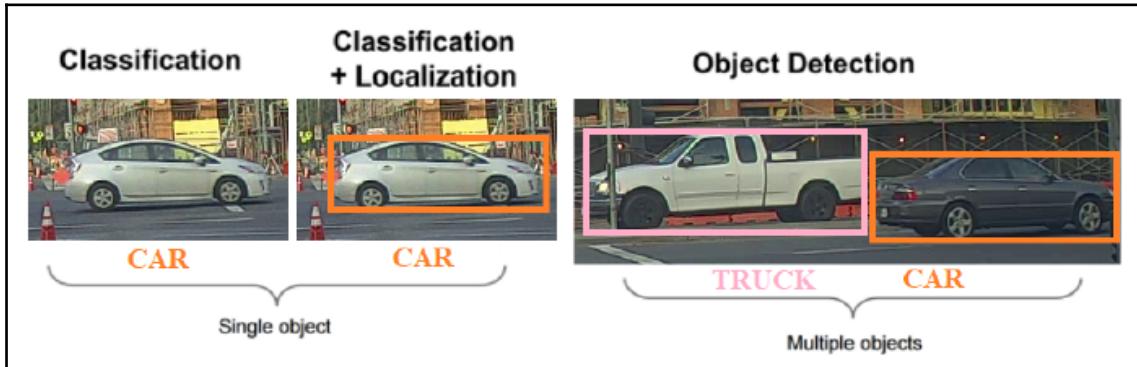


Taken from <https://arxiv.org/pdf/1611.08050.pdf>





Chapter 8: Object Detection in Images



Bounding boxes found by HOG-SVM with mean shift grouping



$$\text{IoU} = \frac{\text{intersection}}{\text{union}} = \frac{B_1 \cap B_2}{B_1 \cup B_2}$$

The diagram illustrates the calculation of Intersection over Union (IoU). It shows two overlapping rectangles, B₁ and B₂. The overlapping area is labeled 'intersection'. The total area covered by either B₁ or B₂ or both is labeled 'union'. The formula for IoU is the intersection divided by the union.

Object Detection: Impact of Deep Learning

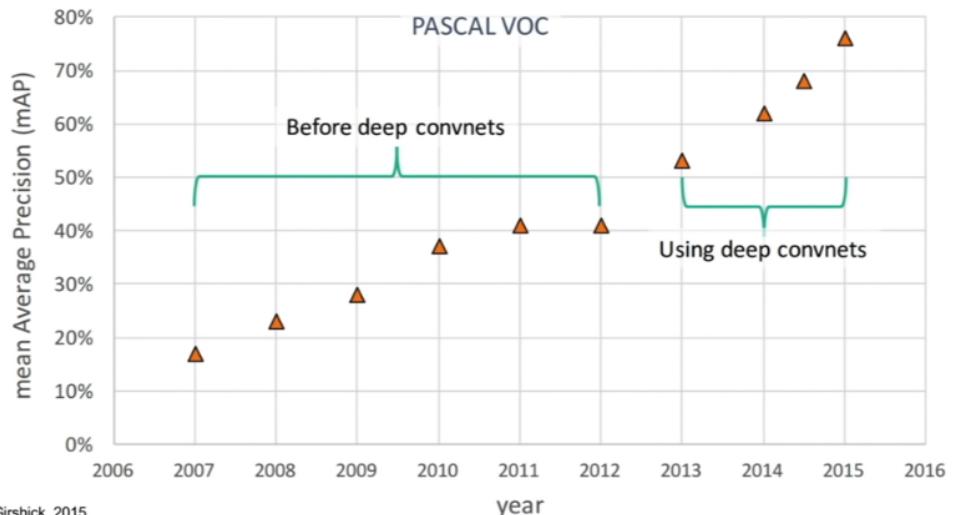
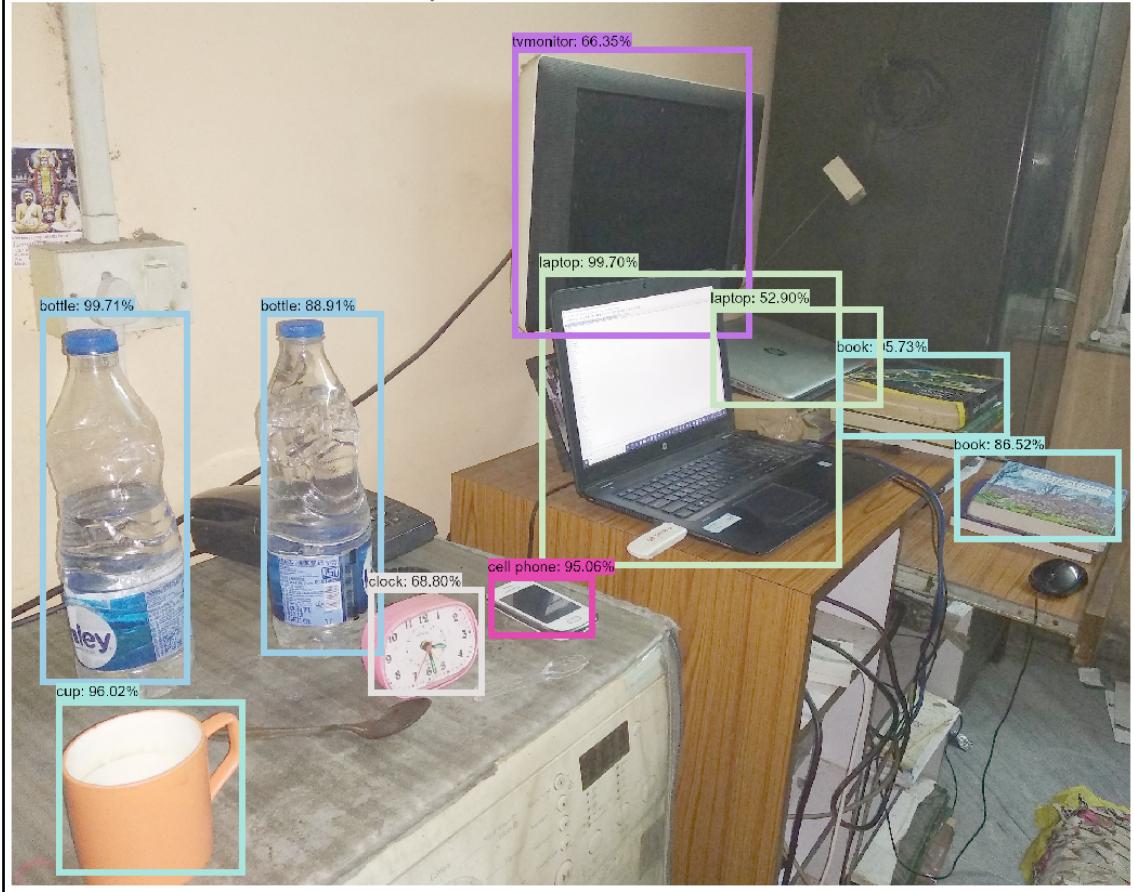
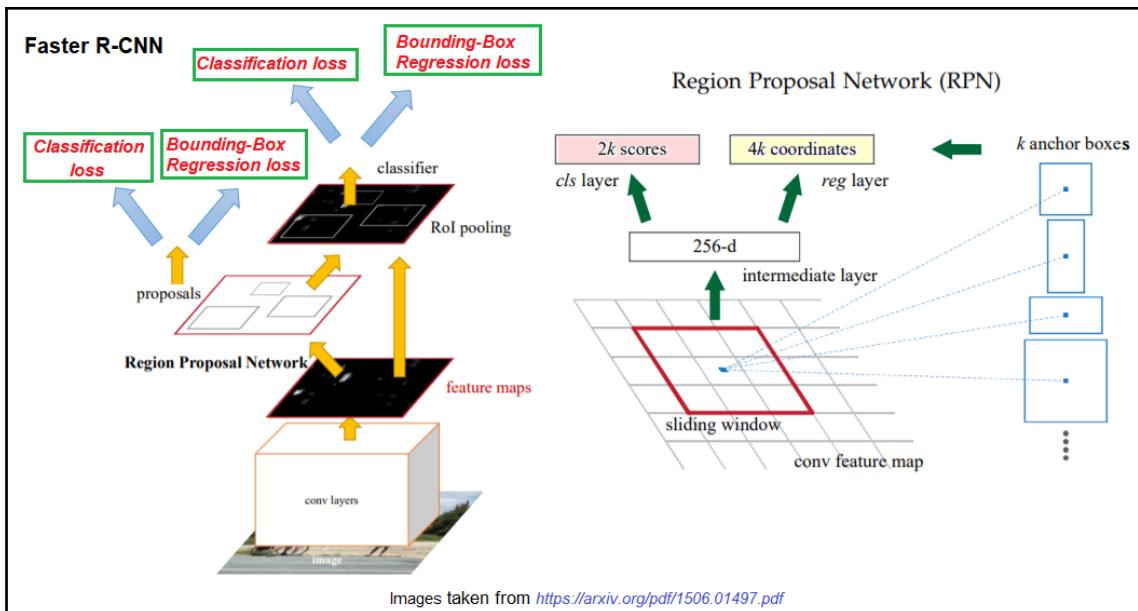


Figure copyright Ross Girshick, 2015.
Reproduced with permission.

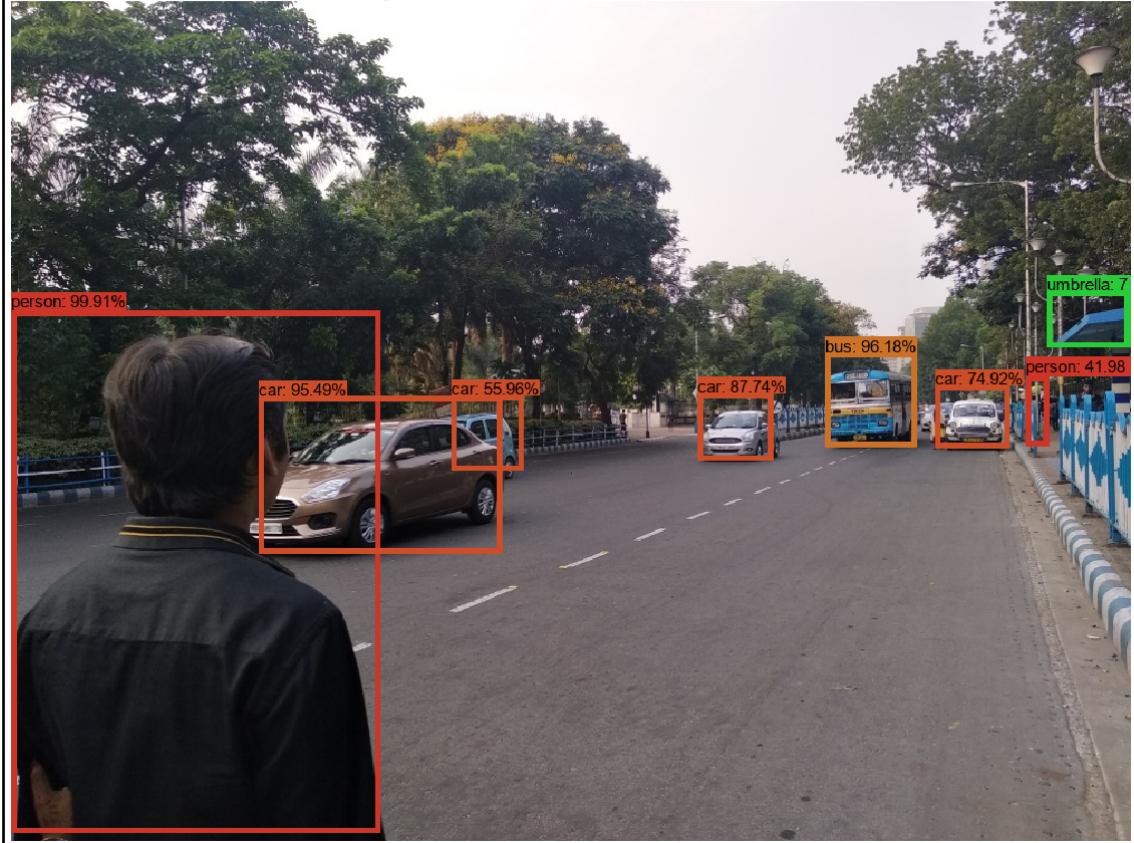
Figure taken from <https://www.youtube.com/watch?v=nDPWywWRlRo> slides

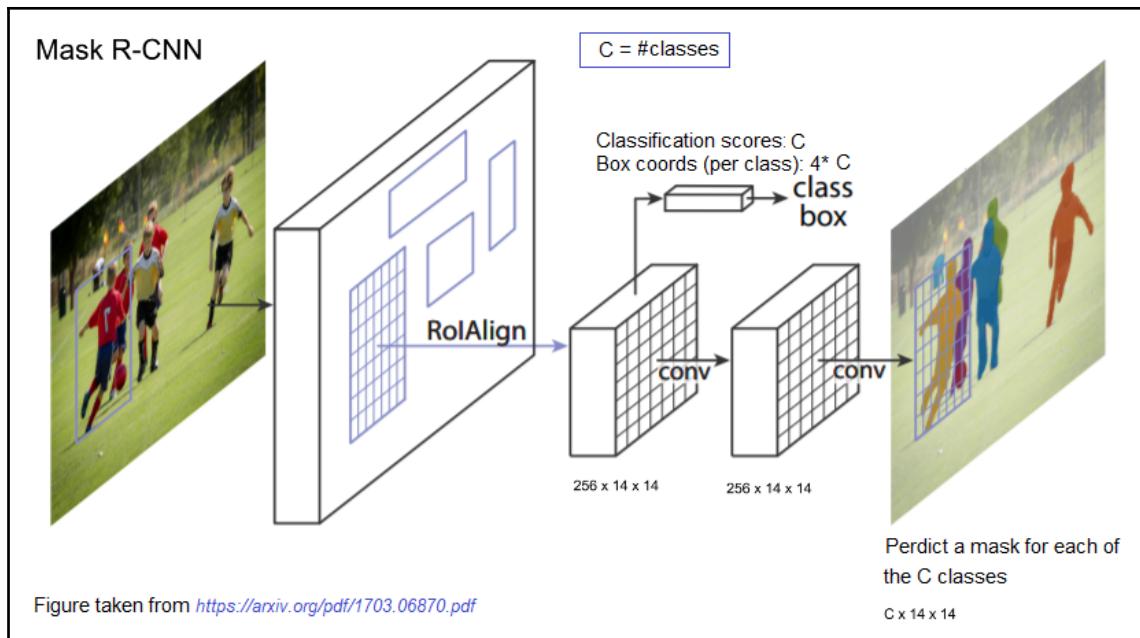
Objects detected with Yolo (v3)





Objects detected with Faster-RCNN





Original Image

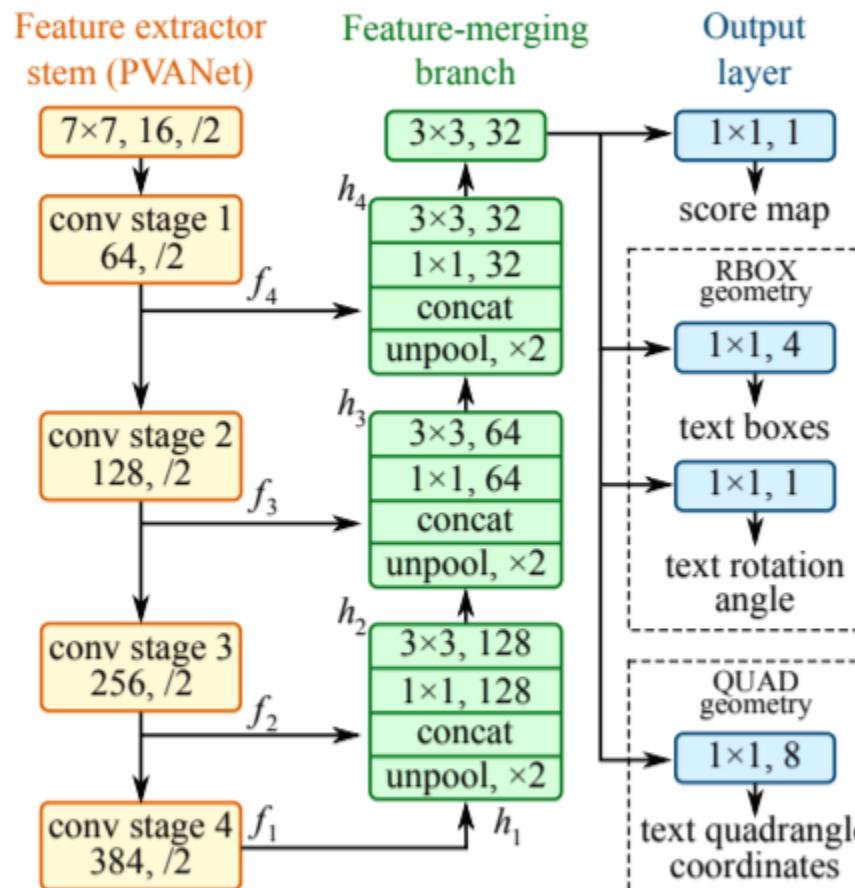


Objects detected with Mask-RCNN



Tracking cars in a video with CSRT MultiTracker





Structure of **EAST** text detection FCN

Figure Taken from <https://arxiv.org/pdf/1704.03155.pdf>

Hands-On

Hands-On

Image

Processing

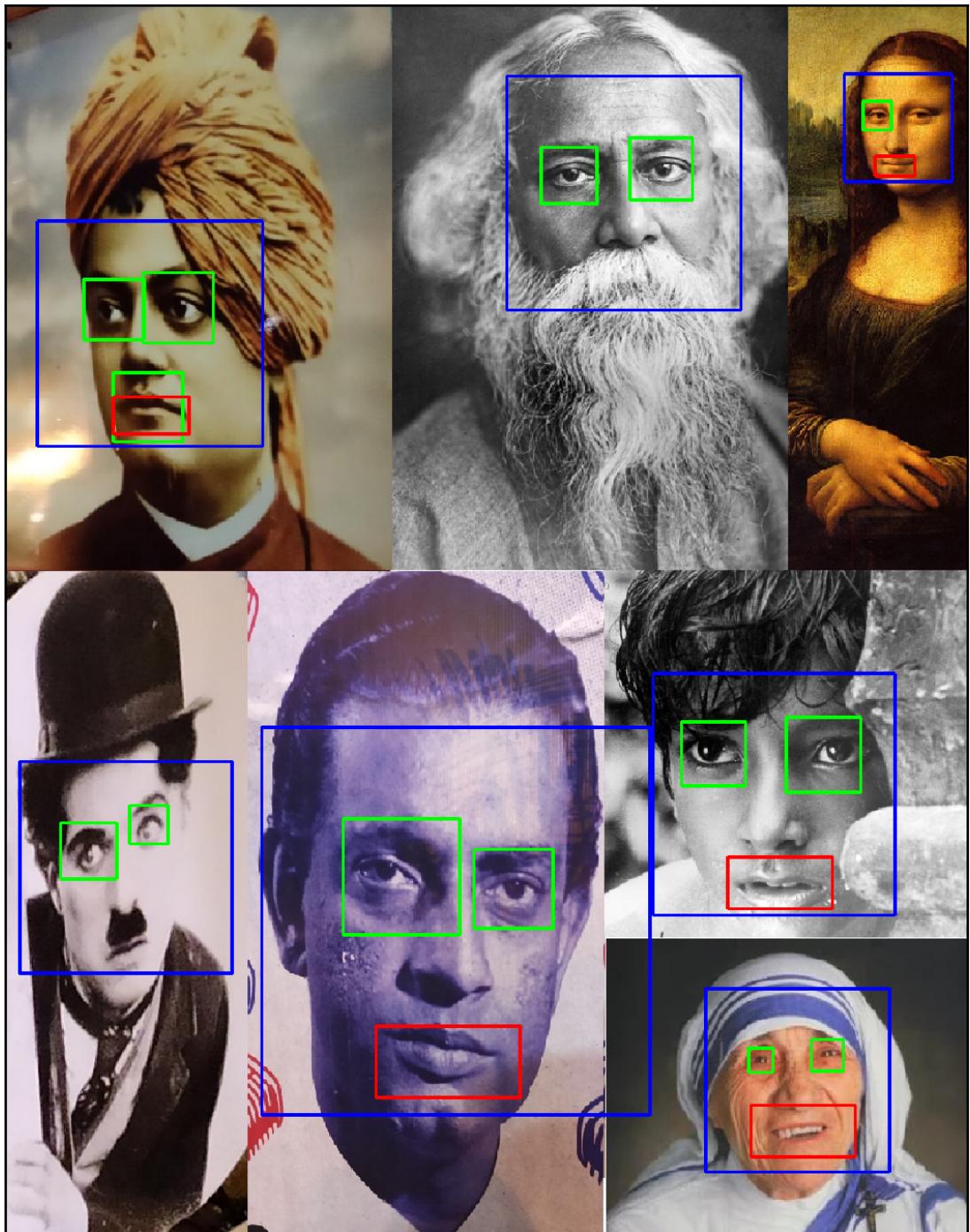
Image Processing

with

Python

with Python





Chapter 9: Face Recognition, Image Captioning, and More

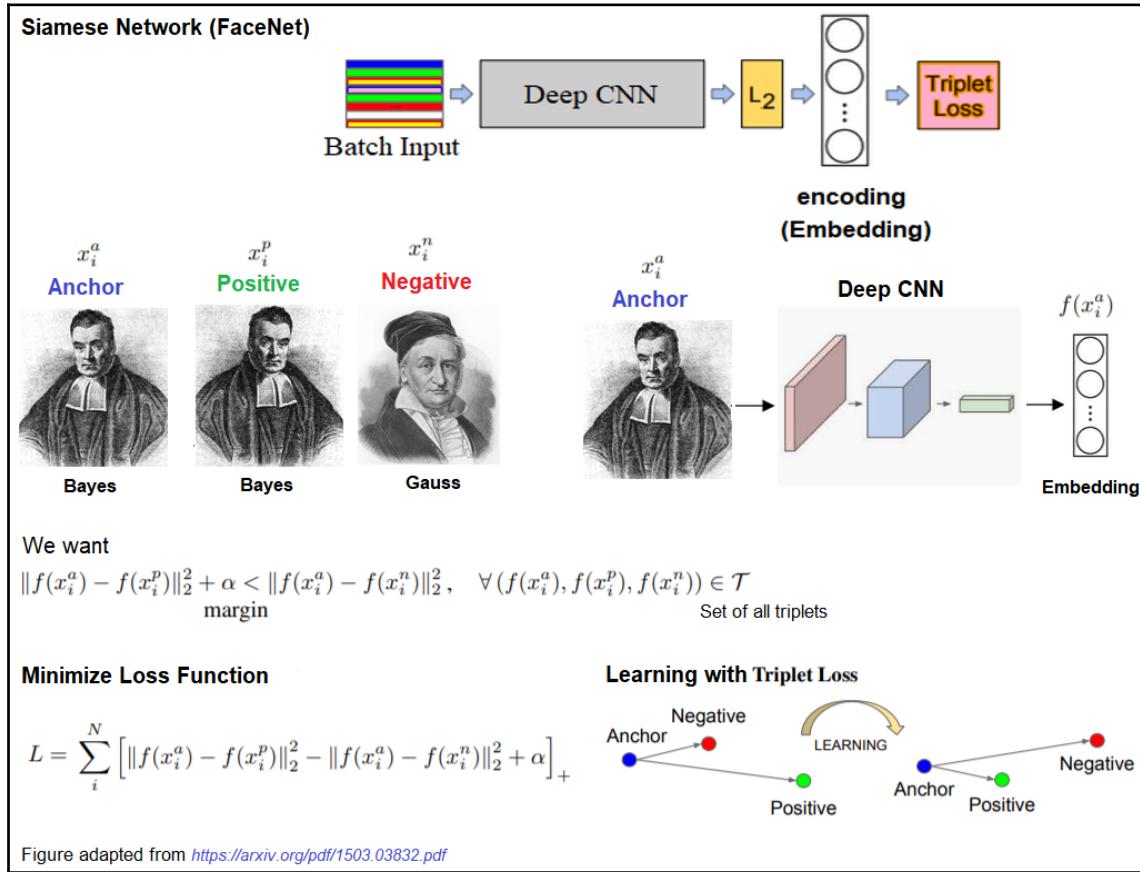
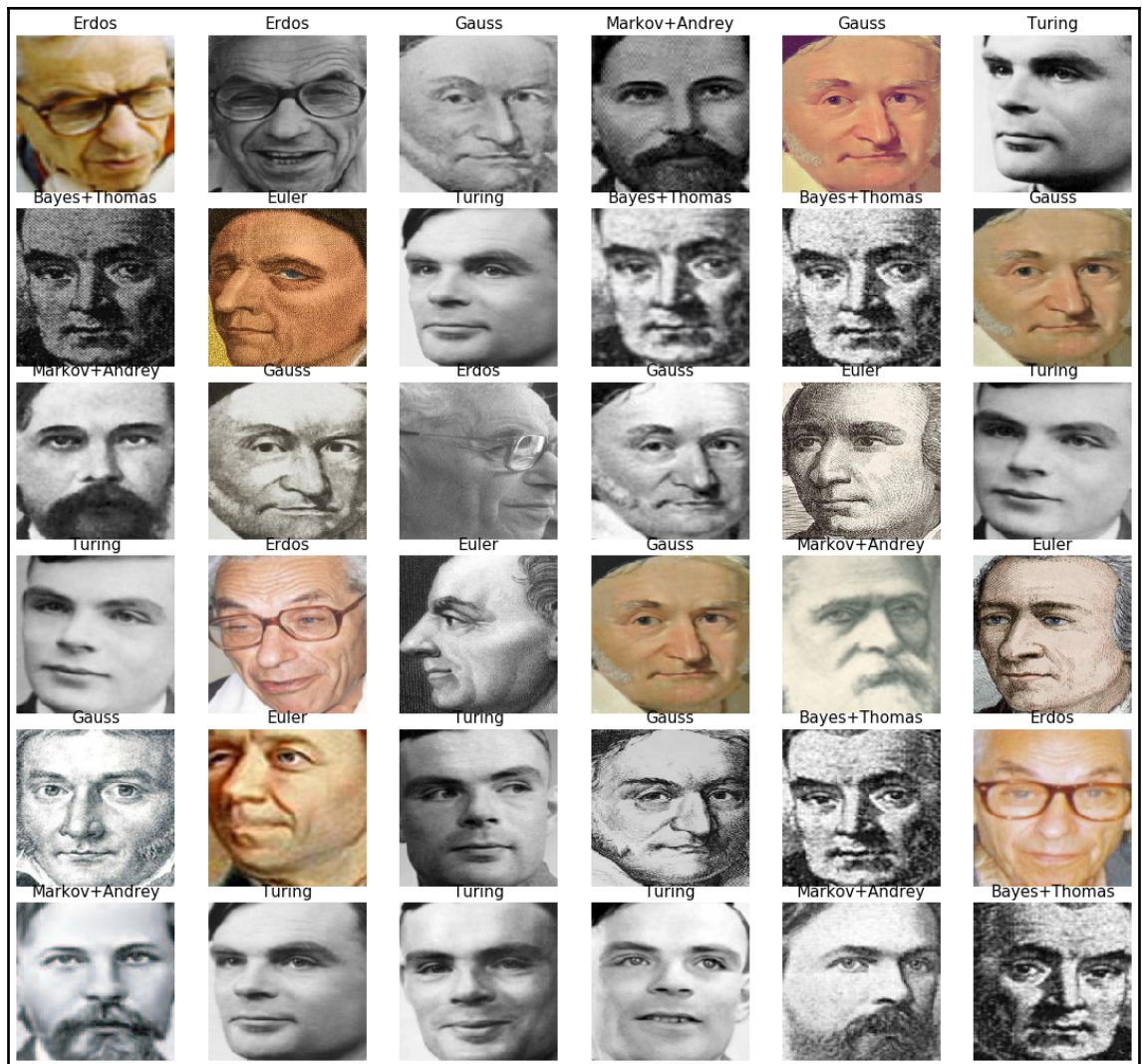


Figure adapted from <https://arxiv.org/pdf/1503.03832.pdf>

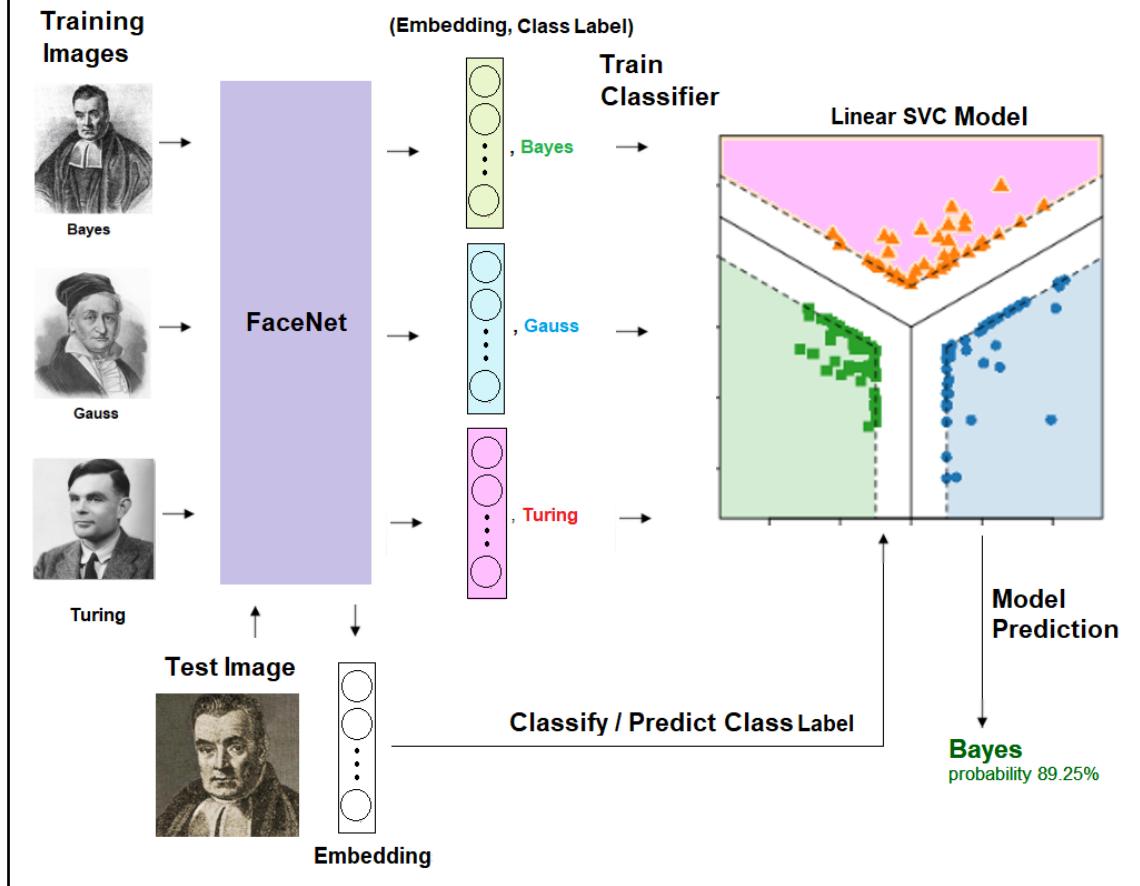




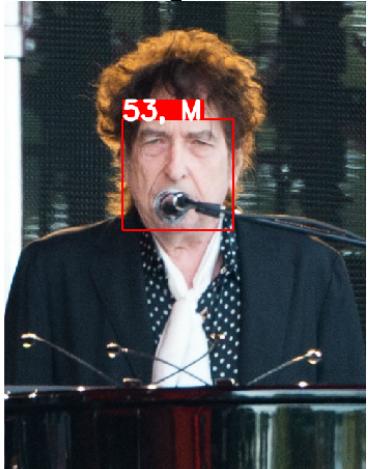
Bayes+Thomas (80.349)



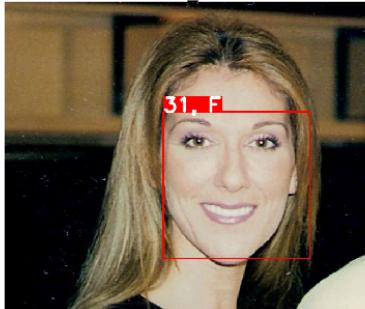
Face Recognition as Classification



bob_dylan



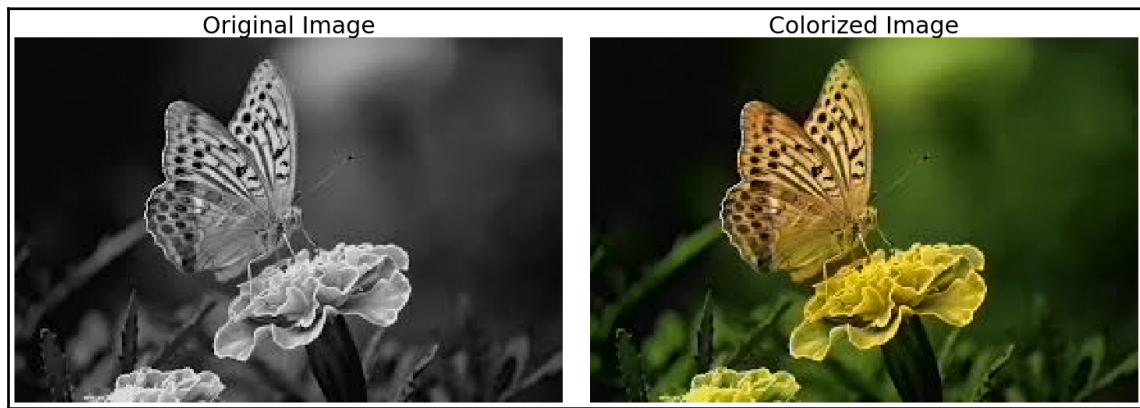
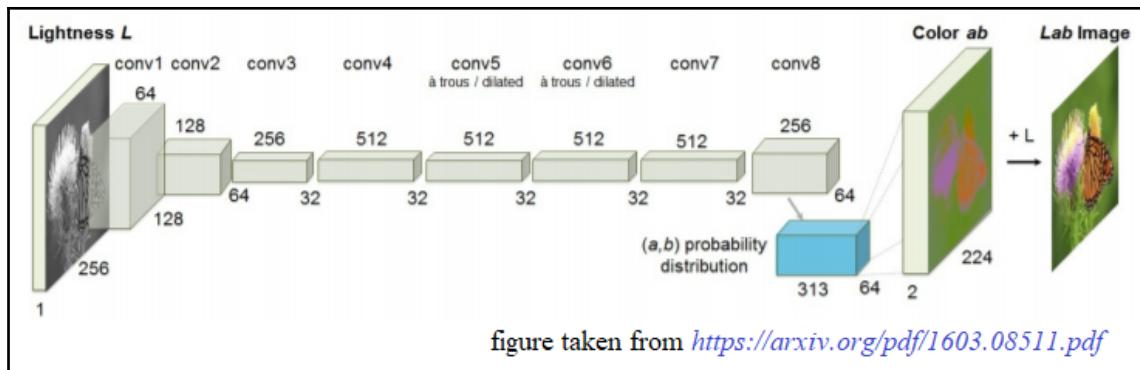
celine dion

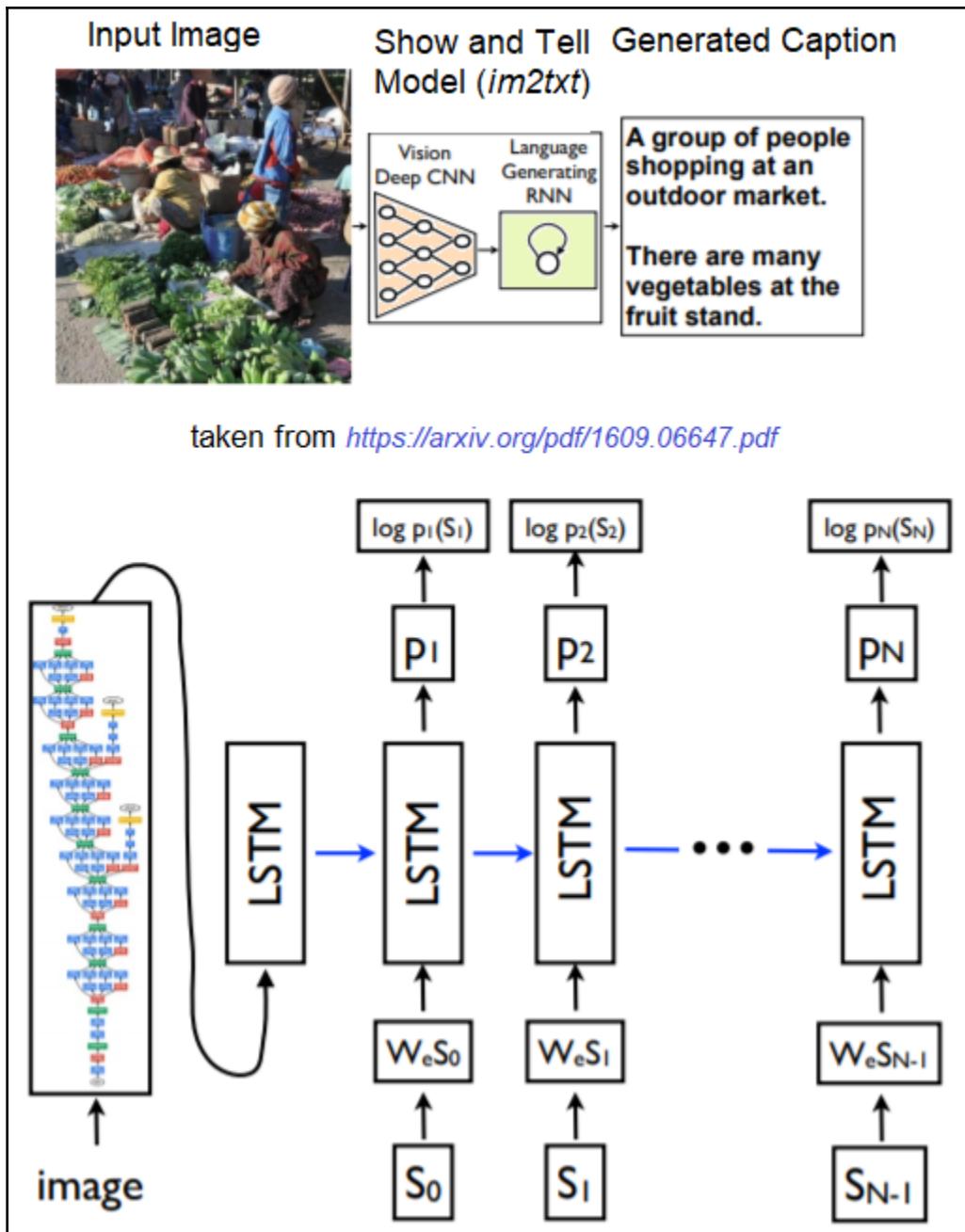


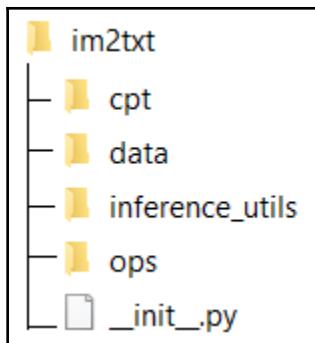
john_lennon











Automatic image captioning with deep learning pretrained model

a man jumping up into the air to catch a frisbee .



a building with a clock on the front of it



a bird perched on top of a bird feeder .



a flock of birds flying over a body of water .



a group of people standing next to a boat .



a little girl holding an umbrella in the water .



a cat sitting on the ground



a group of people standing around a table filled with food .



a group of elephants standing next to each other .



a person on a horse jumping over a hurdle .



a group of people standing on top of a beach .



a group of people walking down a street .



a man standing in a field with a dog .

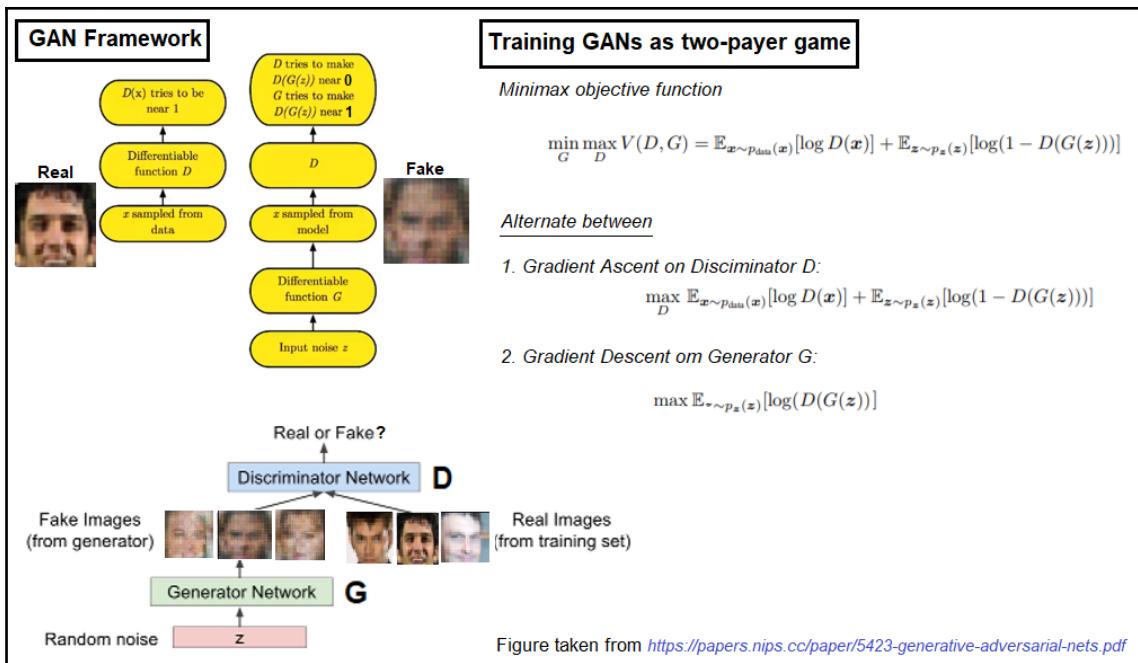


a train station with a train on the tracks .



a tall clock tower with a sky background



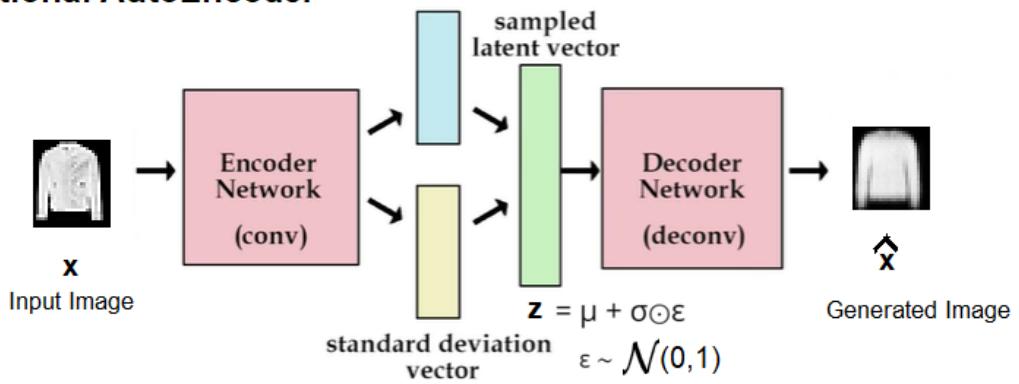




Images generated by GAN (epoch = 10)



Variational AutoEncoder

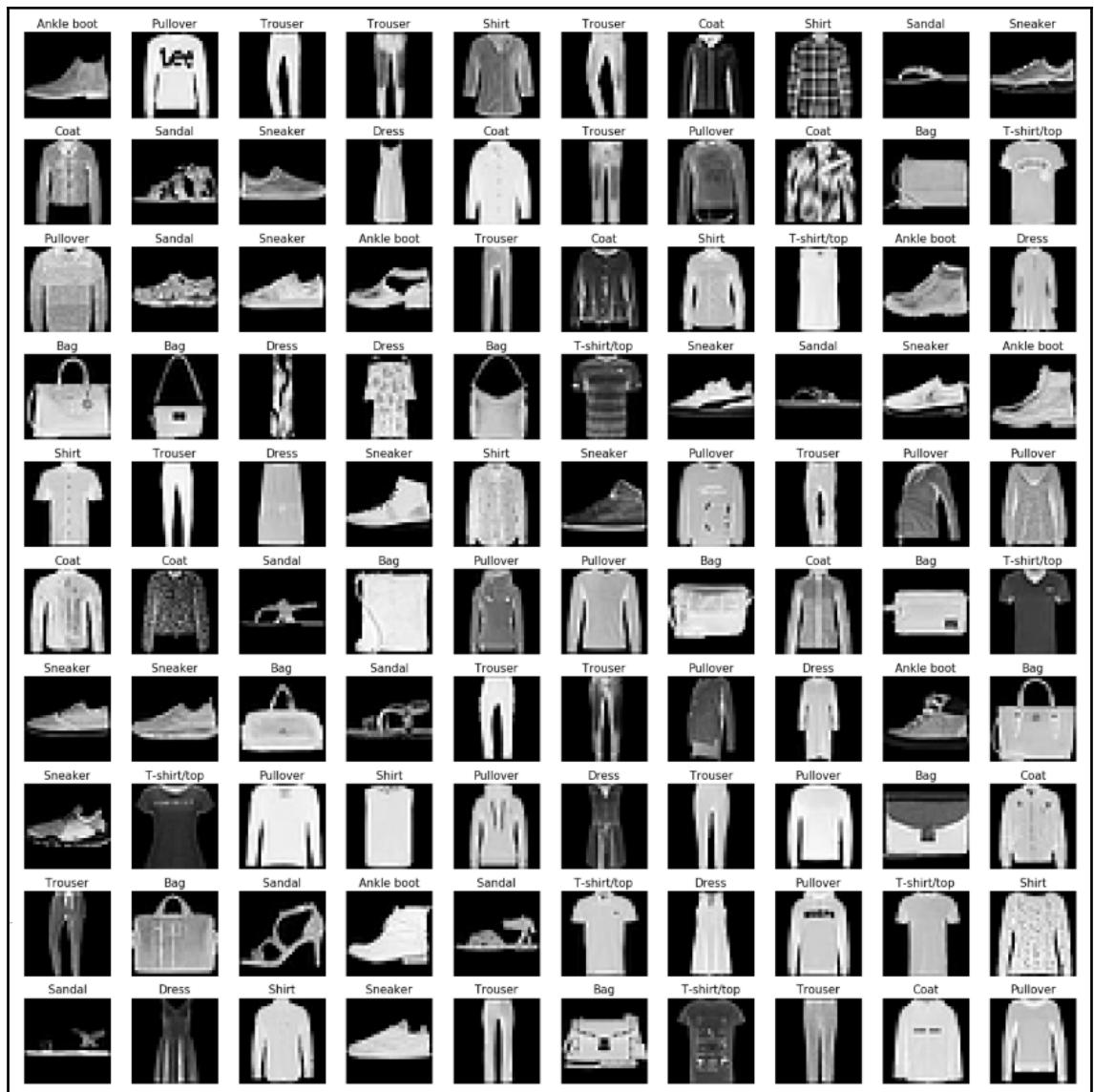


VAE Objective function

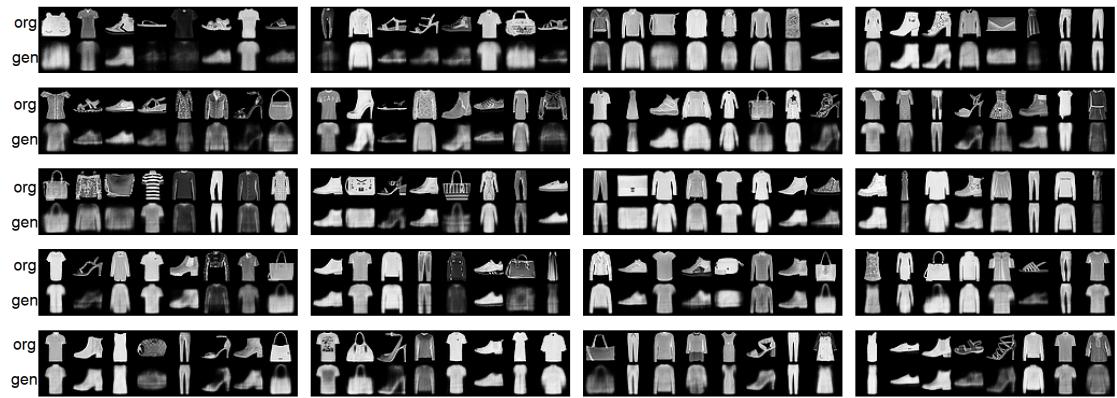
$$\max_{\theta, \phi} \mathcal{L}_{\theta, \phi}(\mathbf{x}) = \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - D_{KL}(q_\phi(\mathbf{z}|\mathbf{x}) \parallel p(\mathbf{z}))$$

variational lower bound likelihood for Reconstruction KL divergence:
 distance to $\mathcal{N}(0,1)$

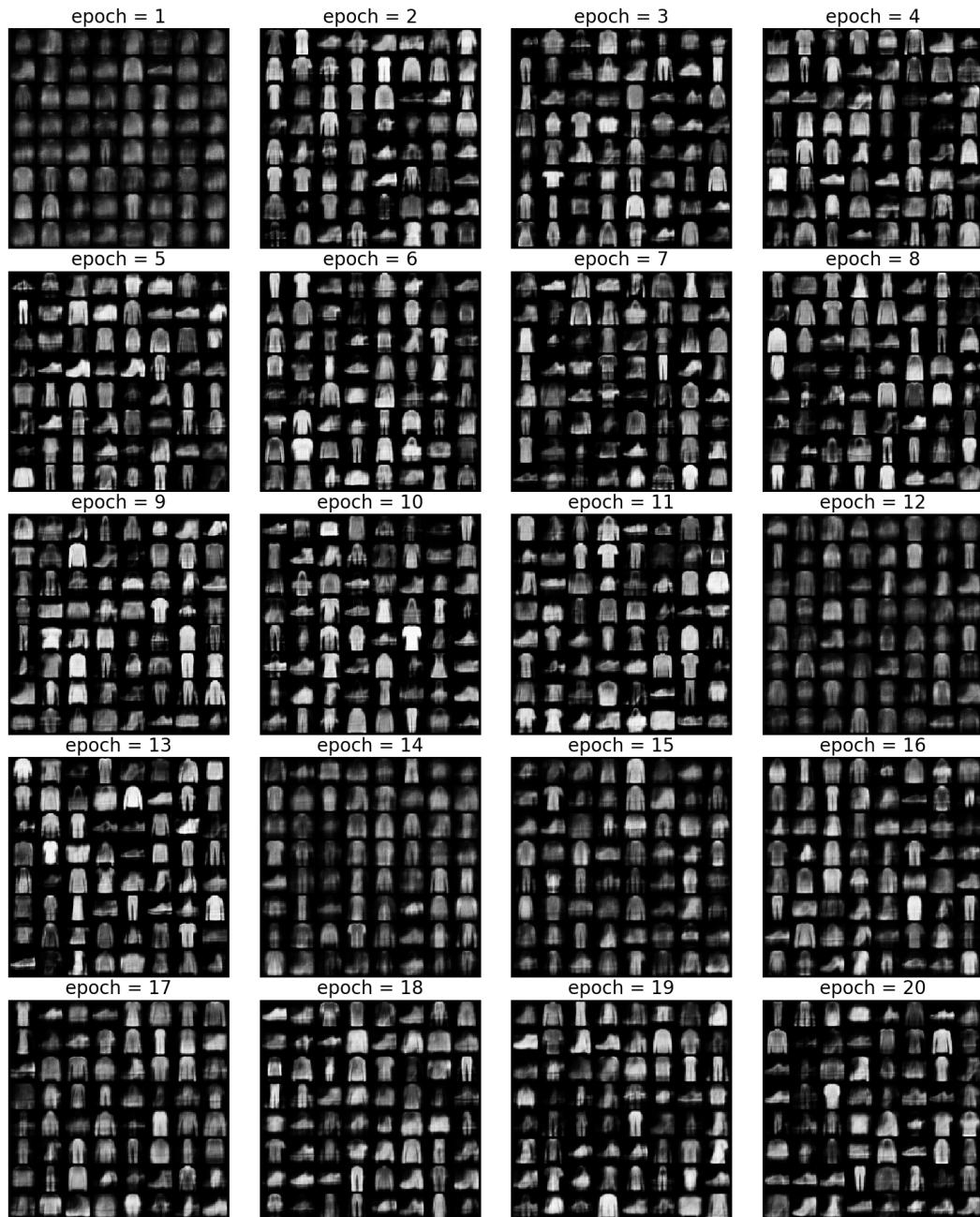
Figure adapted from <https://www.youtube.com/watch?v=9zKuYvjFFS8>

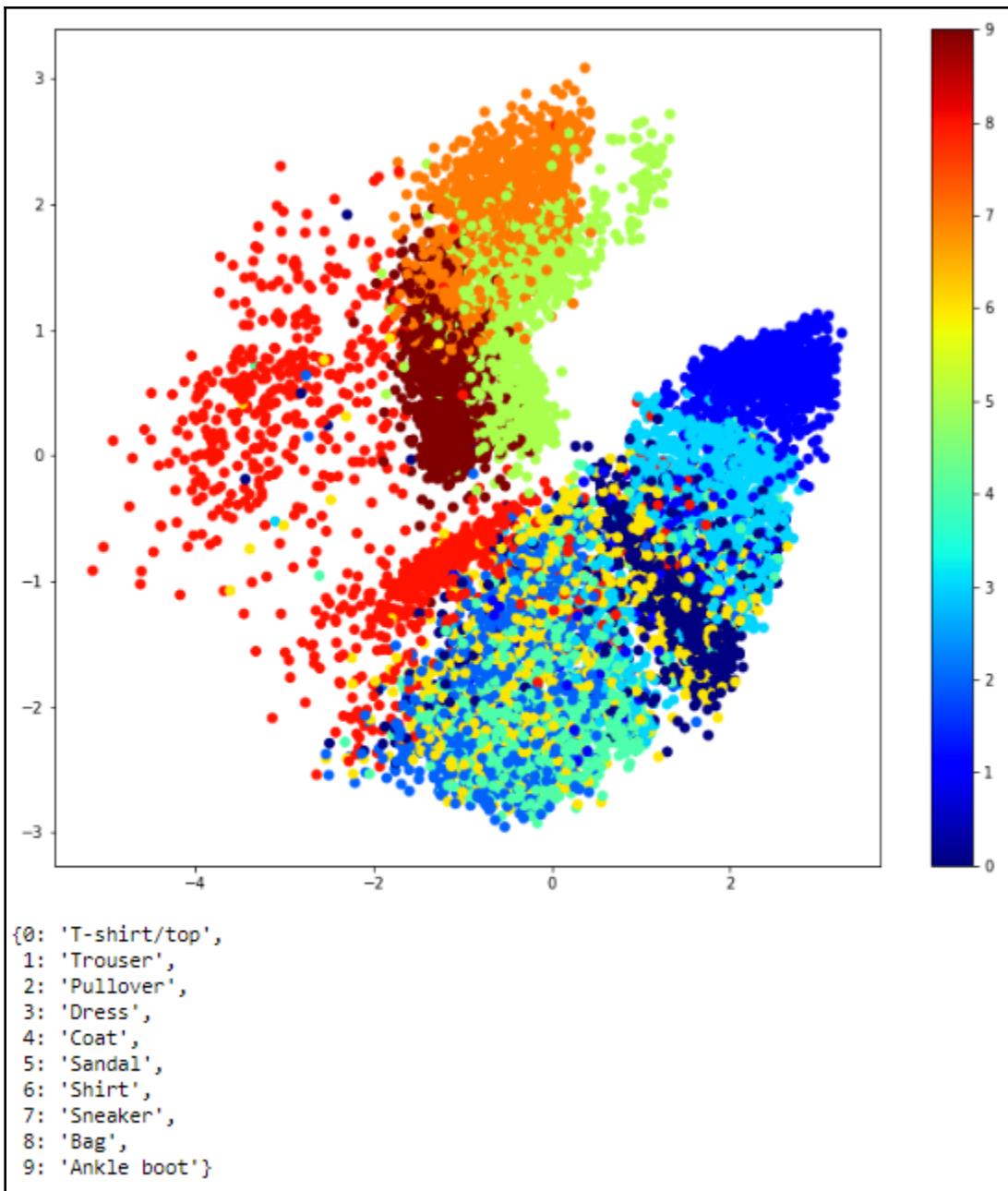


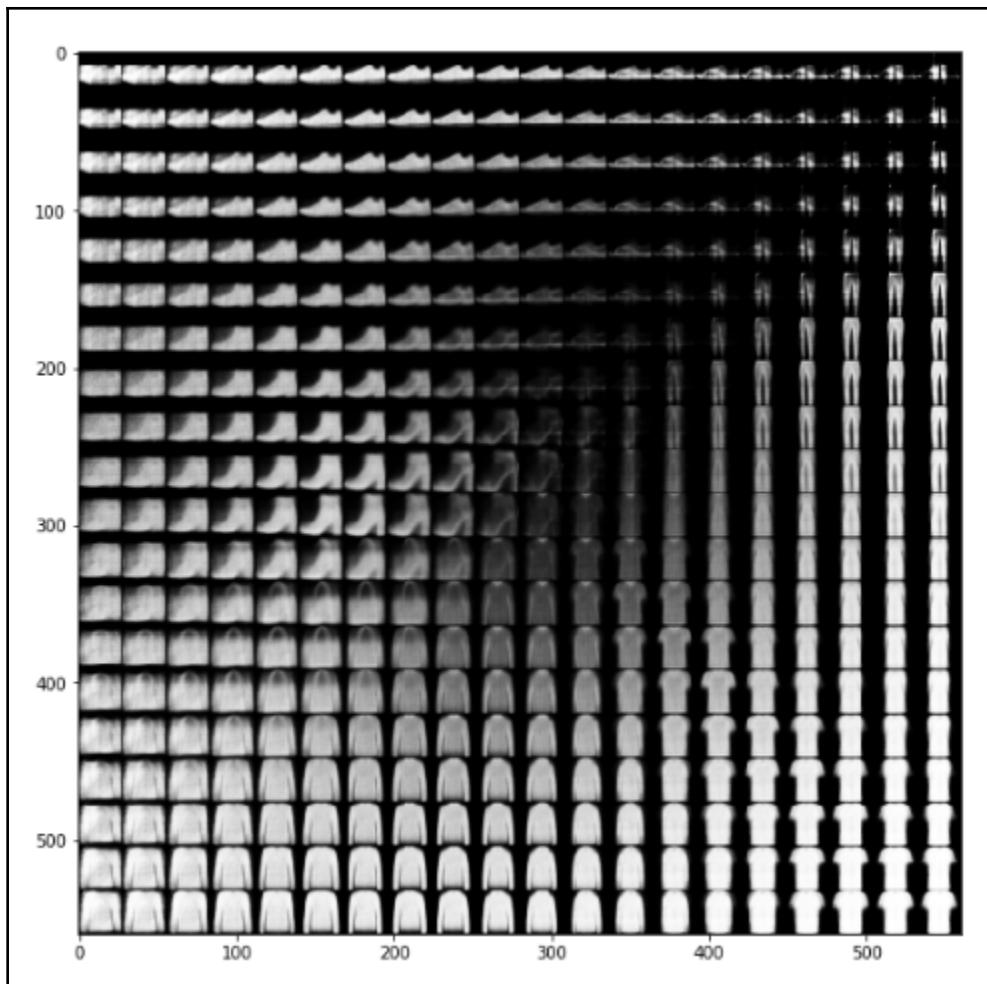
Test images reconstructed with VAE



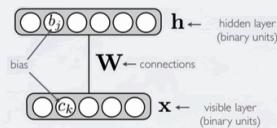
Images generated by VAE after different training epochs







Restricted Boltzmann Machine (RBM)



$$\text{Energy function: } E(\mathbf{x}, \mathbf{h}) = -\mathbf{h}^T \mathbf{W} \mathbf{x} - \mathbf{c}^T \mathbf{x} - \mathbf{b}^T \mathbf{h}$$

$$= -\sum_j \sum_k W_{j,k} h_j x_k - \sum_k c_k x_k - \sum_j b_j h_j$$

$$\text{Joint Distribution: } p(\mathbf{x}, \mathbf{h}) = \exp(-E(\mathbf{x}, \mathbf{h}))/Z$$

partition function (intractable)

Conditional distributions

The diagram shows the conditional distribution of a hidden unit j given the input \mathbf{x} . It consists of two layers: a visible layer x at the bottom and a hidden layer h at the top. The hidden layer h has four units. The probability $p(h_j = 1 | \mathbf{x})$ is calculated as:

$$p(h_j = 1 | \mathbf{x}) = \frac{1}{1 + \exp(-(b_j + \mathbf{W}_{j,:} \cdot \mathbf{x}))}$$

$$= \text{sigm}(b_j + \mathbf{W}_{j,:} \cdot \mathbf{x})$$

A bracket under the term $\mathbf{W}_{j,:} \cdot \mathbf{x}$ is labeled "jth row of \mathbf{W} ".

Taken from https://www.youtube.com/watch?v=p4Vh_zMw-HQ

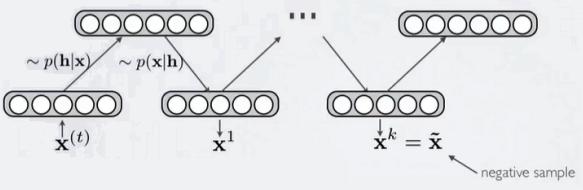
Training objective function

$$\frac{1}{T} \sum_t l(f(\mathbf{x}^{(t)})) = \frac{1}{T} \sum_t -\log p(\mathbf{x}^{(t)})$$

Stochastic gradient descent steps

$$\frac{\partial}{\partial \theta} (-\log p(\mathbf{x}^{(t)})) = E_{\mathbf{h}} \underbrace{\left[\frac{\partial E(\mathbf{x}^{(t)}, \mathbf{h})}{\partial \theta} \middle| \mathbf{x}^{(t)} \right]}_{\text{positive phase}} - E_{\mathbf{x}, \mathbf{h}} \underbrace{\left[\frac{\partial E(\mathbf{x}, \mathbf{h})}{\partial \theta} \right]}_{\text{negative phase}}$$

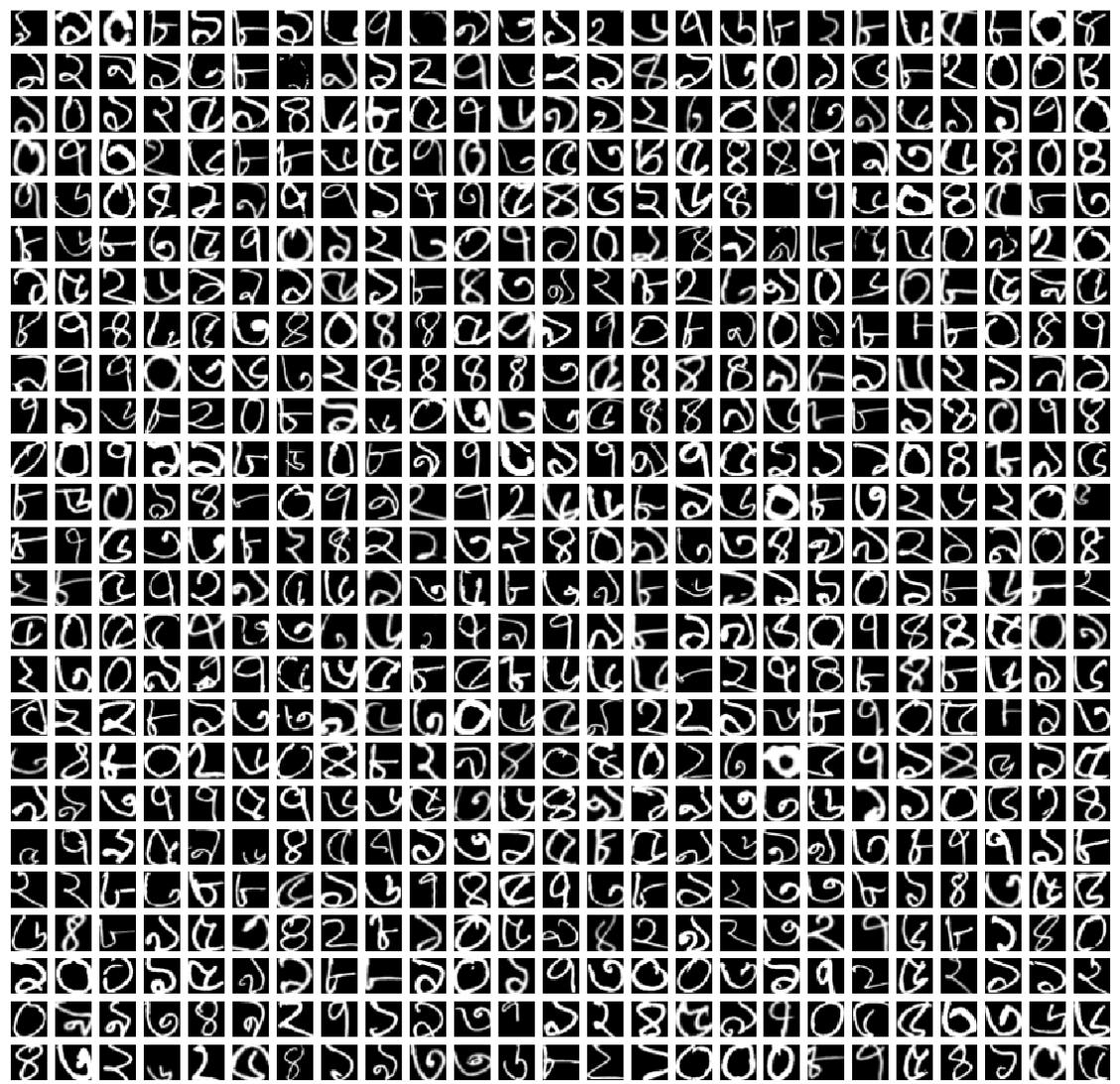
Contrastive Divergence (CD-k)



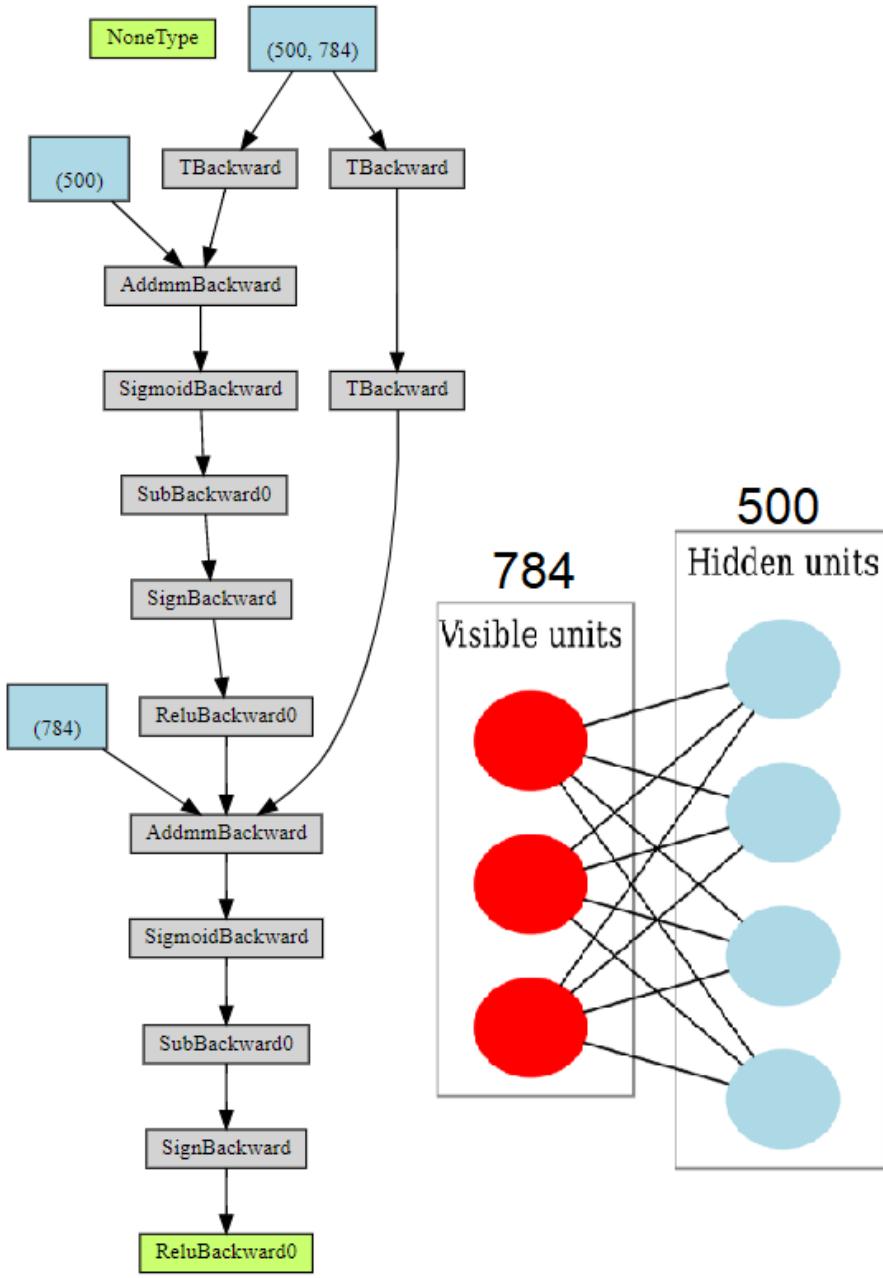
$$E_{\mathbf{h}} \left[\frac{\partial E(\mathbf{x}^{(t)}, \mathbf{h})}{\partial \theta} \middle| \mathbf{x}^{(t)} \right] \approx \frac{\partial E(\mathbf{x}^{(t)}, \tilde{\mathbf{h}}^{(t)})}{\partial \theta}$$

$$E_{\mathbf{x}, \mathbf{h}} \left[\frac{\partial E(\mathbf{x}, \mathbf{h})}{\partial \theta} \right] \approx \frac{\partial E(\tilde{\mathbf{x}}, \tilde{\mathbf{h}})}{\partial \theta}$$

Bengali handwritten digits (train)

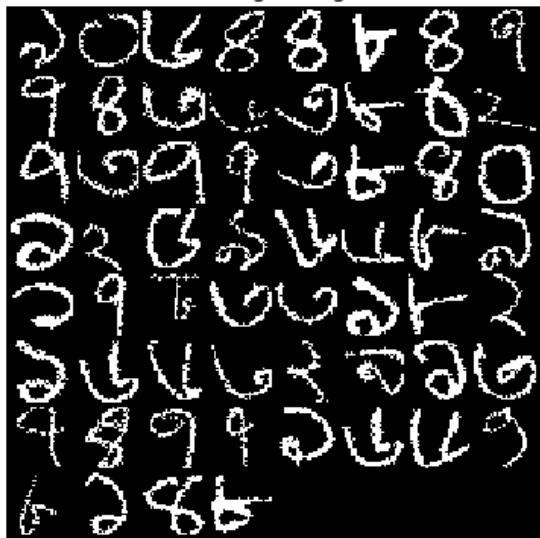


Restricted Boltzman Machine (RBM)

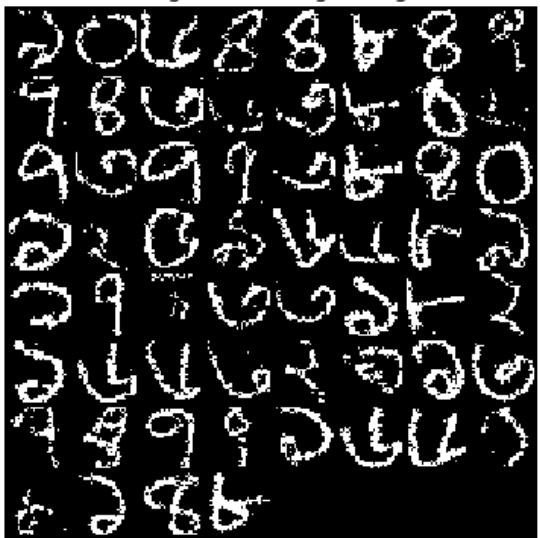


Generating images with RBM (epoch = 20)

Real digit images



RBM-generated digit images



Weights learnt at hidden layer of the RBM

