Statistical Method HW5

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Given the sepal width of ”iris” data, use the following steps to show if the mean of sepal width is 3. Assume the sepal widths are random samples from a normal distribution

```
data(iris)
x = iris$Sepal.Width
```

(a) Use the maximum likelihood estimation method to estimate the model parameters of the normal distribution.

```
set.seed(6121)
likelihood.normal = function(par, data){
  mu = par[1]
  sig = par[2]
  joint = dnorm(data, mean = mu, sd = sig)
  return(-sum(log(joint)))}

opt = optim(c(3,0.1), likelihood.normal, data = x)

mu.hat.mle = opt$par[1]
sig.hat.mle = opt$par[2]
```

```
mu.hat.mle
```

```
## [1] 3.057459
```

```
sig.hat.mle
```

```
## [1] 0.4344073
```

> $\hat{\mu}_{MLE} = 3.057459$, $\hat{\sigma}_{MLE} = 0.4344073$

(b) According to the question ”if the mean of sepal width is 3”, what is the estimate for the quantity of interest?

> We can use $\hat{\mu}_{MLE}$ to estimate μ , which is the mean of sepal width.

(c) Construct the 95% confidence interval for the true quantity of interest by bootstrapping.

```
set.seed(6121)
iter = 1000
np.mu.hat = rep(NA,iter)
np.sig.hat = rep(NA,iter)

for (j in 1:iter){
  dataY = sample(x, length(x), replace = TRUE)
  estY = optim(c(3,0.2), likelihood.normal, data = dataY)
  np.mu.hat[j] = estY$par[1]
  np.sig.hat[j] = estY$par[2]}
```

```
quantile(np.mu.hat, c(0.025, 0.975))
```

```
##     2.5%    97.5%
## 2.991994 3.126761
```

```
quantile(np.sig.hat, c(0.025, 0.975))
```

```
##      2.5%     97.5%
## 0.3811493 0.4825527
```

> The 95% C.I. for the true quantity of interest by bootstrapping is [2.992, 3.127]

(d) Based on the 95% confidence interval in (c), how would you conclude the question ”if the mean of sepal width is 3”?

> We can say that “the mean of sepal width is 3” because 3 is in the interval [2.992, 3.127].

(e) If I use a one-sample t-test to test if $H_0 : \mu = 3$, is the conclusion as the same as the result in (d)?

```
t.test(x, alternative = 'two.sided', mu = 3)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 1.611, df = 149, p-value = 0.1093
## alternative hypothesis: true mean is not equal to 3
## 95 percent confidence interval:
##  2.987010 3.127656
## sample estimates:
## mean of x
##  3.057333
```

> The P-value > 0.05, so we do not reject the H_0. Therefore, we don’t have sufficient evidance to claim that “the mean of sepal width is not 3”, which means “the mean of sepal width is 3” , so the conclusion is as the same as the result in (d).