Math 520 Project 1 **due on April 12, 2017**

Write a one-dimensional Finite Element initial/boundary-value problem solver for the following type of problem:

$$\begin{aligned} u_t - \frac{d}{dx}\left(a(x)\frac{du}{dx}\right) + c(x)u &= f(x,t) \quad 0 < x < L, 0 < t < T \\ u(0,t) = p_0,\, a(x)\frac{du}{dx}\bigg|_{x=L} &= Q_L(t) \quad 0 < t < T \\ u(x,0) &= u_0(x) \quad 0 \le x \le L \end{aligned}$$

We assume that $a(x), c(x)$ and $f(x,t)$ are continuous functions. $a(x) \ge a_0 > 0, c(x) \ge 0$. Use uniform mesh with linear and quadratic elements.
Your function myFE1dibvp should have the following input and output fields:

[uh] = myFE1dibvp(a, c, f, p0, QL, u0, L, T, dt, noOfEle, shapeFn)

where
a, c, f have the type function_handle, and they represent functions $a(x), c(x), f(x,t)$ in the boundary value problem.
p0 is a number and it represents p_0 in the boundary condition.
QL is a function_handle that represents the boundary condition $Q_L(t)$ at $x = L$.
u0 is a function_handle that represents the initial condition function $u_0(x)$.
L is a number that indicates the right end point of the domain $(0, L)$.
T is a number that indicates the ending time.
dt is a number that represents the time step size.
noOfEle is a number that specifies the number of elements (subintervals) the user wants to discretize the domain into.
shapeFn is a number that represents the degree of shape functions to be used. The setting shapeFn$= 1$ indicates that linear shape functions are used, and the setting shapeFn $= 2$ indicates that quadratic shape functions are used.
The output field uh is a cell array that stores the approximated solutions of the exact solution u at every computed time step. Each element in the cell array is a function_handle that represent the approximated solution at a given time step.
Save your function in the file myFE1dibvp.m. Compress all MATLAB files used in this project into one file, and upload it to the Dropbox on Beachboard under Project 1.

In addition, submit a report on the case when
$a(x) = 2 + x$,
$c(x) = x$,
$f(x, t) = (2 + 2t - 4xt - 4x^2 t)\cos x + (1 - 8t - 6xt)\sin x$,
$p_0 = 1$,
$Q_L = 2t(\pi + 2)$,
$u_0(x) = \cos x$,
$L = \pi$,
$T = 1$.
`dt` should be selected appropriately.

The report must include:

1. Derivation of the weak form of the boundary value problem.
2. Description of the flow of the program you wrote with explanation of how you choose `dt`.
3. Verification the exact solution of the initial/boundary value problem is $(1 - 2xt)\cos x$.
4. Figures that show the exact solution and the computed solutions at $t = T$ obtained by discretizing the domain into 16, 32, 64, 128 subintervals and using both linear and quadratic shape functions. Organize your figures to compare the 4 computed solutions using linear shape functions and the exact solution, to compare the 4 computed solutions using quadratic shape functions and the exact solution, and to compare the best computed solutions using the linear and the quadratic shape functions and the exact solution. Explain your observation.
5. Figures that show the errors between the exact solution and the 8 computed solutions at $t = T$. Organize your figures to compare the errors at $t = T$ between the exact solution and the 4 computed solutions using linear shape functions, to compare the errors at $t = T$ between the exact solution and the 4 computed solutions using quadratic shape functions, and to compare the smallest errors obtained by using the linear and the quadratic shape functions. Explain your observation.
6. Figures that show the errors of for the whole time period computed using 128 subintervals. Organize the figures to show the error between the exact solution and the computed solution using linear shape functions in all time steps calculated, and to show the error between the exact solution and the computed solution using quadratic shape functions in all time steps calculated. The error plots are surfaces over $(x, t) \in [0, \pi] \times [0, 1]$.
7. Discussion of the convergence of the method and of your computed solutions.