RE6121011
徐仁瓏

1. (a) $X \sim Poi(\lambda = 3)$

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-3}3^x}{x!}, \quad x = 0, 1, 2, \cdots, \infty$$

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x}{(x-1)!} = e^{-\lambda}\lambda\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda}\lambda e^{\lambda} \text{ (by Maclaurin's series)}$$

$$= \lambda = 3$$

$$var(X) = \lambda = 3$$

(b) $X \sim GEO(p = 1/3)$

$$f(x) = pq^{x-1} = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{x-1}, \quad x = 1, 2, 3, \cdots, \infty$$

$$E(X) = \frac{1}{p} = 3$$

$$var(X) = \frac{q}{p^2} = \frac{2}{3} \times 9 = 6$$

(c) $X \sim N(\mu = 0, \sigma^2 = 1)$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}x^2}, \quad -\infty < x < \infty$$

$$E(X) = \mu = 0$$

$$var(X) = \sigma^2 = 1$$

(d) $X \sim Exp(\lambda = 2)$

$f(x) = \lambda e^{-\lambda x} = 2e^{-2x}$, $x \geq 0$

$E(x) = \frac{1}{\lambda} = \frac{1}{2}$

$Var(x) = \frac{1}{\lambda^2} = \frac{1}{4}$

(e) $X \sim DE(\mu = 0, b = \frac{1}{2})$

$f(x) = \frac{1}{2b} e^{\frac{-|x-\mu|}{b}} = e^{-2|x|}$, $-\infty < x < \infty$

$E(x) = \mu = 0$

$Var(x) = 2b^2 = \frac{1}{2}$

(f) $X \sim Gamma(\alpha = 10, \lambda = \frac{1}{2}) \rightarrow \beta = 2$

$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha} = \frac{x^9 e^{-x/2}}{\Gamma(10)\, 2^{10}} = \frac{x^9 e^{-x/2}}{9! \cdot 2^{10}}$, $x \geq 0$

$E(x) = \alpha\beta = 20$

$Var(x) = \alpha\beta^2 = 40$

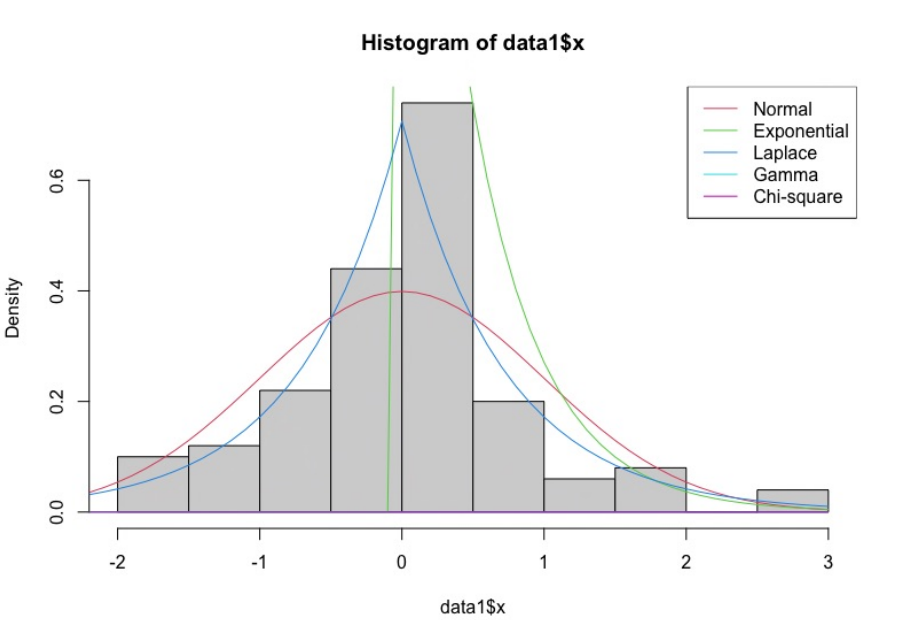
(g) $X \sim \chi^2(k = 20)$

$f(x) = \frac{x^{\frac{k}{2}-1} e^{-x/2}}{\Gamma(\frac{k}{2})\, 2^{k/2}} = \frac{x^9 e^{-x/2}}{\Gamma(10)\, 2^{10}} = \frac{x^9 e^{-x/2}}{9! \cdot 2^{10}}$, $x \geq 0$

$E(x) = k = 20$

$Var(x) = 2k = 40$

2、 dataset 1



從上圖中，可發現 Laplace 分配（$\mu=0, b=\frac{1}{2}$）最為接近，

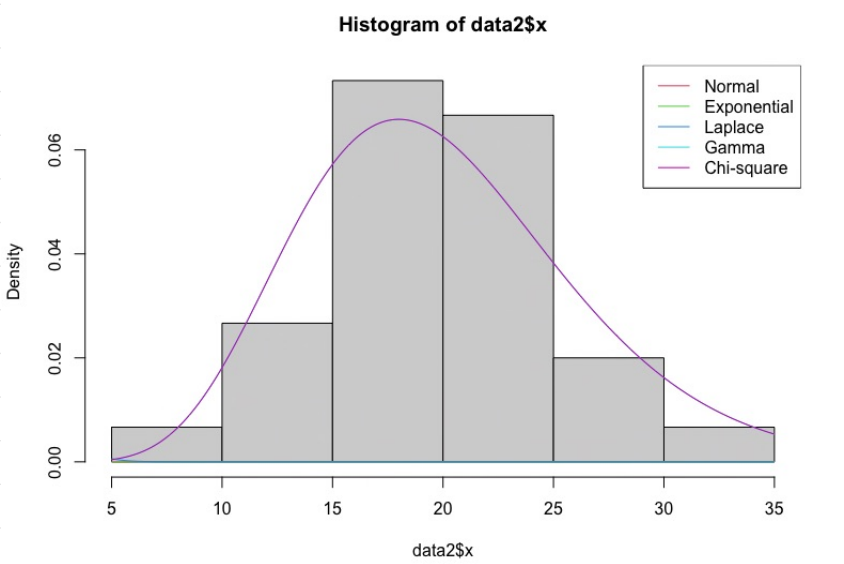
$N(0,1)$ 還須要再集中一点，

$Exp(\lambda=2)$ 範圍須 >0，和此 data 不符，

$Gamma(\alpha=10, \lambda=\frac{1}{2})$ 和 $\chi^2(k=20)$ 其實是同分配，

平均数為 20，遠超过此 data 之範圍

dataset 2



從上圖中，可發現最接近 Gamma($\alpha=10, \lambda=\frac{1}{2}$) 和 $\chi^2(k=20)$，
實際上，他們其實是同一條分配，
$N(0,1)$，$Exp(\lambda=2)$，$DE(\mu=0, b=\frac{1}{2})$ 的平均數
分別為 0，$\frac{1}{2}$，0，遠低於此 data 的範圍，
從上圖來看，他們幾乎疊在 x 軸上。