Project 4: inclined plane with apple

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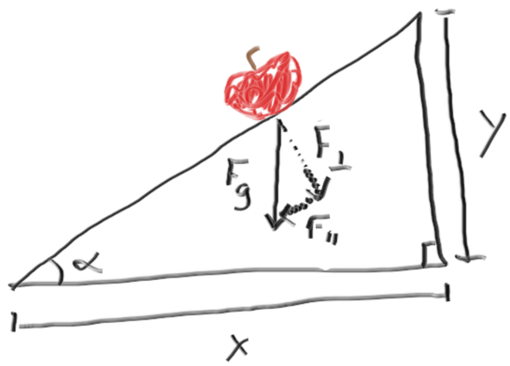
Note: This tutorial is currently only available as a pdf document.

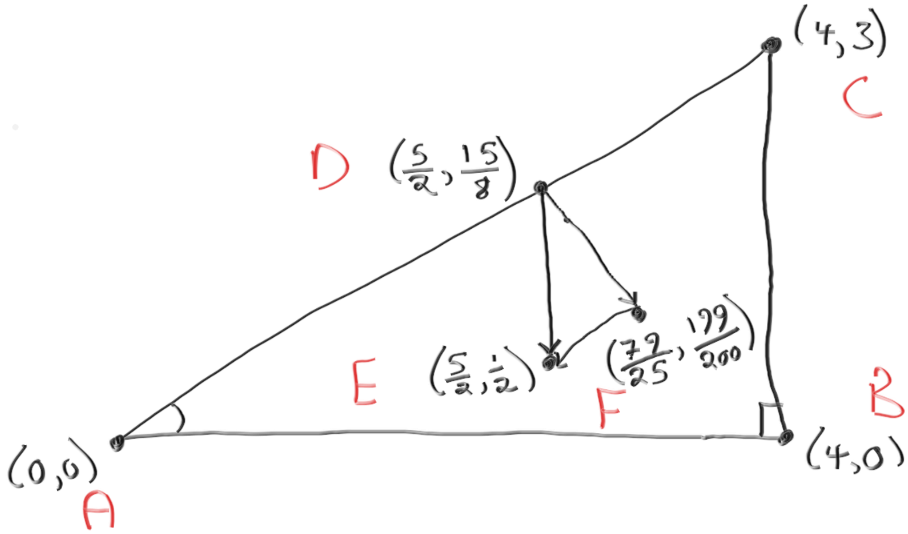
In this project, we are going to draw this free-body diagram of an apple sliding down an inclined plane:



… only we're going to include this photograph of an actual apple:

I found a photograph of an apple on an almost-white background and cropped it a bit so that it will appear to be resting on its base on the inclined plane later.

First, as we did in Project 2, we mark the (x, y) coordinates of the important pieces of the picture:



Some paper-and-pencil (and calculator) work was required to find these coordinates:

- I decided to use a 3-4-5 right triangle, oriented as in my hand-drawn picture and with vertex A at the origin. That gave me points B and C.
- Then I decided to put the apple so that its x-coordinate was 2.5, or $\frac{5}{2}$. Since the line joining points A and C is $y = \frac{3}{4}x$, this meant that the y-coordinate of point D is $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$. **Note:** Ti*k*Z doesn't care whether you enter coordinates as decimals or fractions: putting the coordinates of point D as `(2.5, 1.875)` is fine, and so is `(5/2, 15/8)`.
- I decided that point E should be at height $\frac{1}{2}$, and it should be on the same vertical line as point D, so its coordinates are $(\frac{5}{2}, \frac{1}{2})$.
- To find the coordinates of point F, I first found the equation of the line through $D(\frac{5}{2}, \frac{15}{8})$ perpendicular to the line $y = \frac{3}{4}x$. This turns out to be $y = -\frac{4}{3}x + \frac{125}{24}$. Then I found the line through $E(\frac{5}{2}, \frac{1}{2})$ parallel to the line $y = \frac{3}{4}x$. This came out to be $y = \frac{3}{4}x - \frac{11}{8}$. Then I found the point of intersection of these two lines. This was $(\frac{79}{25}, \frac{199}{200})$, so that was where point F went.

First let's draw the big triangle, the horizontal and vertical measuring bars, and the two little angle markers.

We draw the big triangle with the `\draw` command, using the argument `thick` to make the sides of the triangle a bit thicker than the default thickness. Note that we can draw all three sides of the big triangle in a single `\draw` command:

```
\draw [thick] (0,0) -- (4,0) -- (4,3) -- (0,0);
```

accomplishes the same thing as

```
\draw [thick] (0,0) -- (4,0);
\draw [thick] (4,0) -- (4,3);
\draw [thick] (4,3) -- (0,0);
```

We do a similar thing for the little right triangle corner, only we don't make the lines thick:

```
\draw (3.8,0) -- (3.8, .2) -- (4,.2);
```

A few words about the horizontal and vertical measuring bars for x and y:

- We drew a long line segment for each of these measuring bars, rather than drawing two short segments for each. Later, we'll place nodes with x and y there, and give each of them a white background.
- Rather than draw the tiny endcaps for the x and y line segments "by hand", we used an arrow tip called `Bar` from the `arrows.meta` library, so we need to load this library first. (You could also include the command `\usetikzlibrary{arrows.meta}` in the preamble.) This library has **lots** of fancy, customizable arrow tips. See Section 16.5, Reference: Arrow Tips, from the Ti*k*Z Manual for details.

  Note the syntax for using a fancy arrow tip:

  `\draw [-{Bar}] (0,0) -- (1,0);` produces

  `\draw [{Bar}-] (0,0) -- (1,0);` produces

  `\draw [{Bar}-{Bar}] (0,0) -- (1,0);` produces
- We drew the x and y measurement bars at a distance of 0.3 from the horizontal and vertical legs, respectively, of the big triangle, because … it looked like the right distance. You should adjust your measurement bars (and other details of the picture) to suit your taste.

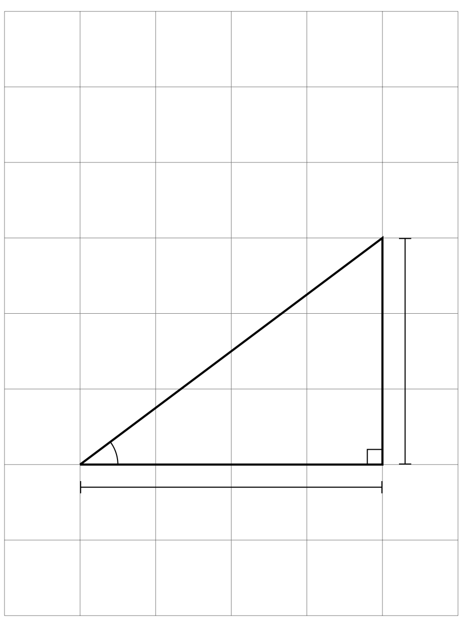
To draw the little arc for the acute angle α, we first had to do a bit of trig: $\tan(\alpha) = \frac{3}{4}$, and α is an acute angle, so $\alpha \approx 36.87°$. Once we know this, we use the `arc` path in the `draw` command. The following command tells Ti*k*Z to sweep out an arc of a circle with radius 0.5 centered at (0,0) from the starting angle of 0 to the ending angle of 36.87°:

```
\draw[radius=0.5 cm] (0.5,0) arc [start angle=0, end angle=36.87];
```

Notice that this command did not actually specify that the center of the circle from which the arc came is (0,0). This is implied: the point (0.5,0) is where the arc begins; the starting angle is 0; and the radius is 0.5 cm. Then to get from the starting point to the center of the circle, we'd go "backwards" a distance of 0.5 cm, putting us at (0,0).

As we did in Project 2, we'll have Ti*k*Z draw a grid to help us make sure we're placing the components of the picture where we intend to place them.

```
\begin{tikzpicture}
% load the arrows library
\usetikzlibrary{arrows.meta}

% help lines (comment out when done)
\draw [help lines] (-1,-2) grid (5,6);

% the triangle for the inclined plane
\draw [thick] (0,0) -- (4,0) -- (4,3) -- (0,0);

% the little right triangle corner
\draw (3.8,0) -- (3.8, .2) -- (4,.2);

% the x measuring bar
\draw [{Bar}-{Bar}] (0,-0.3) --  (4,-0.3);

% the y measuring bar
\draw [{Bar}-{Bar}] (4.3,0) --  (4.3,3);

% the little arc for the angle
\draw [radius = 0.5 cm] (0.5,0) arc
[start angle=0, end angle =  36.87];

\end{tikzpicture}
```

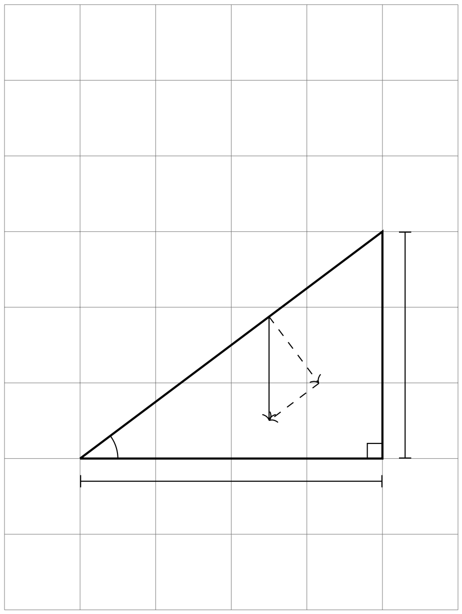


Next we'll draw the force arrows: the downwards arrow for the force of gravity on the apple, and the dotted lines for the components of the gravity force parallel and perpendicular to the inclined plane. (The apple itself we'll leave for last, and we can always re-size it.) I decided to have the downwards (gravity) force arrow go from the apple straight down to where the x-coordinate is 0.5.

To draw the dotted lines, we first needed to do a bit of math. Since the apple will be at $(\frac{5}{2}, \frac{15}{8})$, this vertical arrow will go from $(\frac{5}{2}, \frac{15}{8})$ to $(\frac{5}{2}, \frac{1}{2})$. The line parallel to the inclined plane is then going to have slope $\frac{3}{4}$ and contain the point $(\frac{5}{2}, \frac{1}{2})$, so its equation is $y = \frac{3}{4}x - \frac{11}{8}$. The line from the apple perpendicular to the inclined plane must have slope $-\frac{4}{3}$ and contain the point $(\frac{5}{2}, \frac{15}{8})$, so its equation is $y = -\frac{4}{3}x + \frac{125}{24}$. Finally, the point where these two lines intersect is $(\frac{79}{25}, \frac{199}{200})$, and this is found by setting $\frac{3}{4}x - \frac{11}{8} = -\frac{4}{3}x + \frac{125}{24}$. Notice that we can leave the coordinates of the endpoints of our lines as fractions if we like; Ti*k*Z will do the calculation when it draws the line.

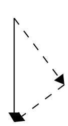
```
\begin{tikzpicture}
% load the arrows library
\usetikzlibrary{arrows.meta}

% help lines (comment out when done)
\draw [help lines] (-1,-2) grid (5,6);

% the triangle for the inclined plane
\draw [thick] (0,0) -- (4,0) -- (4,3) -- (0,0);

% the little right triangle corner
\draw (3.8,0) -- (3.8, .2) -- (4,.2);

% the x measuring bar
\draw [{Bar}-{Bar}] (0,-0.3) --  (4,-0.3);

% the y measuring bar
\draw [{Bar}-{Bar}] (4.3,0) --  (4.3,3);

% the little arc for the angle
\draw [radius = 0.5 cm] (0.5,0) arc
[start angle=0, end angle =  36.87];

% the dashed line pointing about 5:00 from the apple
\draw [dashed, ->] (5/2,15/8) -- (79/25,199/200);

% the line pointing down from the apple
\draw [->] (2.5,15/8) -- (5/2,.5);

% the line parallel to the plane
\draw [dashed, ->] (79/25,199/200) -- (5/2,1/2);
```

```
\end{tikzpicture}
```



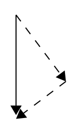
Before we go any further: do those arrows on the little triangle look a little hard to see to you? They do to me, so I'm going to make them look nicer. First of all, Ti*k*Z's default arrow tip looks a bit curvy and insubstantial for this picture. I want the arrow tips to be solid triangles. Looking through the section 16.5 Reference: Arrow Tips in the Ti*k*Z Manual, I find an arrow tip called `Triangle`. Here is what just the little force arrow triangle looks like with the `Triangle` arrow tip substituted for the default one:

```
\[
\begin{tikzpicture}
% the dashed line pointing about 5:00 from the apple
\draw [dashed, -{Triangle}] (5/2,15/8) -- (79/25,199/200);

% the line pointing down from the apple
\draw [-{Triangle}] (2.5,15/8) -- (5/2,.5);

% the line parallel to the plane
\draw [dashed, -{Triangle}] (79/25,199/200) -- (5/2,1/2);
\end{tikzpicture}
\]
```



Now one of the arrows looks great, but the other two are on top of each other, making it hard to distinguish them. One solution to this problem is to have one (or both) arrow stop just slightly short of their destination. Here we've made the downwards-pointing arrow stop a distance of 0.1 above its usual destination.

```
\[
\begin{tikzpicture}
% the dashed line pointing about 5:00 from the apple
\draw [dashed, -{Triangle}] (5/2,15/8) -- (79/25,199/200);

% the line pointing down from the apple
\draw [-{Triangle}] (2.5,15/8) -- (5/2,.5+.05);

% the line parallel to the plane
\draw [dashed, -{Triangle}] (79/25,199/200) -- (5/2,1/2);
\end{tikzpicture}
\]
```



Another possible solution is to leave the arrows in their original positions but use a narrower arrow tip, like `Stealth`:

```
\[
\begin{tikzpicture}
% the dashed line pointing about 5:00 from the apple
\draw [dashed, -{Stealth}] (5/2,15/8) -- (79/25,199/200);

% the line pointing down from the apple
\draw [-{Stealth}] (2.5,15/8) -- (5/2,.5);

% the line parallel to the plane
\draw [dashed, -{Stealth}] (79/25,199/200) -- (5/2,1/2);
\end{tikzpicture}
\]
```

I like this option – using the `Stealth` tip – better, so I've incorporated it into our big picture below.

I've also inserted force arrow labels near the midpoints of the lines using the `node` command, as we did in Project 1. I did some fine-tuning of the placement of the labels using the optional offset to the node placement arguments. For example, the command

```
\node [below right] at (283/100, 299/400) {$F_{||}$};
```

put the label $F_{||}$ a bit below and to the right of the point $(\frac{283}{100}, \frac{299}{400})$. In my opinion, this default `below right` placement put the label a little too far from its line, so I inserted a negative offset:

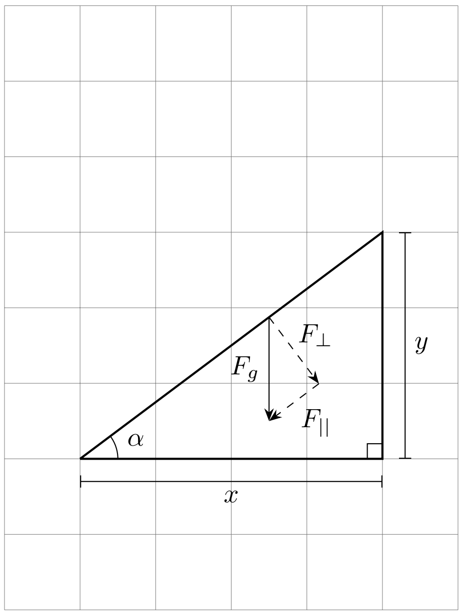
```
\node [below right=-1pt] at (283/100, 299/400) {$F_{||}$};
```

(See Section 17.5.2, Basic Placement Options, in the Nodes and Edges chapter of the Ti*k*Z Manual for more details.)

Here is what we have so far:

```
\begin{tikzpicture}
% load the arrows library
\usetikzlibrary{arrows.meta}

% help lines (comment out when done)
\draw [help lines] (-1,-2) grid (5,6);

% the triangle for the inclined plane
\draw [thick] (0,0) -- (4,0) -- (4,3) -- (0,0);

% the little right triangle corner
\draw (3.8,0) -- (3.8, .2) -- (4,.2);

% the x measuring bar
\draw [{Bar}-{Bar}] (0,-0.3) --  (4,-0.3);

% the y measuring bar
\draw [{Bar}-{Bar}] (4.3,0) --  (4.3,3);

% the little arc for the angle
\draw [radius = 0.5 cm] (0.5,0) arc
[start angle=0, end angle =  36.87];

% the dashed line pointing about 5:00 from the apple, and the label on it
\draw [dashed, -{Stealth}] (5/2,15/8) -- (79/25,199/200);
\node [above right=-2pt] at (283/100, 287/200) {$F_{\perp}$};

% the line pointing down from the apple, and the label on it
\draw [-{Stealth}] (2.5,15/8) -- (5/2,.5);
\node [left] at (5/2, 19/16) {$F_g$};
```

```
% the line parallel to the plane, and the label on it
\draw [dashed, -{Stealth}] (79/25,199/200) -- (5/2,1/2);
\node [below right=-1pt] at (283/100, 299/400) {$F_{||}$};

\end{tikzpicture}
```



Now we just need to place the apple!

Before we go any further, a **NOTE:** If all you want to do it to display a picture amongst your text, there's no need to use Ti*k*Z: LaTeX's `graphicx` package, which includes the `includegraphics` command, is perfectly fine. There are a couple of situations where you'd want to use Ti*k*Z rather than LaTeX's `graphicx` package:

- Use Ti*k*Z when you are including a graphic within a Ti*k*Z picture (so that the graphic has to be properly sized, spaced, layered, and rotated with respect to the rest of the picture), which is what we're doing here.
- Use Ti*k*Z when you wish to use masking (using an image to hide or reveal part of another image) – as far as I know, this isn't possible with just LaTeX, although it might be in the future.

Now let's talk about including images in Ti*k*Z pictures. My apple picture is called `apple.png`, and I've saved it in the same directory on Overleaf as this present document.

Before I can use the apple picture in my Ti*k*Z picture, I need to **declare the image** at the size it'll be used in my Ti*k*Z picture. This I've done by putting `\pgfdeclareimage[height=10mm]{the_apple}{apple}` in the preamble. (Note: you can also just include the image within the tikzpicture environment where you're using it. I've declared it in the preamble here because I'll be using it in several different Ti*k*Z pictures within the same document.)

- I've included the optional `height` argument in my declaration. By setting only the height and not the width (or vice versa), I preserve the aspect ratio of the original picture. I chose 10mm for the height after playing around with re-sizings of the apple compared to the inclined plane picture until I found the size that looked right.
- `the_apple` is the nickname I'm giving to my apple picture – this is how I'll refer to it within a Ti*k*Z picture.
- `apple` is the actual name of the file with my apple picture in it – *without* the file extension.

Having declared the apple picture, I can use it in the `tikzpicture` environment. Here it is, at the size I want it for the big picture, with no rotation:

```
\begin{tikzpicture}
\pgftext[at=\pgfpoint{0cm}{0cm}]{\pgfuseimage{the_apple}}
\end{tikzpicture}
```

Next I need to determine how much to rotate the apple. Some more paper-and-pencil work involving similar triangles told me that I needed to rotate the apple by $\tan^{-1}(\frac{3}{4}) \approx 36.87°$ anticlockwise from the vertical (that's the same angle measure as α), in order that it appear to be sliding on its base down the inclined plane. That's what the next command accomplishes … only what Ti*k*Z is *really* doing is a coordinate transformation. It's rotating the entire plane anticlockwise by 36.87°. (See Section 25.3: Coordinate Transformations in the Ti*k*Z Manual.)

```
\begin{tikzpicture}
\pgftext[rotate=36.87, at=\pgfpoint{0cm}{0cm}]{\pgfuseimage{the_apple}}
\end{tikzpicture}
```

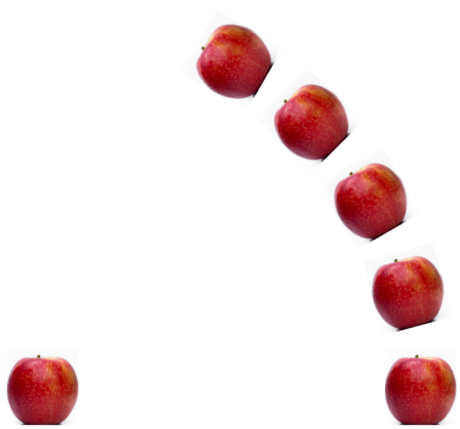
In the following command, the the `at=\pgfpoint{5cm}{0cm}]` part doesn't actually move the resulting picture 5 units to the right, as you might expect; the result looks exactly like the previous picture. This is basically because there is nothing else for `pgf` to refer to when it places the image.

```
\begin{tikzpicture}
\pgftext[rotate=36.87, at=\pgfpoint{5cm}{0cm}]{\pgfuseimage{the_apple}}
```

```
\end{tikzpicture}
```

Now compare this with what happens when we place a sequence of apples at a distance of 5 cm from the origin, with rotations about the origin of 0°, 15°, 30°, 45°, and 60°.

```
\begin{tikzpicture}
% the base apple, sitting at (0,0) with no rotation
\pgftext[rotate=0, at=\pgfpoint{0cm}{0cm}]{\pgfuseimage{the_apple}}
% distance = 5cm, rotation = 0
\pgftext[rotate=0, at=\pgfpoint{5cm}{0cm}]{\pgfuseimage{the_apple}}
% distance = 5cm, rotation = 15
\pgftext[rotate=15, at=\pgfpoint{5cm}{0cm}]{\pgfuseimage{the_apple}}
% distance = 5cm, rotation = 30
\pgftext[rotate=30, at=\pgfpoint{5cm}{0cm}]{\pgfuseimage{the_apple}}
% distance = 5cm, rotation = 45
\pgftext[rotate=45, at=\pgfpoint{5cm}{0cm}]{\pgfuseimage{the_apple}}
% distance = 5cm, rotation = 60
\pgftext[rotate=60, at=\pgfpoint{5cm}{0cm}]{\pgfuseimage{the_apple}}
\end{tikzpicture}
```



This would imply that to get the apple into the right place in our inclined plane picture, we should give it a rotation of 36.87° and place it at a distance of $d((0,0),(2.5,\frac{15}{8}))$ from the origin, like this:

```
\pgftext[rotate=36.87, at=\pgfpoint{3.125cm}{0cm}]{\pgfuseimage{the_apple}}
```

The problem with this is that it puts the line of the inclined plane going right through the apple. (Try it in the finished picture below!)

What I actually did when I placed the apple was to eyeball it. The x-value 3.125 looked OK, but then I needed to move the apple up and to the left.

Fortunately, this was easy to do, because when `pgf` does a rotation, it is really making a coordinate transformation. That is, having rotated 36.87°, we just need to move it up or down with respect to that coordinate transformation. For example, in the command

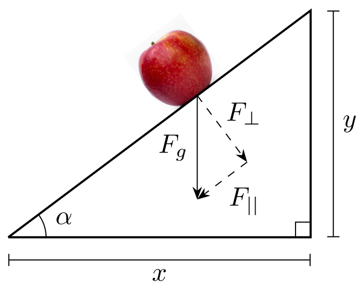
```
\pgftext[rotate=36.87, at=\pgfpoint{3.15cm}{.5cm}]{\pgfuseimage{the_apple}},
```

the .5cm moves the apple "up" in the rotated coordinate system a distance of .5cm. This corresponds to moving the apple in a direction perpendicular to the inclined plane by .5cm.

Finally, we placed the apple "first" in the picture so that it would be "under" the lines for the rest of the inclined plane picture. (This is so that the picture wouldn't cover up some of the width of the lines, and so that the apple would appear to be resting on the inclined plane.)

```
\begin{tikzpicture}
% load the arrows library
\usetikzlibrary{arrows.meta}

% the apple
% (place it first so it goes underneath!)
\pgftext[rotate=36.87, at=\pgfpoint{3.15cm}{.5cm}]{\pgfuseimage{the_apple}}

% help lines (comment out when done)
%\draw [help lines] (-1,-2) grid (5,6);

% the triangle for the inclined plane
\draw [thick] (0,0) -- (4,0) -- (4,3) -- (0,0);

% the little right triangle corner
\draw (3.8,0) -- (3.8, .2) -- (4,.2);

% the x measuring bar, and the x label
\draw [{Bar}-{Bar}] (0,-0.3) --  (4,-0.3);
\node [below] at (2,-0.3) {$x$};

% the y measuring bar, and the y label
\draw [{Bar}-{Bar}] (4.3,0) --  (4.3,3);
\node [right] at (4.3,1.5) {$y$};

% the little arc for the angle,
% and the alpha label
\draw [radius = 0.5 cm] (0.5,0) arc
[start angle=0, end angle =  36.87];
\node [right] at (0.5,0.25) {$\alpha$};
```

```
% the dashed line pointing about 5:00 from the apple, and the label on it
\draw [dashed, -{Stealth}] (5/2,15/8) -- (79/25,199/200);
\node [above right=-2pt] at (283/100, 287/200) {$F_{\perp}$};

% the line pointing down from the apple, and the label on it
\draw [-{Stealth}] (2.5,15/8) -- (5/2,.5);
\node [left] at (5/2, 19/16) {$F_g$};

% the line parallel to the plane, and the label on it
\draw [dashed, -{Stealth}] (79/25,199/200) -- (5/2,1/2);
\node [below right=-1pt] at (283/100, 299/400) {$F_{||}$};

\end{tikzpicture}
```



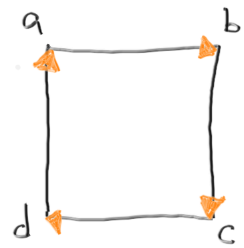
I think this is done! As usual, fine-tune it if you wish, and then try these exercises.

Exercises for Project 4

1. Take a picture with your phone. Save it as a .jpg or.png file, upload it to Overleaf, and put it in the same folder as the one where you're doing these exercises. Declare it at a convenient size, and display it right-side up and then upside down.

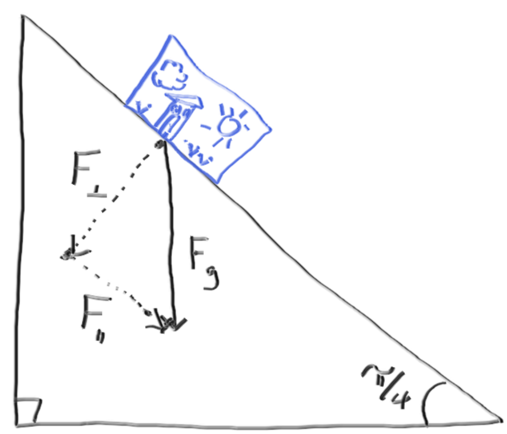
2. Use Ti*k*Z to draw a square about 2 inches on a side. Put your picture from Problem 1 in the middle of it (you may have to change the size at which you declare it).

3. Draw a square – you can use your square from the previous problem. Put vertices at each corner, and label them a, b, c, and d. Next, put fancy arrow tips (from the `arrows.meta` library) on the lines so that they go clockwise around the square. Now you have a directed graph on 4 vertices.



If you want to have the arrows look as they do in the picture, with black lines and colored arrow tips, like this ⟶, the syntax for that is `\draw [-{Triangle[orange]}] (0,0) -- (1,0);`

4. Take the completed inclined plane / apple picture from this project and substitute your picture for the apple. Rotate it so that it looks like it's sliding down the plane, just like we did for the apple.
5. Draw the following inclined plane, including your own picture. Make sure you've got the picture oriented so that it looks like it's sliding down the plane.



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