

## CASE STUDY 1:

### *“FINDING THE WINNING STRATEGY IN A CARD GAME”*

Big Data Analysis

AIT BOUIFADEN Amal

GUERRERO IBERICO Jenny

DE MACEDO Kevin

DAMAS Alexandra



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# INTRODUCTION

Our aim is to play a card game in which the cards are iteratively flipped until we tell the dealer to stop. Then one additional card is flipped.

If that card is red, we win a dollar; otherwise, we lose a dollar.

Our goal is to discover a strategy that best predicts a red card in the deck.

# CASE PRESENTATION AND PROBLEM DEFINITION



52 cards in total

26 red cards

26 black cards

Find out the strategy that best predicts a red card in a shuffled deck.

### III. DATA ANALYTICS APPROACH DEFINITION AND EXAMPLES( WITH THE HELP OF JUPITER)

$$P(A) = \frac{\text{Event seize}}{\text{Sample space seize}}$$

If  $0 < P(\text{redcard}) < 1$  → We have chances to draw a red card.

$$\frac{26}{52} \xrightarrow{\text{Red cards}} \frac{26}{52} \xrightarrow{\text{Total cards}} = 0.5 \text{ we have 50% chances to predict a red card in a shuffled deck.}$$

If  $P(\text{redc} \cup \text{blackc})=1$  → "I bet you that the last card is either red or black"

$$\frac{26}{52} + \frac{26}{52} \xrightarrow{\text{Red + black cards}} \frac{26}{52} \xrightarrow{\text{Total cards}} = 1 \text{ this means that the event is certain, I'll win no matter what}$$

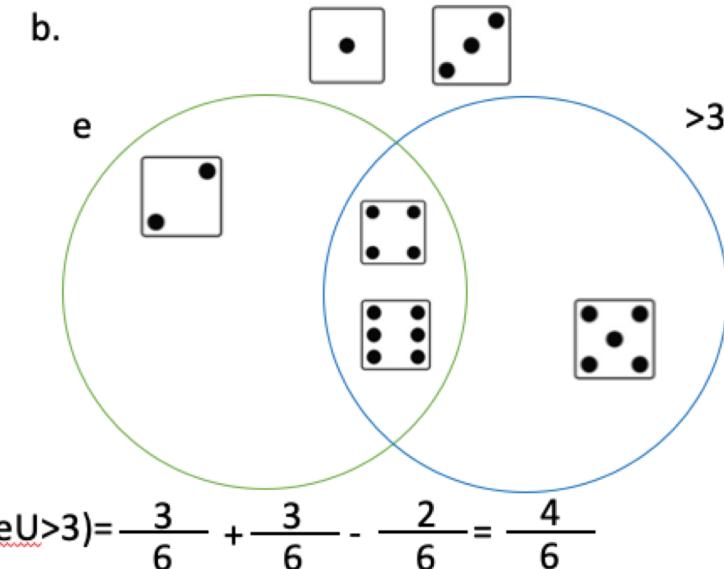
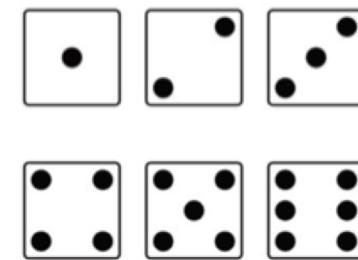
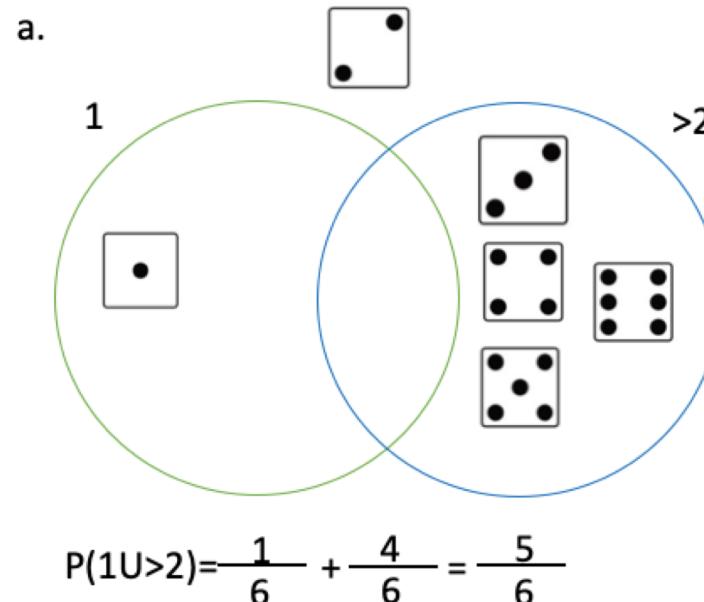
If  $P(\text{greencard})=0$  → "I bet you that the last card is green"

$$\frac{0}{52} \xrightarrow{\text{green cards}} \frac{0}{52} \xrightarrow{\text{Total cards}} = 0 \text{ means that the event is impossible}$$

### III. Data analytics approach definition and examples

A single dice is rolled, what is the probability of

- Obtaining 1 or a digit greater than 2
- Obtaining a number that is even or greater than 3



### III. Data analytics approach definition and examples

We have 7 cards face down, 5 black cards and 2 red cards. What is the probability to pick up 2 red cards by hazard



$$\begin{aligned} P(2 \text{ redcards}) &= P(r) * P(r|r) \\ &= \frac{2}{7} \times \frac{1}{6} \\ &= \frac{2}{42} = 4.76\% \end{aligned}$$

If we pick up a black card. What is the probability that this card could be an even number

$$\begin{aligned} &= \frac{2}{5} \xrightarrow{\text{Even numbers}} \\ &\quad \xrightarrow{\text{Total black cards}} \\ &= 40\% \end{aligned}$$

Plotting probabilities using Matplotlib

Understanding the likelihood of certain cards

Comparing the distribution of cards

Developing strategies

# **SOLUTION DEVELOPMENT AND ILLUSTRATION**

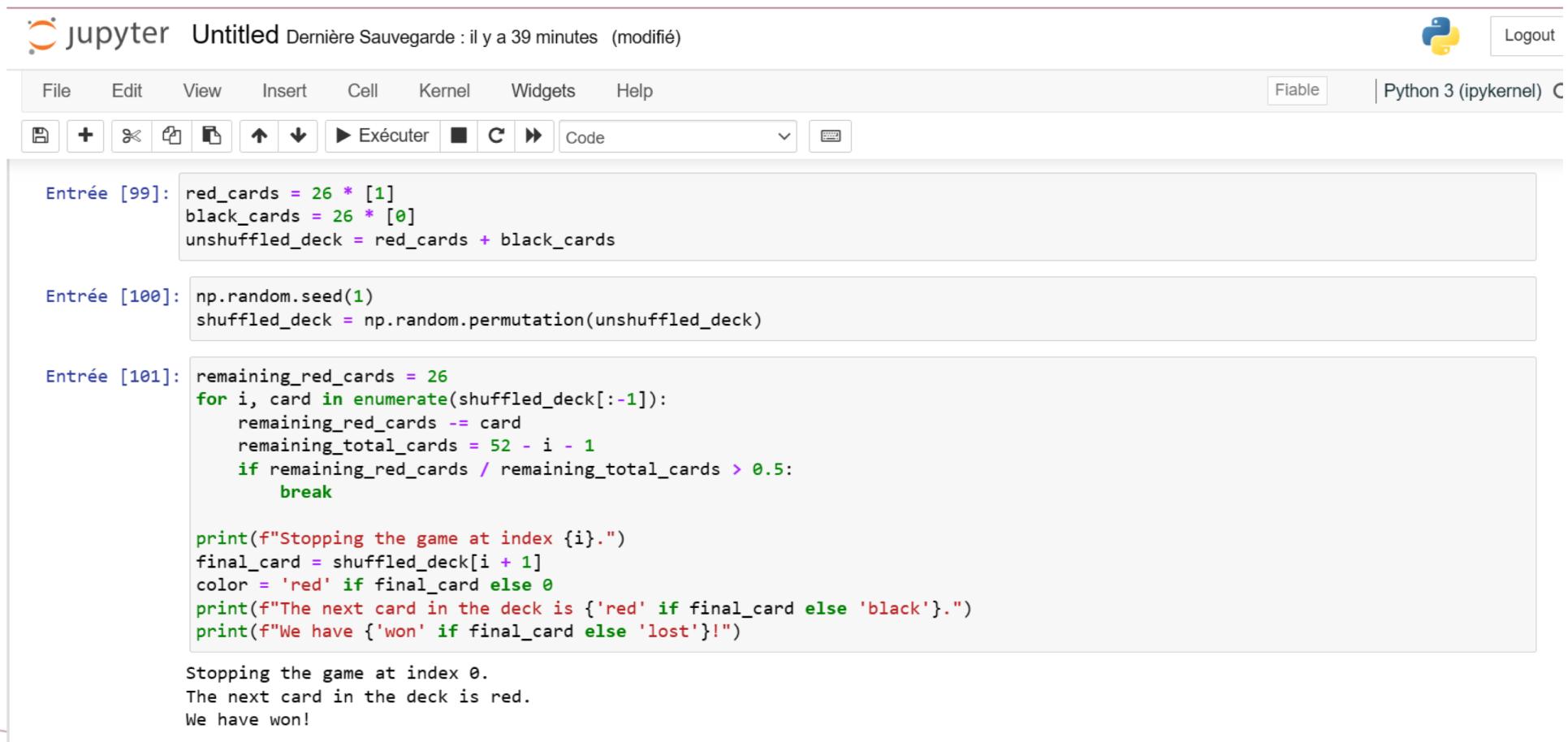
As we say, our aim is to play a card game in which the cards are iteratively flipped until we tell the dealer to stop. Then one additional card is flipped. If that card is red, we win a dollar; otherwise, we lose a dollar. Our goal is to discover a strategy that best predicts a red card in the deck.

At first:

We start by creating a deck holding 26 red cards and 26 black cards. Black cards are represented by 0s, and red cards are represented by 1s.

```
red_cards = 26 * [1]
black_cards = 26 * [0]
unshuffled_deck = red_cards + black_cards
```

# WITH THE HELP OF JUPITER...



The screenshot shows a Jupyter Notebook interface with the following details:

- Title Bar:** jupyter Untitled Dernière Sauvegarde : il y a 39 minutes (modifié)
- Toolbar:** File, Edit, View, Insert, Cell, Kernel, Widgets, Help, Fiable, Python 3 (ipykernel), Logout
- Cells:** Three cells are visible:
  - Entrée [99]:** red\_cards = 26 \* [1]  
black\_cards = 26 \* [0]  
unshuffled\_deck = red\_cards + black\_cards
  - Entrée [100]:** np.random.seed(1)  
shuffled\_deck = np.random.permutation(unshuffled\_deck)
  - Entrée [101]:** remaining\_red\_cards = 26  
for i, card in enumerate(shuffled\_deck[:-1]):  
 remaining\_red\_cards -= card  
 remaining\_total\_cards = 52 - i - 1  
 if remaining\_red\_cards / remaining\_total\_cards > 0.5:  
 break  
  
print(f"Stopping the game at index {i}.")  
final\_card = shuffled\_deck[i + 1]  
color = 'red' if final\_card else 0  
print(f"The next card in the deck is {'red' if final\_card else 'black'}.")  
print(f"We have {'won' if final\_card else 'lost'}!")
- Output:** The output for cell [101] is:

Stopping the game at index 0.  
The next card in the deck is red.  
We have won!

# GENERALIZING THE CARD GAME STRATEGY..

The screenshot shows a Jupyter Notebook interface with the following details:

- Title Bar:** jupyter Untitled Dernière Sauvegarde : il y a 41 minutes (auto-sauvegardé)
- Toolbar:** File, Edit, View, Insert, Cell, Kernel, Widgets, Help; Exécuter, Code dropdown.
- Cell 102 (Entrée [102]):**

```
total_cards = 52
total_red_cards = 26
def execute_strategy(min_fraction_red=0.5, shuffled_deck=None,
                     return_index=False):
    if shuffled_deck is None:
        shuffled_deck = np.random.permutation(unshuffled_deck)

    remaining_red_cards = total_red_cards

    for i, card in enumerate(shuffled_deck[:-1]):
        remaining_red_cards -= card
        fraction_red_cards = remaining_red_cards / (total_cards - i - 1)
        if fraction_red_cards > min_fraction_red:
            break

    return (i+1, shuffled_deck[i+1]) if return_index else shuffled_deck[i+1]
```
- Cell 103 (Entrée [103]):**

```
np.random.seed(0)
observations = np.array([execute_strategy() for _ in range(1000)])
```
- Cell 104 (Entrée [104]):**

```
frequency_wins = observations.sum() / 1000
assert frequency_wins == observations.mean()
print(f"The frequency of wins is {frequency_wins}")
```

The output for Cell 104 is: The frequency of wins is 0.511

# OPTIMIZING STRATEGIES USING THE SAMPLE SPACE FOR A 10-CARD DECK

The screenshot shows a Jupyter Notebook interface with the following details:

- Title Bar:** jupyter Untitled Dernière Sauvegarde : il y a une heure (auto-sauvegardé)
- Toolbar:** File, Edit, View, Insert, Cell, Kernel, Widgets, Help, Exécuter, Code
- Cell 110:** Contains Python code to calculate the probability of a win for a 10-card deck. It defines variables for total cards (10), red cards (5), black cards (5), and an unshuffled deck. It then generates a sample space of permutations, defines a win condition, and prints the probability.
- Output 110:** Probability of a win is 0.5
- Cell 111:** Contains Python code to scan various fractions and return their corresponding probabilities. It defines a scan\_strategies function that iterates over a range of values (50 to 100) to find the lowest and highest probabilities.
- Output 111:** Lowest probability of win is 0.5 and Highest probability of win is 0.5

# PLOTTING STRATEGY OUTCOMES ACROSS A 52-CARD DECK

```
np.random.seed(0)
total_cards = 52
total_red_cards = 26
unshuffled_deck = red_cards + black_cards

def repeat_game_detailed(number_repeats, min_red_fraction):
    execute a
    y across
    _repeats
    ilations.
    where
    rategy
    't halt
    re win
    observations = [execute_strategy(min_red_fraction, return_index=True)
                    for _ in range(num_repeats)]
    successes = [index for index, card, in observations if card == 1] ←
    halt_success = len([index for index in successes if index != 51]) ←
    no_halt_success = len(successes) - halt_success

    failures = [index for index, card, in observations if card == 0] ←
    halt_failure = len([index for index in failures if index != 51]) ←
    no_halt_failure = len(failures) - halt_failure
    result = [halt_success, halt_failure, no_halt_success, no_halt_failure]
    return [r / number_repeats for r in result]

fractions = [value / 100 for value in range(50, 100)]
num_repeats = 50000
result_types = [[], [], [], []]
for fraction in fractions:
    result = repeat_game_detailed(num_repeats, fraction)
    for i in range(4):
        result_types[i].append(result[i])

plt.plot(fractions, result_types[0],
         label='A) Strategy Halts. We Win.')
plt.plot(fractions, result_types[1], linestyle='--',
         label='B) Strategy Halts. We Lose.')
plt.plot(fractions, result_types[2], linestyle=':',
         label='C) No Halt. We Win.')
plt.plot(fractions, result_types[3], linestyle='-.',
         label='D) No Halt. We Lose.')
plt.xlabel('min_red_fraction')
plt.ylabel('Frequency')
plt.legend(bbox_to_anchor=(1.0, 0.5)) ←
plt.show()
```

This list contains all instances of losses.

Scenario where our strategy halts and we win

This list contains all instances of wins.

We return the observed frequencies for all four scenarios.

We scan the scenario frequencies across multiple strategies.

Scenario where our strategy doesn't halt and we lose

Scenario where our strategy halts and we lose

The bbox\_to\_anchor parameter is used to position the legend above the plot to avoid overlap with the four plotted curves.

# EVALUATION AND FEEDBACK: THE OPTIMAL WINNING STRATEGY

```
def optimal_strategy(shuffled_deck):  
    return shuffled_deck[0]
```

This function could be used as a starting point for developing a more sophisticated strategy. For example, in this case it could be modified to return a specific card based on choosing the highest or lowest value card in the deck or to implement a more complex sequence of actions based on the cards drawn so far.





# CONCLUSION

By simplifying our problem, we can utilize sample spaces to yield insights. Sample spaces allow us to test our intuition. If our intuitive solution fails on a toy version of the problem, it is also likely to fail on the actual version of the problem.